Lecture 3:
Space Charge Limited Emission

High Brightness Electron Injectors for Light Sources

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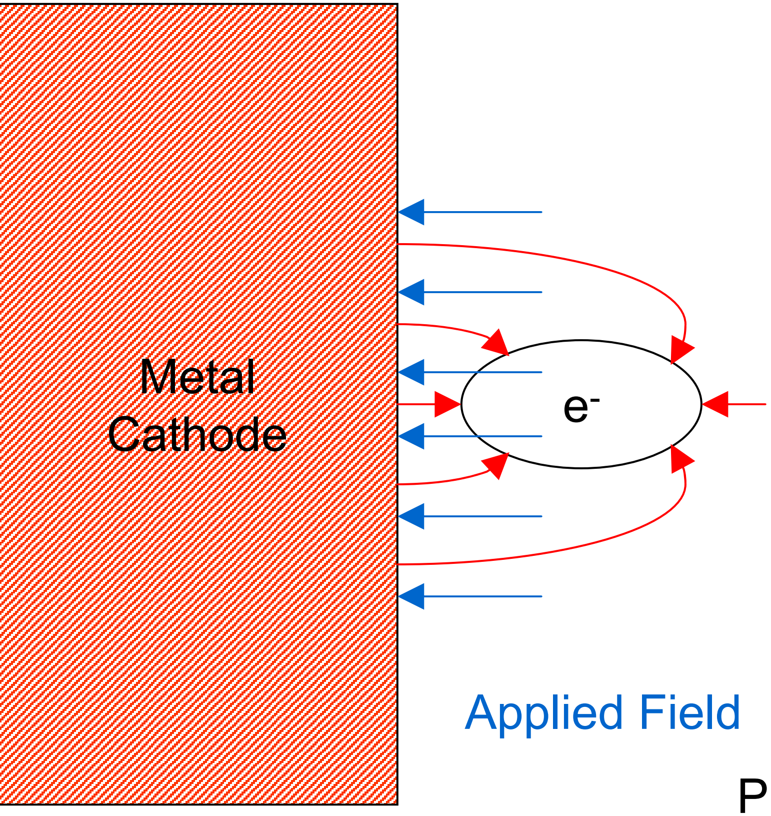
Outline

- Space Charge Field at the Cathode
  - Cigar Beam
  - Pancake Beam
- QE reduction due to Space Charge
  - Space Charge Limit
  - Longitudinal
  - Transverse
- Derive Child-Langmuir Law
  - DC
  - Relativistic correction
  - Pulsed
- Emission
  - Temperature or QE limited
  - Space Charge limited

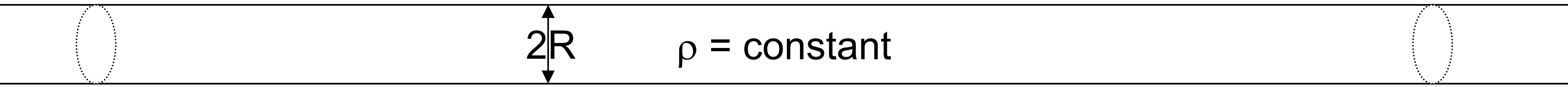
Field at the Cathode



Applied Field + Space Charge Field = Total Field

Principle of Superposition

Infinite Line Charge



$$\int_S D \cdot dS = \int_V \rho dV$$ Gauss's Law

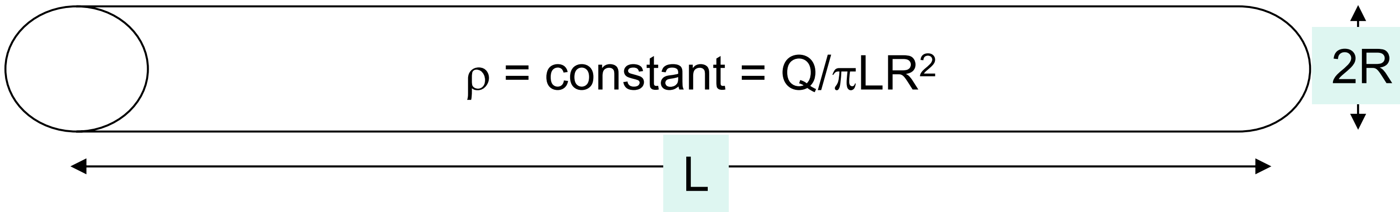
$$\rho_z = \pi R^2 \rho$$ Line Charge Density

$$2\pi r \varepsilon E_r dz = \frac{r^2}{R^2} \rho_z dz \qquad r \le R$$

$$2\pi r \varepsilon E_r dz = \rho_z dz \qquad r \ge R$$

$$E_r = \begin{cases} \dfrac{\rho_z r}{2\pi \varepsilon R^2} & r \le R \\[2ex] \dfrac{\rho_z}{2\pi \varepsilon r} & r \ge R \end{cases}$$

Cigar Beam



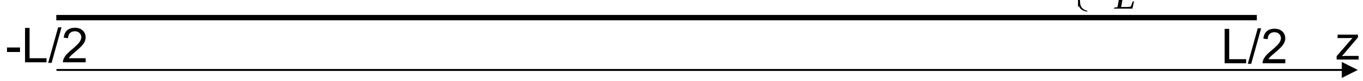
$$\vec{E}(r,z) = \frac{1}{4\pi\varepsilon}\int_V \frac{\vec{a}_s \rho}{s^2} dV = \frac{1}{4\pi\varepsilon}\int_V \frac{\vec{s}\rho}{s^3} dV$$

Coulomb's Law

Cigar Beam II

Model as a line charge (diameter 0) with density

$$\rho_z = \begin{cases} \dfrac{Qr^2}{LR^2} & r \le R \\[2ex] \dfrac{Q}{L} & r > R \end{cases}$$



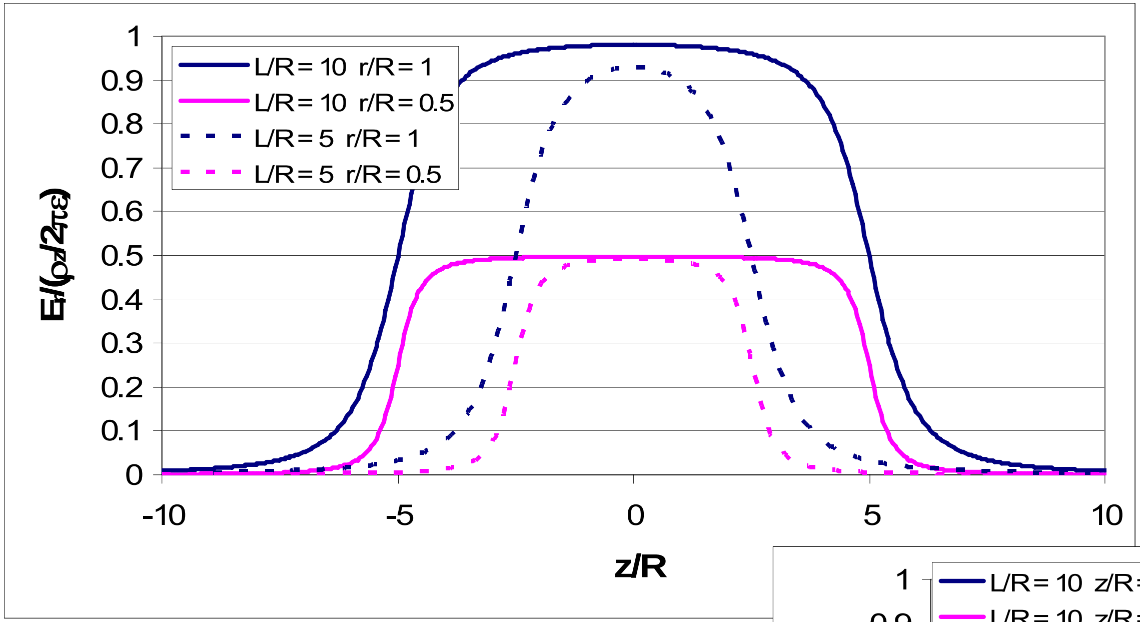
$$E_r(r,z) \approx \int_{-L/2}^{L/2} \frac{Qr^3 dz'}{4\pi\varepsilon LR^2 \left[r^2 + (z'-z)^2 \right]^{3/2}} \qquad r \le R \quad and \quad r << L$$

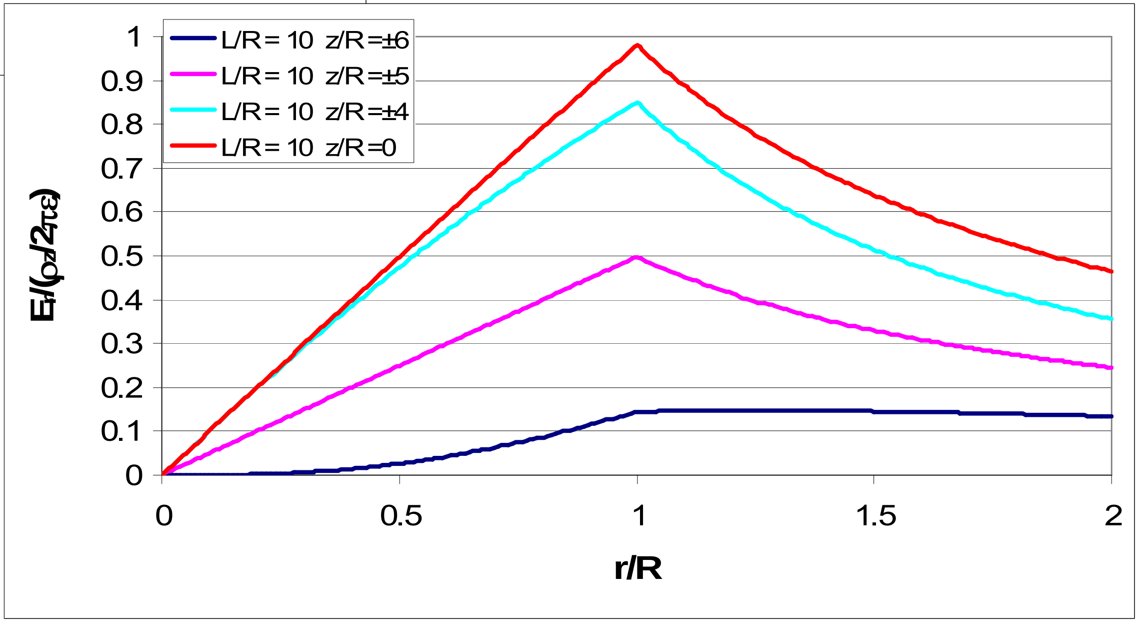
$$\int \frac{a^2 dx}{\left(a^2 + x^2\right)^{3/2}} = \frac{x}{\sqrt{a^2 - x^2}} + C$$

$$E_r(r,z) \approx \frac{Qr}{4\pi\varepsilon LR} \left(\frac{\left(\frac{L}{2R} - \frac{z}{R}\right)}{\sqrt{\left(\frac{r}{R}\right)^2 + \left(\frac{L}{2R} - \frac{z}{R}\right)^2}} + \frac{\left(\frac{L}{2R} + \frac{z}{R}\right)}{\sqrt{\left(\frac{r}{R}\right)^2 + \left(\frac{L}{2R} + \frac{z}{R}\right)^2}} \right) \qquad r \le R \quad and \quad r << L$$

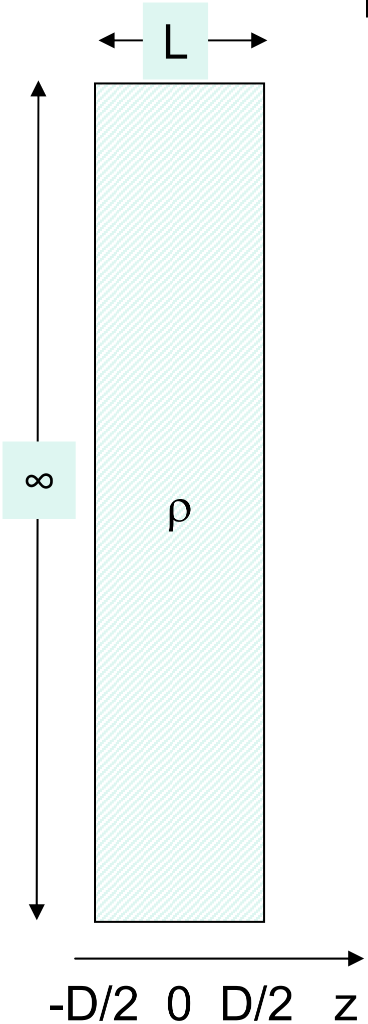
$$E_r(r,z) \approx \frac{QR}{4\pi\varepsilon Lr} \left(\frac{\left(\frac{L}{2R} - \frac{z}{R}\right)}{\sqrt{\left(\frac{r}{R}\right)^2 + \left(\frac{L}{2R} - \frac{z}{R}\right)^2}} + \frac{\left(\frac{L}{2R} + \frac{z}{R}\right)}{\sqrt{\left(\frac{r}{R}\right)^2 + \left(\frac{L}{2R} + \frac{z}{R}\right)^2}} \right) \qquad r > R \quad and \quad r << L$$

Cigar Beam III





Infinite Sheet Charge



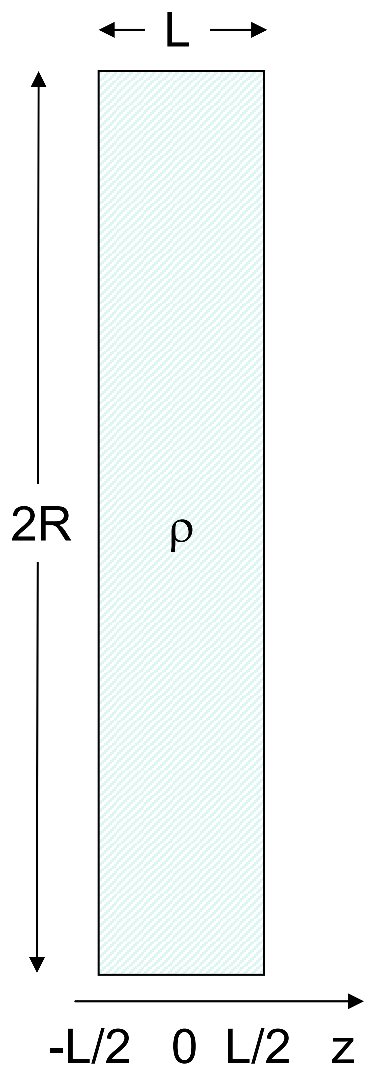
$$\int_S D \cdot dS = \int_V \rho dV \qquad \text{Gauss's Law}$$

$$\pi \varepsilon R^2 E_z = \rho \pi R^2 z$$

$$E_z = \frac{\rho z}{\varepsilon} \qquad -\frac{L}{2} < z < \frac{L}{2}$$

$$E_z = \frac{\rho L}{2\varepsilon} \operatorname{sgn}(z) \qquad |z| > \frac{L}{2}$$

Pancake Beam



$$\int_S D \cdot dS = \int_V \rho dV \qquad \text{Gauss's Law}$$

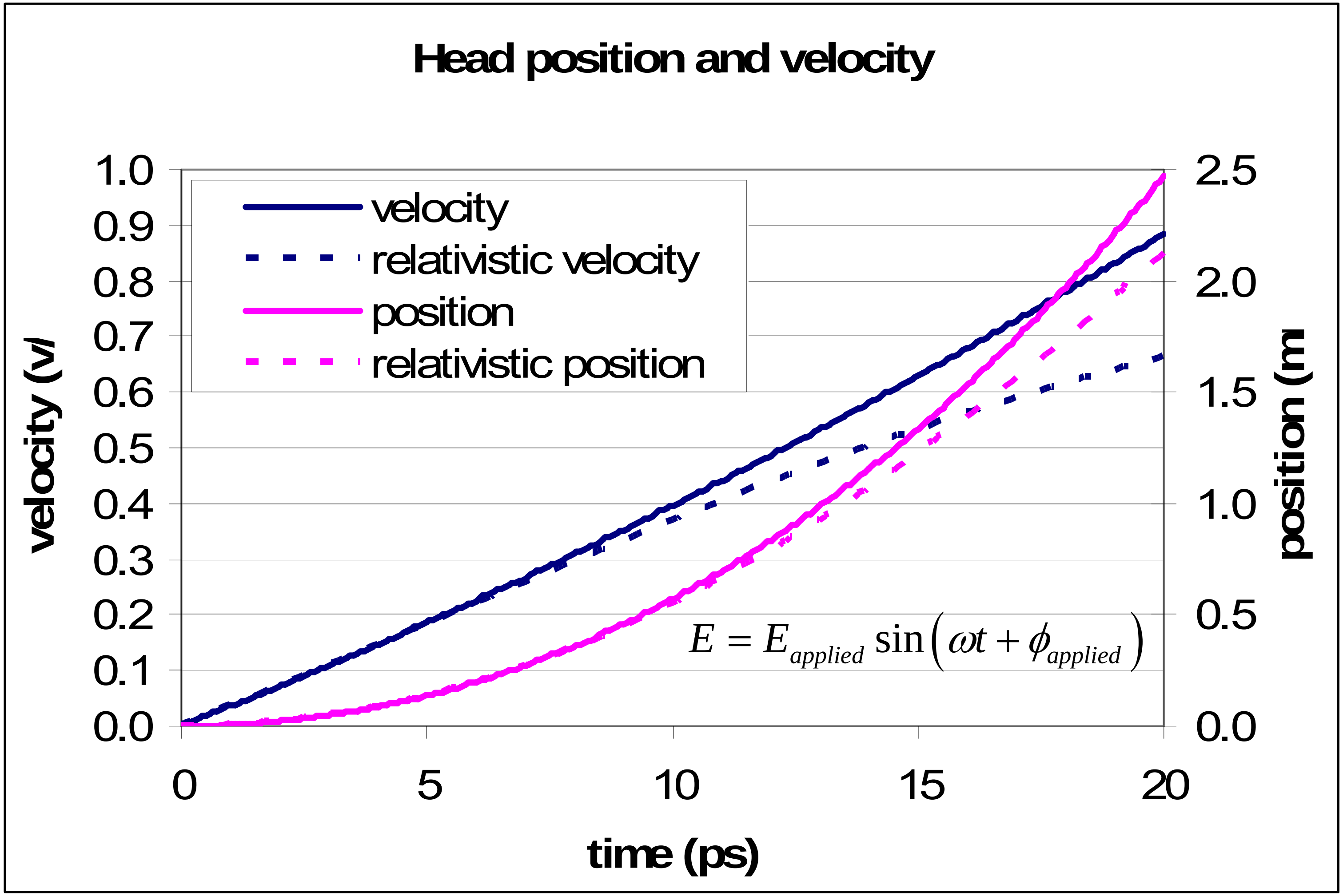
$$\pi \varepsilon R^2 E_z \approx \rho \pi R^2 z \qquad 2R >> L$$

$$E_z \approx \frac{\rho z}{\varepsilon} = \frac{Qz}{\pi \varepsilon R^2 L} \qquad 2R >> L$$

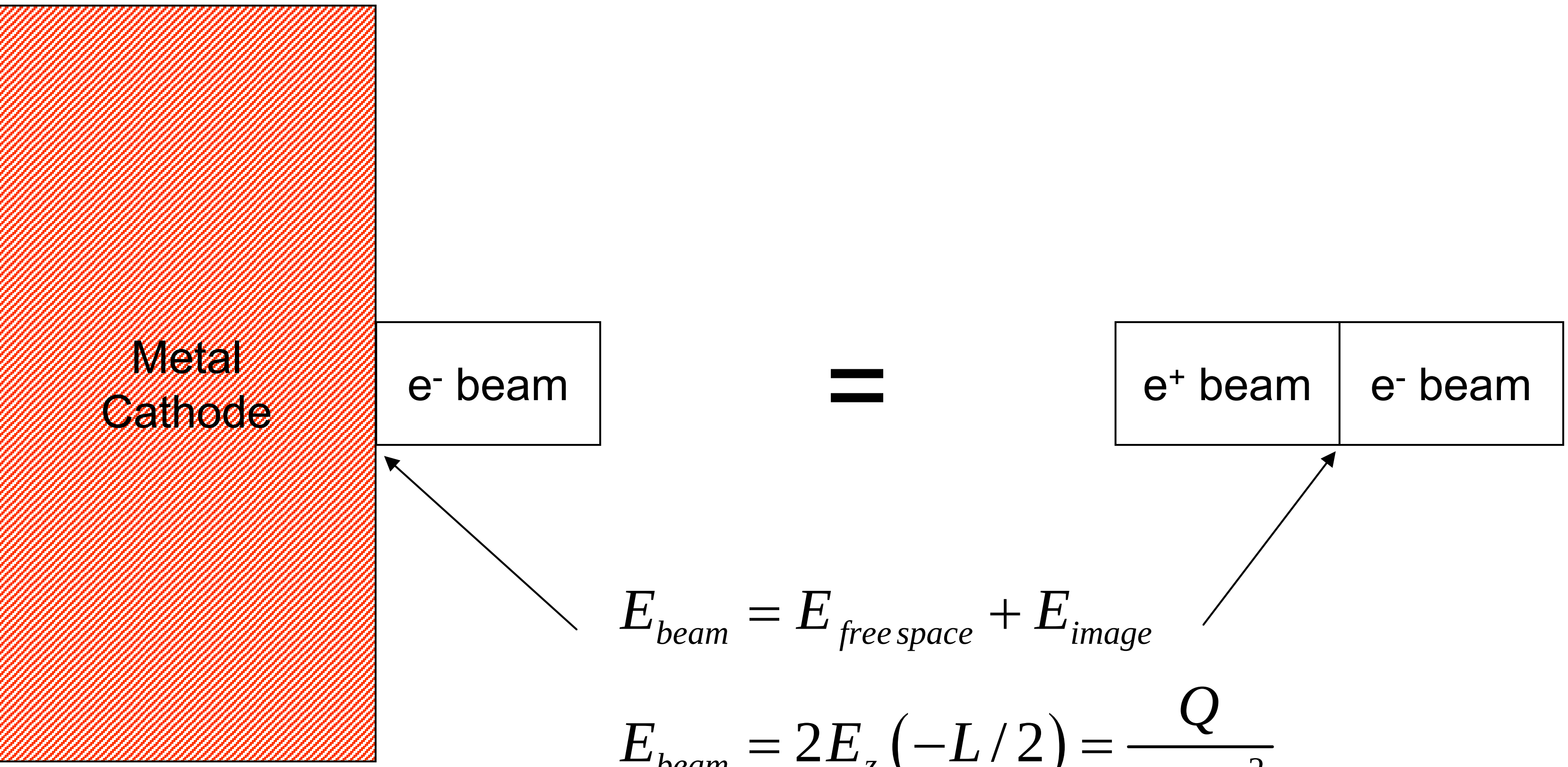
LCLS Beam

- Temporal Flat top Profile
  - Laser pulse length ≈10 ps or 3 mm
- Spatial Flat Top profile
  - R = 1.2 mm
- E-beam
  - Pancake or Cigar?
  - Near the cathode the beam is non-relativistic

What is the Aspect Ratio?



Field at the Cathode



$$E_{beam} = E_{free\,space} + E_{image}$$

$$E_{beam} = 2E_z\left(-L/2\right) = \frac{Q}{\pi\varepsilon_0 R^2}$$

QE(t)

$$QE(t) \approx \eta_0 \left[\frac{hc}{e\lambda} - \varphi_{cathode} + \sqrt{\left(\frac{eE_{total}(t)}{4\pi\varepsilon_0} \right)} \right]^2$$

$$E_{total}(t) = E_{RF}(t) - E_{beam}(t) = E_{applied} \sin\left(\omega t + \theta_{applied}\right) - \frac{Q(t)}{\pi\varepsilon_0 R^2}$$

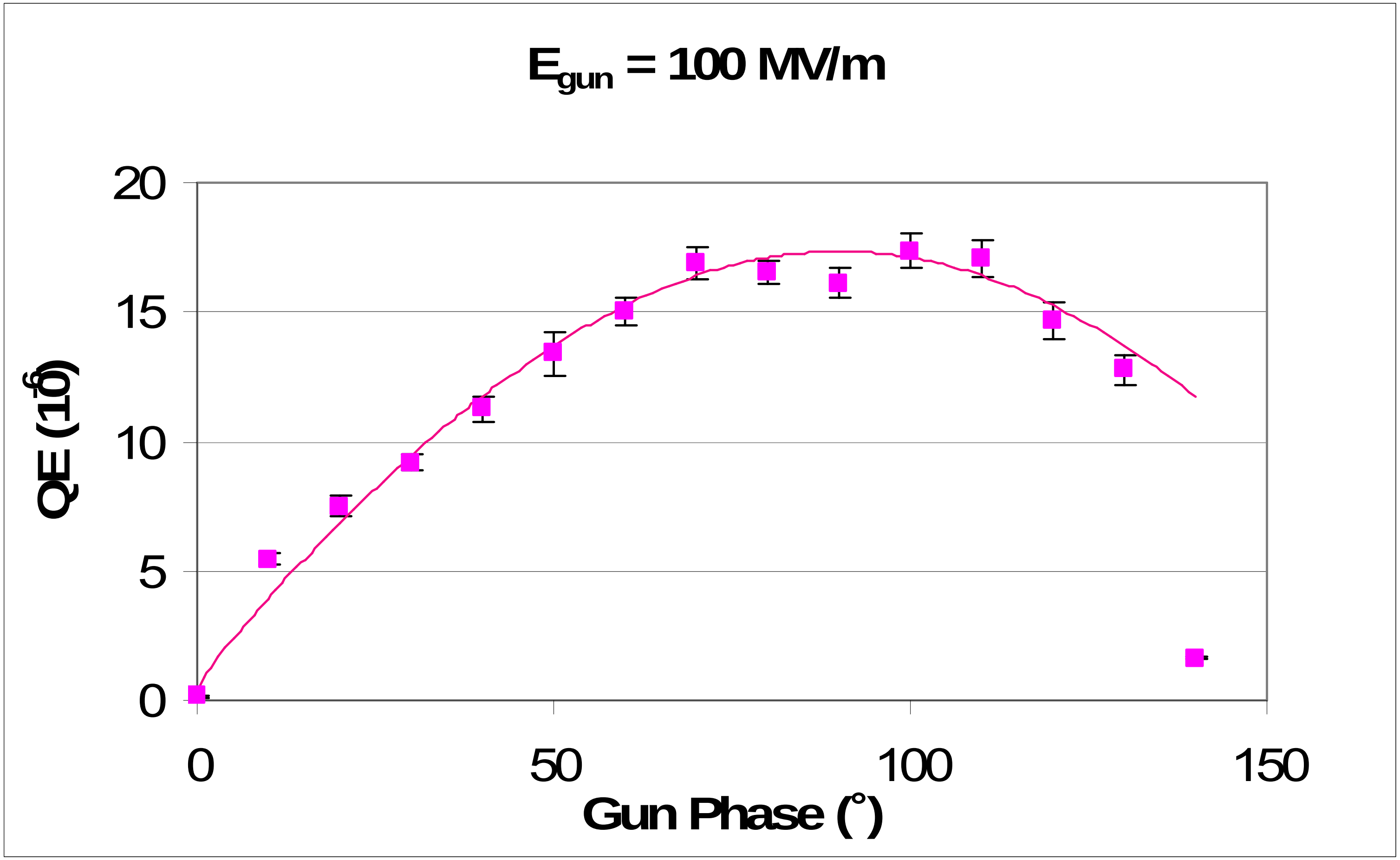
$$Q(t) = \frac{e\lambda}{hc} \int_0^t QE(t') P_{laser}(t') dt'$$

$$Q_{\mathrm{limit}} = \pi\varepsilon_0 R^2 E_{applied} \sin\left(\omega t + \theta_{applied}\right)$$

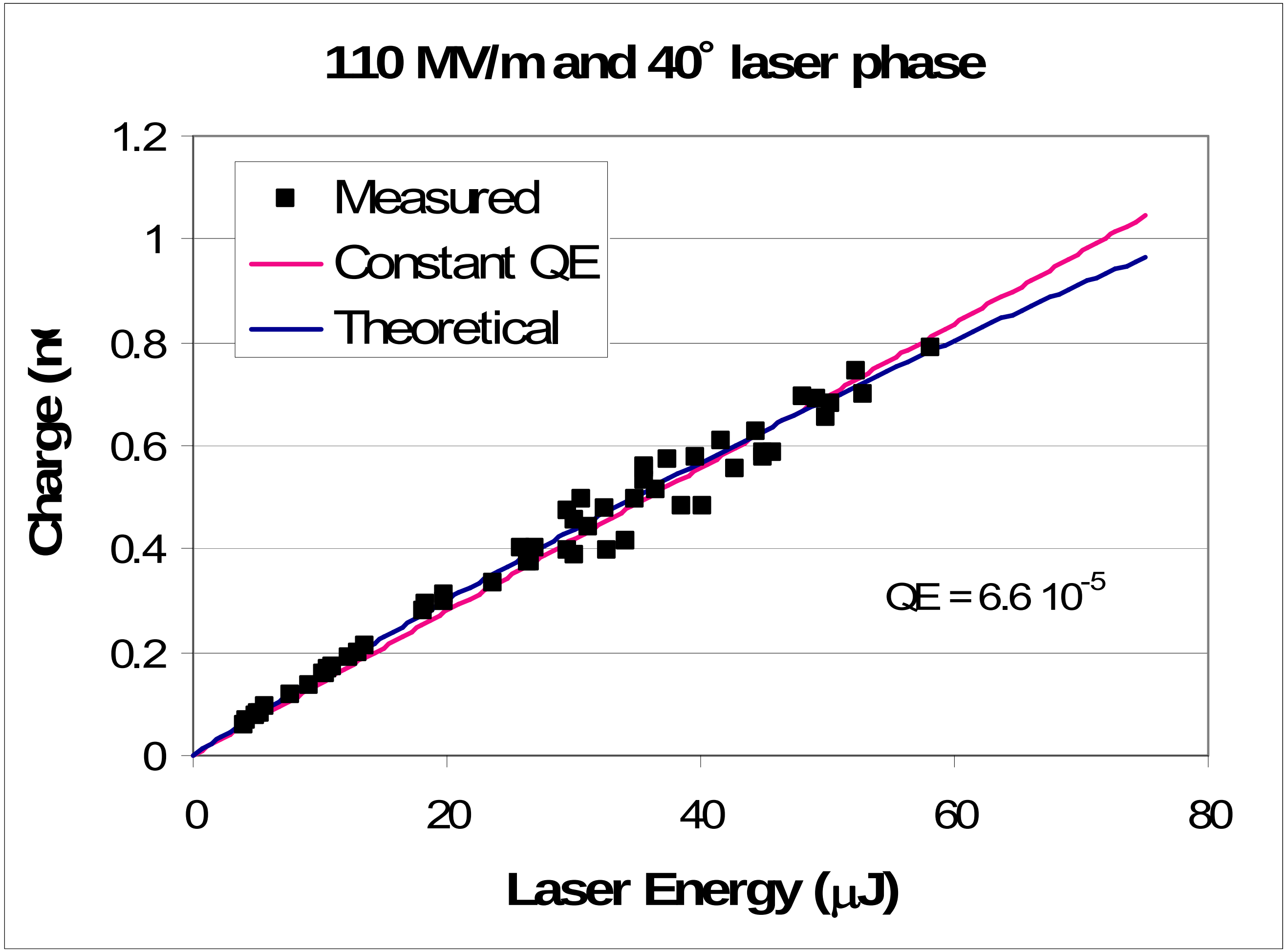
Measured QE

- Measure total charge per pulse
- Measure total laser energy per pulse
- Ratio is the time averaged QE
- Time Dependent QE can be determined using a short laser pulse by measuring QE as a function of $\phi_{applied}$
- Charge Dependent QE can be determined by measuring QE as a function of laser energy

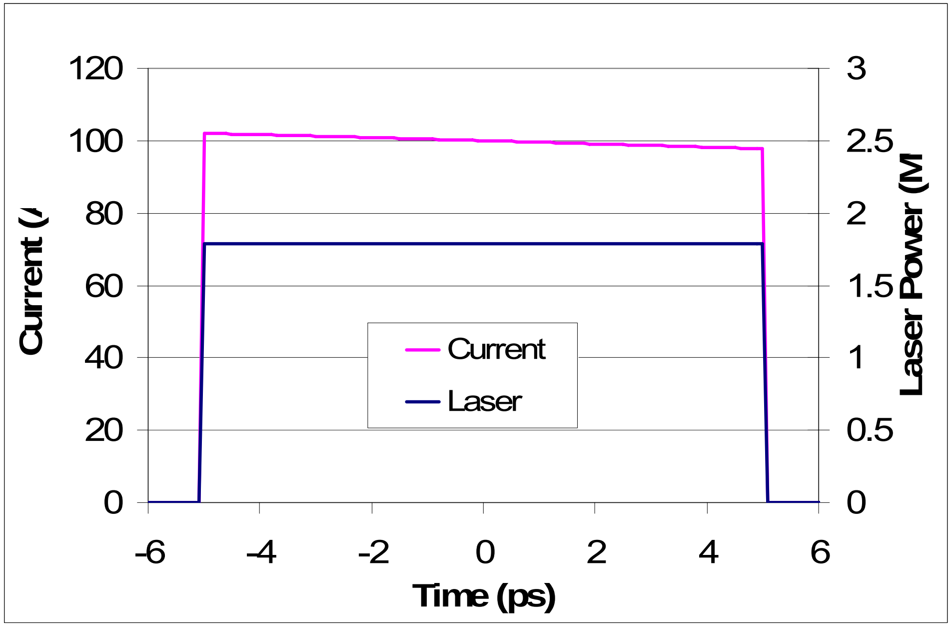
Time (Field) Dependence

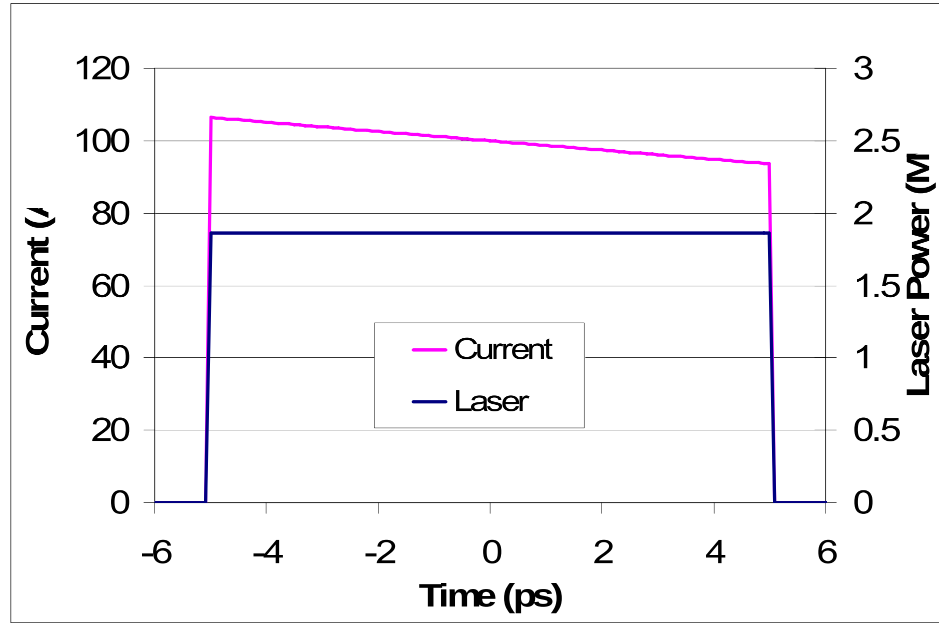


Charge Dependent QE



Simulation





$E_{applied}$ = 120 MV/m
$\phi_{applied}$ = 30°
Q = 1 nC
R = 1.2 mm
λ_0 = 255 nm

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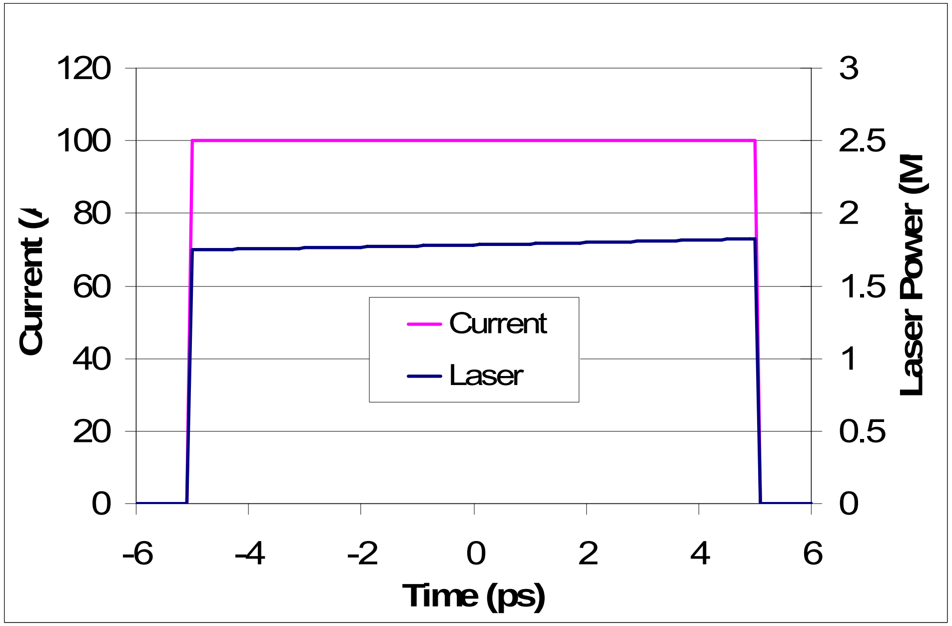
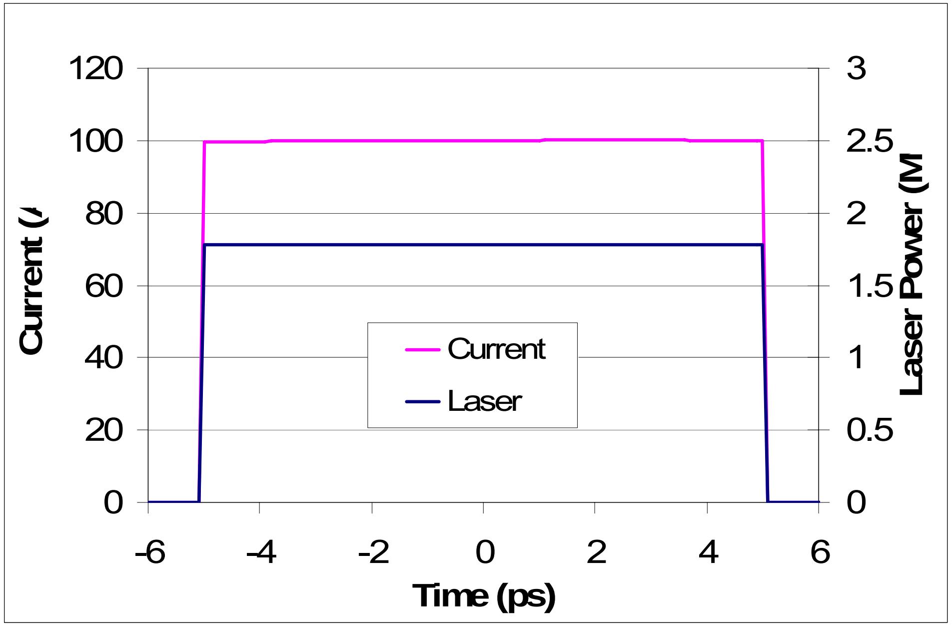
Methods for Generating Flat-Top Electron Pulse

- Choose operating point where decrease of field at the cathode due to space charge almost cancels the increase in field due to RF
- Ramp laser power as a function of time to compensate for change in QE
- Adjust laser wavelength as a function of time (chirp) to compensate change in QE
- Ignore effect (most common solution since the QE dependence on time is not included in simulation codes)

Simulation





$E_{applied}$ = 120 MV/m
$\phi_{applied}$ = 30°
Q = 1 nC
R = 1.2 mm
λ_0 = 255 nm
$d\lambda/dt$ = 0 nm/ps

$E_{applied}$ = 120 MV/m
$\phi_{applied}$ = 30°
Q = 1 nC
R = 1.2 mm
λ_0 = 255 nm
$d\lambda/dt$ = -0.1 nm/ps

Space Charge Diode

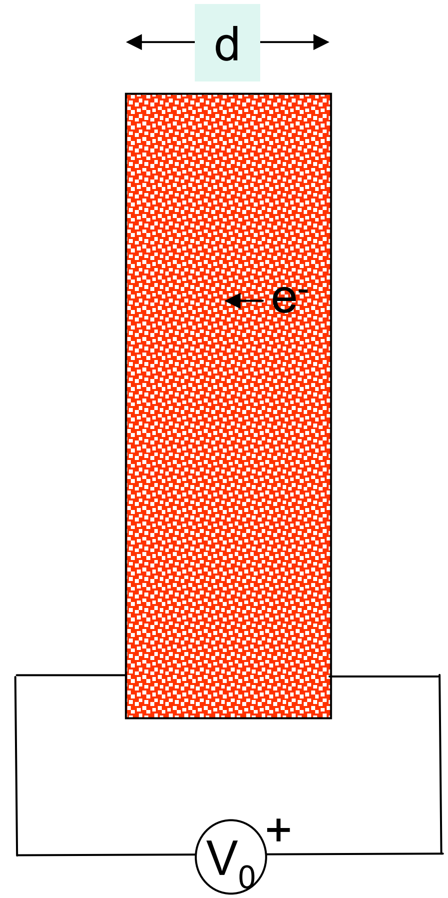
- Planar geometry with potential V_0 across gap of distance d
- Space charge dominated so the field is found by solving Poisson's equation
- Assume steady state (current is independent of position across the gap)

Space Charge Diode non-relativistic

$$\nabla^2 V = \frac{\rho}{\varepsilon_0} = \frac{d^2 V}{dx^2}$$

$$i = \rho v = \rho \sqrt{\frac{2eV}{m}}$$

$$\frac{d^2 V}{dx^2} = \frac{i}{\varepsilon_0} \sqrt{\frac{m}{2eV}}$$



Solution

$$V(x) = V_0 \left(\frac{x}{d} \right)^{4/3}$$

$$\rho(x) = \frac{4\varepsilon_0 V_0}{9d^{4/3}} x^{-2/3}$$

$$i = \frac{4\varepsilon_0}{9} \sqrt{\frac{2e}{m}} \frac{V_0^{3/2}}{d^2}$$

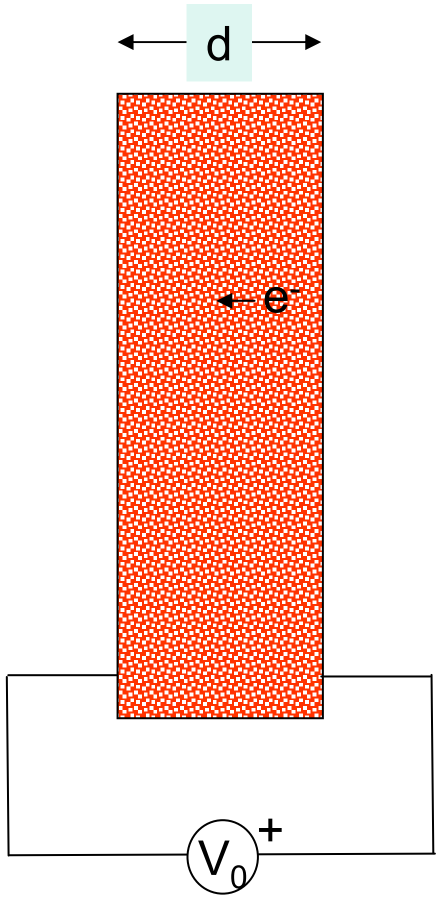
Child-Langmuir Law

Space Charge Diode relativistic

$$\nabla^2 V = \frac{\rho}{\varepsilon_0} = \frac{d^2V}{dx^2}$$

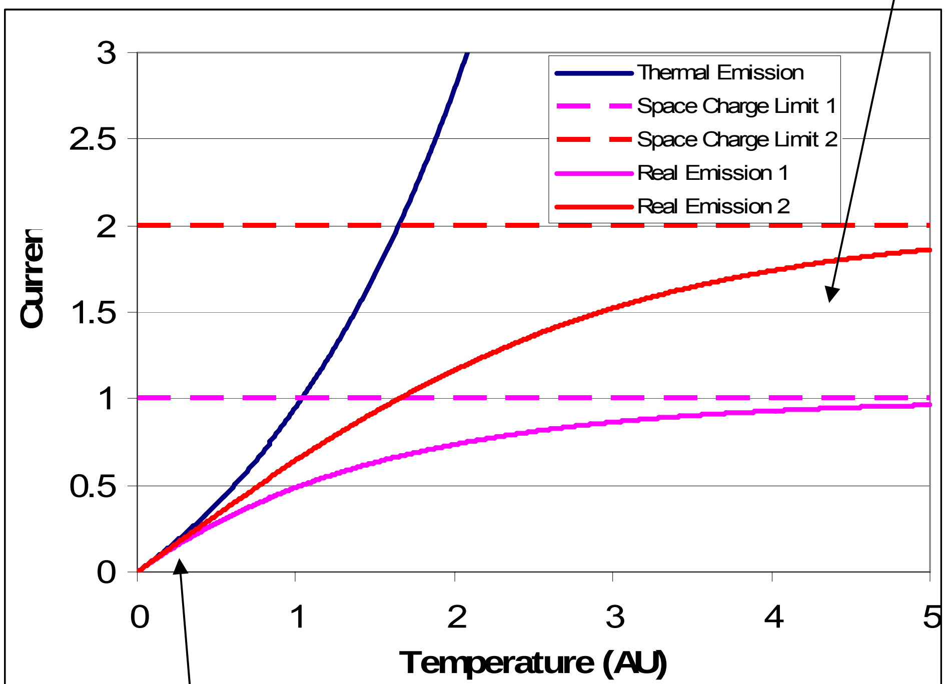
$$i = \rho v$$

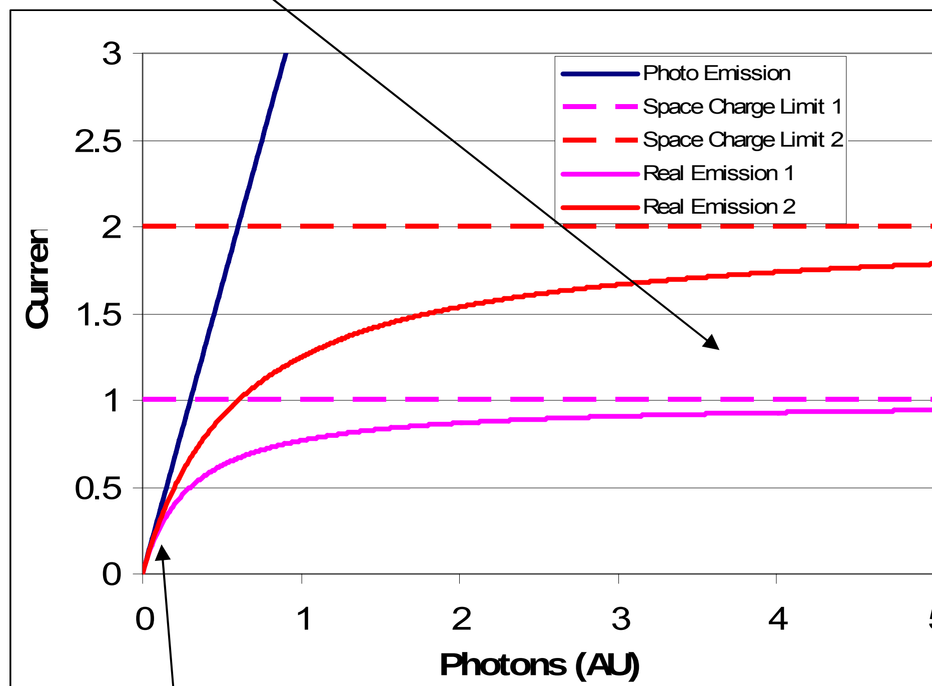
$$\frac{d^2V}{dx^2} = \frac{i}{\varepsilon_0}?$$



Space Charge Limit

space charge limited





temperature limited

photon limited

Home Work #1

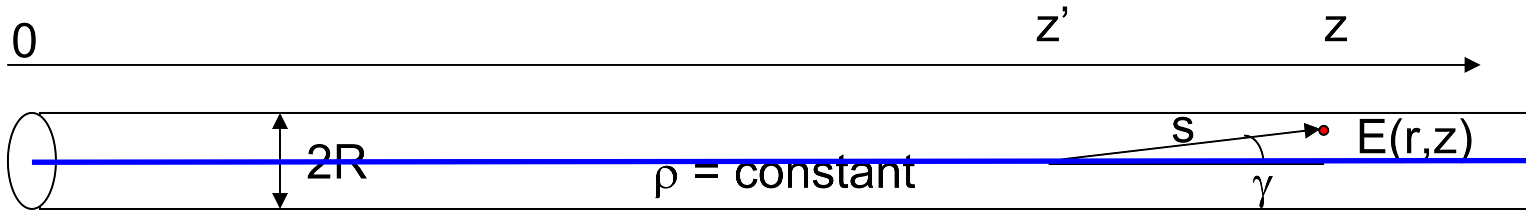
Calculate the QE for the head, core and tail assuming the following parameters. The laser profile is flat-top both temporally and transverse.

- $E_{applied}$ = 100 MV/m
- $\theta_{applied}$ = 40°
- R = 1 mm
- Q = 1.5 nC
- Δt_{laser} = 15 ps
- λ = 266 nm
- $\Phi_{cathode}$ = 4.6 eV
- QE = 10^{-5} with E_{total} = 50 MV/m

Home Work #2

- For the cigar beam, calculate the longitudinal electric field from the radial electric field.

Semi-Infinite Line Charge

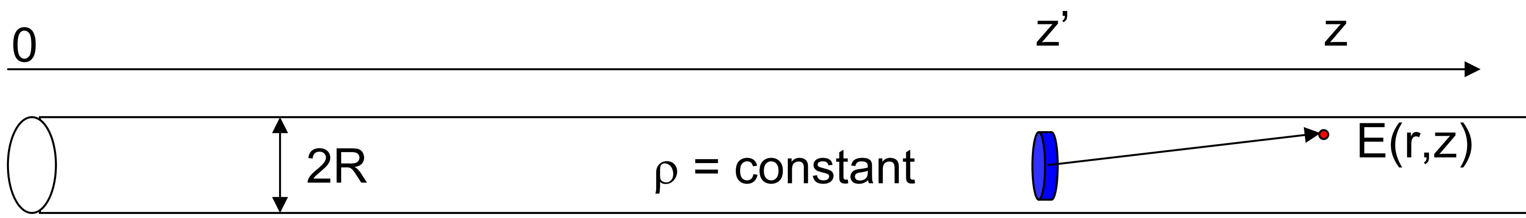


$$\vec{E}(r,z) = \frac{1}{4\pi\varepsilon}\int_V \frac{\vec{a}_s \rho}{s^2} dV = \frac{1}{4\pi\varepsilon}\int_V \frac{\vec{s}\rho}{s^3} dV$$ Coulomb's Law

$$E_r(r,z) \approx \frac{1}{4\pi\varepsilon}\int_0^\infty \frac{\cos\gamma}{s^2}\frac{r^2 \rho_z dz'}{R^2}$$ $r \le R$

Model is equivalent to all the charge within radius r shrunk to a line

Semi-Infinite Line Charge



$$E_r(r,z) \approx \int_0^\infty \frac{\rho_z r^3 dz'}{4\pi\varepsilon R^2 \left[r^2 + (z'-z)^2 \right]^{3/2}} \qquad r \le R$$

$$\int \frac{dx}{\left(a^2+x^2\right)^{3/2}} = \frac{x}{a^2\sqrt{a^2-x^2}}$$

$$E_r(r,z) \approx \frac{\rho_z r}{4\pi\varepsilon R}\left(1+\frac{\left(\frac{z}{R}\right)}{\sqrt{\left(\frac{r}{R}\right)^2+\left(\frac{z}{R}\right)^2}}\right) \qquad r \le R$$