SURFACE IMPEDANCE

Jean Delayen

Thomas Jefferson National Accelerator Facility
Old Dominion University

Definitions

- The electromagnetic response of a metal, whether normal or superconducting, is described by a complex surface impedance, $Z=R+iX$

  R : Surface resistance

  X : Surface reactance

  Both R and X are real

Definitions

For a semi- infinite slab:

$$Z = \frac{E_x(0)}{\int_0^\infty J_x(z)\,dz}$$ Definition

$$= \frac{E_x(0)}{H_y(0)} = i\,\omega\,\mu_0 \frac{E_x(0)}{\partial E_x(z)/\partial z\big|_{z=0_+}}$$ From Maxwell

Definitions

The surface resistance is also related to the power flow into the conductor

$$Z = Z_0\, \vec{S}(0_+)\, /\, \vec{S}(0_-)$$

$$Z_0 = \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2} \simeq 377\Omega$$ Impedance of vacuum

$$\vec{S} = \vec{E} \times \vec{H}$$ Poynting vector

and to the power dissipated inside the conductor

$$P = \tfrac{1}{2}\, R\, H^2(0_-)$$

Normal Conductors (local limit)

Maxwell equations are not sufficient to model the behavior of electromagnetic fields in materials. Need an additional equation to describe material properties

$$\frac{\partial J}{\partial t}+\frac{J}{\tau}=\frac{\sigma}{\tau}E \qquad \Rightarrow \sigma(\omega)=\frac{\sigma_0}{1-i\omega\tau}$$

For Cu at 300 K, $\tau = 3\times 10^{-14}\,\mathrm{sec}$

so for wavelengths longer than infrared $J=\sigma E$

Normal Conductors (local limit)

- **In the local limit** $\vec{J}(z) = \sigma \vec{E}(z)$

- **The fields decay with a characteristic length (skin depth)** $\delta = \left(\frac{2}{\mu_0 \omega \sigma} \right)^{1/2}$

$$E_x(z) = E_x(0) e^{-z/\delta} e^{-iz/\delta}$$

$$H_y(z) = \frac{(1-i)}{\mu_0 \omega \delta} E_x(z)$$

$$Z = \frac{E_x(0)}{H_y(0)} = \frac{(1+i)}{2} \mu_0 \omega \delta = \frac{(1+i)}{\sigma \delta} = (1+i) \left(\frac{\mu_0 \omega}{2\sigma} \right)^{1/2}$$

Normal Conductors (anomalous limit)

- At low temperature, experiments show that the surface resistance becomes independent of the conductivity
- As the temperature decreases, the conductivity σ increases
  - The skin depth decreases $$\delta = \left(\frac{2}{\mu_0 \omega \sigma} \right)^{1/2}$$
  - The skin depth (the distance over which fields vary) can become less then the mean free path of the electrons (the distance they travel before being scattered)
  - The electrons do not experience a constant electric field over a mean free path
  - The local relationship between field and current is not valid $$\vec{J}(z) \neq \sigma \vec{E}(z)$$

Normal Conductors (anomalous limit)

Introduce a new relationship where the current is related to the electric field over a volume of the size of the mean free path (*l*)

$$\vec{J}(\vec{r},t) = \frac{3\sigma}{4\pi l}\int_V d\vec{r}' \, \frac{\vec{R}\left[\vec{R}\cdot\vec{E}(\vec{r}',t-\vec{R}/v_F)\right]}{R^4}e^{-R/l} \quad \text{with} \quad \vec{R} = \vec{r}' - \vec{r}$$

Specular reflection: Boundaries act as perfect mirrors

Diffuse reflection: Electrons forget everything

Normal Conductors (anomalous limit)

- In the extreme anomalous limit

$$\left(\frac{3}{2}\frac{l^2}{\delta_{cl}^2} \gg 1\right)$$

$$\tfrac{9}{8} Z_{p=1} = Z_{p=0} = \left(\frac{\sqrt{3}\,\mu_0^2\,\omega^2 l}{16\pi\sigma}\right)^{1/3}\left(1+i\sqrt{3}\right)$$

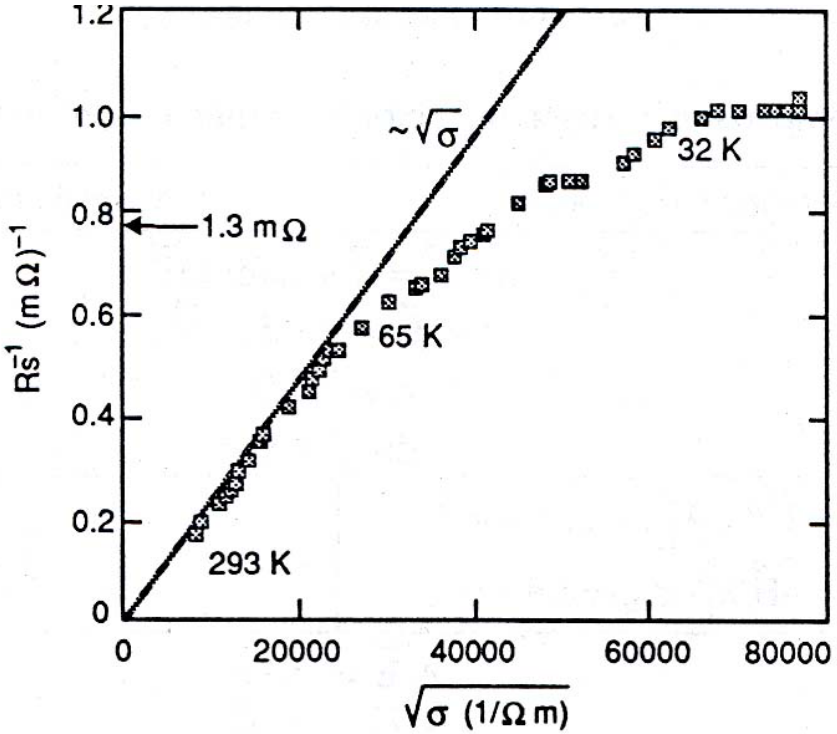


Fig. 2 Anomalous skin effect in a 500 MHz Cu cavity

p : fraction of electrons specularly scattered at surface

$1-p$: fraction of electrons diffusively scattered

Normal Conductors (anomalous limit)

$$R(l \to \infty) = 3.79 \times 10^{-5} \omega^{2/3} \left(\frac{l}{\sigma} \right)^{1/3}$$

For Cu: $l / \sigma = 6.8 \times 10^{-16}\ \Omega \cdot \mathrm{m}^2$

$$\frac{R(4.2\ \mathrm{K}, 500\ \mathrm{MHz})}{R(273\ \mathrm{K}, 500\ \mathrm{MHz})} = \frac{3.79 \times 10^{-5} \omega^{2/3} \left(\frac{l}{\sigma} \right)^{1/3}}{\sqrt{\frac{\mu_0 \omega}{2\sigma}}} \approx 0.12$$

Does not compensate for the Carnot efficiency

Surface Resistance of Superconductors

Superconductors are free of power dissipation in static fields.

In microwave fields, the time-dependent magnetic field in the penetration depth will generate an electric field.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

The electric field will induce oscillations in the normal electrons, which will lead to power dissipation

Surface Impedance in the Two-Fluid Model

In a superconductor, a time-dependent current will be carried by the Copper pairs (superfluid component) and by the unpaired electrons (normal component)

$$J = J_n + J_s$$

$$J_n = \sigma_n E_0 e^{-i\omega t}$$ (Ohm's law for normal electrons)

$$J_s = i\frac{2n_c e^2}{m_e \omega} E_0 e^{-i\omega t}$$ $$(m_e \dot{v}_c = -eE_0 e^{-i\omega t})$$

$$J = \sigma E_0 e^{-i\omega t}$$

$$\sigma = \sigma_n + i\sigma_s$$ with $$\sigma_s = \frac{2n_c e^2}{m_e \omega} = \frac{1}{\mu_0 \lambda_L^2 \omega}$$

Surface Impedance in the Two-Fluid Model

For normal conductors $$R_s = \frac{1}{\sigma\delta}$$

For superconductors

$$R_s = \Re\left[\frac{1}{\lambda_L\left(\sigma_n + i\sigma_s\right)}\right] = \frac{1}{\lambda_L}\frac{\sigma_n}{\sigma_n^2+\sigma_s^2} \simeq \frac{1}{\lambda_L}\frac{\sigma_n}{\sigma_s^2}$$

The superconducting state surface **resistance** is proportional to the normal state **conductivity**

Surface Impedance in the Two-Fluid Model

$$R_s \simeq \frac{1}{\lambda_L} \frac{\sigma_n}{\sigma_s^2}$$

$$\sigma_n = \frac{n_n e^2 l}{m_e v_F} \propto l \;\exp\left[-\frac{\Delta(T)}{kT}\right] \qquad \sigma_s = \frac{1}{\mu_0 \lambda_L^2 \omega}$$

$$\boxed{R_s \propto \lambda_L^3 \, \omega^2 \, l \exp\left[-\frac{\Delta(T)}{kT}\right]}$$

This assumes that the mean free path is much larger than the coherence length

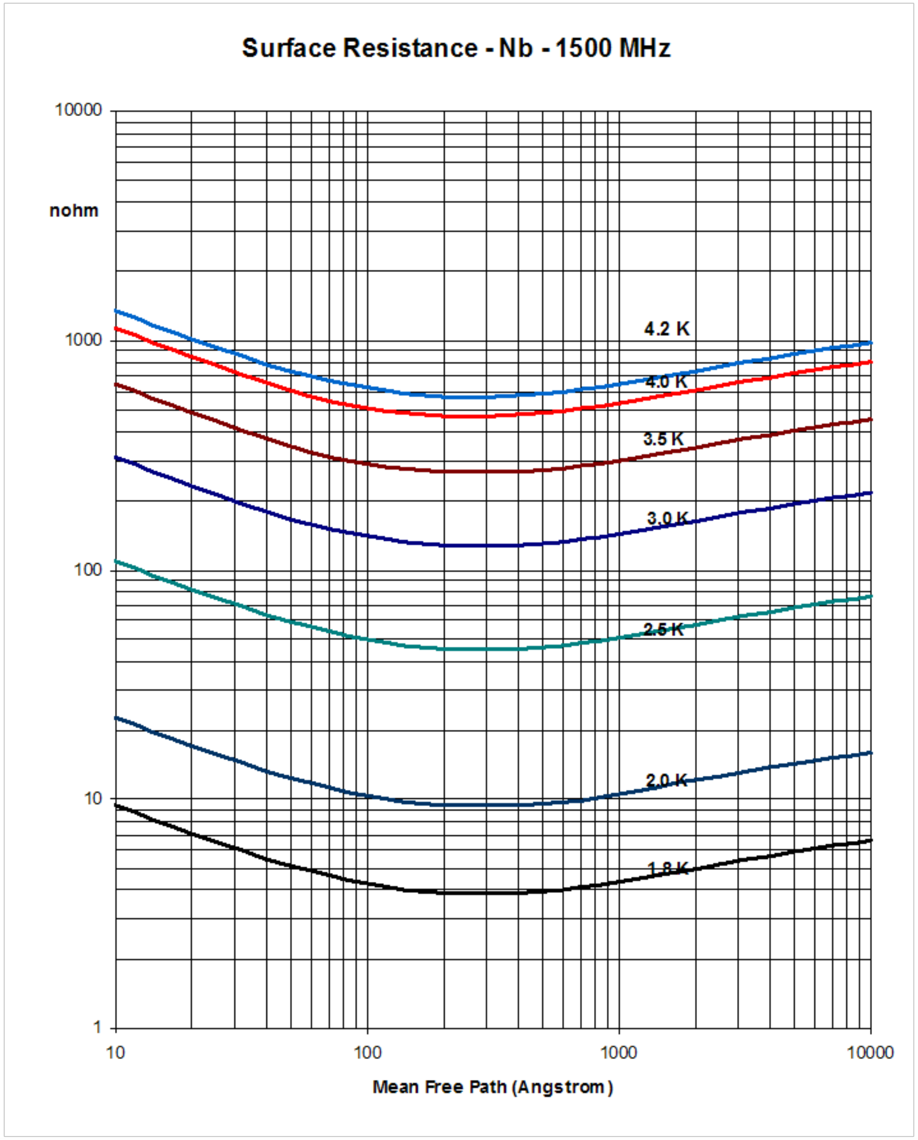
Surface Impedance in the Two-Fluid Model

For niobium we need to replace the London penetration depth with

$$\Lambda = \lambda_L \sqrt{1 + \xi / l}$$

As a result, the surface resistance shows a minimum when $\xi \approx l$

Surface Resistance of Niobium



Electrodynamics and Surface Impedance in BCS Model

$$H_0\phi + H_{ex}\ \phi = i\hbar\frac{\partial\phi}{\partial t}$$

$$H_{ex} = \frac{e}{mc}\sum A\left(r_i,t\right)p_i$$

H_{ex} is treated as a small perturbation $\qquad H_{rf} << H_c$

There is, at present, no model for superconducting surface resistance at high rf field

$$J \propto \int\frac{R\left[R\cdot A\right]I\left(\omega,R,T\right)e^{-\frac{R}{l}}}{R^4}dr$$

similar to Pippard's model

$$J\left(k\right) = -\frac{c}{4\pi}K\left(k\right)A\left(k\right)$$

$K\left(0\right)\neq 0:$ Meissner effect

Surface Resistance of Superconductors

Temperature dependence

–close to T_c :

dominated by change in $\lambda(t)$ $\quad \dfrac{t^4}{\left(1-t^2\right)^{3/2}}$

–for $T < \dfrac{T_c}{2}$:

dominated by density of excited states $\sim e^{-\Delta/kT}$

$$R_s \sim \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{\mathrm{kT}}\right)$$

Frequency dependence

ω^2 is a good approximation

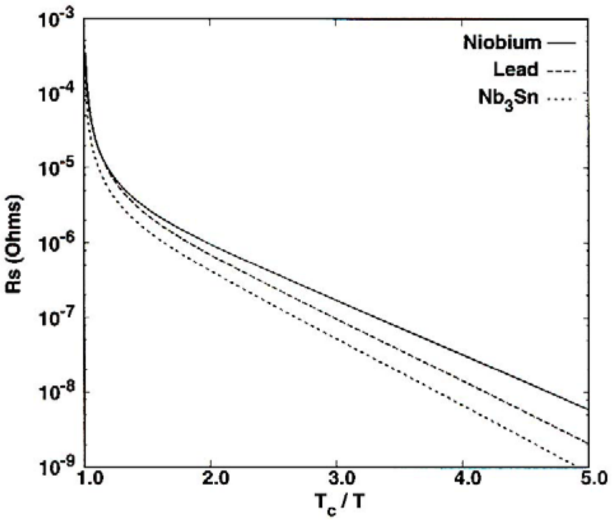


Figure 4.5: Theoretical surface resistance at 1.5 GHz of lead, niobium and Nb_3Sn as calculated from program [94]. The values given in Table 4.1 were used for the material parameters.

Surface Resistance of Superconductors

- The surface resistance of superconductors depends on the frequency, the temperature, and a few material parameters
  - Transition temperature
  - Energy gap
  - Coherence length
  - Penetration depth
  - Mean free path

- A good approximation for $T<T_c/2$ and $\omega<<\Delta/h$ is

$$R_s \sim \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{kT}\right) + R_{res}$$

Surface Resistance of Superconductors

$$R_s \sim \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{kT}\right) + R_{res}$$

In the dirty limit $\quad l \ll \xi_0 \qquad R_{BCS} \propto l^{-1/2}$

In the clean limit $\quad l \gg \xi_0 \qquad R_{BCS} \propto l$

R_{res}:

- Residual surface resistance
- No clear temperature dependence
- No clear frequency dependence
- Depends on trapped flux, impurities, grain boundaries, …

Surface Resistance of Superconductors

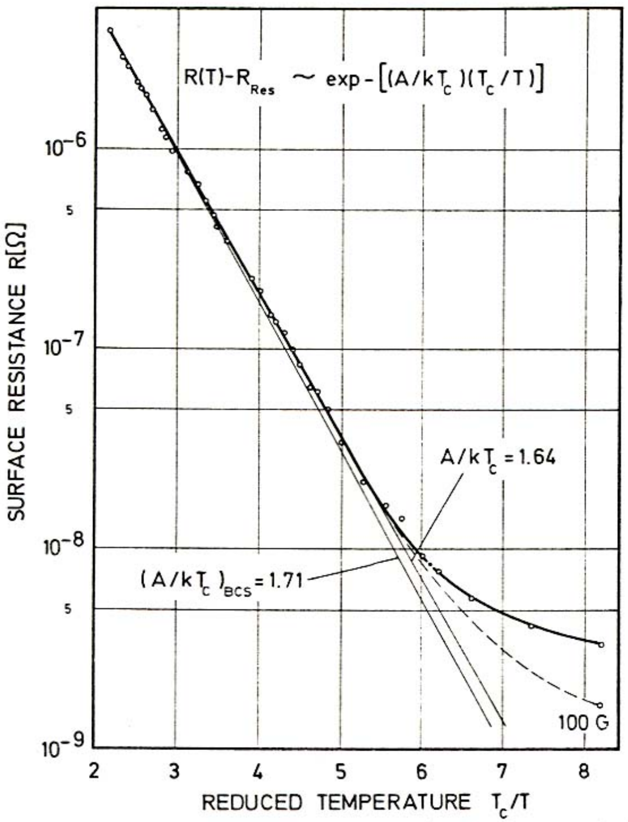


Fig. 2. Temperature dependence of surface resistance of niobium at 3.7 GHz measured in the TE_{011} mode at $H_{rf} \simeq 10$ G. The values computed with the BCS theory used the following material parameters:

$T_c = 9.25$ K; $\lambda_L(T=0, l=\infty) = 320$ Å;
$\Delta(0)/kT = 1.85$; $\xi_F(T=0, l=\infty) = 620$ Å; $l = 1000$ Å or 80 Å.

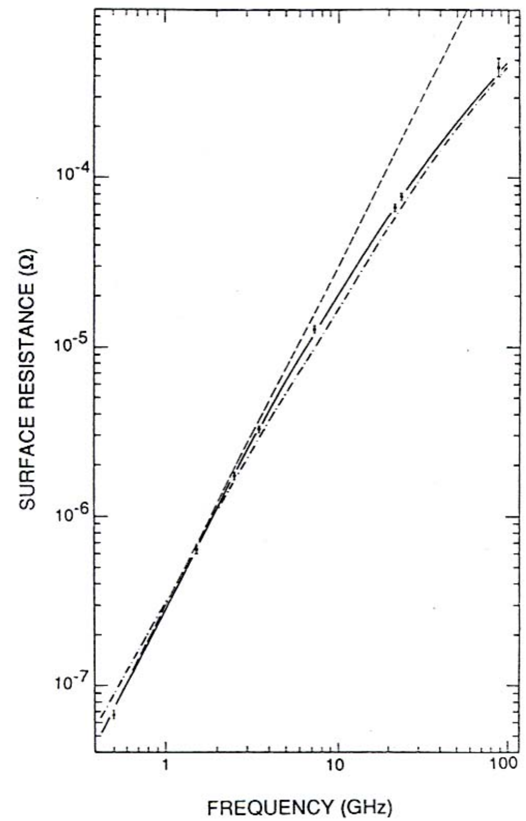
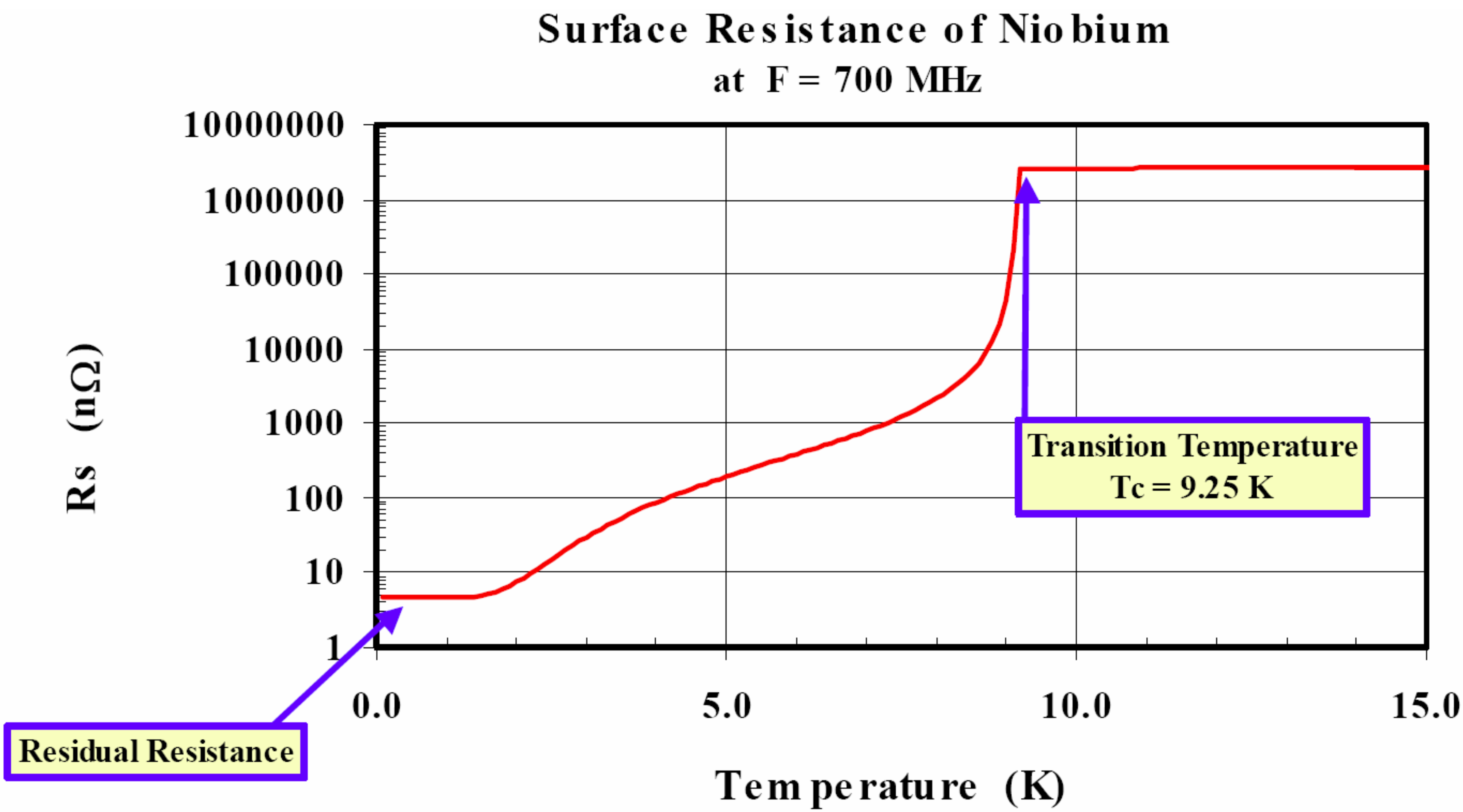
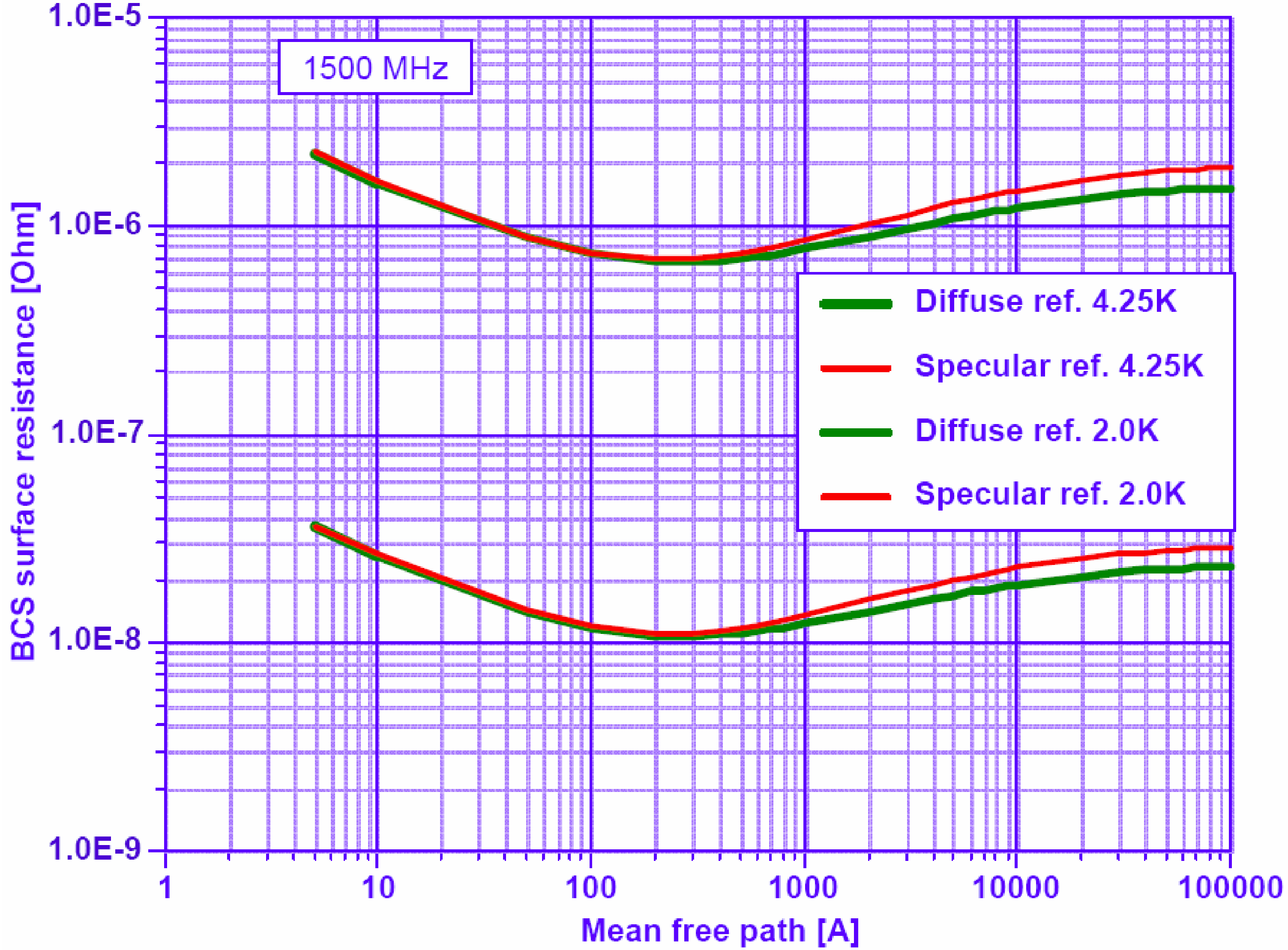


Fig. 5. The surface resistance of Nb at 4.2 K as a function of frequency [62,63]. Whereas the isotropic BCS surface resistance (– · – ·) resulted in $R \propto \omega^{1.8}$ around 1 GHz, the measurements fit better to ω^2 (– – –). The solid curve, which fits the data over the entire range, is a calculation based on the smearing of the BCS density-of-states singularity by the energy gap anisotropy in the presence of impurity scattering [61]. The authors thank G. Müller for providing this figure.

Surface Resistance of Niobium



Surface Resistance of Niobium



Super and Normal Conductors

- Normal Conductors
  - Skin depth proportional to $\omega^{-1/2}$
  - Surface resistance proportional to $\omega^{1/2 \rightarrow 2/3}$
  - Surface resistance independent of temperature (at low T)
  - For Cu at 300K and 1 GHz, R_s=8.3 mΩ

- Superconductors
  - Penetration depth independent of ω
  - Surface resistance proportional to ω^2
  - Surface resistance strongly dependent of temperature
  - For Nb at 2 K and 1 GHz, $R_s \approx 7$ nΩ

However: do not forget Carnot