Resonant Cavities

Accelerator and Beam Diagnostics
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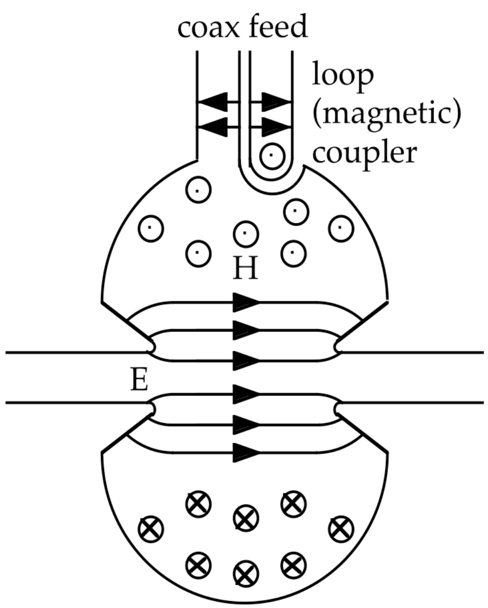
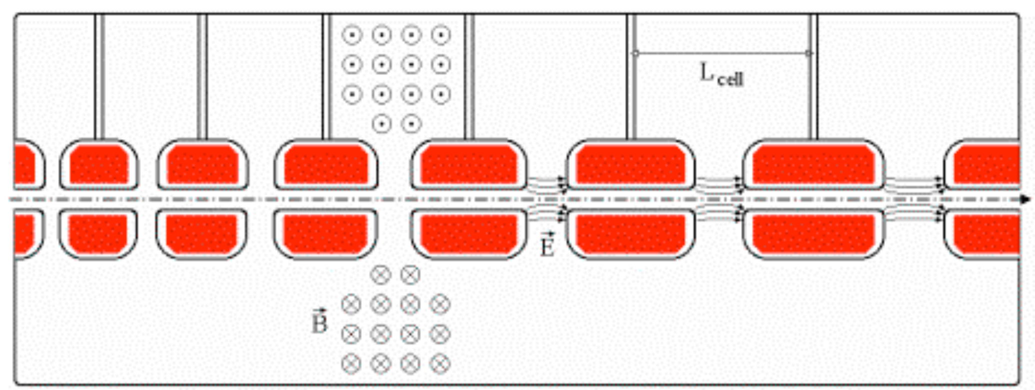
Overview

- Introduction
- Parallel resonant circuit description
  - Transmission measurements (S21)
  - Reflection measurements (S11)
  - Quality factor
- Resonant modes
  - Monopole and dipole modes
- Real cavity design
- Bead pull measurements

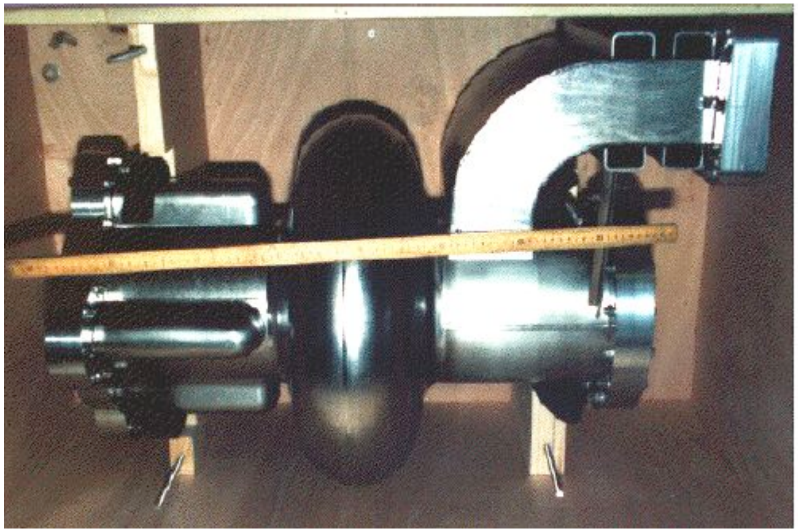
RF Cavities

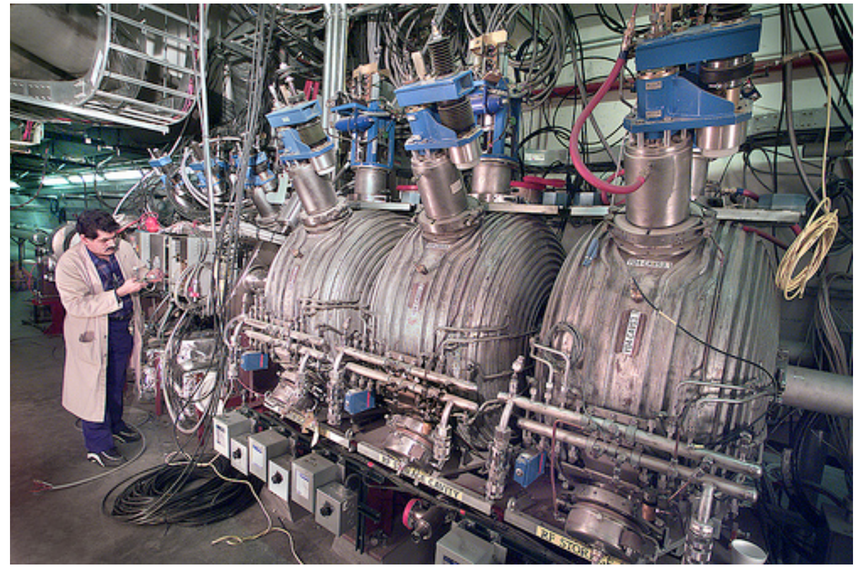
- Radiofrequency (RF) cavities are used to
  - Store energy for accelerating beam
  - Extract beam energy (for example as a beam pickup)
  - Modulate beam energy or position as a beam kicker

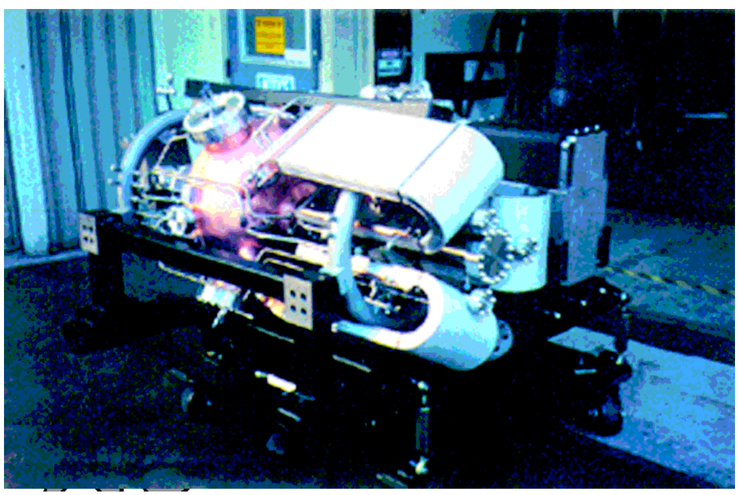




Example RF Cavities







Accelerating Cavity

Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

An accelerating cavity needs to provide an electric field E longitudinal with the velocity of the particle

Magnetic fields provide deflection but no acceleration

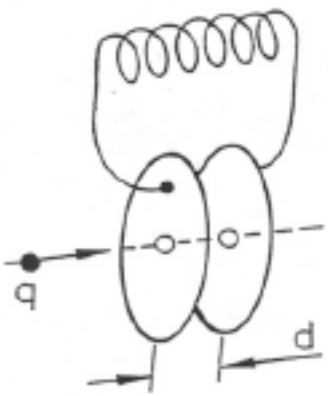
DC electric fields can provide energies of only a few MeV

Higher energies can be obtained only by transfer of energy from traveling waves →resonant circuits

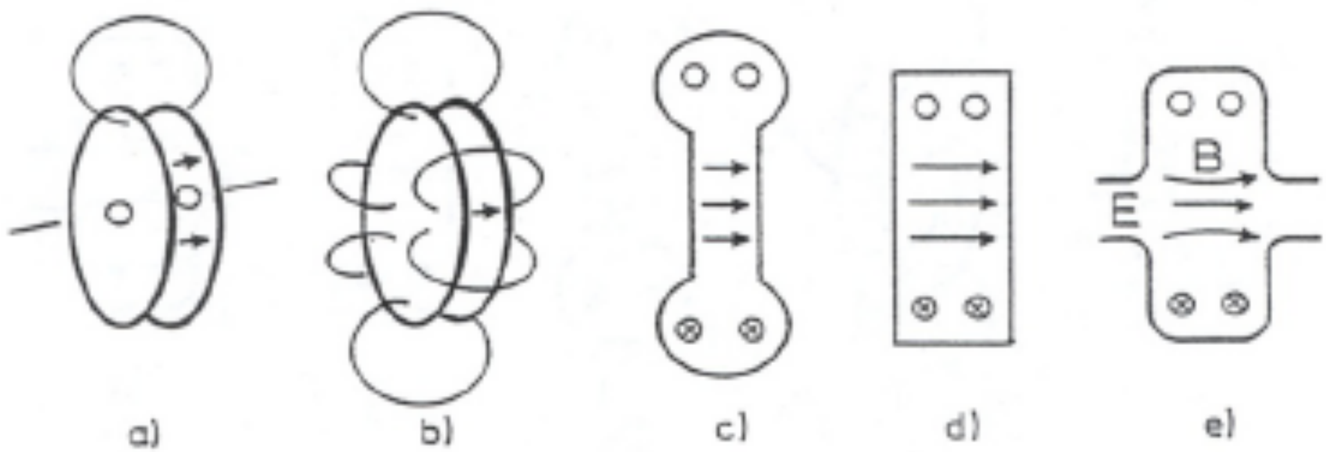
Transfer of energy from a wave to a particle is efficient only is both propagate at the same velocity

Energy Gain $$E = \frac{1}{L}\int E_z(z)\cos(\omega z / \beta c)dz$$

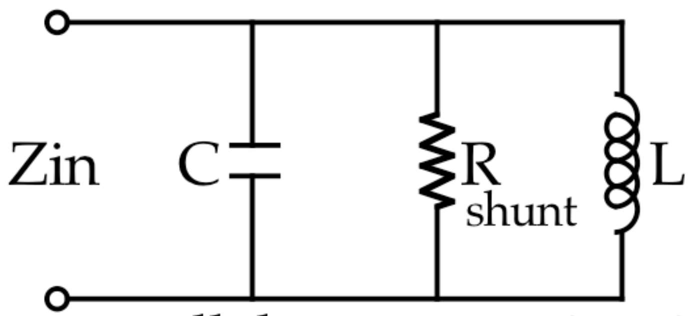
Parallel Resonant Circuit Model



- Imagine two capacitive plates with a parallel inductor.
- This creates a resonator with resonant frequency $\omega_o = \frac{1}{\sqrt{LC}}$
- If the inductor becomes many single loops of wire, this eventually becomes an accelerating cavity



Parallel Resonant Circuit Model



parallel resonant circuit

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1}$$

$$= \frac{R}{1 + jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

$$\approx \frac{R}{1 + jQ2\left(\frac{\delta\omega}{\omega_o}\right)}$$

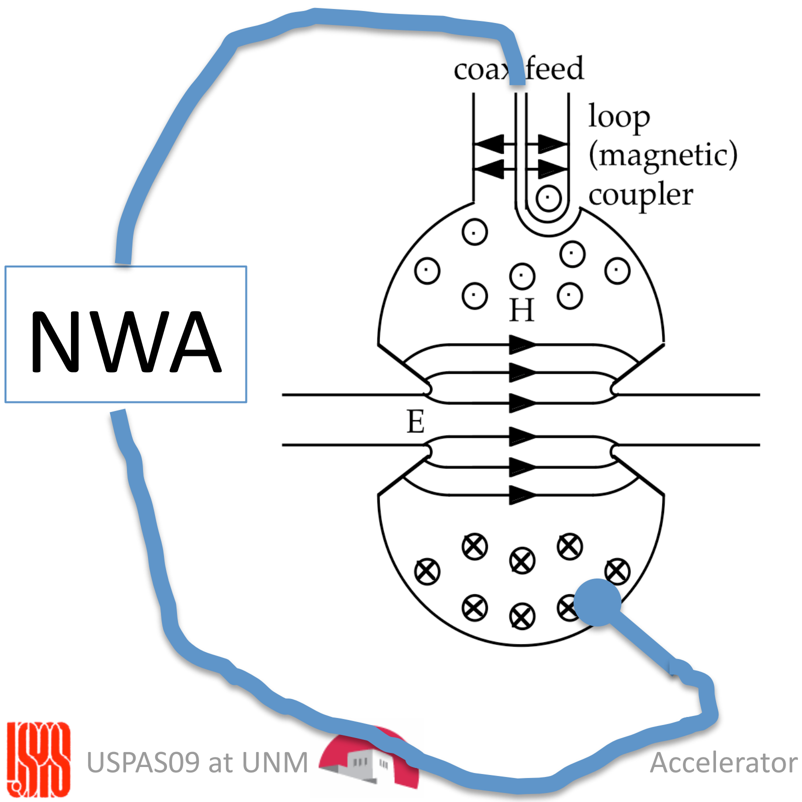
- Treat impedance of an isolated cavity mode as a parallel LRC circuit
- Voltage gained is given by $P_{loss} = \frac{1}{2}\frac{V^2}{R_s}$

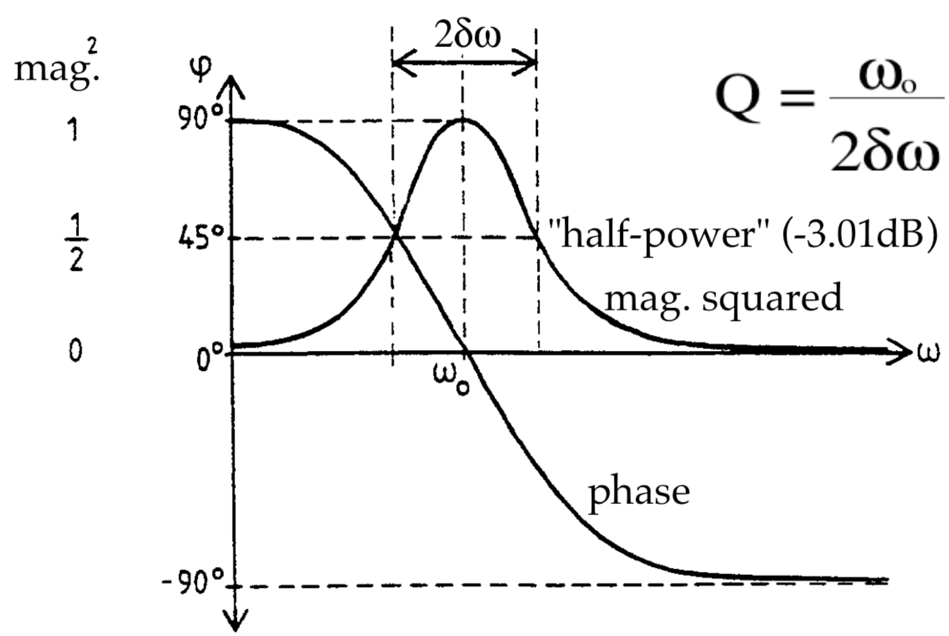
$$\frac{R}{Q} = \sqrt{\frac{L}{C}} = \frac{1}{\omega C} = \omega L$$

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad Q = \frac{R}{\omega_o L} = \omega_o RC$$

Transmission (S21) measurement

- Mode resonant frequency and Q can be measured in a transmission measurement.
- This is usually done with a network analyzer coupled to two cavity probes.





Quality Factor

Quality Factor Q_0:

$$Q_0 \equiv \frac{\text{Energy stored in cavity}}{\text{Energy dissipated in cavity walls per radian}} = \frac{\omega_0 U}{P_{diss}}$$

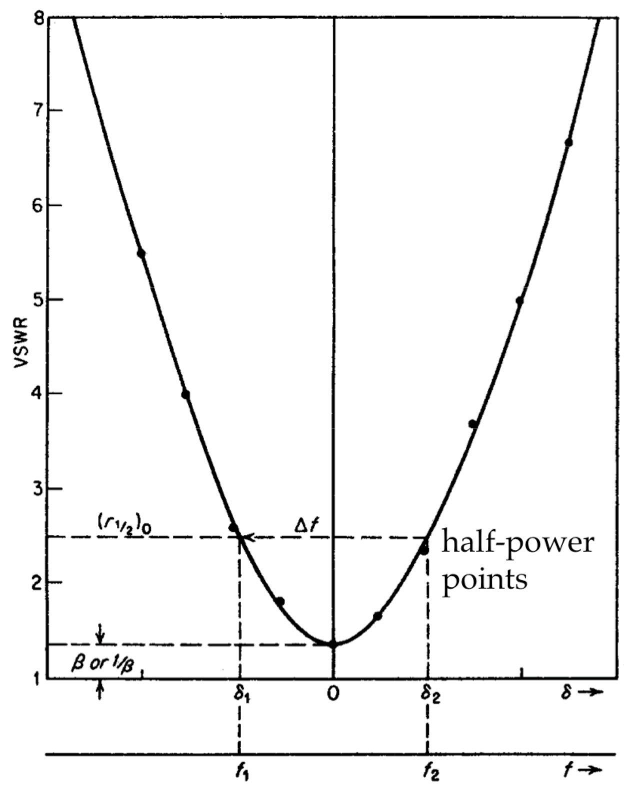
$$= \omega_0 \tau_0 = \frac{\omega_0}{\Delta\omega_0}$$

$$Q_0 = \frac{\omega\mu_0}{R_s} \frac{\int_V dV\, |\mathbf{H}|^2}{\int_A da \left|\mathbf{H}_{||}\right|^2}$$

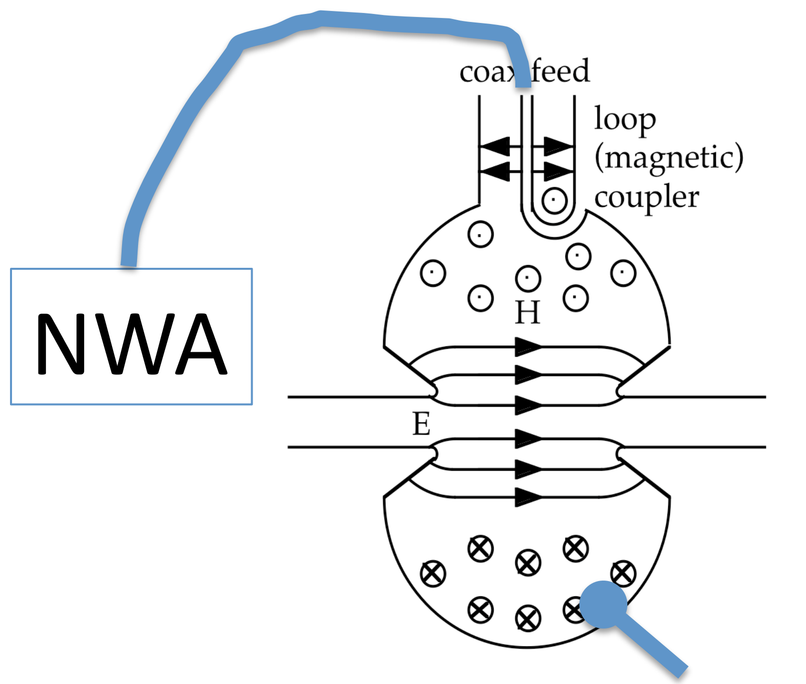
Lower surface resistance gives higher Q. For a given R/Q, this gives higher R. Lower external power is required for a given voltage V.

Reflection (S11) near resonance

- The cavity frequency, Q, and coupling factor can be measured in reflection

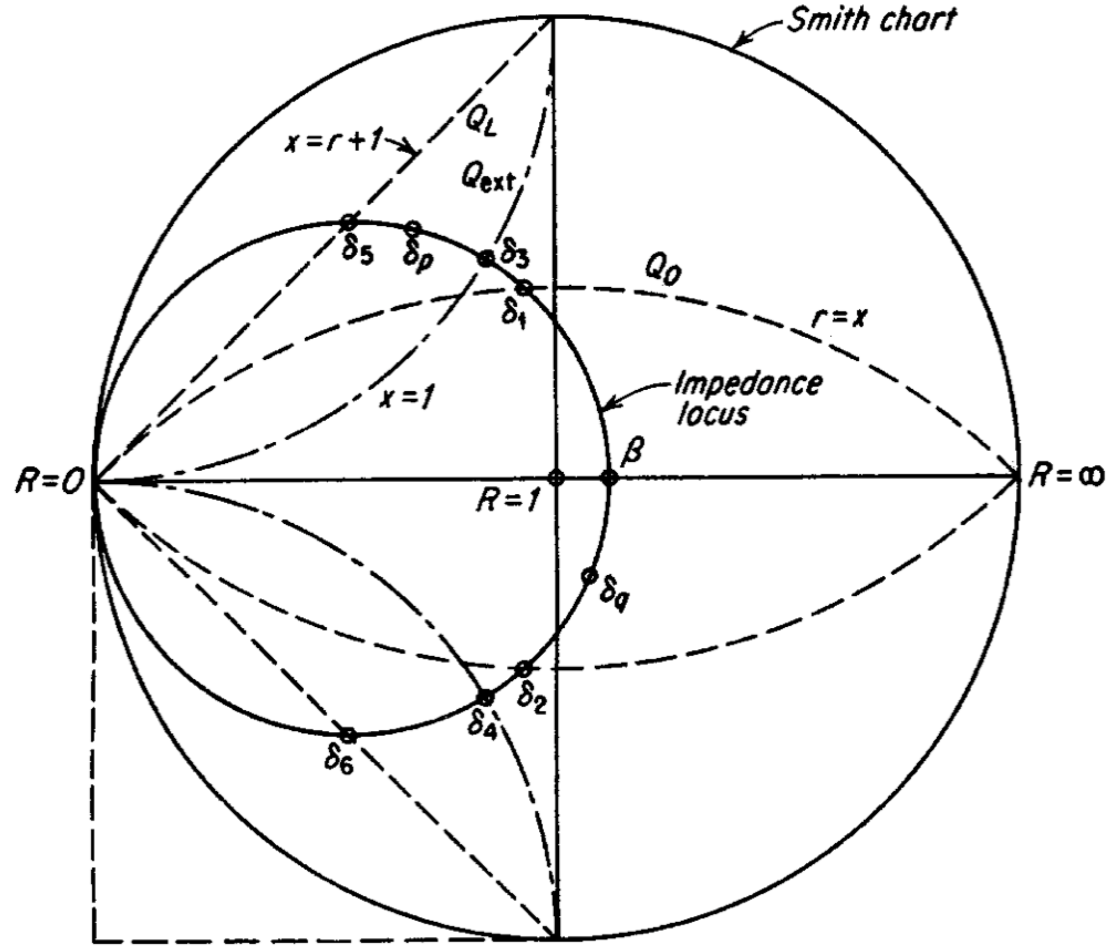


VSWR close to resonance



Smith Chart View

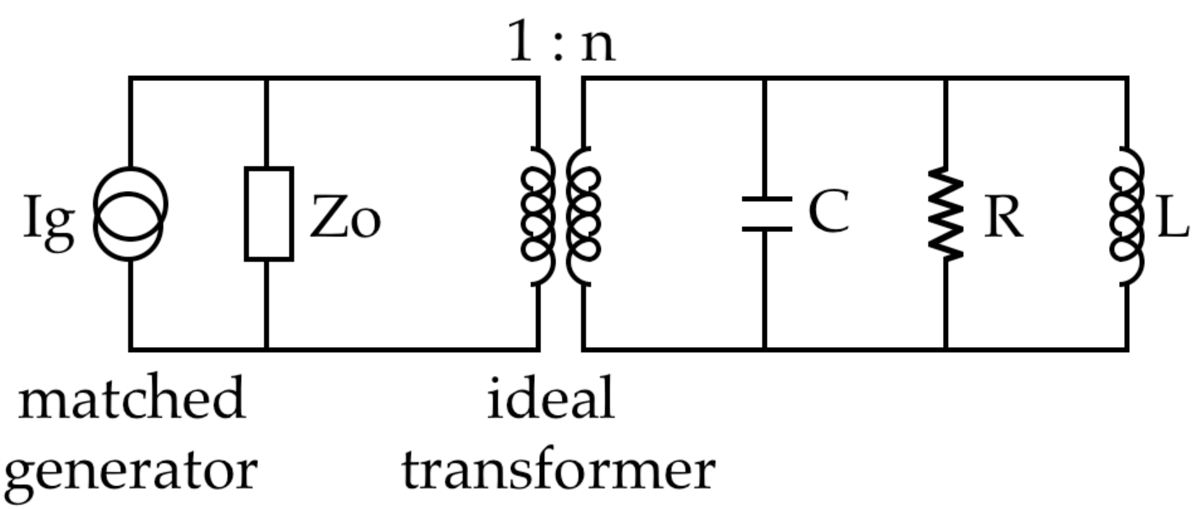
- The Smith Chart (Phillip Smith) is a polar display of the complex reflection coefficient i.e. phase and amplitude)
- A resonator appears as a circle in this format



Identification of the half-power points from the Smith chart.

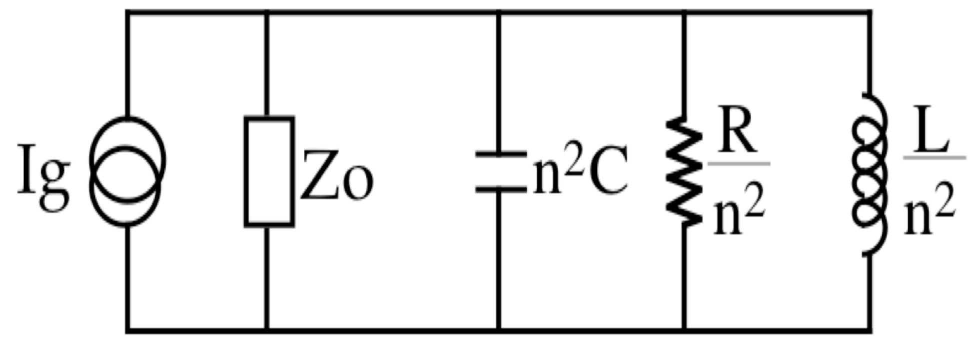
Cavity Coupling

- To accelerate beam, we have to power the cavity.
- Typical power sources include klystrons, tetrodes, etc.
- It is most efficient to match the load (cavity+beam) to the power source.
- Model the generator as a matched current source and coupling as a n turn transformer.

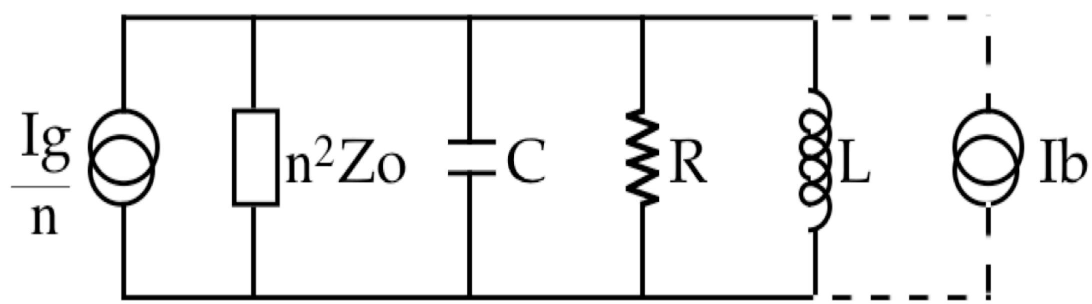


Coupling to external generator/load

- Transform the cavity impedance to the load



- Transform the generator current to the load



Cavity Coupling

- This model allows efficient power transfer to the cavity/beam.
- Some definitions

$$\beta = \frac{\text{power loss in ext. cct}}{\text{power loss in cavity}} = \frac{Q_o}{Q_{ext}} = \frac{R}{n^2 Z_o}$$

Coupling beta

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}$$

Loaded Q

$$Q_o = (1+\beta) Q_L$$

Loaded Q

Optimal coupling (no beam)

$$\beta = 1 \quad Q_L = \frac{Q_o}{2} \qquad n^2 = \frac{R}{Z_o}$$

Resonant Modes

Electromagnetic modes satisfy Maxwell equations

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix} = 0$$

With the boundary conditions (assuming the walls are made of a material of low surface resistance)

no tangential electric field $\quad \vec{n}\times\vec{E} = 0$

no normal magnetic field $\quad \vec{n}\bullet\vec{H} = 0$

Resonant Modes

Assume everything $\sim e^{-i\omega t}$

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix} = 0$$

For a given cavity geometry, Maxwell equations have an infinite number of solutions with a sinusoidal time dependence

For efficient acceleration, choose a cavity geometry and a mode where:

- Electric field is along particle trajectory
- Magnetic field is 0 along particle trajectory
- Velocity of the electromagnetic field is matched to particle velocity

Example: PEP-II Cavity

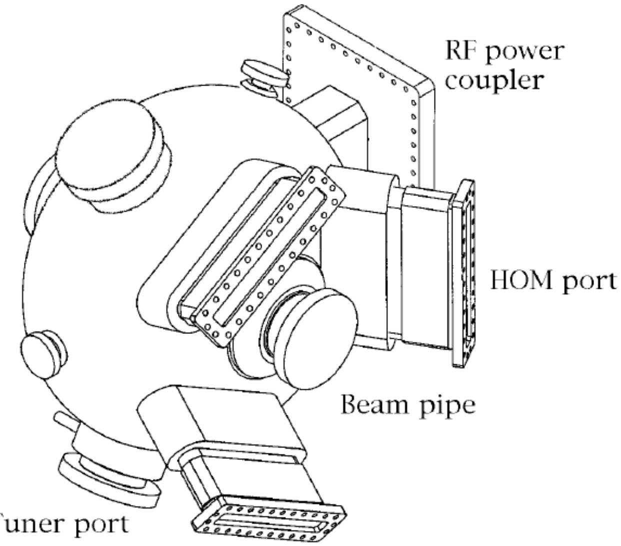
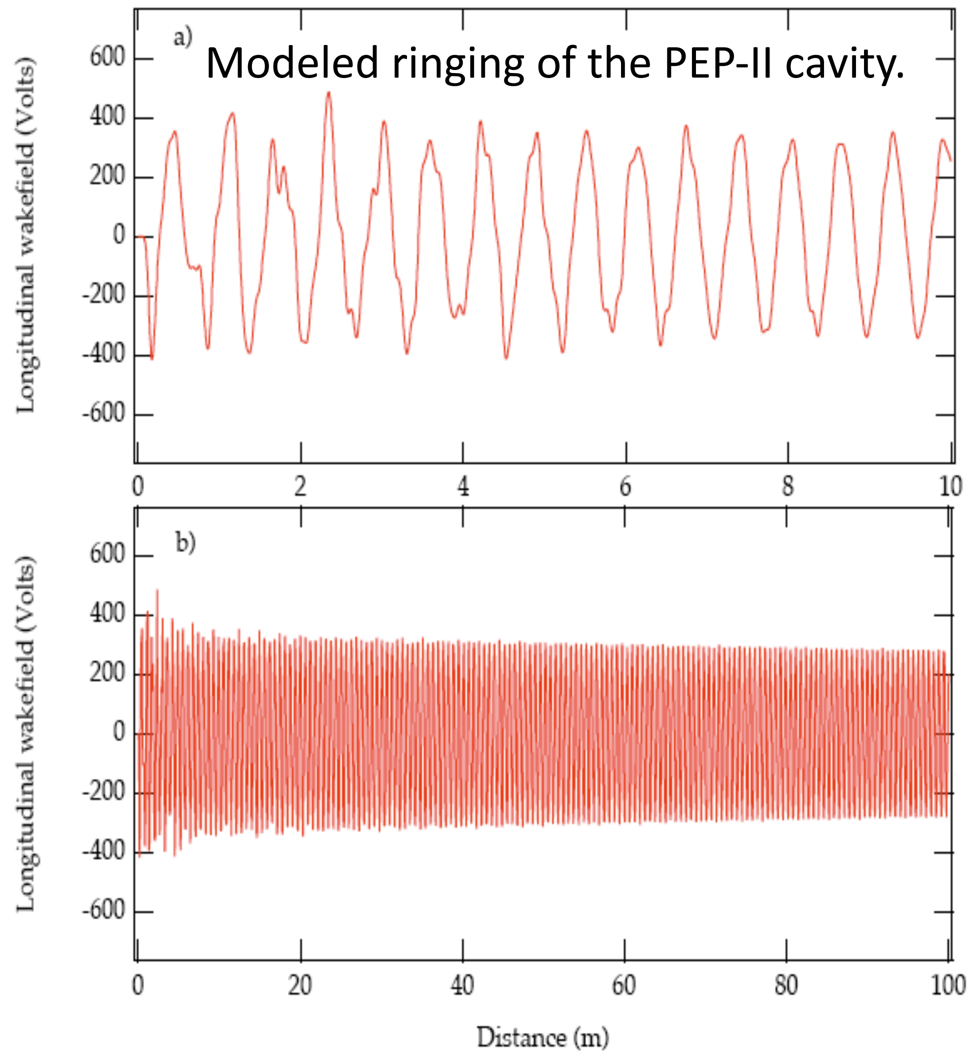


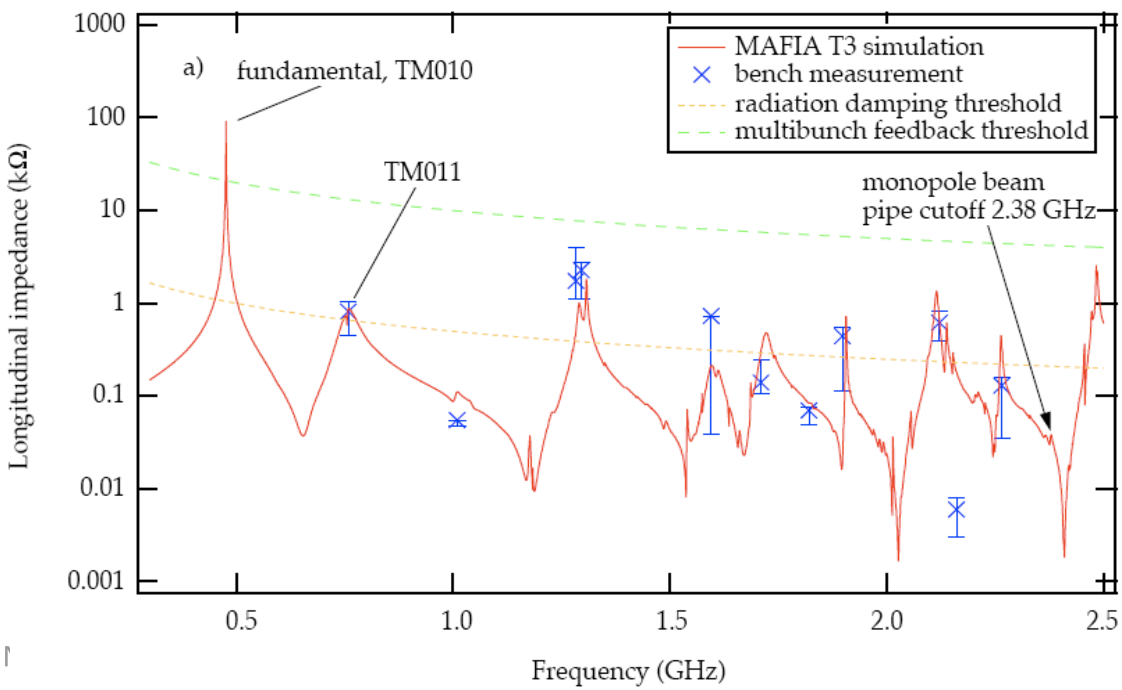
FIG. 1. CAD drawing of the PEP-II cavity. The waveguides are designed to couple to the cavity HOMs.

- Cavities have large number of resonant modes.
- Frequencies extend up to cutoff frequency of the beam pipe (TE or TM)
- Ringing is usually dominated by cavity fundamental mode.

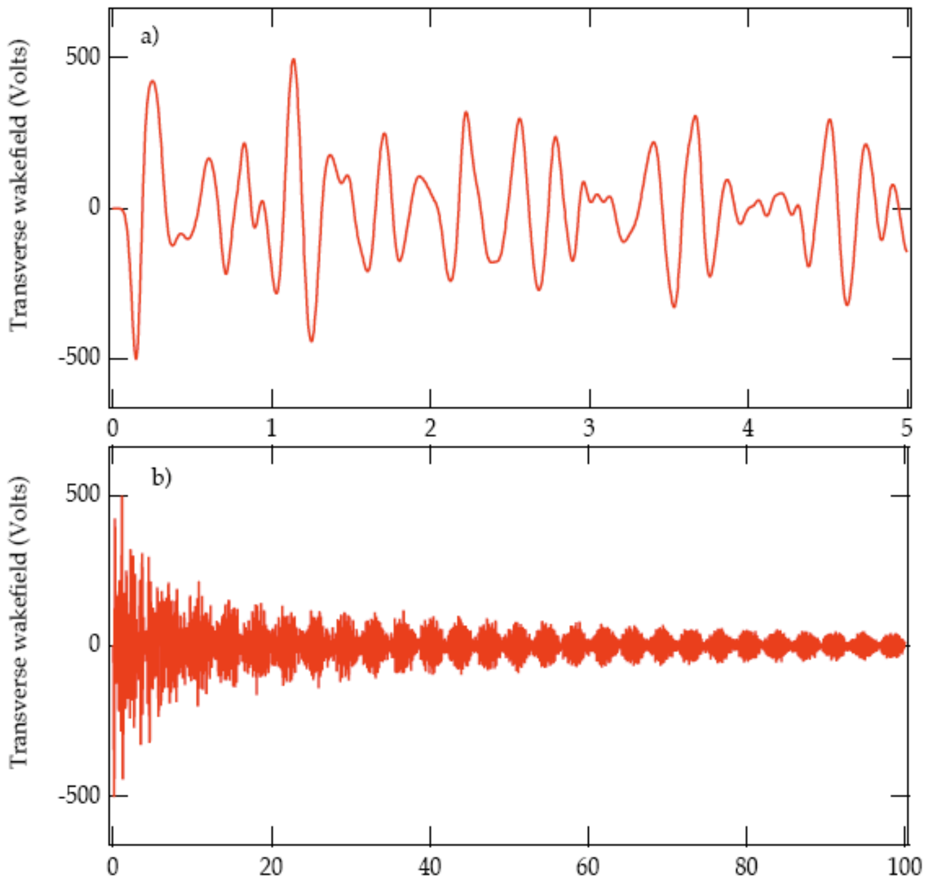


PEP-II Impedance

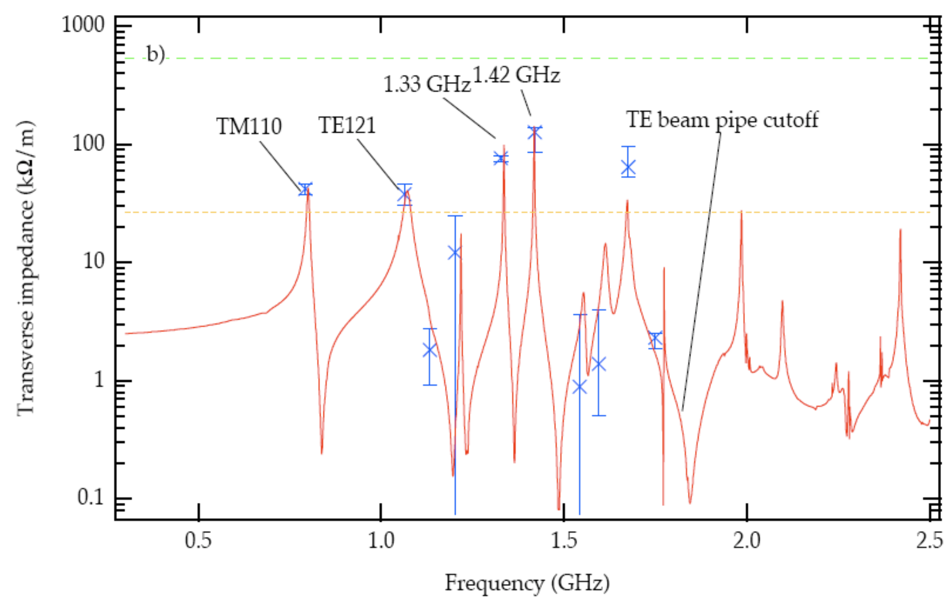
- The time domain wake in frequency domain is called the beam impedance
- Each mode can be defined by a frequency, Q, and shunt impedance
- Usually higher order modes are not wanted and some attempt is made to damp them.



PEP-II Transverse Wake

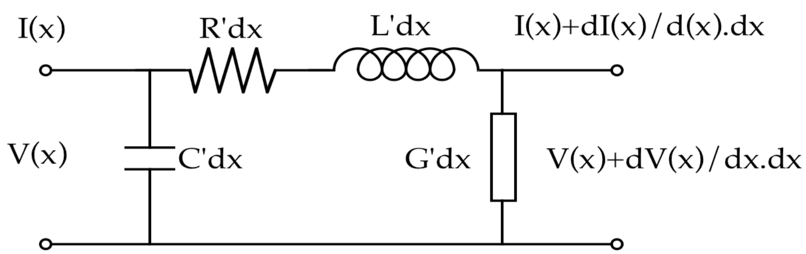


- A similar situation occurs for cavity modes which can give transverse beam deflections.

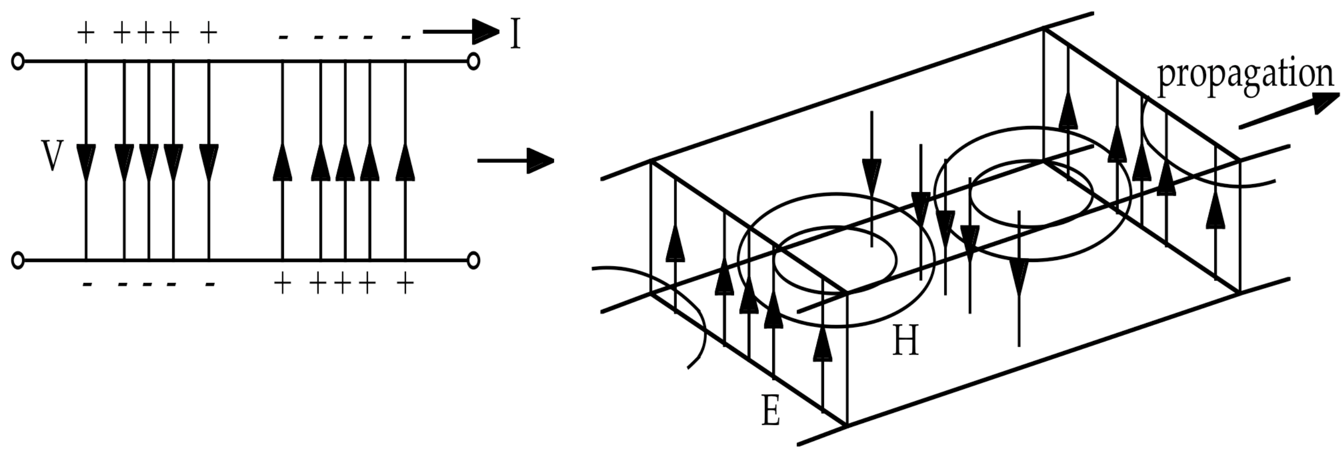


Cavity modes: transmission line analogy

- Consider the equivalent circuit for a transmission line

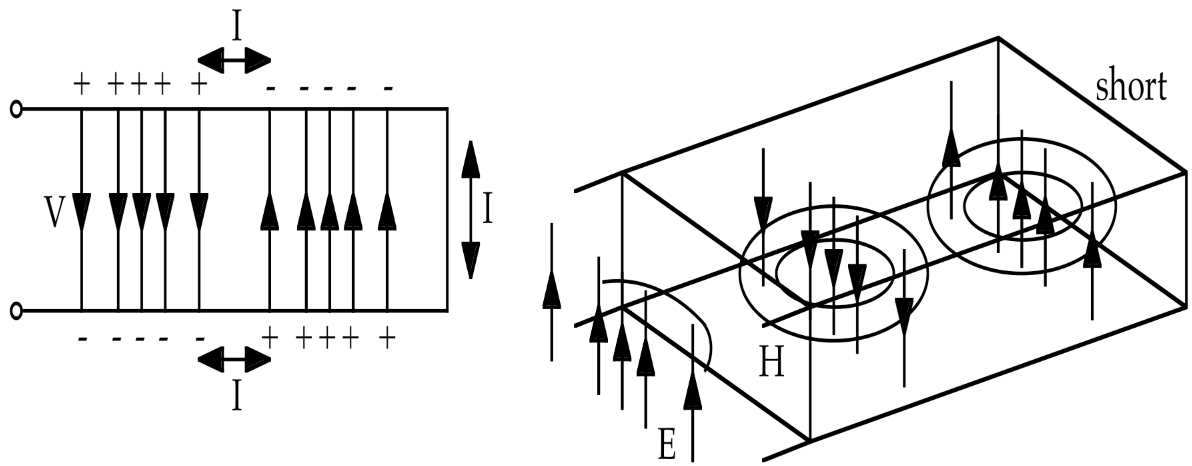


- This gives a field pattern

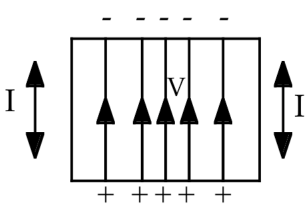


Cavity modes: transmission line analogy

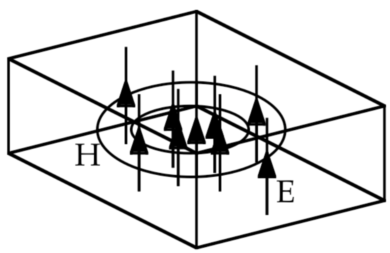
- If we short one end of the line, forward and reflected waves give a standing wave



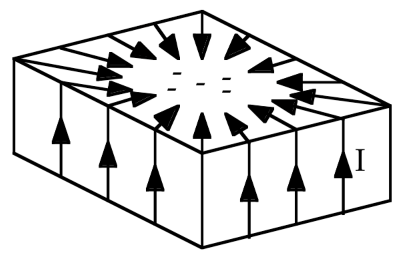
- If we short the other end of the line, we make a cavity



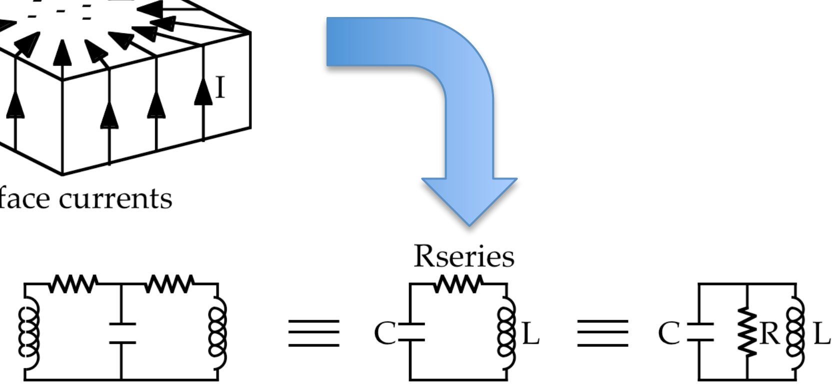
shorted line



standing wave



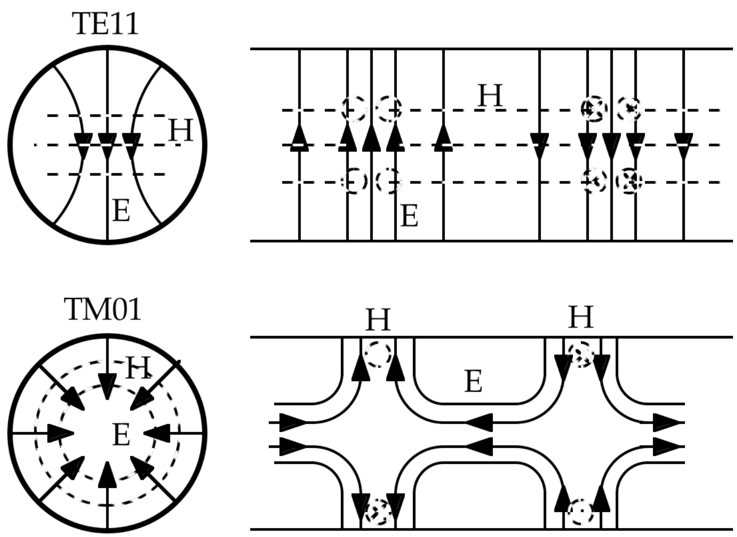
surface currents



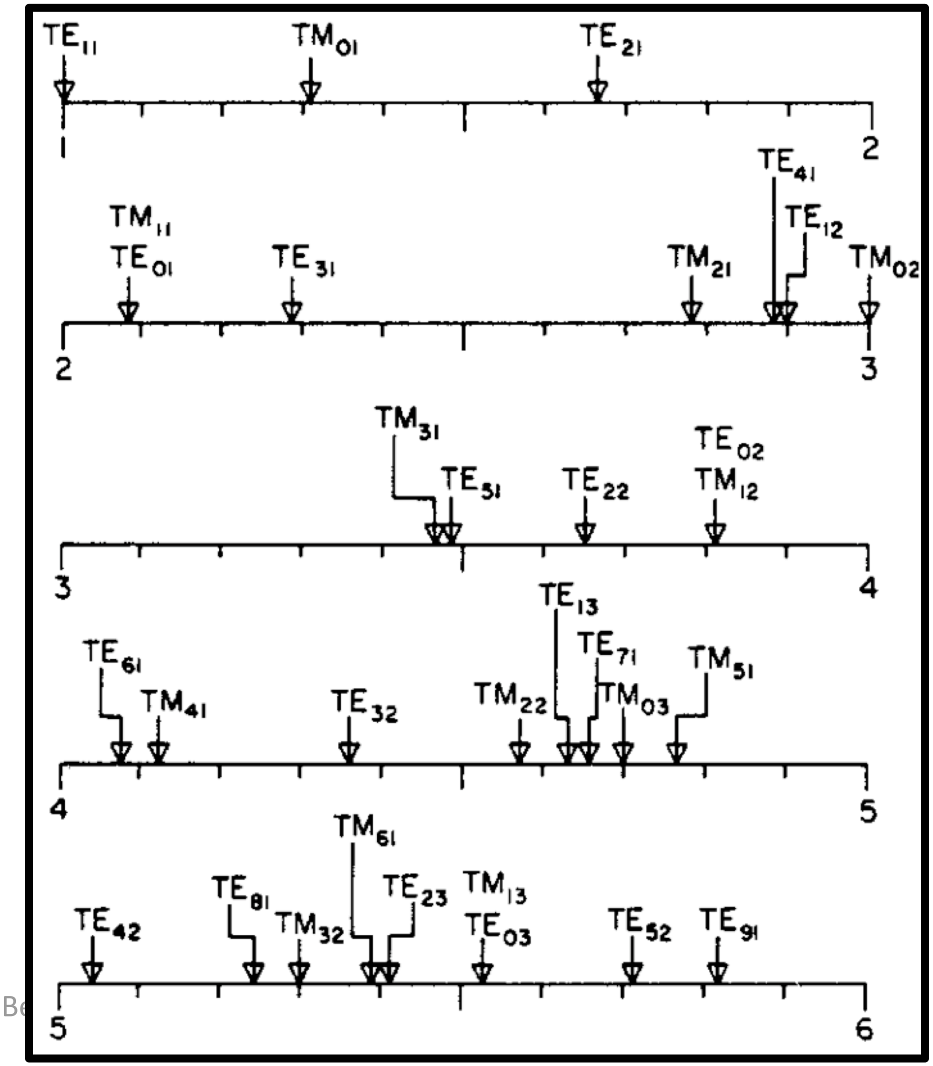
equivalent circuit series circuit parallel circuit

Pillbox cavities

- Consider the two lowest modes of a cylindrical waveguide (TE=transverse electric field, TM=transverse magnetic field)



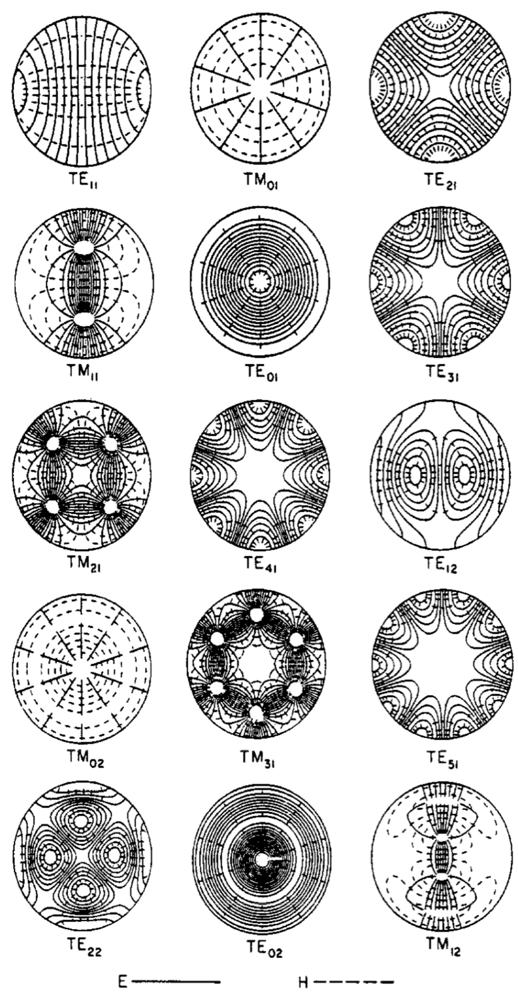
First two modes in circular waveguide

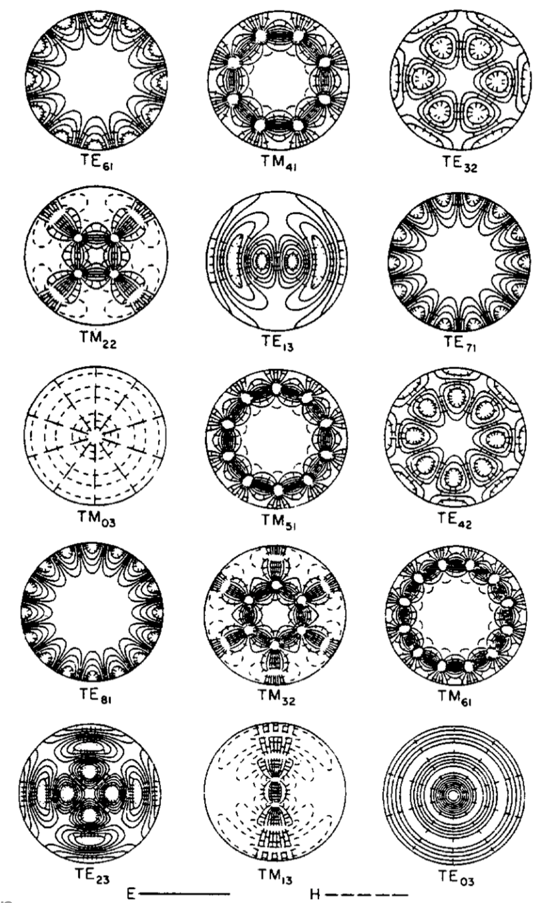


Cut-off frequencies for modes in circular waveguide, normalized to that of the lowest mode (TE11).

Pillbox Cavities

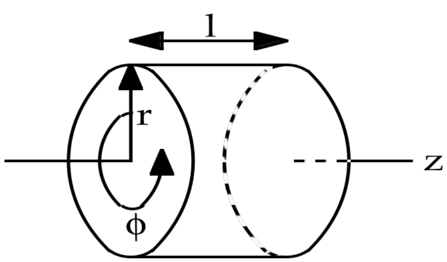
- Many different modes; only TM modes have field along beam direction and can interact with beam





Pillbox Cavities

- The transverse variations of the longitudinal field are solutions of Maxwell's equations within a circular boundary condition and are Bessel functions of the first kind.



pillbox cavity

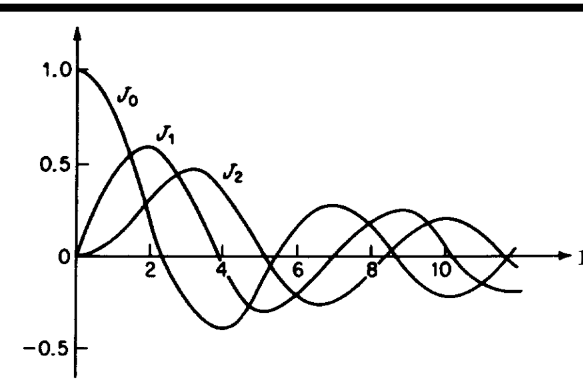
$$E_z(r,\phi) = E_o J_m(k_{mn} r)\cos m\phi$$

J_m are the first order Bessel functions
$k_{mn} = x_{mn}/r$ is the transverse wave number
x_{mn} are the roots of the Bessel functions J_m

- For TM_{mnz} modes the fields are:

$$E_z(r,z,t,\ \phi) = E_o J_m(\frac{X_{mn}}{a} r) e^{j\omega t} \cos(m\phi)\cos(k_z z)$$

$$H_\phi(r,z,t,\ \phi) = H_o J'_m(\frac{X_{mn}}{a} r) e^{j\omega t} \cos(m\phi)\cos(k_z z)$$



Low-order Bessel functions of the first kind

Monopole modes

- Modes which have no azimuthal variation are labelled “monopole” modes and TM modes of this type have longitudinal electric field on axis and thus can interact strongly with the beam.
- The radial distribution of E_z follows J_0, where the zeros satisfy the boundary condition that $E_z = 0$ at the conducting wall at radius a. Similarly H_f and E_r (if present) follow J'_0 and are zero in the center and have a finite value at the wall.

For TM_{oni} modes:

$$E_z = E_o J_o(k_{on} r)\cos(k_z z) \text{ where } k_{on} = x_{on}/a \text{ and } k_z = i\pi/\text{length } (i \geq 0)$$

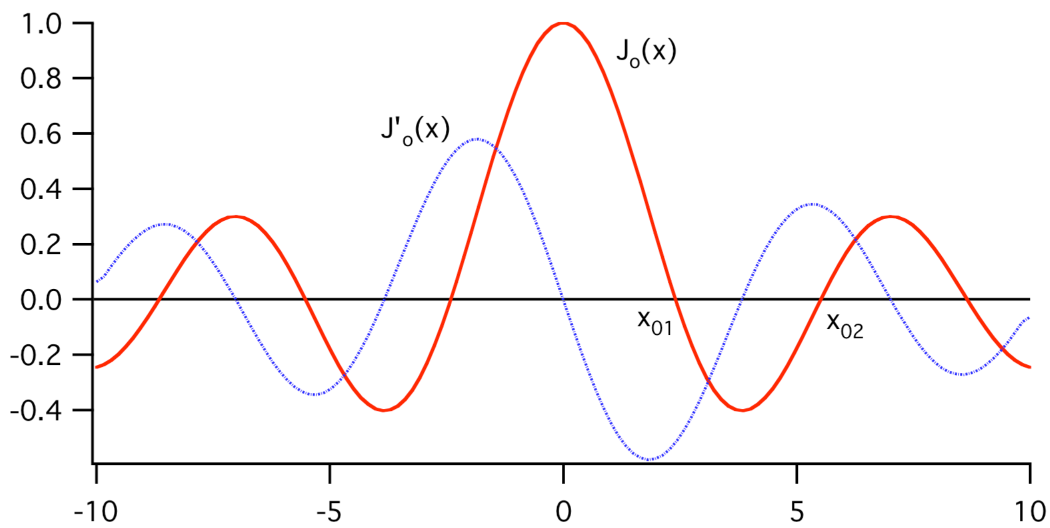
$$H_\phi = H_{\phi o} J'_o(k_{on} r)\cos(k_z z)$$

$$E_r = E_{ro} J'_o(k_{on} r)\sin(k_z z)$$

$x_{o1} = 2.405$

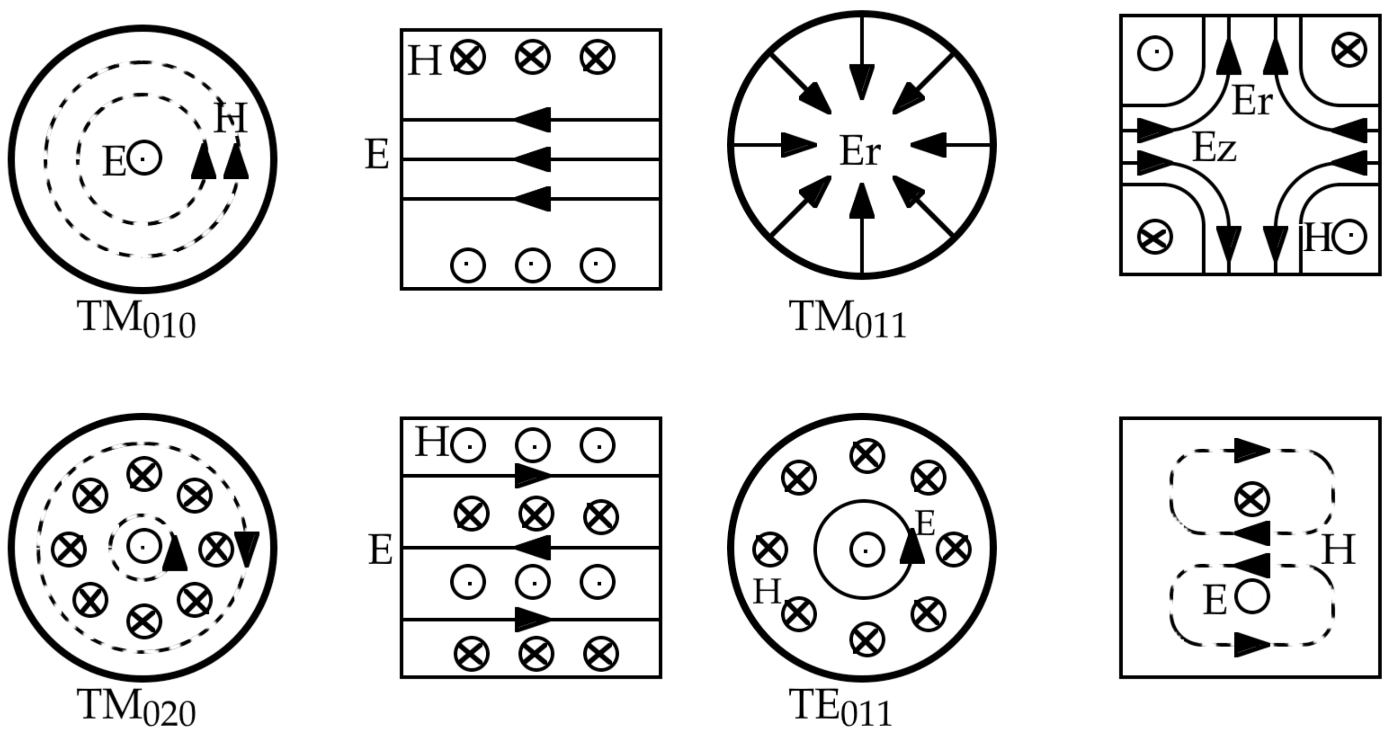
$x_{o2} = 5.520$

$x_{o3} = 8.654$



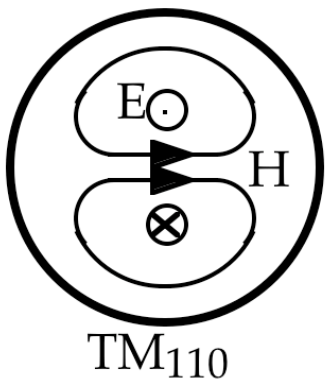
Monopole modes

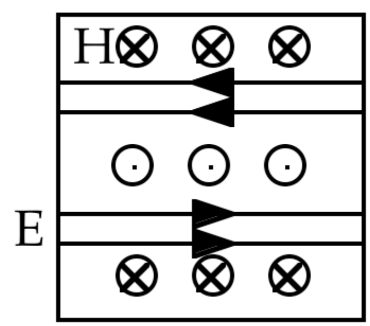
- A few example field patterns



Dipole modes

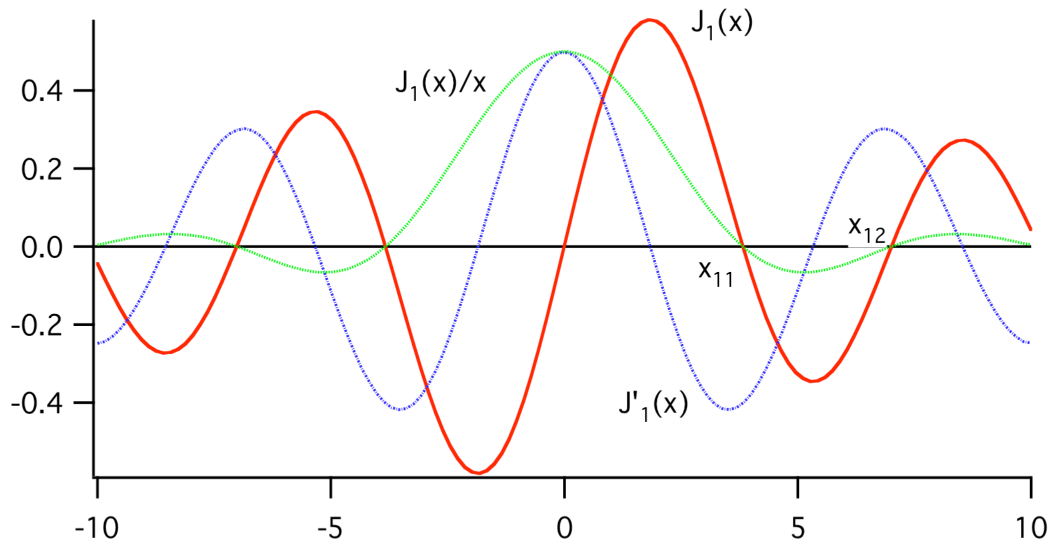
- Characterized by one full period of variation around the azimuth (m=1)
- For TM modes this means there is no longitudinal field on axis and that the field strength grows linearly with radius close to the center, with opposite sign either side of the axis.
- This transverse gradient to the longitudinal field gives rise to a transverse voltage kick which is proportional to the beam current and the beam offset.
- Therefore, no transverse kick to the beam without a longitudinal field gradient (Panofsky-Wenzel Theorem)





Dipole Modes

- Field distributions for dipole modes



For TM_{1ni} modes:

$$E_z = E_o J_1(k_{1n} r)\cos(\phi)\cos(k_z z) \text{ where } k_{1n} = x_{1n}/a \text{ and } k_z = i\pi/\text{length } (i \geq 0)$$

$$H_\phi = H_{\phi o} J_1'(k_{1n} r)\cos(\phi)\cos(k_z z) \qquad x_{11} = 3.383171$$

$$|H_r| = H_{ro}\frac{J_1'}{r}(k_{1n} r)\sin(\phi)\cos(k_z z) \qquad x_{12} = 7.01559$$

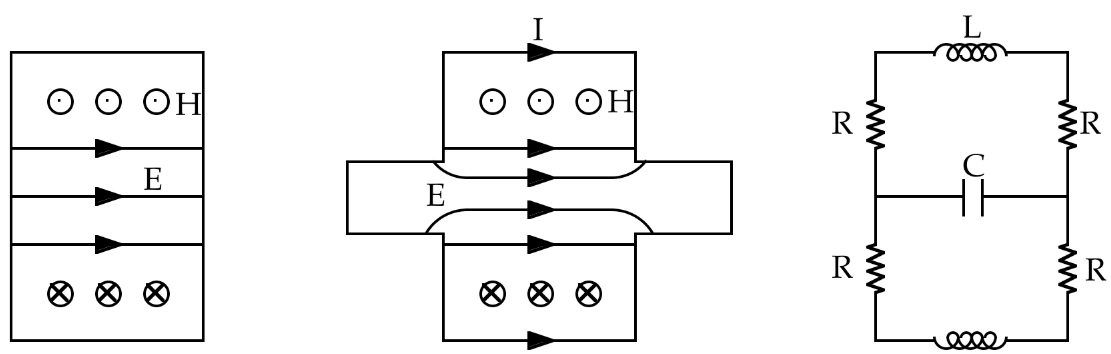
$$x_{13} = 10.17347$$

Higher-order modes (m>1):

- For modes with higher azimuthal order (m=2, quadrupole, m=3, sextupole etc.), the fields close to the beam axis become progressively weaker as the stored energy is concentrated towards the outer edge of the cavity.
- Modes with even m have no sign reversal across the axis and have a small transit time factor. Modes with odd m may couple weakly to transverse motion of the beam but are generally not problematic.

Real Cavities

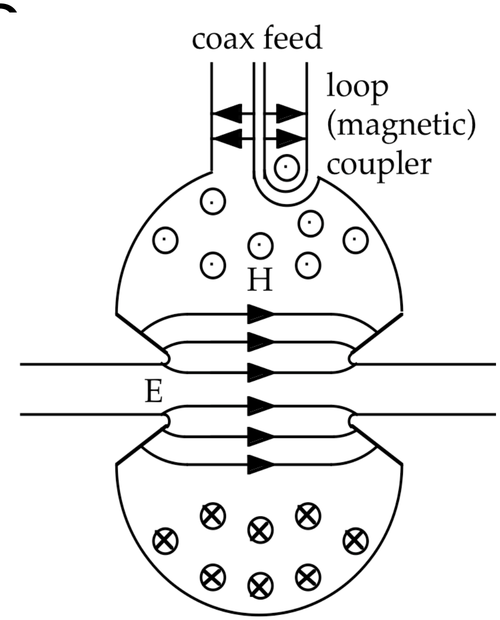
- In practice the simple pillbox cavity shape can be improved upon to maximize the shunt impedance for acceleration.
- The shape must also be modified to allow passage of the beam and addition of one or more couplers



pillbox | pillbox + beam pipes | distributed L,C, wall loss

Normal Conducting Cavities

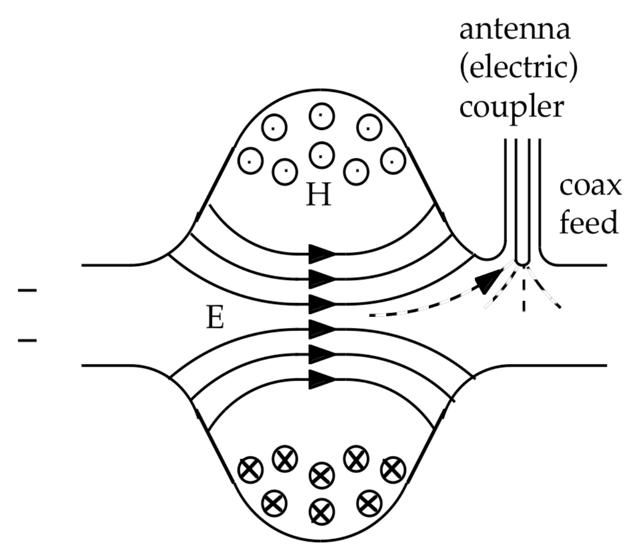
- Since R/Q=sqrt(L/C), maximize L and minimize C, while keeping the optimum interaction length (max. T), and maximum
- The “nose-cone” or re-entrant cavity increases the volume occupied by the magnetic field and the surface area carryin the current and decreasing the surface are in the capacitive region (nose tips).
- Limiting factors to achievable gradient are wall-power dissipation and E-field strength at the nose-tips.



"nose-cone" copper cavity

Super Conducting Cavities

- Smooth shape is determined by the need to avoid field emission from the surface. R/Q is low but Q is very high
- Gradient limited by
  - Field emission from surface impurities
  - Ultimately limited by surface magnetic field



"bell-shaped" superconducting cavity

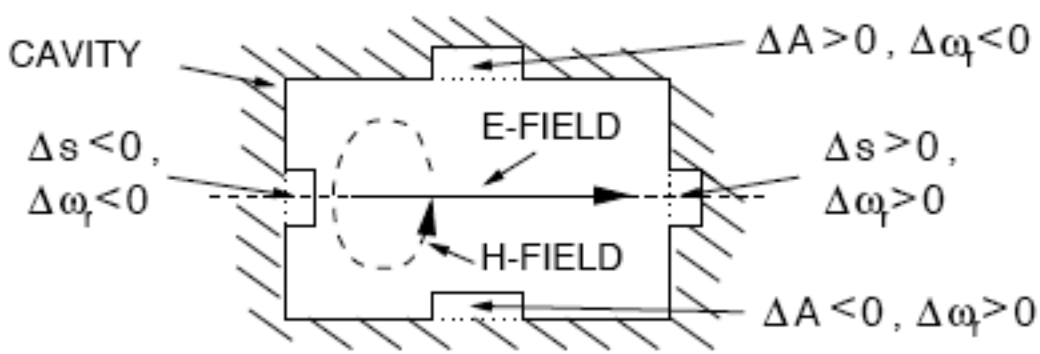
Cavity Tuning

- Mode resonant frequency can be adjusted by introducing perturbations
- Effect depends on whether perturbation affects electric or magnetic fields (change in capacitance or inductance).

$$(L + \Delta L) = \frac{\mu H}{I}(A + \Delta A).$$

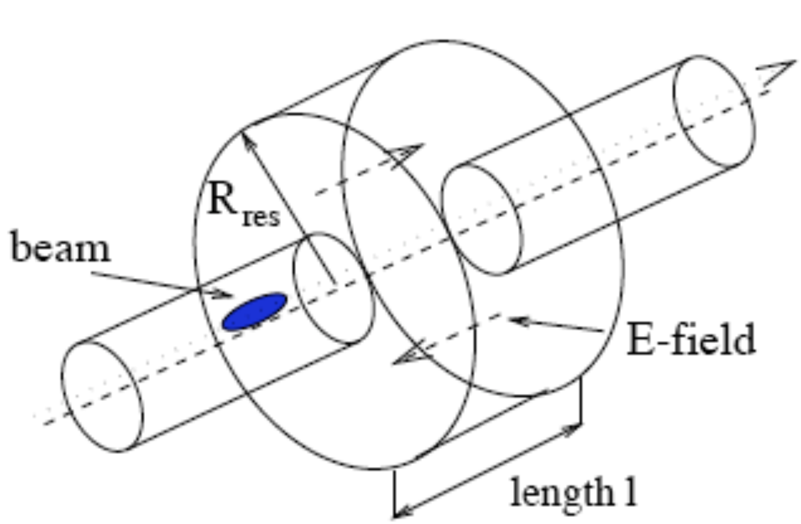
$$(C + \Delta C) = \frac{Q}{E(s + \Delta s)}.$$

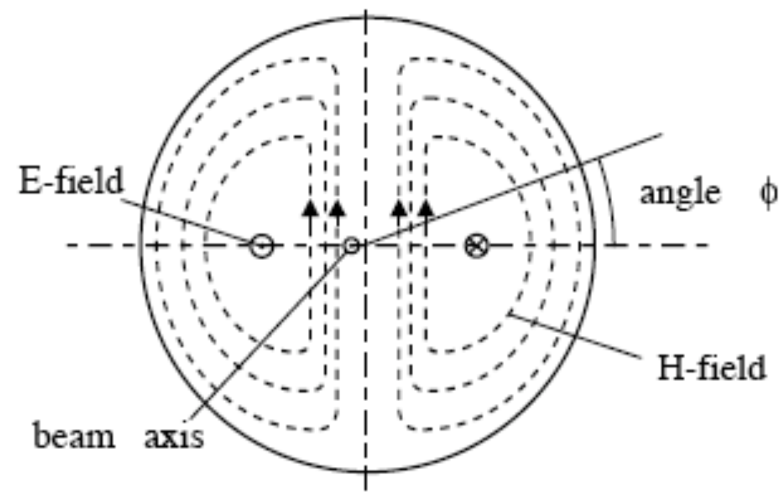
$$\omega_r + \Delta\omega_r = [(L + \Delta L)(C + \Delta C)]^{-\frac{1}{2}}$$



Cavity BPMs

- Use the linear dependence of excited field in the m=1 dipole mode as a measure of beam position.
- Full (x,y) measurement requires each mode polarization, usually achieved with two cavities.





Cavity BPM Sensitivity

- Dipole mode parameters (assume pillbox)

$$Q_{110} = \frac{1}{2\pi} \cdot \frac{a_{11}}{1 + (R_{\text{res}} \cdot l^{-1})} \cdot \frac{\lambda_{110}}{\delta} \qquad \text{with} \qquad \delta = \sqrt{\frac{1}{\pi \cdot f_{110} \cdot \mu \cdot \kappa}}$$

$$\left(\frac{R}{Q}\right)_{110} = \frac{(V_{110}^{\max})^2}{2\omega_{110} \cdot W_{110}} = \frac{2 \cdot Z_0 \cdot l \cdot (J_1^{\max})^2 \cdot T_{tr}{}^2}{\pi \cdot R_{\text{res}} \cdot J_0^2(a_{11}) \cdot a_{11}} \approx 130.73 \cdot \frac{l}{R_{\text{res}}} \cdot T_{tr}{}^2$$

- Dipole mode sensitivity for offset dx

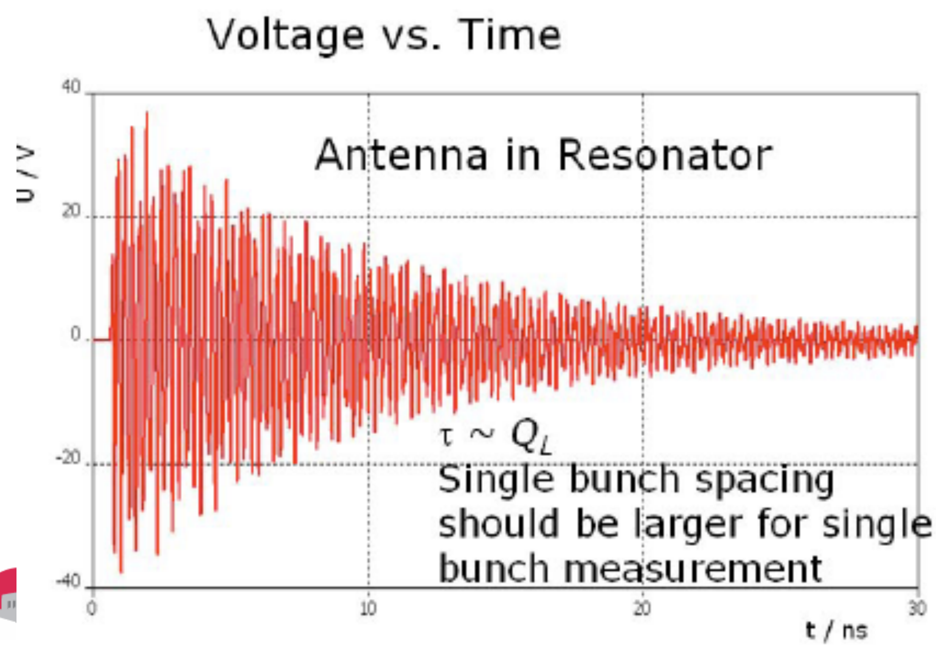
$$V_{110}^{in}(\delta x) = \left(\frac{R}{Q}\right)_{110} \cdot \omega \cdot q \cdot \left\langle \frac{a_{11} \cdot \delta x}{2 \cdot J_1^{max} \cdot R_{\text{res}}} \right\rangle = \delta x \cdot q \cdot \frac{l \cdot T_{tr}{}^2}{R_{\text{res}}{}^3} \cdot 0.2474 \left[\frac{\text{Vm}}{\text{pC}}\right]$$

Cavity BPM Signals

- For a single bunch passage, the cavity will "ring down" with resonant frequency and decay time.

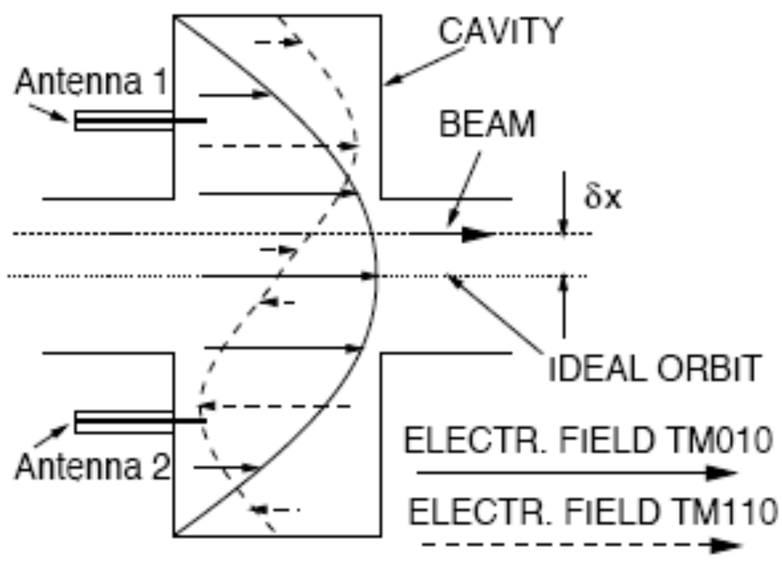
$$V_{110}(t_b) = V_{110}(t=0) \cdot \exp\left(-\frac{1}{2} \cdot t_b \cdot \frac{\omega_{110}}{Q_\mathrm{L}}\right)$$

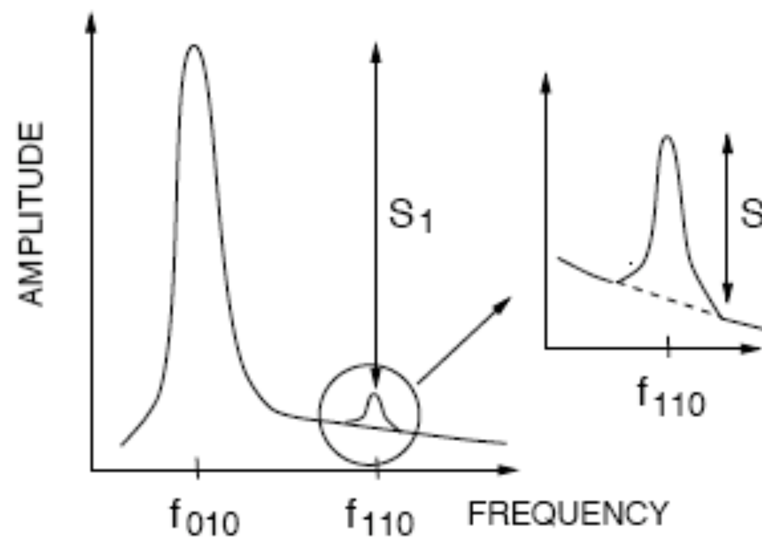
$$V_{110}^{out}(\delta x) = \left(\frac{R}{Q}\right) \omega q \sqrt{\frac{50\ \Omega}{Q_\mathrm{L}}} M_b \frac{\beta}{1+\beta} = V_{110}^{in}(\delta x) \left(\frac{R}{Q}\right)_{110}^{-\frac{1}{2}} \sqrt{\frac{50\ \Omega}{Q_\mathrm{L}}} \sqrt{1 - \frac{Q_\mathrm{L}}{Q_0}}$$



Common mode signals

- Since the field maximum of the common modes is on the cavity axis, they will be excited much stronger than the TM_{110} by a beam near the axis
- Due to their finite Q, all modes have field components even at the TM_{110}-mode frequency
- There is a 90 degree phase shift between the monopole and dipole excitation.





Cavity BPM Signal Processing

- One example of a typical Cavity BPM signal path
  - Dipole cavity gives offset signal. Magic-T used to subtract monopole mode
  - Separate monopole cavity used to produce reference signal

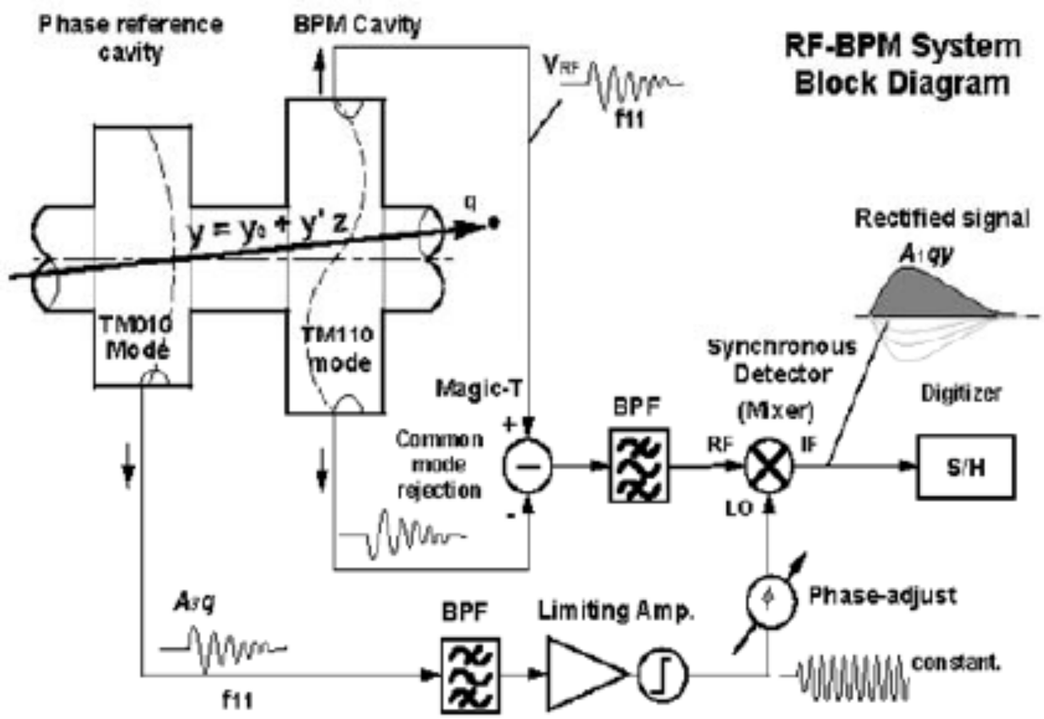


Fig. 2. Simplified RF-BPM Diagram.

Example: LCLS Cavity BPMs

- Taken from DIPAC 2009 Talk by Steve Smith

Cavity BPM Requirements

- Undulator orbit critical
  - Must keep electrons and photons coincident
  - to fraction of beam size
  - over distance > gain length

Parameter	Requirement	Conditions
Resolution	< 1 micron	200 pC < Q < 1 nC Over ± 1 mm range
Offset Stability	< ±1 micron	1 hour ± 1 mm range, 20 C ± 0.56 C
	< ±3 microns	24 hour ± 1 mm range, 20 C ± 0.56 C
Gain Stability	± 10 %	± 1 mm range 20 C ± 0.56 C
Aperture	10 mm	

SLAC

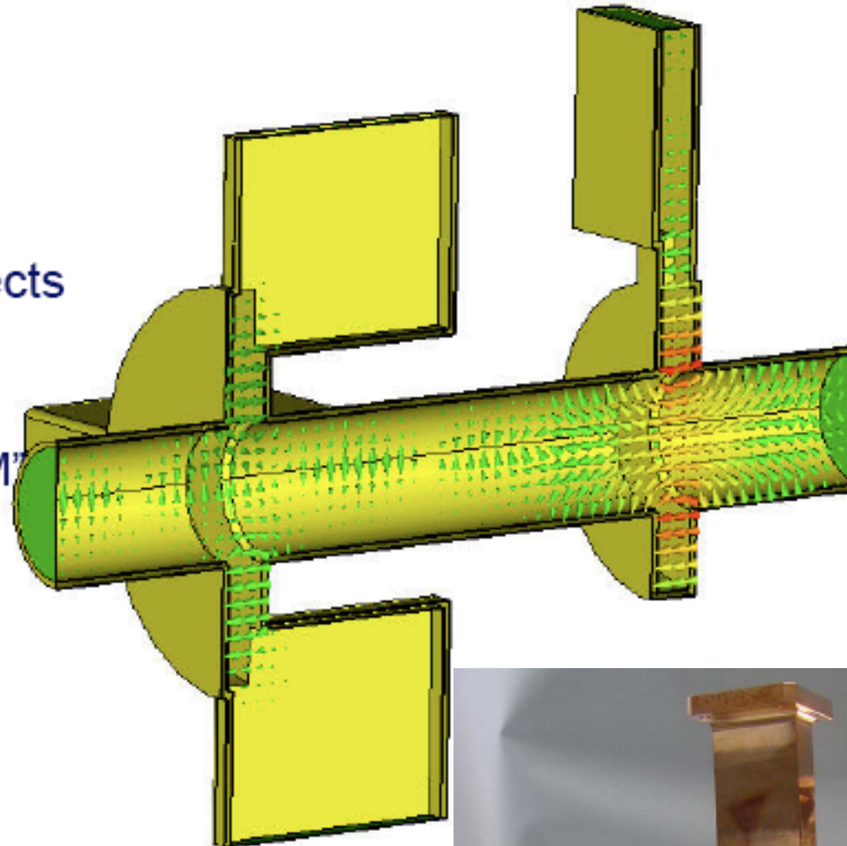
Steve Smith

R. Lill, S. Hoobler, R. Johnson, W.E. Norum, L. Morrison, N. Sereno, S. Smith, T. Straumann, G. Waldsmith, D. Walters, A. Young, D. Anderson, V. Smith, R. Traller, + many others

Design

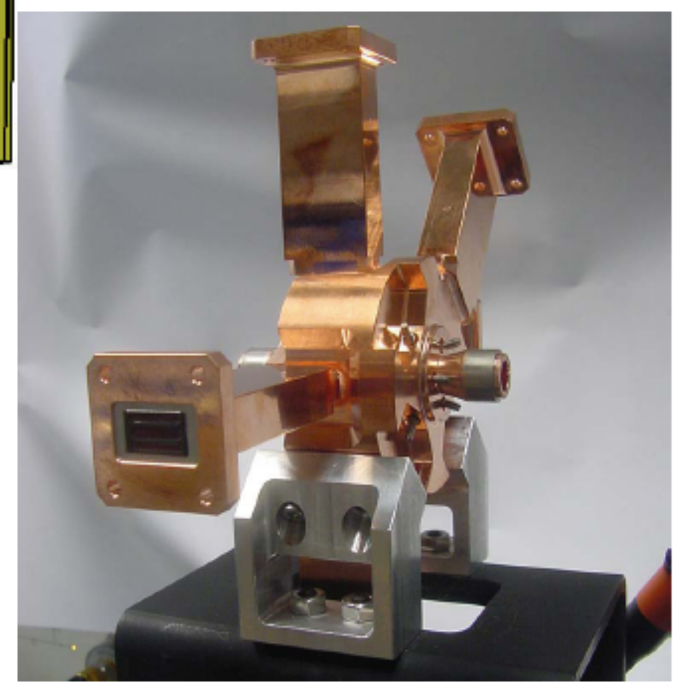
Concepts

- Avoid the monopole mode
- Cavity-waveguide coupler rejects monopole mode by symmetry
  - Zenghai Li (PAC 2003)
  - T. Shintake, "Comm-free BPM"
  - V. Balakin (PAC 1999)
- Predecessor at KEK's ATF
  - 16 nm resolution in test beam Walston, (NIM 2007)



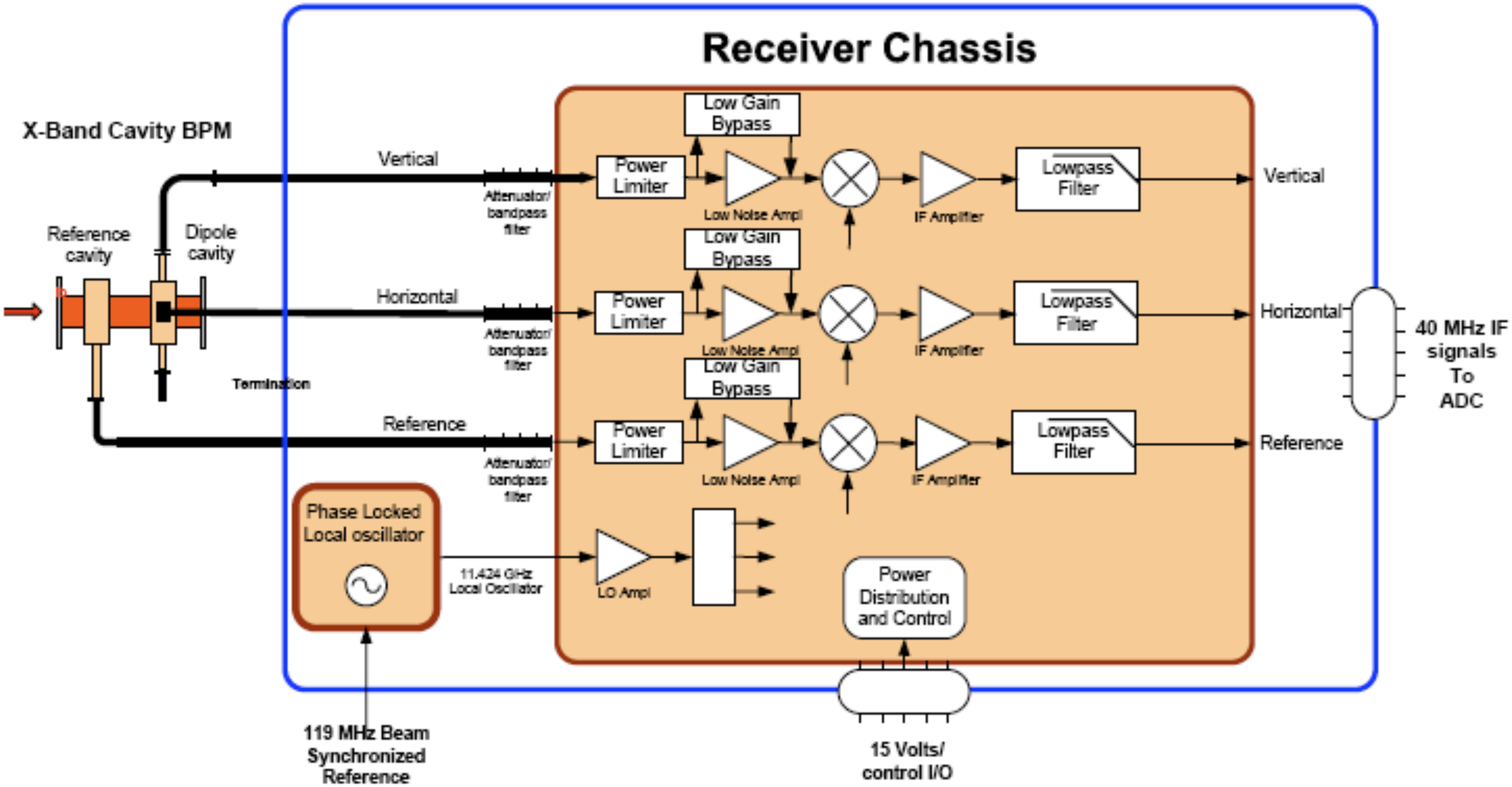
Choices

- Single, degenerate X&Y cavity
- Reference cavity per BPM



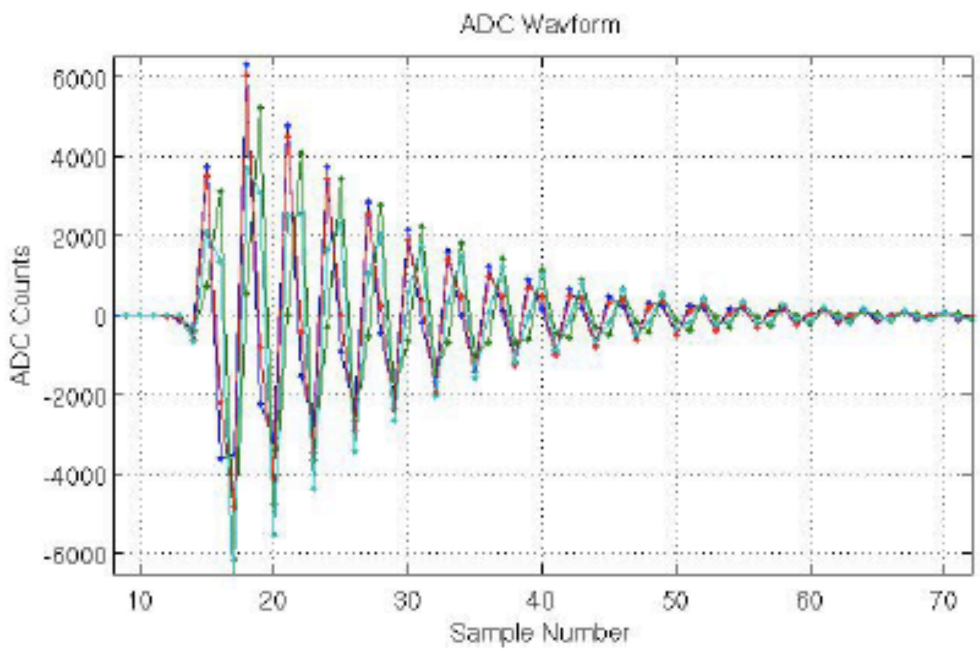
Receiver

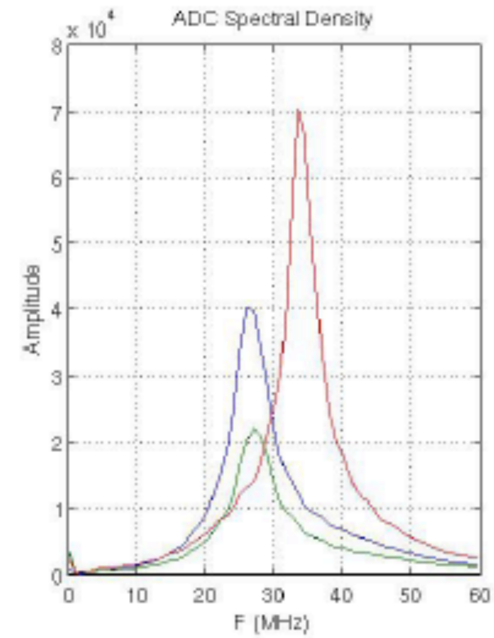
- Downconverts X-band to ~40 MHz IF
- Mounted on Undulator stand

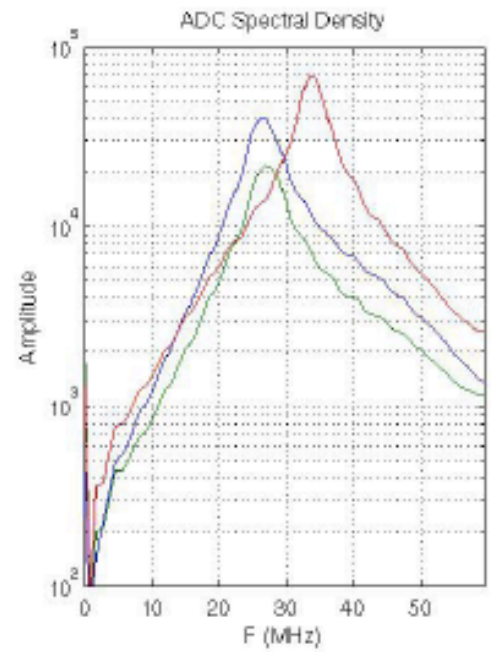


Waveform / Spectrum

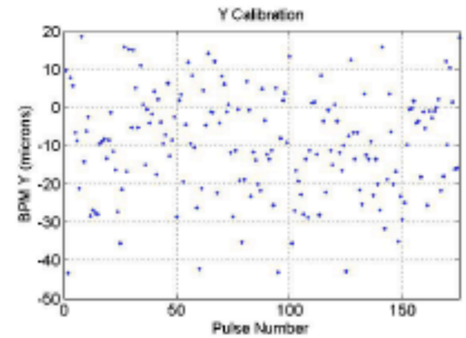
- Cavity IF waveform sampled at 119 MHz
- 16 bit digitizer
- Extract amplitude, phase of
  - X, Y, Reference







Calibration



Move BPM

Measure complex amplitudes

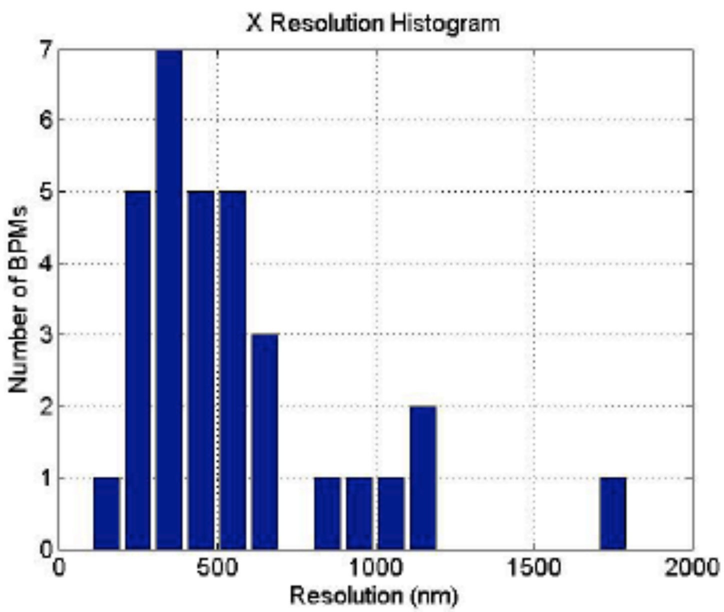
X, Y, Ref

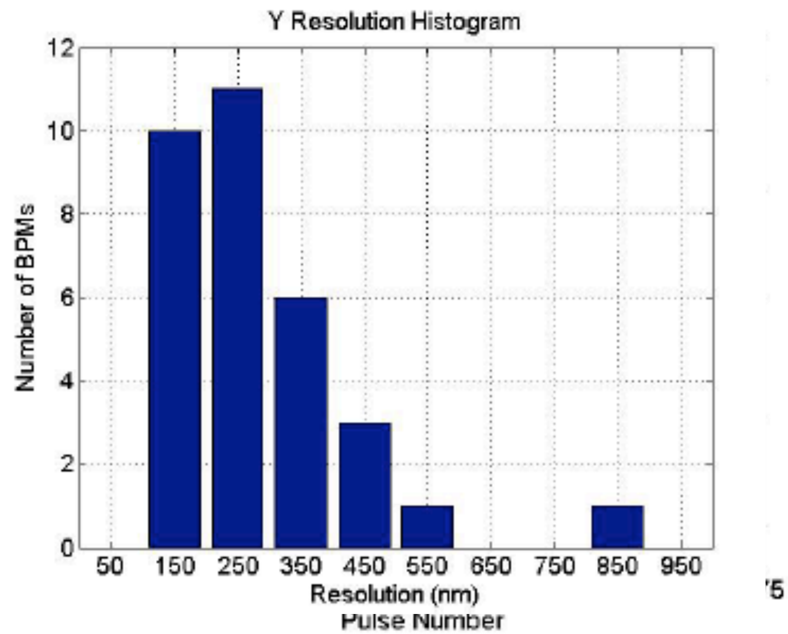
Normalized amplitude = Position/Reference (Complex)

Remove beam jitter using adjacent upstream BPMs

Fit complex normalized amplitude to mover position

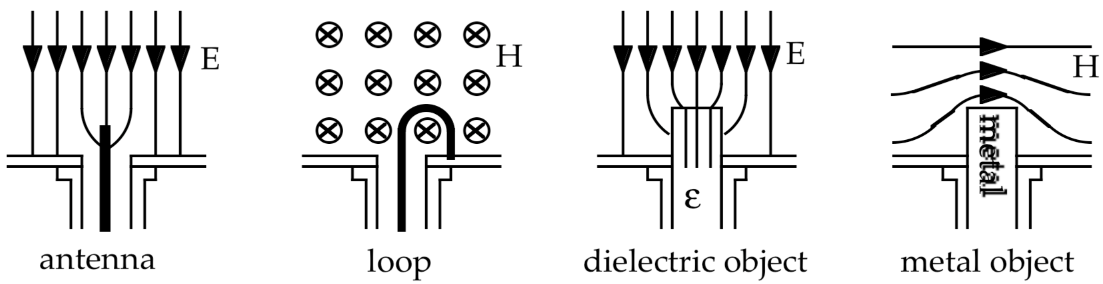
Repeat for off-axis component



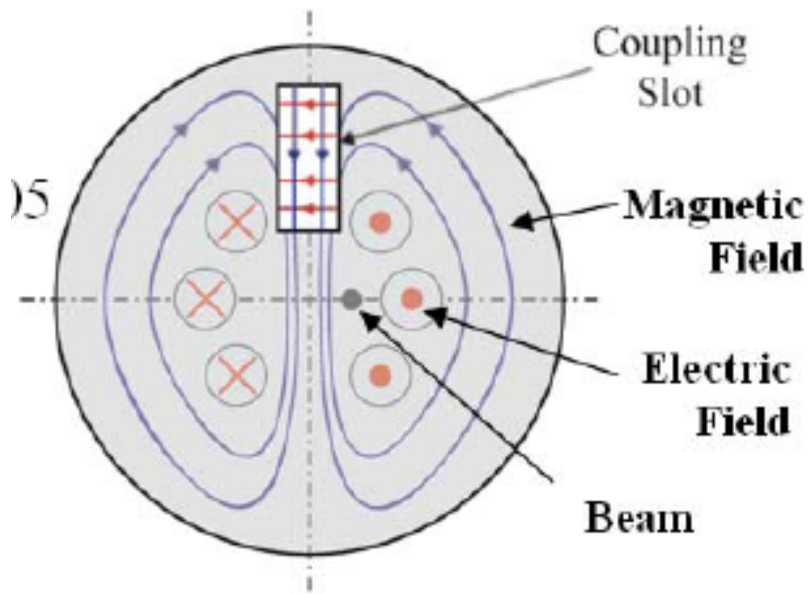


Field Couplers and Probes

- Antennae and loops couple to electric and magnetic fields
- Coupling is determined by the overlap of the antenna field pattern and the mode field distribution.

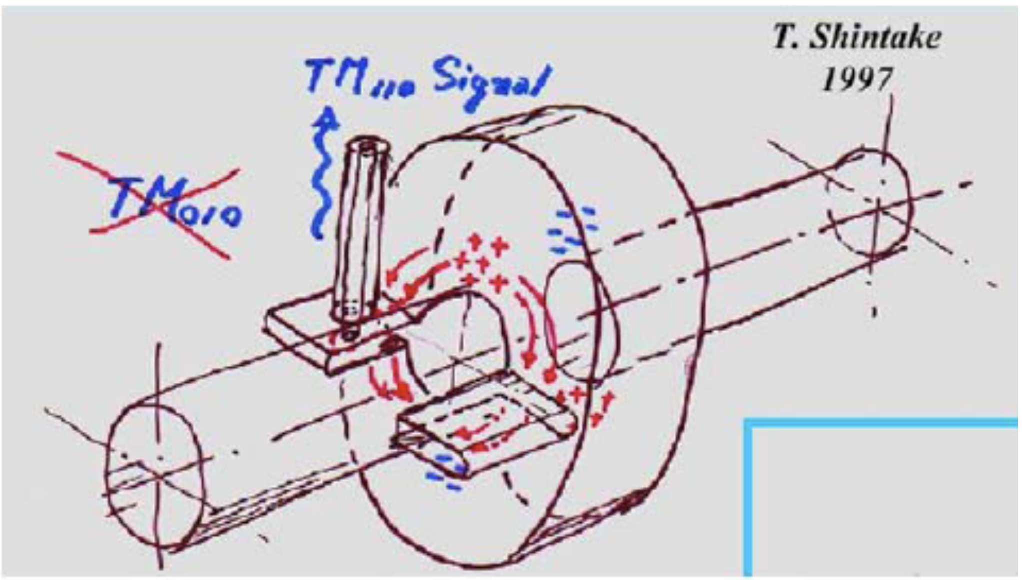


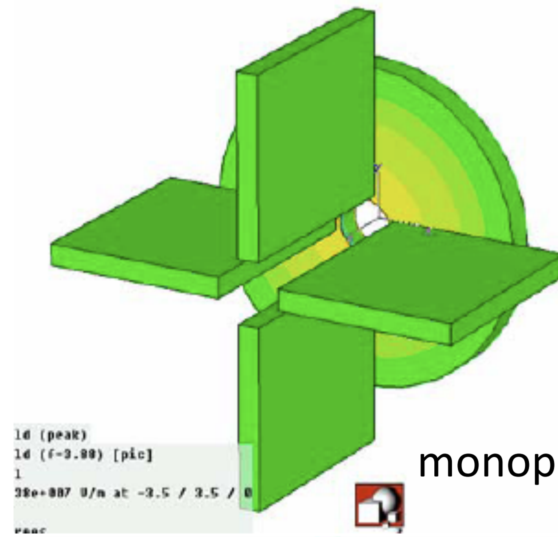
- Apertures couple to the TE10 mode of an external waveguide.
- Coupling is determined by the overlap of the waveguide field pattern and the mode field distribution.
- Coupling frequency must be above TE10 cutoff



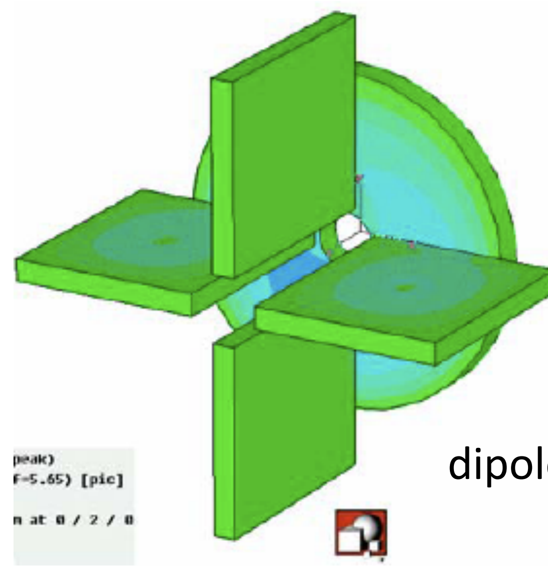
Example: Cavity BPM TM_{010} rejection

- Use waveguide coupler location to access TM110 mode and reject TM010 mode





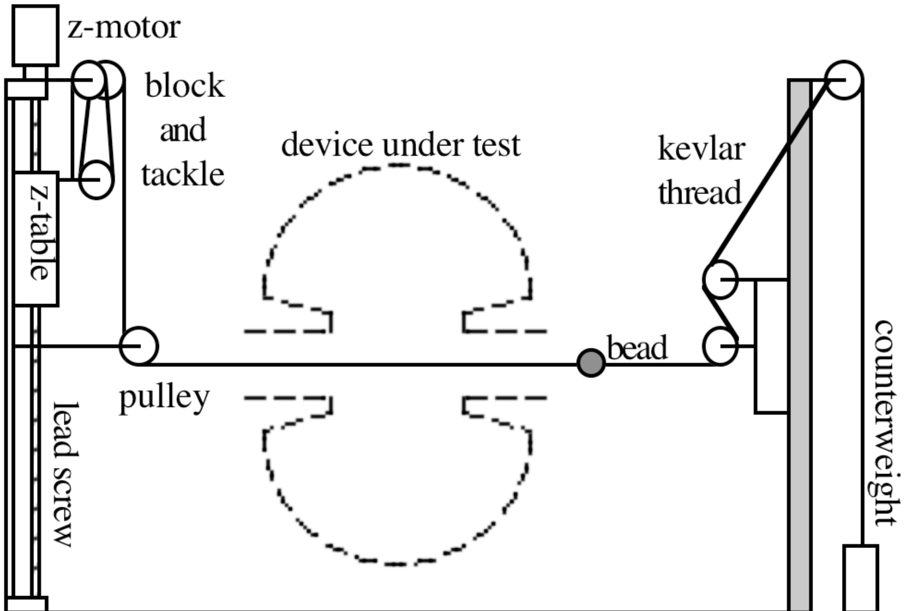
monopole



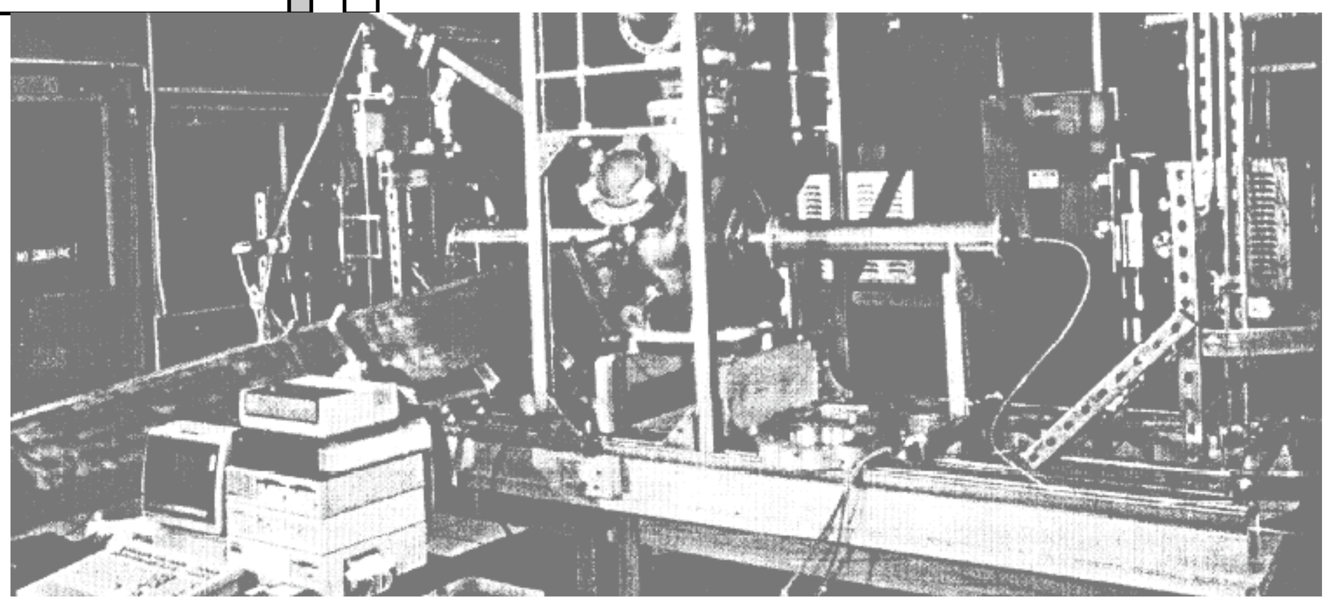
dipole

- Microwave Studio simulation of coupling to monopole and dipole modes

Cavity Bead-pull Measurement



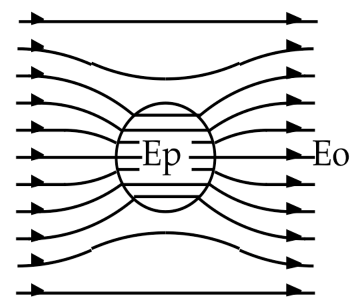
- Use a bead (metallic or dielectric) to perturb the field along the cavity axis.
- The frequency shift is proportional to the field value.



Bead Perturbation

- Slater has shown that the change in resonant frequency upon introducing an object into the cavity field is proportional to the relative change in stored energy

$$\frac{\Delta\omega}{\omega} = \frac{\Delta U_M - \Delta U_E}{U} = \frac{\int_{\Delta V} \left(\mu H^2 - \varepsilon E^2\right) dV}{\int_{V} \left(\mu H^2 + \varepsilon E^2\right) dV}$$



$$E_p = \frac{3E_o}{\varepsilon_r + 2}$$

perturbation of a uniform E-field by a dielectric bead

- For the case of a small non-conducting sphere, radius r, where the unperturbed field may be considered uniform over a region larger than the bead, it can be shown that:

$$\frac{\Delta\omega}{\omega} = \frac{\Delta U}{U} = -\frac{\omega\pi r^3}{PQ}\left[\varepsilon_o \frac{\varepsilon_r - 1}{\varepsilon_r + 2} E_o^2 + \mu_o \frac{\mu_r - 1}{\mu_r + 2} H_o^2\right]$$

Bead Perturbation

- For a dielectric bead (μ_r = 1) the expression reduces to:

$$\frac{\Delta\omega}{\omega} = -\frac{\pi r^3}{U}\left[\varepsilon_o \frac{\varepsilon_r - 1}{\varepsilon_r + 2} E_o^2\right]$$

- For a metal bead ($\varepsilon_r \rightarrow \infty$, $\mu_r \rightarrow 0$):

$$\frac{\Delta\omega}{\omega} = -\frac{\pi r^3}{U}\left[\varepsilon_o E_o^2 - \frac{\mu_o}{2} H_o^2\right]$$

- A metallic bead can be used to measure the electric field if the magnetic field is known to be zero (e.g.: on axis of a monopole mode), and gives a larger frequency shift than common dielectric materials such as Teflon (ε_r = 2.08) or Alumina (ε_r = 9.3).
- Shaped beads such as needles or disks can be used to enhance the perturbation and give directional selectivity. The enhancement or "form factor" can be calculated for ellipsoids or calibrated in a known field.

Bead pull: Calculation of R, R/Q

- By mapping the longitudinal distribution of E_z and integrating, the cavity shunt impedance can be determined

$$RT^2 = \frac{(VT)^2}{2P} = \frac{\left[\int E_z(z) e^{j\omega\frac{z}{v}} dz\right]^2}{2P}$$

where

$$E^2 = -\frac{\Delta\omega P Q(\varepsilon_r + 2)}{\omega^2 \pi r^3 \varepsilon_o(\varepsilon_r - 1)}$$

- Therefore

$$RT^2 = -\frac{Q(\varepsilon_r+2)}{\omega\pi r^3 \varepsilon_o(\varepsilon_r - 1)} \cdot \frac{\left[\int \sqrt{\frac{\Delta\omega}{\omega}}(z)\left(\cos\frac{\omega z}{c} + j\sin\frac{\omega z}{c}\right)dz\right]^2}{2}$$

- If the cavity is symmetric in z and t=0 at z=0 in the center,

$$\frac{RT^2}{Q} = -\frac{(\varepsilon_r+2)}{4\pi^2 f r^3 \varepsilon_o(\varepsilon_r - 1)} \cdot \left[\int \sqrt{\frac{\Delta f}{f}}(z)\left(\cos\frac{2\pi f z}{c}\right)dz\right]^2$$

Summary

- RF cavities are a critical aspect of any accelerator.
- They are also one of the more fascinating aspects of accelerators (to me!)
- This lecture is just a taste of the many interesting aspects of RF cavities. Topics not covered include:
  - RF power sources
  - Interaction with beam (Robinson instabilities)
  - Coupled bunch instabilities
  - Cavity mode damping
  - …..

References

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  - "RF Superconductivity," Jean Delayen
- Cavity BPMs
  - "Cavity Beam Position Monitors", R. Lorenz, Beam Instrumentation Workshop'98
  - "Cavity BPM Designs, Related Electronics And Measured Performances", D. Lipka, Beam Instrumentation Workshop 2009
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