The Fringe Pattern of a Synchrotron-Light Interferometer

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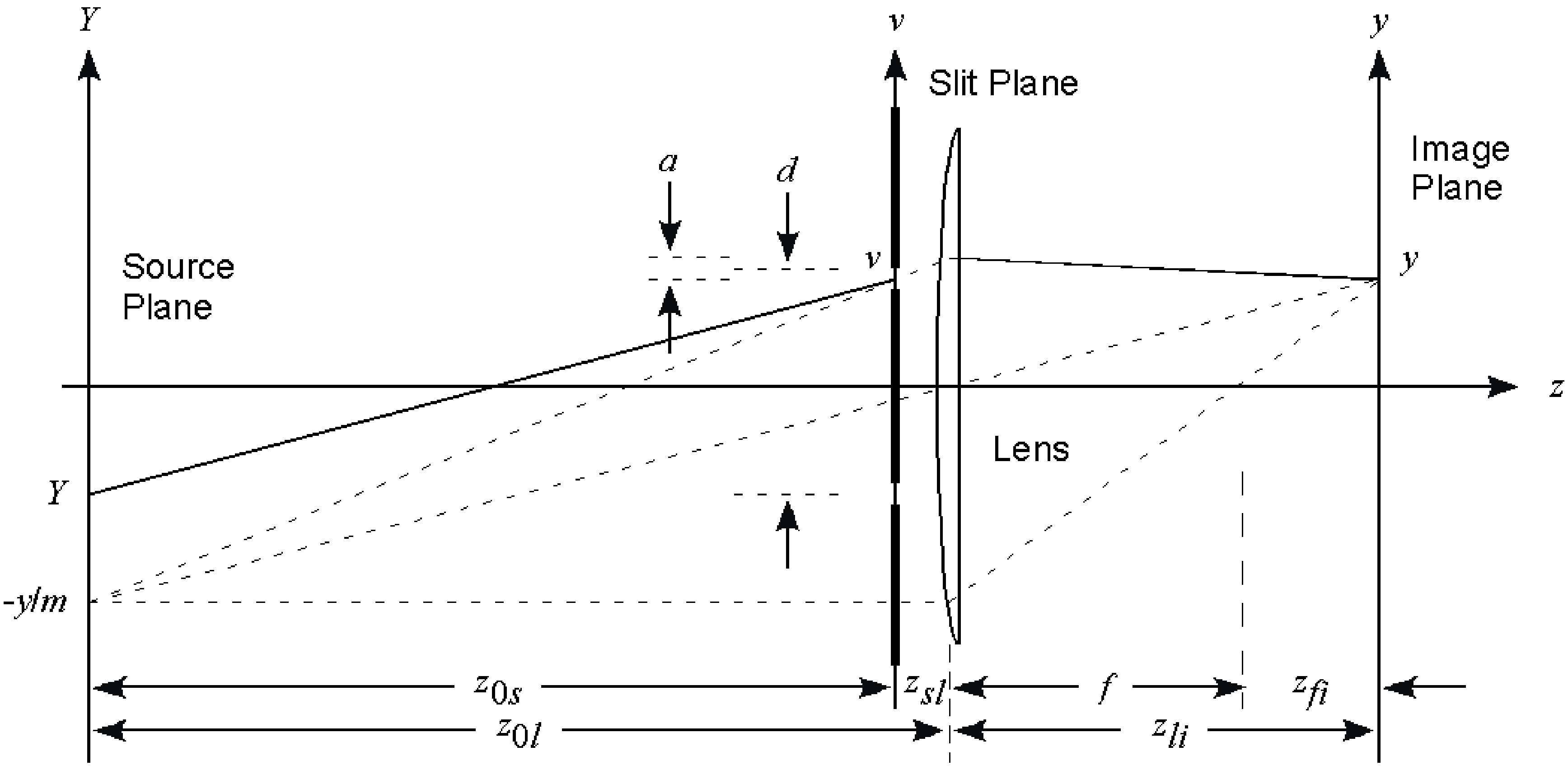
Derivation of the Interference Pattern

- Assumptions:
  - One dimensional (*y*)
  - Two narrow, parallel slits of width *a*
  - Small slit separation (center to center) $d > a$
  - Slits are far from the source: $z_{0s} \gg d$
  - Small source size σ_0, but comparable to slit width
  - Nearly monochromatic light, due to a bandpass filter

Case 1: Monochromatic Point Source

- Consider a point source at Y on the (X,Y) plane.
- The light is monochromatic, with $\lambda = 2\pi/k$
- We calculate the intensity at a point y on the image plane (x,y).
- The two slits are on the (u,v) plane.
- The electric field at y is found using a Fraunhofer diffraction integral over the slit plane.
  - We will need the difference in the length of the optical path for all the rays leaving Y and arriving at y.

Layout of the Two-Slit Interferometer



Reference Path

- Compare all paths to a reference path that:
  - Leaves the source plane at $-y/m$
    - Here m is the magnification of the lens: $m = z_{li}/z_{0l}$
  - Is imaged geometrically to y on the image plane.
- All paths from $-y/m$ to y are *equal* in length.
  - A fundamental property of geometric imaging.
  - Applies to the imaged ray passing through the slit at v.
- Consider a ray leaving $Y \neq -y/m$ that diffracts in the slit at v with an angle that brings it to y.
  - Its path matches the imaging path from $-y/m$ to v to Y *after* the slit…but not before.
    - This lets us compute the path difference Δs from the reference.

Difference in the Optical Path Length

$$\Delta s = \sqrt{(v-Y)^2 + {z_{0s}}^2} - \sqrt{\left(v + \frac{z_{0l}}{z_{li}} y\right)^2 + {z_{0s}}^2}$$

$$\approx \frac{1}{2z_{0s}}\left[-2v\left(Y + \frac{z_{0l}}{z_{li}} y\right) + Y^2 - \left(\frac{z_{0l}}{z_{li}} y\right)^2\right]$$

$$= \frac{1}{2z_{0s}}\left[-2v\left(Y + \frac{fy}{z_{fi}}\right) + Y^2 - \left(\frac{fy}{z_{fi}}\right)^2\right]$$

$$= -(gv + h)$$

where $\frac{1}{f} = \frac{1}{z_{0l}} + \frac{1}{z_{li}} = \frac{1}{z_{0l}} + \frac{1}{f + z_{fi}}$

Fraunhofer Diffraction Integral

$$E(y,Y,k) = \int_{\text{slits}} \mathrm{e}(v)\frac{1}{s_{Y,v}}\exp\left[iks_{Y,v} + iks_{v,y}\right]dv$$

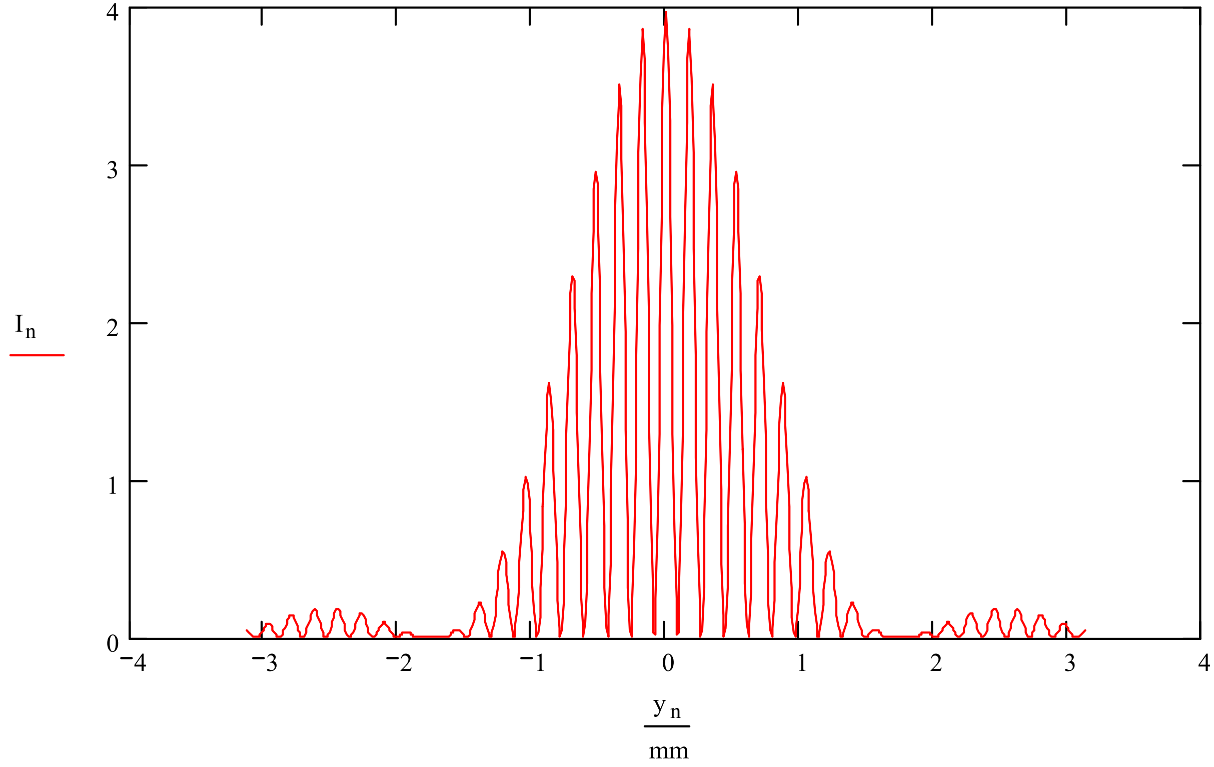
$$= \int_{\text{slits}} \sqrt{A(v)}\exp\left[ik\Delta s + i\phi(v)\right]dv$$

$$= ae^{-ikh}\,\text{sinc}\left(\frac{kga}{2}\right)\left(\sqrt{A_1}e^{-ikgd/2} + \sqrt{A_2}e^{ikgd/2+i\phi_0}\right)$$

$$I(y,Y,k) = a^2\,\text{sinc}^2\left(\frac{kga}{2}\right)\left[A_1 + A_2 + 2\sqrt{A_1A_2}\cos\left(kgd + \phi_0\right)\right]$$

- Envelope of single-slit diffraction: $a^2\,\text{sinc}^2\left(\frac{kga}{2}\right)$
- Individual slits: A_1, A_2
- Interference between slits: $\cos\left(kgd + \phi_0\right)$
- Phase offset between slits: ϕ_0

Fringes of a Monochromatic Point Source



Fringe pattern of a monochromatic point source at 450 nm, using parameters for the PEP-II HER interferometer at SLAC. $A_1 = A_2 = 1$, $d = 5$ mm, $a = 0.5$ mm.

Diffraction from an Extended Source #1

- Replace point source at Y with a Gaussian: $$\frac{1}{\sqrt{2\pi}\sigma_0}\exp\left[-\frac{1}{2}\left(\frac{Y-Y_0}{\sigma_0}\right)^2\right]$$
- Source consists of independent electrons: Incoherent.
  - Integrate the intensity, not the electric field.
- Compare arguments: $\operatorname{sinc}^2(kga/2)$ versus $\cos(kgd+\phi_0)$
  - Recall that: $$g=\frac{1}{z_{0s}}\left(Y+\frac{fy}{z_{fi}}\right)\sim\frac{1}{z_{0s}}\left(\sigma_0+\frac{fy}{z_{fi}}\right)$$
  - For d=5 mm, a=0.5 mm, λ=450 nm, σ_0=0.3 mm, z_{0s}=10 m:
    - $k\sigma_0 a/(2z_{0s}) = 0.105$ Small compared to zero of sinc at π
    - $k\sigma_0 d/z_{0s} = 2.09$ Significant change in cosine phase

Diffraction from an Extended Source #2

- We remove the sinc from the integral over the source, getting:

$$I(y,k) = a^2 \mathrm{sinc}^2\left[\frac{ka}{2z_{0s}}\left(Y_0 + \frac{fy}{z_{fi}}\right)\right]$$

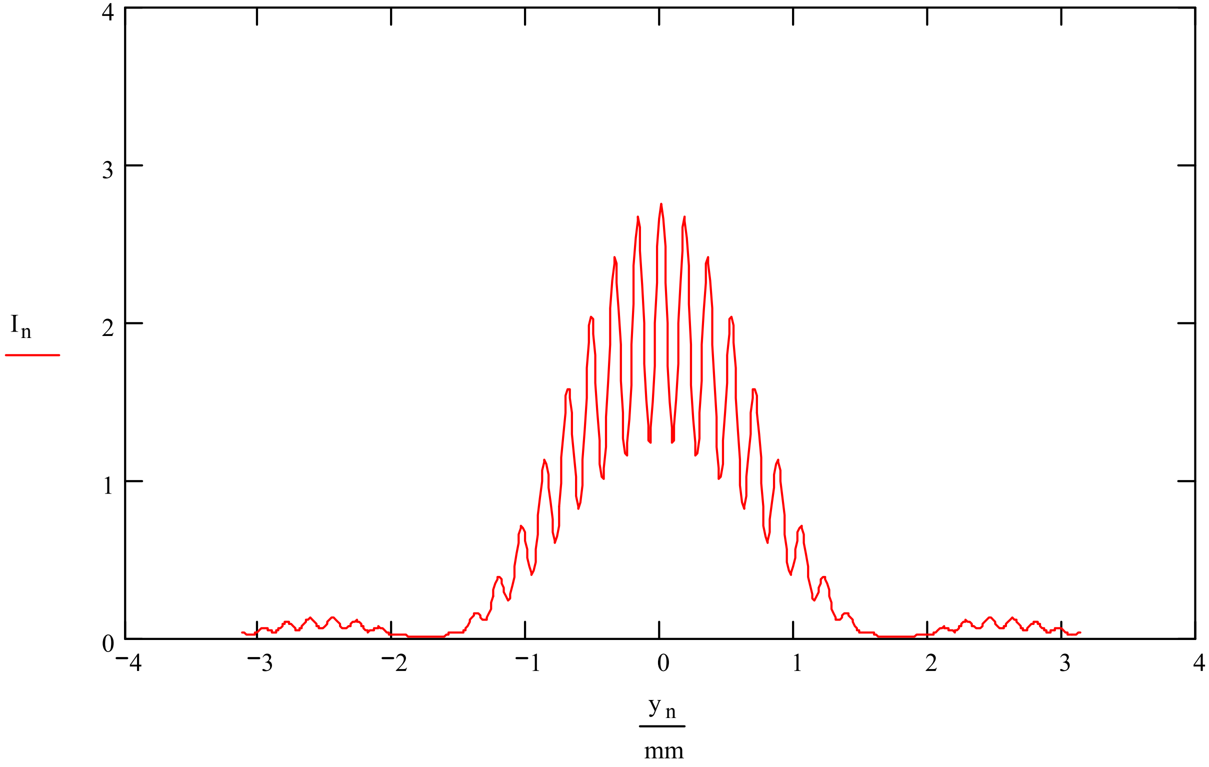
$$\bullet\left\{A_1 + A_2 + \frac{2\sqrt{A_1A_2}}{\sqrt{2\pi}\sigma_0}\int \exp\left[-\frac{1}{2}\left(\frac{Y-Y_0}{\sigma_0}\right)^2\right]\cos\left[\frac{kd}{z_{0s}}\left(Y+\frac{fy}{z_{fi}}\right)+\phi_0\right]dY\right\}$$

$$= a^2 \mathrm{sinc}^2\left[\frac{kg_0a}{2}\right]\left\{A_1 + A_2 + 2\sqrt{A_1A_2}\exp\left[-\frac{1}{2}\left(\frac{k\sigma_0 d}{z_{0s}}\right)^2\right]\cos\left(kg_0d+\phi_0\right)\right\}$$

where

$$g_0 = \frac{1}{z_{0s}}\left(Y_0 + \frac{fy}{z_{fi}}\right) = \frac{fy}{z_{0s}z_{fi}} - \theta_0 = \frac{y}{f + z_{fi}(1 - z_{sl}/f)} - \theta_0$$

Fringes of an Extended Source



Fringe pattern of an extended source, with σ_0=0.2 mm

Quasi-Monochromatic Light #1

- Pass synchrotron light through a narrow Gaussian filter: $$\frac{1}{\sqrt{2\pi}\sigma_k}\exp\left[-\frac{1}{2}\left(\frac{k-k_0}{\sigma_k}\right)^2\right]$$
- Compare arguments: $\operatorname{sinc}^2(kg_0 a/2)$ versus $\cos(kg_0 d+\phi_0)$
  - Recall that: $$g_0=\frac{y}{f+z_{fi}(1-z_{sl}/f)}-\theta_0$$
    - Essentially g_0 is a scaled vertical coordinate with an offset θ_0.
  - Compare effect of σ_k to an argument of π, where sinc = 0.
  - For $\Delta\lambda = 30$ nm FWHM:
    - $\pi\sigma_k/k_0 = 0.09$ Small
- Again we can remove the sinc from the integration over the bandpass filter.

Quasi-Monochromatic Light #2

$$I(y) = a^2 \mathrm{sinc}^2\left[\frac{k_0 g_0 a}{2}\right]$$

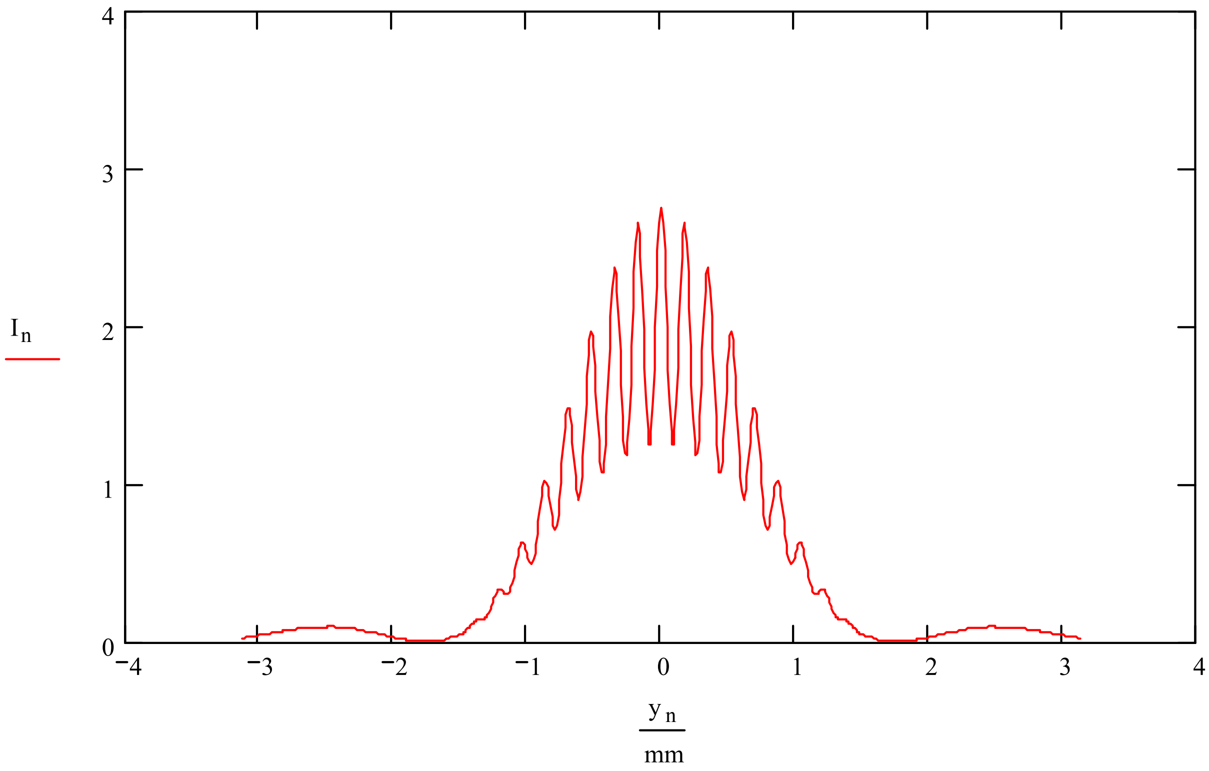
$$\bullet\left\{A_1 + A_2 + \frac{2\sqrt{A_1 A_2}}{\sqrt{2\pi}\sigma_k}\int \exp\left[-\frac{1}{2}\left(\frac{k-k_0}{\sigma_k}\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{k\sigma_0 d}{z_{0s}}\right)^2\right]\cos\left(kg_0 d + \phi_0\right)dk\right\}$$

$$= a^2 \mathrm{sinc}^2\left[\frac{k_0 g_0 a}{2}\right]\left\{A_1 + A_2\right.$$

$$\left. + \frac{2\sqrt{A_1 A_2}}{\sqrt{1+\left(\frac{\sigma_k \sigma_0 d}{z_{0s}}\right)^2}}\exp\left[-\frac{\left(\frac{k_0\sigma_0 d}{z_{0s}}\right)^2 + \left(\sigma_k g_0 d\right)^2}{2\left[1+\left(\frac{\sigma_k\sigma_0 d}{z_{0s}}\right)^2\right]}\right]\cos\left[\frac{k_0 g_0 d}{1+\left(\frac{\sigma_k\sigma_0 d}{z_{0s}}\right)^2} + \phi_0\right]\right\}$$

Beam size — Bandwidth

Fringes with a Bandpass Filter

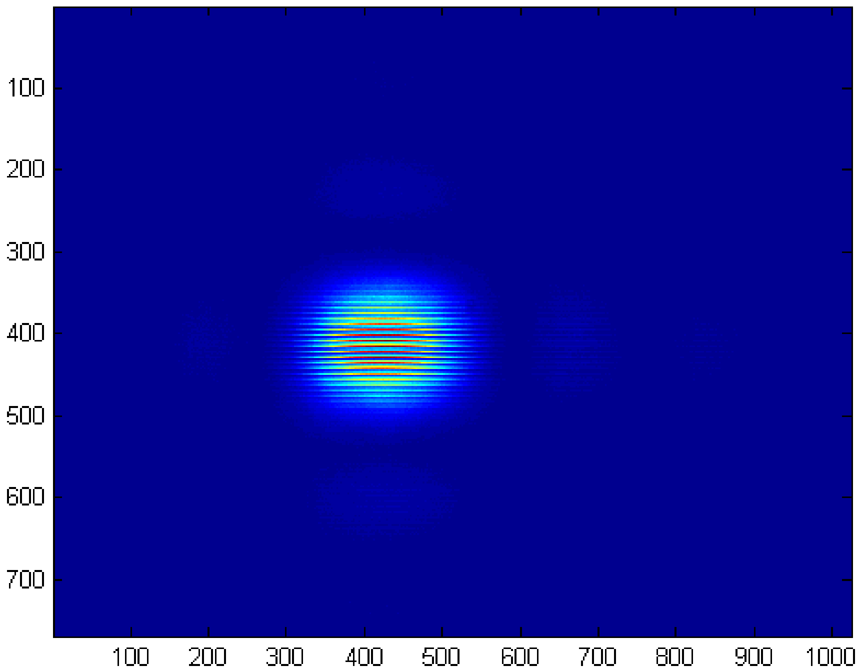


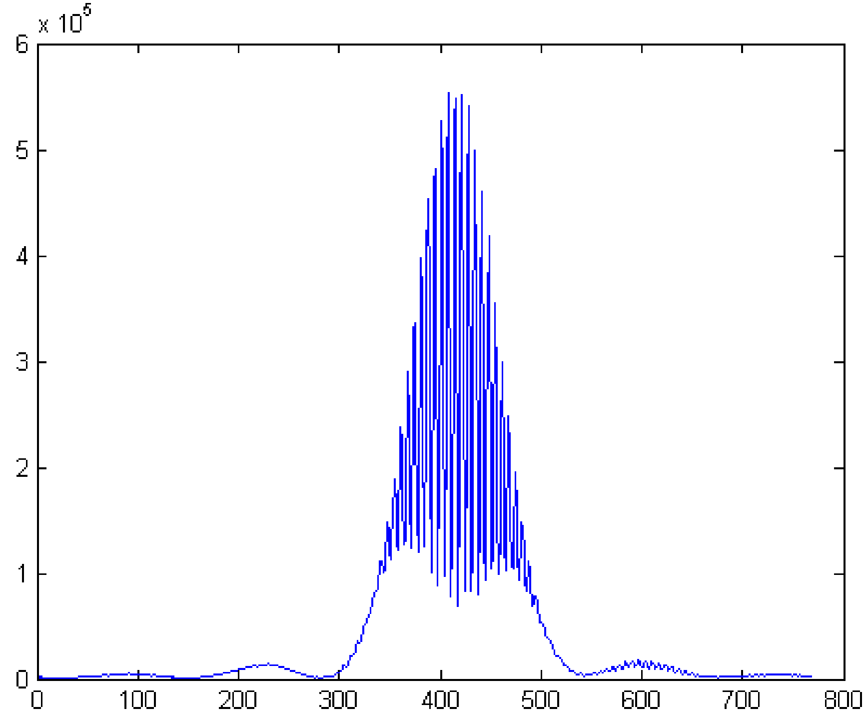
Fringe pattern using a Gaussian bandpass filter with a full width at half maximum (FWHM) of $\Delta\lambda = 30$ nm.

Compare to van Cittert and Zernicke

- We previously looked at the derivation of van Cittert and Zernicke (VCZ). Is this approach the same?
- VCZ:
  - The "degree of coherence" $\mu(A_1,A_2)$ between two apertures due to the finite source.
    - VCZ says that, for a narrow-band source, this expression resembles the diffraction pattern on the slit plane due to the source.
  - The intensity pattern on the camera is then determined from the single-aperture patterns and μ.
- My approach changes the order, but reaches the same result:
  - First do the full calculation through two slits to the camera for a monochromatic point source.
  - Next include the finite size.
  - Finally add the narrow bandwidth.

Interferometry on SPEAR-3

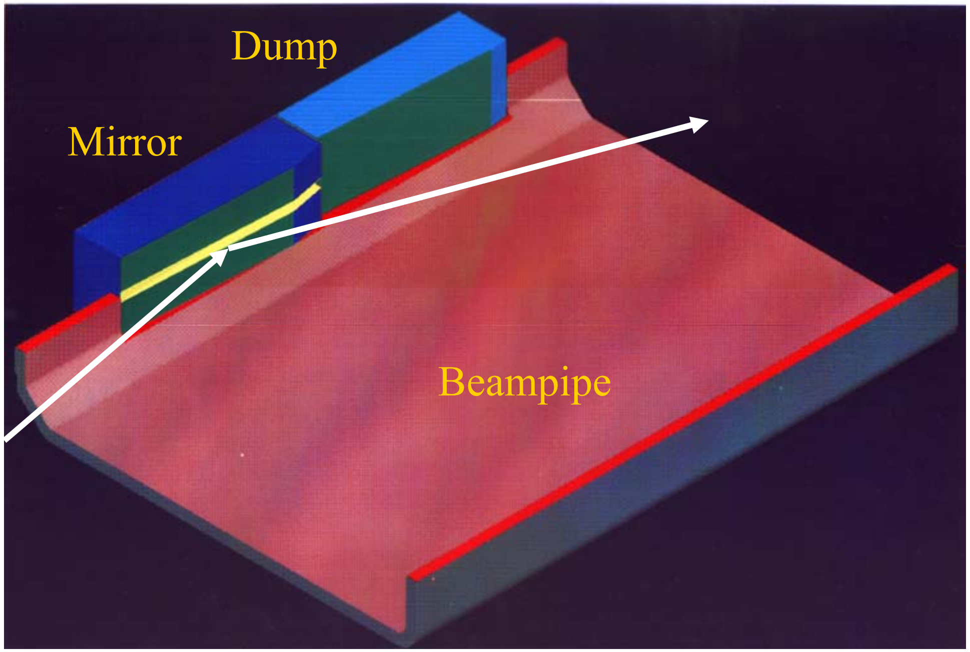


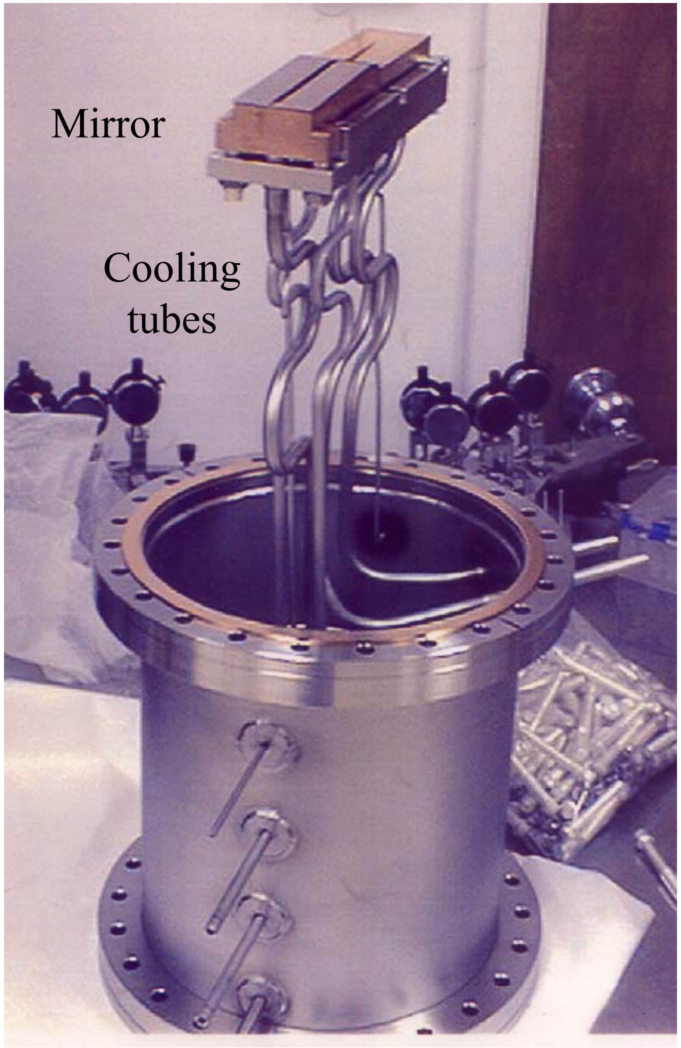


Distortion of PEP's First Mirror

- The first mirror in PEP-II takes a huge heat load, even with grazing incidence to spread out the heat.
  - Extensive (and stiff) stainless-steel water-cooling tubes on the rear add mechanical stress.
- A slot along the midplane of the mirror is meant to allow the narrow and hot x-ray fan to bypass the mirror and hit a separate dump.
  - Some folding of the mirror about the slot.
- Thermal and mechanical stresses reduce image quality and would decrease fringe contrast in an interferometer.

First Mirror (M1) in the HER of PEP-II

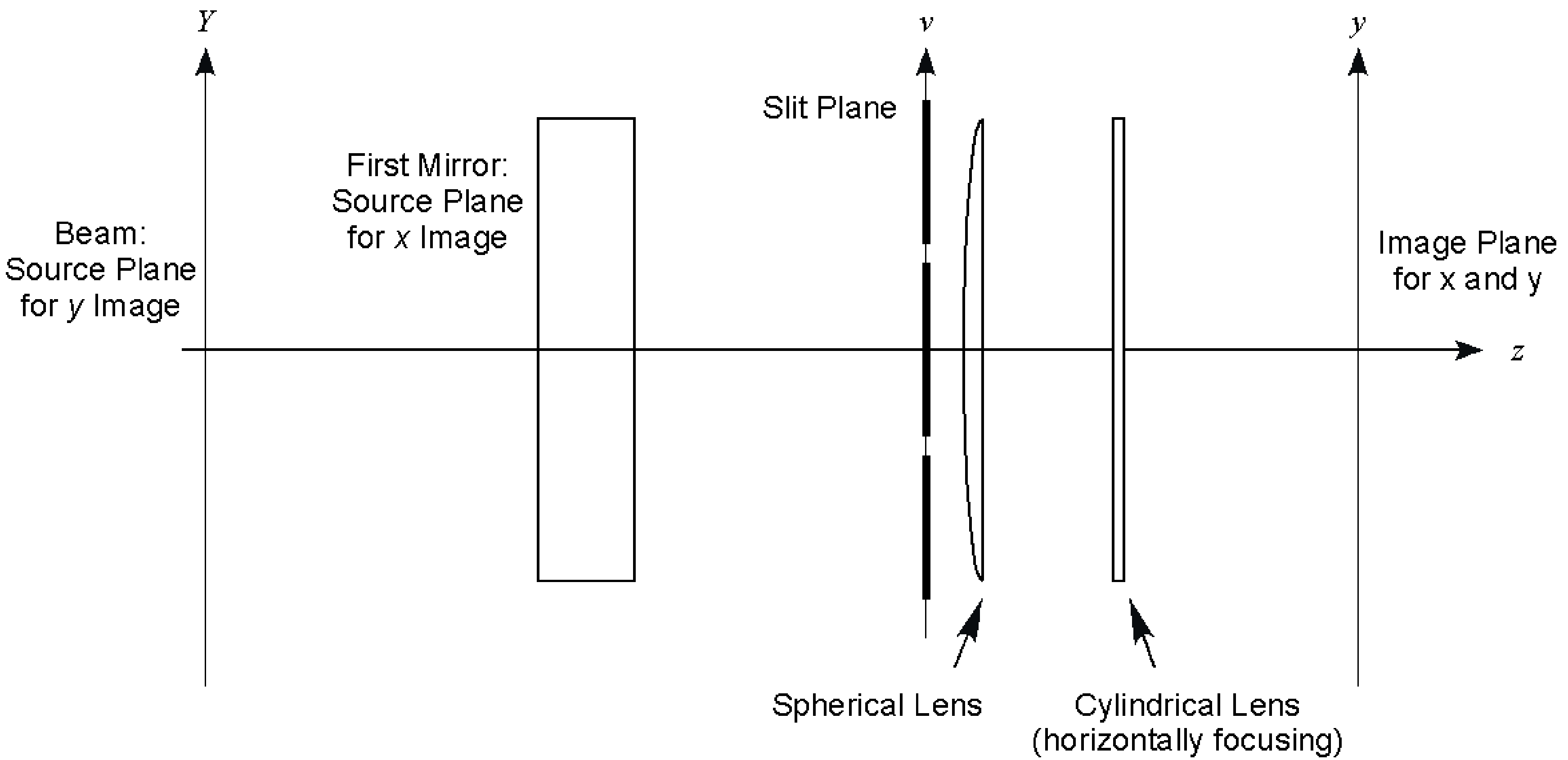




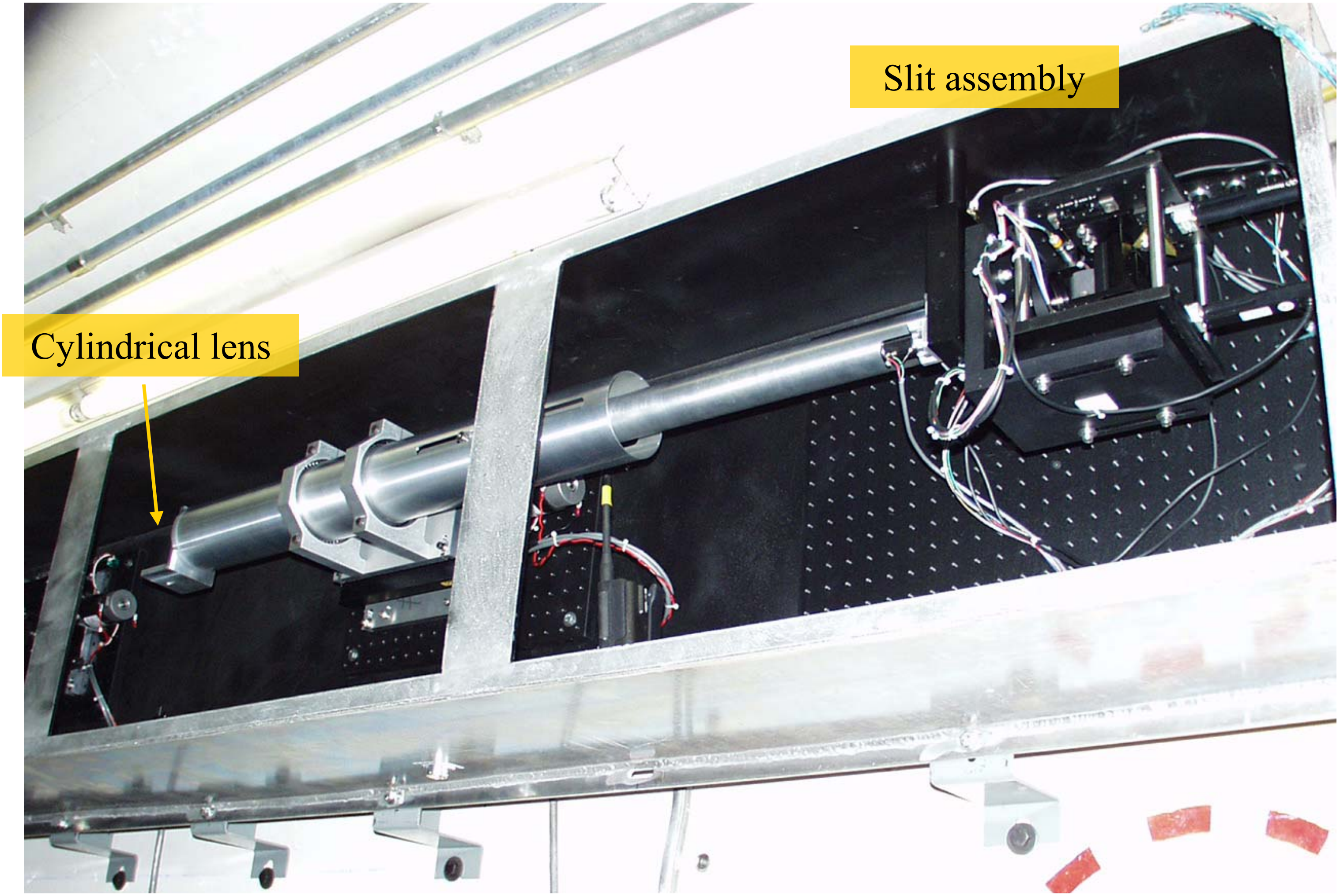
Compensation with a Cylindrical Lens

- Interferometer slits pass light from two thin horizontal stripes along M1.
  - Little of M1's surface contributes…and we can reduce this more.
- Fringes from a vertical interferometer measurement form a series of parallel horizontal lines.
  - Beam size is calculated from the vertical intensity variation.
    - Beam is imaged through the slits onto the camera.
  - The direction along the stripe is less interesting.
    - Change the focal length horizontally to image M1, not the beam.
    - Insert a cylindrical lens to shorten the focal length.
    - Position along each stripe corresponds to an *x* coordinate of M1.
    - Computer selects *x* with best fringe visibility from the video.
- The interferometer uses only two small rectangles, selected for fringe quality, on M1's surface.

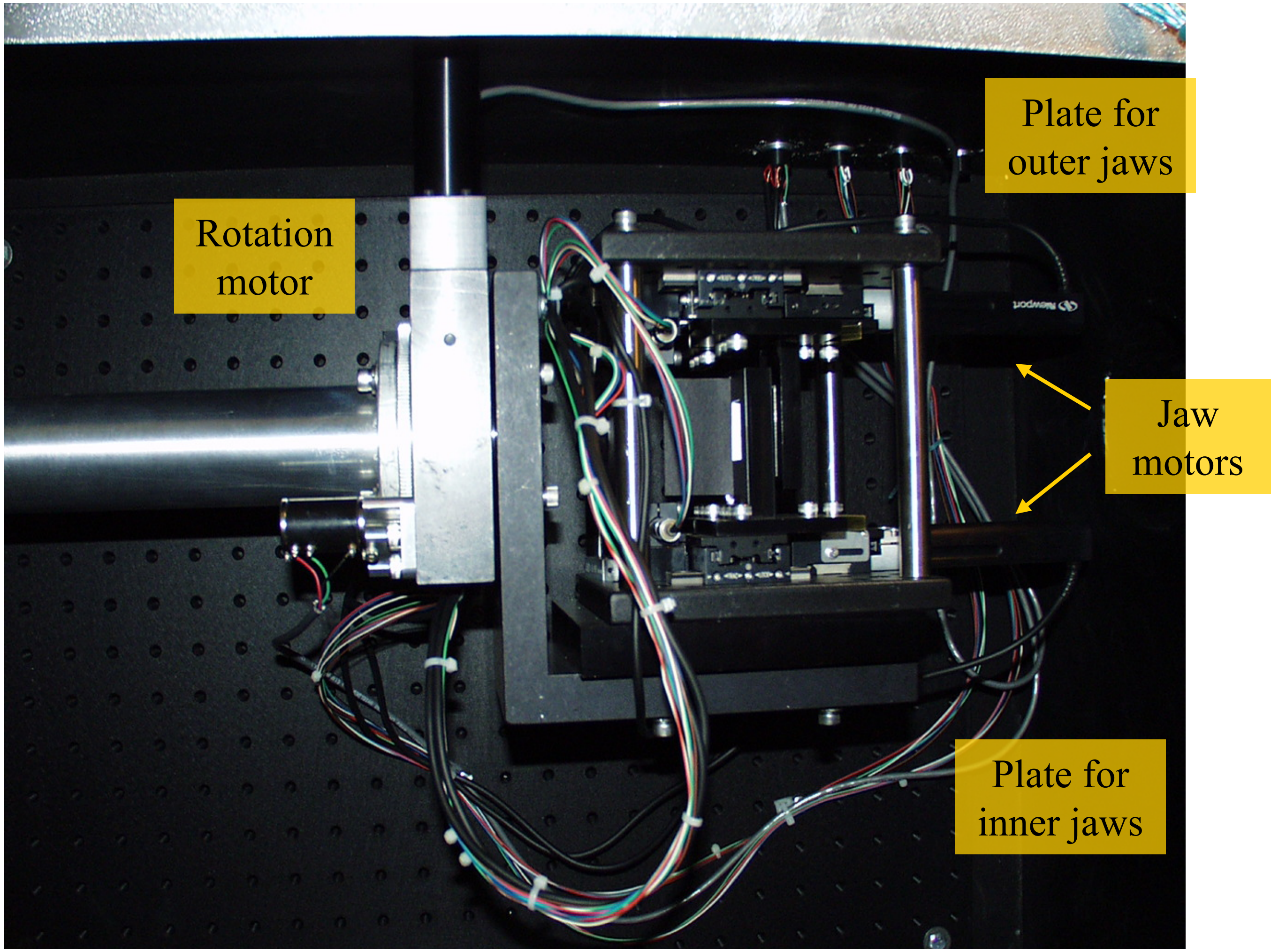
Adding the Cylindrical Lens



LER Interferometer on Tunnel Wall

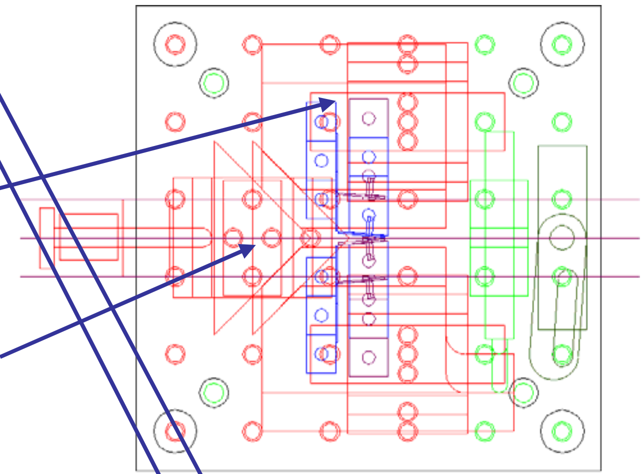


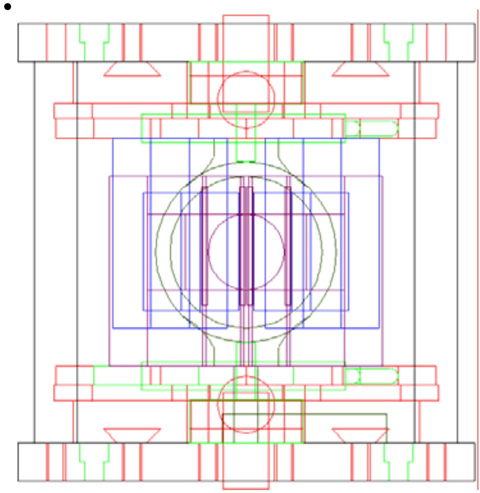
Double-Slit Assembly

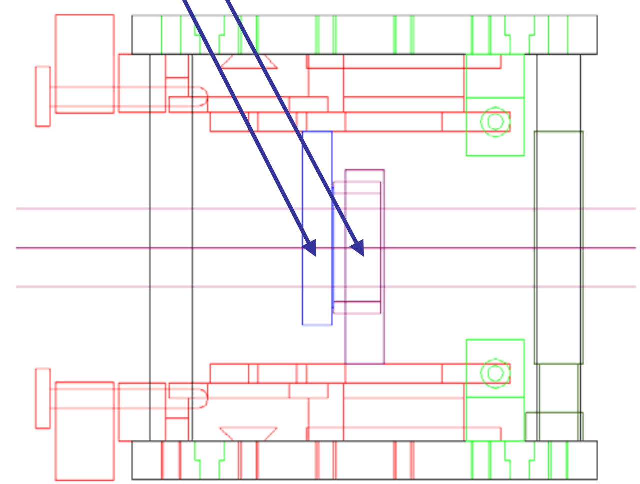


Sketch of Slit Assembly

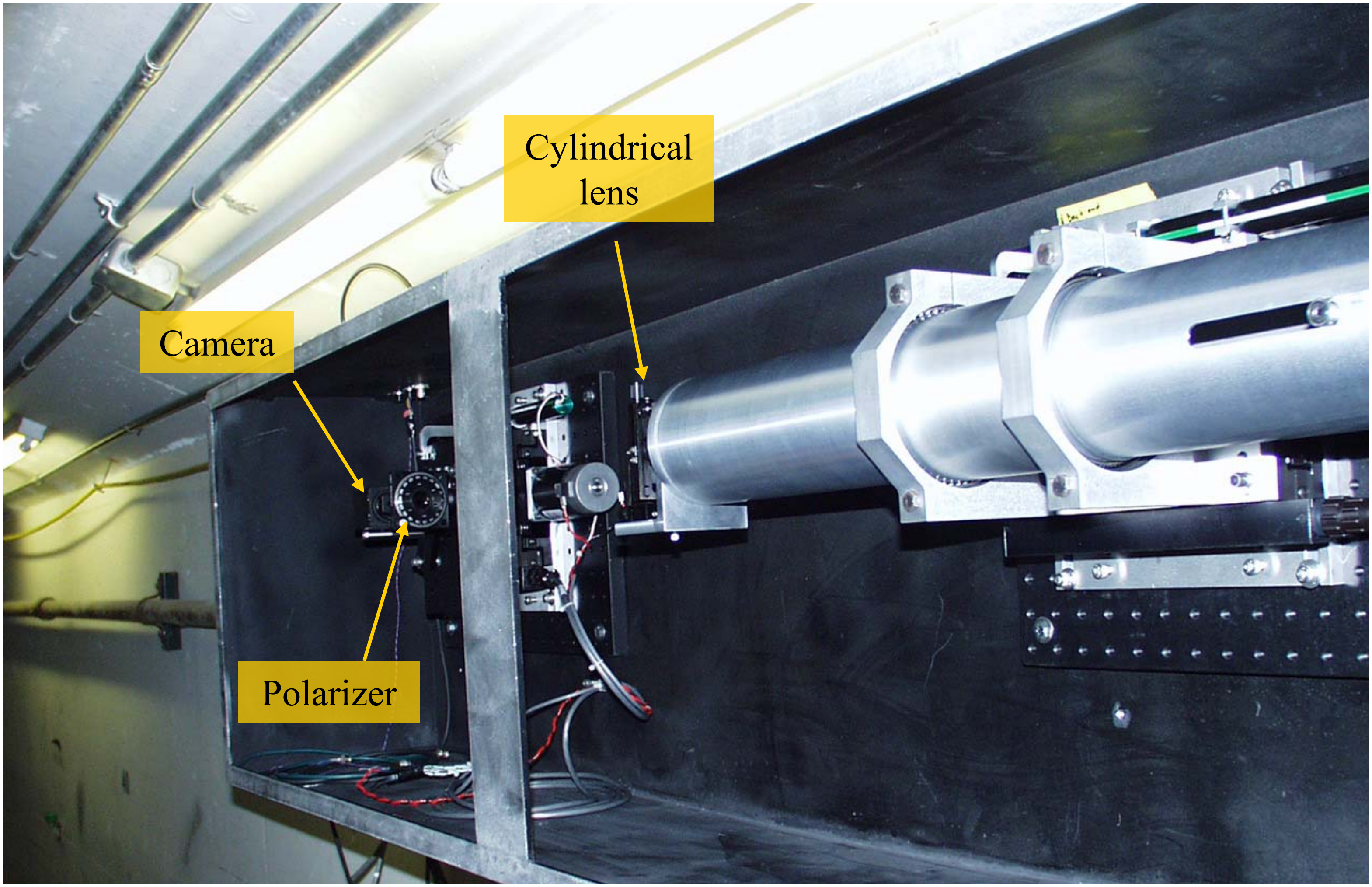
- Inner jaws mounted from lower plate.
- Outer jaws mounted from upper plate.
- Each jaw is on a translation stage with no motor, moving transversely.
- A third, motorized stage on each plate moves longitudinally, spreading the jaws with a wedge.



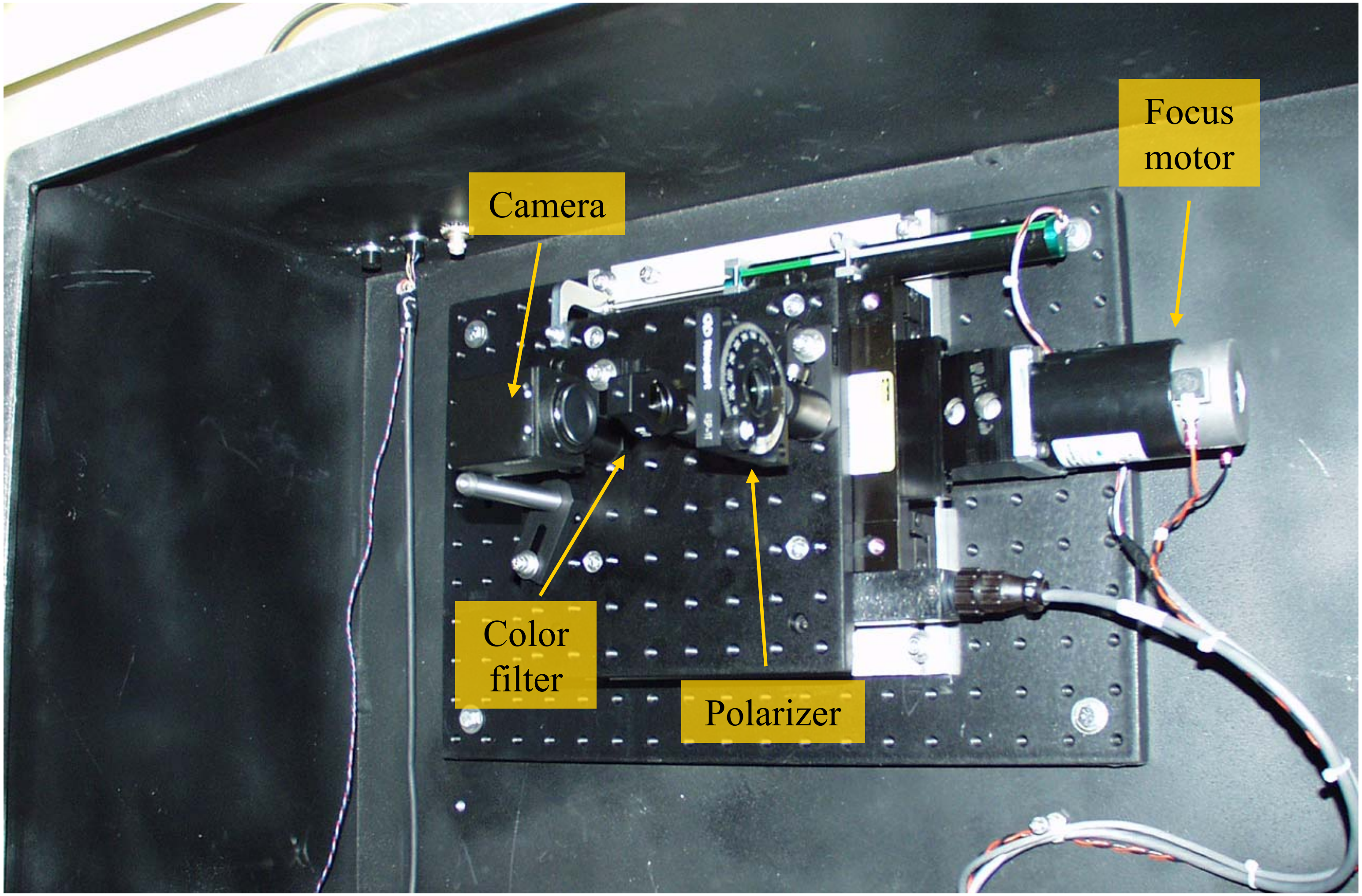




Cylindrical Lens to Camera



Polarizer, Filter, and Camera



PEP-II Interferometer Software