ELECTRONIC DAMPING OF MICROPHONICS IN SUPERCONDUCTING CAVITIES

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Outline

- Introduction – Microphonics
- Electronic Damping
- Amplitude and Phase feedback method
- Extension with Frequency feedback method

Introduction – Microphonics

- Superconducting cavities has a high susceptibility to external vibrations and electromagnetic radiation.
- Microphonics – Is the result of external vibrations and pressure that cause a change in the cavity frequency.
- Is a critical issue if the change in cavity frequency exceeds the bandwidth, which leads to a perturbation in amplitude and phase in the accelerating field.
- Frequency changes can be only controlled by supplying rf power.
- Measured microphonic noise levels ~ 10 Hz

Ponderomotive Effects

- Ponderomotive effects : Change in cavity frequency caused by the electromagnetic field – radiation pressure.

- Superconducting cavity is considered as a mechanical system, with infinite number of mechanical modes of vibration.

- Frequency shift $\Delta\ \omega_\mu$ caused by mechanical mode μ of vibrations

$$\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\Delta\dot{\omega}_\mu + \Omega_\mu^2 \Delta\omega_\mu = -k_\mu \Omega_\mu^2 V^2 + n(t)$$

Ω_μ - Linear frequency for the mode μ

k_μ - Lorentz coefficient for the mode μ

$n(t)$ - Driving term for microphonics

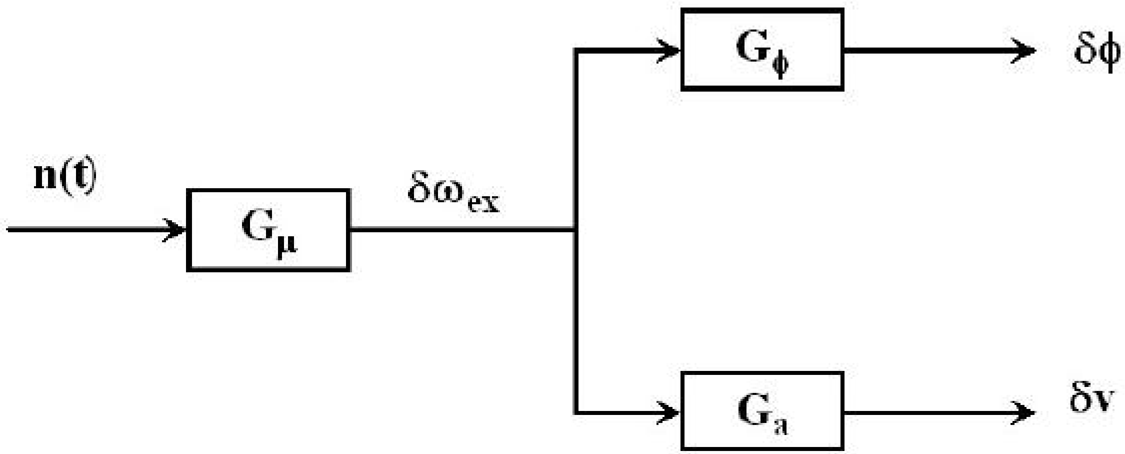
Final Requirement

- Transfer function representation of the field errors.

  $\Delta\omega_{ex}$ – Change in cavity frequency

  $\delta\upsilon$ – Amplitude error

  $\delta\varphi$ – Phase error



- The transfer functions G_a and G_φ are properties of the cavity and of the rf control system.

Electronic Damping of Microphonics

- Reduction of microphonics by modulation of the cavity field which induces the ponderomotive forces that counteract the effect of microphonic vibrations on the cavity frequency.

- The model used is the superconducting accelerating cavity operated in Self Excited Loop (SEL) with phase and amplitude feedback.

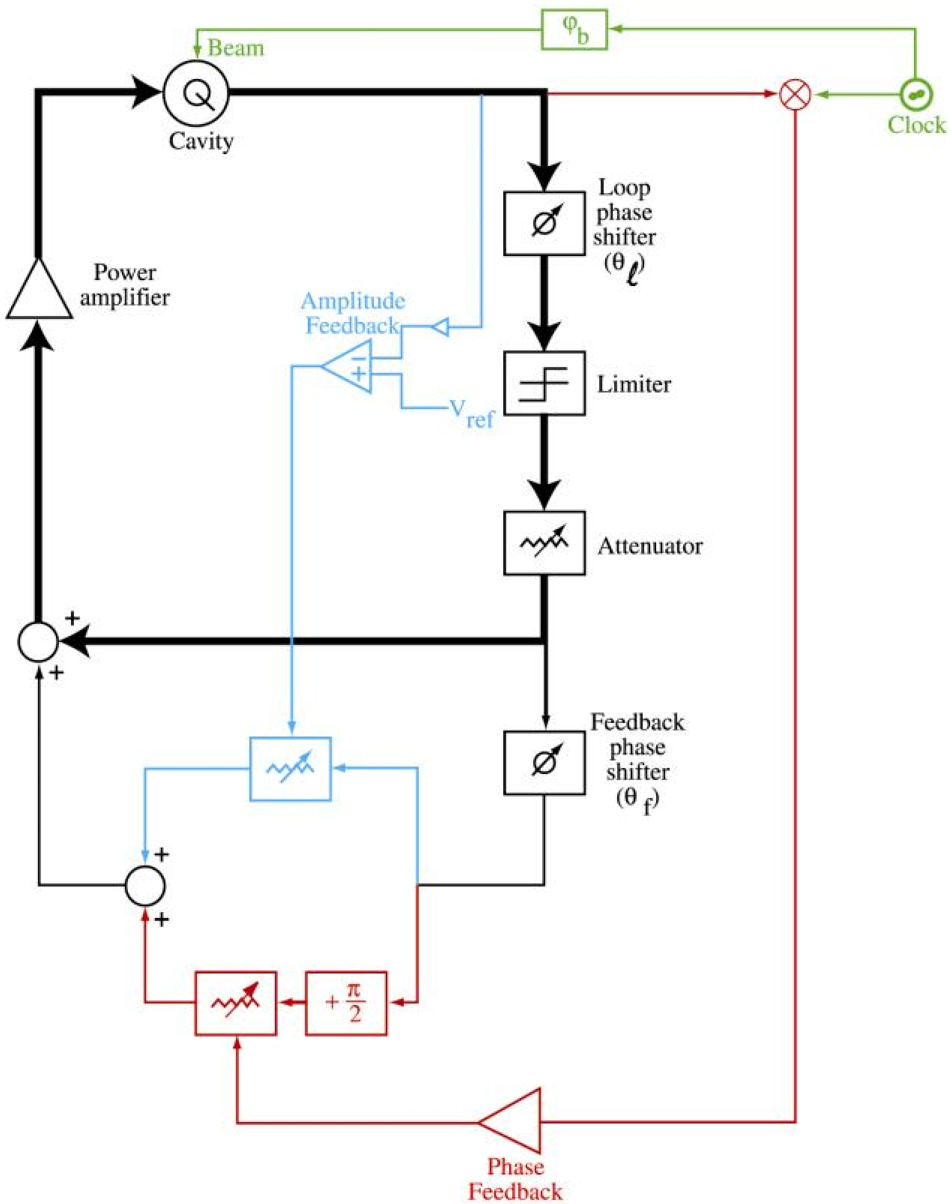
- SEL operates in the unlocked state where the loop frequency automatically tracks the resonator frequency.

Self Excited Loop

- The loop will oscillate at the frequency

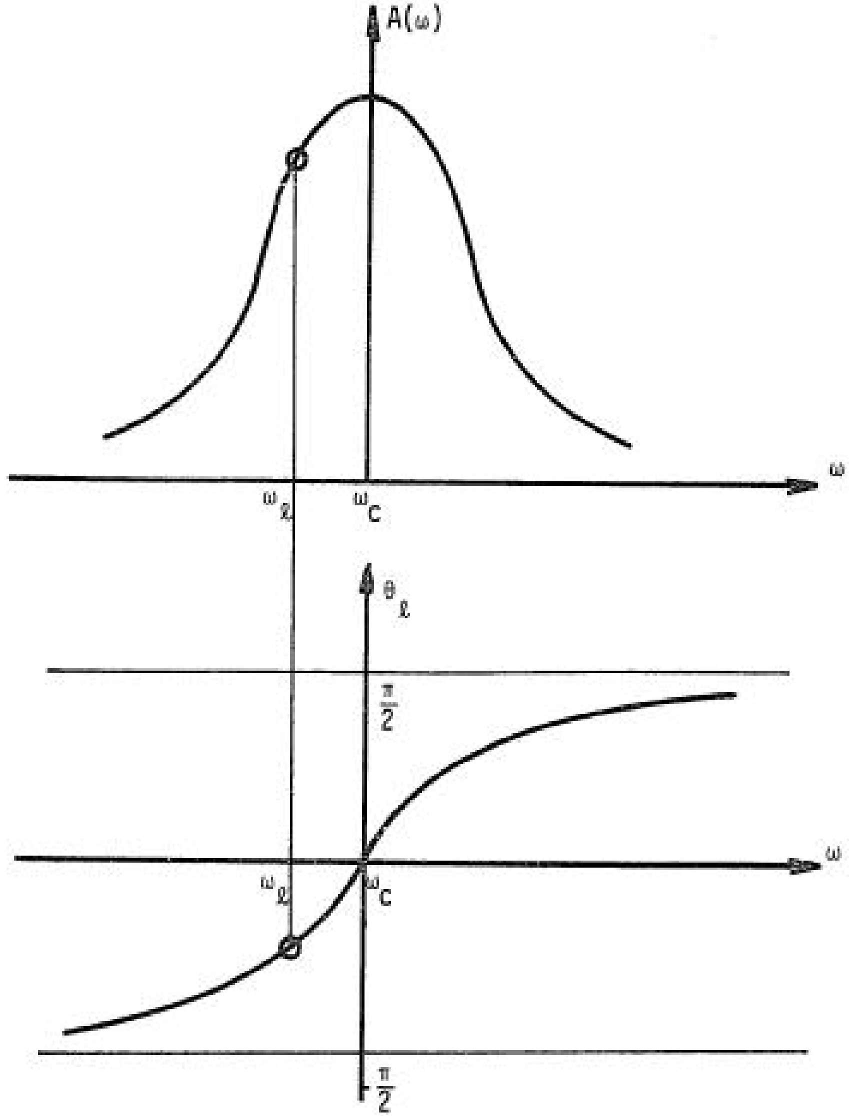
$$\omega = \omega_c \left(1 + \frac{\tan \theta_l}{2Q} \right)$$

- Amplitude feedback is provided by adding a signal in phase which is controlled by the amplitude error.
- Phase feedback is provided by adding a signal in quadrature which is controlled by the phase difference between the resonator and an external reference frequency ω_r.



J. R. Delayen, “Phase and Amplitude Stabilization of Beam-Loaded Superconducting Resonators”, Proc. Linac 92, p. 371

- To loop is operated at resonance ($\theta_l = 0$) and in the low frequency side control ($\theta_f < 0$), that gives a small amount of coupling between the phase and amplitude.



J. R. Delayen, "Phase and Amplitude Stabilization of Superconducting Resonators", Ph.D. thesis, California Institute of Technology, 1978.

Resonator Field

- The differential equation for the resonator field

$$\ddot{v} + 2\frac{1+\beta}{\tau_0}\dot{v} + \omega_c^2 v = \frac{2}{\tau_0}\dot{v}_g - \frac{2}{\tau_0}\dot{v}_b$$

$$v_g = V_{p0} 2\beta^{1/2}\left[1+\exp(i\theta_f)(\Delta v_g + i\Delta t)\right]\exp\left[i(\theta_l + \alpha)\right]$$

$$v_b = V_b \exp\left[i(\omega_r t + \varphi_b)\right] \qquad V_b = \frac{i_b R_{sh}}{2} \; and \; V_{p0} = \sqrt{R_{sh} P_{inc}}$$

i_b – beam current

R_{sh} – resonator shunt impedance

β – coupling coefficient

τ_o – intrinsic decay time

P_{inc} – power driving the resonator

Δv_g – additional in phase signal providing amplitude feedback

Δt – additional in quadrature signal providing phase feedback

Steady state condition

- Resonator field $v = V\exp(i\alpha)$

  V – real amplitude the resonator field

  α – absolute phase

- Neglect the changes of amplitude and frequency during one rf period

- Initially consider that at the steady state
  - The beam is off
  - The loop is unlocked

- When the loop is unlocked and the beam is off the steady state amplitude is found from the above equations

$$V_0 = \frac{2\beta^{1/2}}{1+\beta} V_{p0} \cos\theta_l$$

Steady state mode

- When the beam is on and loop is locked to the frequency ω_r the amount of steady state amplitude and phase feedbacks which are required to still maintain the amplitude V_o in the resonator are

$$\Delta v_{g0} = \cos\theta_l \cos(\theta_l + \theta_f)\left[\frac{b}{1+\beta}(1+y_0 y) + (y_r - y_l)y\right]$$

$$\Delta t_0 = \cos\theta_l \cos(\theta_l + \theta_f)\left[\frac{b}{1+\beta}(y_0 - y) + (y_r - y_l)\right]$$

$$y_o = \tan\theta_o$$

$$y_l = \tan\theta_l$$

$$y = \tan(\theta_l + \theta_f)$$

$$y_r = \frac{\tau_o(\omega_r - \omega_c)}{1+\beta}$$

- Ratio of power absorbed by the beam to power dissipated in the cavity

$$b = \frac{V_b \cos\varphi_o}{V_o}$$

Deviations from Steady state mode

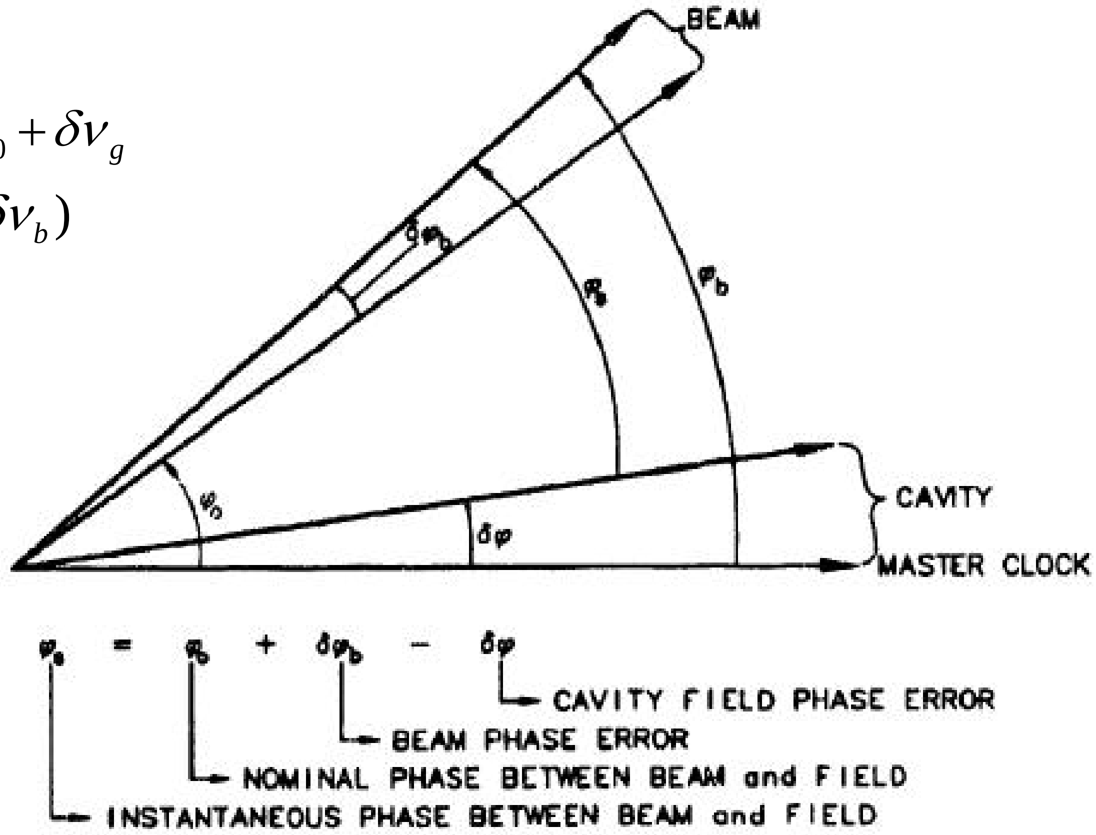
- Loop equations linearized around the steady state

$$V = V_0(1+\delta v), \Delta v_g = \Delta v_{g0} + \delta v_g$$

$$\Delta t = \Delta t_0 + \delta t, V_b = V_{b0}(1+\delta v_b)$$

$$\varphi_s = \varphi_0 + \delta\varphi_b - \delta\varphi$$

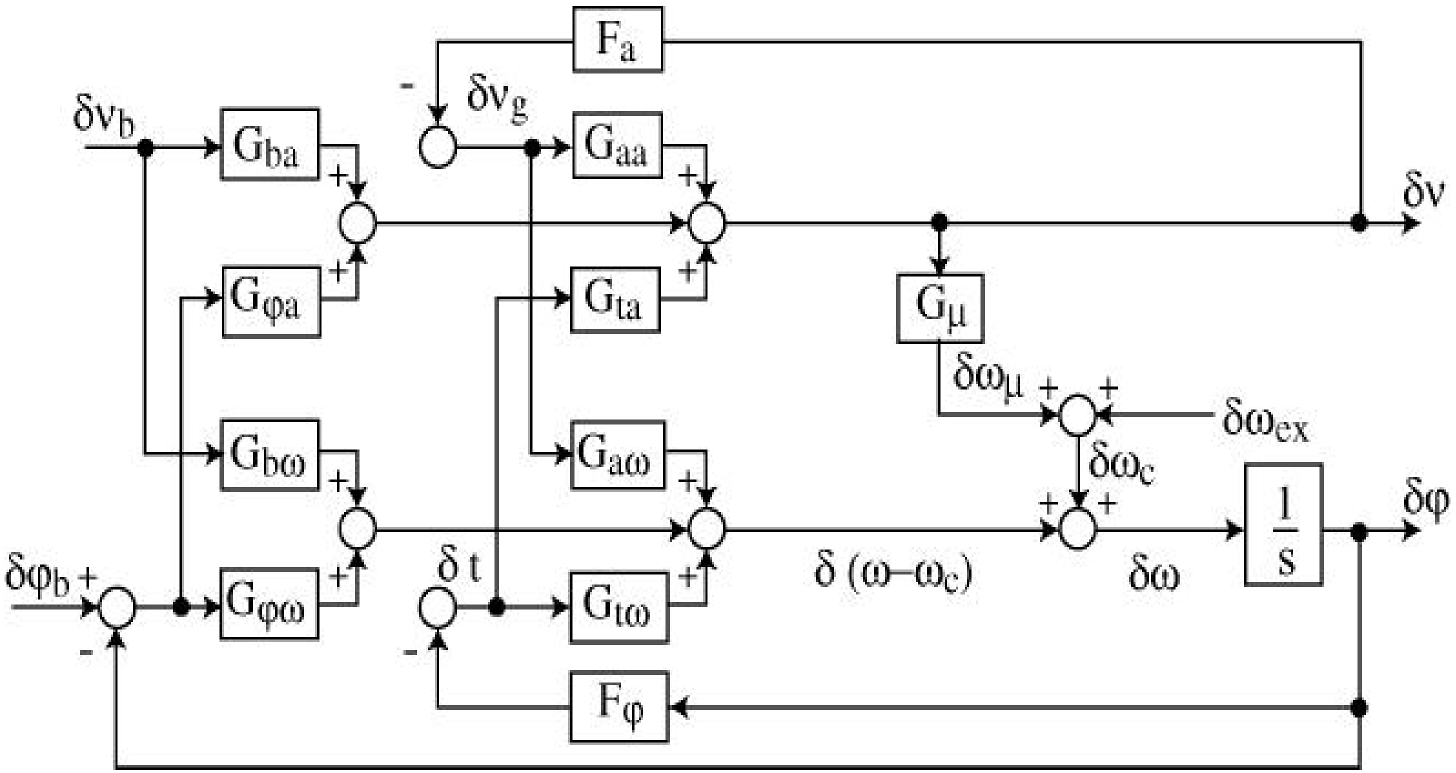
$$\omega = \omega_r + \delta\omega - \delta\omega_\mu - \delta\omega_{ex}$$



Phase relationship between the beam and the cavity field.

J. R. Delayen, “Phase and Amplitude Stabilization of Beam-Loaded Superconducting Resonators”, Proc. Linac 92, p. 371

Transfer function representation of SEL



$$\delta\omega_c = \delta\omega_{ex} + \delta\omega_\mu$$

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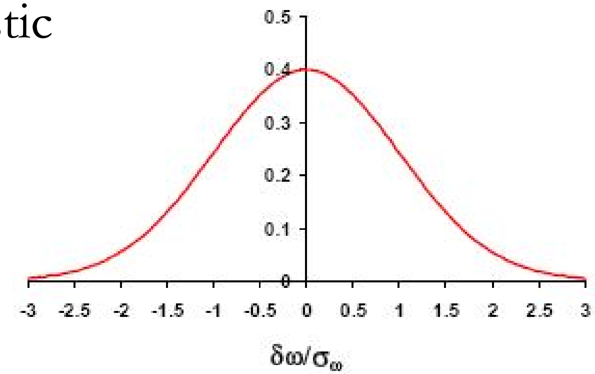
Residual errors

- Consider $n(t)$ to be a white noise stationary stochastic process of Spectral Density A^2.

$$p(\delta\omega) = \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\delta\omega}{\sigma_\omega} \right)^2 \right]$$



- Then the mean square values of fluctuations in frequency due to microphonics, field amplitude and phase are,

$$< \delta\omega_{ex}^2 > = A^2 \int_{-\infty}^{\infty} d\omega \left| G_\mu(i\omega) \right|^2 = A^2 \frac{\pi\tau_\mu}{2\Omega_\mu^2}$$

$$< \delta v^2 > = < \delta\omega_{ex}^2 > \frac{2\Omega_\mu^2}{\pi\tau_\mu} \int_{-\infty}^{\infty} d\omega \left| G_\mu(i\omega) G_a(i\omega) \right|^2$$

$$< \delta\varphi^2 > = < \delta\omega_{ex}^2 > \frac{2\Omega_\mu^2}{\pi\tau_\mu} \int_{-\infty}^{\infty} d\omega \left| G_\mu(i\omega) G_\phi(i\omega) \right|^2$$

Performance of Stabilization

- The mean square errors $<\delta \nu^2>$ and $<\delta \varphi^2>$
- Calculated with
  - No beam loading
  - Loop phase adjusted so the unlocked cavity operates on resonance ($\theta_l = 0$)
  - Small feedback angle ($\theta_f << 1$)
  - Large proportional feedback gains (k_a, $k_\varphi >> 1$)

$$<\delta \nu^2> = \frac{\tau^2 <\delta \omega_{ex}^2>}{(k_a+1)^2}\left[\theta_f\right]^2$$

$$<\delta \varphi^2> = \frac{\tau^2 <\delta \omega_{ex}^2>}{k_\varphi^2}\left[1 + \theta_f k_\mu V_o^2 \frac{2\tau}{k_a+1}\left(1 - \frac{\tau_\mu}{2}\tau\Omega_\mu^2 \frac{k_\varphi + k_a + 1}{k_\varphi (k_a+1)}\right)\right]$$

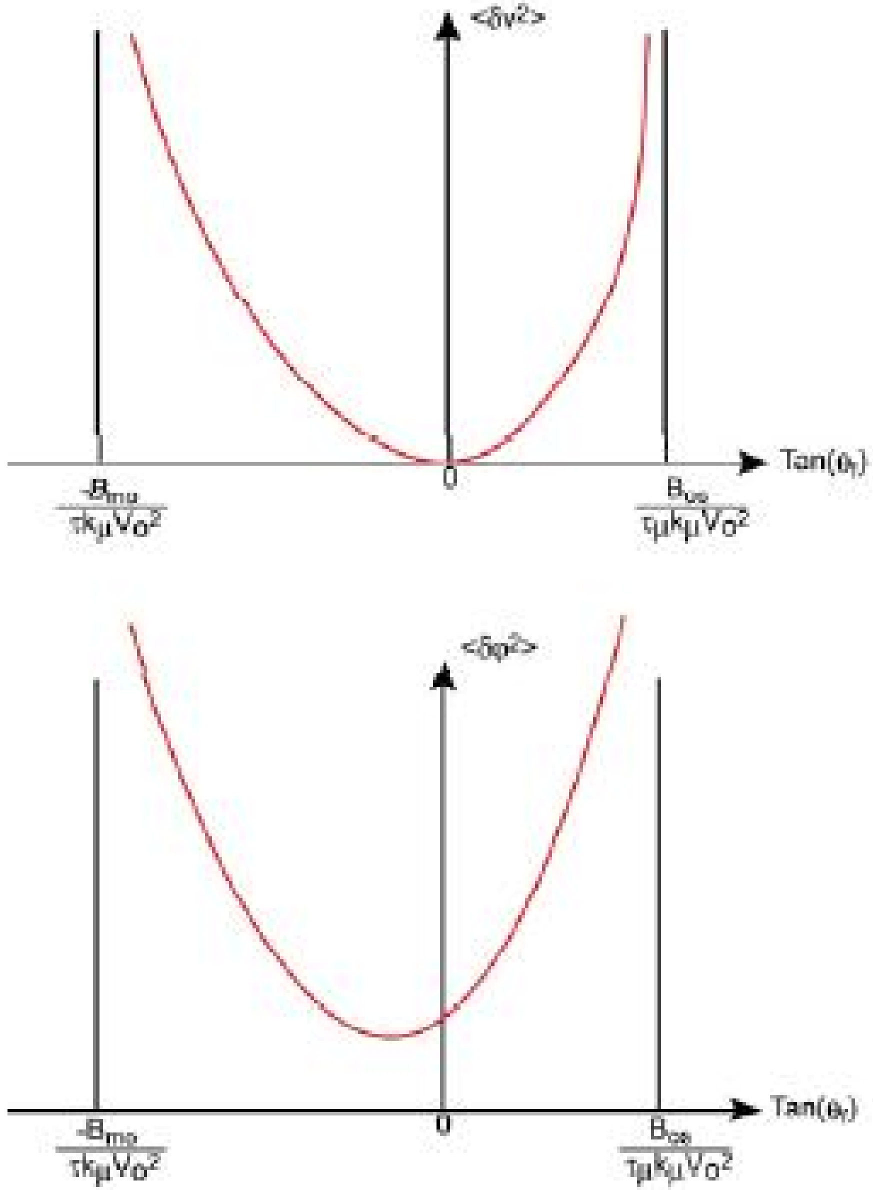
- In the absence of feedback phase shift, microphonics do not contribut to an amplitude error.

- But has a residual rms phase error.

$$< \delta\varphi^2 >^{1/2} = k_\varphi^{-1}\tau < \delta\omega_{ex}^2 >^{1/2}$$

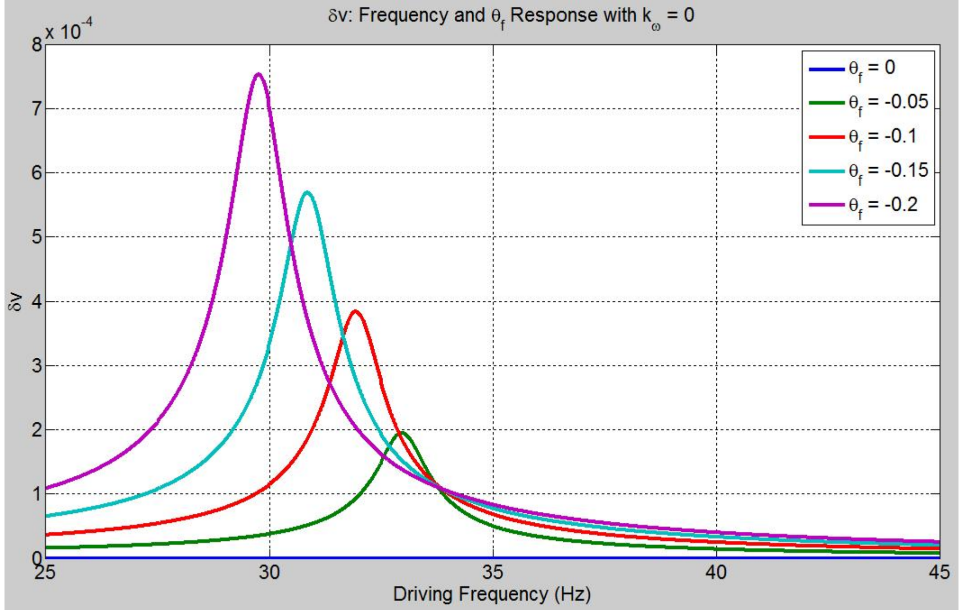
- Around $\theta_f = 0$ the amplitude error is quadratic while the phase error is linear in θ_f.

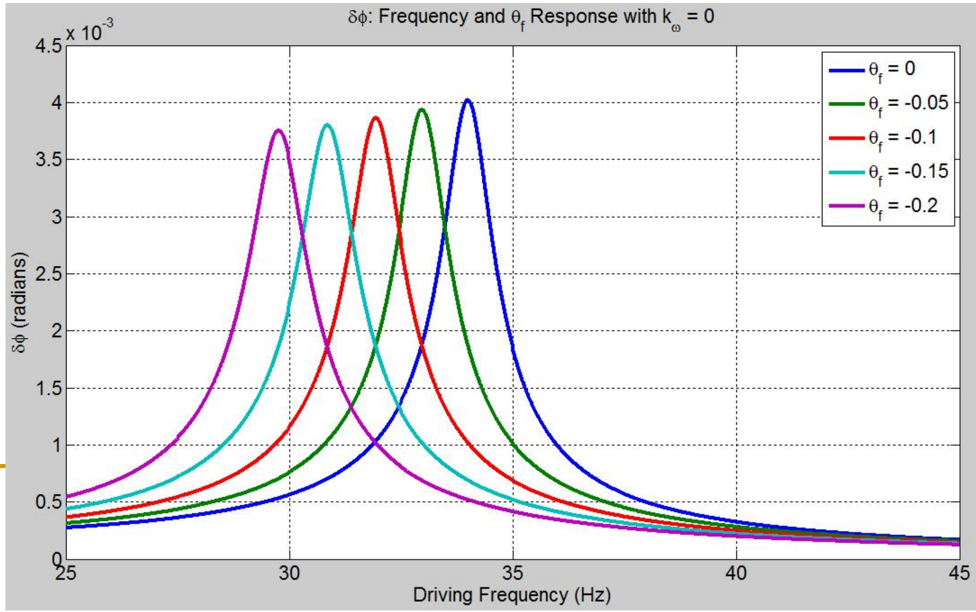
- This suggests that, if one is willing tc accept a small amount of amplitude error, the phase error can be reducec by introducing a phase shift in the feedback signals($\theta_f \neq 0$).



J. R. Delayen, "Electronic Damping of Microphonics in Superconducting Cavities", Proc. PAC 01, p. 1146

Residual errors as a function of driving frequency





A. Hofler, J.R. Delayen, “Simulation of Electronic Damping of Microphonic Vibrations in Superconducting Cavities”, Proc. PAC05

Damping by frequency feedback

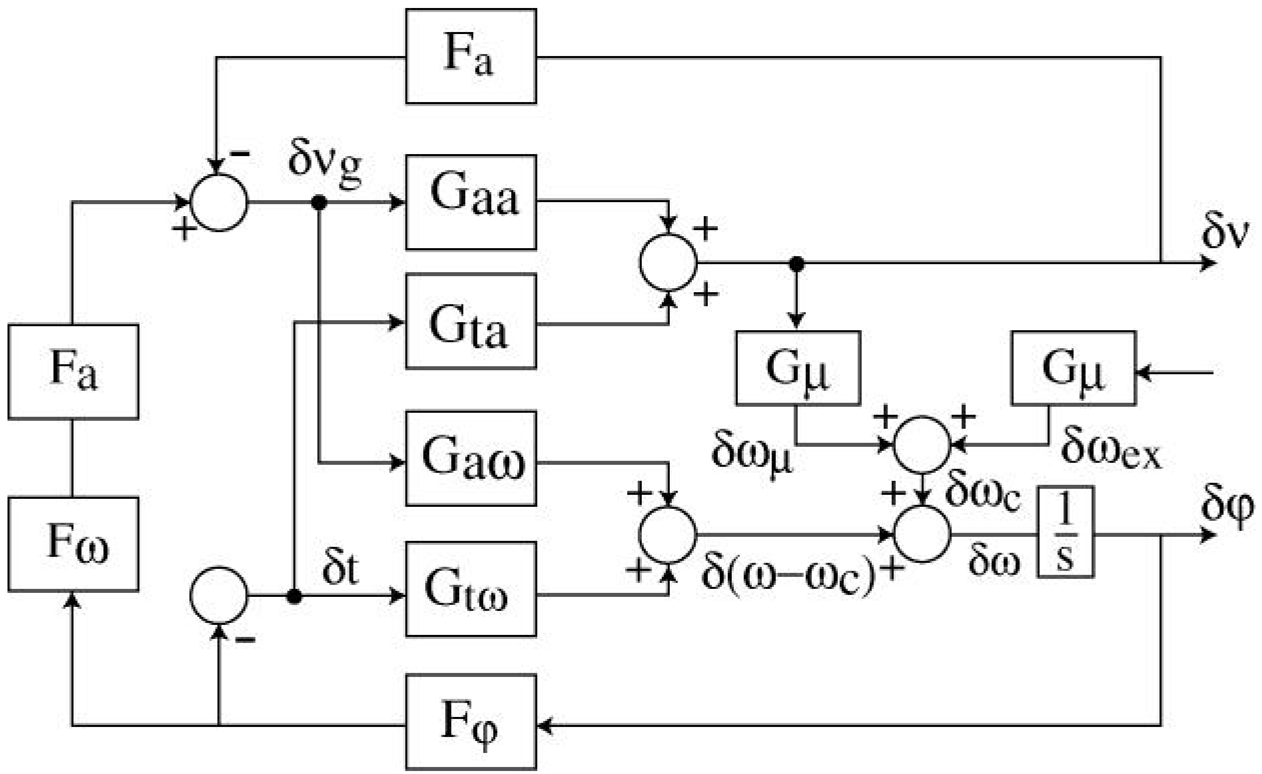
- The effectiveness of microphonics damping reduces as the feedback gain is increased.
- A more effective way to damp microphonics is to modulate the amplitude reference on purpose by an amount dependent on the instantaneous frequency offset between the cavity and the master reference, and with the appropriate phase shift in order to act as a damping mechanism.
- In the absence of beam loading, and with no feedback phase shift ($\theta_f = 0$), the signal driving the resonator is

$$\delta v_g = -F_a \delta v = -F_a \frac{(V-E)}{E} \qquad V = V_0(1+\delta v) \qquad V_g = V_{go}\left[1+\delta v_g + i\delta t\right]$$

- A modulation of the amplitude reference introduces an additional term in the signal driving the resonator

$$E = E_o\left(1+\delta e\right) \qquad \delta v_g(s) = -F_a \delta v(s) + \delta e(s) F_a$$

Transfer function diagram for Damping by frequency feedback



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Damping by frequency feedback

- Thus δt is an appropriate signal to provide a modulation because of the amplitude reference:

$$\delta e(s) = -F_\omega \delta t(s) = \delta\varphi(s) F_\varphi F_\omega$$

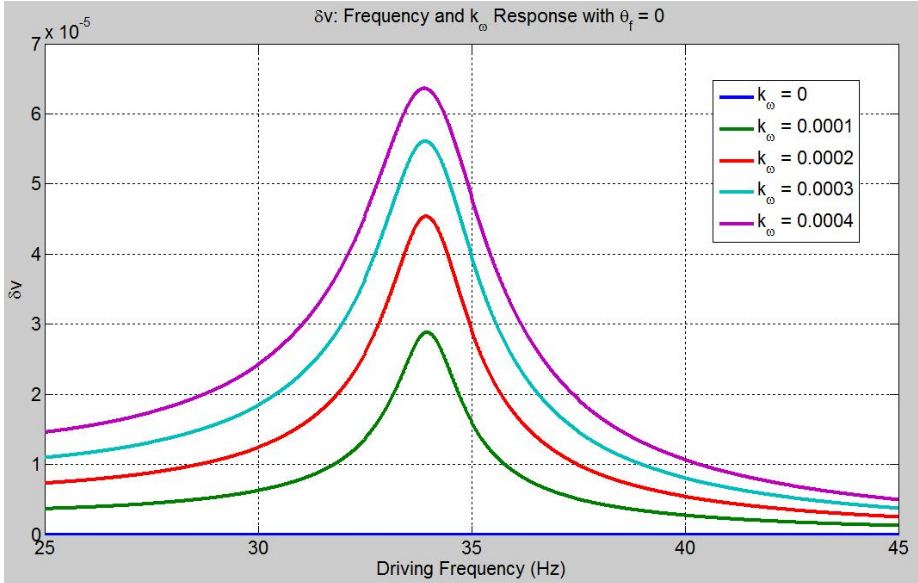
F_ω – Frequency feedback transfer function

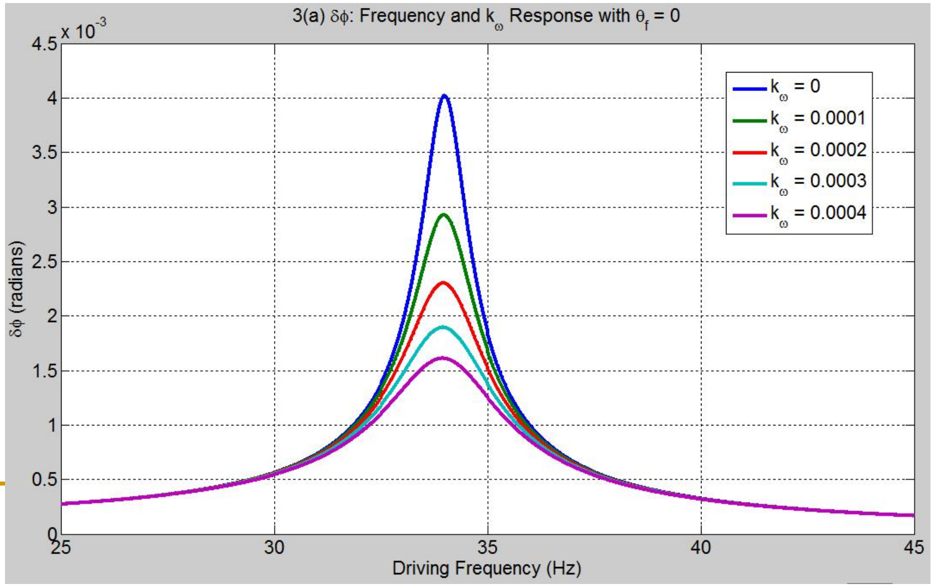
- When the phase feedback gain (k_φ) is sufficiently high, the "in quadrature" feedback signal δt is directly proportional to the instantaneous phase error which, is proportional to the instantaneous difference between cavity and reference frequency.
- For a effective damping mechanism, the frequency feedback needs to introduce a $\pi/2$ phase shift between the frequency error and the amplitude modulation. For this reason, a good choice for F_ω is an integral-type feedback of the form:

$$F_\omega = -k_\omega \frac{\Omega_\mu}{s}$$

Residual errors

- Phase error $$<\delta\varphi^2>=\frac{\tau^2<\delta\omega_{ex}^2>}{k_\varphi^2}\frac{1}{1+k_\omega\tau\tau_\omega\Omega_\mu k_\mu V_o^2}$$
- Amplitude error $$<\delta v^2>=\tau^2<\delta\omega_{ex}^2>\frac{k_\omega^2}{1+k_\omega\tau\tau_\omega\Omega_\mu k_\mu V_o^2}$$





A. Hofler, J.R. Delayen, “Simulation of Electronic Damping of Microphonic Vibrations in Superconducting Cavities”, Proc. PAC05

Summary

- SEL stabilizes the rf fields in superconducting cavities when microphonics and ponderomotive effects are present.

- This study gives the analytical models of electronic damping of microphonics in superconducting cavities with no beam loading.

- Simulations have demonstrated the effectiveness of damping by frequency feedback.

- Future study involves the study of rf field stabilization with beam loading, for low velocity applications.

References

- J.R. Delayen, “Ponderomotive Instabilities and Microphonics – A Tutorial”
- J. R. Delayen, “Phase and Amplitude Stabilization of Beam-Loaded Superconducting Resonators”, Proc. Linac 92, p. 371
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THANK YOU