



Argonne
NATIONAL
LABORATORY

... for a brighter future



U.S. Department
of Energy

UChicago ►
Argonne_{LLC}



A U.S. Department of Energy laboratory
managed by UChicago Argonne, LLC

Laser applications for accelerators

Laser Basics

Yuelin Li
Accelerator Systems Division
Argonne National Laboratory
ylli@aps.anl.gov

USPAS 2008 Summer Session, Annapolis,
June 23-27, 2008

Content

- Laser and accelerator history
- Map of laser application in accelerators
- Laser basics
 - Rate equations
 - Laser configurations
 - Gaussian beam optics and ABCD law
 - Laser cavity and laser modes
- Laser configurations
 - Mode-locking and q-switch
 - MOPA
 - CPA and dispersion
- Laser materials
- Other lasers
 - Semiconductor lasers
 - Fiber lasers
- Frequency conversion and short wavelength lasers

Content

- Laser and accelerator history
- Map of laser application in accelerators
- Laser basics
 - Rate equations
 - Laser configurations
 - Gaussian beam optics and ABCD law
 - Laser cavity and laser modes
- Laser configurations
 - Mode-locking and q-switch
 - MOPA
 - CPA and dispersion
- Laser materials
- Other lasers
 - Semiconductor lasers
 - Fiber lasers
- Frequency conversion and short wavelength lasers

Lasers and accelerators at birth

Ancient: a cave man's bow

.....

1929, Cyclotron, Lawrence

1939, Nobel Prize, Lawrence



Ancient: Let there be light

.....

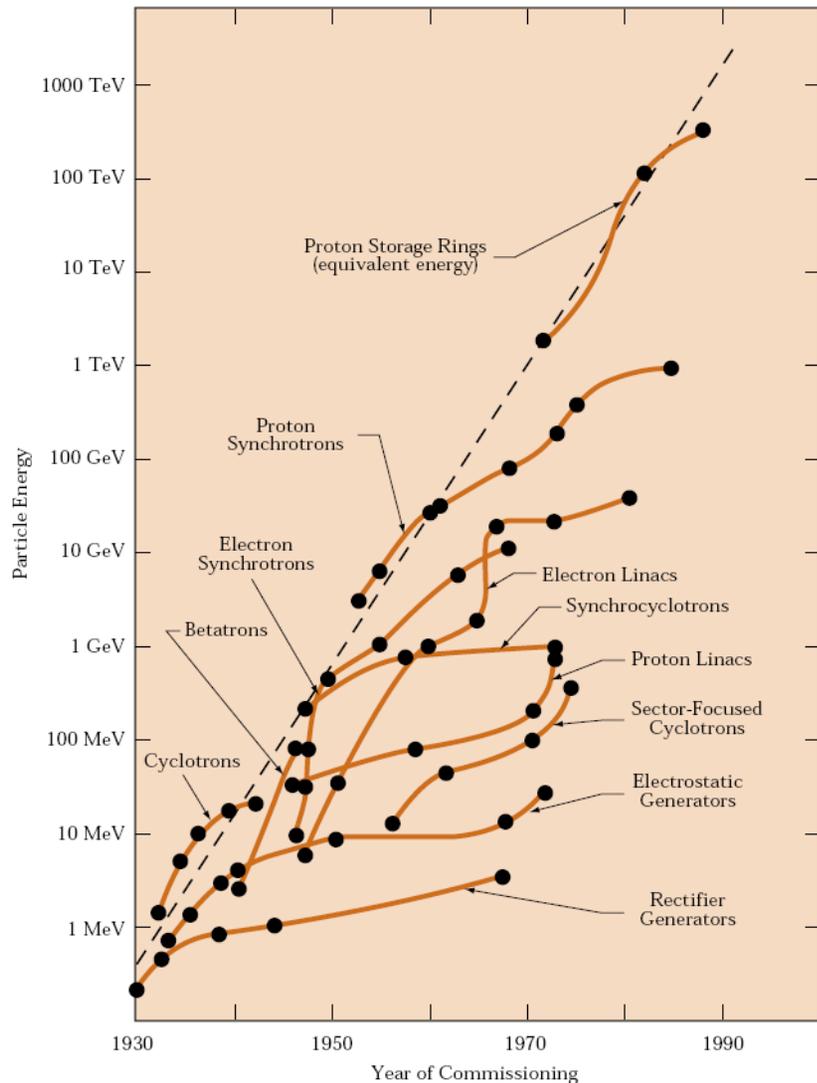
1917, theory of stimulated radiation by Einstein

1960, flash-lamp pumped ruby, Dr. Mainman

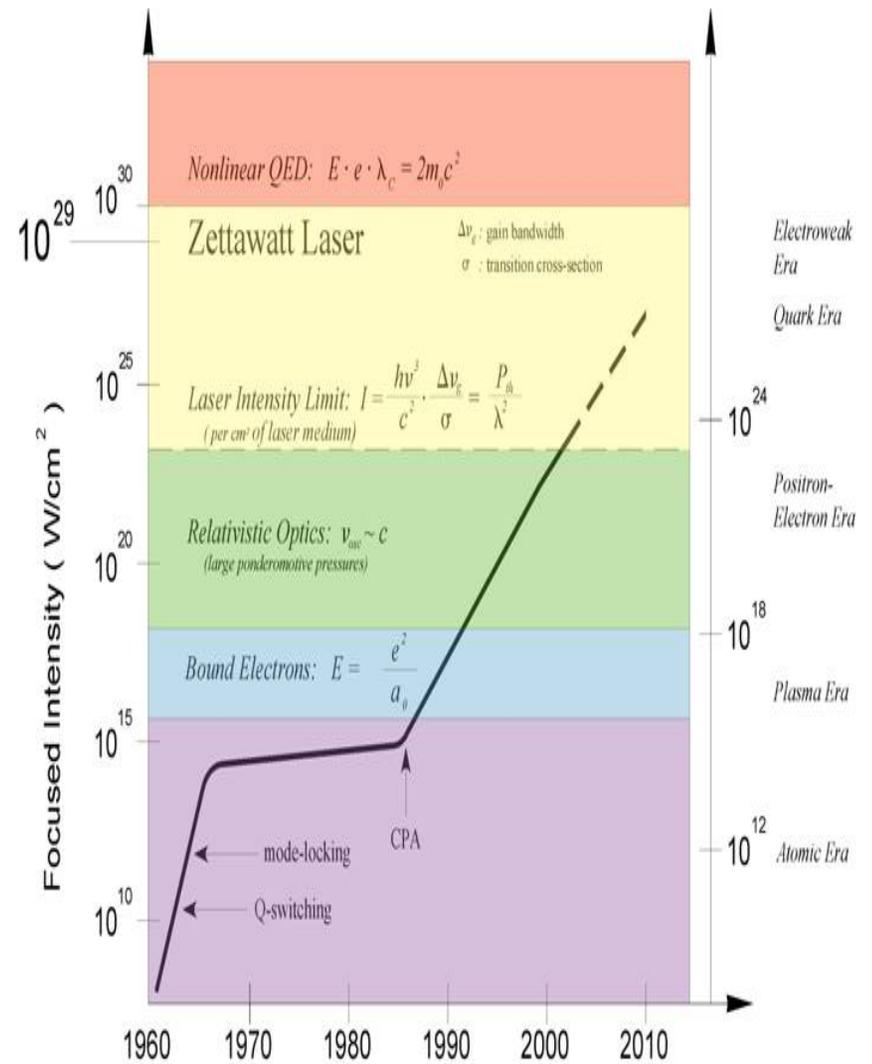
1964, Nobel Prize, Towne, Basov, and Prokhorov

Today's lasers and accelerators

Aiming for higher energy/intensity and better control



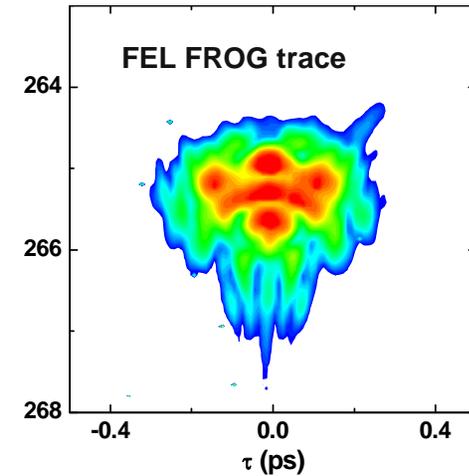
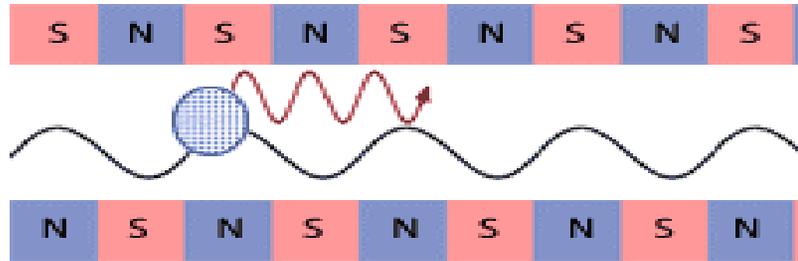
(Panofsky, <http://www.slac.stanford.edu/pubs/beamline>)



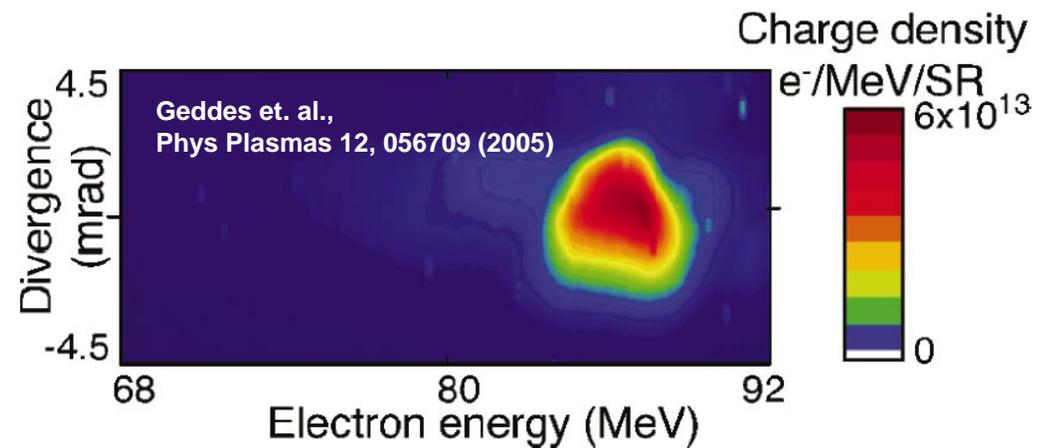
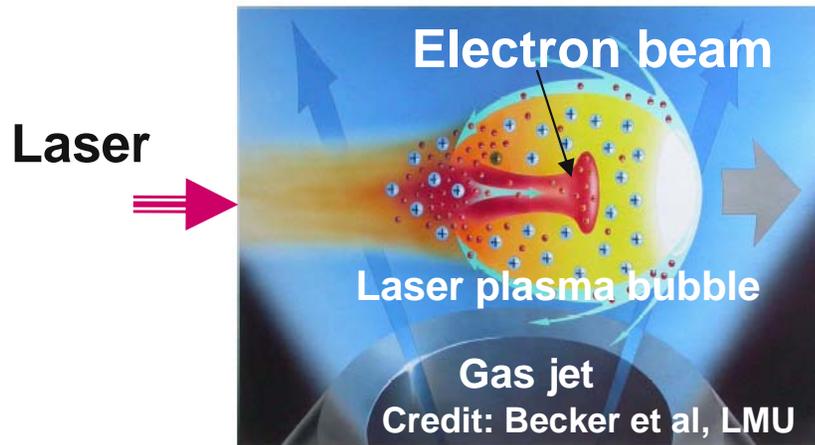
(<http://www.eecs.umich.edu/CUOS>)

Newest developments

■ Accelerator-generated lasers: a free-electron laser



■ Laser as an accelerator: a compact accelerator

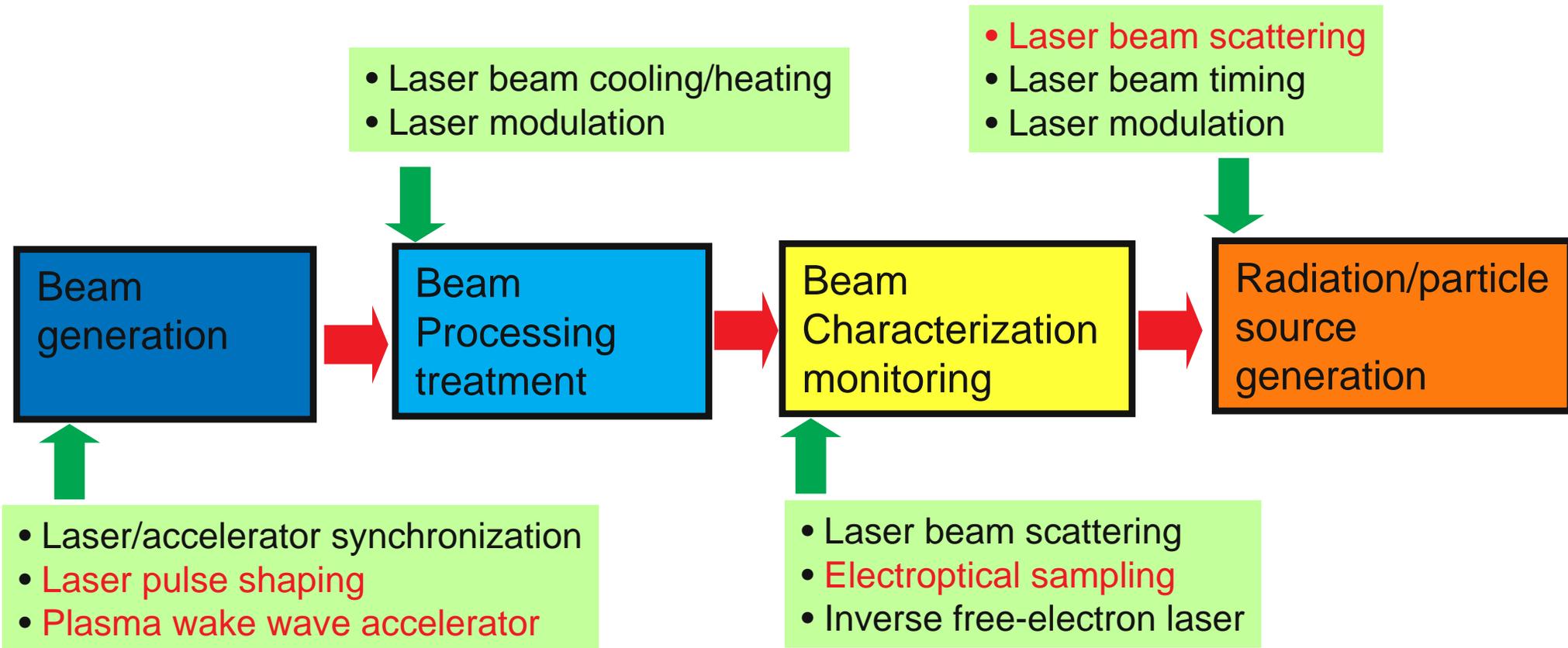


S.P.D. Mangles et al. Nature 431, 535 (2004); C.G.R. Geddes et al. Nature 431, 538 (2004); J. Faure et al. Nature 431, 541 (2004)

Content

- Laser and accelerator history
- Map of laser application in accelerators
- Laser basics
 - Rate equations
 - Laser configurations
 - Gaussian beam optics and ABCD law
 - Laser cavity and laser modes
- Laser configurations
 - Mode-locking and q-switch
 - MOPA
 - CPA and dispersion
- Laser materials
- Other lasers
 - Semiconductor lasers
 - Fiber lasers
- Frequency conversion and short wavelength lasers

A map for laser applications in accelerators



Content

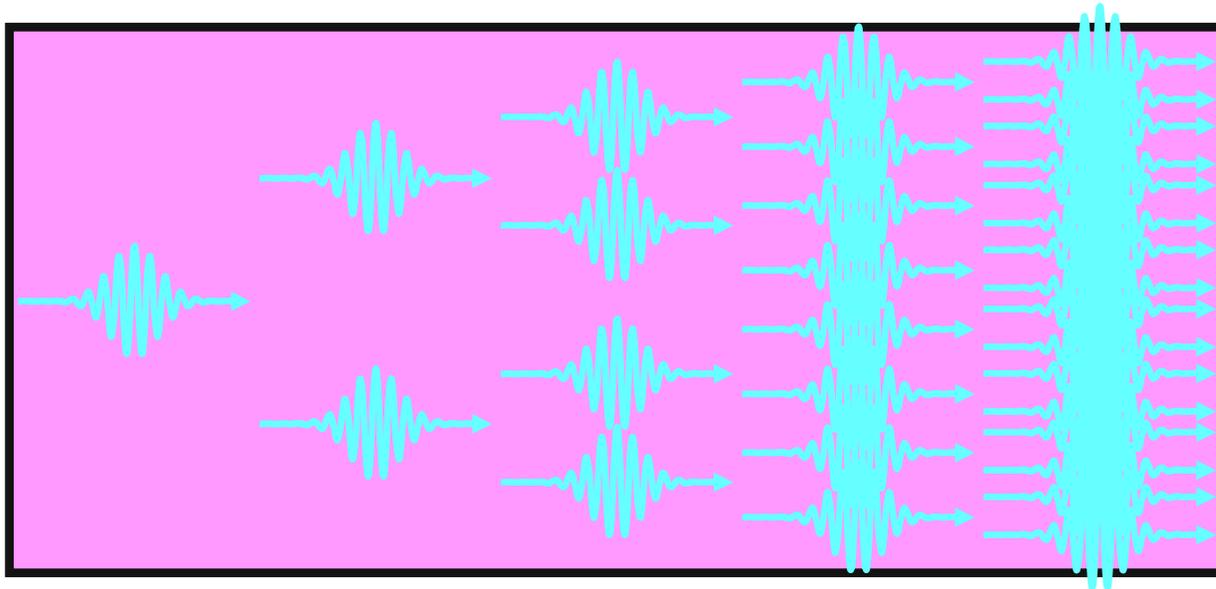
- Laser and accelerator history
- Map of laser application in accelerators
- Laser basics
 - Rate equations
 - Laser configurations
 - Gaussian beam optics and ABCD law
 - Laser cavity and laser modes
- Laser configurations
 - Mode-locking and Q-switch
 - MOPA
 - CPA and dispersion
- Laser materials
- Other lasers
 - Semiconductor lasers
 - Fiber lasers
- Frequency conversion and short wavelength lasers

Laser

Light Amplification by Stimulated Emission of Radiation

If a medium has many excited molecules, one photon can become many.

Excited medium

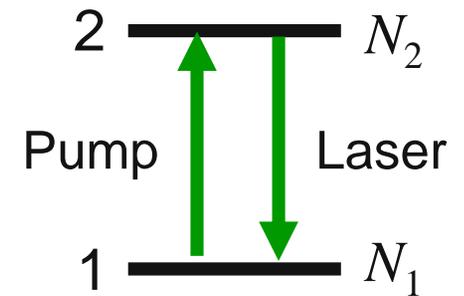


This is the essence of the laser. The factor by which an input beam is amplified by a medium is called the **gain** and is represented by G .

Credit: R. Trebino

USPAS, 2008

Rate equations for a two-level system



Rate equations for the densities of the two states:

$$\frac{dN_2}{dt} = \underset{\substack{\text{Absorption} \\ \uparrow \\ \text{Pump intensity}}}{BI(N_1 - N_2)} - \underset{\substack{\text{Stimulated emission} \\ \downarrow}}{AN_2} - \underset{\substack{\text{Spontaneous} \\ \text{emission}}}{AN_2}$$

$$\frac{dN_1}{dt} = BI(N_2 - N_1) + AN_2$$

$$\Rightarrow \frac{d\Delta N}{dt} = -2BI\Delta N + 2AN_2 \quad \leftarrow \begin{aligned} 2N_2 &= (N_1 + N_2) - (N_1 - N_2) \\ &= N - \Delta N \end{aligned}$$

$$\Rightarrow \frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

If the total number of molecules is N :

$$N \equiv N_1 + N_2$$

$$\Delta N \equiv N_1 - N_2$$

Credit: R. Trebino

Why inversion is impossible in a two-level system

$$\frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -2BI\Delta N + AN - A\Delta N$

$$\Rightarrow (A + 2BI)\Delta N = AN$$

$$\Rightarrow \Delta N = AN / (A + 2BI)$$

$$\Rightarrow \Delta N = N / (1 + 2BI / A)$$

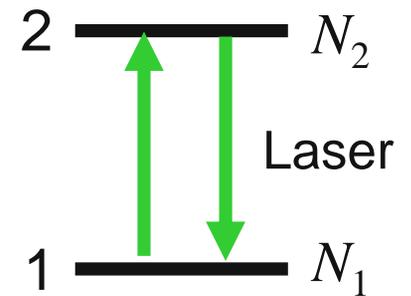
$$\Rightarrow \Delta N = \frac{N}{1 + I / I_{sat}}$$

where: $I_{sat} = A / 2B$

I_{sat} is the **saturation intensity**.

ΔN is **always** positive, no matter how high I is!

It's impossible to achieve an inversion in a two-level system!

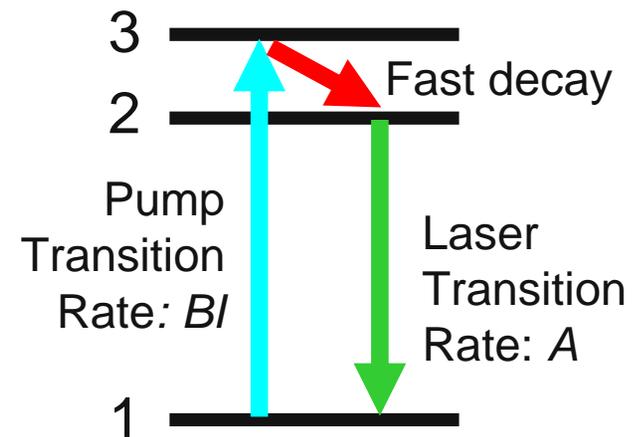


Credit: R. Trebino

USPAS, 2008

A 3-level system

Assume we pump to a state 3 that rapidly decays to level 2.



$$\frac{dN_2}{dt} = BIN_1 - AN_2$$

Spontaneous emission

$$\frac{dN_1}{dt} = -BIN_1 + AN_2$$

Absorption

The total number of molecules is N :

$$N \equiv N_1 + N_2$$

$$\Delta N \equiv N_1 - N_2$$

Level 3 decays fast and so is zero.

$$\frac{d\Delta N}{dt} = -2BIN_1 + 2AN_2$$

$$2N_2 = N - \Delta N$$

$$2N_1 = N + \Delta N$$

$$\Rightarrow \frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N$$

Credit: R. Trebino

Population inversion, 3 level system

$$\frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -BIN - BI\Delta N + AN - A\Delta N$

$$\Rightarrow (A + BI)\Delta N = (A - BI)N$$

$$\Rightarrow \Delta N = N(A - BI)/(A + BI)$$

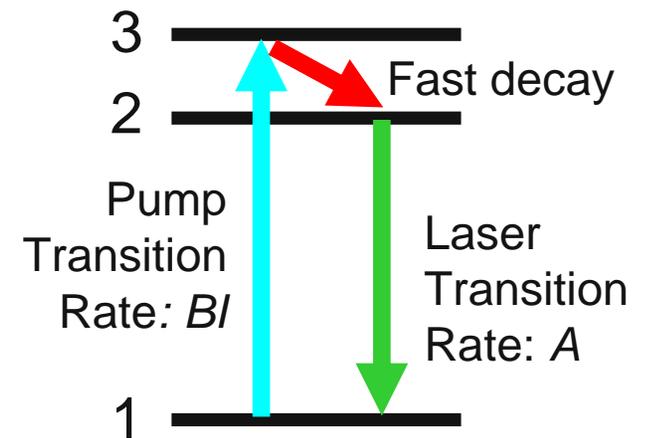
$$\Rightarrow \Delta N = N \frac{1 - I/I_{sat}}{1 + I/I_{sat}}$$

where: $I_{sat} = A/B$

I_{sat} is the **saturation intensity**.

Now if $I > I_{sat}$, ΔN is negative!

$$\text{Gain: } g \propto -\sigma\Delta N$$



Credit: R. Trebino

Rate equations for a four-level system

Now assume the lower laser level 1 also rapidly decays to a ground level 0.

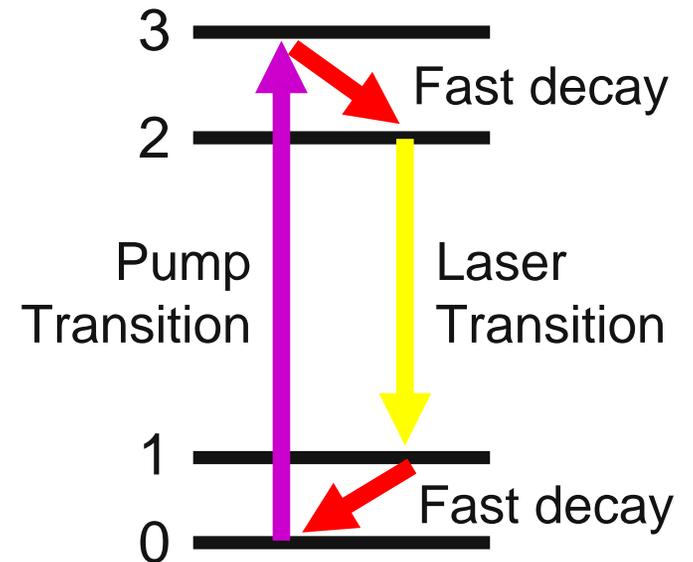
As before:
$$\frac{dN_2}{dt} = BIN_0 - AN_2$$

$$\frac{dN_2}{dt} = BI(N - N_2) - AN_2$$

Because $N_1 \approx 0$, $\Delta N \approx -N_2$

$$-\frac{d\Delta N}{dt} = BIN + BI\Delta N + A\Delta N$$

At steady state: $0 = BIN + BI\Delta N + A\Delta N$



The total number of molecules is N :

$$N \equiv N_0 + N_2$$

$$N_0 = N - N_2$$

Credit: R. Trebino

Population inversion in a four-level system (cont'd)

$$0 = BIN + BI\Delta N + A\Delta N$$

$$\Rightarrow (A + BI)\Delta N = -BIN$$

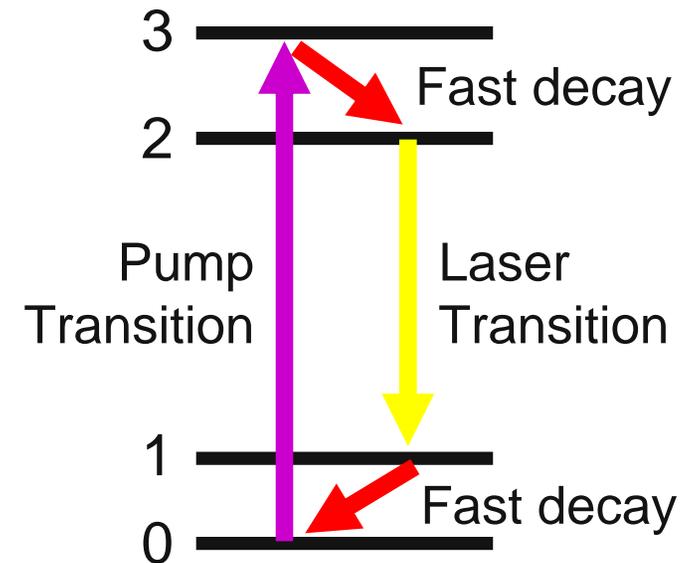
$$\Rightarrow \Delta N = -BIN / (A + BI)$$

$$\Rightarrow \Delta N = -(BIN / A) / (1 + BI / A)$$

$$\Rightarrow \Delta N = -N \frac{I / I_{sat}}{1 + I / I_{sat}} \quad \text{where: } I_{sat} = A / B$$

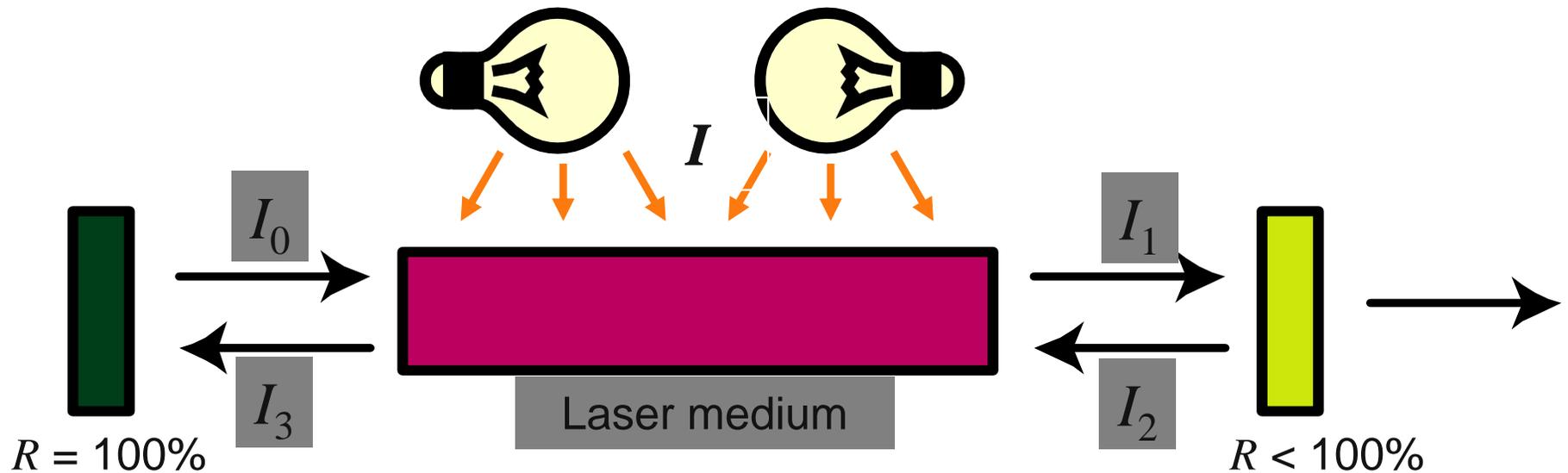
I_{sat} is the **saturation intensity**.

Now, ΔN is negative—always!



Credit: R. Trebino

How to build a laser



- Laser medium
 - Depends on wavelength, pulse duration, power
- Pump it: ASE
 - Multimode in time and space
- Add resonator: laser oscillation
 - Mode selection

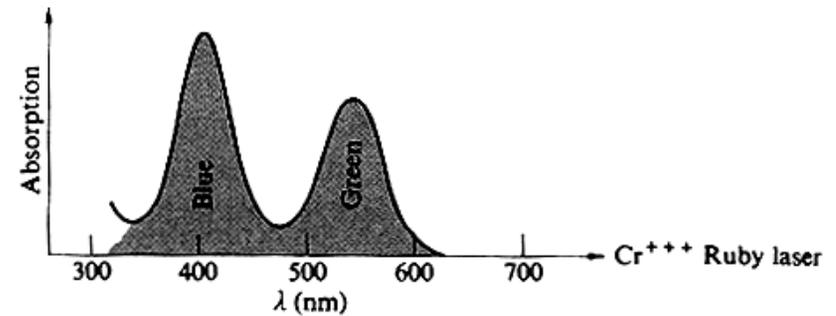
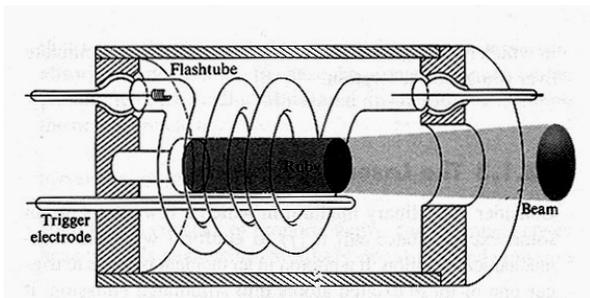
Credit: R. Trebino

USPAS, 2008

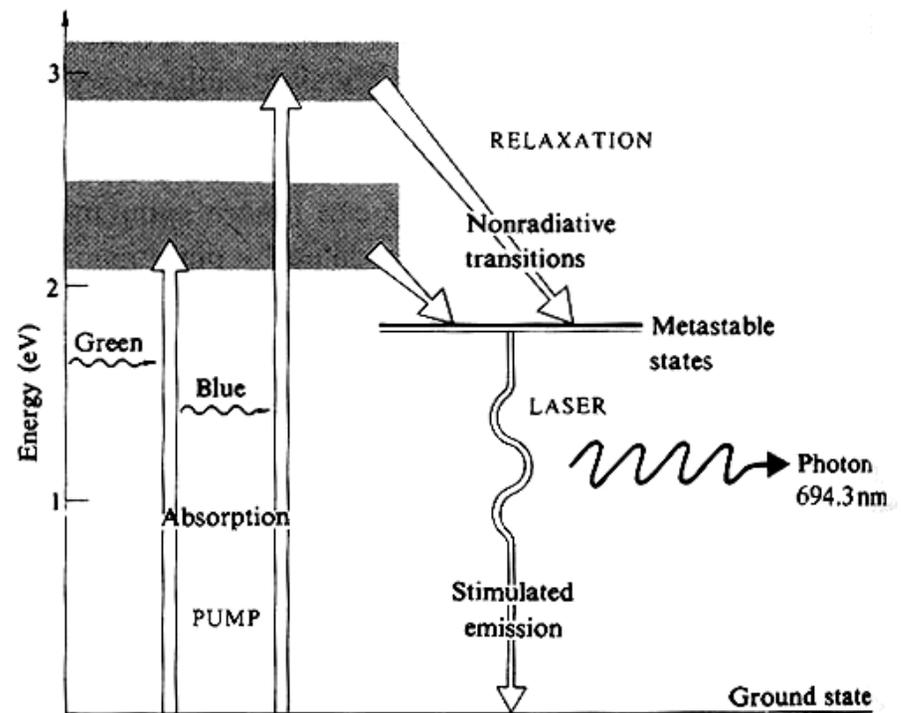
The historic ruby laser

Invented in 1960 by Ted Maiman at Hughes Research Labs, it was the first laser.

Ruby is a three-level system, so you have to hit it hard.



(a)



(b)

Credit: R. Trebino

USPAS, 2008

Helmholtz Equation: Gaussian beam optics

Wave equation

$$\left(\nabla^2 - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} \right) E(x, y, z, t) = 0$$

$$E(x, y, z, t) = A(x, y, z) e^{i(k_z z - \omega t)}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2} + \cancel{\frac{\partial^2}{\partial z^2}} \right) A - \cancel{k_z^2} A + i2k_z \frac{\partial}{\partial z} A + \cancel{\frac{n^2 \omega^2}{c^2}} A = 0$$

Paraxial condition

$$\frac{\partial^2}{\partial z^2} A \ll \frac{\partial}{\partial z} A, \quad k_z^2 = \frac{n^2 \omega^2}{c^2} = n \frac{2\pi}{\lambda}$$

Paraxial Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) A + i2k_z \frac{\partial}{\partial z} A = 0$$

Gaussian beam optics

Paraxial Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) A + i2k_z \frac{\partial}{\partial z} A = 0$$

Fourier transform

$$a(k_x, k_y, z) = \iint A(x, y, z) e^{-i(k_x x + k_y y)} dx dy$$

$$\rightarrow A(x, y, z) = \frac{1}{4\pi^2} \iint a(k_x, k_y, z) e^{i(k_x x + k_y y)} dk_x dk_y$$

Thus $\frac{1}{4\pi^2} \iint \left[(-k_x^2 - k_y^2) a + i2k_z \frac{\partial}{\partial z} a \right] e^{i(k_x x + k_y y)} dk_x dk_y = 0$

$$\rightarrow \frac{\partial}{\partial z} a = -i \frac{k_x^2 + k_y^2}{2k_z} a$$

$$\rightarrow a(k_x, k_y, z) = a_0(k_x, k_y, 0) e^{-i \frac{k_x^2 + k_y^2}{2k_z} z}$$

Gaussian beam optics

Let

$$A(x, y, 0) = A_0 e^{-\frac{x^2+y^2}{w_0^2}} = A_0 e^{-\frac{r^2}{w_0^2}}.$$

Fourier transform

$$\begin{aligned} a(k_x, k_y, 0) &= \iint A_0 e^{-\frac{x^2+y^2}{w_0^2}} e^{-i(k_x x + k_y y)} dk_x dk_y \\ &= \pi w_0^2 A_0 e^{-\frac{k_x^2+k_y^2}{4} w_0^2} \end{aligned}$$

Thus

$$\begin{aligned} a(k_x, k_y, z) &= a_0(k_x, k_y, 0) e^{-i\frac{k_x^2+k_y^2}{2k_z} z} \\ &= \pi w_0^2 A_0 e^{-\frac{k_x^2+k_y^2}{4} w_0^2} e^{-i\frac{k_x^2+k_y^2}{2k_z} z} \\ &= \pi w_0^2 A_0 e^{-\frac{w_0^2}{4}(k_x^2+k_y^2) \left(1+i\frac{z}{z_0}\right)} \end{aligned}$$

Rayleigh length (diffraction length)

$$z_0 = \frac{k_z w_0^2}{2} = \pi \frac{w_0^2}{\lambda}$$

Gaussian beam optics

Inverse Fourier transform

$$\begin{aligned}
 A(x, y, z) &= \frac{1}{4\pi^2} \iint a(k_x, k_y, z) e^{i(k_x x + k_y y)} dk_x dk_y \\
 &= \frac{1}{4\pi^2} \iint \pi w_0^2 A_0 e^{-\frac{w_0^2}{4}(k_x^2 + k_y^2) \left(1 + i\frac{z}{z_0}\right)} e^{i(k_x x + k_y y)} dk_x dk_y \\
 &= \frac{1}{4\pi} w_0^2 A_0 \iint e^{-\frac{(w_0 \sqrt{1 + iz/z_0})^2}{4}(k_x^2 + k_y^2)} e^{i(k_x x + k_y y)} dk_x dk_y \\
 &= \frac{A_0}{1 + iz/z_0} e^{-\frac{x^2 + y^2}{w_0^2 (1 + iz/z_0)}} = \frac{A_0}{\sqrt{1 + z^2/z_0^2}} e^{-i \tan^{-1} \frac{z}{z_0}} e^{-\frac{x^2 + y^2}{w(z)^2} \left(1 - i\frac{z}{z_0}\right)} \\
 &= \frac{A_0 w_0}{w(z)} e^{-\frac{x^2 + y^2}{w(z)^2} \left(1 - i\frac{z}{z_0}\right) - i\eta} = \frac{A_0 w_0}{w(z)} e^{-\frac{x^2 + y^2}{w(z)^2}} e^{i k \frac{x^2 + y^2}{2R(z)} - i\eta}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{1 + iz/z_0} &= \frac{1}{1 + z^2/z_0^2} \left(1 - i\frac{z}{z_0}\right) \\
 &= \frac{1}{\sqrt{1 + z^2/z_0^2}} e^{-i \tan^{-1} \frac{z}{z_0}}
 \end{aligned}$$

$$w(z) = w_0 \left(1 + \frac{z^2}{z_0^2}\right)^{1/2} \quad R(z) = z \left(1 + \frac{z_0^2}{z^2}\right) \quad \eta = \tan^{-1} \frac{z}{z_0}$$

↑
Beam radius

↑
Wave front radius of curvature

Gaussian optics: summary

- Gaussian distribution is the solution of paraxial Helmholtz equation
- TM00 mode

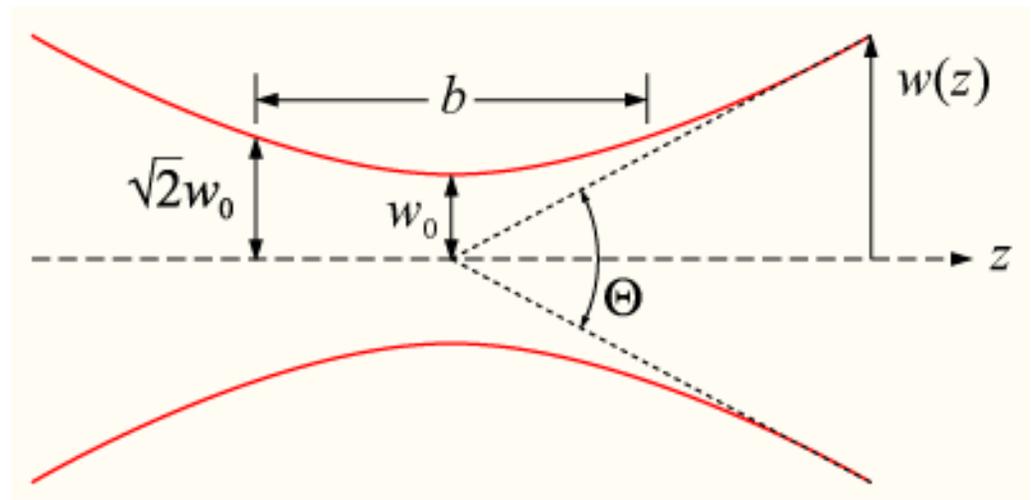
$$E(r, z) = E_0 \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w^2(z)}\right),$$

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2},$$

$$z_0 = \frac{\pi w_0^2}{\lambda},$$

$$w_0 = \frac{2\lambda}{\pi\Theta},$$

$$b = 2z_0.$$



w_0 : beam waist
 z_0 : Rayleigh range
 b : confocal parameter

Beam propagation: ABCD matrices

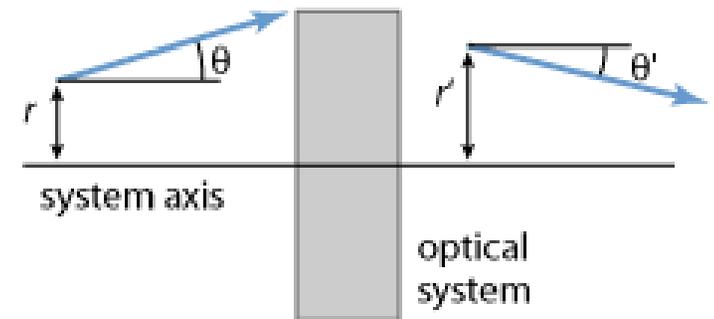
$$\begin{pmatrix} r' \\ \theta' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r \\ \theta \end{pmatrix}$$

For a thin lens

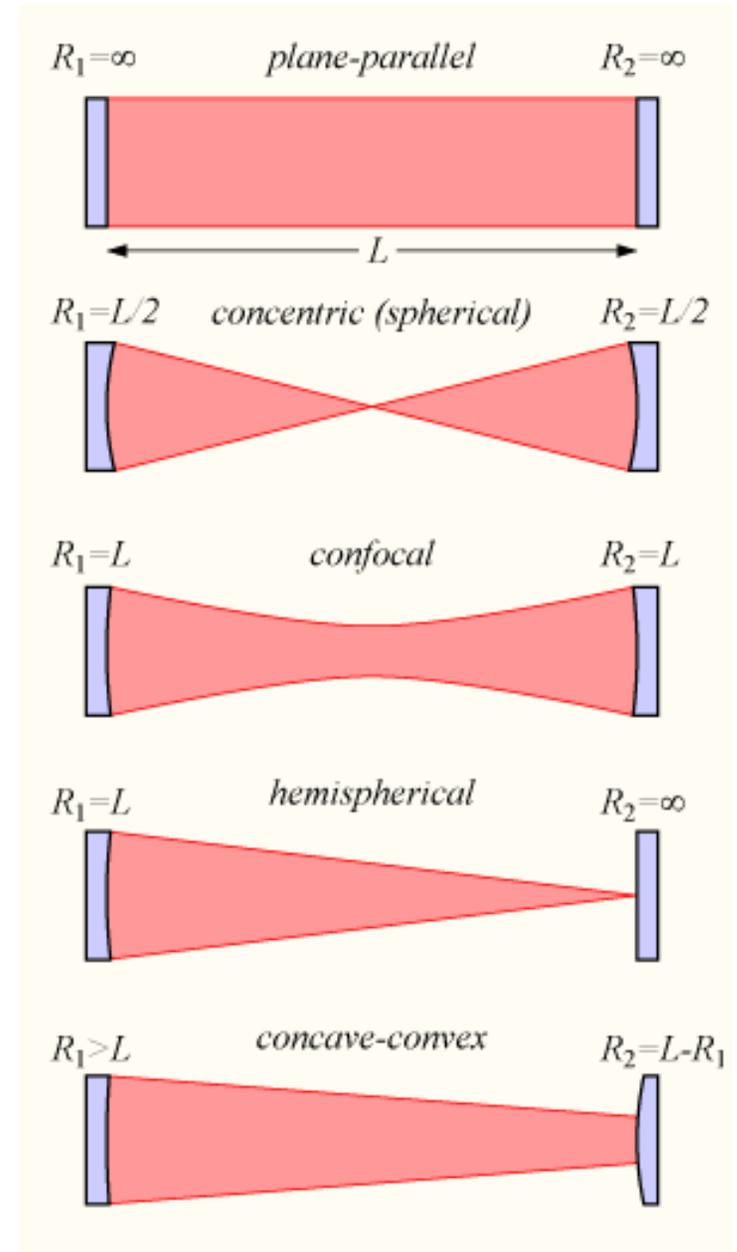
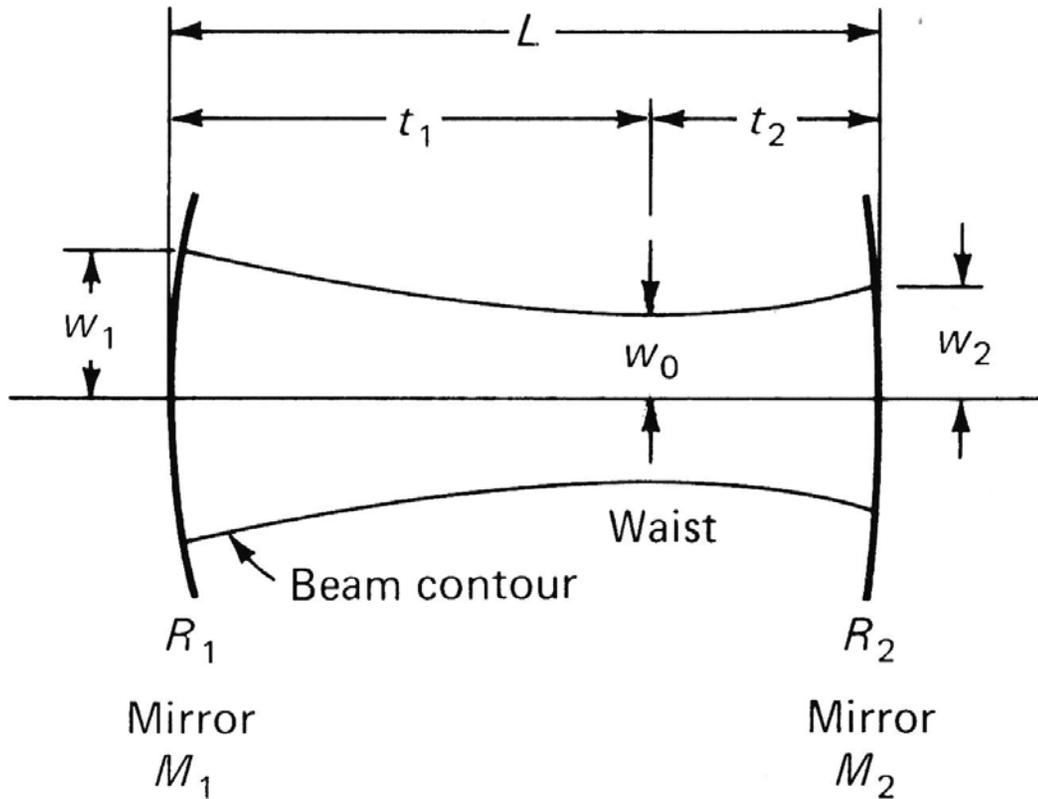
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

For free space (drift space)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

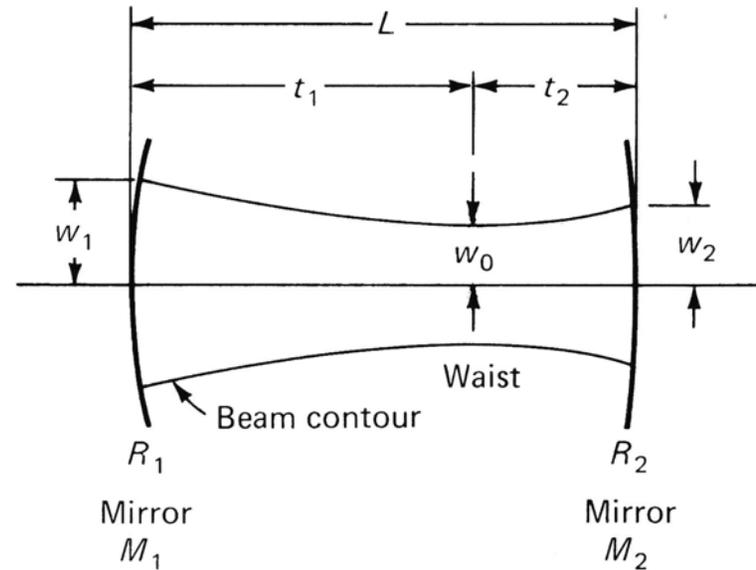
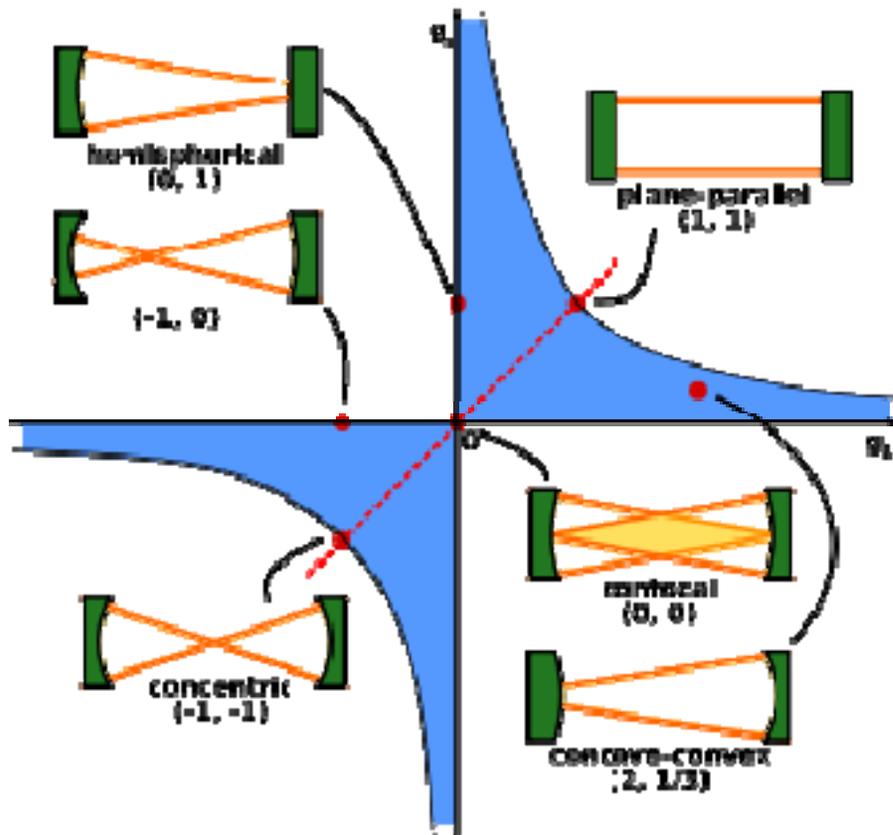


Cavity: Optical resonators



Credit: W. Koechner: Solid State Laser engineering,
Credit: Wikipedia

Stability of laser resonators



$$0 < \left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right) < 2,$$

$$g_1 = 1 - \frac{L}{R_1},$$

$$g_2 = 1 - \frac{L}{R_2}.$$

Credit: W. Koechner: Solid State Laser engineering,
Credit: Wikipedia

High order modes

$$E(x, y, z) = E_0 \frac{w_0^2}{w^2(z)} H_n \left(\frac{\sqrt{2}x}{w(z)} \right) H_m \left(\frac{\sqrt{2}y}{w(z)} \right) e^{-\frac{x^2+y^2}{w^2(z)}} e^{-i \left[k \frac{x^2+y^2}{2R(z)} - (1+n+m)\eta(z) \right]}$$

Hermite polynomials

$$H_0(x) = 1$$

$$H_1(x) = 2x$$

$$H_2(x) = 4x^2 - 2$$

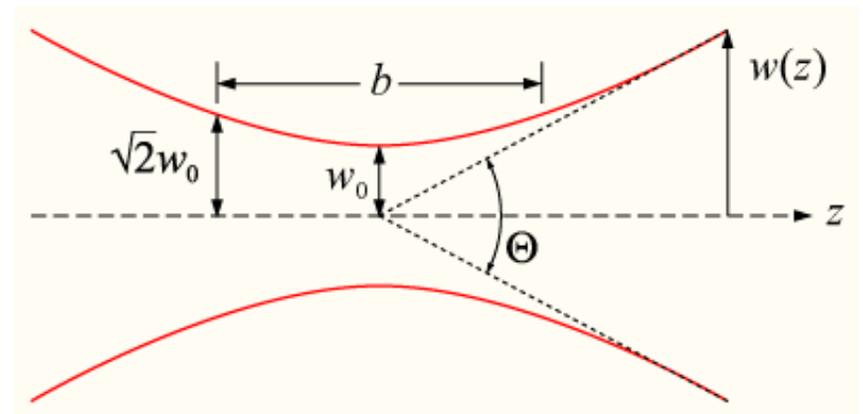
$$H_3(x) = 8x^3 - 12x$$

...

The M^2 factor

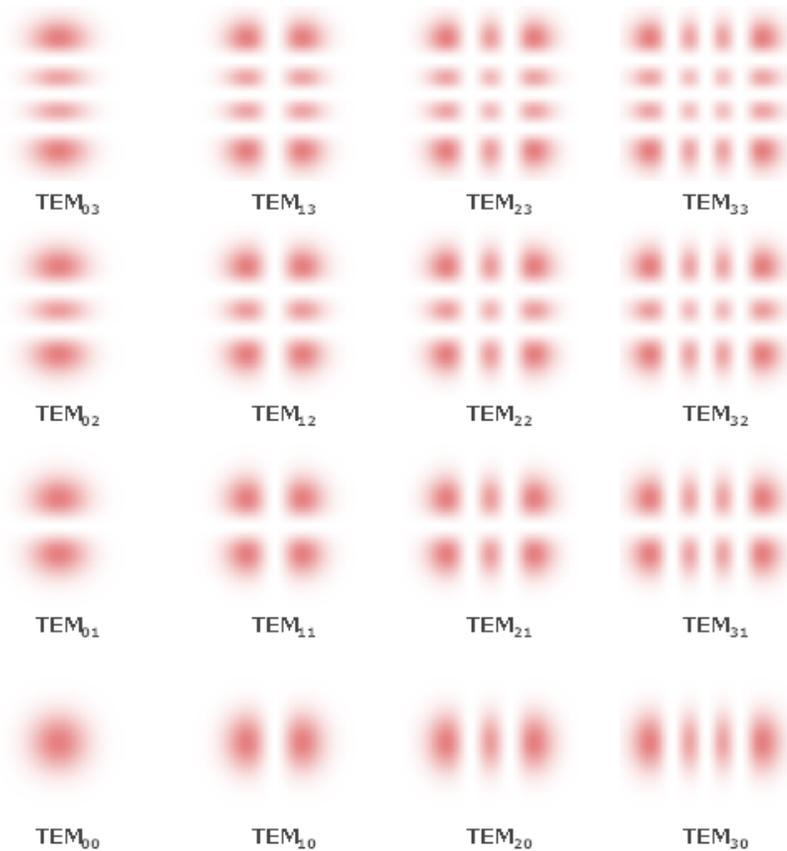
$$\theta = \frac{\Theta}{2} = M^2 \frac{\lambda}{\pi w_0}$$

$M^2=1$ =diffraction limited; $M^2 > 1$, M^2 times diffractions limited

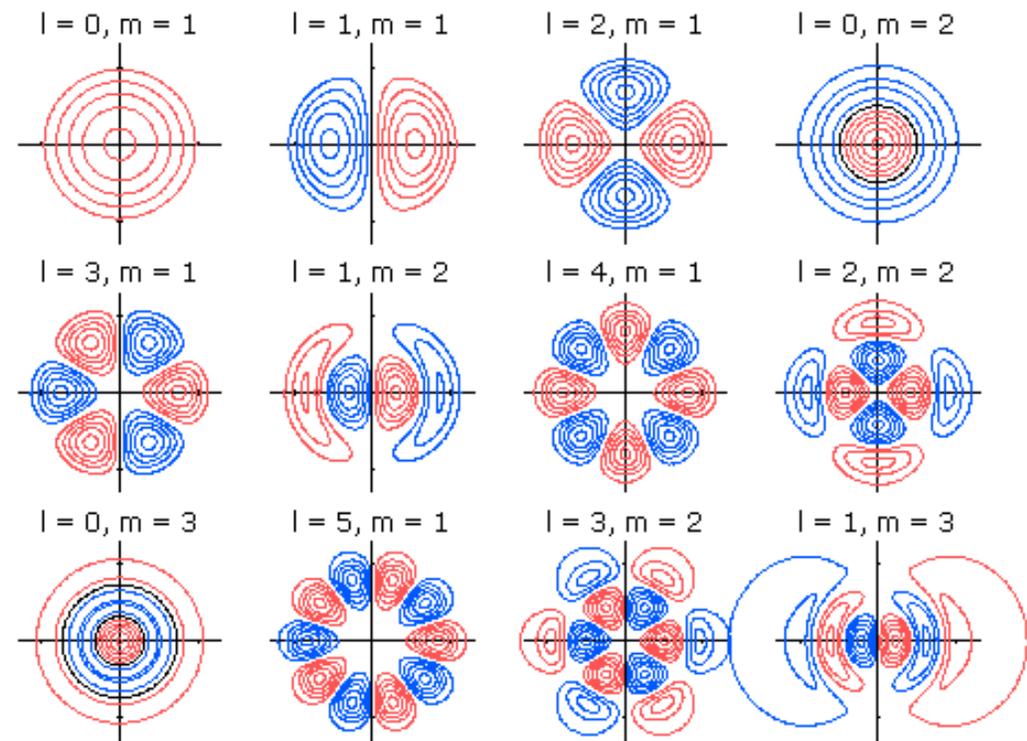


High order modes

Hermite-Gaussian modes Intensity profile

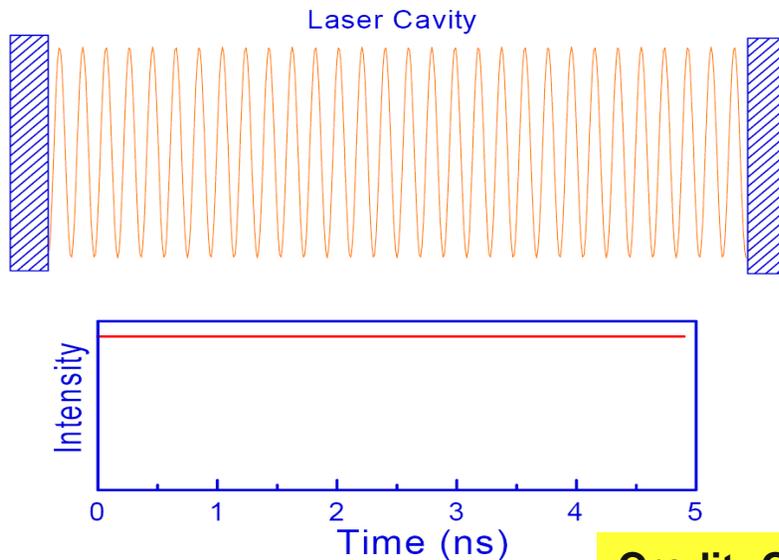


Guided modes in fibers Field

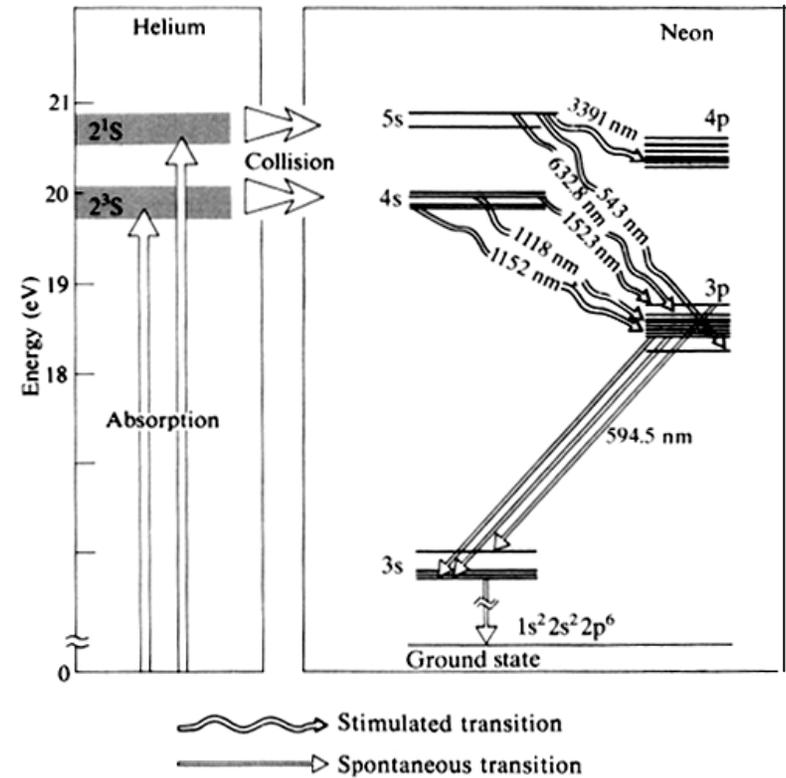


Encyclopedia of Laser Physics and Technology
http://www.rp-photonics.com/higher_order_modes.html

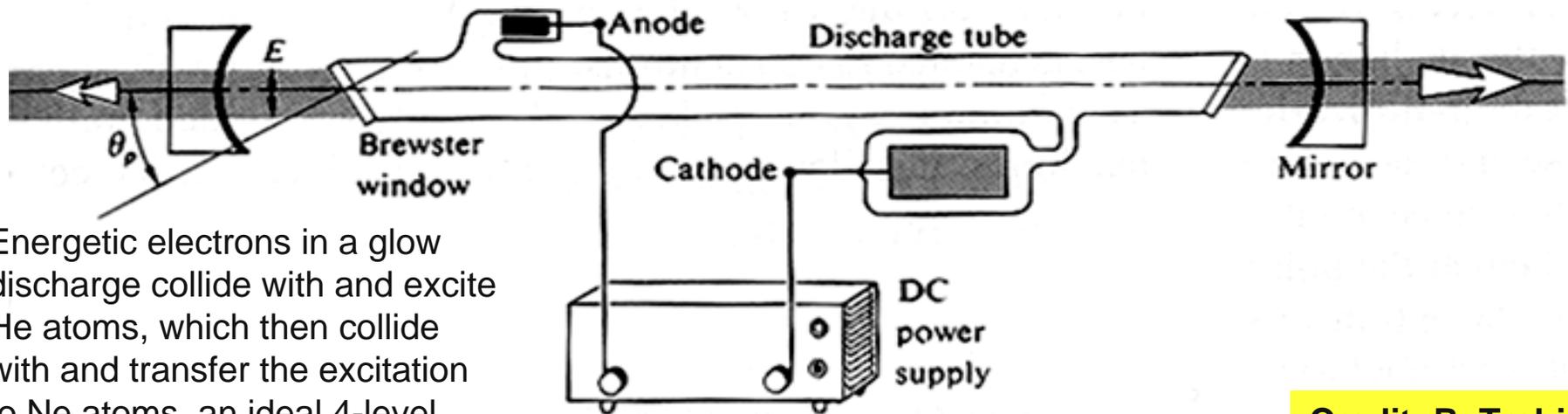
A cavity and laser oscillator



Credit: Cundiff, UCB



Resonant condition for cavity: $L=n\lambda$



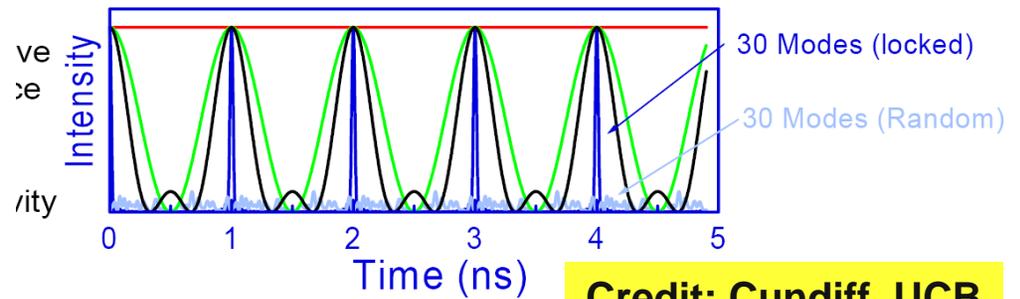
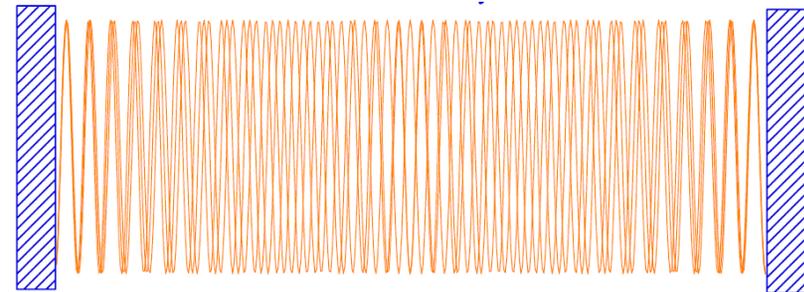
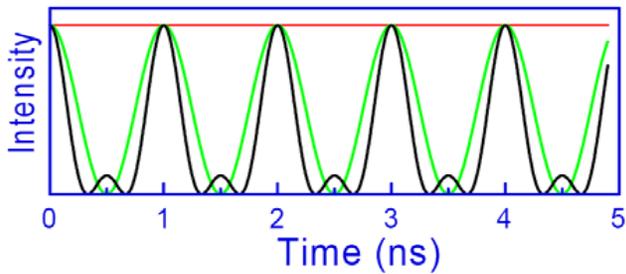
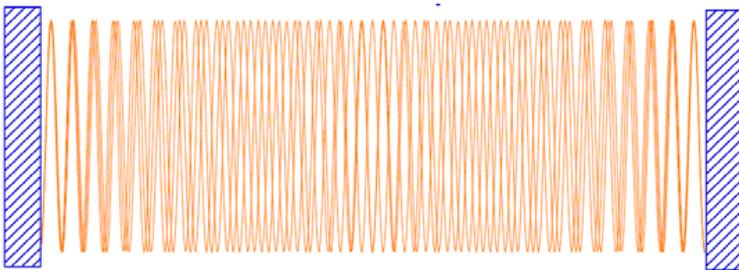
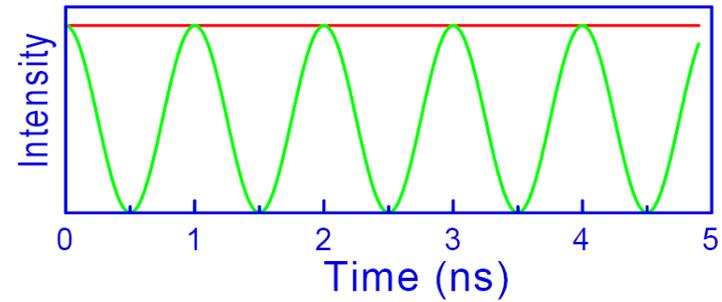
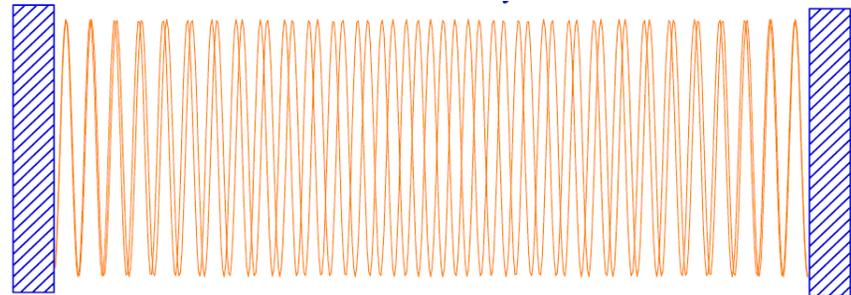
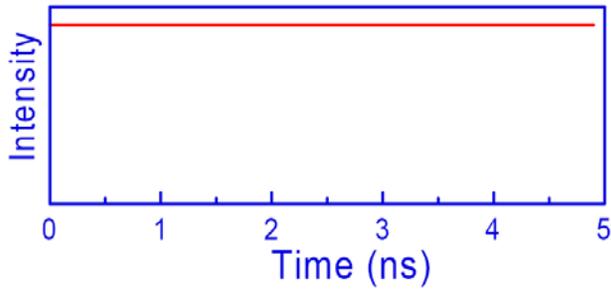
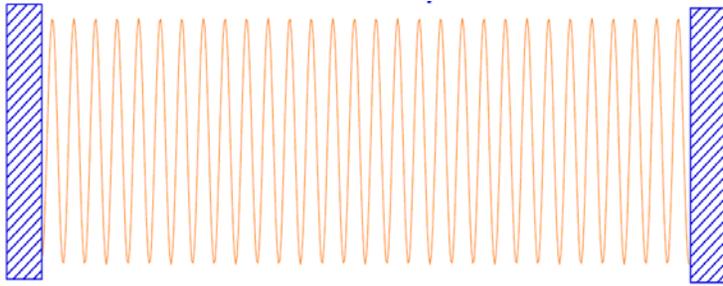
Energetic electrons in a glow discharge collide with and excite He atoms, which then collide with and transfer the excitation to Ne atoms, an ideal 4-level system.

Credit: R. Trebino

Content

- Laser and accelerator history
- Map of laser application in accelerators
- Laser basics
 - Rate equations
 - Laser configurations
 - Gaussian beam optics and ABCD law
 - Laser cavity and laser modes
- Laser configurations
 - Mode-locking and Q-switch
 - MOPA
 - CPA and dispersion
- Laser materials
- Other lasers
 - Semiconductor lasers
 - Fiber lasers
- Frequency conversion and short wavelength lasers

Mode locking: what



Credit: Cundiff, UCB

USPAS, 2008

Mode locking: how

Introduce amplitude or phase modulation/control

Active mode locking

- Acousto-optic modulator, drive with RF

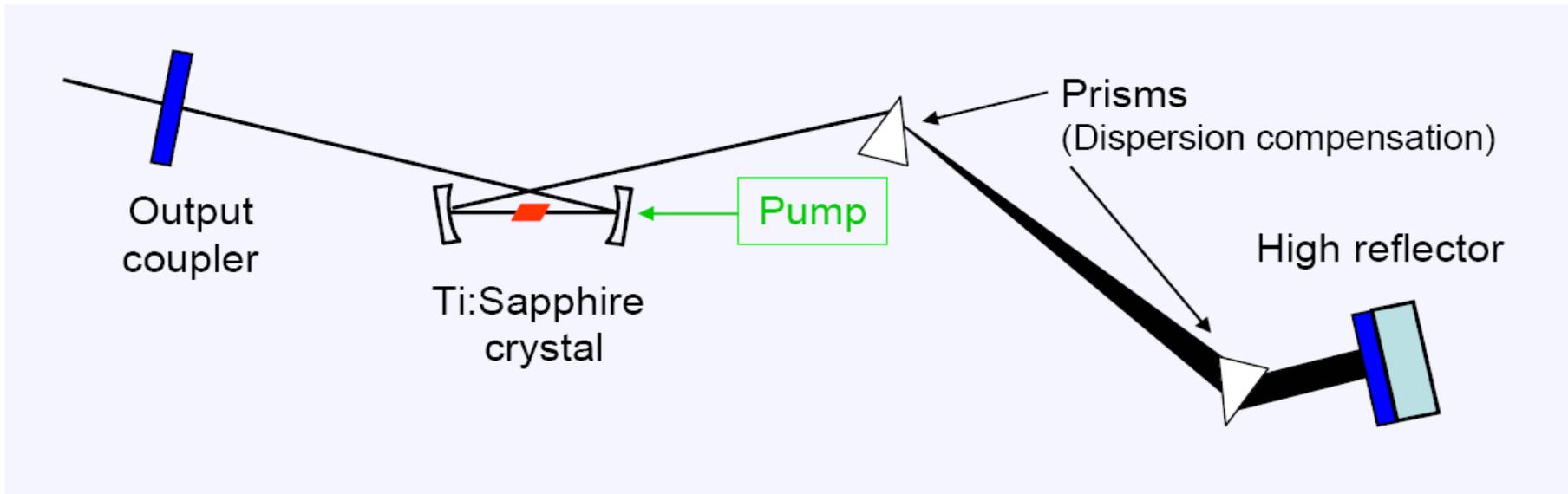
Passive mode locking

- Saturable absorption
- Nonlinear lensing + aperture
- Nonlinear polarization rotation + polarizer

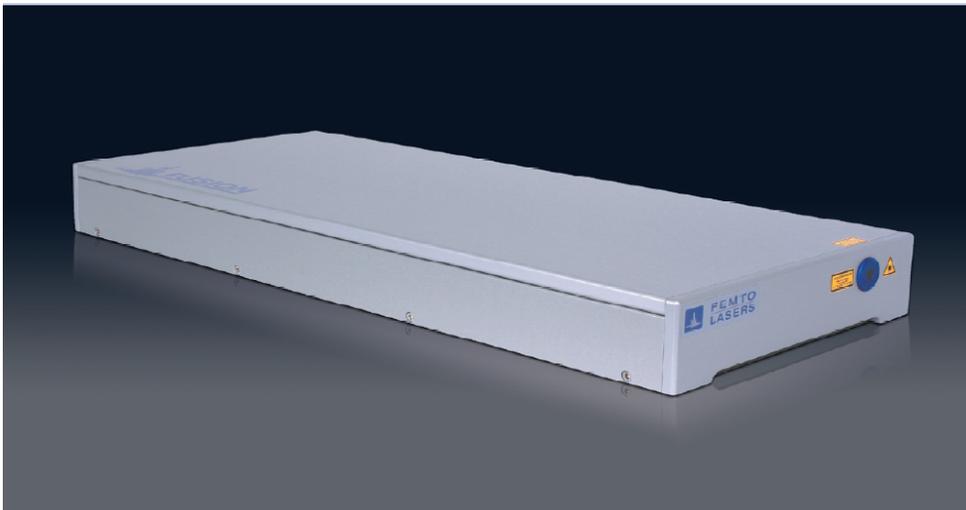
Mode locking: result

- Shorter pulse, high intensity, larger bandwidth
- Single mode
- Accurate timing at round trip time

Ti: Sapphire oscillator: an example



M.T. Asaki, et al, Opt. Lett. 18, 977 (1993)



Femtolasers: Fusion (28"x12"x3")

Pulse duration	< 10 fs
Bandwidth (FWHM) @ 800 nm	> 100 nm
Mode locked output power (av.)	150 - 500 mW
Output energy @ 75 MHz	2 - 6.5 nJ
Peak power @ 75 MHz	200 - 650 kW

Q-switch

Q factor of a resonator

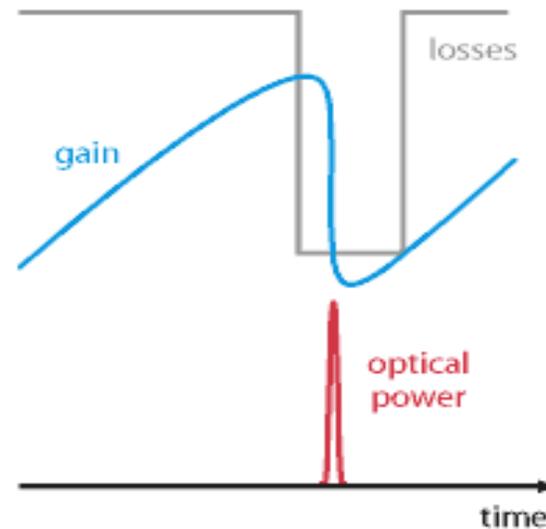
$$Q = \nu T \frac{2\pi}{L}$$

T: round trip time; ν : optical frequency;
L: fraction power loss per round trip

Q switch: reducing the loss

Active: acousto-optical, electro-optical, and mechanical

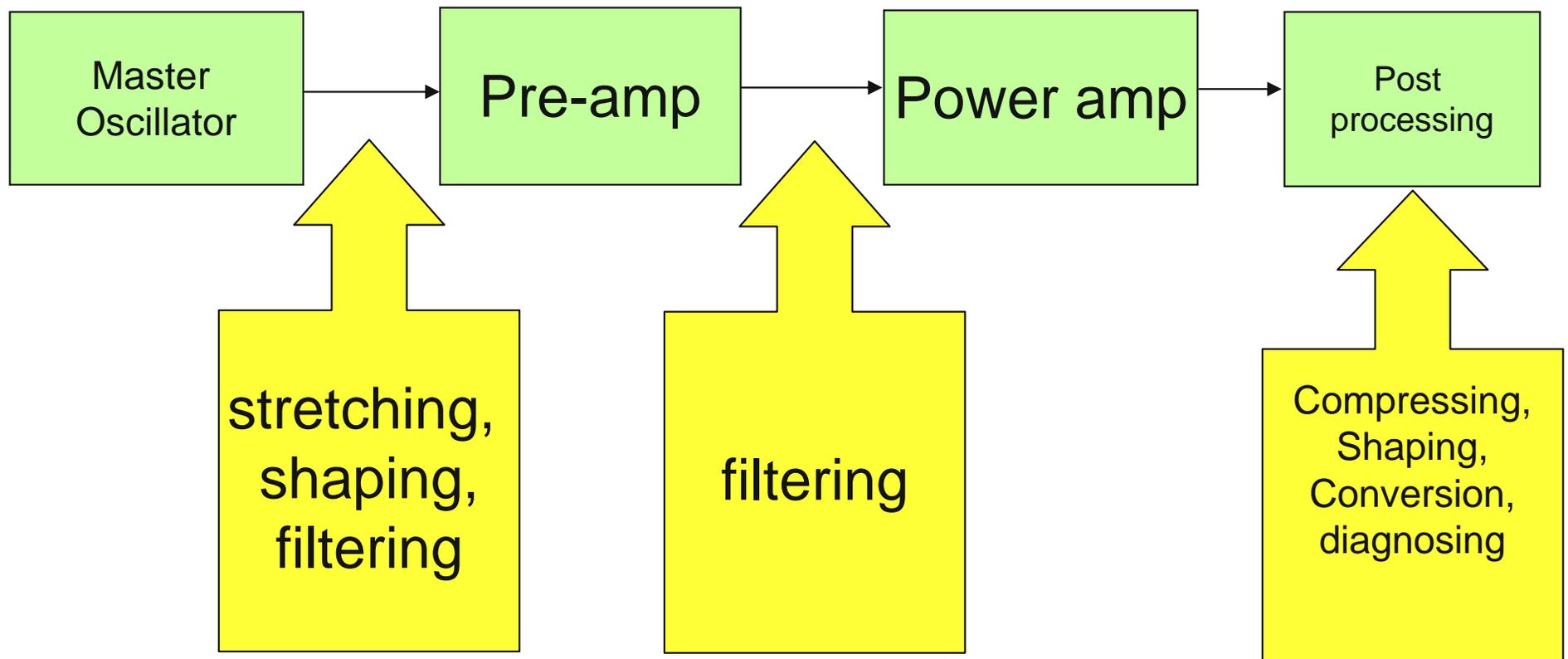
Passive: saturable absorbers



A MOPA system

Master Oscillator – Power Amplifier

- An oscillator usually does not have enough energy, thus needs amplification
- A MOPA is expected to carry over the characteristics of an OSC
- Pulse duration is limited at 10 ps due to damage



A MOPA example: Flash drive laser

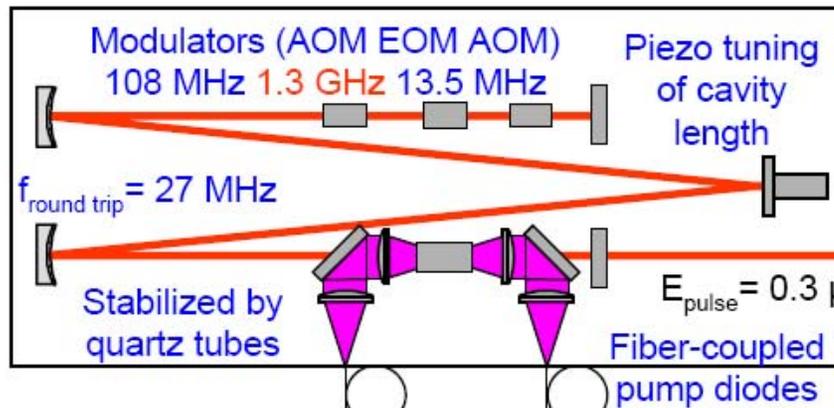


Laser System Overview

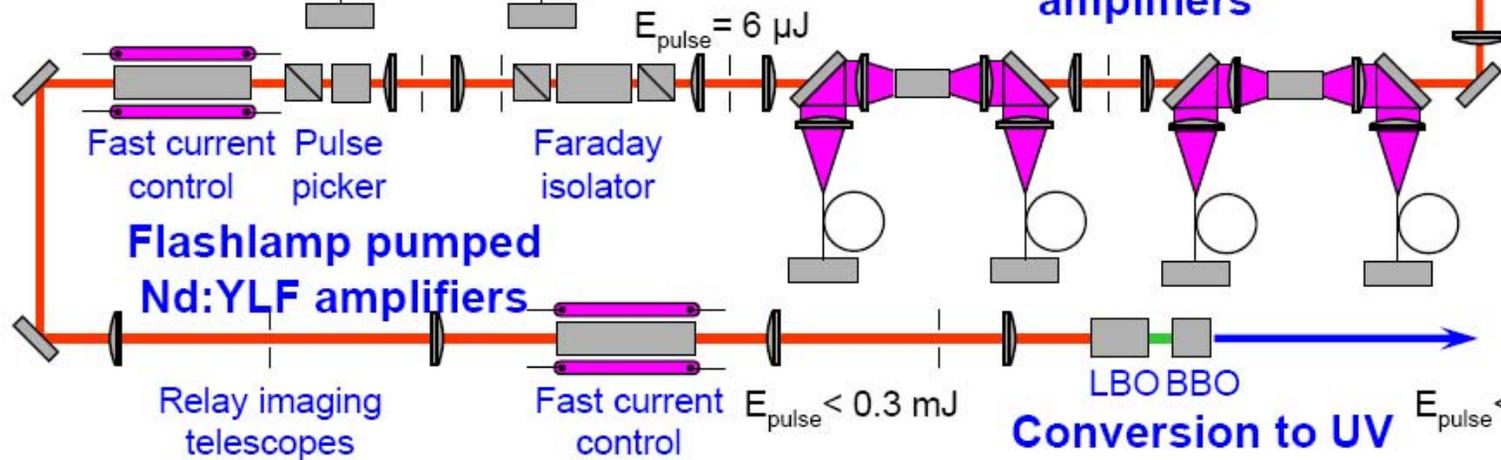
VUV-FEL
Vacuum-Ultraviolet
Free-Electron Laser

In cooperation of DESY and Max-Born-Institute, Berlin,
I. Will et al., NIM A541 (2005) 467,
S. Schreiber et al., NIM A445 (2000)

Diode-pumped Nd:YLF Oscillator



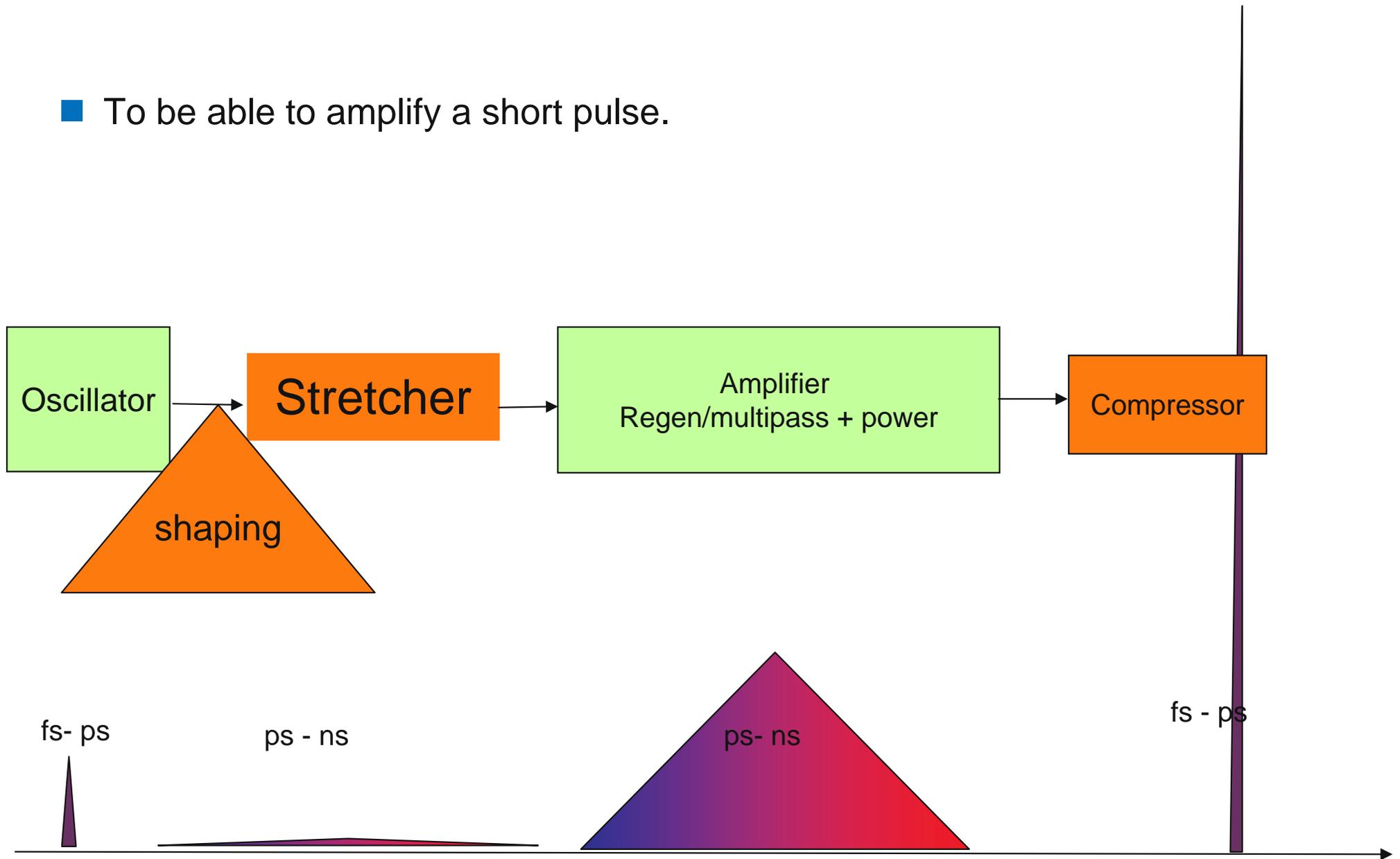
Diode pumped Nd:YLF amplifiers



LBOBBO
Conversion to UV $E_{\text{pulse}} < 50 \mu\text{J}$

Chirped pulse amplification

- To be able to amplify a short pulse.



A CPA example

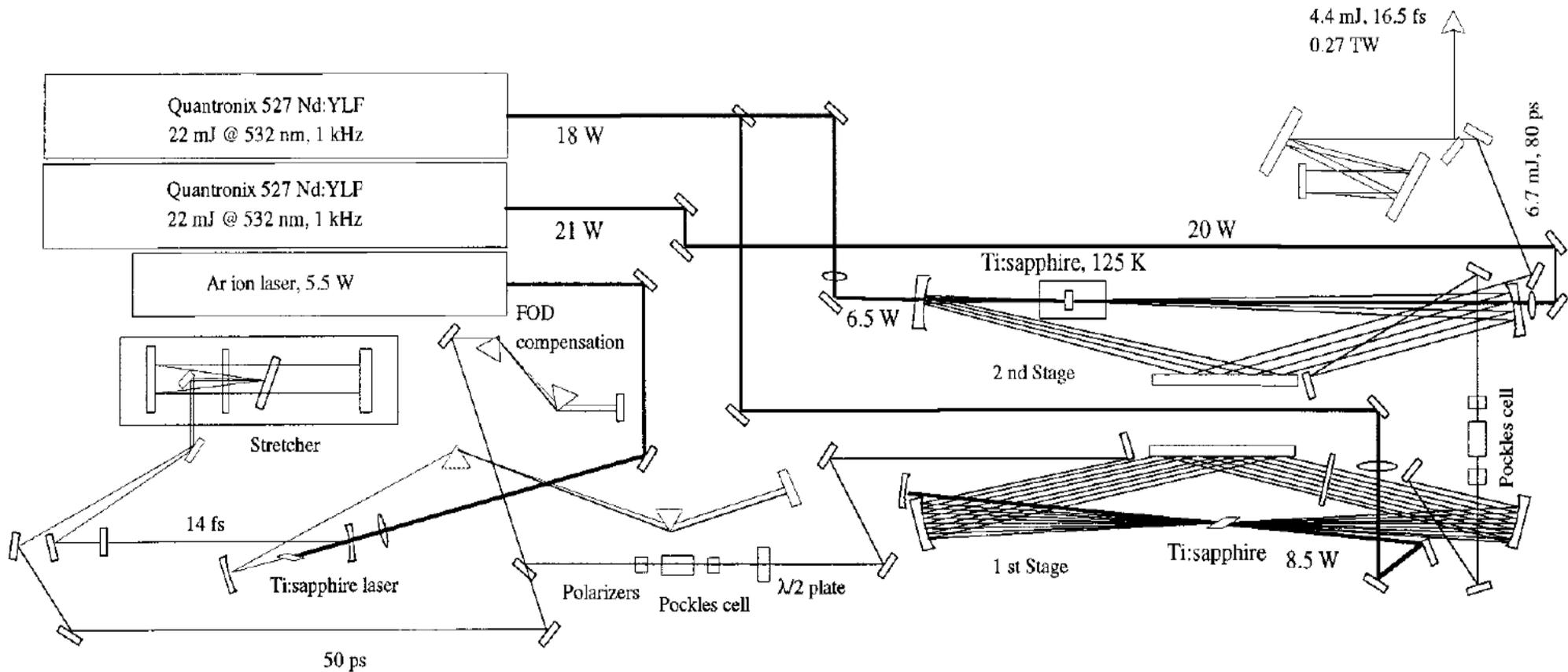


FIG. 7. Schematic diagram of a kHz repetition rate, 0.2 TW Ti:sapphire CPA system.

S. Backus et al, Rev. Sci. Instrum. 69, 1207 (1998).

How to stretch/compress a pulse: adding second order phase

- A transform-limited Gaussian pulse in the time and frequency domain

$$E(t) = E_0 \exp\left[-\frac{1}{2}\left(\frac{t}{\tau}\right)^2\right] \Leftrightarrow E(\omega) = E_0 \exp\left[-\frac{1}{2}\left(\frac{\omega}{\Delta\omega}\right)^2\right]$$

- Add a second order phase, which is to 'chirp' a pulse

$$E'(\omega) = E_0 \exp\left[-\frac{1}{2}\left(\frac{\omega}{\Delta\omega}\right)^2 + ia\omega^2\right]$$

- What is the new pulse form in the time domain? How does the frequency change as function of time? (Home work)

S. Backus et al, Rev. Sci. Instrum. 69, 1207 (1998).

Stretcher and compressor

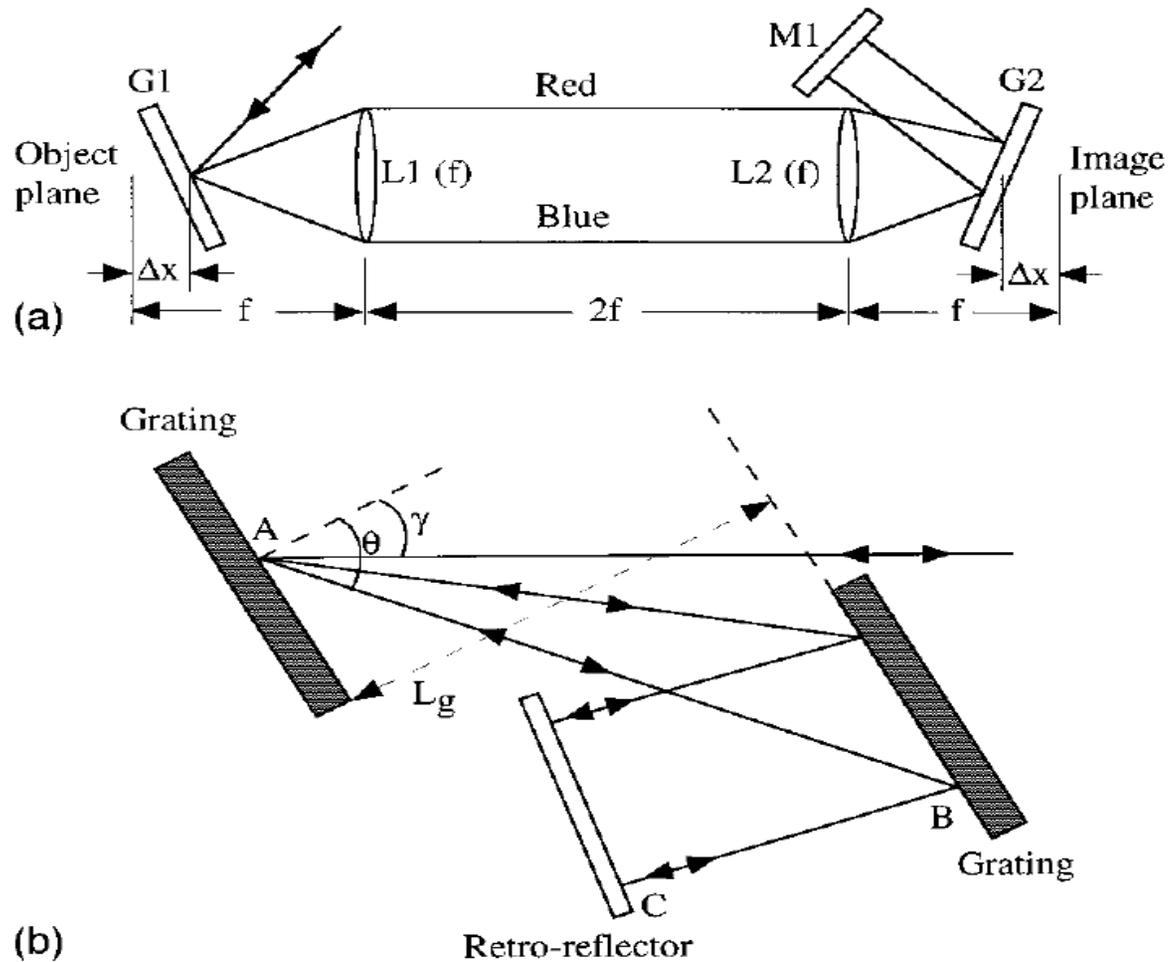


FIG. 3. Schematic diagrams of (a) a pulse stretcher and (b) a pulse compressor.

S. Backus et al, Rev. Sci. Instrum. 69, 1207 (1998).

Stretcher and compressor

TABLE I. Expressions for the linear, quadratic, and cubic phase introduced by grating stretchers; compressors, prism pairs, and materials found in a typical amplifier.

Order	Material	Grating pair compressor/stretcher	Prism pair
GVD	$\frac{d^2\phi_m(\omega)}{d\omega^2} = \frac{\lambda^3 L_m}{2\pi c^2} \frac{d^2 n(\lambda)}{d\lambda^2}$	$\frac{d^2\phi_c(\omega)}{d\omega^2} = \frac{\lambda^3 L_g}{\pi c^2 d^2} \left[1 - \left(\frac{\lambda}{d} - \sin \gamma \right)^2 \right]^{-3/2}$	$\frac{d^2\phi_p(\omega)}{d\omega^2} = \frac{\lambda^3}{2\pi c^2} \frac{d^2 P}{d\lambda^2}$
TOD	$\frac{d^3\phi_m(\omega)}{d\omega^3} = -\frac{\lambda^4 L_m}{4\pi^2 c^3} \left(3 \frac{d^2 n(\lambda)}{d\lambda^2} + \frac{\lambda d^3 n(\lambda)}{d\lambda^3} \right)$	$\frac{d^3\phi_c(\omega)}{d\omega^3} = -\frac{6\pi\lambda}{c} \frac{d^2\phi_c(\omega)}{d\omega^2} \left(\frac{1 + \frac{\lambda}{d} \sin \gamma - \sin^2 \gamma}{\left[1 - \left(\frac{\lambda}{d} - \sin \gamma \right)^2 \right]} \right)$	$\frac{d^3\phi_p(\omega)}{d\omega^3} = \frac{-\lambda^4}{4\pi^2 c^3} \left(3 \frac{d^2 P}{d\lambda^2} + \lambda \frac{d^3 P}{d\lambda^3} \right)$
FOD	$\frac{d^4\phi_m(\omega)}{d\omega^4} = \frac{\lambda^5 L_m}{8\pi^3 c^4} \left(12 \frac{d^2 n(\lambda)}{d\lambda^2} + 8\lambda \frac{d^3 n(\lambda)}{d\lambda^3} + \lambda^2 \frac{d^4 n(\lambda)}{d\lambda^4} \right)$	$\frac{d^4\phi_c(\omega)}{d\omega^4} = \frac{6d^2}{c^2} \frac{d^2\phi_c(\omega)}{d\omega^2} \left(\frac{80 \frac{\lambda^2}{d^2} + 20 - 48 \frac{\lambda^2}{d^2} \cos \gamma + 16 \cos 2\gamma - 4 \cos 4\gamma + \frac{32\lambda}{d} \sin \gamma + \frac{32\lambda}{d} \sin 3\gamma}{\left(-8 \frac{\lambda}{d} + \frac{4d}{\lambda} + \frac{4d}{\lambda} \cos 2\gamma + 32 \sin \gamma \right)^2} - \frac{d^3\phi_c(\omega)}{d\omega^3} \frac{6\pi\lambda}{c} \frac{1 + \lambda/d \sin \gamma - \sin^2 \gamma}{(1 - (\lambda/d - \sin \gamma)^2)} \right)$	$\frac{d^4\phi_p(\omega)}{d\omega^4} = \frac{\lambda^5}{8\pi^3 c^4} \left(12 \frac{d^2 P}{d\lambda^2} + 8\lambda \frac{d^3 P}{d\lambda^3} + \lambda^2 \frac{d^4 P}{d\lambda^4} \right)$

$P(\lambda) = L_p \cos \beta(\lambda)$
 $\beta(\lambda) = -\arcsin(n_p(\lambda) \sin \alpha(\lambda))$
 $+ \arcsin[n_p(\lambda_r) \sin \alpha(\lambda_r)]$
 $\alpha(\lambda) = \xi$
 $- \arcsin[\sin \theta_b(\lambda)]/n_p(\lambda)$
 $\theta_b(\lambda) = \arctan[n_p(\lambda)]$

S. Backus et al, Rev. Sci. Instrum. 69, 1207 (1998).

Dispersions

TABLE II. Sample values of dispersion for material (1 cm), grating pairs, and prism pairs at 800 nm wavelength.

Optical element	GVD $d^2\varphi/d\omega^2$ (fs ²)	TOD $d^3\varphi/d\omega^3$ (fs ³)	FOD $d^4\varphi/d\omega^4$ (fs ⁴)
Fused silica	361.626	274.979	-114.35
BK7	445.484	323.554	-98.718
SF18	1543.45	984.277	210.133
KD*P	290.22	443.342	-376.178
Calcite	780.96	541.697	-118.24
Sapphire	581.179	421.756	-155.594
Sapphire at the Brewster angle	455.383	331.579	-114.912
Air	0.0217	0.0092	2.3×10^{-11}
Compressor: 600 ℓ /mm, $L = 1$ cm, 13.89°	-3567.68	5101.21	-10226
Prism pair: SF18	-45.567	-181.516	-331.184

S. Backus et al, Rev. Sci. Instrum. 69, 1207 (1998).

Content

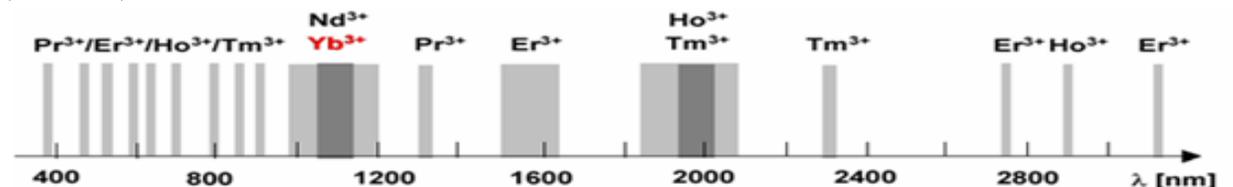
- Laser and accelerator history
- Map of laser application in accelerators
- Laser basics
 - Rate equations
 - Laser configurations
 - Gaussian beam optics and ABCD law
 - Laser cavity and laser modes
- Laser configurations
 - Mode-locking and q-switch
 - MOPA
 - CPA and dispersion
- Laser materials
- Other lasers
 - Semiconductor lasers
 - Fiber lasers
- Frequency conversion and short wavelength lasers

Laser materials

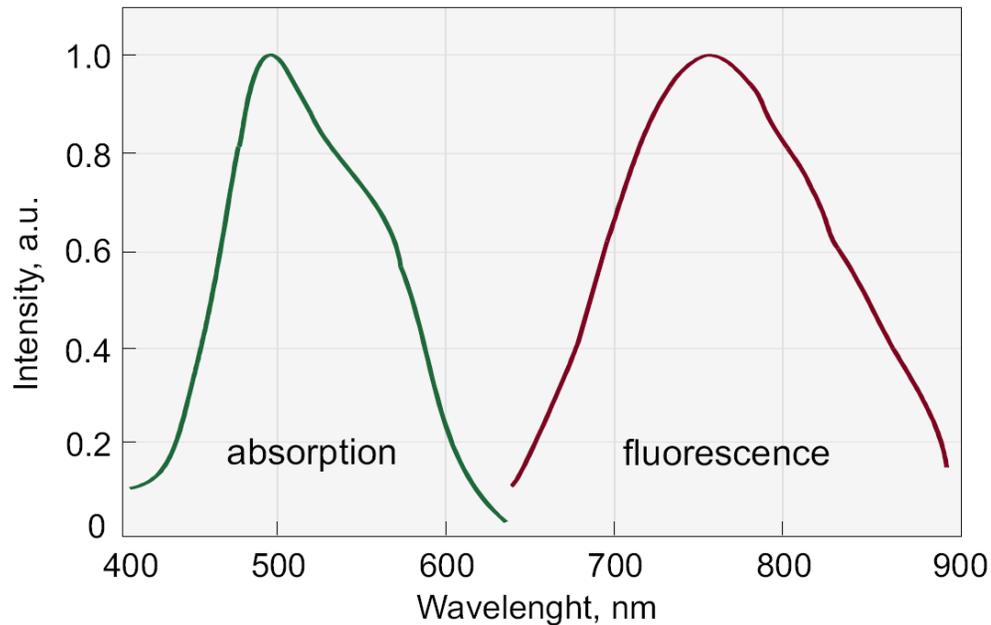
- What we care
 - Lasing mechanism: four-level systems is always preferred
 - Lasing wavelength: tunable is better
 - Lasing bandwidth: bigger is better but not always
 - Pump requirement: visible preferred
 - Gain lifetime: longer is better
 - Damage threshold: higher is better
 - Saturation flux: higher is better
 - Heat conductivity: as high as possible
 - Thermal stability: as small as possible
 - Form: solid is always preferred

Laser materials

- Host + active ions
- Host
 - Crystalline solids (Sapphire, Garnets, Fluoride, Aluminate, etc.)
 - *Difficult to grow to large size*
 - *Narrow line width thus lower lasing threshold, and narrow absorption band*
 - *Good thermal conductivity*
 - Glass (property varies by make, and processing)
 - *Easy to make in large size and large quantities*
 - *No well defined bonding field thus larger line width and higher lasing threshold, large absorption band*
 - *Lower thermal conductivity thus severe thermal birefringence and thermal lensing, lower duty cycle*
- Active ions
 - Rare earth ions: No. 58-71, most importantly, Nd^{3+} , Er^{3+} ...
 - Transition metals: Ti^{3+} , Cr^{3+} ,



Laser material: Ti: Saaphire ($\text{Al}_2\text{O}_3:\text{Ti}^{3+}$)

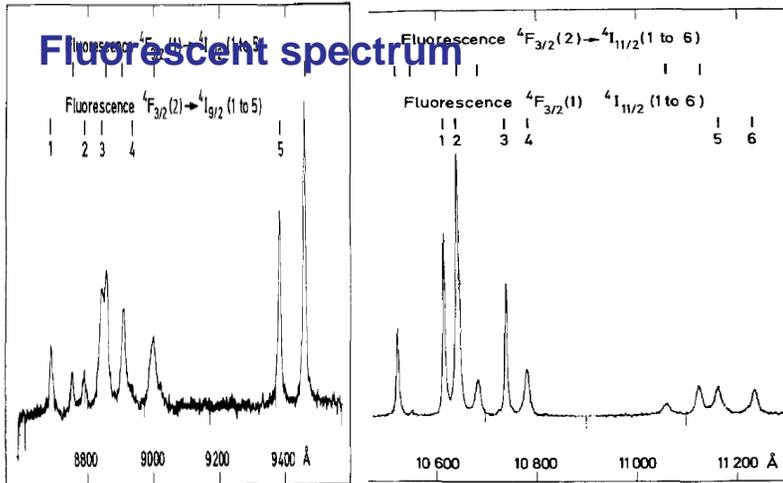
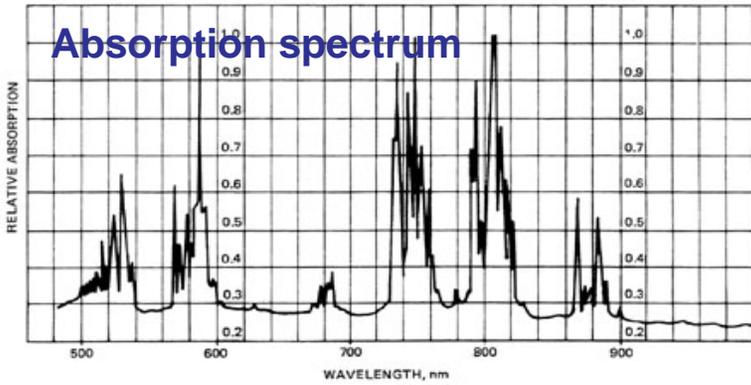


MATERIAL PHYSICAL AND LASER PROPERTIES

Chemical formula	$\text{Ti}^{3+}:\text{Al}_2\text{O}_3$
Crystal structure	Hexagonal
Lattice constants	$a=4.748, c=12.957$
Density	3.98 g/cm^3
Mohs hardness	9
Thermal conductivity	$0.11 \text{ cal}/(\text{C} \times \text{sec} \times \text{cm})$
Specific heat	0.10 cal/g
Melting point	$2050 \text{ }^\circ\text{C}$
Laser action	4-Level Vibronic
Fluorescence lifetime	$3.2 \mu\text{sec}$ (T=300K)
Tuning range	660–1050 nm
Absorbtion range	400–600 nm
Emission peak	795 nm
Absorption peak	488 nm
Refractive index	1.76 @ 800 nm

- Giving shortest pulse so far
- Wonderful tunability
- Good thermal properties
- Short gain lifetime (has to be pumped by a ns-pulsed green laser)

Laser material (Nd:YAG)



PROPERTIES OF 1.0% Nd:YAG AT 25°C

Formula	$Y_{2.97}Nd_{0.03}Al_5O_{12}$
Crystal structure	Cubic
Density	4.55 g/cm ³
Melting point	1970 °C
Mohs hardness	8.5
Transition	$^4F_{3/2} \rightarrow ^4I_{11/2}$ @ 1064 nm
Fluorescence lifetime	230 μ s for 1064 nm
Thermal conductivity	0.14 Wcm ⁻¹ K ⁻¹
Specific heat	0.59 Jg ⁻¹ K ⁻¹
Thermal expansion	6.9×10^{-6} °C ⁻¹
$\partial n/\partial t$	7.3×10^{-6} °C ⁻¹
Young's modulus	3.17×10^4 Kg/mm ²
Poisson ratio	0.25
Thermal shock resistance	790 Wm ⁻¹
Refractive index	1.818 @ 1064 nm

- High saturation flux
- Narrow bandwidth (0.15 nm), thus long pulse (>10 ps), high gain
- Good thermal properties
- Long gain lifetime (diode pump)

Laser material: (Nd:Glass)

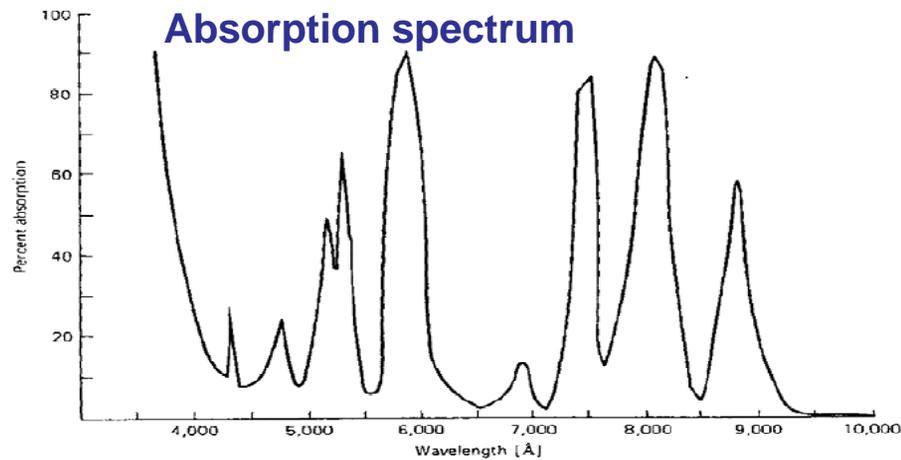


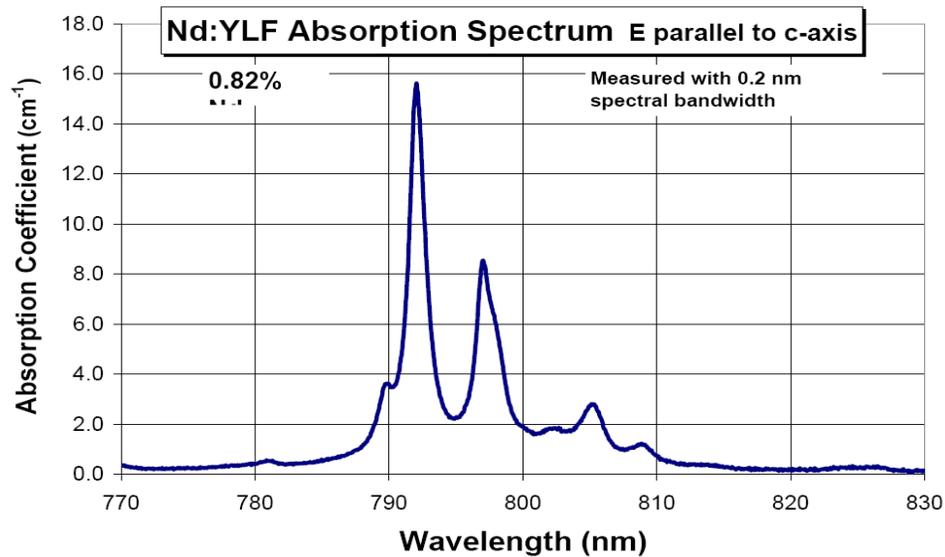
Fig. 2.9. Absorption versus wavelength of Nd: glass. (Material: ED-2; thickness: 6.3 mm)



Chemical formula	Nd: Y ₃ Al ₅ O ₁₂
Weight % Nd	0.725
Atomic % Nd	1.0
Nd atoms/cm ³	1.38 × 10 ²⁰
Melting point	1970 C
Knoop hardness	1215
Density	4.56 g/cm ³
Rupture stress	1.3–2.6 × 10 ³ kg/cm ³
Modulus of elasticity	3 × 10 ³ kg/cm ²
Thermal expansion coefficient	
[100] orientation	8.2 × 10 ⁻⁶ C ⁻¹ , 0–250 C
[110] orientation	7.7 × 10 ⁻⁶ C ⁻¹ , 10–250 C
[111] orientation	7.8 × 10 ⁻⁶ C ⁻¹ , 0–250 C
Linewidth	4.5 Å
Stimulated emission cross section	
R ₂ – Y ₃	σ ₂₁ = 6.5 × 10 ⁻¹⁹ cm ²
4F _{3/2} – ⁴ I _{11/2}	σ ₂₁ = 2.8 × 10 ⁻¹⁹ cm ²
Spontaneous fluorescence lifetime	230 μs
Photon energy at 1.06 μm	hν = 1.86 × 10 ⁻¹⁹ J
Index of refraction	1.82 (at 1.0 μm)
Scatter losses	α _{sc} ≈ 0.002 cm ⁻¹

- High saturation flux
- large bandwidth (20 nm), thus short pulse (<1 ps),
- Poor thermal
- Long gain lifetime (diode pump)
- Only for big lasers now (PW or MJ)

Laser Material: Nd:YLF



- High saturation flux
- Narrow bandwidth (0.15 nm), thus long pulse (>10 ps), high gain
- Good thermal properties
- Long gain lifetime (diode pump)
- Difficult to handle

Physical Properties					
Chemical Formula	LiY _{1.0-x} Nd _x F ₄				
Lattice Parameters	a=5.16Å b=10.85Å				
Crystal Structure	Tetragonal				
Space Group	I4 ₁ /a				
Nd atoms/cm ³	1.40x10 ²⁰ atoms/cm ³ for 1% Nd doping,				
Mohs Hardness	4 ~ 5				
Melting Point	819°C				
Density	3.99 g/cm ³				
Modulus of Elasticity	85 GPa				
Thermal Expansion Coefficient	8.3x10 ⁻⁶ /k ⊥c, ac=13.3x10 ⁻⁶ /k c				
Thermal Conductivity Coefficient	0.063 W/cm K				
Specific Heat	0.79 J/g K				
Optical Properties					
Transparency Region	180nm to 6.7μm				
Peak Simulation Emission Cross Section	1.8x10 ⁻¹⁹ cm ² (E c) at 1.047μm 1.2x10 ⁻¹⁹ cm ² (E ⊥ c) at 1.053μm				
Spontaneous Fluorescence Lifetime	485μs for 1% Nd doping				
Scatter Losses	<0.2% / cm				
Peak Absorption Coefficient	α=10.8cm ⁻¹ (792.0 nm E c) α=3.59cm ⁻¹ (797.0 nm E ⊥ c)				
Refractive Indices	Wavelength (nm)	n _e	n _o		
	262	1.485	1.511		
	350	1.473	1.491		
	525	1.456	1.479		
	1050	1.448	1.470		
	2065	1.442	1.464		
Sellmeier Equations	n ₁ ² (λ) = A + Bλ ² /(λ ² -C) - Dλ ² /(λ ² -E)				
	A	B	C	D	E
	n _o 3.38757	0.70757	0.00931	0.18849	50.99741
	n ₂ 1.31021	0.84903	0.00876	0.53607	134.9566

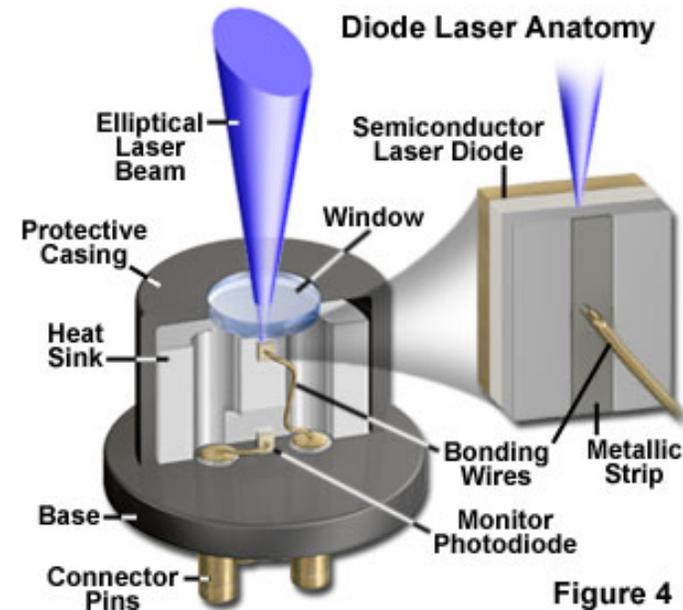
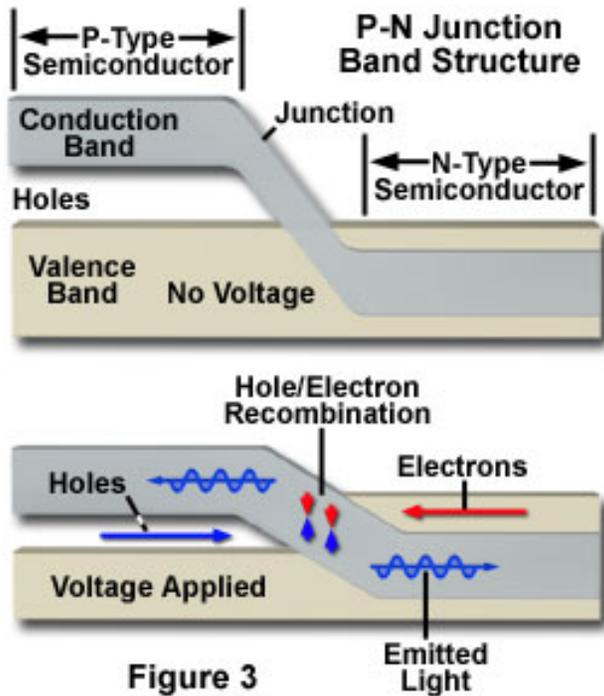
Laser materials: summary

	Ti:Sa $\text{Al}_2\text{O}_3:\text{Ti}$	Nd:YAG $\text{Y}_{3.0-x}\text{Nd}_x\text{Al}_5\text{O}_{12}$	Nd:Glass (Kigre Q-88) $\text{Y}_3\text{Al}_5\text{O}_{12}:\text{Nd}$	Nd:YLF $\text{LiY}_{1.0-x}\text{Nd}_x\text{F}_4$
Fluorescence life time (μs)	3.2	230	330	485
Peak wavelength (nm)	780	1064	1054	1047,1053
Line width (nm)	220	0.15	22	1
Emission cross section (10^{-19} cm^2)	3	6.5	0.4	1.8
Saturation flux (J/cm^2)	0.9	0.6	4.5	.43
Thermal conductivity ($\text{w cm}^{-1} \text{ K}^{-1}$)	0.5	0.14	0.0084	0.06
Thermal expansion coef ($10^{-6}/^\circ\text{C}$)		7.5	10	10
n	1.76	1.8	1.55	1.5
dn/dT ($10^{-6}/^\circ\text{C}$)		7.3	-0.5	

Content

- Laser and accelerator history
- Map of laser application in accelerators
- Laser basics
 - Rate equations
 - Laser configurations
 - Gaussian beam optics and ABCD law
 - Laser cavity and laser modes
- Laser configurations
 - Mode-locking and q-switch
 - MOPA
 - CPA and dispersion
- Laser materials
- Other lasers
 - Semiconductor lasers
 - Fiber lasers
- Frequency conversion and short wavelength lasers

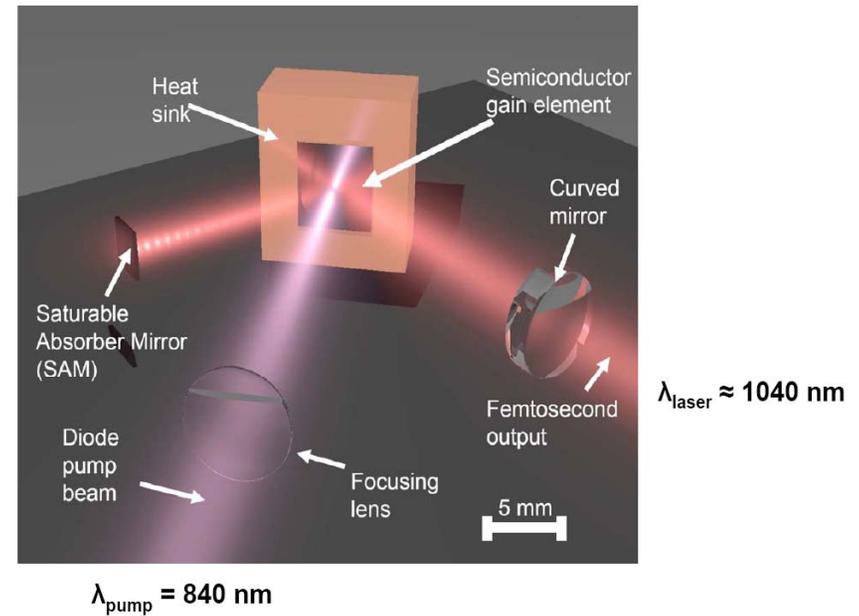
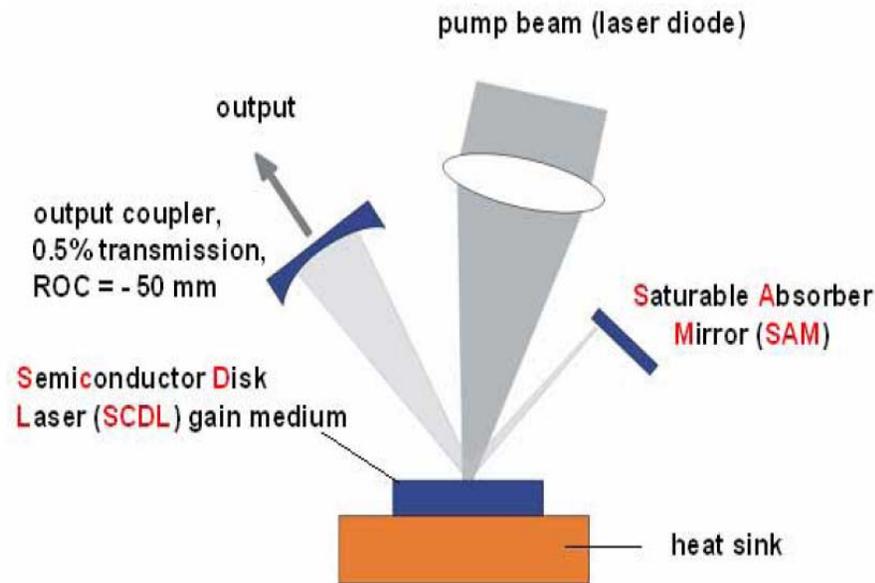
Semiconductor laser: laser diode



- Convert current into light, can be tuned by junction temperature
- Used mostly for pumping other lasers, also CD and DVD players
- VCSELs and VECSELs (vertical cavity surface emission lasers and vertical external cavity surface emission lasers)
- Also for seeding pulse fiber lasers (next page)

Credit: <http://www.olympusmicro.com/>

Mode-locked semiconductor lasers: (using V)



290-fs pulses from a semiconductor disk laser

Peter Klopp¹, Florian Saas¹, Martin Zorn², Markus Weyers², and Uwe Griebner^{1*}

¹Max-Born-Institute, Max-Born-Strasse 2A, D-12489 Berlin, Germany

²Ferdinand-Braun-Institute, Gustav-Kirchhoff-Straße 4, D-12489 Berlin, Germany

*Corresponding author: griebner@mbi-berlin.de

Opt. Express 16, 5770 (2008)

- Can achieve sub picosecond pulse duration
- 500 mW power
- Widely tunable
- Very high rep rate, up to 100 GHz

Pump for lasers

■ Flash lamps:

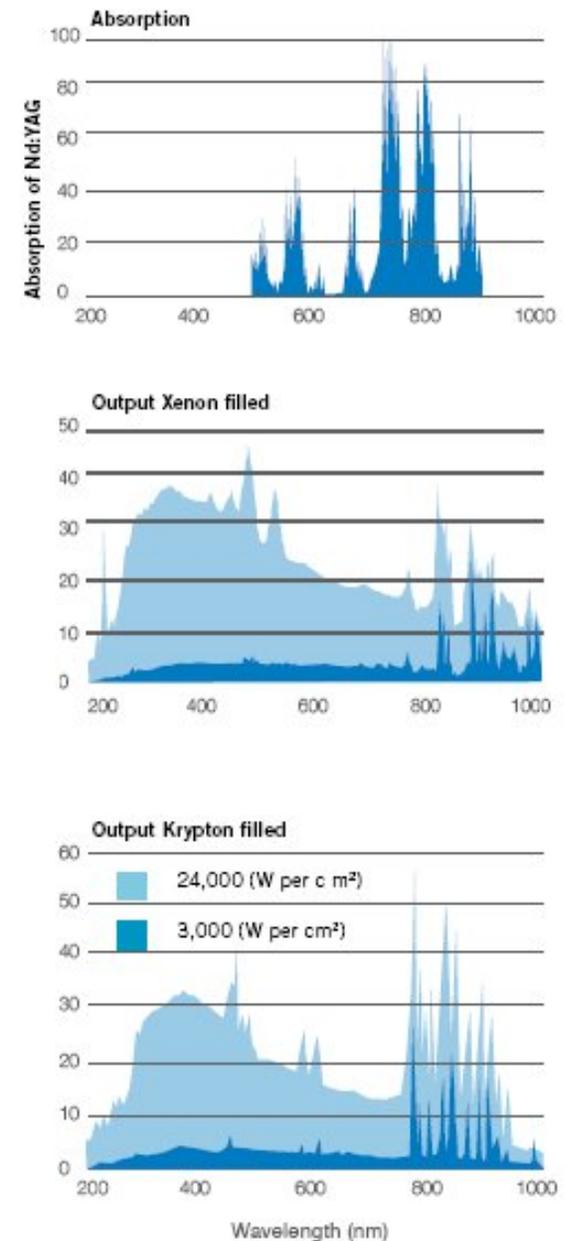
- Converts electrical power to light
- Cheap, low pumping efficiency, and poor stability.
- Still opted for situations when high energy capacity is the key (NIF), suitable for ruby laser, Nd:Glass, Nd:YAG, Nd:YLF, etc.

■ Laser diodes:

- converts electrical current into light
- High efficiency, high stability especially in CW mode.
- Opted now for most off the shelf KHz system and fiber lasers

■ Pump lasers (Nd:YAG or Nd: YLF)

- Both pumped by flash lamps and diodes
- Normally for Ti: Sapphire system.



Fiber lasers

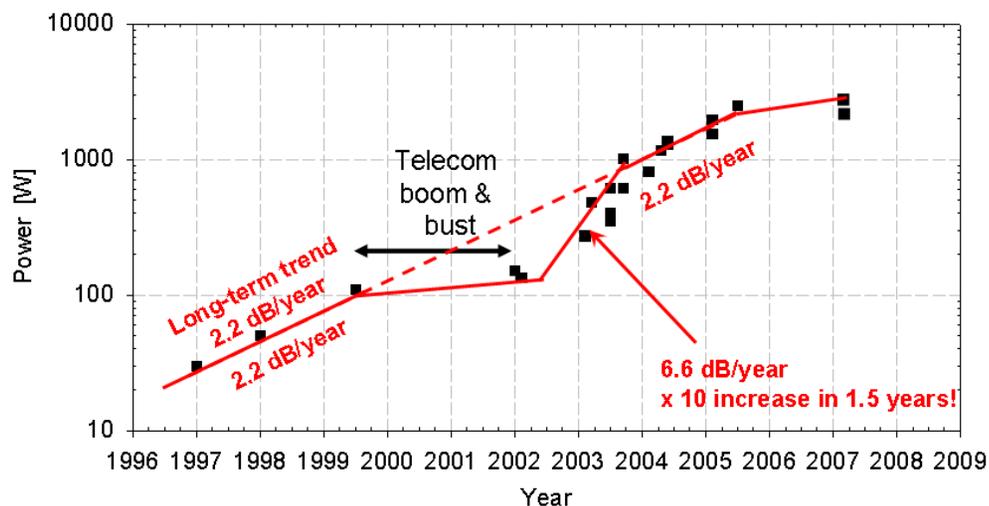
Power progress in fiber laser sources

Average power over 2 kW

limited by available pump



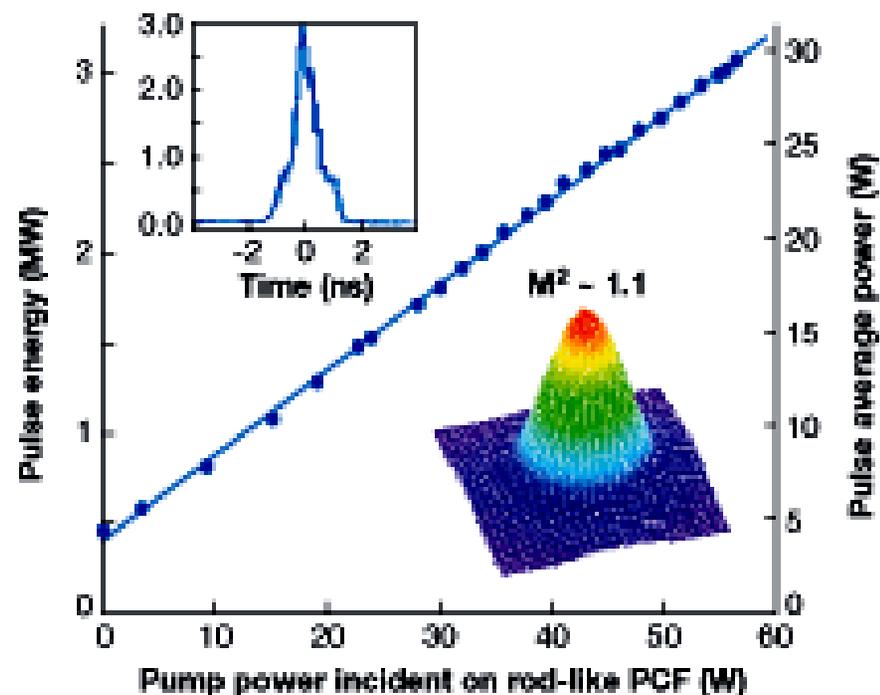
Ultra-high powers from fibres



Wavelength ~ 1.1 μm

Credit: David Richardson

Peak Power over 1 MW



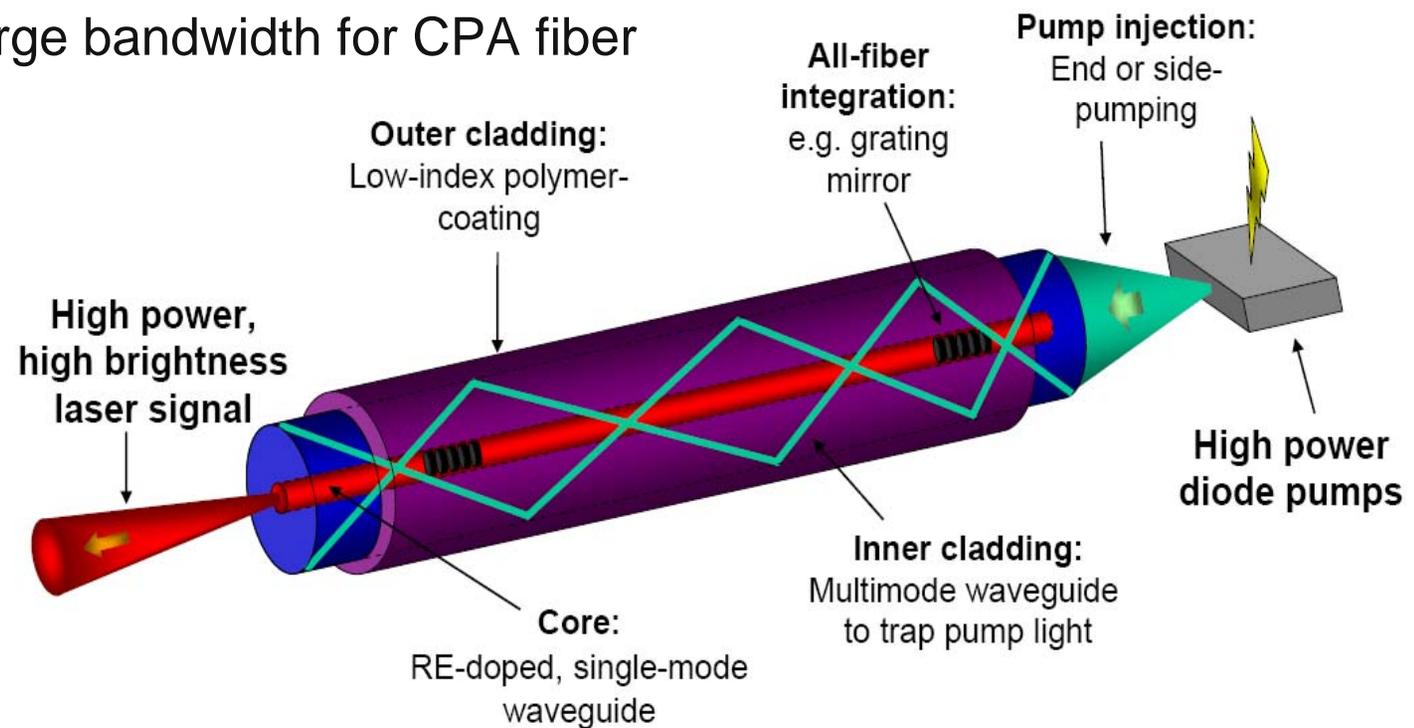
Teodoro et al, Laser Focus World, Nov. 2006

E. Snitzer, "Neodymium glass laser," Proc. of the Third International conference on Solid Lasers, Paris, page 999 (1963).

C.J. Koester and E. Snitzer, "Amplification in a fiber laser," Appl. Opt. 3, 10, 1182 (1964).

Laser configuration: Fiber lasers

- Can be a straight MOPA or a CPA
- Low threshold, high efficiency, high rep rate, high power
- No thermal problem, good stability
- Relatively large bandwidth for CPA fiber



Rare-earth-doped core converts multimode pump energy to high brightness, *diffraction-limited*, signal beam

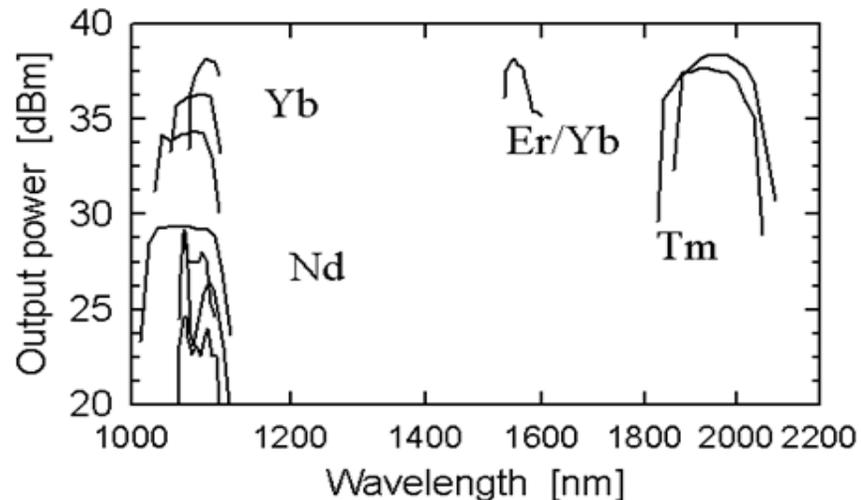
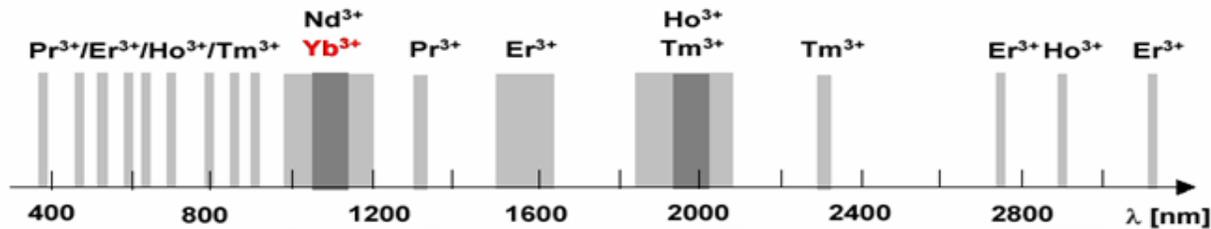
Credit: David Richardson

J. Limpert et al., 'High-power ultrafast fiber laser systems,' IEEE Xplore 12, 233 (2006).

Fiber laser wavelength



Wavelength Coverage



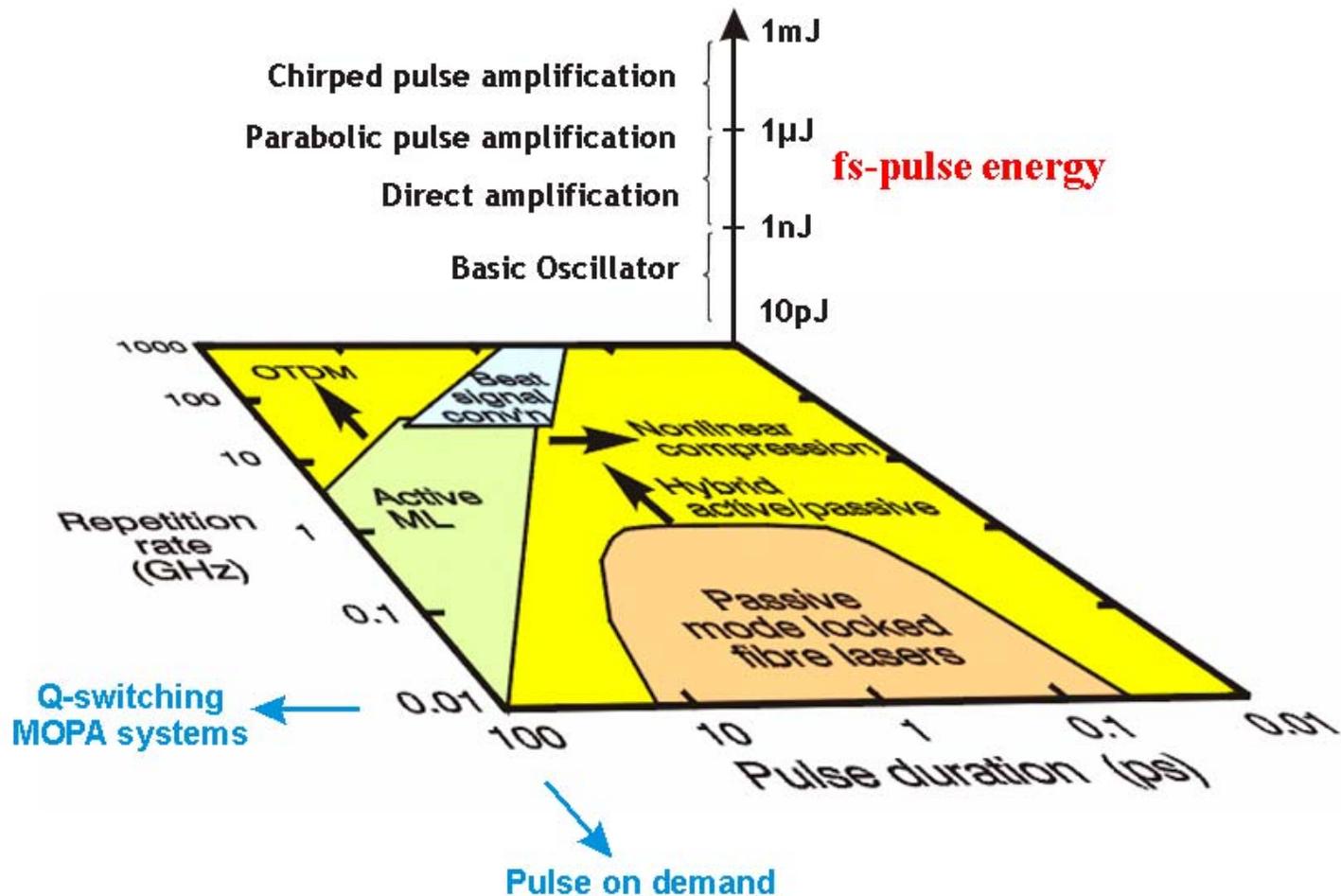
- Many RE transitions but most not good in silica
- Nd, Yb, Er, Tm most attractive for high power operation
- Raman gain for other wavelengths

Credit: David Richardson

Fiber laser capabilities



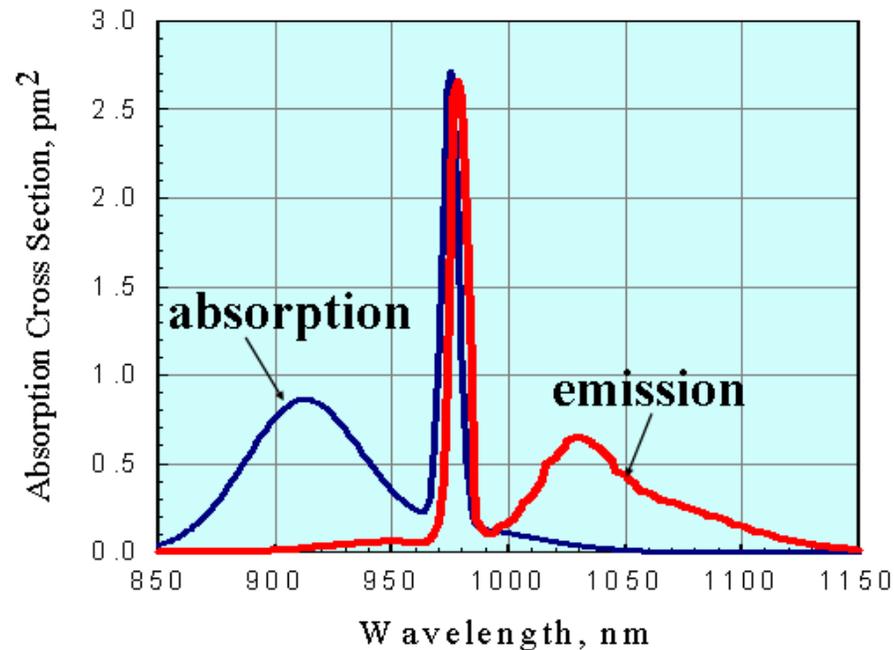
Operating regimes of fibre based ultrashort pulse sources



Credit: David Richardson

Fiber laser architecture

Yb-doped fibres for cladding pumping



Light

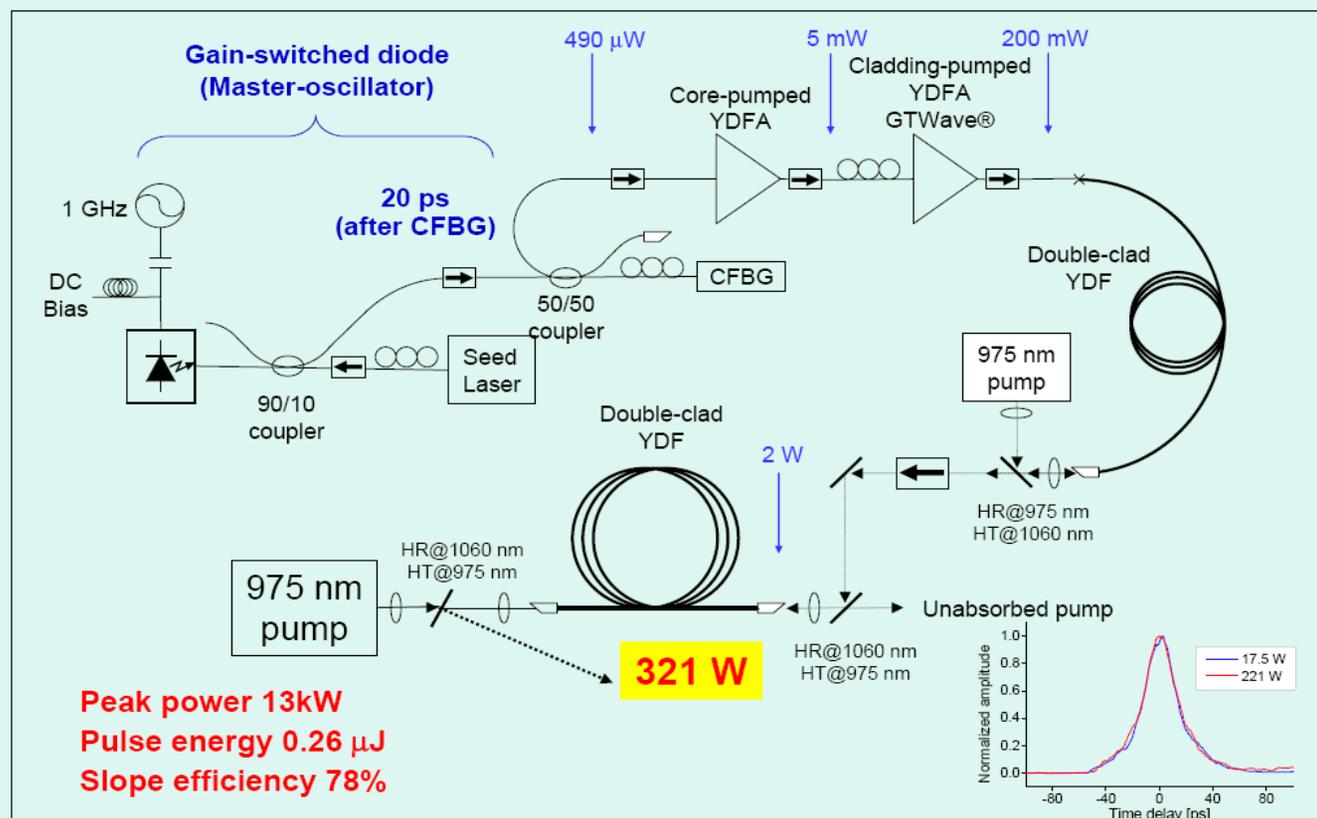
- Pump bands at 915nm and 976nm
- Broad gain bandwidths around 1060nm
- Small quantum defect and high efficiency (~85%)

Credit: David Richardson

A short pulse MOPA fiber laser example

1 GHz, 20 ps, 321 W average and 13 kW peak power

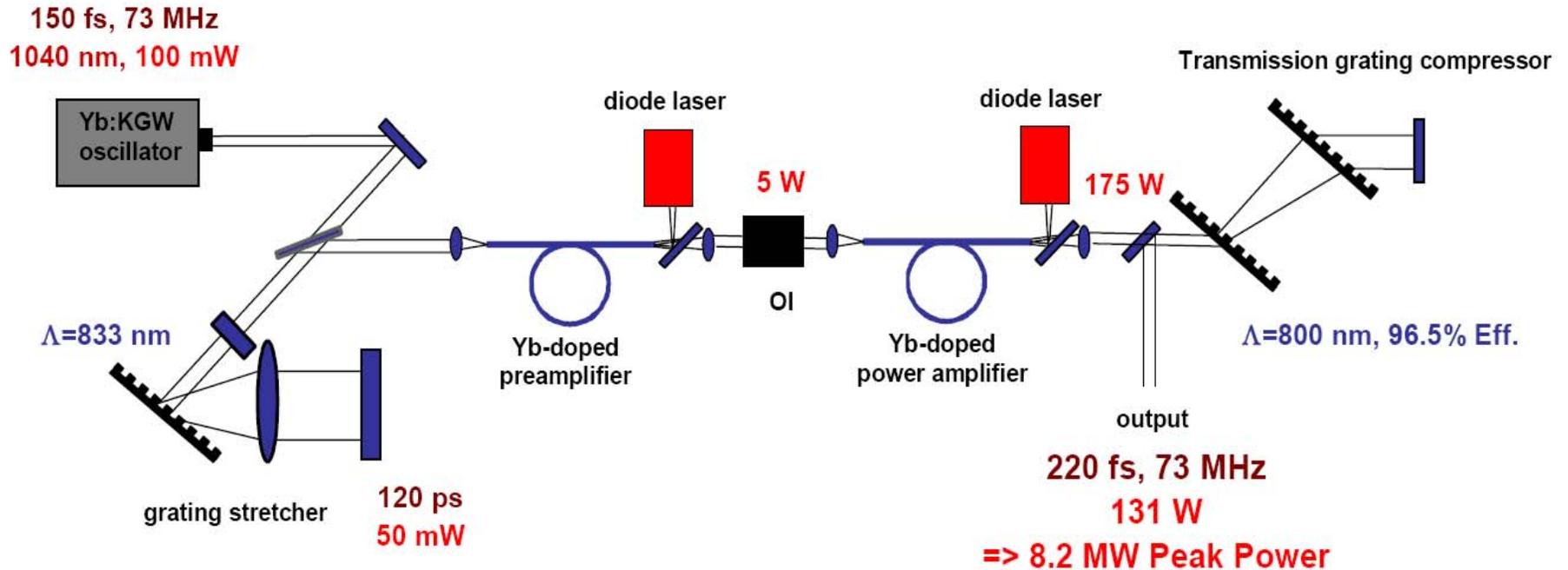
321 W average power, 1 GHz, 20 ps, 1060 nm pulsed fibre MOPA source



Dupriez et al, <http://www.ofcnfoec.org/materials/PDP3.pdf>

A CPA fiber laser example

73 MHz, 220 fs, 131 W average and 8.2 MW peak power



Roeser et al., Opt. Lett. 30, 2754 (2005)

Fig. 9: Schematic setup of the high average power fiber CPA system.

1 μ m, 1 mJ, 1 ps, 50 kHz has been achieved, F. Roeser et al, Opt. Lett. 32, 3294 (2007)

J. Limpert et al., 'High-power ultrafast fiber laser systems,' IEEE Xplore 12, 233 (2006).

Content

- Laser and accelerator history
- Map of laser application in accelerators
- Laser basics
 - Rate equations
 - Laser configurations
 - Gaussian beam optics and ABCD law
 - Laser cavity and laser modes
- Laser configurations
 - Mode-locking and q-switch
 - MOPA
 - CPA and dispersion
- Laser materials
- Other lasers
 - Semiconductor lasers
 - Fiber lasers
- Frequency conversion and short wavelength lasers

Maxwell equation in a medium

The induced polarization, P , contains the effect of the medium:

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= 0 & \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} &= 0 & \vec{\nabla} \times \vec{B} &= \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \frac{\partial \vec{P}}{\partial t}\end{aligned}$$

The induced polarization in Maxwell's Equations yields another term in the wave equation:

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2}$$

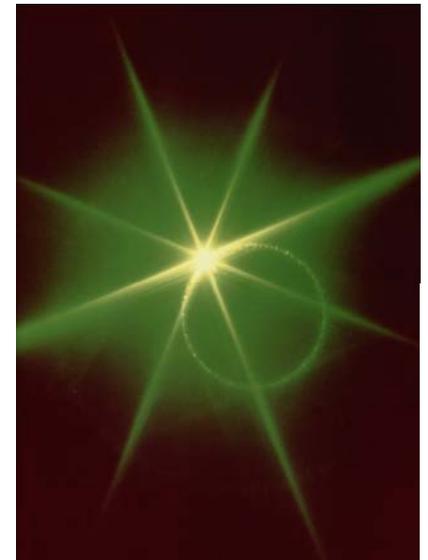
As we've learned, this is the "Inhomogeneous Wave Equation."
The polarization is the driving term for a new solution to this equation.

Credit: R. Trebino

Maxwell equation in a nonlinear medium

Nonlinear optics is what happens when the polarization is the result of higher-order (nonlinear!) terms in the field:

$$\mathcal{P} = \epsilon_0 \left[\chi^{(1)} \mathcal{E} + \chi^{(2)} \mathcal{E}^2 + \chi^{(3)} \mathcal{E}^3 + \dots \right]$$



What are the effects of such nonlinear terms?
Consider the second-order term:

Since $\mathcal{E}(t) \propto E \exp(i\omega t) + E^* \exp(-i\omega t)$,

$$\mathcal{E}(t)^2 \propto E^2 \exp(2i\omega t) + 2|E|^2 + E^{*2} \exp(-2i\omega t)$$

$2\omega = 2\text{nd harmonic!}$

Harmonic generation is one of many exotic effects that can arise!

Credit: R. Trebino

USPAS, 2008

Sum and difference frequency generation

Suppose there are two different-color beams present:

$$E_1 \exp(i\omega_1 t) + E_1^* \exp(-i\omega_1 t) \quad E_2 \exp(i\omega_2 t) + E_2^* \exp(-i\omega_2 t)$$

So:

$$\begin{aligned} E(t)^2 \propto & E_1^2 \exp(2i\omega_1 t) + E_1^{*2} \exp(-2i\omega_1 t) \\ & + E_2^2 \exp(2i\omega_2 t) + E_2^{*2} \exp(-2i\omega_2 t) \\ & + 2E_1 E_2 \exp[i(\omega_1 + \omega_2)t] + 2E_1^* E_2^* \exp[-i(\omega_1 + \omega_2)t] \\ & + 2E_1 E_2^* \exp[i(\omega_1 - \omega_2)t] + 2E_1^* E_2 \exp[-i(\omega_1 - \omega_2)t] \\ & + 2|E_1|^2 + 2|E_2|^2 \end{aligned}$$

2nd-harmonic gen

2nd-harmonic gen

Sum-freq gen

Diff-freq gen

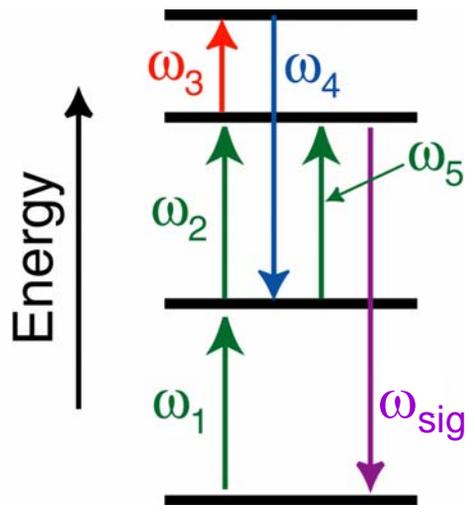
dc rectification

Note also that, when ω_i is negative inside the exp, the E in front has a $*$.

Credit: R. Trebino

USPAS, 2008

Conservation laws and phase matching



Energy must be conserved:

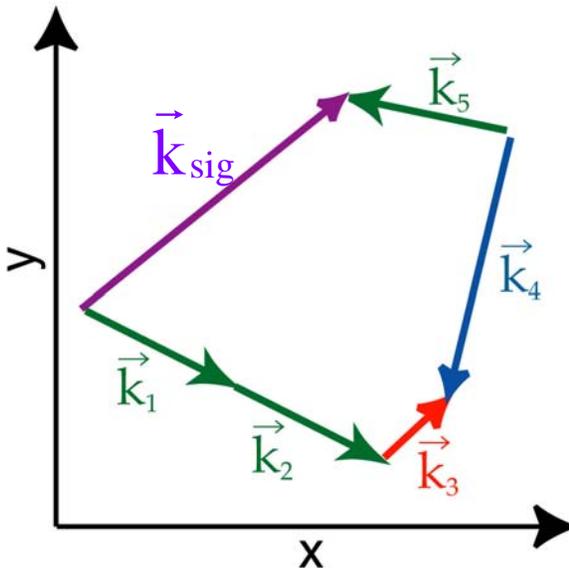
$$\omega_1 + \omega_2 + \omega_3 - \omega_4 + \omega_5 = \omega_{sig}$$

Momentum must also be conserved:

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{k}_4 + \vec{k}_5 = \vec{k}_{sig}$$

Unfortunately, \vec{k}_{sig} may not correspond to a light wave at frequency ω_{sig} !

Satisfying these two relations simultaneously is called "**phase-matching.**"



Credit: R. Trebino

USPAS, 2008

Phase matching and conversion efficiency

- Small signal second harmonic conversion efficiency

$$I_{2\omega} = C^2 L^2 I_{\omega}^2 \frac{\sin^2 \frac{\Delta k L}{2}}{\left(\frac{\Delta k L}{2}\right)^2} = C^2 L^2 I_{\omega}^2 \operatorname{sinc}^2 \frac{\pi L}{2l_c},$$

$$C^2 = \frac{8\pi^2 d_{\text{eff}}^2}{\epsilon_0 c \lambda_0^2 n_0^3} = 5.46 \frac{d_{\text{eff}}}{\lambda_0 n^{3/2}}$$

$\Delta k = 4\pi/\lambda_1(n_1 - n_2)$: difference in wave number

L : crystal length

$l_c = \pi/\Delta k$, coherence length (when phase is matched)

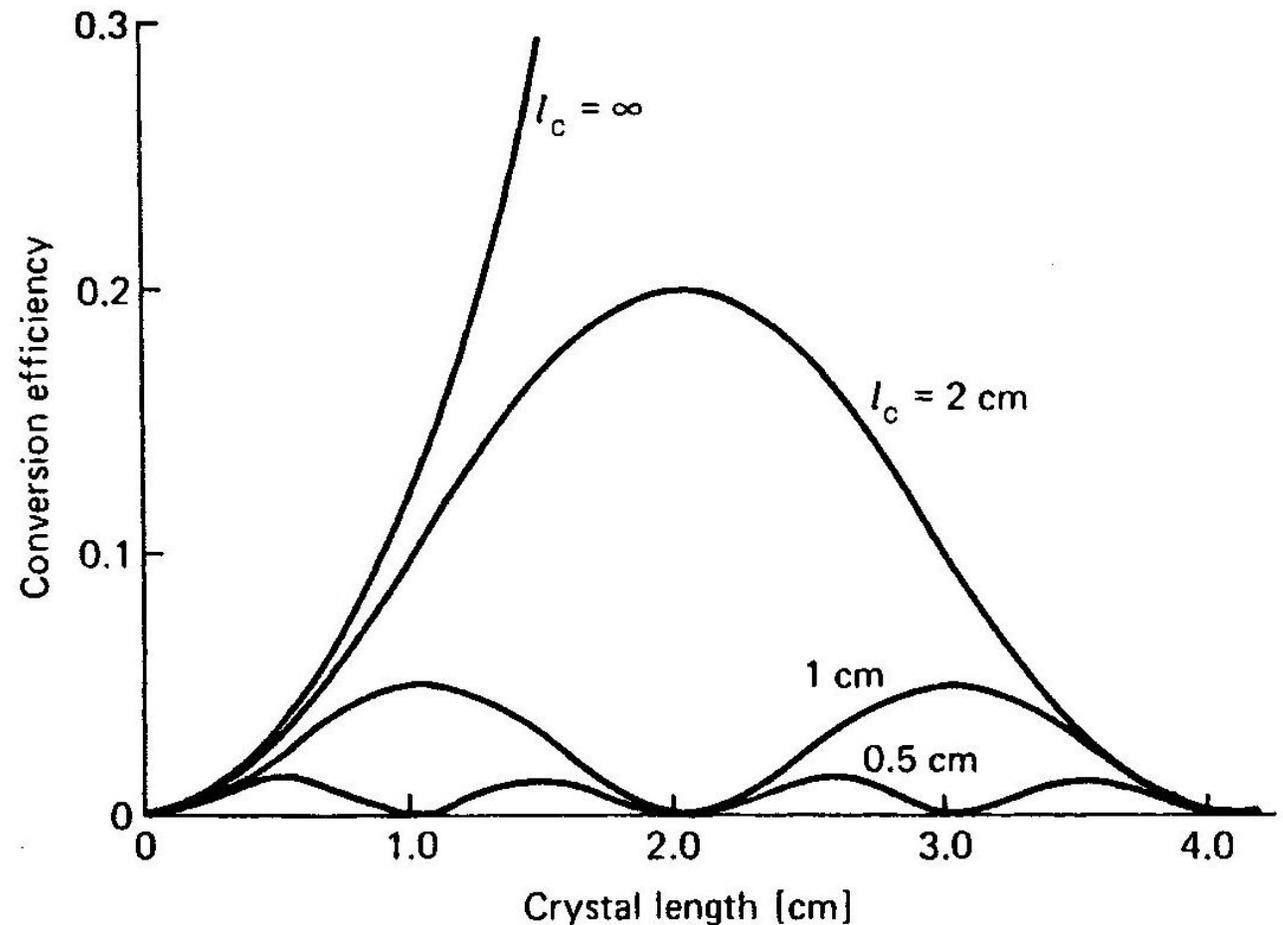
d_{eff} : effective nonlinear coefficient, in m/V

Credit: W. Koechner: Solid State Laser engineering,

Conversion efficiency at small signal

- The small signal conversion efficiency and effect of phase mismatch

$$\eta = C^2 L^2 \sin^2 c^2 \frac{\pi L}{2l_c}$$

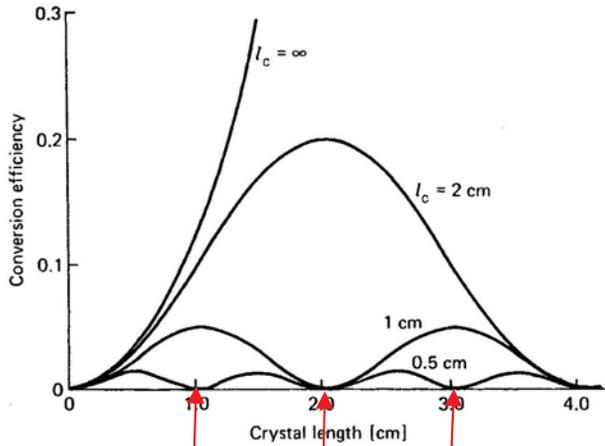


Credit: W. Koechner: Solid State Laser engineering,

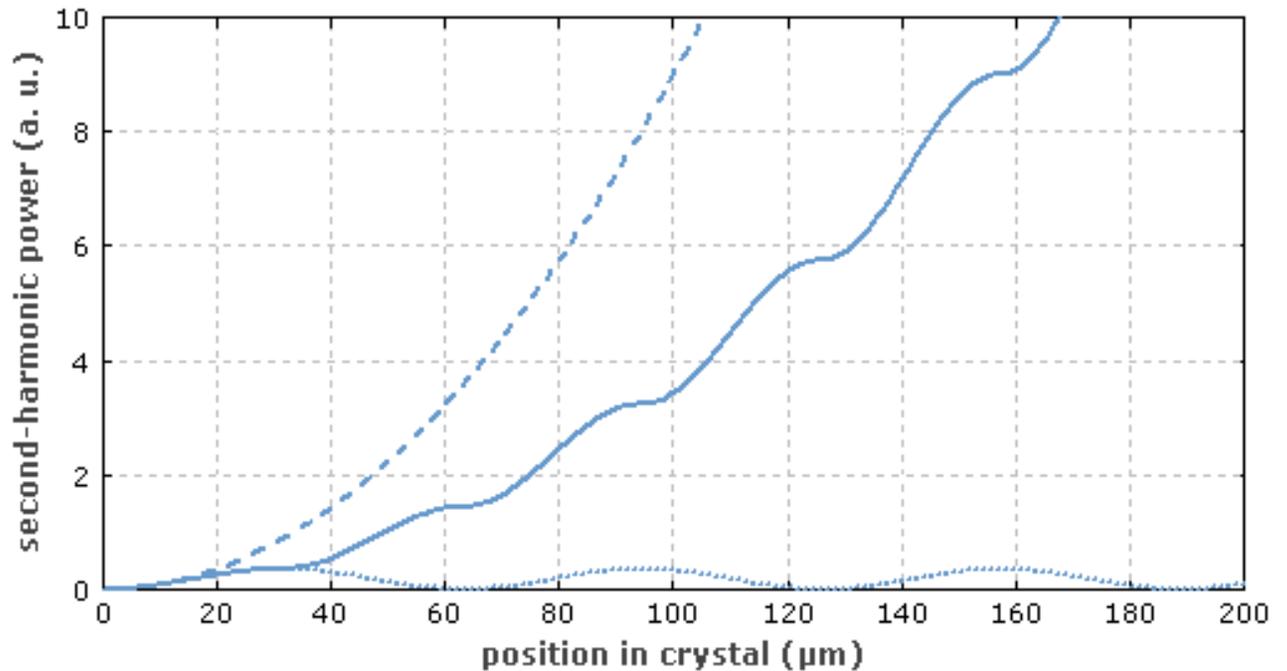
Quasi phase matching

■ Quasi-phase matching

Achieve phase matching by modulating the spatial nonlinear property



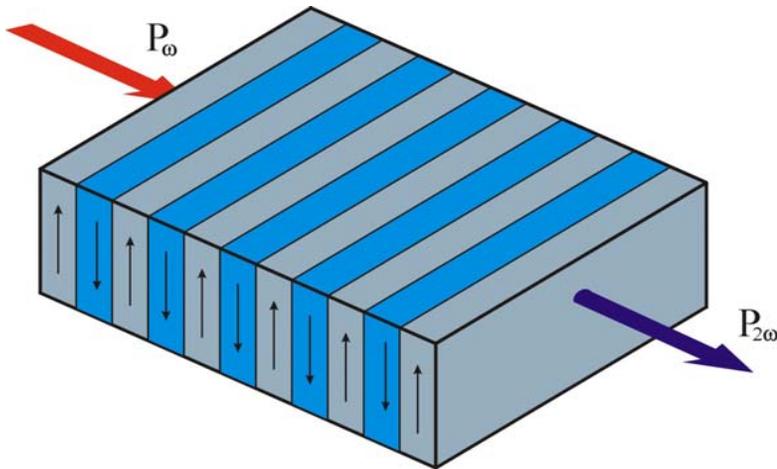
Tune the interaction here



Encyclopedia of Laser Physics and Technology
<http://www.rp-photonics.com>

Quasi-phase matching: Periodically poling crystals

- The most popular technique for generating quasi-phase-matched crystals is periodic poling of ferroelectric nonlinear crystal materials



Wavelength Range: 400-4000 nm

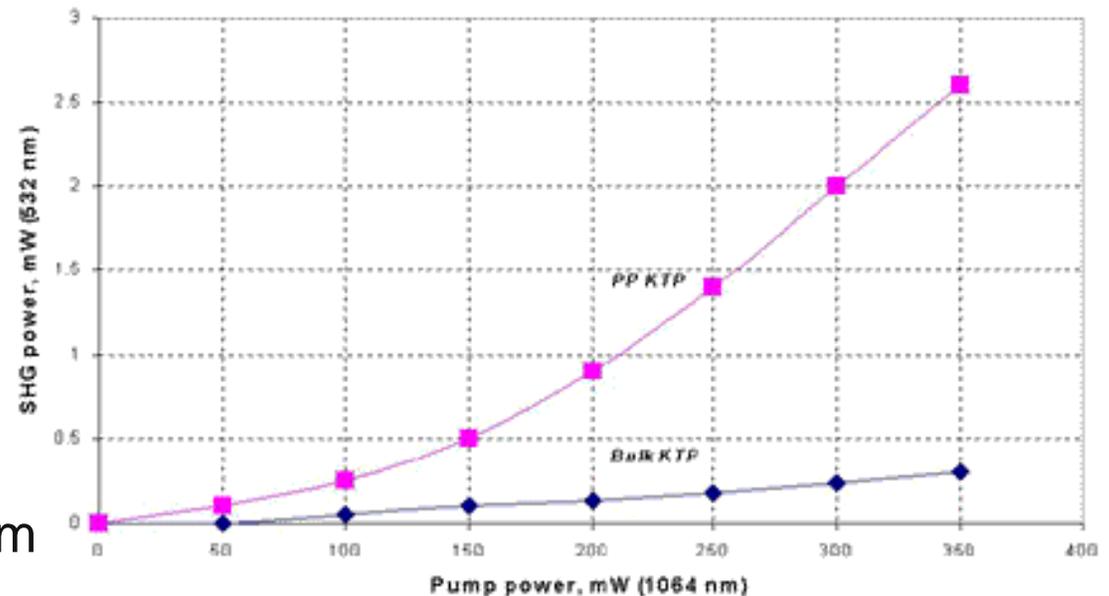
Dimensions:

Thickness: up to 1 mm

Width (typical): 2 mm

Length: up to 30 mm

Second Harmonic Generation in periodically poled and bulk KTP crystals (L=10 mm)



Credit: www.raicol.com/products.asp

Pump depletion and signal saturation

- Phase match, with pump depletion

$$I_{2\omega} = I_{\omega} \tanh^2 \left(CLI_{\omega}^{1/2} \sin c \frac{\pi L}{2l_c} \right),$$

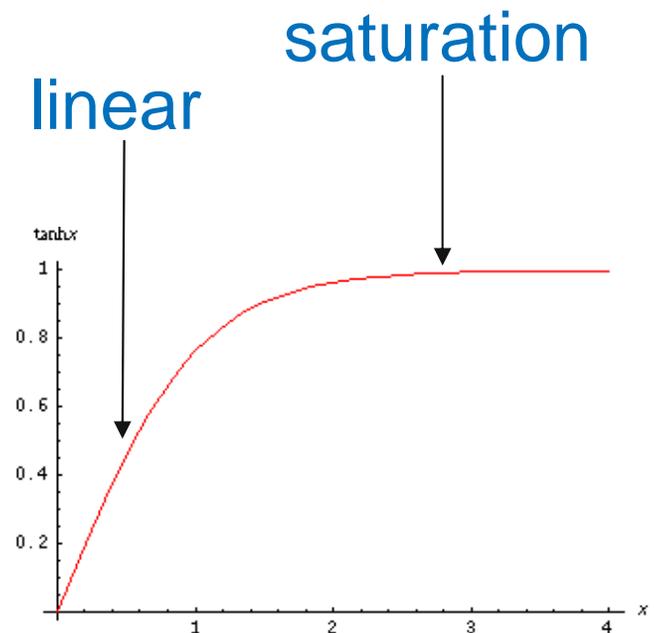
- Consider frequency quadrupling
 - At low pump power, highly nonlinear

$$I_{2\omega} \propto I_{\omega}^2$$

$$\Rightarrow I_{4\omega} \propto I_{2\omega}^2 \propto I_{\omega}^4$$

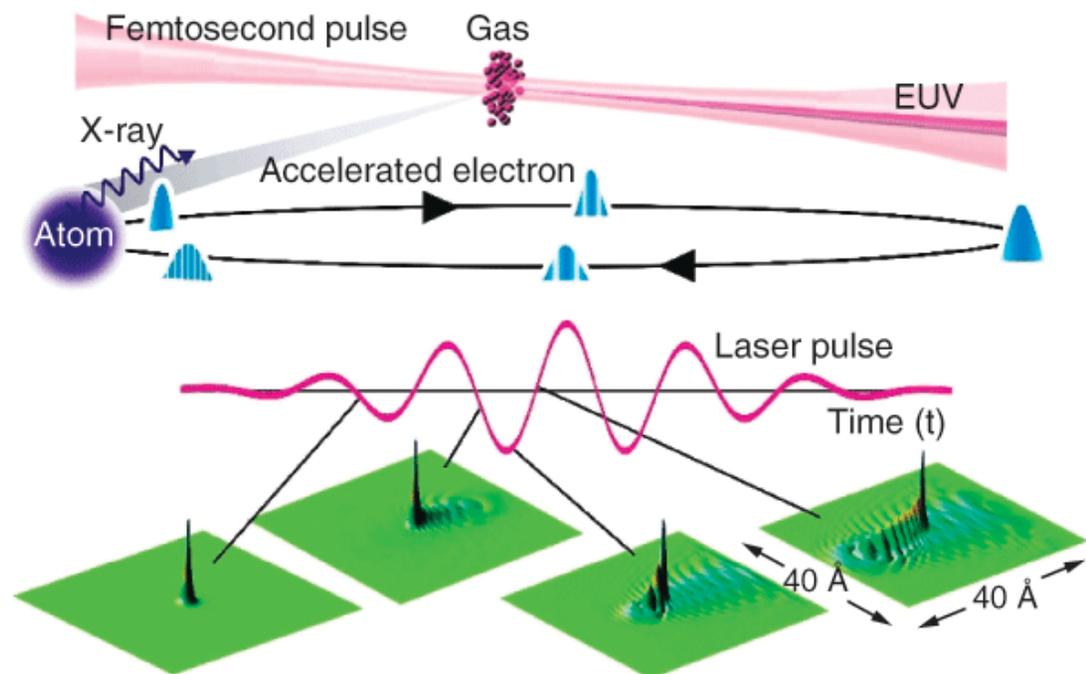
- Consider frequency quadrupling
 - At high pump power, linear, desired

$$I_{2\omega} \propto I_{\omega}$$
$$\Rightarrow I_{4\omega} \propto I_{2\omega} \propto I_{\omega}$$



Frequency conversion: High order harmonics generation

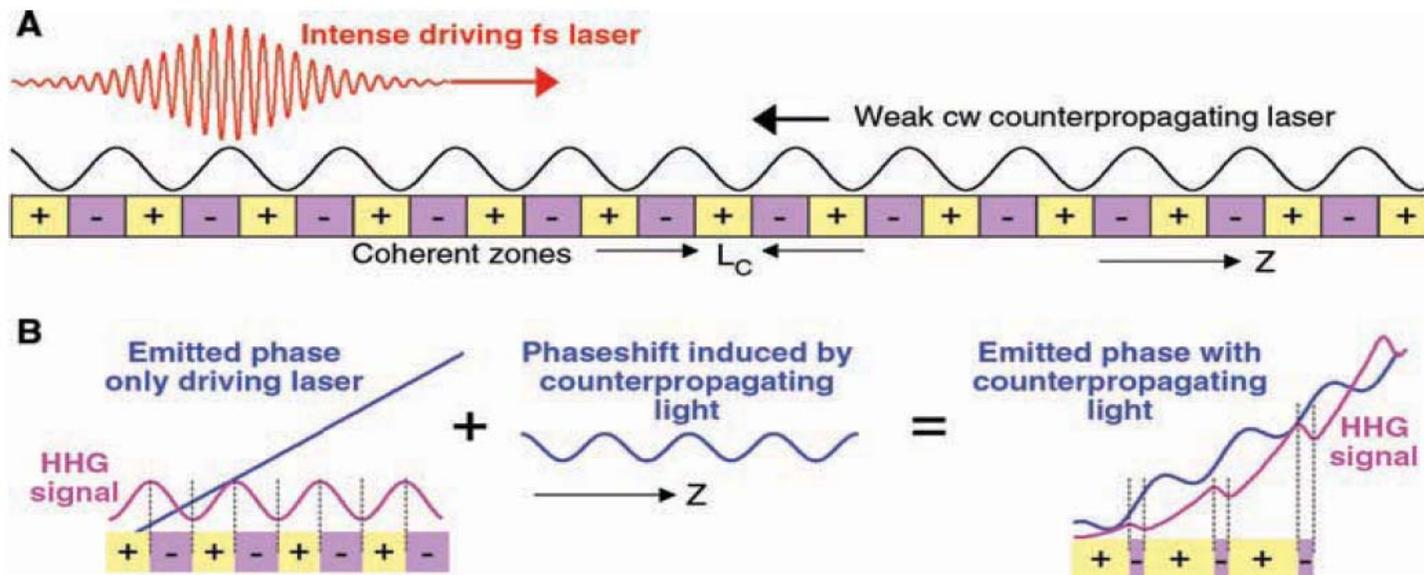
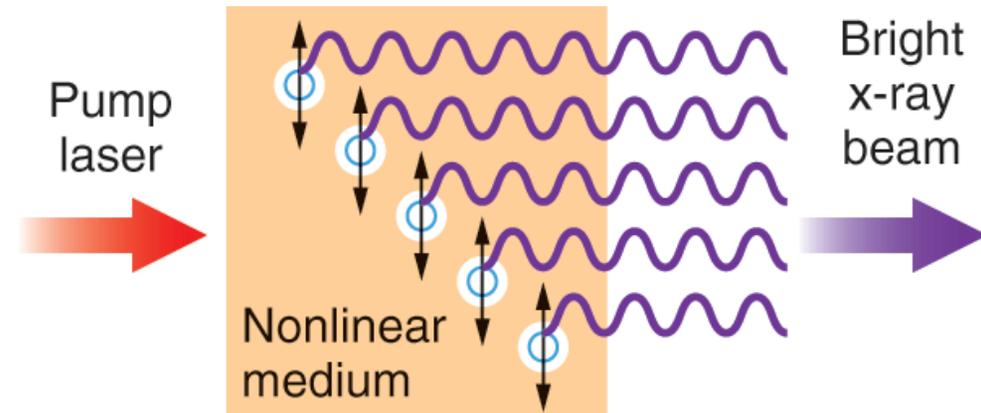
- Atoms as the nonlinear media
- Emission due to multiple photon absorption of electrons bounded to an atom
- Carry over the spatial coherence of the driving laser
- Of short pulse duration comparable or shorter than the drive laser up to attoseconds
- Currently at ~ 10 nm.



H. Kaptyn et al., 'Harnessing Attosecond Science in the Quest for Coherent X-rays,' *Science* 317, 775 (2007).

High order harmonics generation: phase matching

- Phase mismatching due to group velocity difference or continuous signal emission or group velocity mismatch
- Can be accomplished by modulating the interaction via modulated laser propagation or counter propagating beams

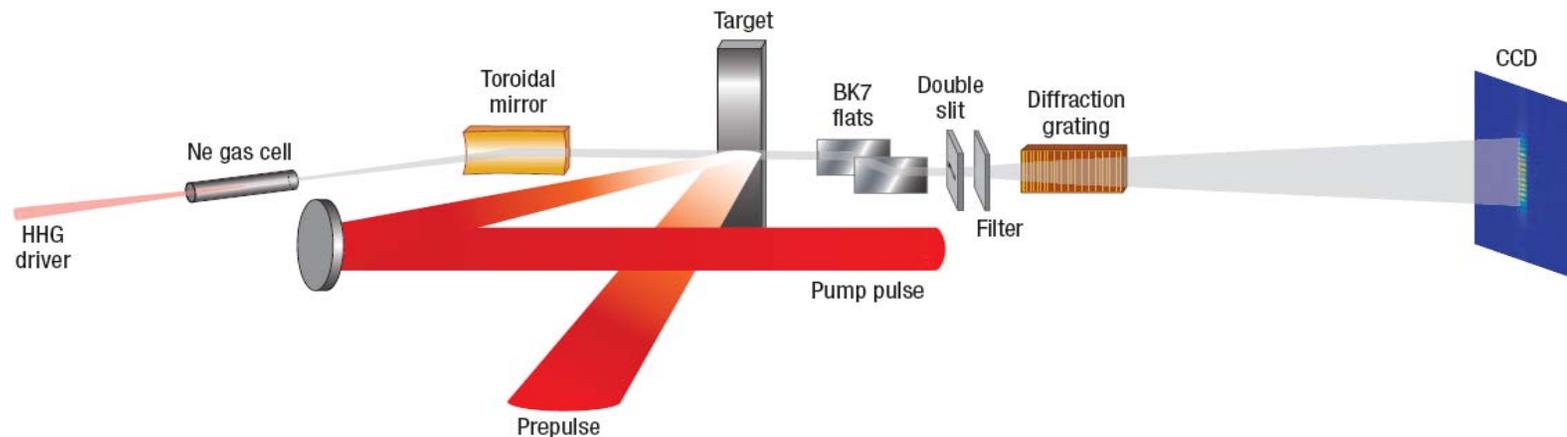


H. Kaptyn et al., 'Harnessing Attosecond Science in the Quest for Coherent X-rays,' Science 317, 775 (2007).

Short wavelength lasers

■ Plasma base soft-x-ray lasers

- Still a traditional laser based on population inversion, but in hot dense plasma to accommodate the high energy difference between atomic levels, wavelength from 1 nm to 100 nm.
- Can be pumped by laser, discharge, and bombs
- Challenges: tunability and capability for shorter wavelengths, stabilities
- J. Hecht, “The history of X-ray lasers,” *Optics & Photonics News* 19, 26 (2008); Y. Wang et al., *Nature Photonics* 2, 94 (2008)



Short wavelength lasers

- X-ray Free electron lasers: 0.1 nm, high brightness, high transverse coherence, etc.
- Design with cavity going on
- Many projects going on

