Lecture 15: Current Topics II Injector Physics and Design

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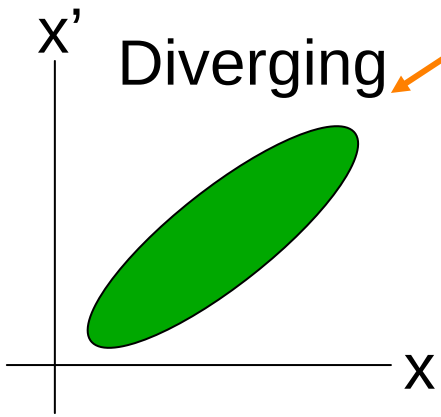
Correlated Beams

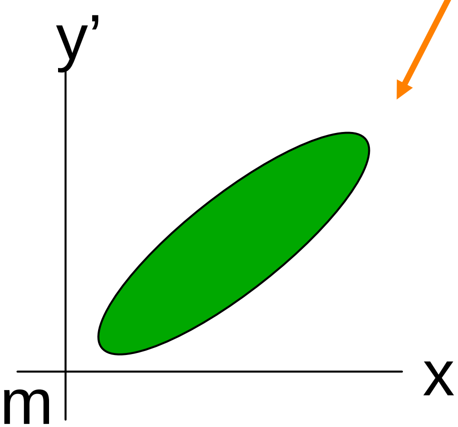
The beam matrix gives us a complete description of the lowest order correlations that exist within a beam.

$$\Sigma = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle & \langle xz \rangle & \langle xz' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle & \langle x'z \rangle & \langle x'z' \rangle \\ \langle xy \rangle & \langle x'y \rangle & \langle y^2 \rangle & \langle yy' \rangle & \langle yz \rangle & \langle yz' \rangle \\ \langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'^2 \rangle & \langle y'z \rangle & \langle y'z' \rangle \\ \langle xz \rangle & \langle x'z \rangle & \langle yz \rangle & \langle y'z \rangle & \langle z^2 \rangle & \langle zz' \rangle \\ \langle xz' \rangle & \langle x'z' \rangle & \langle yz' \rangle & \langle y'z' \rangle & \langle zz' \rangle & \langle z'^2 \rangle \end{bmatrix}$$

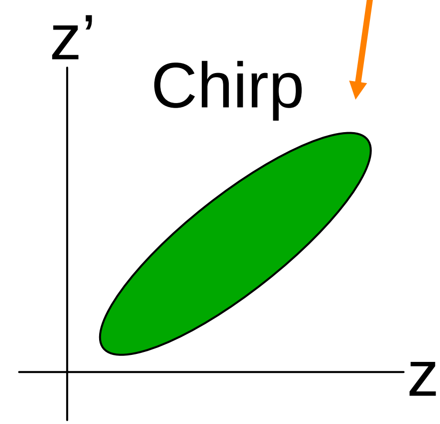
Diverging



Angular momentum



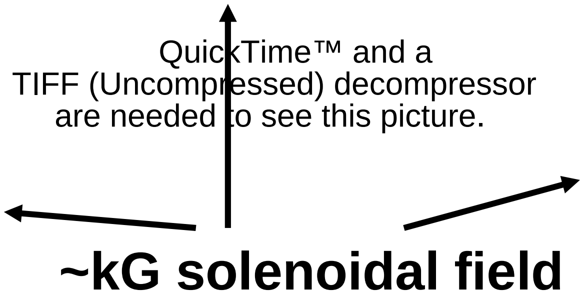
Chirp



Applications

- High energy physics - low vertical emittance
- Nuclear physics - electron cooling
- Ultrashort pulse x-rays
- Improving FEL gain

Electron Cooling - Low energy



Low energy design restrictions:
- <1GeV/nucleon
- <0.5MeV electron
- ~1kG continuous solenoid magnetic field in cooling region

The cooling rates are inversely proportional to a relative electron-ion velocity cubed - any coherent angle above the thermal level dramatically depresses the cooling process.

Inside the cooling solenoid, the beam is required to be calm, i.e., not to have any angles in excess of the thermal ones (assumed to be negligible).

Magnetized (‘calm’) electron beam state is characterized by beam spot size :: Larmor radius >>1 and the beam electrons follow the magnetic field lines of flux.

Electron Cooling - High energy

At higher nucleon and electron beam energies, the continuous solenoid field becomes problematic and we look for a lumped element approach - that still produces a ‘calm’ or magnetized beam with good overlap on the hadron beam.

Conservation of p_θ for the electron beams lets us tune the beam profile to match the requirements for cooling.

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a is the beam edge radius; initial conditions refer to the cathode

What is going on with the electrons?

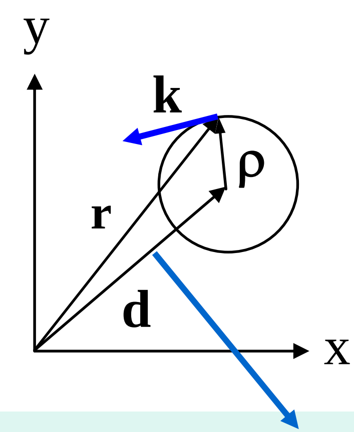
Shift from Cartesian to Drift-Cyclotron motion

Drift motion (large) ~ beam spot size and solenoid field

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} \qquad \vec{d} = \vec{r} - \vec{\rho} = \begin{pmatrix} x \\ y \end{pmatrix} - \frac{\beta_s}{2}\begin{pmatrix} y' \\ -x' \end{pmatrix} \qquad \beta_s = \frac{eB_z}{2\gamma\beta mc}$$

Cyclotron motion (small) ~ thermal emittance

$$\vec{k}_\perp = \gamma\beta \begin{pmatrix} x' \\ y' \end{pmatrix}$$



4D Transverse emittance in normal mode coordinates:

$$\varepsilon_{4D}^2 = \frac{1}{4}\left\langle d^2 \right\rangle \left\langle k_\perp^2 \right\rangle = \varepsilon_{drift}\varepsilon_{cyclotron}$$

D

$$\varepsilon_{drift} >> \varepsilon_{cyclotron}$$

Extending this treatment to generic lattices

Can we build an adaption scheme that transforms an electron beam in a storage ring (which has much larger horizontal emittance than vertical emittance) to a distribution without transverse angles for efficient hadron cooling schemes at higher energies?

Consider a general optical lattice that correlates the vertical phase space coordinates to the horizontal ones in a particular way:

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'Vortex state'

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Injecting this vortex beam into a solenoid field tuned to QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture. eliminates the transverse velocity and we have a calm beam for cooling.

Skew Quad Lattice can do it!

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4x4 uncoupled transform matrix for x and y motions.
Block of normal quads and drifts!

Rotate by 45° to form a block of skew quads and drifts:

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3 quad magnets!!

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This lattice turns any flat distribution into a vortex state.

‘Vortex’ States

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We have found a simple optical system that transforms a flat distribution (no vertical or no horizontal emittance) into a pure vortex state.

3 Simple Steps to a Flat Beam

Let's run this transformation backwards!

1. **Induce canonical angular momentum in the beam.**
   - Put a magnetic field on the cathode.
   - Creates correlations between horizontal and vertical phase spaces, tuned by the solenoid field strength.

2. **Pass the beam through the solenoid fringe field and drift.**
   - Convert cyclotron-drift motion to superposition of vortex states.
   - Manipulate correlations and prepare for injection in to the Adapter lattice.
     - Sets up skew velocity components for skew quads to act on.
   - Gives emittance compensation schemes the space to act.

3. **Transport beam through the Adapter lattice.**
   - De-correlates the coupled phase spaces.
   - One emittance is very large, one is very small. Geometric average is invariant!!

Flat Beams - New Regime For Photoinjectors

To produce flat beams with the current approach requires us to operate the photoinjector in a new configuration. A strong magnetic solenoid field is applied to the cathode.

The evolution of the launched beam is dominated now by both angular momentum and space charge forces.

The beam dynamics within the rf gun is significantly different than previous designs.

At cathode

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Adapter exit

Obtaining Vertical Emittances << Initial Thermal Emittance

The horizontal to vertical emittance ratio is related to the initial angular momentum and the initial thermal emittance:

$$\frac{\varepsilon_x}{\varepsilon_y} = 2\frac{\left(eB_z\right)^2}{\left(2\gamma\beta mc\right)^2}\frac{\left\langle x_0^2+y_0^2\right\rangle}{\left\langle x_0'^2+y_0'^2\right\rangle} = \frac{1}{2}\left(\frac{1}{2}\frac{eB_z}{mc}\right)^2\frac{\left\langle r_0^2\right\rangle^2}{\varepsilon_{thermal}^2} = \frac{1}{2}\frac{\left\langle p_\theta / mc\right\rangle^2}{\varepsilon_{thermal}^2}$$

To make this ratio large requires:

$$\left\langle\frac{p_\theta}{mc}\right\rangle = \frac{1}{2}\frac{eB_z}{mc}\left\langle r_0^2\right\rangle > \sqrt{2}\quad \varepsilon_{thermal} \propto \sqrt{\left\langle r_0^2\right\rangle}\sqrt{\left\langle r_0'^2\right\rangle}$$

The inequality can be satisfied easily by increasing the spot size at the cathode or the magnetic field at the cathode, or both.

Simple Model of the Beam Dynamics – Envelope Description From 2nd-order Moments

- Beam Envelope equation of motion:

$$\sigma_r'' + \sigma_r'\left(\frac{\gamma'}{\gamma\beta^2}\right) + k_{eff}^2\sigma_r - \frac{\kappa_s}{\sigma_r\gamma^3\beta^3} - \frac{\varepsilon_{nr}^2}{\sigma_r{}^3\gamma^2\beta^2} - \left(\frac{\langle p_\theta\rangle}{mc\gamma\beta}\right)^2\frac{1}{\sigma_r{}^3} = 0.$$

- Canonical angular momentum:

$$p_\theta = -\frac{1}{2}eB_{z0}r_0^2$$

- 4D Emittance:

$$\varepsilon_{nr}^2 = \varepsilon_{4D} = (\gamma\beta)^2\left[\langle r^2\rangle\langle r'^2 + (r\theta')^2\rangle - \langle rr'\rangle^2 - \langle r^2\theta'\rangle^2\right]$$

Beam matrix and Correlations

- 4D Beam Sigma matrix

$$\Sigma = \begin{pmatrix} \langle XX^T \rangle & \langle XY^T \rangle \\ \langle YX^T \rangle & \langle YY^T \rangle \end{pmatrix} = \begin{pmatrix} \Sigma_X & \Sigma_X C \\ C^T \Sigma_X & \Sigma_Y \end{pmatrix}$$

- Beam is initially correlated by solenoid field

$$C_0 = \begin{pmatrix} 0 & -\frac{2}{eB_0} \\ \frac{eB_0}{2} & 0 \end{pmatrix}$$

- Correlation matrix propagates through beam line

$$C_0 \Rightarrow C = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ \frac{1+a^2}{b} & -a \end{pmatrix}$$

Unimodular
Skew-symmetric

Analytical result for 3-skew quadrupole adapter

q1 D1 q2 D2 q3 DT=D1+D2

1. Correlation matrix at channel entrance $C_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

- Quad strengths determined by solving $(A-B)+(A+B)C_1 = 0$

$$q_1 = \pm\sqrt{\frac{-D_1 a + b - D_1 D_T c + D_T d}{D_1 D_T b}},$$

$$q_2 = -\frac{b + D_T d}{D_1 D_2 (1 + q_1 b)},$$

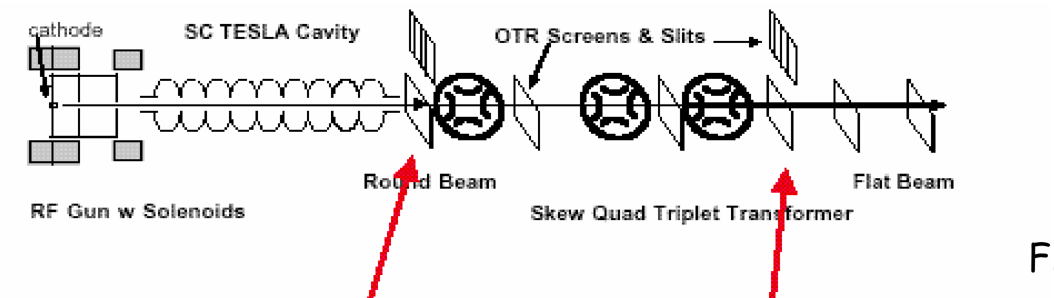
$$q_3 = -\frac{q_1 + q_2 + D_1 q_1 q_2 a - c}{1 + (D_T q_1 + D_2 q_2) a + D_1 D_2 q_2 (q_1 + c)}.$$

(+) for Horizontal beam
(-) for Vertical beam

Many different emittances to track

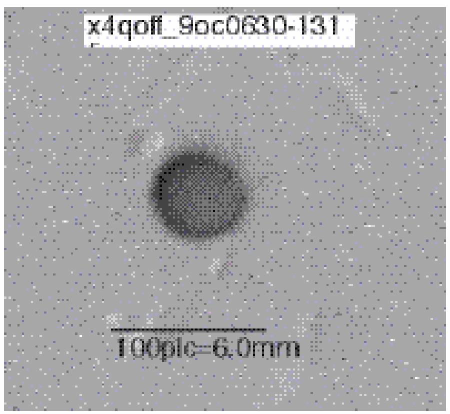
- For a beam with initial canonical angular momentum, the normal modes in the RF gun, GTL, and Linac are the drift and cyclotrons modes.
- The radial emittance of the round beam is important to track for space charge induced growth and emittance compensation.
- In the Adapter beamline, rotational symmetry is broken and the normal modes of interest are the horizontal and vertical.
- In a perfect system, the total 4D emittance is conserved and the Adapter converts the former set of normal modes into the latter.
  Drift -> Horizontal, Cyclotron -> Vertical

Emittance Ratios of 50:1 have been measured

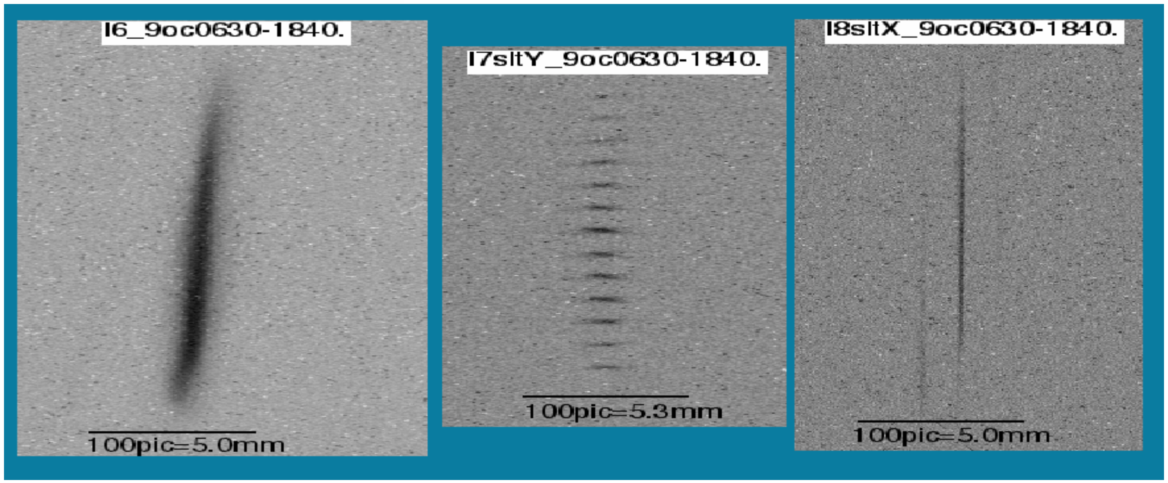


(2001)

Flat beam measurements



Round beam image on fluorescent screen



Flat beam image on fluorescent screen

Beam image through slits for emittance measurement

Horizontal emittance ~ 0.9 mm-mrad @ 1nC (Vertical emittance ~ 45 mm-mrad). Horizontal measurement resolution-limited by finite CCD pixel size (~1 μm).

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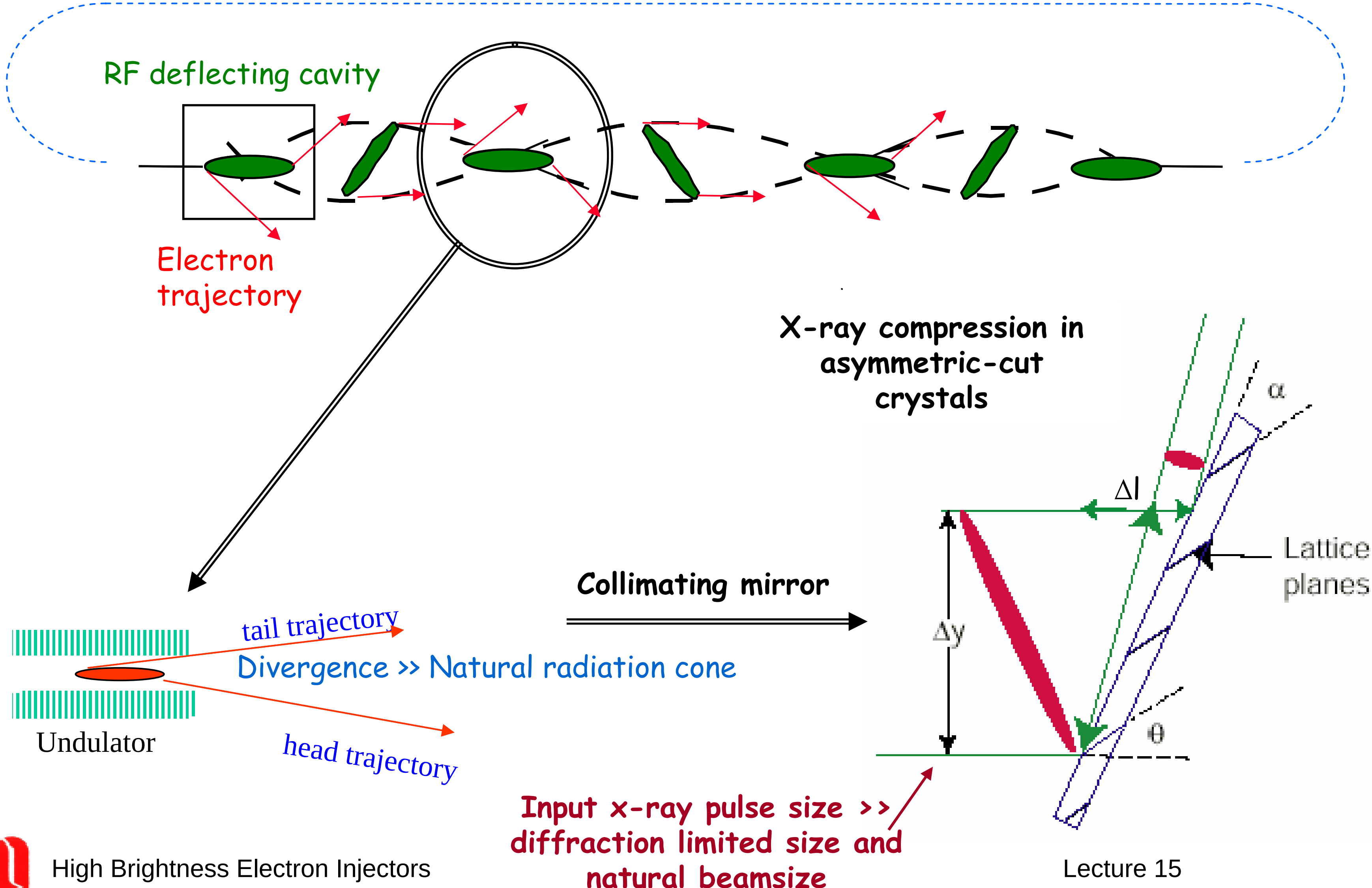
Short-pulse hard x-ray scheme



FEL needs very bright electron beam…

$$\varepsilon_N < \gamma \frac{\lambda_r}{4\pi}$$

transverse emittance: $\varepsilon_N \approx 0.2\ \mu$m at 1 Å, 15 GeV

$$\sigma_\delta < \rho \approx \frac{1}{4}\left(\frac{1}{2\pi^2}\frac{I_{pk}}{I_A}\frac{\lambda_u^2}{\beta\varepsilon_N}\left(\frac{K}{\gamma}\right)^2\right)^{1/3}$$

energy spread:

$\sigma_\delta \approx 0.01\%$ at I_{pk} = 4 kA, $K \approx 3.5$, $\lambda_u \approx 3$ cm, …

Long. emittance requirement is: $\gamma\sigma_z\sigma_\delta \equiv \gamma\varepsilon_z \approx$ **60 μm**

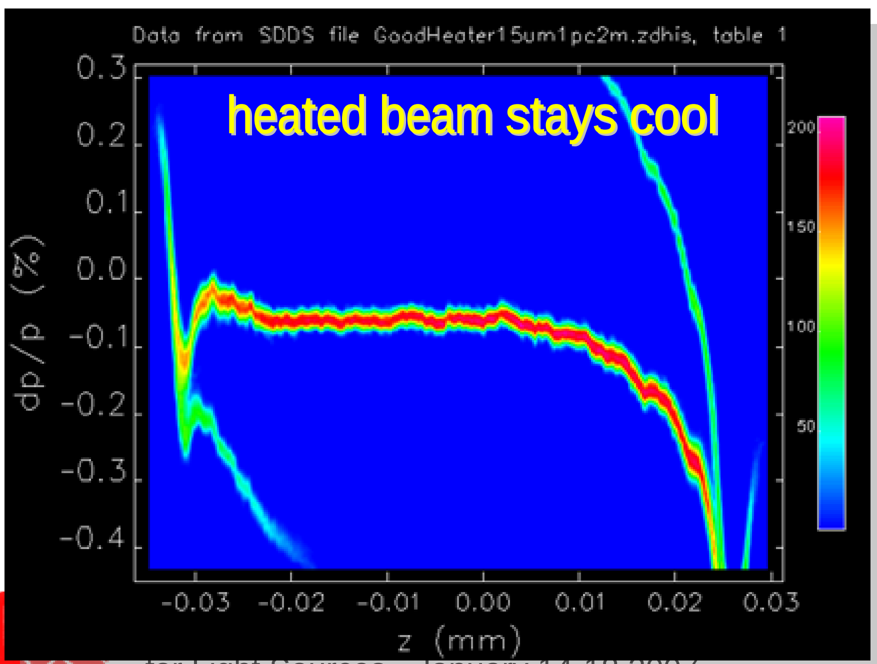
RF gun produces both $\gamma\varepsilon_x \sim \gamma\varepsilon_z \sim$ few μm

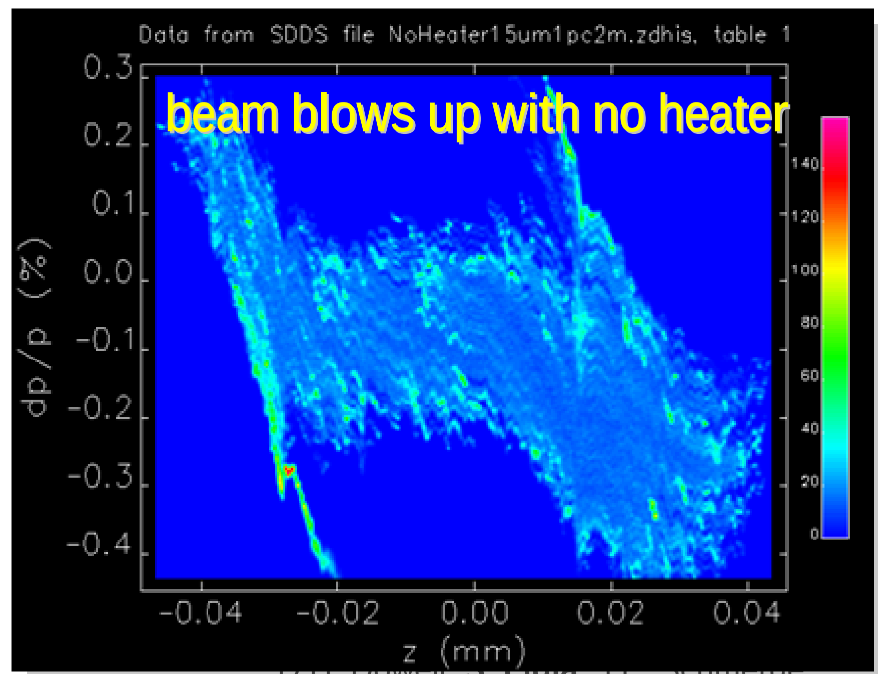
Can we reduce $\gamma\varepsilon_x$ at the expense of $\gamma\varepsilon_z$?

Intrinsic energy spread is <u>too</u> small to be a benefit for x-ray SASE FEL

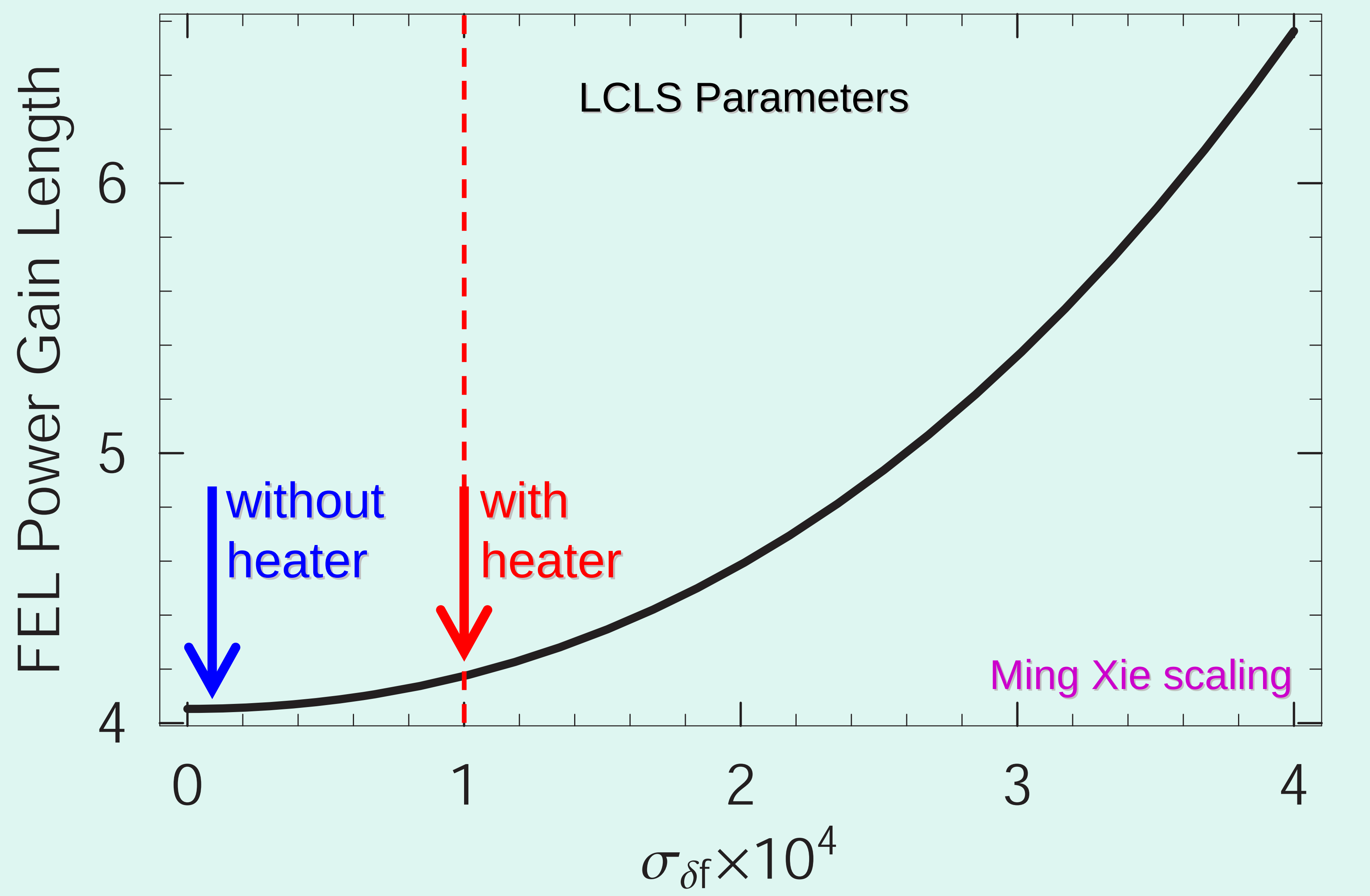
A beam heater is required, which Landau damps the CSR/LSC micro-bunching instability

Final long. phase space at 14 GeV for initial modulation of 1% at λ = 15 μm



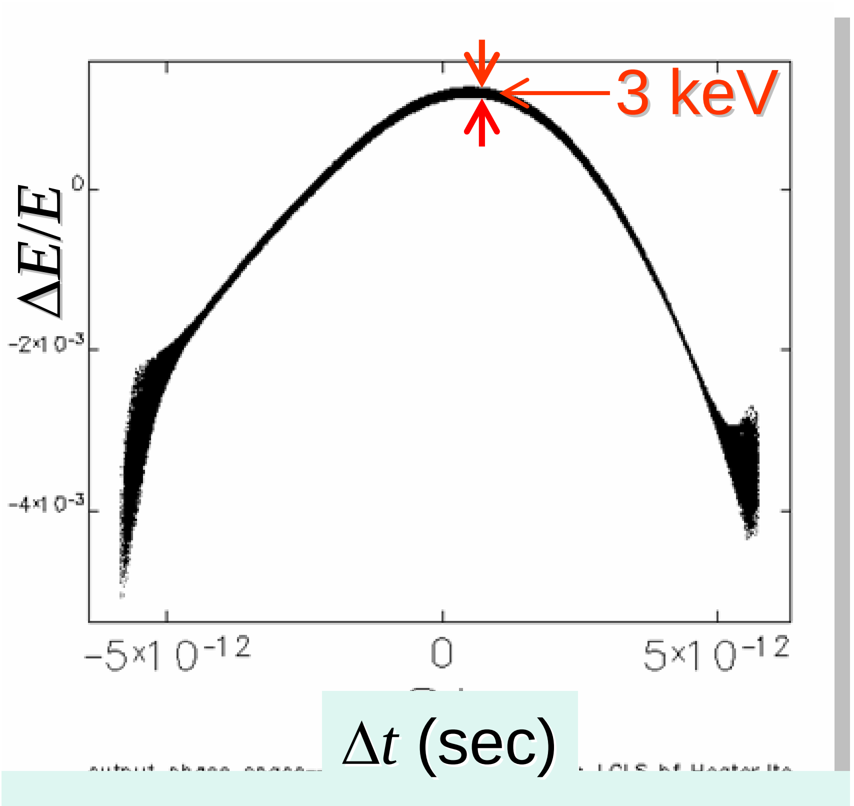


Very Small Energy Spread is Wasted

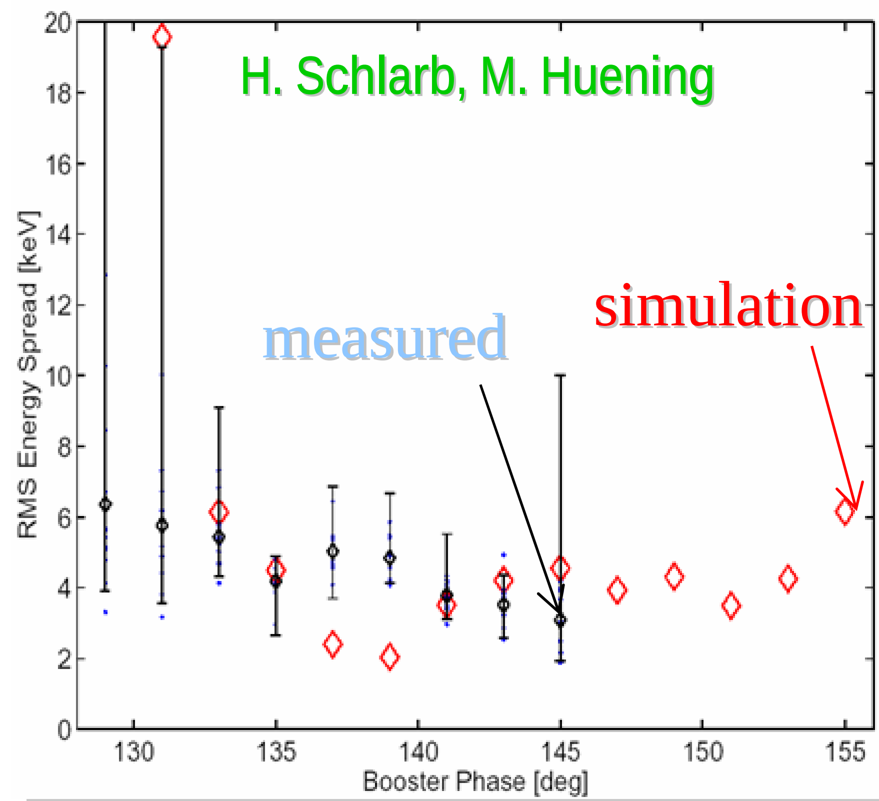


How cold is the photo-injector beam?

Parmela Simulation



TTF measurement



3 keV, accelerated to 14 GeV, and compressed ×36 ⇒

$3/14\times10^{6}\times36 < 1\times10^{-5}$

Too small to be useful in FEL (no effect on FEL gain when $<1\times10^{-4}$)

Longitudinal-Transverse correlation

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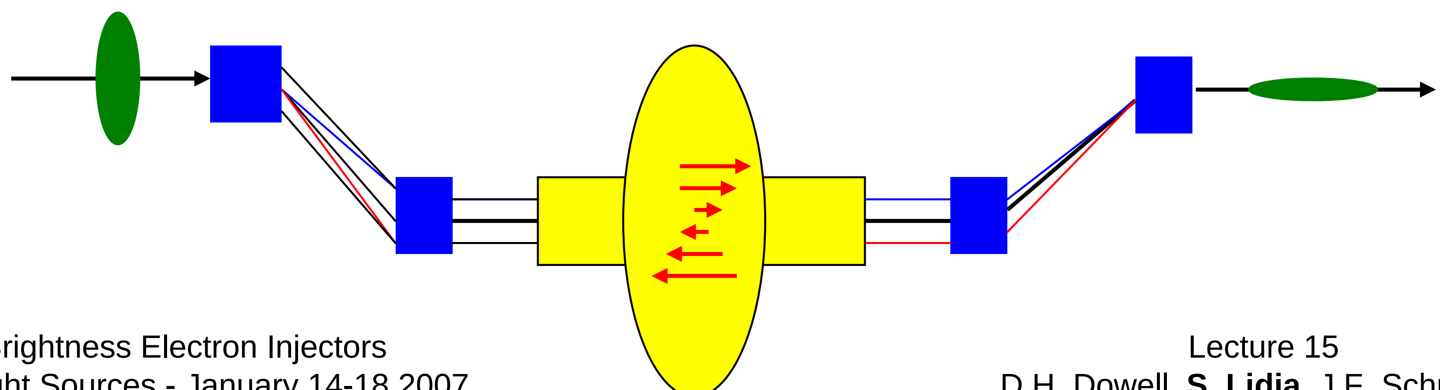
Smaller normalized transverse emittances can be obtained at the expense of the longitudinal emittance.

Place a TM_{110} cavity in a magnetic chicane at the location of large dispersion. This correlates the betatron amplitude with the energy bunch length.



Dipole mode deflecting cavity

Energy change from
the electric field

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Transverse kick from the
magnetic field

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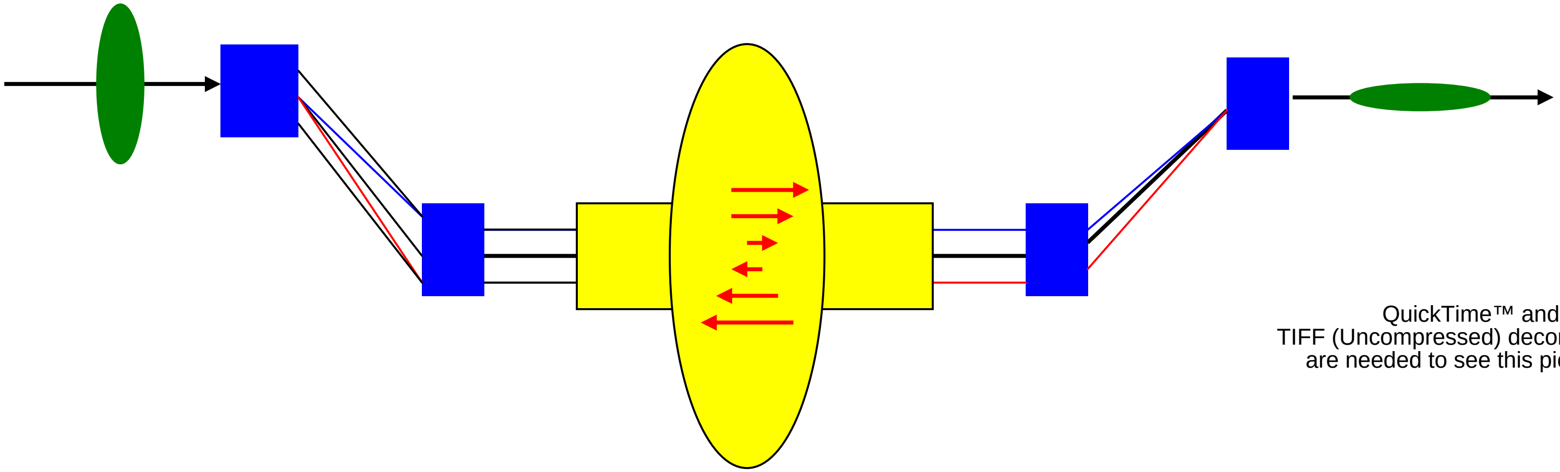
Transverse to Longitudinal Exchange

Small energy spread
at entrance



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Horizontal beam size is (transverse) emittance dominated, and shows little dispersion (since the energy spread is small).
Cavity introduces (linear) longitudinal energy variation across transverse spot.

Horizontal-Longitudinal Phase Space Manipulation

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Full emittance exchange

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Complete emittance exchange

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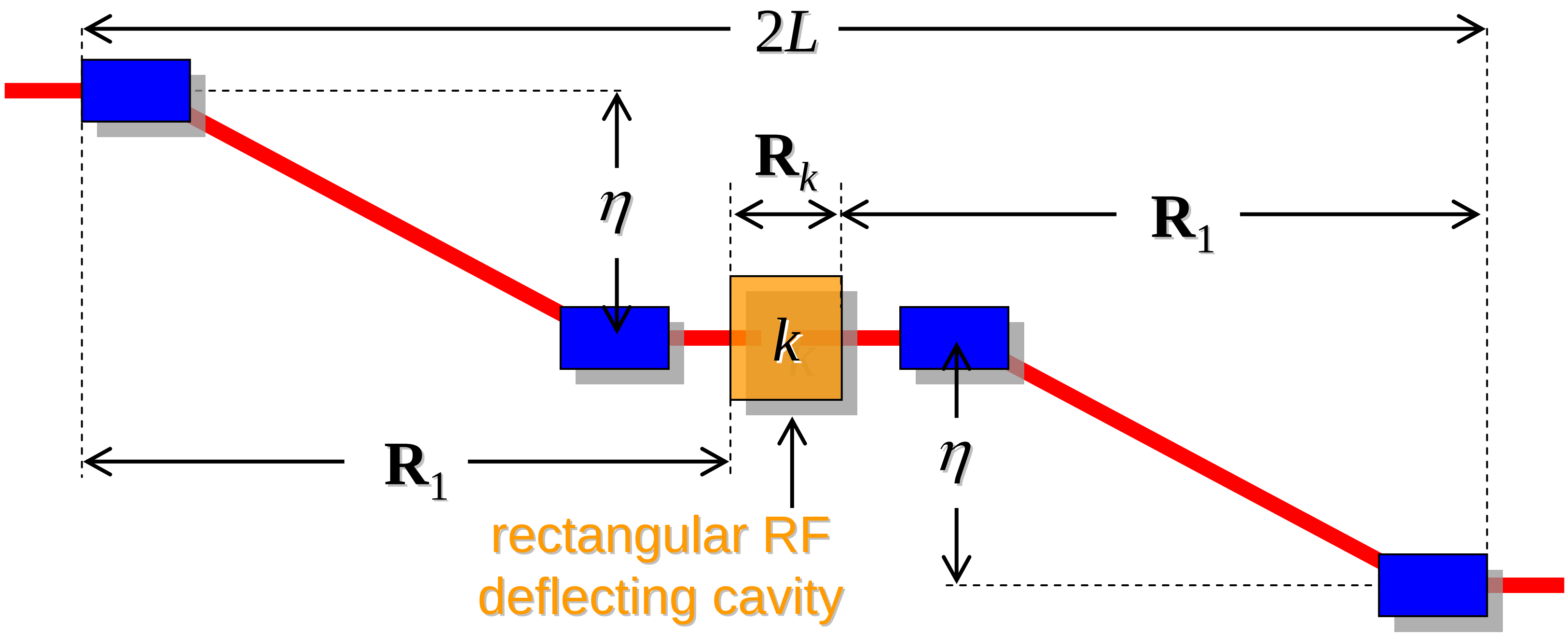
Improved Emittance Exchanger



$\xi \equiv R_{56}$ of dog-leg **x, z mapping (ignore y coordinate here)**

$$\mathrm{R}_1 = \begin{pmatrix} 1 & L & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathrm{R}_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{pmatrix}, \quad \mathrm{R} = \mathrm{R}_1 \mathrm{R}_k \mathrm{R}_1$$

Cornacchia, Emma, PRST AB 5, 084001 (2002) – system modified by K.-J. Kim

Full Emittance Exchange

If RF deflector voltage is set to: $k = -1/\eta$

$$\mathbf{R} = \begin{pmatrix} 0 & 0 & kL & \eta + kL\xi \\ 0 & 0 & k & k\xi \\ k\xi & \eta + kL\xi & 0 & 0 \\ k & kL & 0 & 0 \end{pmatrix}$$

and transverse (bend-plane) and longitudinal emittances are completely exchanged.

$$\epsilon_x = \epsilon_{z0}$$
$$\epsilon_z = \epsilon_{x0}$$

A complete, 6D emittance transform

We can use emittance exchange in sequence with flat beam production to make a beam with larger longitudinal emittance and very small transverse emittance in both planes.

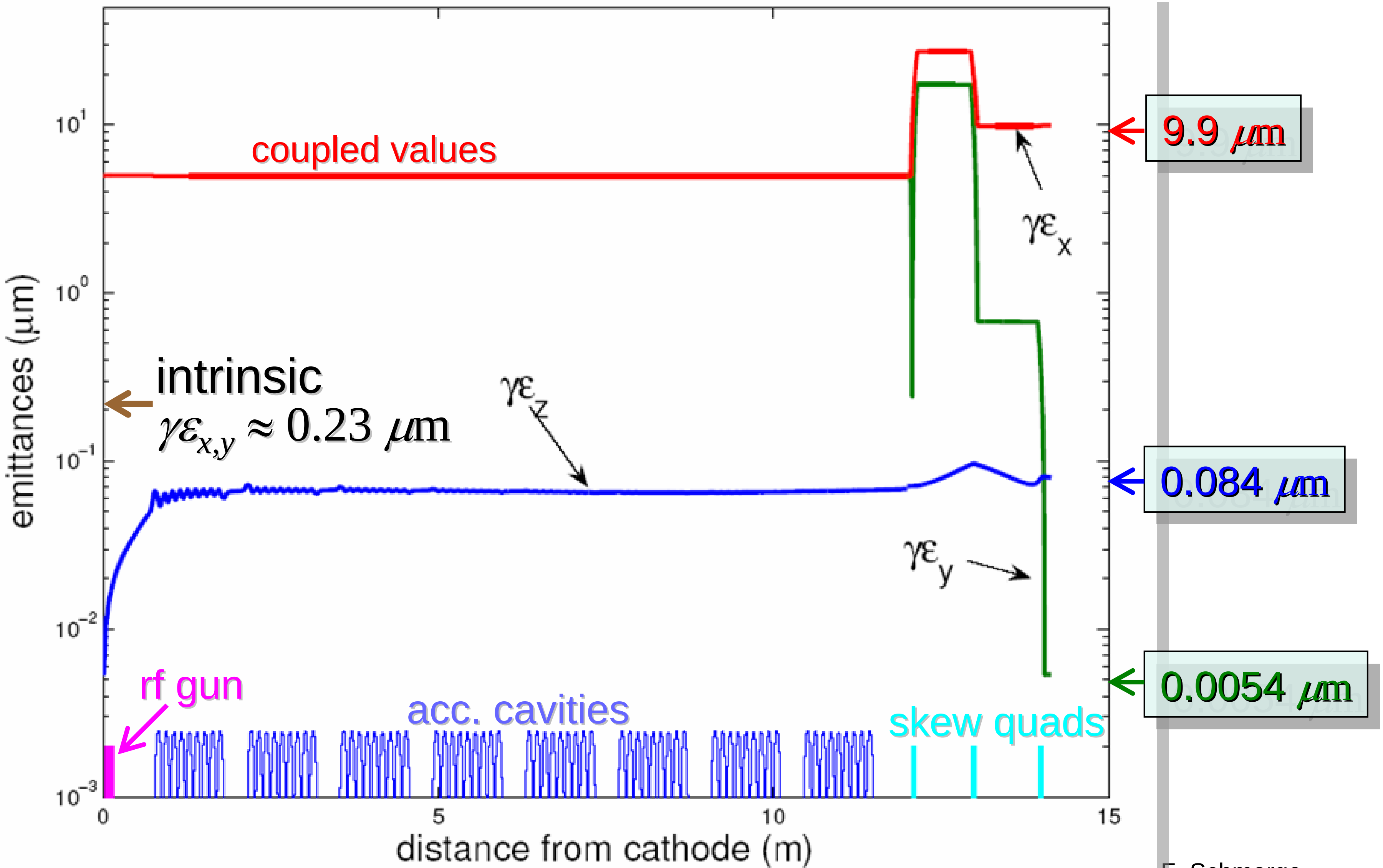
ASTRA and *Impact-T* simulations

- 2.6-GHz, 1.5-cell RF gun
- 138 MV/m peak E-field in gun
- 8 TESLA SC-cavities at 1.3 GHz (216 MeV)
- round-to-flat converter of 3 skew quads
- 3D oblate ellipsoid laser pulse
- 0.6 μm per mm radius thermal emittance
- 36 MV/m peak E-field in TESLA cavities
- 300 μm rms laser spot size on cathode
- 50 μm rms bunch length (80 fs laser pulse)
- 20 pC bunch charge (34 A)

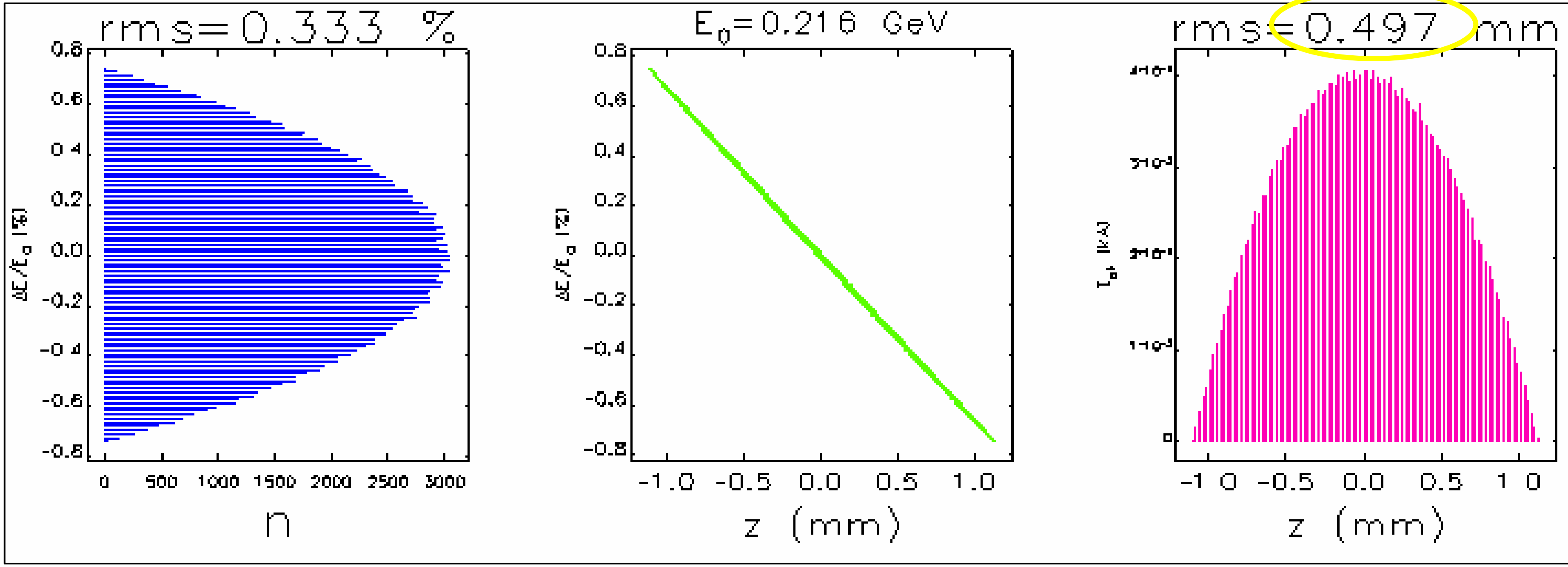
Emittance Levels from Simulation

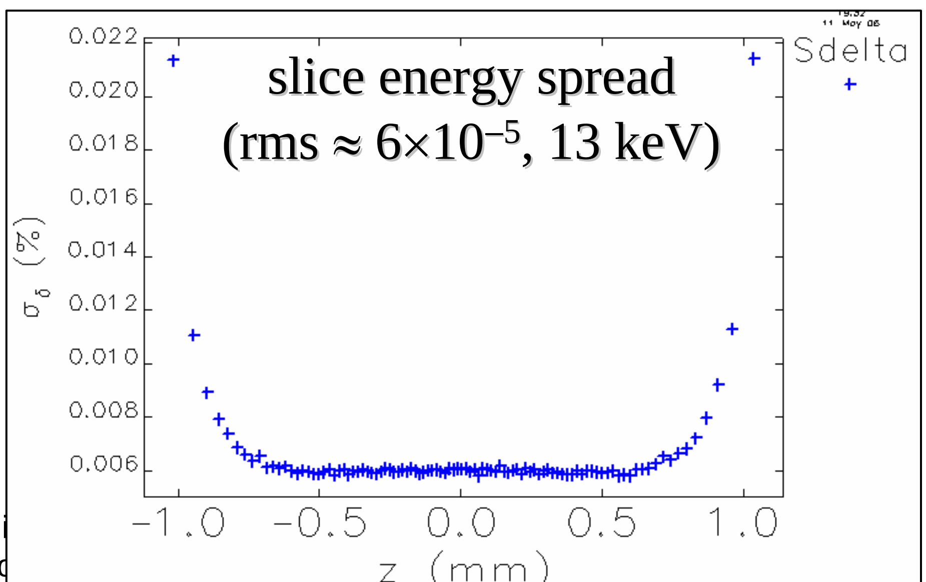
- **Thermal 2D**: $(0.6\ \mu\text{m/mm})\times(0.3\ \text{mm}) \approx \mathbf{0.23\ \mu m}$
- **Transverse 4D**: $(0.23\ \mu\text{m})^2 \approx \mathbf{0.053\ \mu m^2}$
- **After skew 4D**: $(9.9\ \mu\text{m})\times(0.0054\ \mu\text{m}) \approx \mathbf{0.053\ \mu m^2}$
- **Longitudinal 2D**: $\mathbf{0.080\ \mu m}$

Evolution of Transverse Emittances Along Photo-Injector Beamline (to 216 MeV)



Longitudinal Distributions After Exchanger (no CSR)

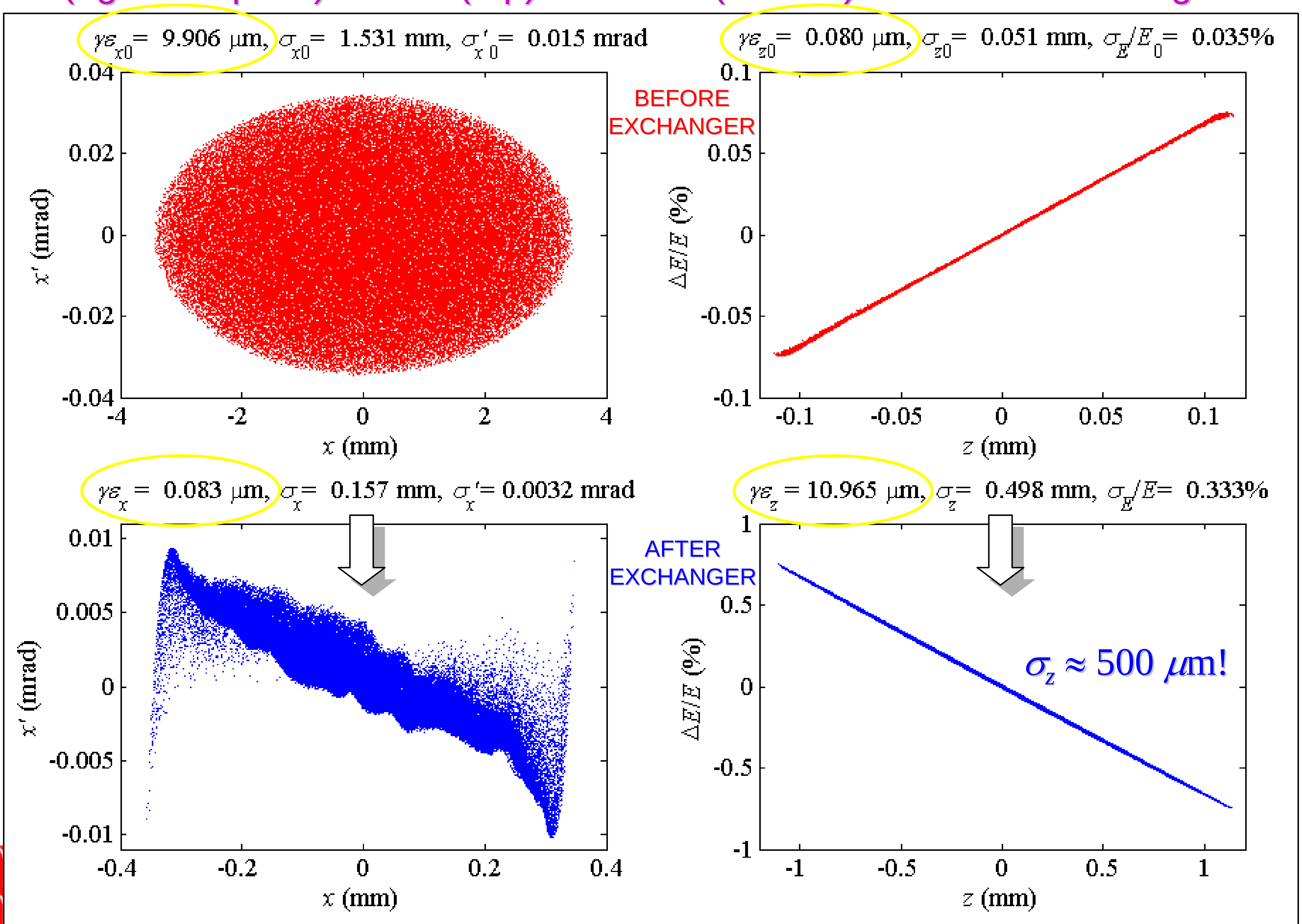




$$E_0 = 216\ \mathrm{MeV}$$

$$\gamma\varepsilon_z = 9.92\ \mu\mathrm{m}$$
$$\gamma\varepsilon_y = 0.0054\ \mu\mathrm{m}$$
$$\gamma\varepsilon_x = 0.084\ \mu\mathrm{m}$$

Transverse phase space (left two plots) and longitudinal phase space (right two plots) before (top) and after (bottom) emittance exchange.



Wakefield compensation by tailored slice current profile

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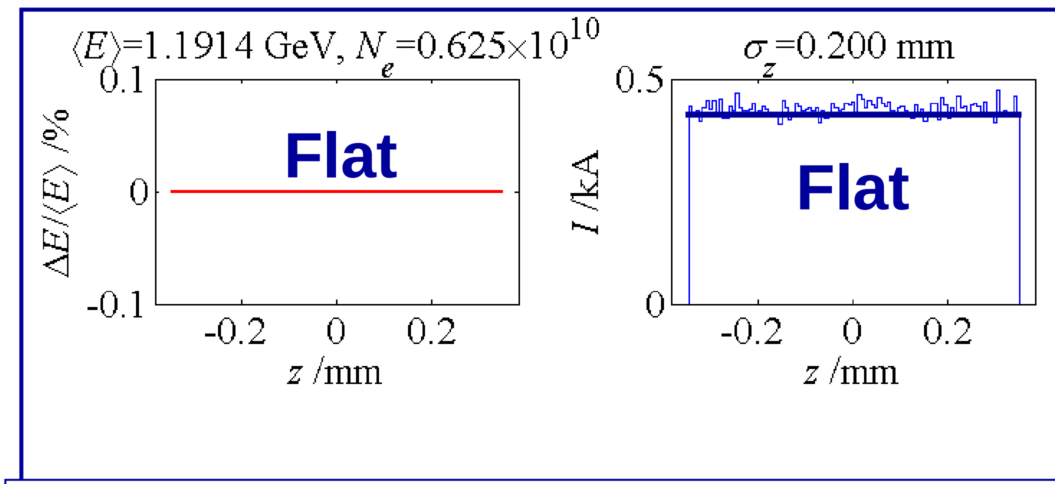
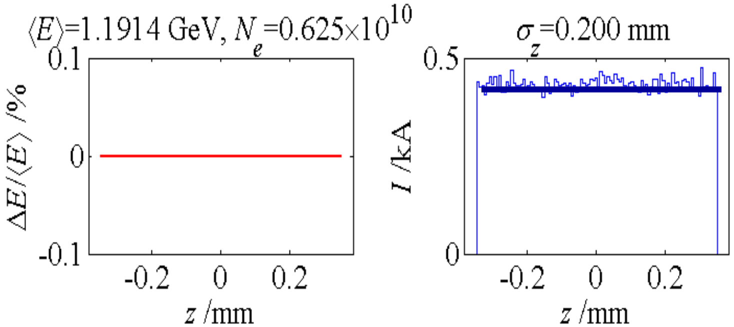


Seeded, harmonic cascade FELs flat current and flat energy beams

A distribution at the beginning of the accelerator that will evolve into flat-flat distribution can be found using reverse tracking*

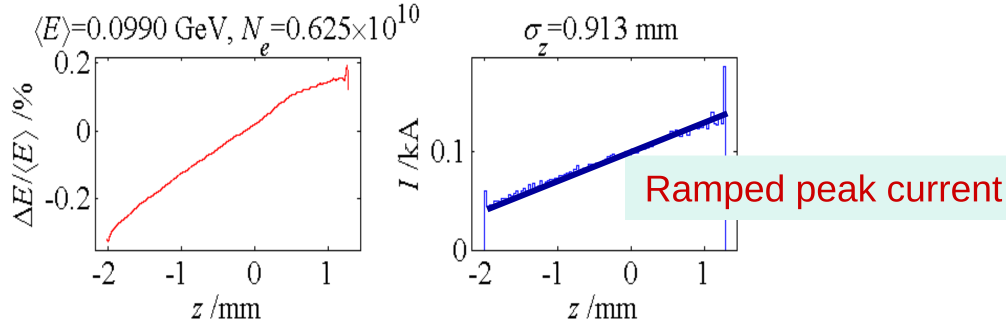
Reverse tracking: use of wakefields

Begin tracking



end of accelerator

End tracking

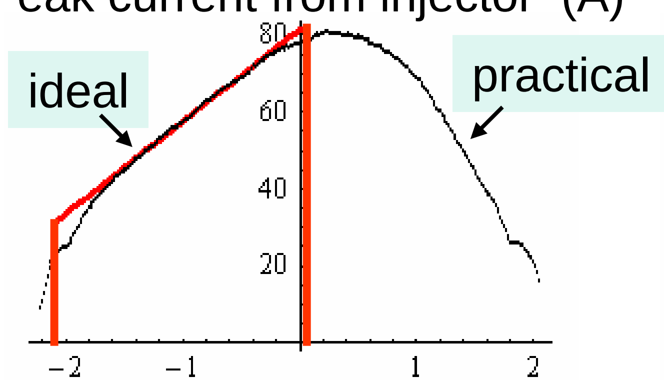


start of accelerator

LiTrack: no LSC no CSR

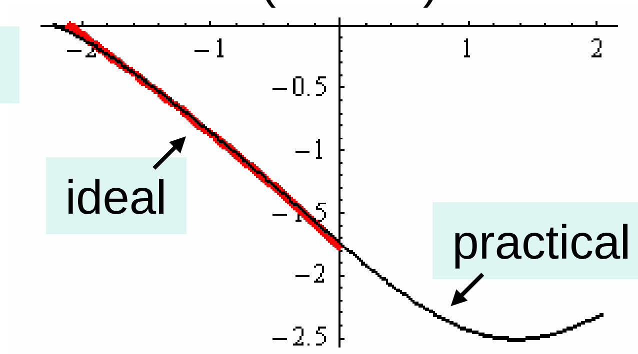
Wake potential: $W(s) = -\int_s^\infty w(s-s')\lambda_z(s')ds'$ where $w(s) = A\dfrac{Z_0 c}{\pi a^2} L \exp\left(-\sqrt{s/s_0}\right)$

Peak current from injector (A)



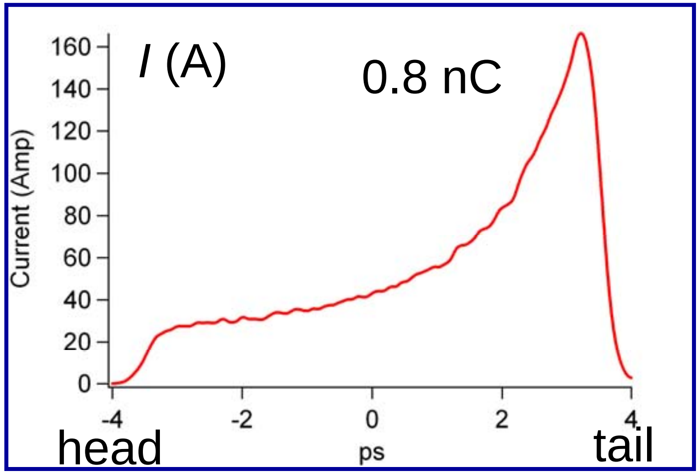
s (mm)

W (MV/nC)



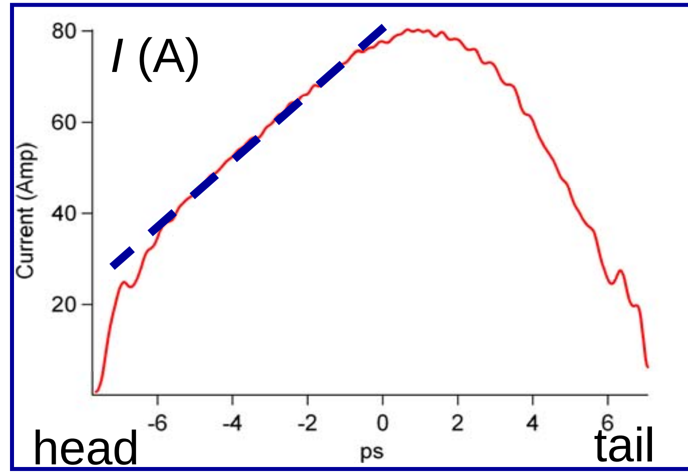
s (mm)

Photocathode laser is used to shape the electron distribution in the e-gun

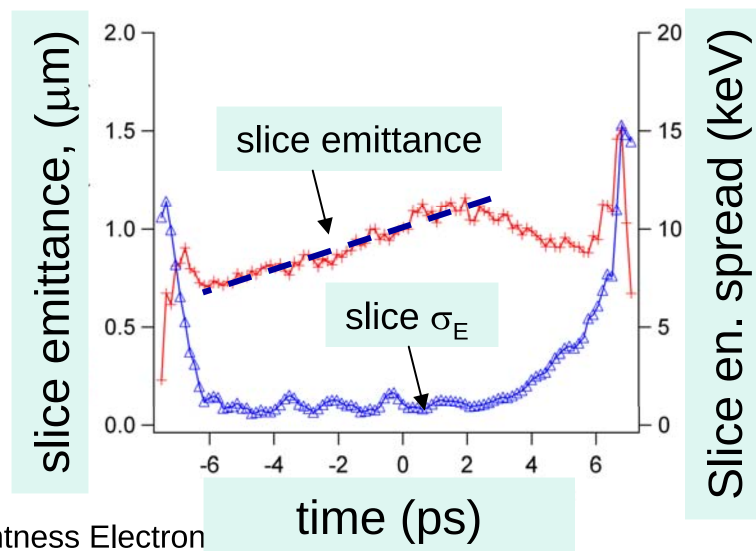


Next to the cathode

space
charge



At the end of the injector at 100 MeV



For HC FEL
larger emittance in the tail may not be a problem and smaller emittance at the head can be beneficial

Discussion

Photoinjectors produce very bright beams.

We can manipulate the phase space of the beam to better match the requirements of our application.

It helps to pay attention to what our colleagues in other areas of beam physics are doing.