Generating THz in Storage Rings.
Part I

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Why Studying CSR?

Coherent Synchrotron Radiation (CSR) has been matter of great interest and study in the last years:

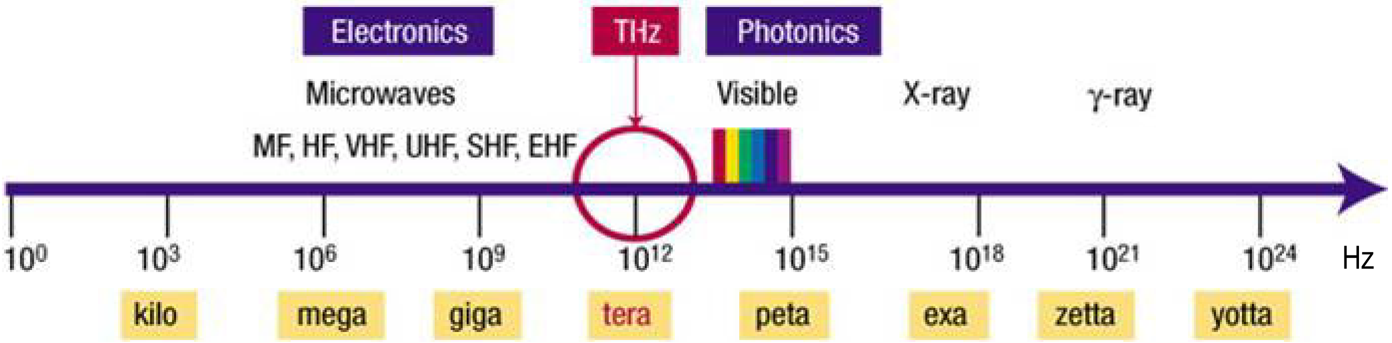
- **As something to carefully avoid or at least control in every short bunch high charge accelerator where CSR can jeopardize the performances (linear colliders, short pulses synchrotron radiation sources, damping rings, ...);**

- **As a powerful diagnostic for bunch compressors in free electron lasers (FEL) (FLASH, LCLS, FERMI, …);**

- **But also as a ‘dream’ for potential revolutionary synchrotron radiation (SR) source in the THz frequency range;**

The "Terahertz Gap"

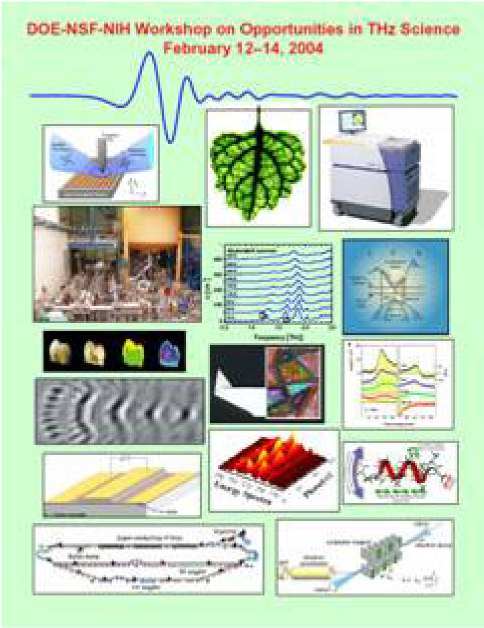
Scarcity of broadband powerful source in such a region of the spectrum



1 THz
= 4.1 meV
= 33 cm^{-1}
= 300 μm

THz Science: collective excitations, protein motions & dynamics, superconductor gaps, magnetic resonances, terabit wireless, medical imaging, security screening, detecting explosives & bio agents …

"DOE-NSF-NIH Workshop on Opportunities in THz Science"
February 12-14, 2004
http://www.science.doe.gov/bes/reports/abstracts.html#THz



Why CSR from Storage Rings

- High Stability

- Many users capability

- Multicolor experiments capability

- Capability of "exotic" experiments (femtoslicing, stacking, ...)

- **Non interceptive radiation processes are required. Synchrotron and edge radiation most efficient**

Two Lectures

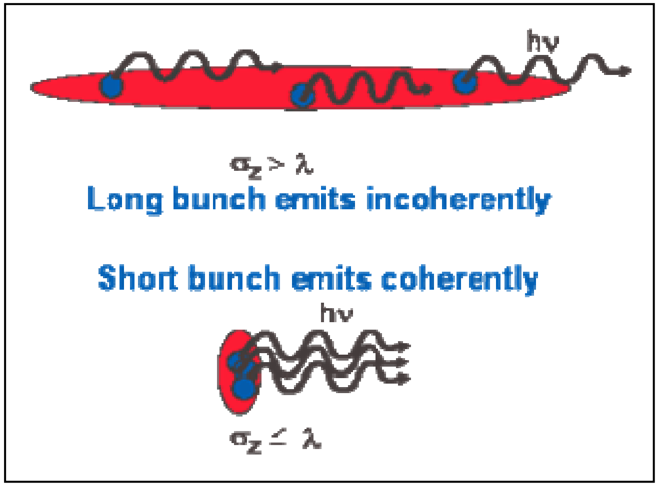
- **Part I**

  CSR in storage rings by "short" bunches

- **Part II**

  CSR in storage rings: alternative schemes

CSR Basics



The power spectrum of the radiation from a bunch with N particles is given by:

$$\frac{dP}{d\omega} = \frac{dp}{d\omega}\left\{\underbrace{N\left[1-g(\omega)\right]}_{P_{SR}\propto N \text{ incoherent}} + \underbrace{N^2 g(\omega)}_{\boxed{P_{CSR}\propto N^2 \text{ coherent}}}\right\}$$

Single particle power spectrum for the radiating process under consideration (including shielding effects)

$P_{SR} \propto N$ *incoherent*

$P_{CSR} \propto N^2$ *coherent*

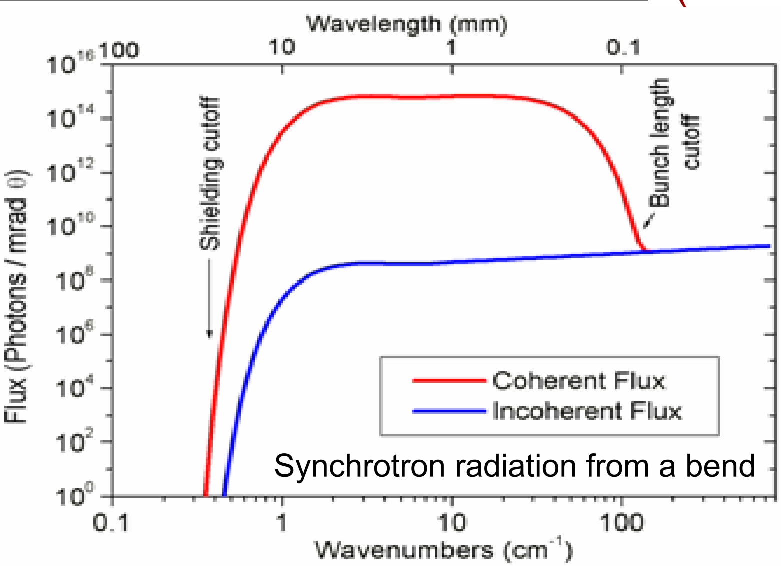
CSR for: $g(\omega) \geq 1/N$

$$g(\omega) = \left|\int_{-\infty}^{\infty} dz\, S(z)\, e^{i\omega\cos(\theta)z/c}\right|^2$$

$$0 \leq g(\omega) \leq 1$$

$\theta \equiv$ *observation angle*

Normalized Bunch Longitudinal Distribution



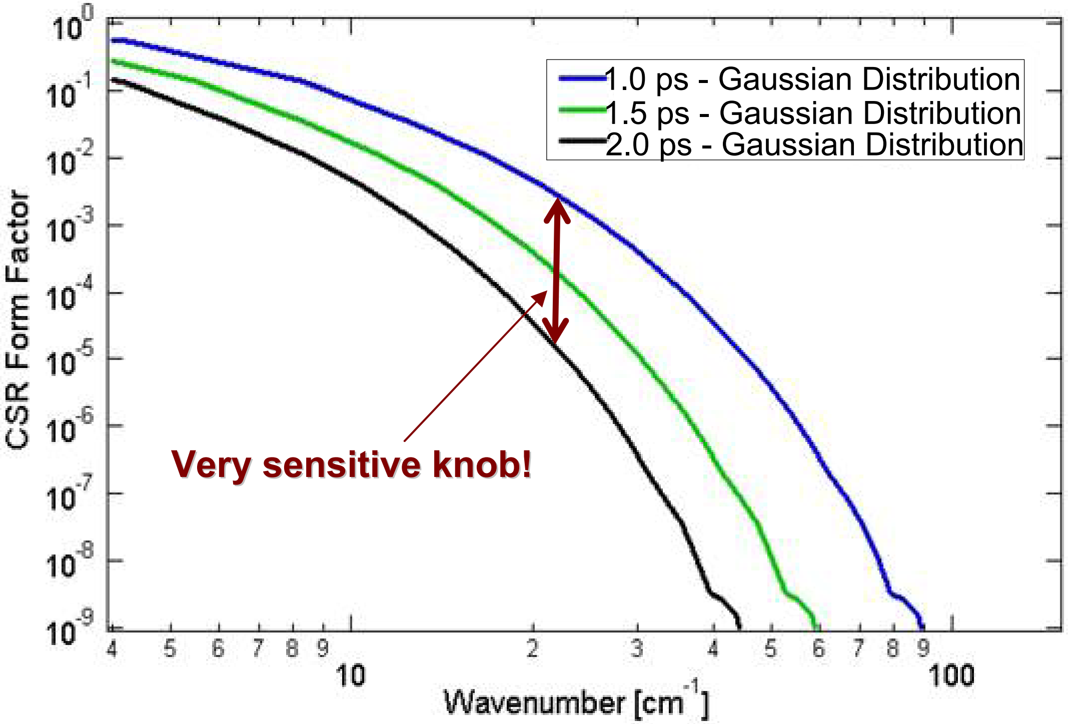
The CSR factor $g(\omega)$ determines the high frequency cutoff for CSR, while the vacuum chamber (shielding) defines the low frequency one.

CSR Form Factor vs. Bunch Length

$$g(\omega)=\left|\int_{-\infty}^{\infty} dz\, S(z)\, e^{i\omega z/c}\right|^{2}$$

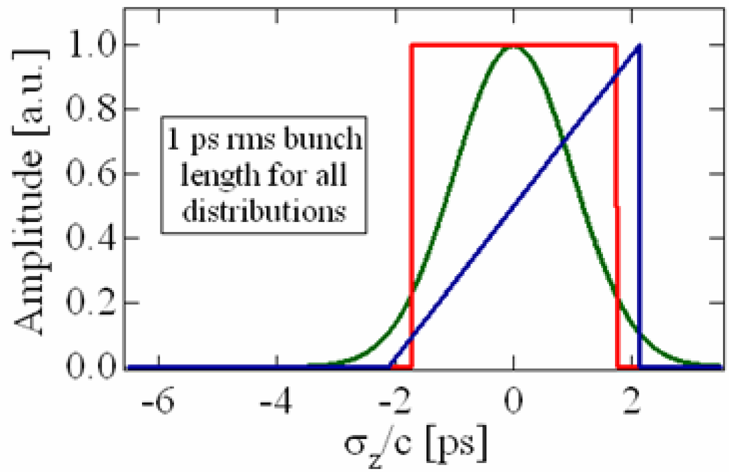
$$S(z)=\frac{1}{\sqrt{2\pi}\, c\,\sigma_{\tau}}\, e^{-\frac{z^{2}}{2c^{2}\sigma_{\tau}^{2}}}$$

$$g(\omega)=e^{-\omega^{2}\sigma_{\tau}^{2}}$$

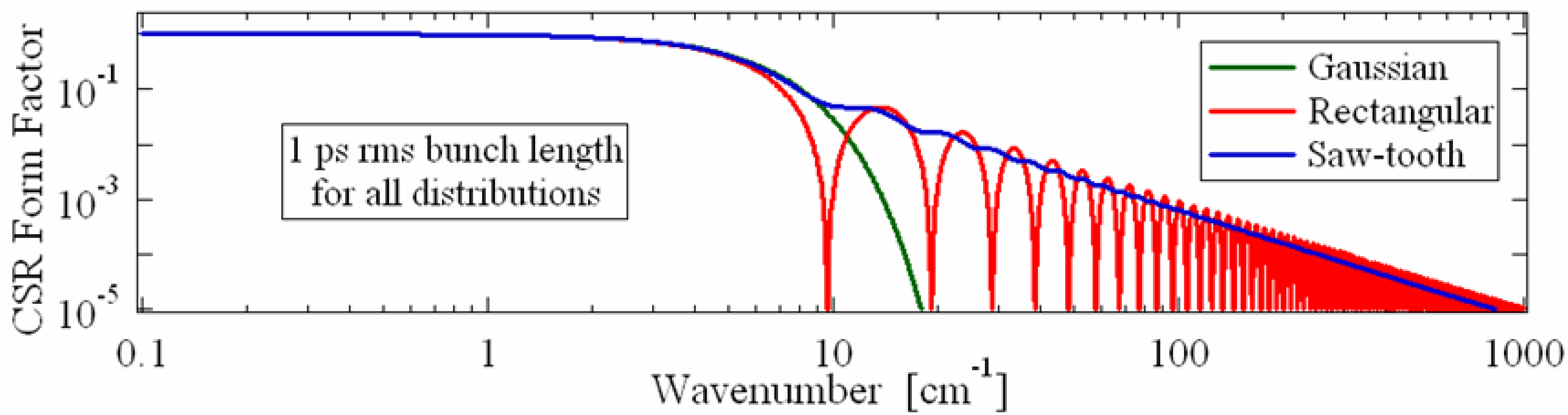


To extend the CSR spectrum towards higher frequencies the bunches must be shortened.

CSR Form Factor vs. Longitudinal Bunch Distribution

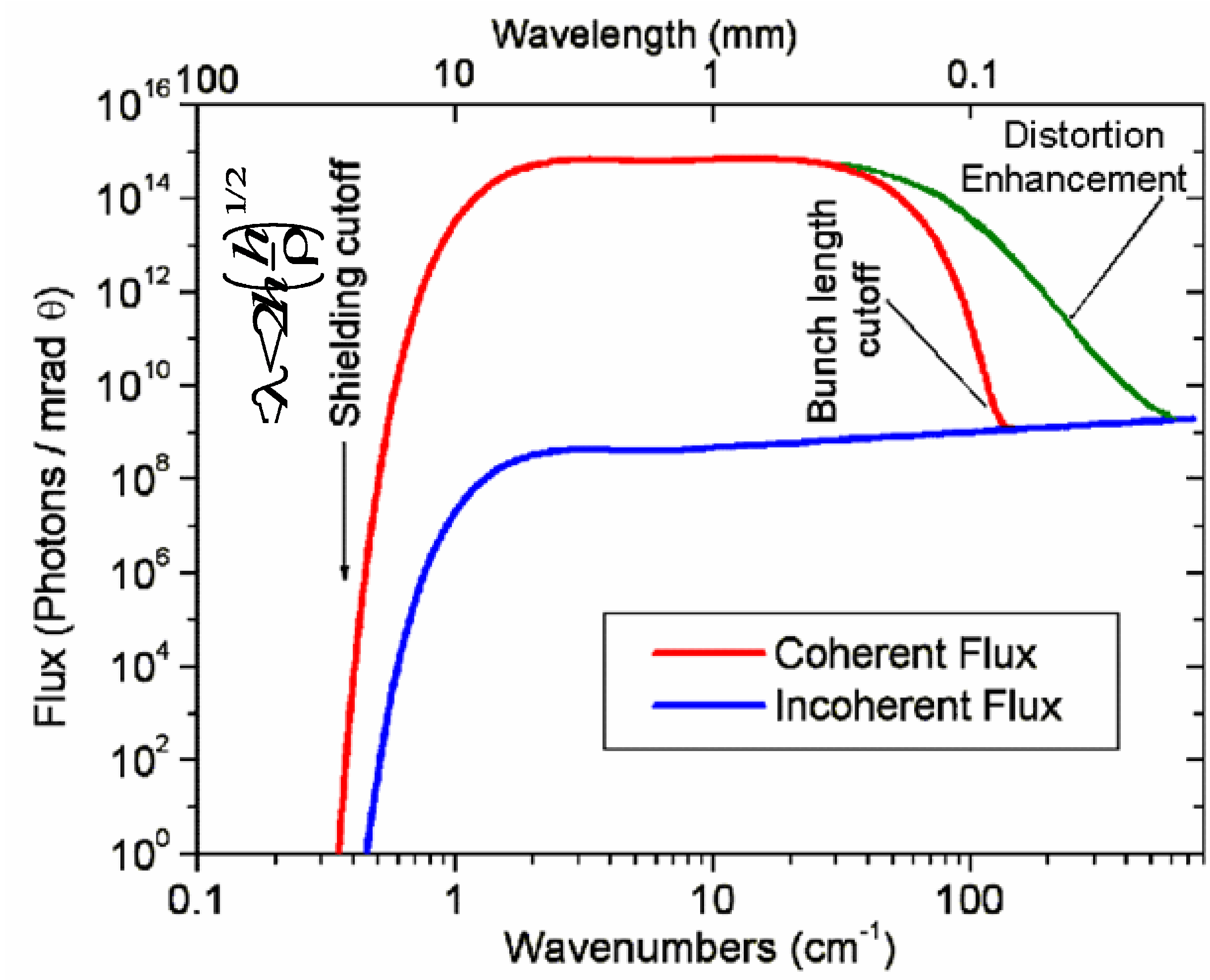


$$g(\omega) = \left| \int_{-\infty}^{\infty} dz\, S(z) e^{i\omega z/c} \right|^2$$



To extend the CSR spectrum towards higher frequencies: the 'saw-tooth' distribution is the best.

The CSR Spectrum from a Storage Ring Dipole



The "Cocktail" for a Good THz Source

Extend the vacuum chamber cutoff towards wavelengths as long as possible.

Shorten the bunches as much as possible.

Find a mechanism for generating sharp edged distributions (saw-tooth like possibly).

And of course, put as much particles as possible in the bunch.

The Vacuum Chamber Cutoff

Using the parallel plate model for representing the vacuum chamber effect for the case of synchrotron radiation from a bend, one can find the cutoff frequency:

$$\boxed{\lambda_{Cutoff} \sim 2h\left(\frac{h}{\rho}\right)^{1/2}}$$

$h \equiv vacuum\ chamber\ half-gap$

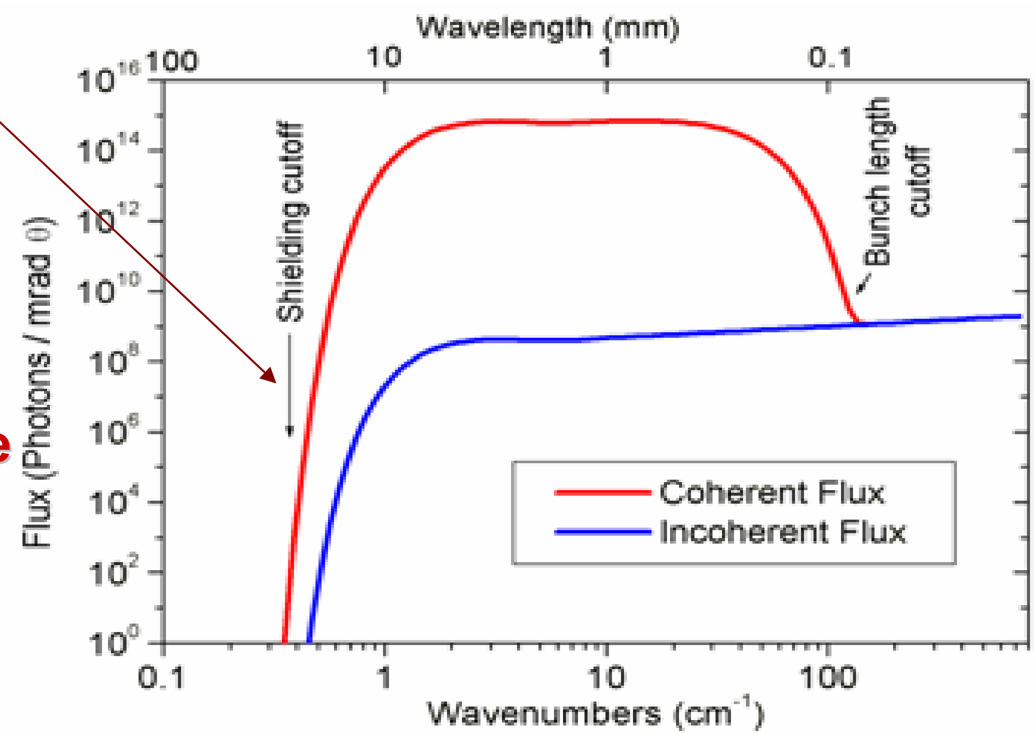
$\rho \equiv bending\ radius$

The radiation can be extracted from the vacuum chamber only if :

$$\boxed{\lambda < \lambda_{Cutoff}}$$

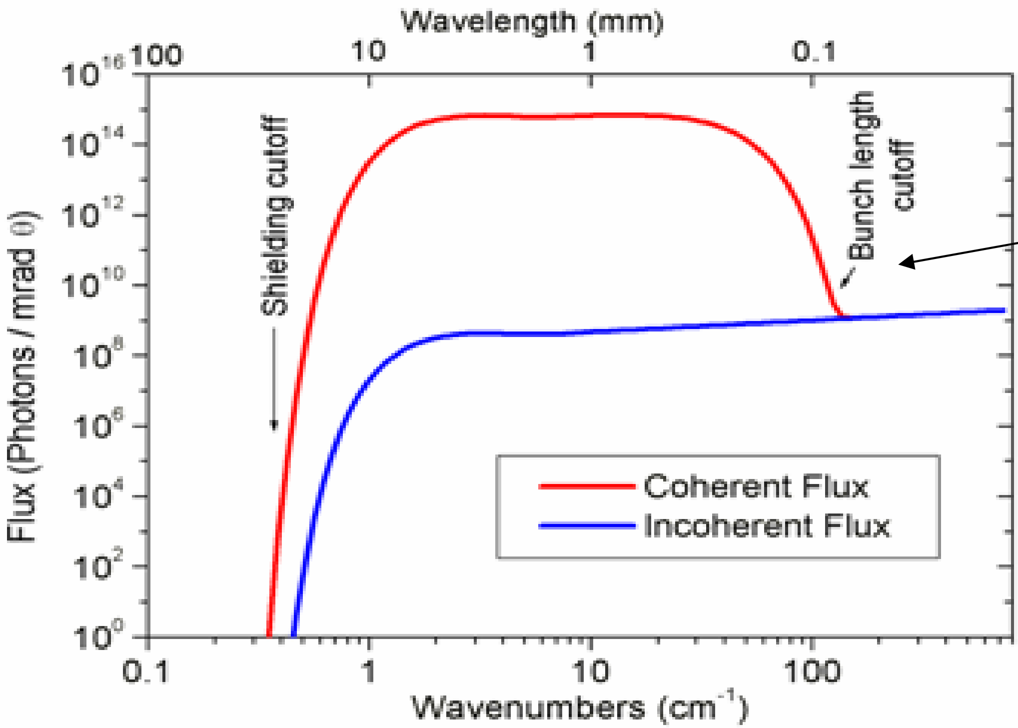
So, to extend the spectrum towards longer wavelengths large vacuum chamber gaps and small bending radius are required.

$w \equiv wavenumber \equiv 1/\lambda$



Shortening the Bunch

For a storage ring in the linear regime and for currents below the threshold for the microwave instability, the bunch is Gaussian with length:



$$\sigma_\tau \propto \left(\frac{\alpha_C E^3}{V_{RF} f_{RF}} \right)^{\frac{1}{2}}$$

$\alpha_C \equiv$ *momentum compaction*

$V_{RF} \equiv$ *peak RF voltage*

$f_{RF} \equiv$ *RF frequency*

$E \equiv$ *beam energy*

So it is natural to try shortening the bunch using those knobs.

Unfortunately, we will see that for short bunches (~ ps) the situation becomes a little bit more complicated...

How to Generate non-Gaussian Bunches

Non-linearities in the longitudinal dynamics:

RF non-linearities

Lattice non-linearities

Requires only a finite momentum spread value

And/Or

Wakefields:

CSR Impedance

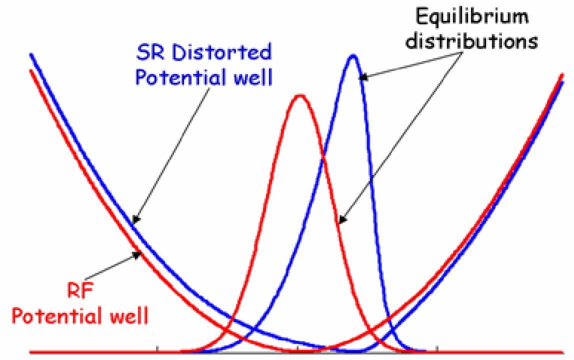
Vacuum Chamber Impedance

Requires some current to be effective

RF Non-linearities

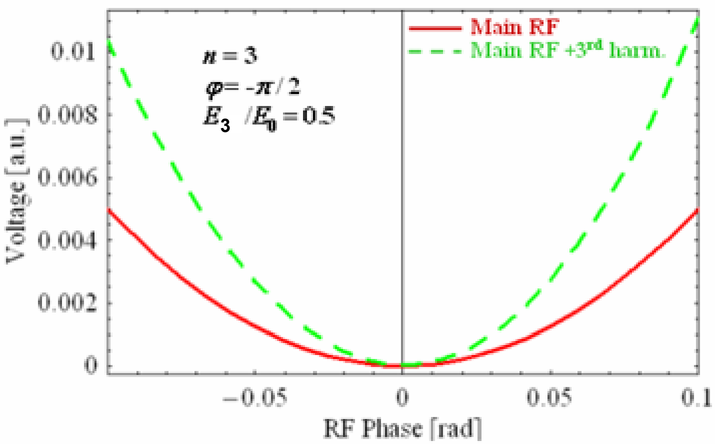
With a single frequency RF cavity with sufficient field amplitude, the potential well is with good approximation parabolic and the bunch is Gaussian at equilibrium.

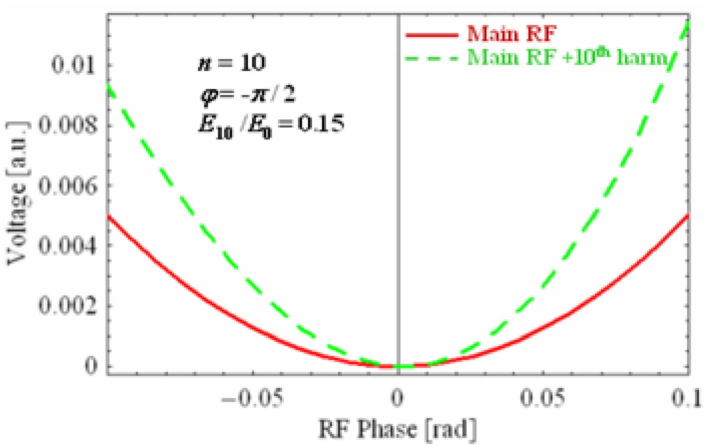
In principle, by using additional high-harmonic cavities one could obtain more complex shapes for the potential well generating the non-Gaussian distributions we are interested to.



This is a very complex scheme to realize, and so far people has only added one higher harmonic cavity in rings. For this case:

$$V(z) = -E_0 \frac{c}{\omega}\cos\left(\frac{\omega}{c}z\right) - E_n \frac{c}{n\omega}\cos\left(\frac{n\omega}{c}z + \varphi\right)$$





Very difficult and not very efficient!

Possible Distorting Knobs

Non-linearities in the longitudinal dynamics:

~~RF non-linearities~~

Lattice non-linearities

↔ **Requires only a finite momentum spread value**

And/Or

Wakefields:

CSR Impedance

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↔ **Requires some current to be effective**

Lattice Non-Linearities

The single particle longitudinal dynamics in a storage ring is defined by the focusing strength of the RF cavity (ies) and by the lattice characteristics.

The lattice component can be represented by the momentum compaction α_C of the ring, defined by:

$$\frac{\Delta L}{L_0} = \alpha_\beta + \left(\alpha_C - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p_0}$$

where L_0 is the ring length, γ is the nominal energy in rest mass units, p_0 is the particle nominal momentum, and if ε is the beam emittance:

$$\alpha_\beta \equiv betatron\ motion\ term \qquad \alpha_\beta = f(lattice)\varepsilon^{1/2} \qquad and\ is\ usually\ negligible$$

$$\boxed{\Rightarrow \quad \frac{\Delta L}{L_0} \cong \left(\alpha_C - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p_0}}$$

where this relation defines the orbit length variation for an off-momentum particle.

The momentum compaction is a function of the lattice parameters and in the more general case can be a nonlinear function of the relative momentum difference:

$$\alpha = \alpha_1 + \alpha_2 \frac{\Delta p}{p_0} + \alpha_3 \left(\frac{\Delta p}{p_0}\right)^2 + \ldots.$$

We want to investigate if a non-linear momentum compaction can generate the strongly non-Gaussian distributions we are interested to.

Nonlinear Phase Space an Extreme Simulation Example

$$E_0 = 1.7\ GeV,\ f_{RF} = 500\ MHz,\ V_{RF} = 1.3\ MV,\ L = 240\ m$$

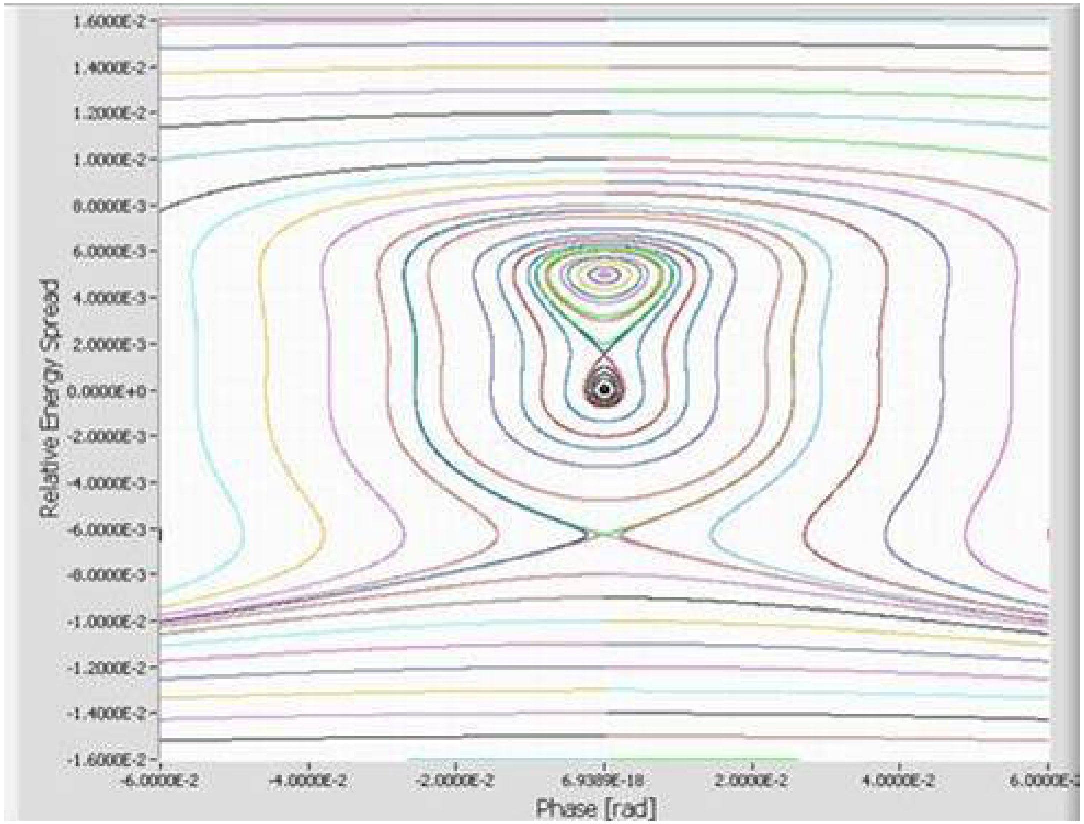
$$\boxed{\alpha_1 = 3\times10^{-6},\ \ \alpha_2 = -1\times10^{-3},\ \ \alpha_3 = -5\times10^{-3},\ \ \alpha_4 = 1.5\times10^{1}}$$

Strongly nonlinear!

The simulation is performed without damping, and the figure shows the longitudinal phase space trajectories

Two stable "buckets" with different energy are clearly visible.

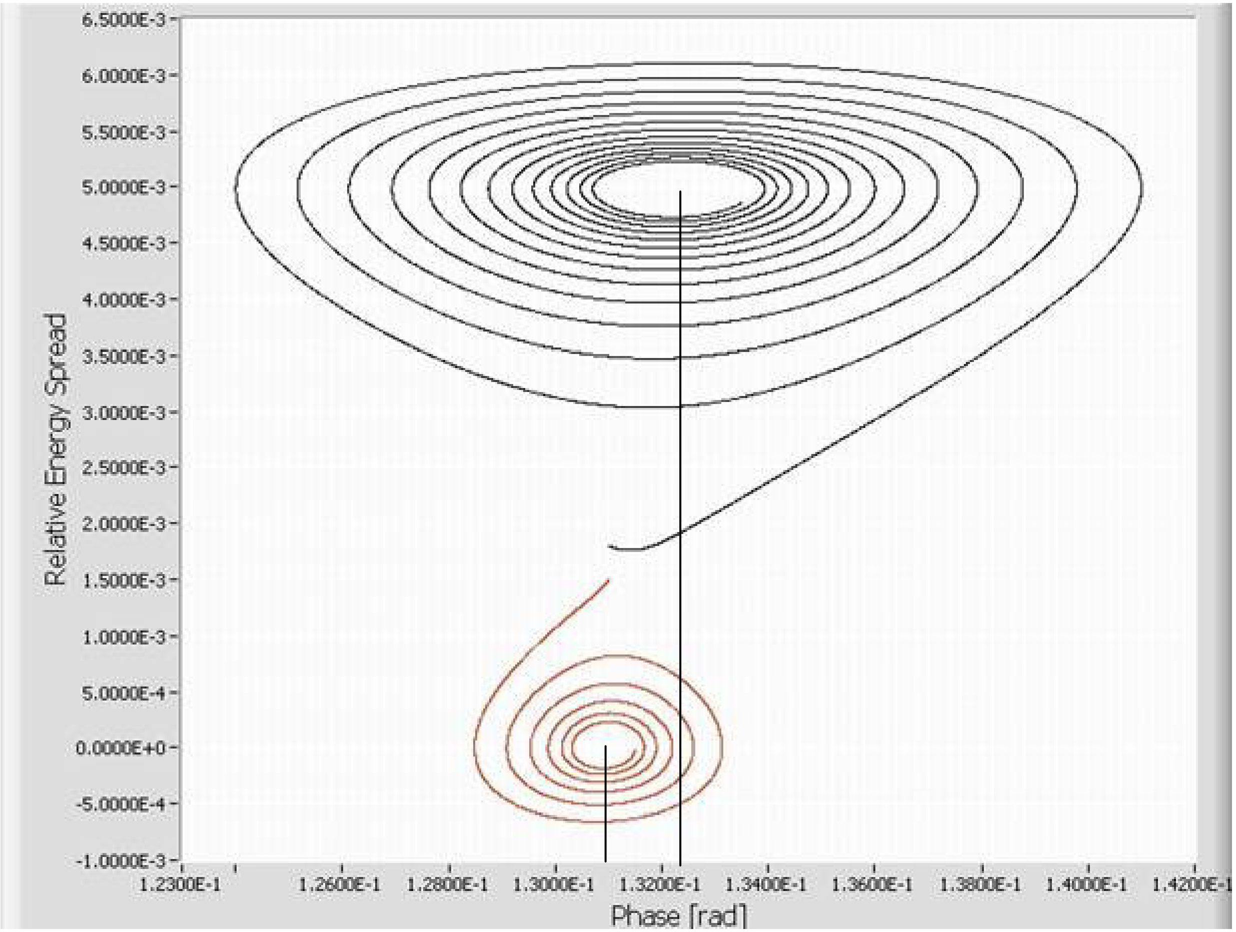
The bunch distribution is given by the projection of the phase space on the phase axis. The figure shows perfect symmetry respect to the zero phase and so symmetric bunch distributions...



But...

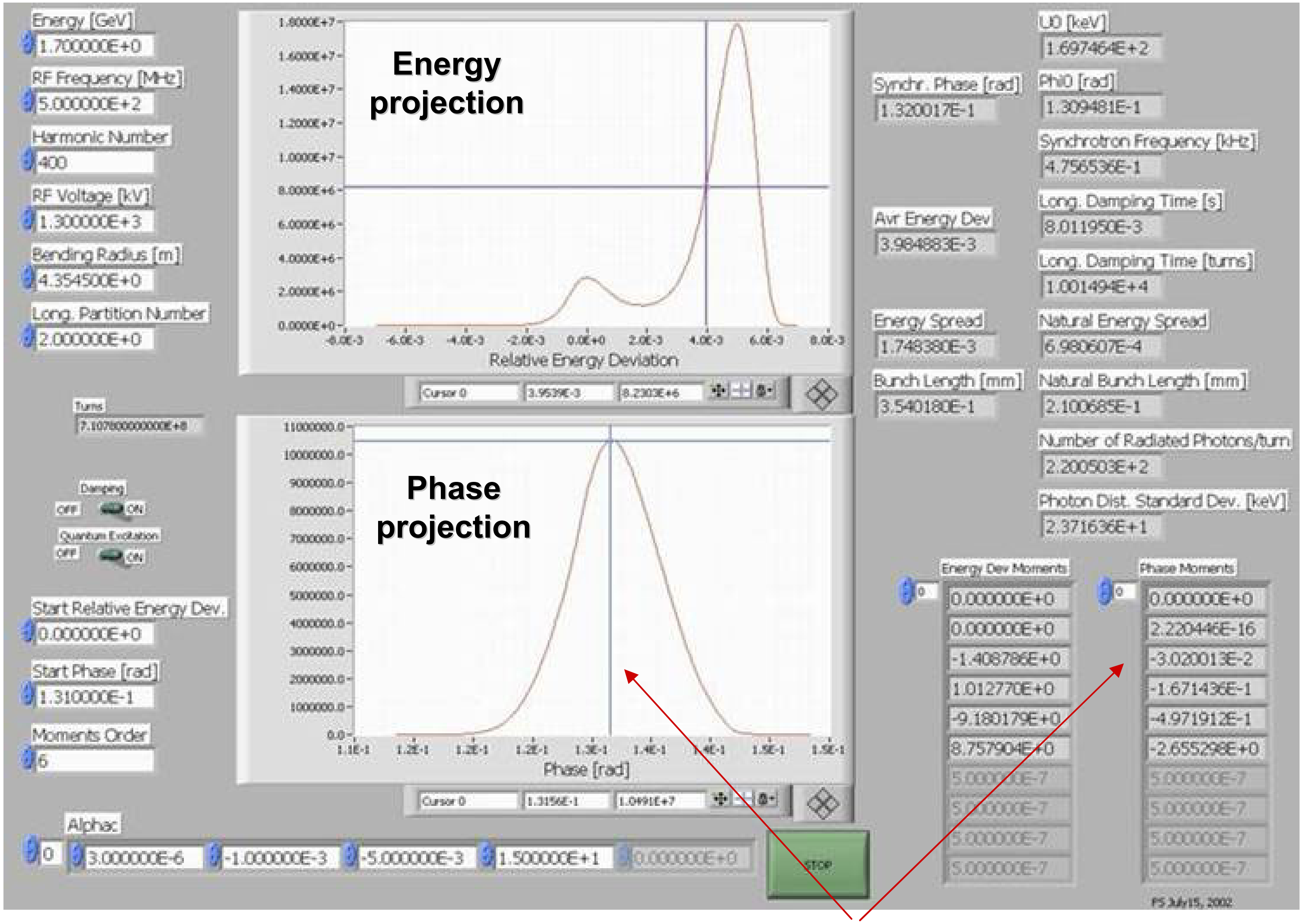
SR Energy Losses Effect

But when the radiation damping is switched on, the "centers" of the two buckets move to different synchronous phases for compensating for the different synchrotron radiation losses and the symmetry is broken!



The synchrotron radiation losses break the symmetry of the phase plane and allow for asymmetric distributions.

Simulated Distribution



The distribution moments show that even for this extreme case, the distribution is only slightly asymmetric.

Possible Distorting Knobs

Non-linearities in the longitudinal dynamics:

~~RF non-linearities~~

~~Lattice non-linearities~~

↔ **Requires only a finite momentum spread value**

And/Or

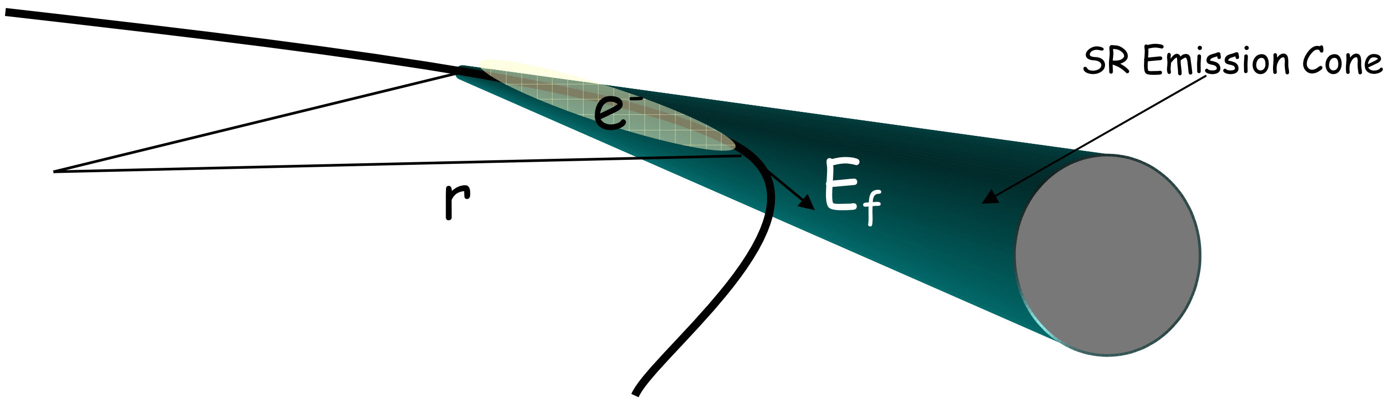
Wakefields:

CSR Impedance

Vacuum Chamber Impedance

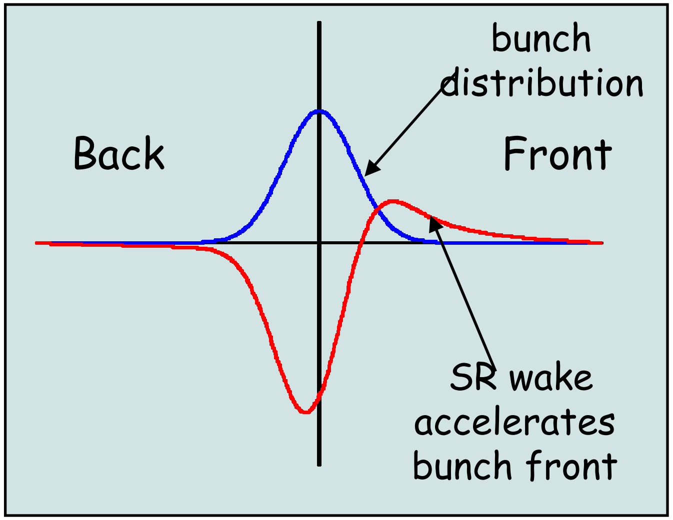
↔ **Requires some current to be effective**

The Synchrotron Radiation Wake



Because of the curved trajectory of the beam, the photons radiated from particles in the tail of the bunch catch up with the particles in the head.

The curved trajectory also allows for the electric field of these photons to assume a component parallel to the motion direction of the particles in the head and therefore to change their energy.



The Analytical Expression for the SR Wakefield

J.B. Murphy, S. Krinsky, R. Gluckstern , *Particle Accelerators* **57**, 9 (1997)

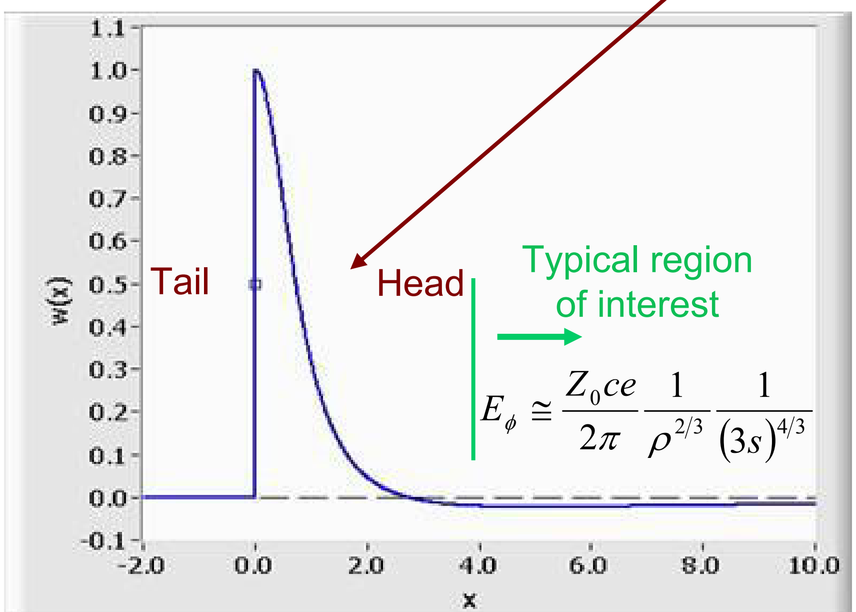
$$if \quad \frac{s}{2\rho} \lesssim \frac{h}{\rho} << 1 \qquad \gamma^2 \frac{h}{\rho} >> 1 \qquad [SI\ Units] \qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\ \Omega \equiv vacuum\ impedance$$

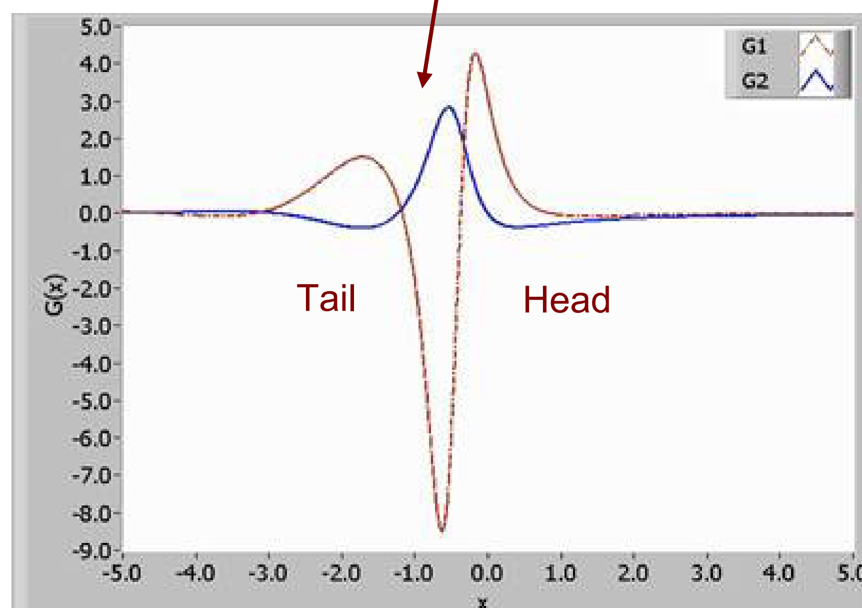
$$E_s \cong \frac{Z_0 ce}{\pi}\left[\cancel{\frac{1}{16}\frac{1}{\rho^2\gamma^2}\frac{\cos(s/2\rho)}{\sin^2(s/2\rho)}} - \frac{1}{3}\frac{\gamma^4}{\rho^2} w\left(3\gamma^3 \frac{s}{2\rho}\right) + \cancel{\frac{1}{16}\frac{\rho}{\gamma^2 h^3} G_1\left(\frac{s}{2}\frac{\rho^{1/2}}{h^{3/2}}\right)} + \frac{1}{8}\frac{1}{h^2} G_2\left(\frac{s}{2}\frac{\rho^{1/2}}{h^{3/2}}\right)\right]$$

Coulomb

Free Space

Infinite parallel plate shield



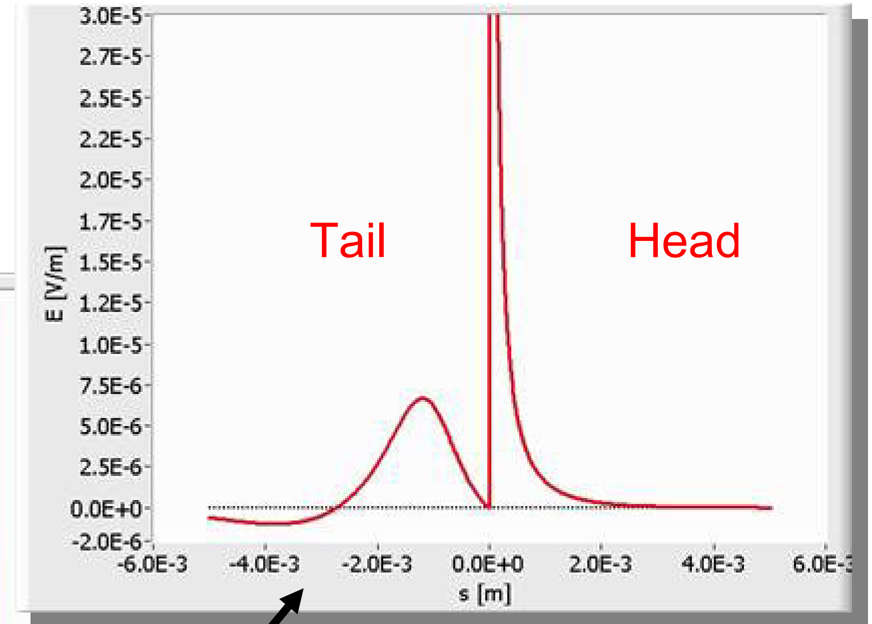


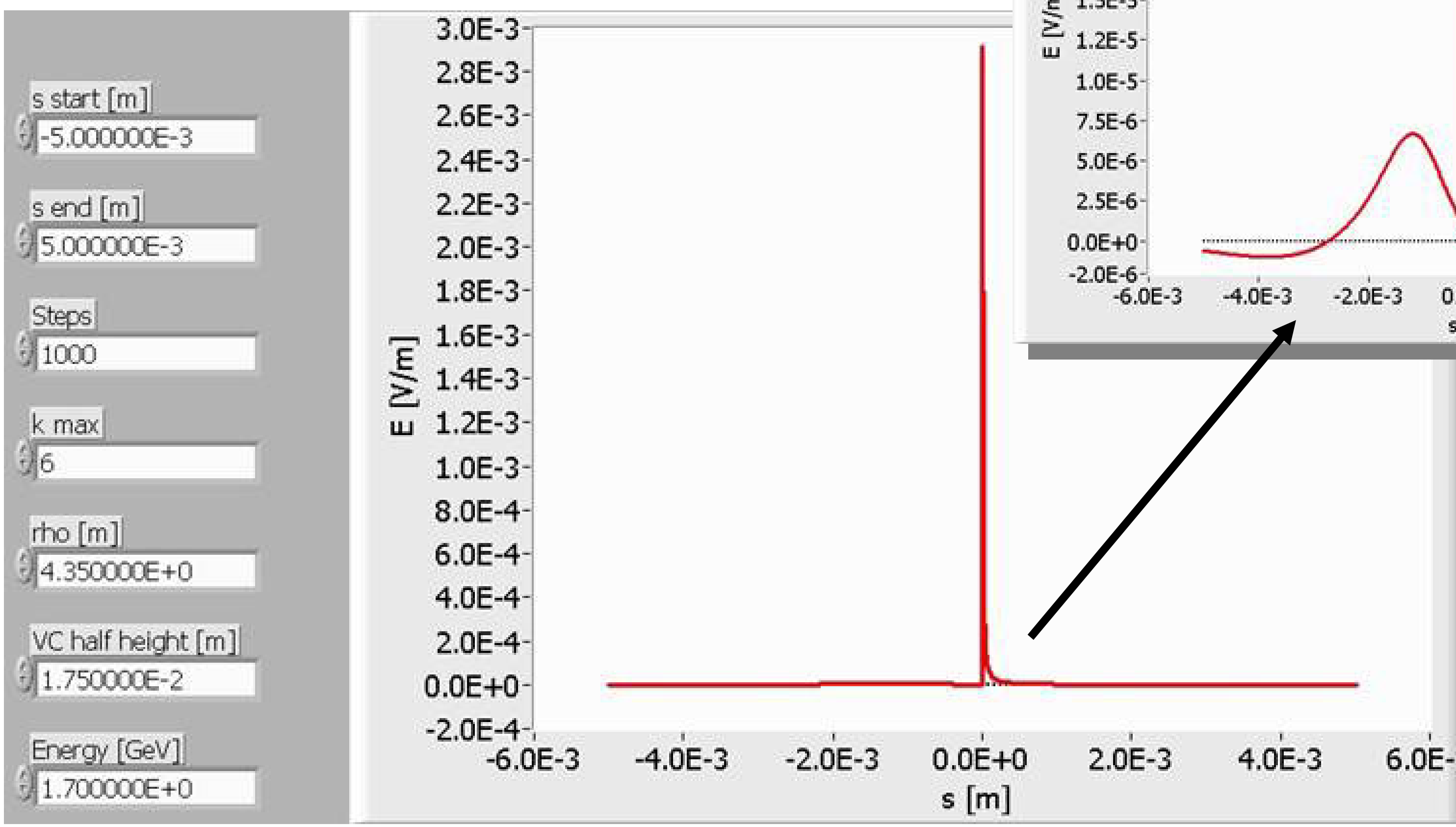
Example of SR Wakefield

BESSY II case: $E = 1.7$ GeV

$\rho = 4.35$ m

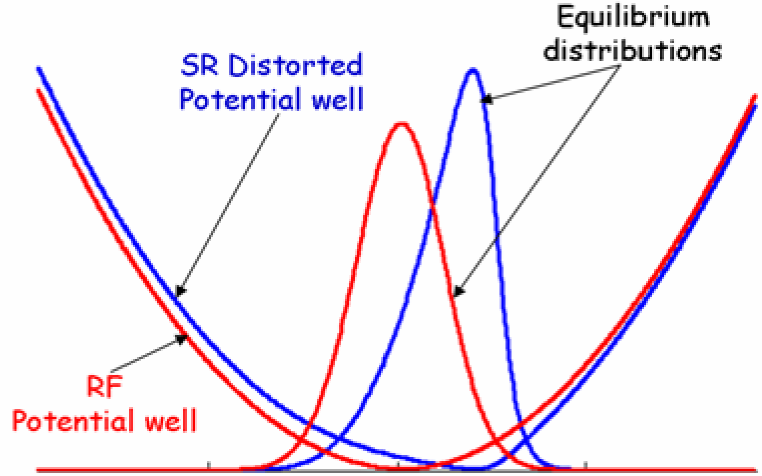
$h = 1.75$ cm





Potential Well Distortion Due to the SR wake

The strongly nonlinear SR wake generates a distortion of the parabolic potential well due to the RF cavity, and the bunch assumes non-Gaussian equilibrium distributions.

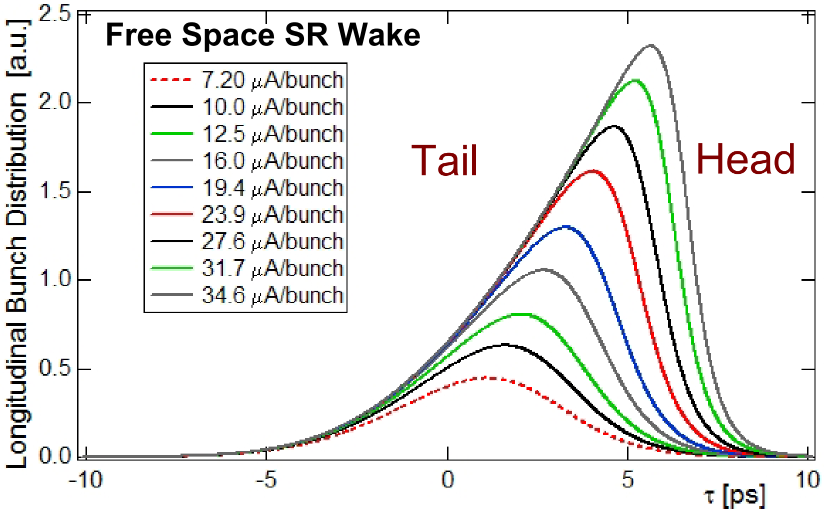


The current distribution $I(s)$ can be calculated by the Haissinski Equation:

$$I(s) = K\, e^{-\frac{s^2}{2\sigma_{z0}^2} - \frac{c}{2\pi f_{RF} V_{RF} \sigma_{z0}^2} \int_{-\infty}^{\infty} I(s-s')S(s')ds'}$$

$$S(s) = \frac{2\pi\rho}{ec} \int_{-\infty}^{s} E_s(s')ds'$$

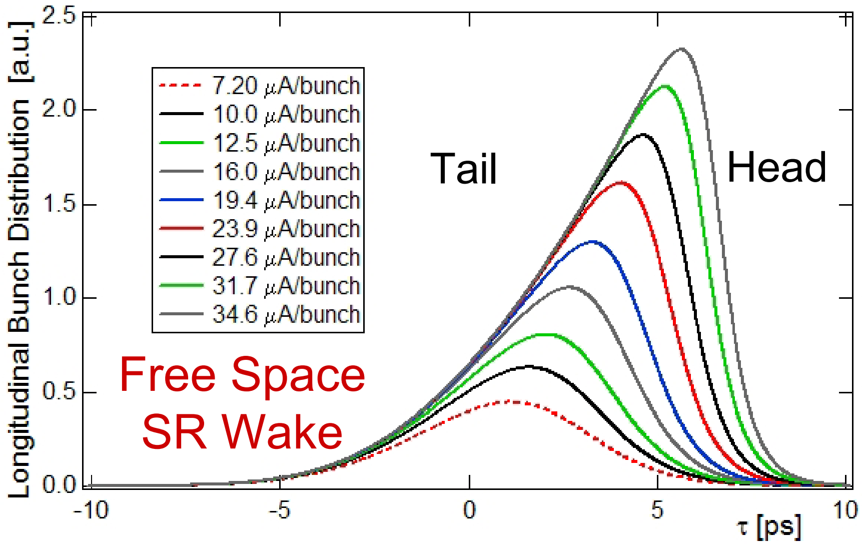
where $S(s)$ is the **Step Function Wake** and σ_{zo} is the natural bunch length.

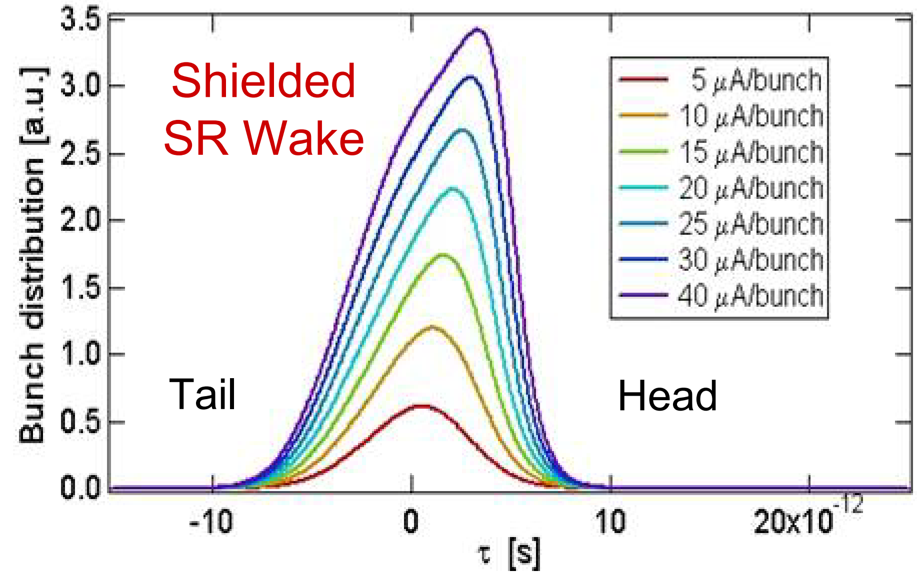


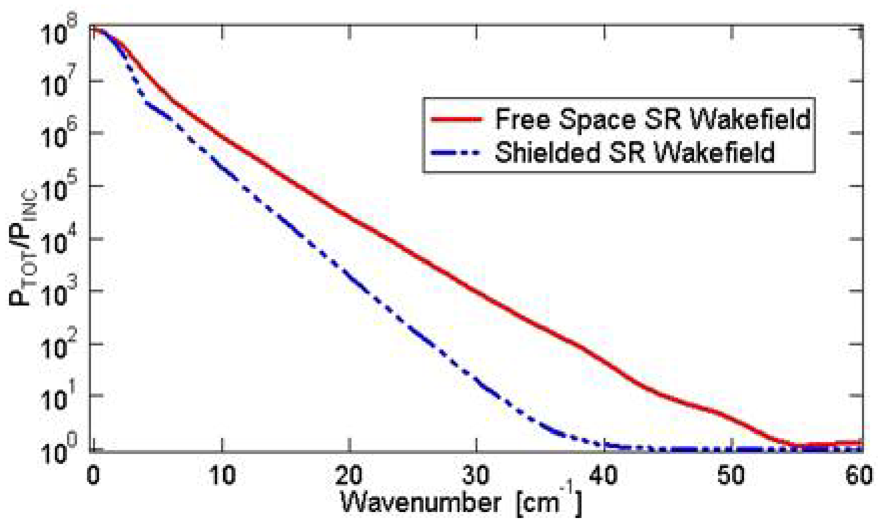
The free space SR wake generates the saw-tooth like distributions we were looking for!

(Bane, Krinsky and Murphy AIP Proc. **367**, 1995)

Vacuum Chamber Shielding







The effect of the vacuum chamber shielding is to reduce the CSR emitted power

In a CSR optimized source the shielding effects must be minimized

A 'Free Space' SR Ring

J.B. Murphy, S. Krinsky, R. Gluckstern , *Particle Accelerators* **57**, 9 (1997)

The vacuum chamber shielding terms in the SR wakefield become negligible when:

$$\Sigma = \frac{\sigma_z}{2} \frac{\rho^{1/2}}{h^{3/2}} \lesssim 0.2 \quad \Rightarrow \quad \Sigma \propto \frac{\sigma_z}{\lambda_{Cutoff}}$$

For a given bunch length σ_z, a proper choice of the bending radius ρ and of the vacuum chamber half-height h allows to make the shielding effects negligible.

Other Wakefields: Resistive Wall

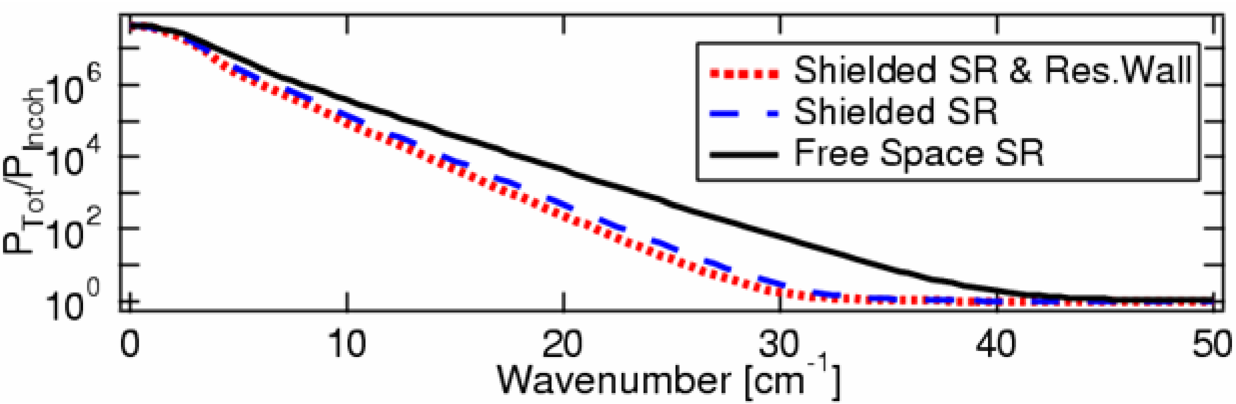
Long Range Resistive Wall wakefield (SI units - parallel plate model):

$$E_{RW} = \begin{cases} \dfrac{Z_0 ec}{4\pi\sqrt{2\pi}} \dfrac{1}{h^2} \left(-\dfrac{s_0}{s} \right)^{3/2} & s < 0 \quad downstream \quad particle \\ 0 & s \geq 0 \quad upstream \quad particle \end{cases} \qquad for \ bunch \ length \ \ \sigma_z >> s_0$$

$$s_0 = \left(\frac{2h^2}{Z_0 \sigma_C} \right)^{1/3} \equiv characteristic \ length, \ \ \sigma_C \equiv conductivity, \ \ h \equiv chamber \ \ half-distance$$

For example: K. Bane, M. Sands "Micro Bunches Workshop" AIP Conf. Proceedings 367 (1995).

This wake can be added to the others in the calculation of the equilibrium distribution by the Haissinski equation. From the distribution the CSR factor and spectrum are then readily evaluated.



The example shows that the resistive wall can reduce the CSR intensity. In general larger gap (once more) and high conductivity chambers are preferred.

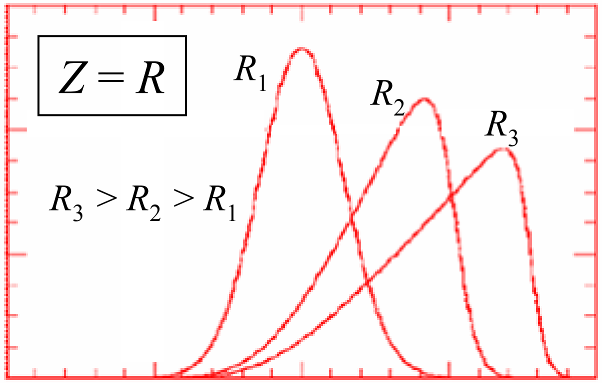
Other Wakefields: "Geometric" Wakes

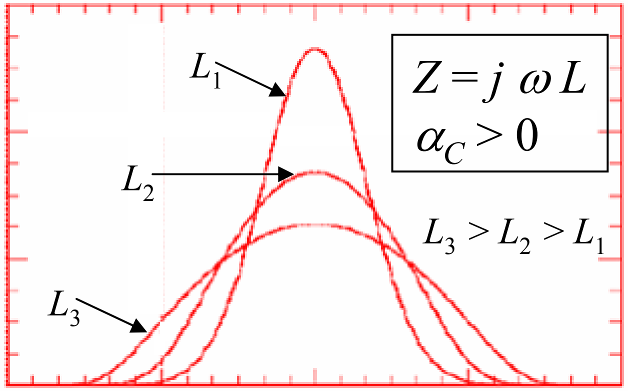
The effect of the wake fields due to the vacuum chamber of an accelerator are usually modeled by using the broadband impedance model:

$$\boxed{Z = R + iX(\omega)}$$

where the reactive part can be either capacitive or inductive depending on the frequency.

- **The real (resistive) part of the impedance generates *asymmetric non-Gaussian* distributions and *bunch lengthening*. The *bunch center of mass moves towards a different RF phase* to compensate for the wake induced energy losses.**

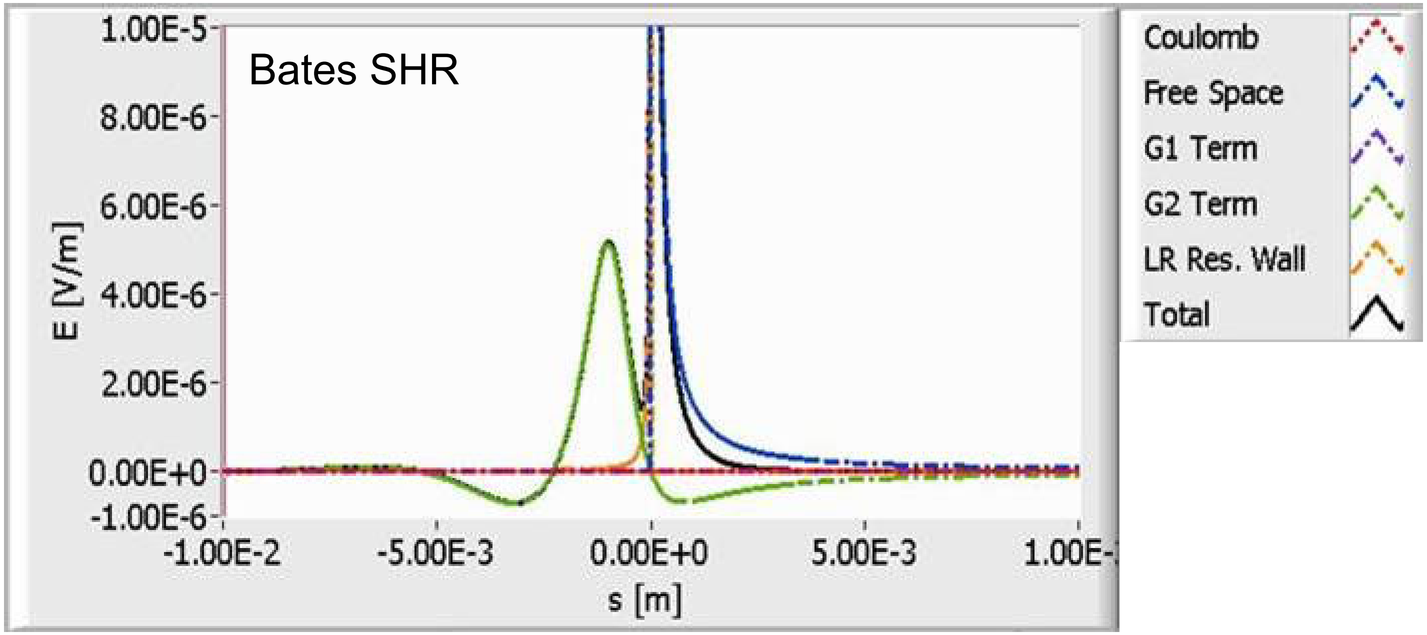




- **The imaginary (reactive) part of the impedance generates *symmetric non-Gaussian* bunch distributions. The bunch *center of mass does not move* (no energy losses). It generates *bunch lengthening or shortening*.**

In the short bunch regime of our interest, the effect of these wakes is usually negligible.

The Typical Wakefield



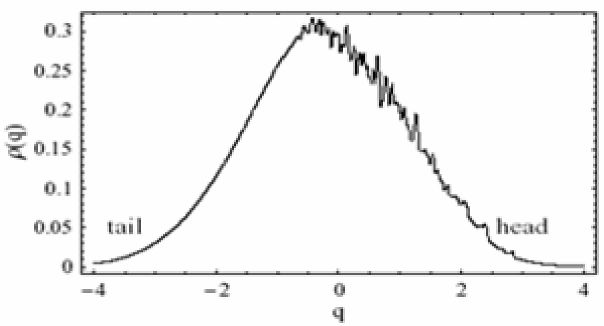
- **Synchrotron Radiation Wakes Included (Free Space and shielded G1 and G2)**
- **Long Range Resistive Wall Included**
- **Coulomb Wake not Included (negligible)**
- **Vacuum Chamber Geometric Wakes not Included (negligible)**

Increasing the Current per Bunch.

The bunch distribution in a real ring is never completely smooth and shows a modulated profile that changes randomly with time (noise).

These micro-structures usually have characteristic length << than the bunch length and radiate CSR. The wakefield from this radiation modulates the energy of neighbor particles that starts to move inside the bunch due to the longitudinal dispersion of the accelerator. Part of these particles moves in the direction that increases the size of the radiating micro-structure, and thus increasing the CSR intensity creating a gain mechanism for the process .

Above a certain current threshold, this gain becomes large enough to sustain the micro-bunching process and to generate an exponential growth of the micro-structure amplitude (up to saturation in the non linear regime of the instability).



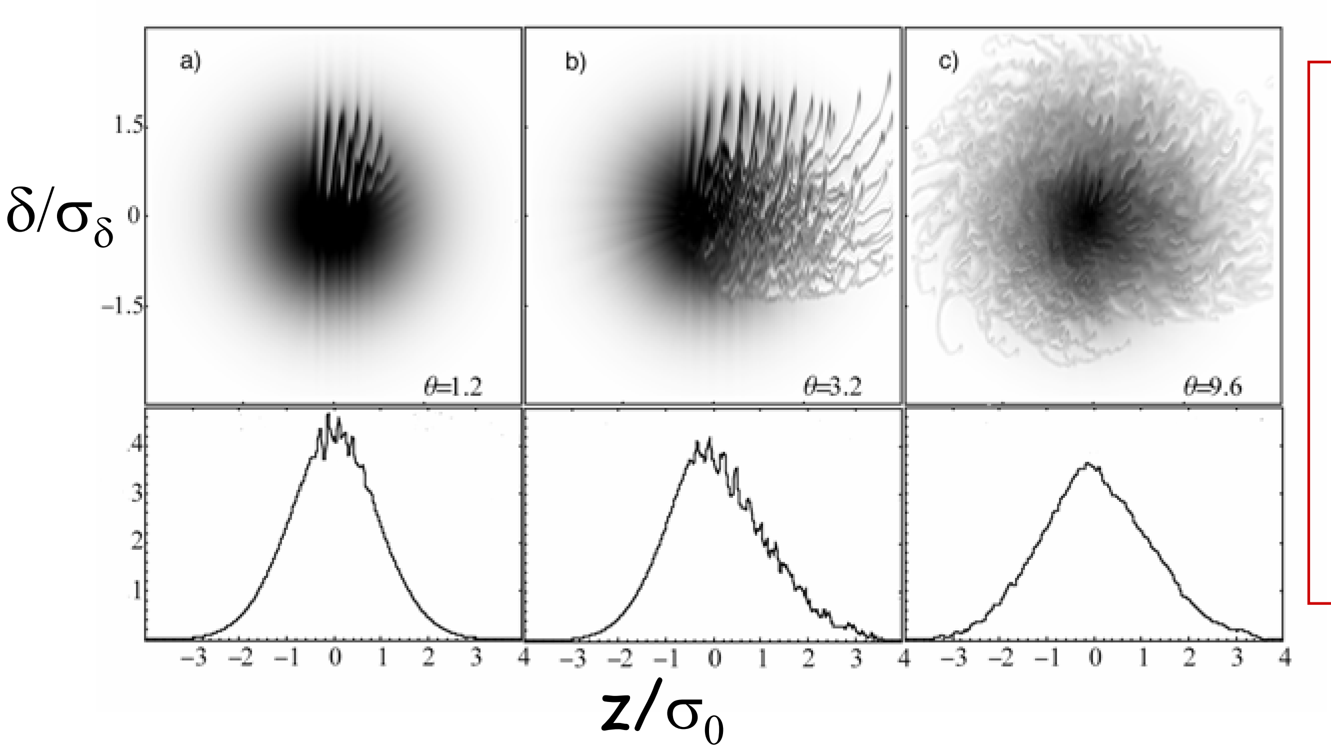
Simulated instability showing the microbunching.
Venturini, WarnocK SLAC

Such an instability, often referred as the **micro-bunching instability (MBI)**, is nothing else that a SASE process in the THz regime.

The MBI, for the case of storage rings, was predicted by Sam Heifets and Gennady Stupakov (**PRST-AB 5, 054402, 2002**) and simulated by Marco Venturini and Bob Warnock (**PRL 89, 224802, 2002**)

MBI Simulations

Small perturbations to the bunch density can be amplified by the interaction with the radiation. Instability occurs if growth rate is faster than decoherence from bunch energy spread.



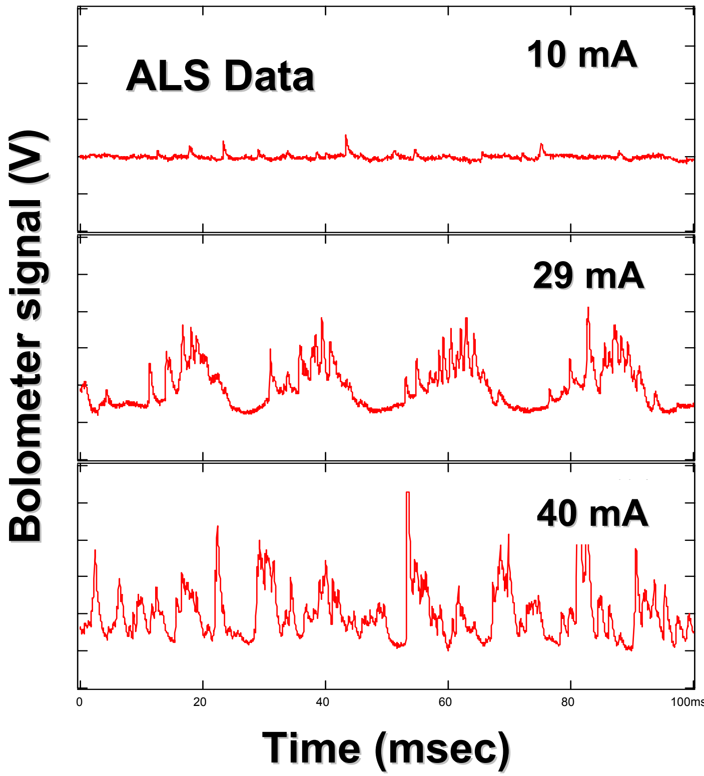
Nonlinear effects cause the instability to saturate. Radiation damping damps the increased energy spread and bunch length, resulting in a pulsing 'sawtooth' instability.

Simulation by Marco Venturini

Terahertz CSR Bursts

According to what said before, the presence of the micro-bunching instability should be associated with the emission of random "burst" of CSR.

In many electron storage rings around the world, strong random pulses ("bursts") of CSR in the THz frequency range were observed for high single bunch current.



A. Anderson et al.:, Opt. Eng. **39**, 3099, (2000).

G.L. Carr et al.:, NIMA **463**, 387, (2001).

M. Abo-Bakr et al.:, EPAC2000 Proceedings.

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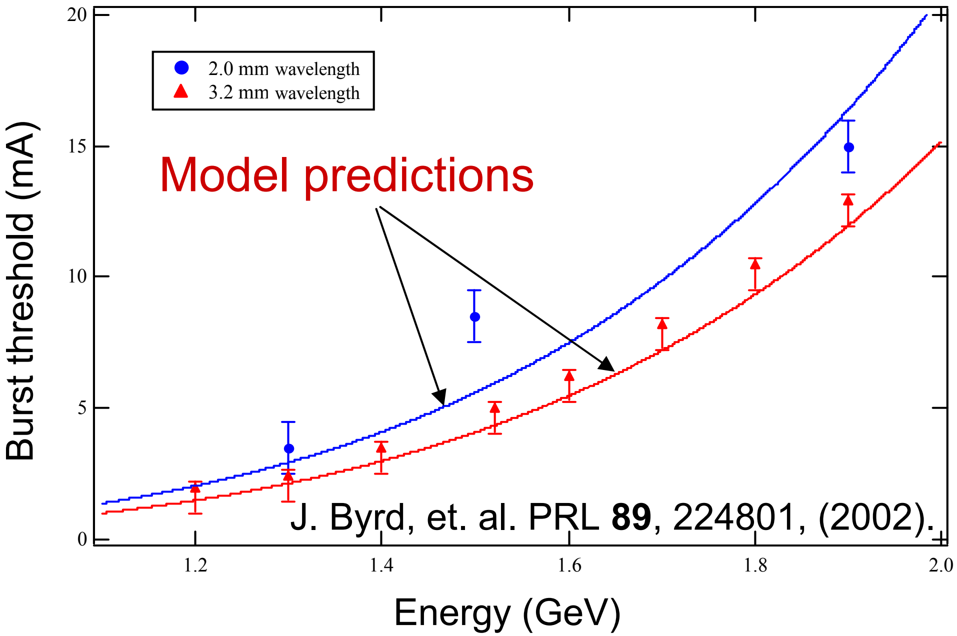
CSR Instability Experimental Verification

Experiments at the ALS provided the experimental confirmation that the THz CSR bursts were associated with the MBI.



The instability thresholds predicted by the Heifets-Stupakov model for the instability were in agreement with the measured thresholds

The MBI dramatically limits the maximum stable single bunch current in the short bunch regime!

The beam becomes unstable if the single bunch current is larger than (SI Units):

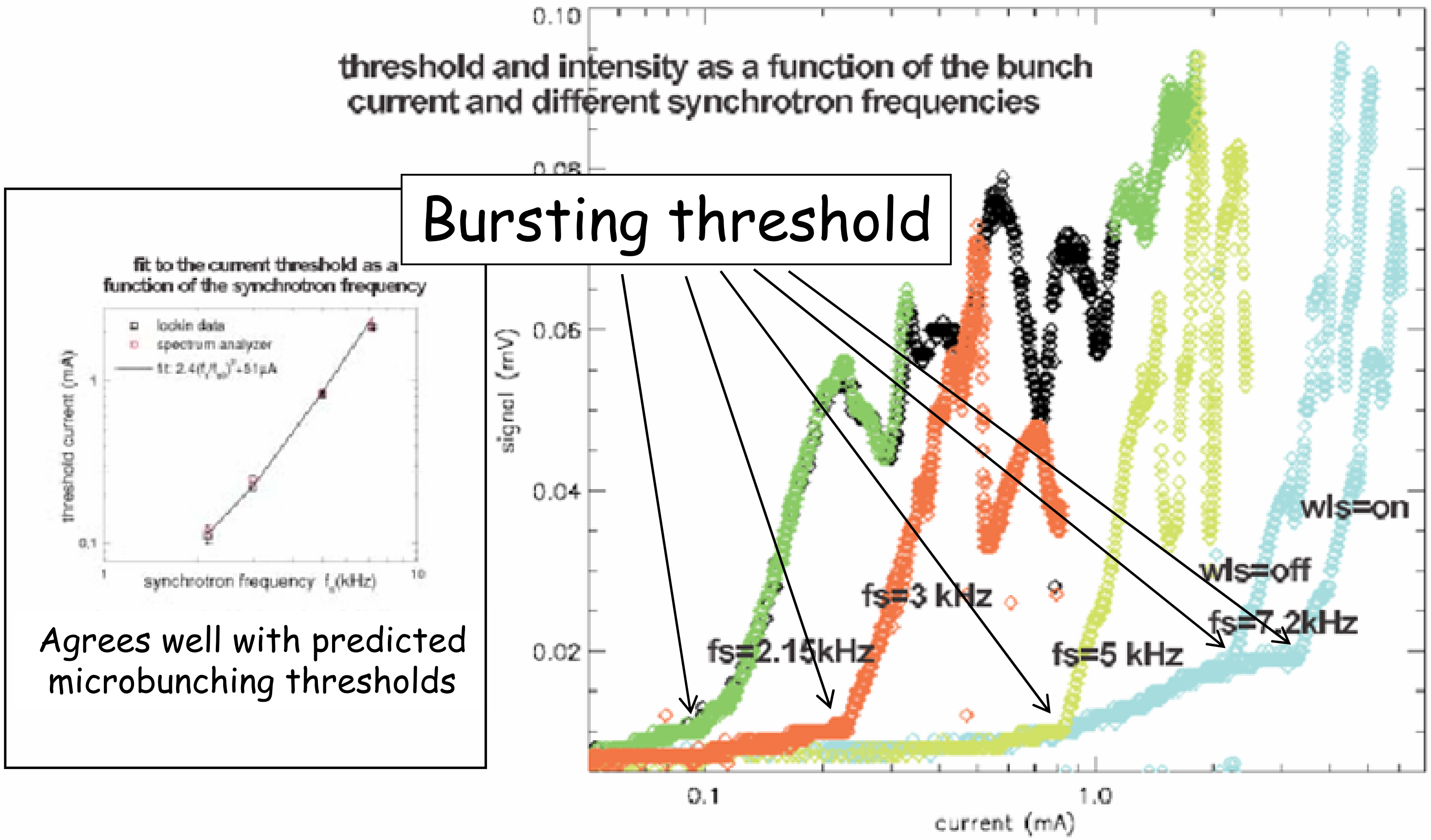
$$\boxed{I_{MBI}=\frac{\pi^{1/6}}{\sqrt{2}}\frac{ec}{r_0}\frac{\gamma}{\rho^{1/3}}\alpha_C\delta_0^2\sigma_z\frac{1}{\lambda^{2/3}}}=\frac{\sqrt{2}\,\pi^{7/6}e^2}{m_0c^3r_0}\frac{hV_{RF}f_0^2}{\rho^{1/3}}\frac{\sigma_z^3}{\lambda^{2/3}}=K\frac{1}{h^{1/2}f_0\left(V_{RF}\cos\varphi_s\right)^{1/2}}\frac{\alpha_C^{3/2}}{\rho^{11/6}J_s^{3/2}}\frac{\gamma^{9/2}}{\lambda^{2/3}}$$

$$K=\frac{m_0^{1/2}e^{1/2}c^3C_q^{3/2}}{2\pi^{1/3}r_0}\cong 2.956\times10^{-4}\ [SI\ Units],\ m_0\equiv electron\ mass,\ \delta_0\equiv natural\ energy\ spread,\ J_s\equiv long.\ partition\ \ number$$

$$f_0\equiv revolution\ \ frequency,\ \ r_0\equiv classical\ electron\ radius\cong 2.82\times10^{-15}\,m,\ \ C_q=3.8319\times10^{-13}\,m,\ \ \varphi_0\equiv sync.\ phase$$

CSR Instability Results Confirmed in Several Rings

Single bunch CSR-signal at 1.25 MHz and ~5Hz bandwidth

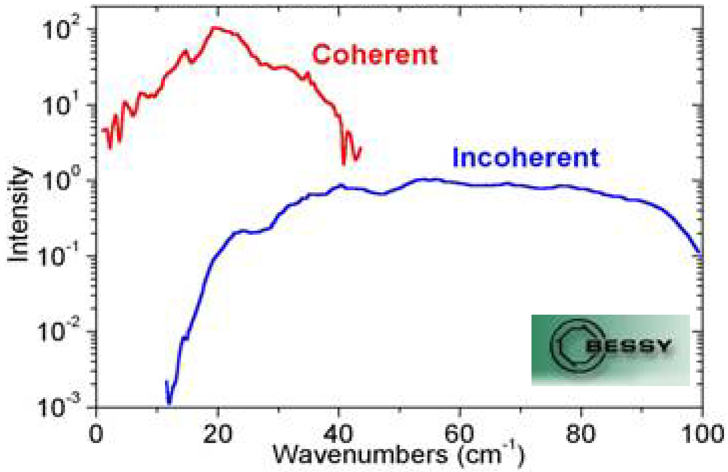


BESSY II results: courtesy of G. Wuestefeld

The BESSY II Results

In 2002, The BESSY-II group provided the first evidence of stable CSR in a storage ring.

Abo-Bakr *et al.*, PRL 88, 254801 (2002),
and M. Abo-Bakr *et al.*, Phys. Rev. Lett. 90, 094801 (2003)



Very interesting characteristics of the BESSY results were:

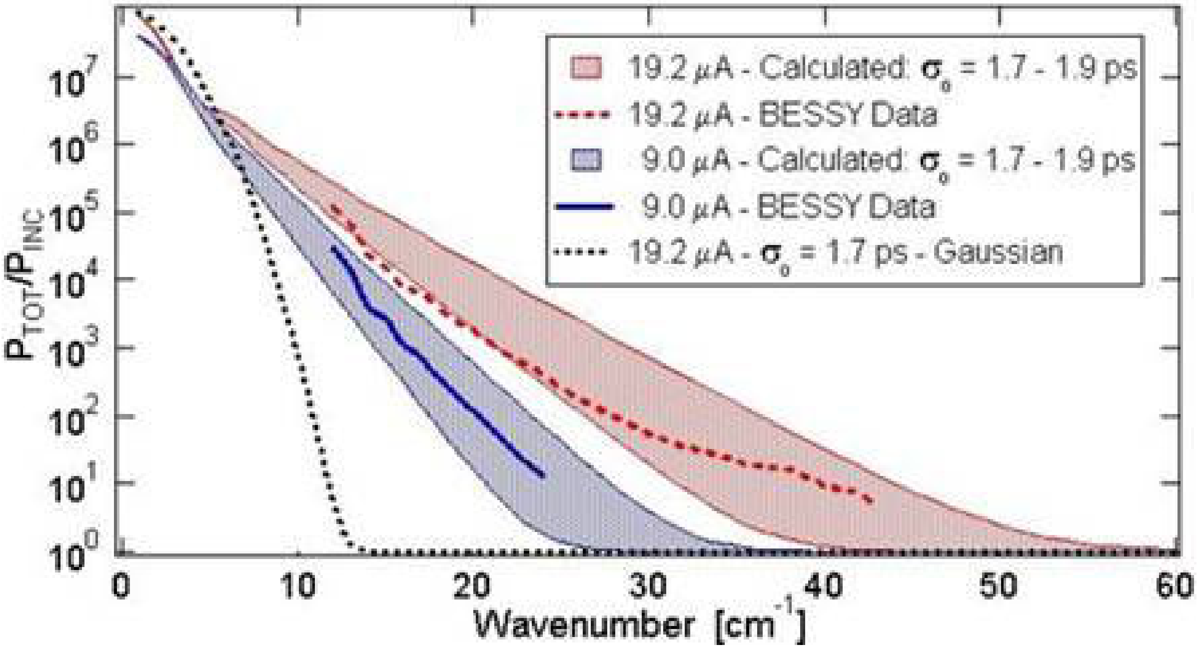
- a very stable CSR flux (no presence of bursts),

- an impressive power radiated in the THz region,

- and a spectrum significantly broader than the one expected for a Gaussian distribution their bunch length.

Can This Model Describe Reality?

The figure shows the model predictions compared to the BESSY II data. Shielding and resistive wall contributions were included. The "width" of the model predictions accounts for the indeterminacy on the knowledge of the BSSY II machine parameters.



The understanding of the physics behind the BESSY results showed the dominant role played in the short bunch regime by the SR wake and allowed to develop a model for optimizing a storage ring as a <u>stable</u> source of THz CSR.

Such a model has been used for calculating the CSR performance of a number of existing storage rings (DAΦNE, Bates, SPEAR, …) and also for designing a storage ring completely optimized for the generation of stable CSR in the THz frequency range (CIRCE, later in the lecture).

(F. Sannibale *et al.*, PRL **93**, 094801, 2004.)

Creating an Optimized Source

We now have all the information required for optimizing a storage ring as a source of stable CSR in the THz frequency range. We learned that:

- **The spectral bandwidth of the CSR is determined by the bunch length and longitudinal distribution.** Short asymmetric equilibrium distributions with a sharp edge generated by the SR wakefield significantly extend the bandwidth.
- **The maximum current per bunch is limited by the MBI.** In order to obtain a stable CSR emission, the current per bunch must be maintained below the instability threshold.
- **Shielding effects due to the vacuum chamber need to be carefully minimized.** A criterion was given that showed that by using a large gap vacuum chamber and a small bending radius the shielding can be made negligible.
- **Resistive wall impedance needs to be minimized.** A large gap-high conductivity vacuum chamber makes the effect negligible. The "geometric" vacuum chamber impedance has usually a very small effect in the short bunch regime.

In what follows, we will make the (realistic) assumption of a storage ring where the vacuum chamber has been properly designed in order to make the shielding and vacuum chamber impedance effects negligible. For such a ring only the **free space synchrotron radiation wakefield** needs to be considered.

We also assume linear RF focusing.

The Optimized Source: The Equilibrium Distribution

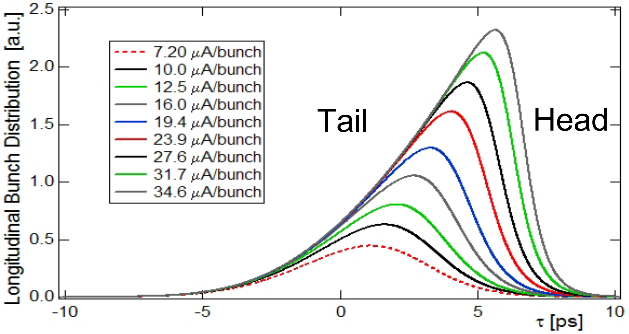
$$S(s) \cong \begin{cases} -Z_0\left(\dfrac{\rho}{3}\right)^{1/3} s^{-1/3} & s > 0 \\ 0 & s \le 0 \end{cases}$$

Free Space SR Step Function Response (wake function of a unit step - SI units)

The Haissinski equation for the free space SR case assumes the shape:

$$y_\kappa(x) = \kappa \exp\left[-\frac{x^2}{2} + \mathrm{sgn}(\alpha_C)\int_0^\infty y_\kappa(x-z) z^{1/3} dz\right]$$

$$x = z/\sigma_{z0}, \quad y_\kappa = \left(Z_0 c/\dot{V}_{RF}\right)\left(\rho/\sigma_{z0}^4\right), \quad \kappa \equiv normalization\ parameter$$



The figure shows an example of equilibrium distributions obtained by solving this equation, and from them the number of particles per bunch is derived:

$$\boxed{N = C\left(\frac{B}{E}\right)^{1/3} f_{RF} V_{RF} \sigma_{z0}^{7/3} F(\kappa)}$$

$$with \quad F(\kappa) = \int_{-\infty}^{\infty} y_\kappa(x) dx$$

E = beam energy
B = dipole field in the bending magnet
f_{RF} = RF frequency
V_{RF} = peak RF voltage
σ_{z0} = Natural bunch length

$$C = \left(2\pi 3^{1/3}\right)/\left(e^{2/3} c^{5/3} Z_0\right) \cong 6.068 \times 10^{-4}\ [SI\ Units],$$

The factor F is proportional to the distribution integral.
F also indicates the bunch distortion: the larger the more distorted is the bunch.

K. Bane, S. Krinsky, J.B. Murphy, *Microbunches Workshop,* Upton NY 1995

The Optimized Source: Accounting for the MBI

We already saw that the MBI sets a limit to the maximum current per bunch. This limit can be written as:

$$N \le N_{MBI} = A\left(\frac{B}{E}\right)^{1/3} f_{RF} V_{RF} \frac{\sigma_{z0}^{3}}{\lambda^{2/3}}$$

$$A = \left(2^{1/2}\pi^{7/6}e^{4/3}\right)/\left(r_0 m_0 c^{8/3}\right) \cong 4.528\times 10^{-3}\ [SI\ units]$$

And by comparing N_S with the number of particles per bunch that we previously calculated

$$N = C\left(\frac{B}{E}\right)^{1/3} f_{RF} V_{RF} \sigma_{z0}^{7/3} F$$

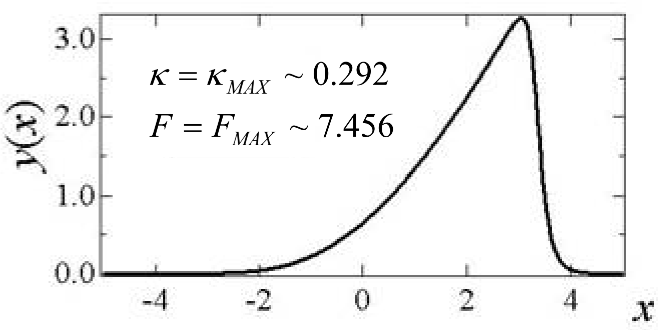
we obtain the stability condition: $F \le F_{MAX} = G\left(\frac{\sigma_{z0}}{\lambda}\right)^{2/3}$ $\quad G = 2^{3/2}\pi^{7/6}/3^{1/3} \cong 7.456$

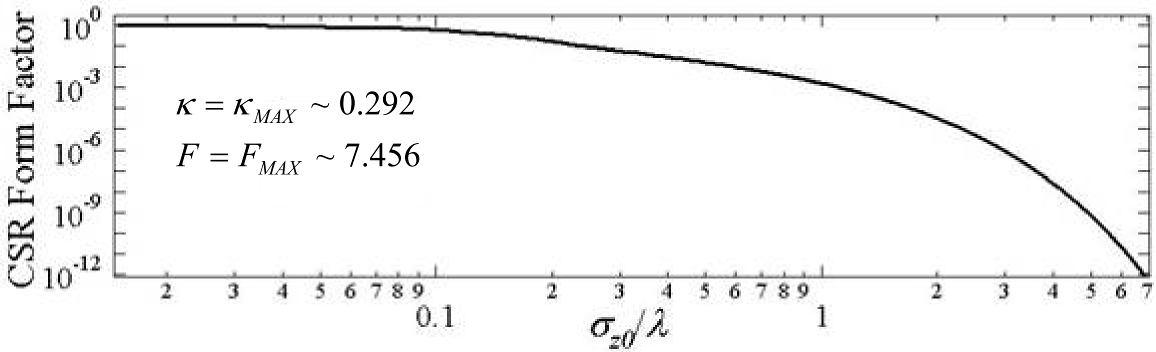
Experimental results at ALS and at BESSY II have shown that the first unstable mode for the MBI shows up when $\lambda \sim \sigma_{z0}$

$$\Rightarrow \quad \boxed{F_{MAX} \sim G} \quad \Rightarrow \quad \boxed{N_{MAX} \sim C\left(\frac{B}{E}\right)^{1/3} f_{RF} V_{RF} \sigma_{z0}^{7/3} G}$$

The Optimized Source: The CSR Bandwidth

We can now calculate the maximum stable distribution distortion for $F = F_{MAX}$ and from that the CSR form factor $g(\lambda)$ for this case:





$$g(\lambda) = \frac{1}{F_{MAX}^2} \left| \int_{-\infty}^{\infty} y_{\kappa_{MAX}}(x) \exp\left(i 2\pi \frac{\sigma_{z0}}{\lambda} \right) dx \right|^2$$

The last equation shows that CSR form factor is fully defined by the choice of the natural bunch length σ_{z0}:

$$\sigma_{z0} = c \left(C_q \frac{m_0 c^2}{2\pi e} \frac{\alpha_C}{h f_0^2 V_{RF}} \frac{\gamma^3}{J_s \rho} \right)^{1/2}$$

We already showed that the low frequency roll-off is instead defined by the vacuum chamber cutoff

$$\lambda_{\min} < 2 h_{Gap} \left(\frac{h_{Gap}}{\rho} \right)^{1/2}$$

Optimizing a CSR Source: The Radiated Power

For $N\, g(\lambda) \gg 1$ $\Rightarrow$ $dP/d\lambda = dp/d\lambda\, g(\lambda)\, N^2$

For synchrotron radiation and for wavelengths much longer than the critical wavelength, the power radiated by a single electron in a ring with length L, is given by:

$$\frac{dp}{d\lambda} \cong \left(\frac{2^{10}\pi^7 c^8}{3e}\right)^{1/3} \frac{r_0 m_0}{\Gamma(1/3)} \frac{1}{L} \left(\frac{E}{B}\right)^{1/3} \frac{1}{\lambda^{7/3}}$$

Using the last expression, the one for N_{MAX}, and assuming N_b bunches we obtain the maximum power radiated by a storage ring.

$$\boxed{\left(\frac{dP}{d\lambda}\right)_{MAX} \sim D\, \frac{N_b}{L} \left(\frac{B}{E}\right)^{1/3} \left(f_{RF} V_{RF}\right)^2 \left(\frac{\sigma_{z0}^2}{\lambda}\right)^{7/3} G^2 g(\lambda)}$$

$$D = \left(\frac{2^{16} 3\, \pi^{13}}{e^5 c^2}\right)^{1/3} \frac{r_0 m_0}{Z_0^2 \Gamma(1/3)} \cong 2.642 \times 10^{-21}\ [SI\ units]$$

Optimizing a Source: In Summary

The design of an optimized CSR THz source should probably start by deciding the desired bandwidth for the coherent radiation.

We saw how this choice imposes constraints on the vacuum chamber gap and on the dipole bending radius (low frequency cutoff) and also imposes the value for the natural bunch length σ_{z0} (high frequency cutoff).

The selection of σ_{z0} allows also to define the vacuum chamber characteristics necessary to make the effect of shielding and of resistive wall and geometric wakefields negligible.

The total power radiated within the selected coherent bandwidth can be maximized by the proper choice of the machine parameters according to:

$$\left(\frac{dP}{d\lambda}\right)_{MAX} \propto \frac{N_b}{L}\left(\frac{B}{E}\right)^{1/3}\left(f_{RF}V_{RF}\right)^2$$

The momentum compaction does not appear among the above parameters but plays a fundamental role. It is used for maintaining σ_{z0} constant while freely changing the other quantities.

Optimization Tradeoffs

Extreme maximization of B and V_{RF} requires superconductive systems with high cost impact

Too low energies should be avoided for accelerator physics reasons: poor lifetime, increased sensitivity to instabilities, ...
And also because the dependency of the CSR Power on the energy is weak.

In a small ring straight sections are short. Reasonably long straight sections allow interesting upgrades using insertion devices.
Additionally, the control of the machine parameters, is extremely important for the CSR tuning. Space for several families of quadrupoles and sextupoles is required

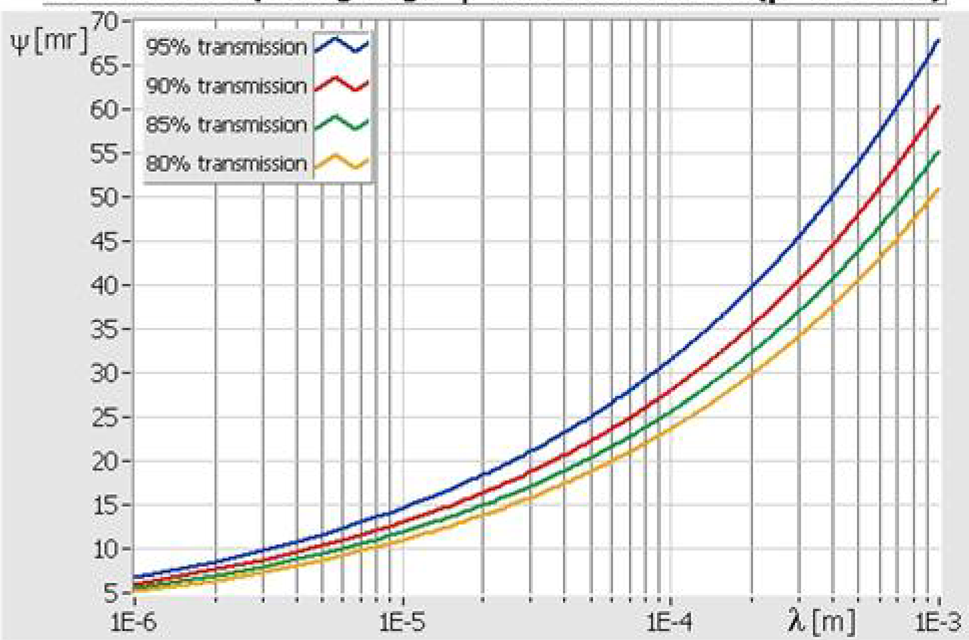
...

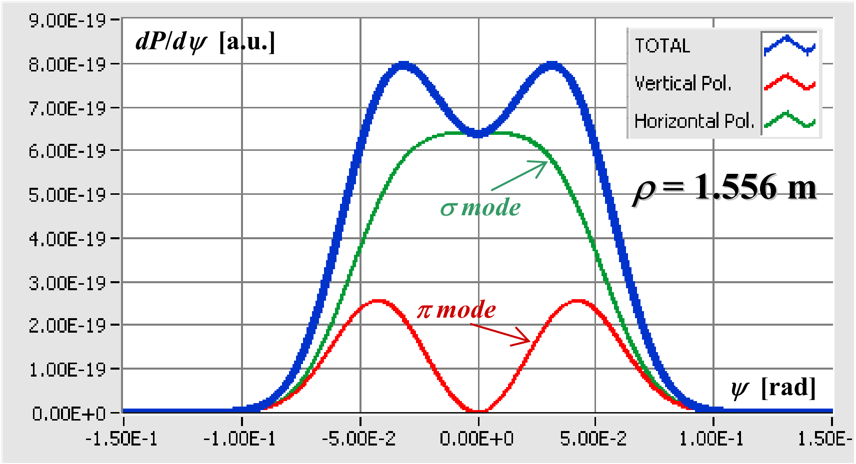
THz Synchrotron Radiation Divergence

$$\lambda_C = \frac{4}{3}\pi\frac{\rho}{\gamma^3} \qquad \psi_{Typ} \approx \frac{1}{\gamma}\left(\frac{\lambda}{\lambda_C}\right)^{1/3} \;\; if \;\; \lambda >> \lambda_C$$

$$\boxed{\psi_{Typ} \approx \left(\frac{3}{4\pi}\,\frac{\lambda}{\rho}\right)^{1/3}}$$

IR Radiation Opening Angle ψ and Transmission (ρ=1.556 m)

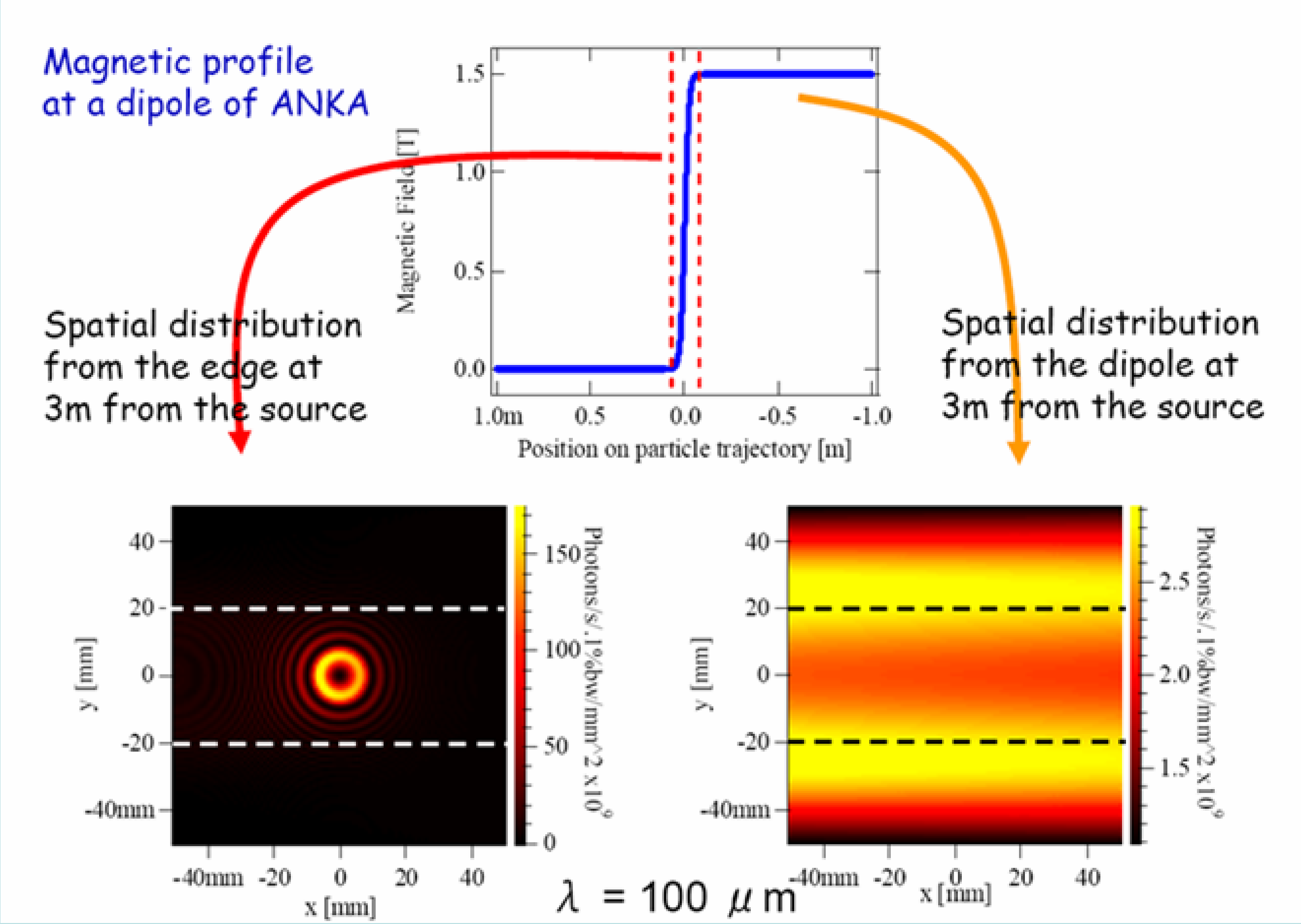




$$\frac{P_\sigma}{P_\pi} \cong 3$$

Very large divergence! The beamlines must be designed with very large acceptance to efficiently extract the radiation and to avoid undesired interference issues.

Edge vs. Synchrotron Radiation



Y-L. Mathis - UVSOR Workshop on THz CSR (Sept. 07)

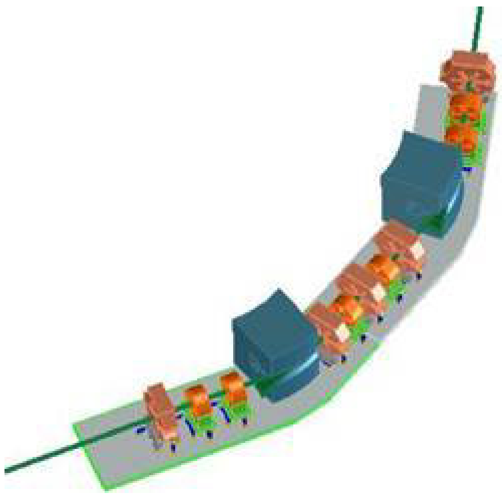
ESRF, ANKA, SOLEIL

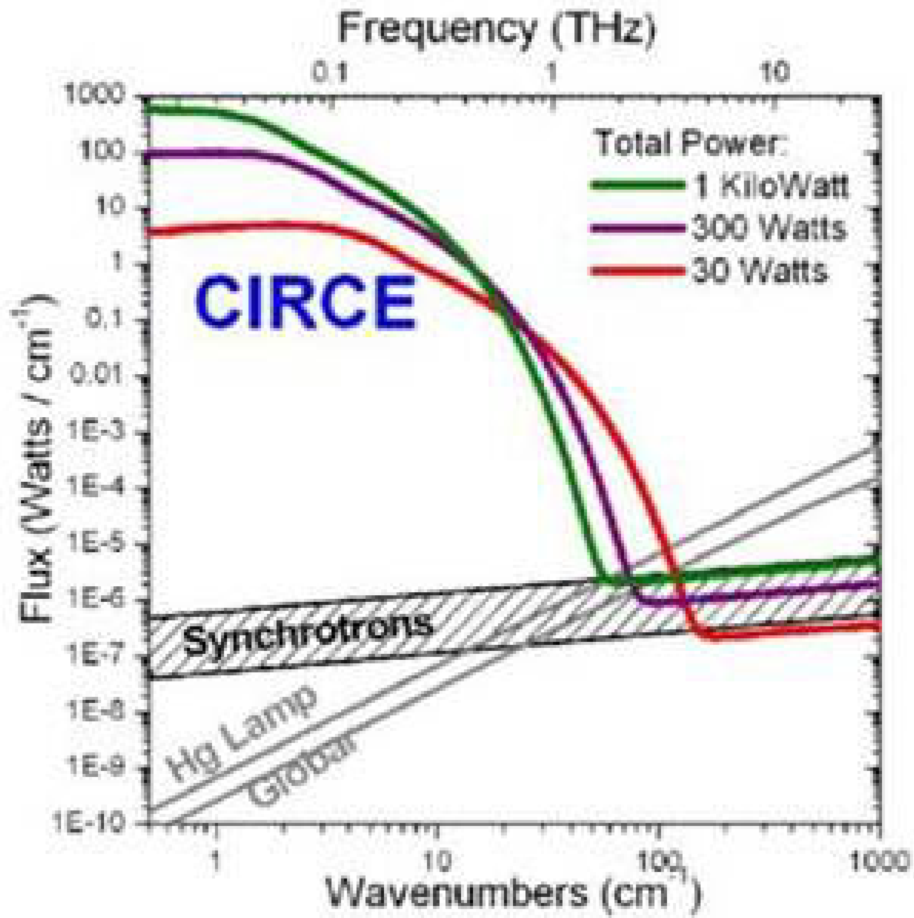
Edge radiation allows for smaller acceptance beamlines.

The case of CIRCE
The Coherent InfraRed CEnter

10^6 - 10^8 power gain with respect to the ALS BL 1.4 at the maximum current!

L= 66 m
Normal Conductive RF
Normal Conductive Dipoles
DBA with Control up to 3rd order α_C





CIRCE Performance Table

Horizontal Acceptance = 300 mrad
Power integrated between 1 and 100 cm^{-1}

$E = 600$ MeV $\quad f_{RF} = 1.5$ GHz
$V_{RF} = 0.6$ MV $\quad L = 66$ m
$h = 330 \quad \rho = 1.335$ m

	rms pulse length [ps]	Total power [w]	Pulse peak power [kw]	Energy per pulse [nJ]	Total current [mA]	Current per bunch [μA]	Particles per bunch	Momentum Compaction
Mode 1	**1.0**	**2.04**	**0.54**	**1.36**	**8.0**	**24.0**	**$3.3\ 10^{7}$**	**$2.4\ 10^{-4}$**
Mode 2	**2.0**	**15.1**	**2.0**	**10.0**	**35**	**106**	**$1.5\ 10^{8}$**	**$8.6\ 10^{-4}$**
Mode 3	**3.0**	**47.8**	**4.3**	**32.0**	**90**	**272**	**$3.7\ 10^{8}$**	**$1.9\ 10^{-3}$**

With Superconductive RF: $V_{RF} \sim 1.5$ MV

- **Power & Energy increase a factor 6.25**
- **Currents increase a factor 2.5**
- **Momentum compaction increases a factor 2.5**

CIRCE Lattice

CIRCE Parameters (NC RF):

$E = 600$ MeV	$f_{RF} = 1.5$ GHz
$V_{RF} = 0.6$ MV	$U_0 = 8.62$ kV
$I_{total} = 8\text{-}90$ mA	$I_{bunch} = 24\text{-}270$ μA
$L = 66$ m	$\#\ buckets = 330$
$\sigma_{\tau 0} = 1\text{-}3$ ps	$\sigma_\delta = 4.5\ \ 10^{-4}$
$\alpha = 2\ \ 10^{-3} - 2\ \ 10^{-4}$	$\rho = 1.335$ m
$2h = 4$ cm	$\Sigma = 0.06 - 0.18$

CIRCE Parameters (SC RF):

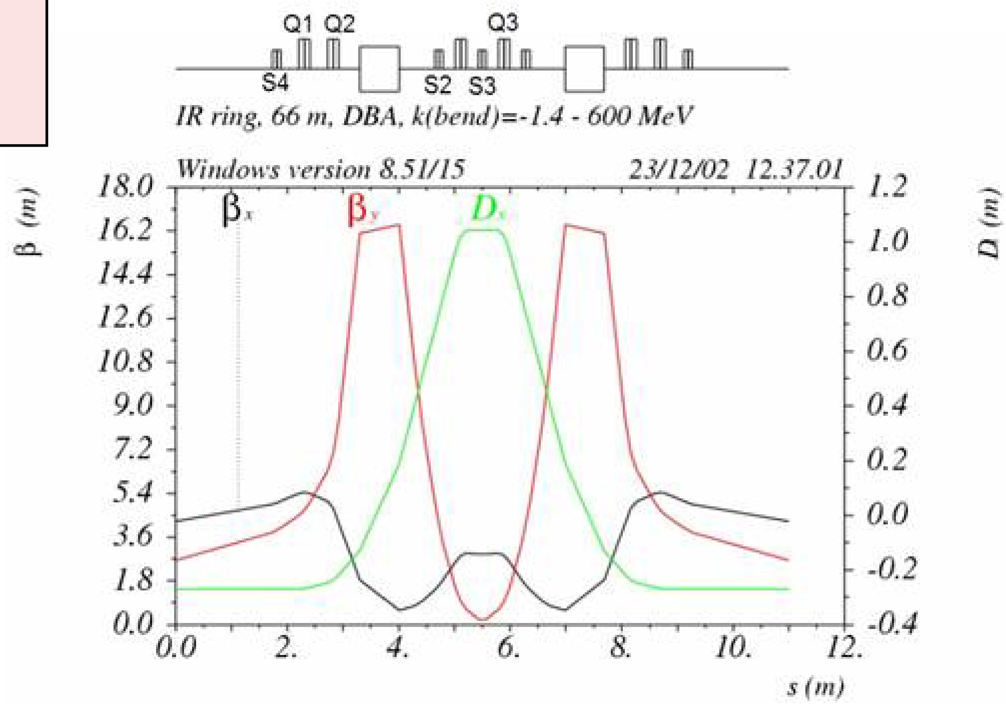
Same as the normal conductive case but:

$V_{RF} = 1.5$ MV

$I_{total} = 20\text{-}225$ mA $\qquad I_{bunch} = 60\text{-}675$ μA

$\alpha = 10^{-2} - 10^{-3}$

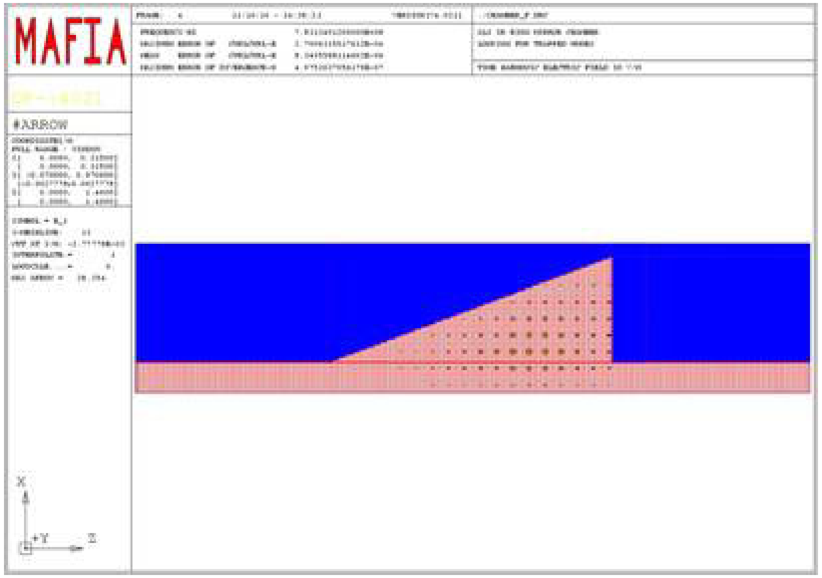
- **Periodicity = 6**
- **DBA lattice 50 nm emittance (diffraction limited in far-infrared)**
- **Variable momentum compaction with 3rd order correction**
- **Magnets pre-aligned on girders**
- **Shielding fits directly over magnets (i.e. no tunnel access)**

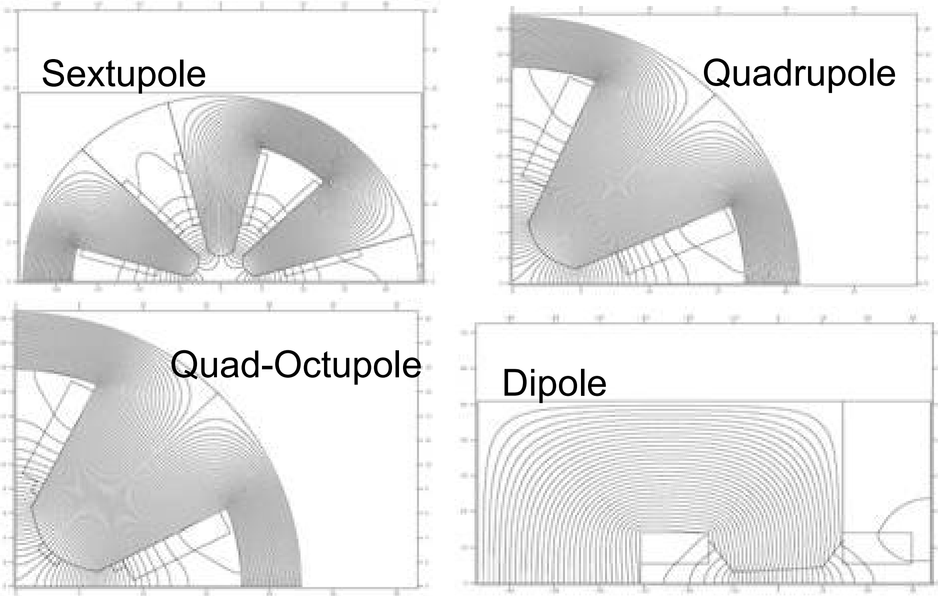


CIRCE Engineering Examples

- **Built Prototype of the (very!) large acceptance (300 x 140 mrad^2) dipole vacuum chamber**

- **Performed RF Measurements for High Order Modes (HOM)**
- **Defined Efficient Scheme for HOM Damping**





- **All Magnets Designed**

Used References

Microbunching instability (MBI):

- Sam Heifets and Gennady Stupakov, PRST-AB **5**, 054402, 2002
- Marco Venturini and Bob Warnock, PRL **89**, 224802, 2002
- J.M.Byrd *et al*., PRL **89**, 224801, 2002

Stable CSR Model:

- K. Bane, S. Krinsky, J.B. Murphy, *Microbunches Workshop,* Upton NY 1995 AIP Proc. **367**, 1995.
- F. Sannibale *et al*., PRL **93**, 094801, 2004.
- F. Sannibale *et al*., ICFA Beam Dynamics-Newsletter **35**, 2004

CIRCE:

- J. M. Byrd, *et al*., Infrared Physics & Technology **45** (2004) 325-330.
- J. Byrd, *et al*., 9th European Particle Accelerator Conference, Lucerne, Switzerland, July 2004. LBNL-55603.

**CSR in Storage Rings: ICFA Beam Dynamics-Newsletter 35, 2004
(http://icfa-usa.jlab.org/archive/newsletter.shtml)**

Homework

Assuming that you need a storage ring-based CSR THz source with a given spectrum, calculate a set of parameters that maximizes the radiated power within the desired bandwidth. Explain the reasons behind your choices.

Physical Constants (SI Units)

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
speed of light in vacuum	c, c_0	299 792 458	$\mathrm{m\,s^{-1}}$	(exact)
magnetic constant	μ_0	$4\pi \times 10^{-7}$	$\mathrm{N\,A^{-2}}$	
		$= 12.566\,370\,614... \times 10^{-7}$	$\mathrm{N\,A^{-2}}$	(exact)
electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817... \times 10^{-12}$	$\mathrm{F\,m^{-1}}$	(exact)
Newtonian constant of gravitation	G	$6.674\,28(67) \times 10^{-11}$	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$	1.0×10^{-4}
Planck constant	h	$6.626\,068\,96(33) \times 10^{-34}$	$\mathrm{J\,s}$	5.0×10^{-8}
$h/2\pi$	$\hbar$	$1.054\,571\,628(53) \times 10^{-34}$	$\mathrm{J\,s}$	5.0×10^{-8}
elementary charge	e	$1.602\,176\,487(40) \times 10^{-19}$	C	2.5×10^{-8}
magnetic flux quantum $h/2e$	Φ_0	$2.067\,833\,667(52) \times 10^{-15}$	Wb	2.5×10^{-8}
conductance quantum $2e^2/h$	G_0	$7.748\,091\,7004(53) \times 10^{-5}$	S	6.8×10^{-10}
electron mass	m_e	$9.109\,382\,15(45) \times 10^{-31}$	kg	5.0×10^{-8}
proton mass	m_p	$1.672\,621\,637(83) \times 10^{-27}$	kg	5.0×10^{-8}
proton-electron mass ratio	m_p/m_e	$1836.152\,672\,47(80)$		4.3×10^{-10}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5376(50) \times 10^{-3}$		6.8×10^{-10}
inverse fine-structure constant	α^{-1}	$137.035\,999\,679(94)$		6.8×10^{-10}
Rydberg constant $\alpha^2 m_e c/2h$	R_∞	$10\,973\,731.568\,527(73)$	$\mathrm{m^{-1}}$	6.6×10^{-12}
Avogadro constant	N_A, L	$6.022\,141\,79(30) \times 10^{23}$	$\mathrm{mol^{-1}}$	5.0×10^{-8}
Faraday constant $N_A e$	F	$96\,485.3399(24)$	$\mathrm{C\,mol^{-1}}$	2.5×10^{-8}
molar gas constant	R	$8.314\,472(15)$	$\mathrm{J\,mol^{-1}\,K^{-1}}$	1.7×10^{-6}
Boltzmann constant R/N_A	k	$1.380\,6504(24) \times 10^{-23}$	$\mathrm{J\,K^{-1}}$	1.7×10^{-6}
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3 c^2$	σ	$5.670\,400(40) \times 10^{-8}$	$\mathrm{W\,m^{-2}\,K^{-4}}$	7.0×10^{-6}

From: http://physics.nist.gov