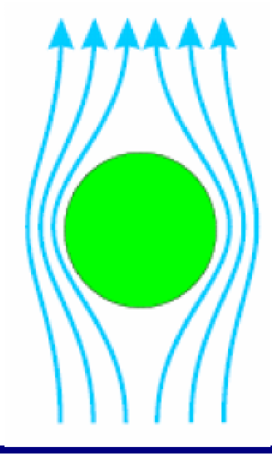
S. Belomestnykh

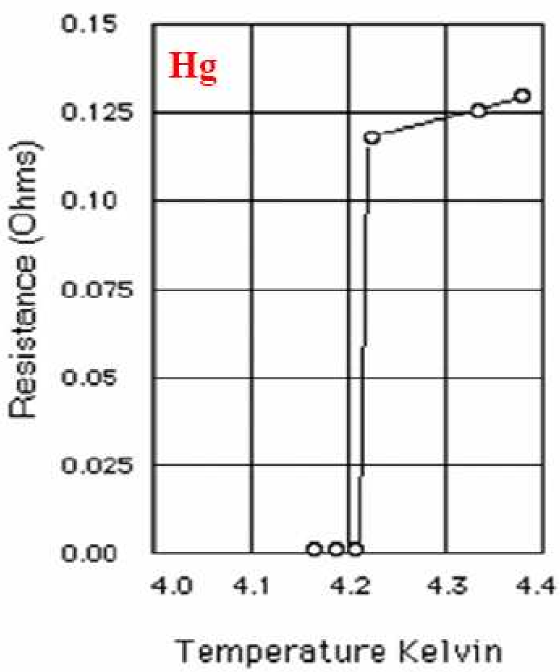
Superconducting RF for storage rings, ERLs, and linac-based FELs:

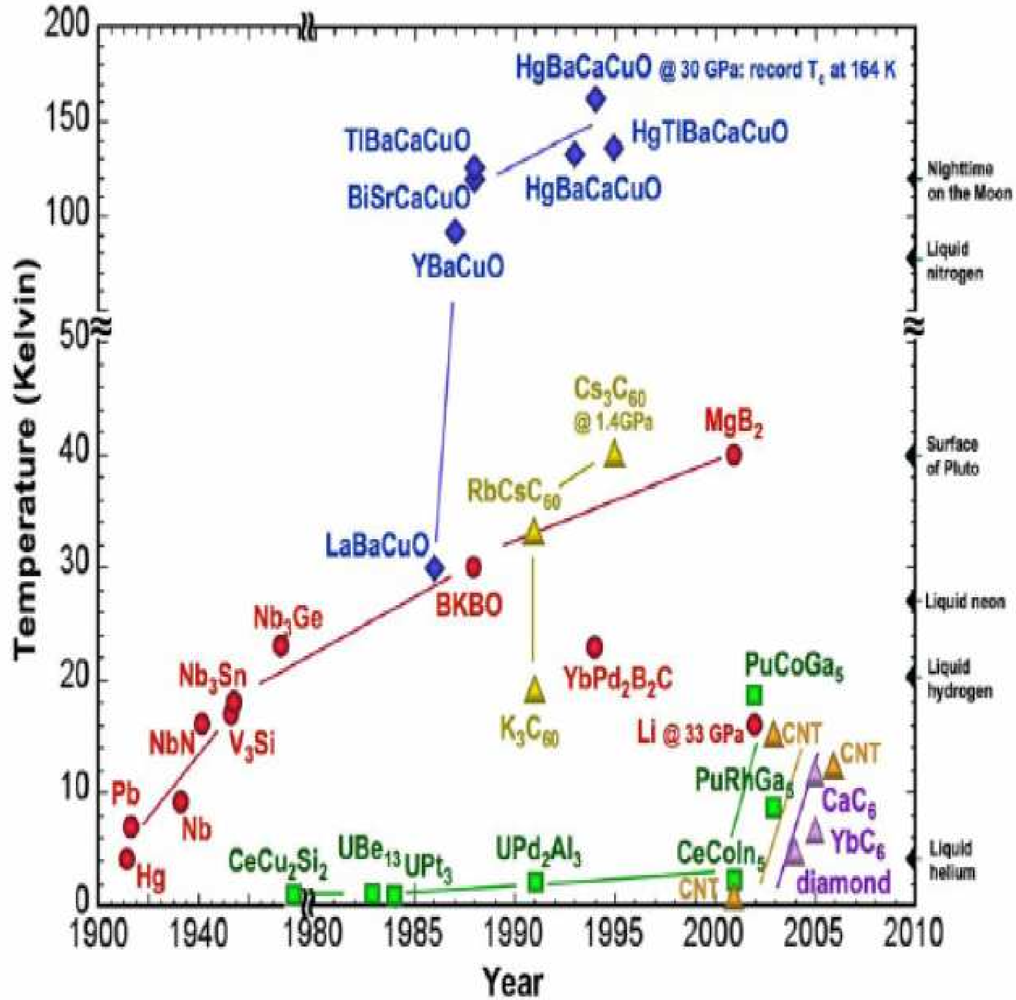
- **Lecture 3** *Basic concepts of RF superconductivity: RF losses and related figures of merits, Q vs E*



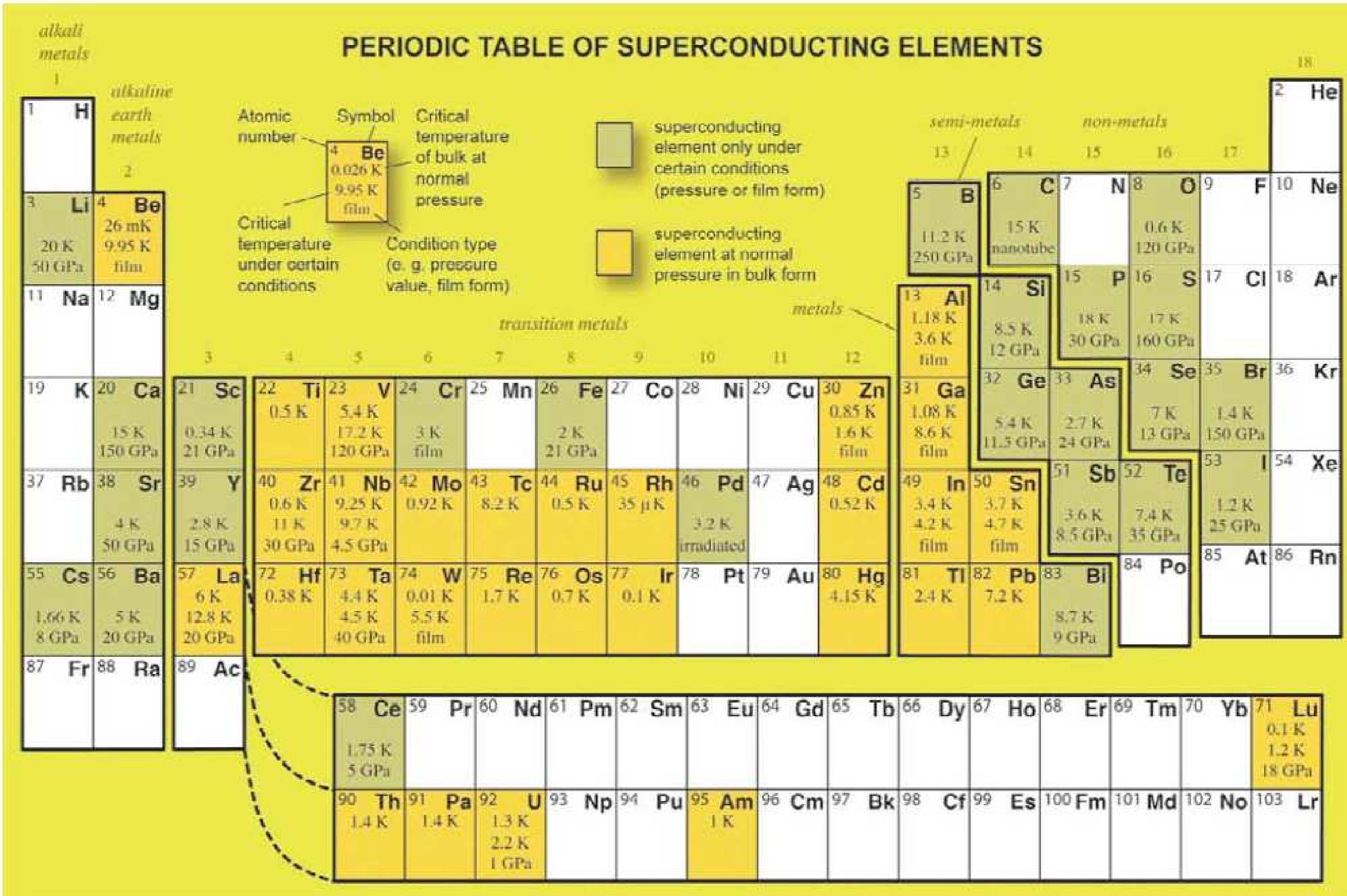
Discovery of superconductivity

Discovered in 1911 by Heike Kamerlingh Onnes and Giles Holst after Onnes was able to liquify helium in 1908. Nobel prize in 1913





Superconducting elements

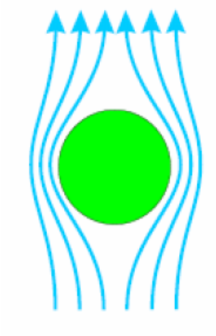


Superconducting state

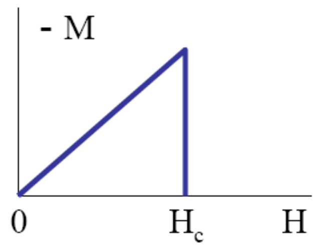
- The superconducting state is characterized by the critical temperature T_c and field H_c

$$H_c(T) = H_c(0) \cdot \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

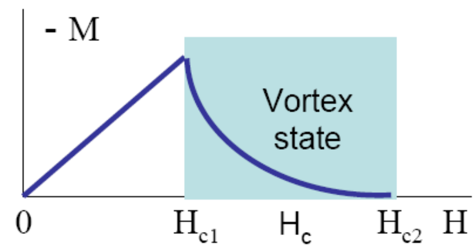
- The external field is expelled from a superconductor if $H_{ext} < H_c$ for Type I superconductors.
- For Type II superconductors the external field will partially penetrate for $H_{ext} > H_{c1}$ and will completely penetrate at H_{c2}



Superconductor in Meissner state = ideal diamagnetic



Complete Meissner effect in type-I superconductors

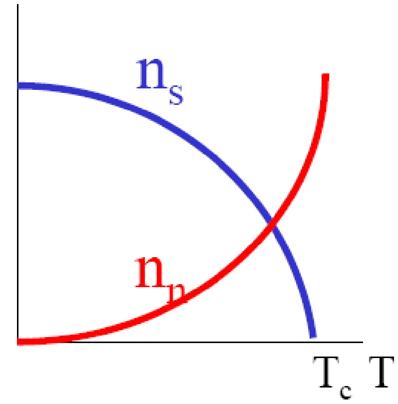


High-field partial Meissner effect in type-II superconductors

- Type-I: Meissner state B = H + M = 0 for $H < H_c$; normal state at $H > H_c$
- Type-II: Meissner state B = H + M = 0 for $H < H_{c1}$; partial flux penetration for $H_{c1} < H < H_{c2}$; normal state for $H > H_{c2}$

London equations (1935)

- Two-fluid model: coexisting SC and N "liquids" with the densities $n_s(T) + n_n(T) = n$.
- Electric field E accelerates only the SC component, the N component is short circuited.
- Second Newton law for the SC component: $m d\mathbf{v}_s/dt = e\mathbf{E}$ yields the first London equation:



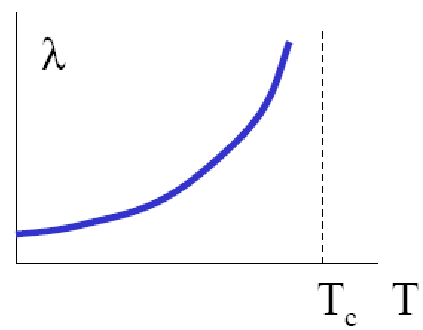
$$dJ_s/dt = (e^2 n_s/m)\mathbf{E} \quad \Longleftrightarrow \quad \mathbf{J} = \sigma \mathbf{E}$$

(ballistic electron flow in SC) (viscous electron flow in metals)

- Using the Maxwell equations, $\nabla\times \mathbf{E} = -\mu_0 \partial_t \mathbf{H}$ and $\nabla\times \mathbf{H} = \mathbf{J}_s$ we obtain the second London equation:

$$\lambda^2 \nabla \mathbf{H} - \mathbf{H} = 0$$

- London penetration depth: $\boxed{\lambda = \left(\frac{m}{e^2 n_s(T) \mu_0} \right)^{1/2}}$



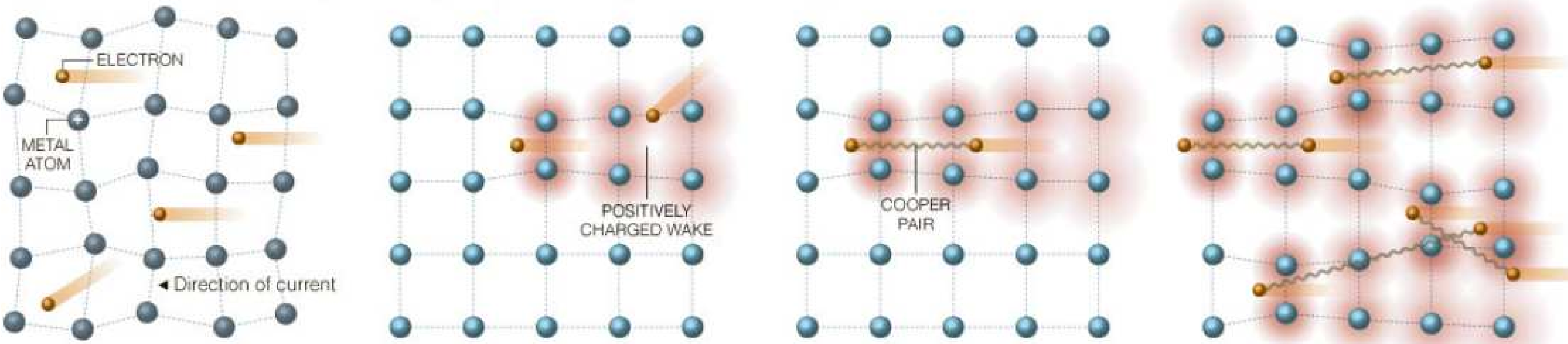
50+ years of BCS theory

Bardeen-Cooper-Schrieffer (BCS) theory (1957). Nobel prize in 1972

January 7, 2008

Low-Temperature Superconductivity

December was the 50th anniversary of the theory of superconductivity, the flow of electricity without resistance that can occur in some metals and ceramics.



ELECTRICAL RESISTANCE

Electrons carrying an electrical current through a metal wire typically encounter resistance, which is caused by collisions and scattering as the particles move through the vibrating lattice of metal atoms.

CRITICAL TEMPERATURE

As the metal is cooled to low temperatures, the lattice vibration slows. A moving electron attracts nearby metal atoms, which create a positively charged wake behind the electron. This wake can attract another nearby electron.

COOPER PAIRS

The two electrons form a weak bond, called a Cooper pair, which encounters less resistance than two electrons moving separately. When more Cooper pairs form, they behave in the same way.

SUPERCONDUCTIVITY

If a pair is scattered by an impurity, it will quickly get back in step with other pairs. This allows the electrons to flow undisturbed through the lattice of metal atoms. With no resistance, the current may persist for years.

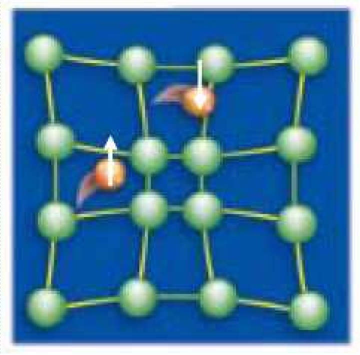
Sources: Oak Ridge National Laboratory; Philip W. Phillips

JONATHAN CORUM/THE NEW YORK TIMES

BCS theory

- **Attraction** between electrons with antiparallel momenta **k** and spins due to exchange of lattice vibration quanta (phonons)
- Instability of the normal Fermi surface due to bound states of electron (Cooper) pairs
- Bose condensation of overlapping Cooper pairs in a coherent superconducting state.
- Scattering on electrons does not cause the electric resistance because it would break the Cooper pair

The strong overlap of many Cooper pairs results in the macroscopic phase coherence



What is the phase coherence?

Incoherent (normal) crowd: each electron for itself

Phase-coherent (superconducting) condensate of electrons

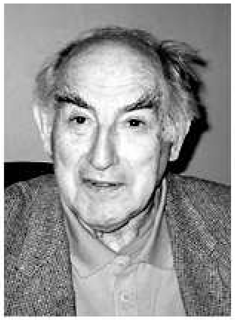
GL theory

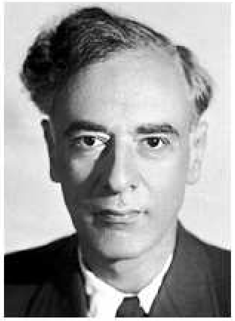
- the linear London equations

$$\boxed{\frac{\partial \vec{J}_s}{\partial t} = -\frac{\vec{E}}{\lambda^2 \mu_0}, \qquad \lambda^2 \nabla^2 \vec{H} - \vec{H} = 0}$$

along with the Maxwell equations describe the electrdynamics of SC at all T if:

- J_s is much smaller than the depairing current density J_d
- the superfluid density n_s is unaffected by current



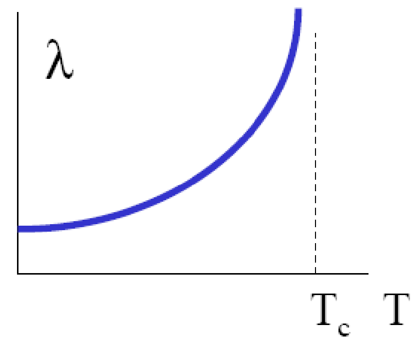


- Generalization of the London equations to nonlinear problems
- Phenomenological Ginzburg-Landau theory (1950, Nobel prize 2003) was developed before the microscopic BCS theory (1957).
- GL theory is one of the most widely used theories

Fundamental lengths and GL parameter

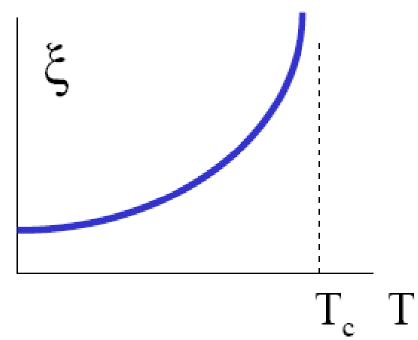
- **Magnetic London penetration depth:**

$$\lambda(T) = \left(\frac{m\beta}{2e^2 \mu_0 \alpha_0} \right)^{1/2} \sqrt{\frac{T_c}{T_c - T}}$$



- **Coherence length – a new scale of spatial variation of the superfluid density $n_s(r)$ or superconducting gap $\Delta(r)$:**

$$\xi(T) = \left(\frac{\hbar^2}{4m\alpha_0} \right)^{1/2} \sqrt{\frac{T_c}{T_c - T}}$$

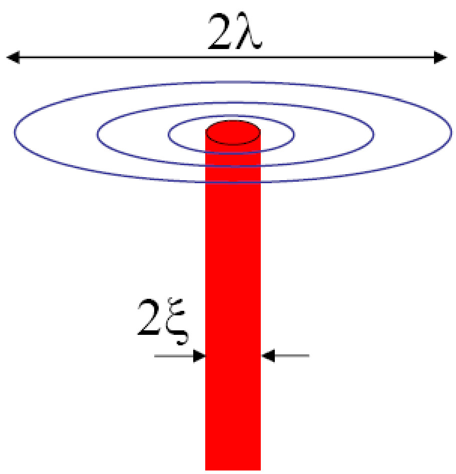


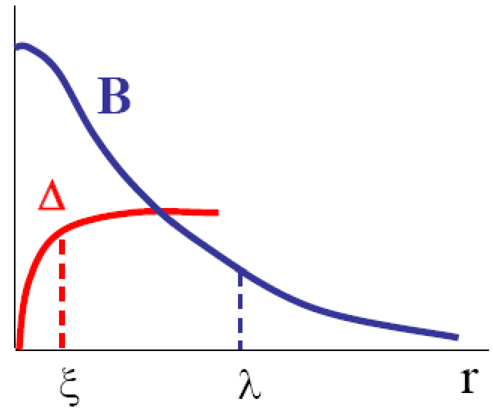
- **The GL parameter $\kappa = \lambda/\xi$ is independent of T.**
- **Critical field $H_c(T)$ in terms of λ and ξ:**

$$B_c(T) = \frac{\phi_0}{2\sqrt{2}\pi\xi(T)\lambda(T)}$$

Vortex state

Single vortex line





- Small core region $r < \xi$ where $\Delta(r)$ is suppressed
- Region of circulating supercurrents, $r < \lambda$.
- Each vortex carries the flux quantum ϕ_0

Important lengths and fields

- Coherence length ξ and magnetic (London) penetration depth λ

$$B_{c1} = \frac{\phi_0}{2\pi\lambda^2}\left(\ln\frac{\lambda}{\xi} + 0.5\right), \qquad B_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda\xi}, \qquad B_{c2} = \frac{\phi_0}{2\pi\xi^2}$$

Type-II superconductors: $\lambda/\xi > 1/\sqrt{2}$: For clean Nb, $\lambda \approx 40$ nm, $\xi \approx 38$ nm

What happens if AC field is applied?

Two fluid model considers both superconducting and normal conducting components:

- At $0 < T < T_c$ not all electrons are bonded into Cooper pairs. The density of *unpaired,* "normal" electrons is given by the Boltzman factor

$$n_{\text{normal}} \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

where 2Δ is the energy gap around Fermi level between the ground state and the excited state.

- Cooper pairs move without resistance, and thus dissipate no power. In DC case the lossless Cooper pairs short out the field, hence the normal electrons are not accelerated and the SC is lossless even for $T > 0$ K.
- The Cooper pairs do nonetheless have an inertial mass, and thus they cannot follow an AC electromagnetic fields instantly and do not shield it perfectly. A residual EM field remains and acts on the unpaired electrons as well, therefore causing power dissipation.
- We expect the surface resistance to drop exponentially below T_c.
- From previous lecture, recollect the Ohm's low. For the nearly-free electron model

$$\mathbf{j} = \sigma\mathbf{E} = \frac{n_n e^2 \tau}{m}\mathbf{E}$$

where τ is the average time between collisions.

- Between scattering events the electrons gain velocity

$$\Delta\mathbf{v} = \frac{-e\mathbf{E}\tau}{m}$$

Current in two-fluid model

- To calculate the surface impedance of a superconductor, one must take into account the "superconducting" electrons n_s in the two-fluid model
- There is no scattering, thus $\mathbf{j}_s = -n_s e \mathbf{v}$ and one can get the first London equation

$$m\frac{\partial \mathbf{v}}{\partial t} = -e\mathbf{E} \quad \Rightarrow \quad \frac{\partial \mathbf{j}_s}{\partial t} = \frac{n_s e^2}{m}\mathbf{E}$$

- In an RF field one gets

$$\mathbf{j}_s = -i\frac{n_s e^2}{m\omega}\mathbf{E} = -i\sigma_s\mathbf{E} \quad \text{or} \quad \mathbf{j}_s = \frac{-i}{\omega\mu_0\lambda_L^2}\mathbf{E}$$

- One can notice that the effective scattering time for a superconductor is the RF period divided by 2π.
- The total current is simply a sum of currents due to two "fluids":

$$\mathbf{j} = \mathbf{j}_n + \mathbf{j}_s = (\sigma_n - i\sigma_s)\mathbf{E}$$

- Thus one can apply the same treatment to a superconductor as was used for a normal conductor before with the substitution of the newly obtained conductivity.

Surface impedance of superconductors

- The surface impedance

$$Z_s = \sqrt{\frac{\omega\mu_0}{2\sigma}}(1+i) \Rightarrow \sqrt{\frac{\omega\mu_0}{2(\sigma_n - i\sigma_s)}}(1+i)$$

- The penetration depth

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} \Rightarrow \frac{1}{\sqrt{\pi f \mu_0 (\sigma_n - i\sigma_s)}}$$

- Note that $1/\omega$ is of the order of 100 ps whereas the relaxation time for normal conducting electrons if of the order of 10 fs. Also, $n_s >> n_n$ for $T << T_c$, hence $\sigma_n << \sigma_s$.
- Then

$$\delta \approx (1+i)\lambda_L \left(1 + i\frac{\sigma_n}{2\sigma_s}\right) \quad \text{and} \quad H_y = H_0 e^{-x/\lambda_L} e^{-ix\sigma_n/2\sigma_s\lambda_L}$$

- The fields decay rapidly, but now over the London penetration depth, which is much shorter than the skin depth of a normal conductor.
- For the impedance we get

$$Z_s \approx \sqrt{\frac{\omega\mu_0}{\sigma_s}}\left(\frac{\sigma_n}{2\sigma_s} + i\right) \qquad X_s = \omega\mu_0\lambda_L \qquad R_s = \frac{1}{2}\sigma_n\omega^2\mu_0\lambda_L^3$$

BCS surface resistivity

- Let us take a closer look at the surface impedance

$$Z_s \approx \sqrt{\frac{\omega\mu_0}{\sigma_s}}\left(\frac{\sigma_n}{2\sigma_s}+i\right) \qquad X_s = \omega\mu_0\lambda_L \qquad R_s = \frac{1}{2}\sigma_n\omega^2\mu_0\lambda_L^3$$

- One can easily show that $X_s >> R_s \rightarrow$ the superconductor is mostly reactive.
- The surface resistivity is proportional to the conductivity of the normal fluid! That is if the normal-state resistivity is low, the superconductor is more lossy. Analogy: a parallel circuit of a resistor and a reactive element driven by a current source. Observation: lower Q for cavities made of higher purity Nb.
- Calculation of surface resistivity must take into account numerous parameters. Mattis and Bardeen developed theory based on BCS, which predicts

$$R_{BCS} = A\frac{\omega^2}{T}e^{-\left(\frac{\Delta}{k_B T_c}\right)\frac{T_c}{T}},$$

where A is the material constant.

- While for low frequencies ($\leq$ 500 MHz) it may be efficient to operate at 4.2 K (liquid helium at atmospheric pressure), higher frequency structures favor lower operating temperatures (typically superfluid LHe at 2 K, below the lambda point, 2.172 K).
- Approximate expression for Nb: $R_{BCS} \approx 2\times10^{-4}\left(\frac{f\,[\mathrm{MHz}]}{1500}\right)^2\frac{1}{T}e^{\left(\frac{-17.67}{T}\right)}[\mathrm{Ohm}]$

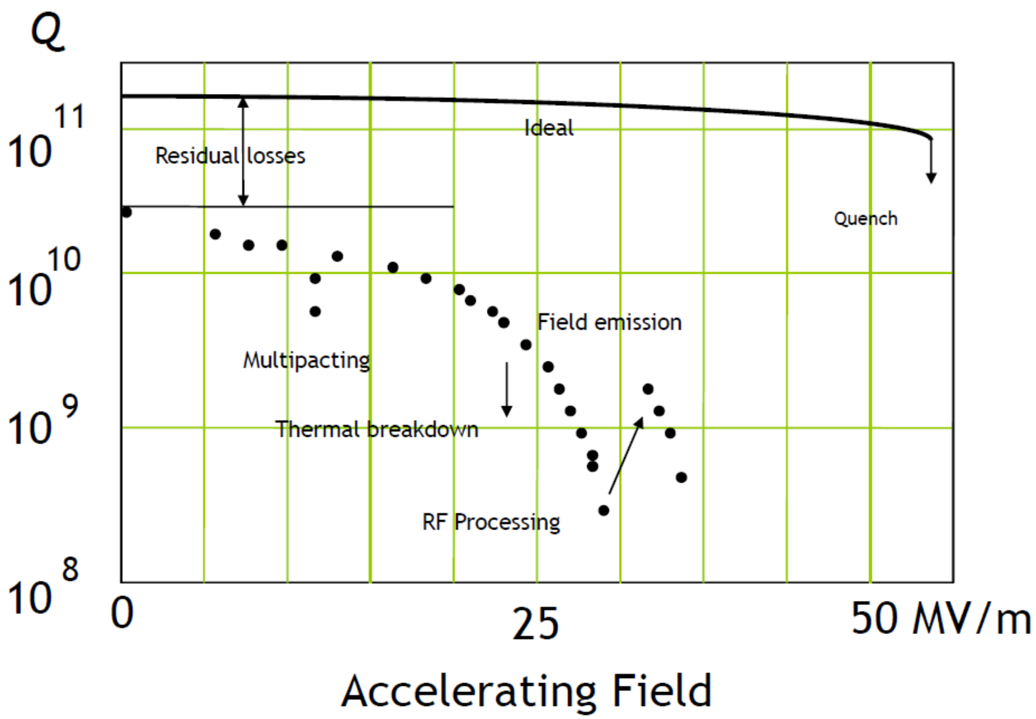
Why Nb?

	Type	T_c	H_{c1}	H_c	H_{c2}	Fabrication
	-	K	Oe	Oe	Oe	-
Nb	II	9.25	1700	2060	4000	bulk, film
Pb	I	7.20	-	803	-	electroplating
Nb_3Sn*	II	18.1	380	5200	250000	film
MgB_2	II	39.0	300	4290		film
Hg	I	4.15	-	411/339	-	-
Ta	I	4.47	-	829	-	-
In	I	3.41	-	281.5	-	-

**) Other compounds with the same β-tungsten or A15 structure are under investigation as well.*

- High critical temperature (cavities with High-T_c sputter coatings on copper have shown much inferior performance in comparison to niobium cavities) $\rightarrow$ lower RF losses $\rightarrow$ smaller heat load on refrigeration system.
- High RF critical field, which of the order of H_c. Strong flux pinning associated with high H_{c2} is undesirable as it is coupled with losses due to hysteresis. Hence a 'soft' superconductor must be used.
- Good formability is desirable for ease of cavity fabrication. Alternative is a thin superconducting film on a copper substrate.
- Pure niobium is the best candidate, although its critical temperature Tc is only 9.25 K, and the thermodynamic critical field about 200 mT. Nb_3Sn with a critical temperature of 18.1 K looks more favorable at first sight, however the gradients achieved in Nb_3Sn coated niobium cavities were always below 15 MV/m, probably due to grain boundary effects in the Nb3Sn layer. For this reason niobium is the preferred superconducting material.

Q vs E: real world



- ► It is customary to characterize performance of superconducting cavities by plotting dependence of their quality factor on either electric field (accelerating or peak surface) or peak magnetic field.
- ► Q vs E plots is a "signature" of cavity performance.
- ► At low temperatures measured Q is lower than predicted by BCS theory.
- ► There are several mechanisms responsible for additional losses. Some of them are well understood and preventable, some are still under investigation.

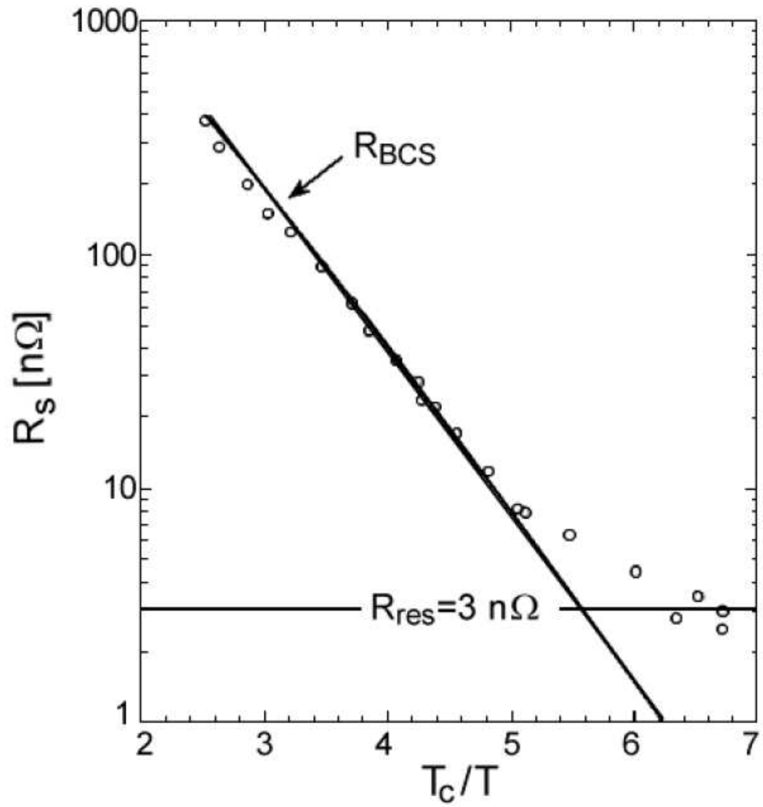
Residual surface resistivity

- At low temperatures the measured surface resistivity is larger than predicted by theory:

$$R_s = R_{BCS}(T) + R_{res}$$

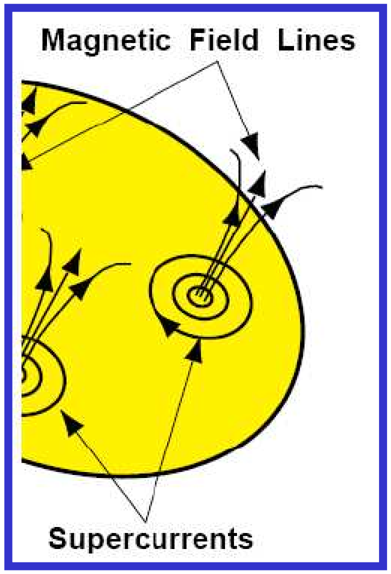
where R_{res} is the temperature independent residual resistivity. It can be as low as 1 nOhm, but typically is ~10 nOhm.

- Characteristics:
  - no strong temperature dependence
  - no clear frequency dependence
  - can be localized
  - not always reproducible
- Causes for this are:
  - magnetic flux trapped in at cooldown
  - dielectric surface contaminations (chemical residues, dust, adsorbates)
  - NC defects & inclusions
  - surface imperfections
  - hydrogen precipitates



Trapped magnetic flux

- Ideally, if the external magnetic fiels is less than H_{c1}, the DC flux will be expelled due to Meissner effect. In reality, there are lattice defects and other inhomogeneities, where the flux lines may be "pinned" and trapped within material.



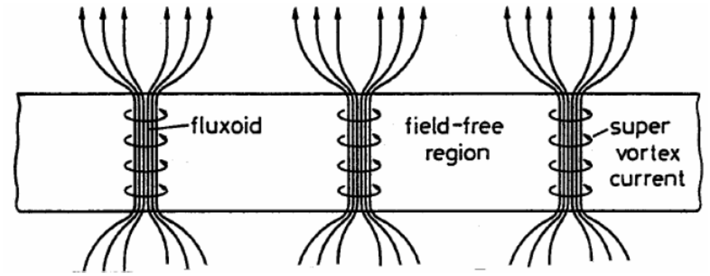
- The resulting contribution to the residual resistance

$$R_{mag} = \frac{H_{ext}}{2H_{c2}} R_n$$

- For high purity (RRR=300) Nb one gets

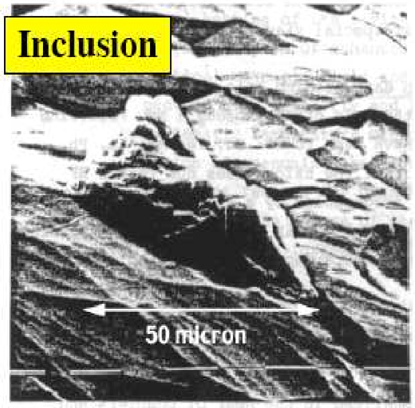
$$R_{mag} = 0.3(n\Omega) H_{ext}(mOe)\sqrt{f(GHz)}$$

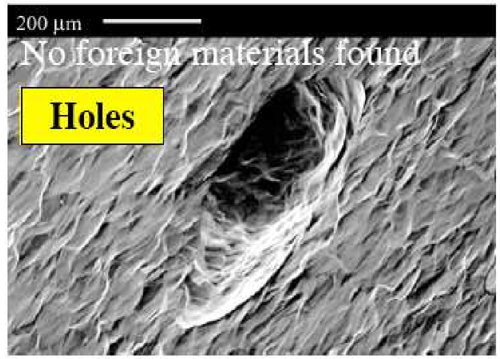
- Earth's field is 0.5 G, which produces residual resistivity of 150 nOhm at 1 GHz and $Q_0 < 2\times10^9$
- Hence one needs magnetic shielding around the cavity to reach quality factor in the 10^{10} range.
- Usually the goal is to have residual magnetic field of less than 10 mG.



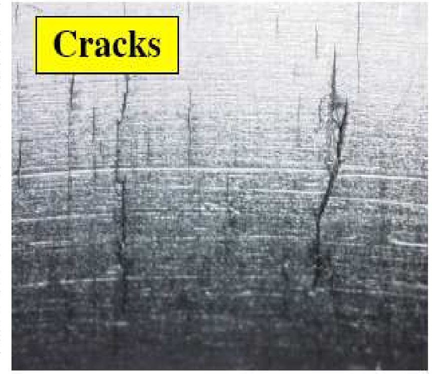
Cornell University
Laboratory for Elementary-Particle Physics

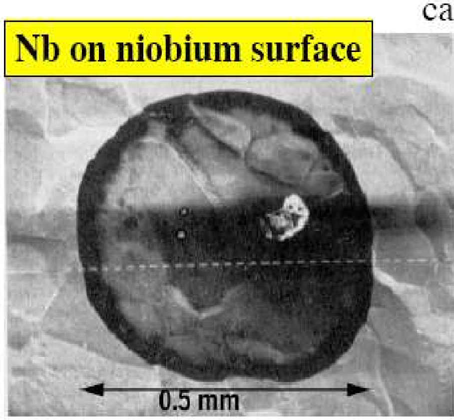
Examples of surface defects

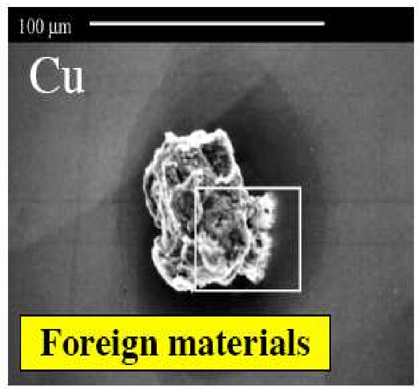


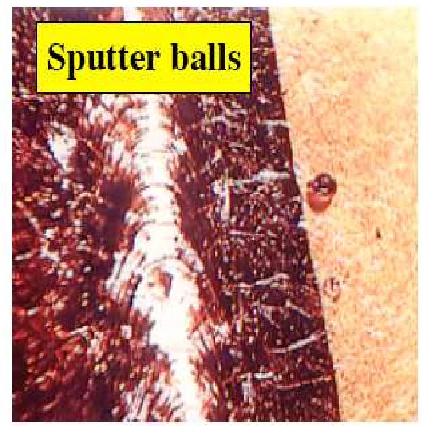


Surface defects, holes can also cause TB







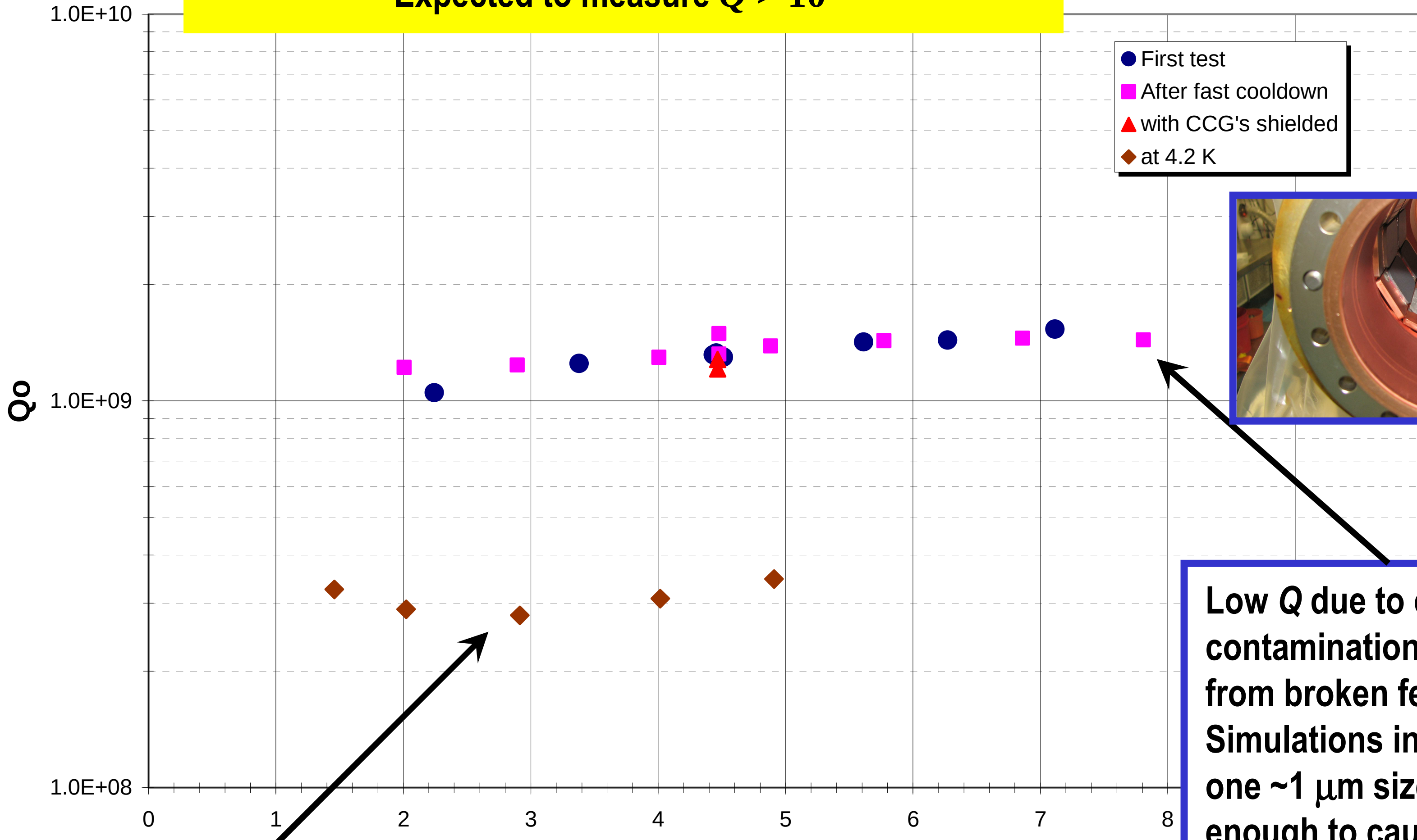


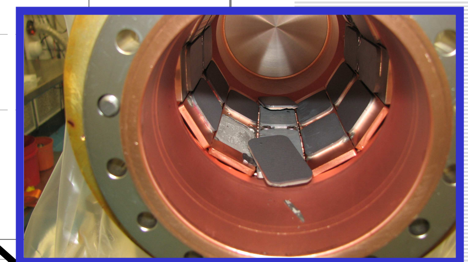
Surface defects can cause:

- Enhanced residual losses
- Premature quench
- Field emission

Surface contamination

**1.3 GHz two-cell cavity with attached ferrite beam loads.
Expected to measure $Q > 10^{10}$**



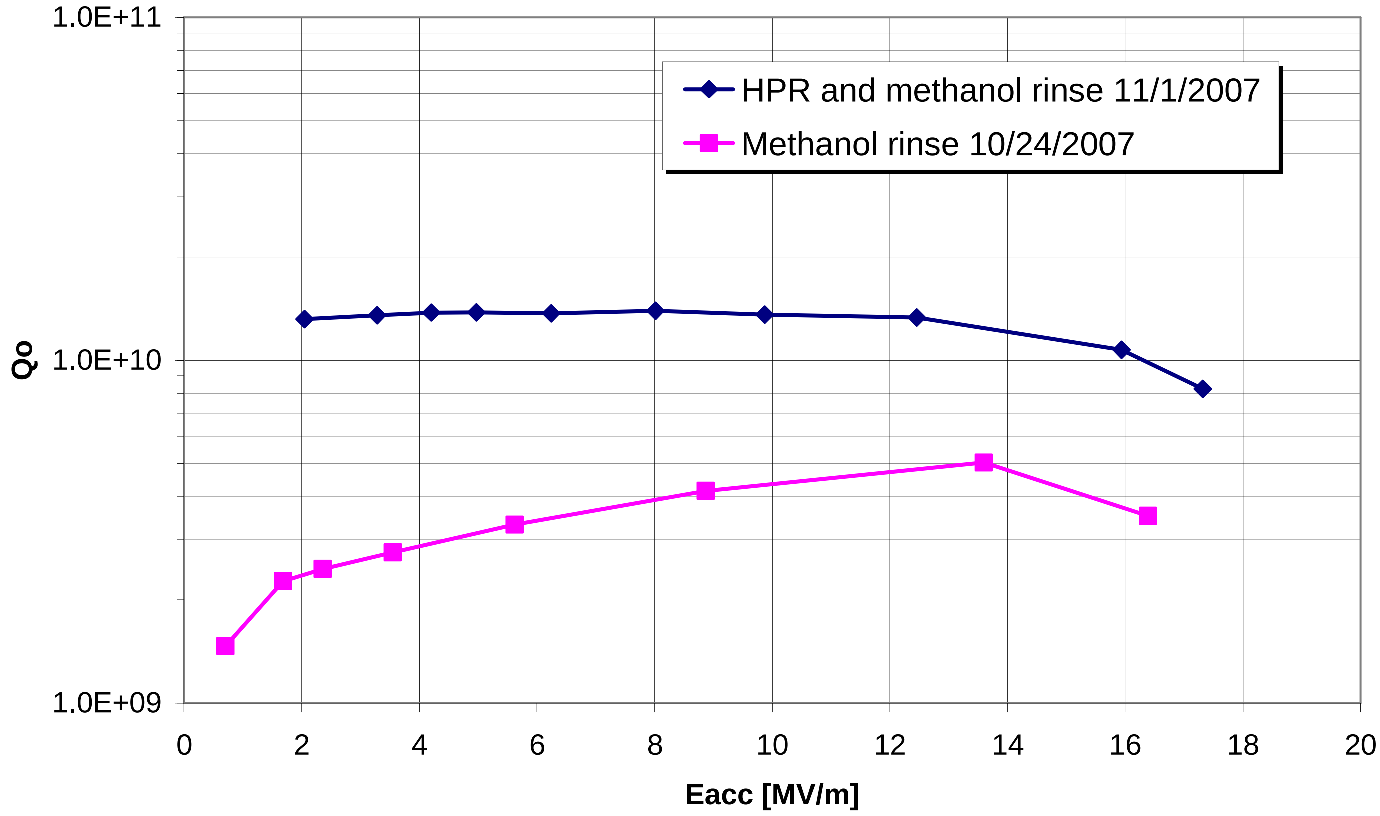


Low Q due to cavity contamination by dust from broken ferrite tiles. Simulations indicate that one ~1 μm size particle is enough to cause this!

***Q* is consistent with BCS at 4.2 K**

Cavity re-test

- **Re-tested the cavity after removing ferrite and rinsing it with methanol: *Q* was still low.**
- **HPR'd the cavity and tested again: *Q* is back to normal.**

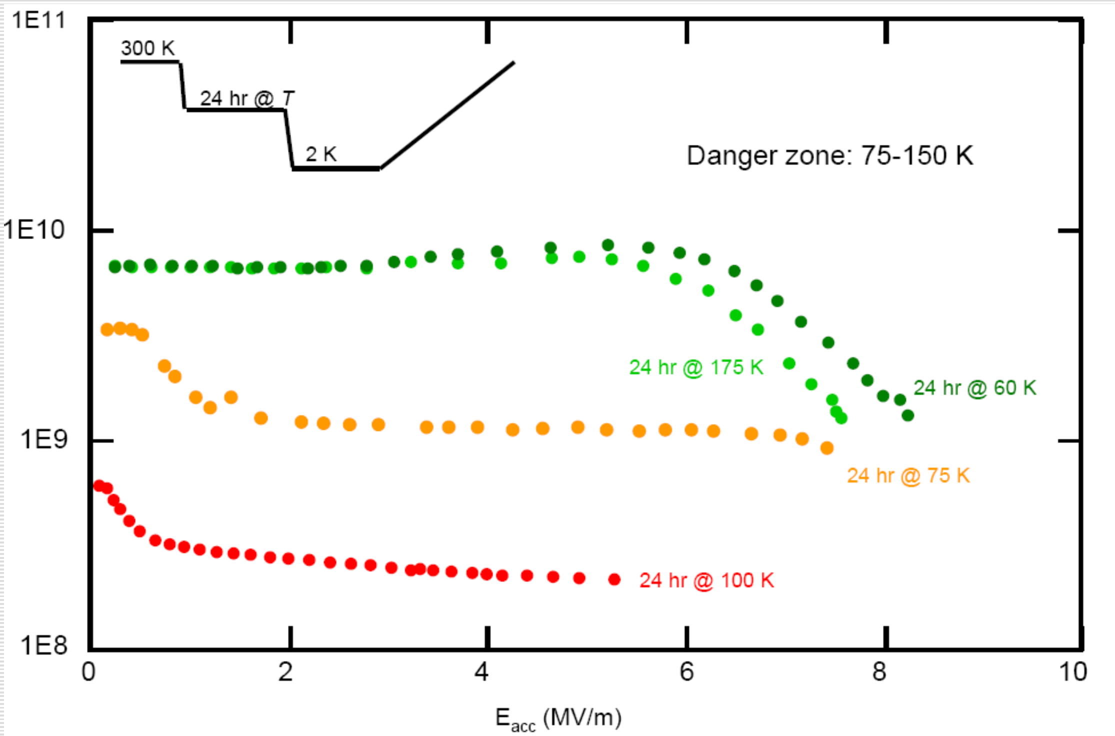


Q disease

The hydrogen dissolved in bulk niobium can under certain conditions during cooldown precipitate as a lossy hydride at the niobium surface. It has poor superconducting properties: Tc = 2.8 K and Hc = 60 Oe. This is known as the "*Q*-diseas." At temperatures above 150 K too high concentration of hydrogen is required to form the hydride phase ($10^3 – 10^4$ ppm). However, in the temperature range from 75 to 150 K the required hydrogen concentrations drops to as low as 2 wt ppm while its diffusion rate remains significant. This is the danger zone.

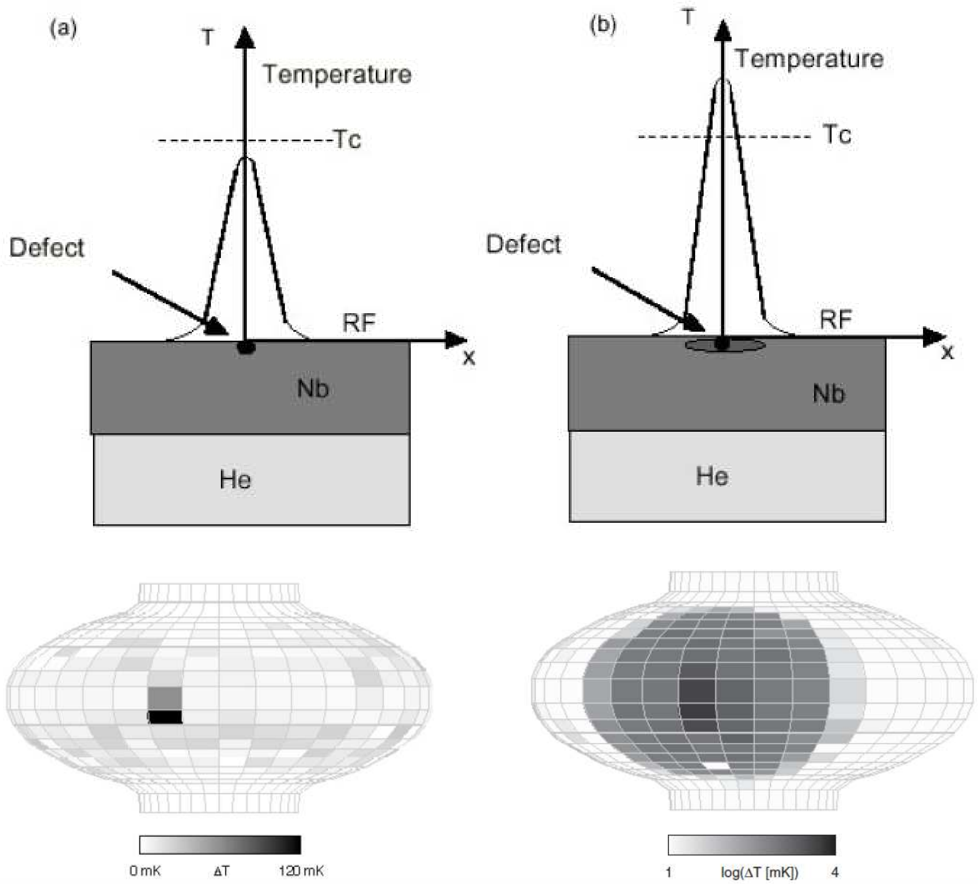
Mitigation:

- rapid cooldown through the danger temperature zone;
- degassing hydrogen by heating the Nb cavity in vacuum of better than 10^{-6} Torr at 600°C for 10 hrs or at 800°C for 1 to 2 hrs.;
- keep the acid temperature below 15°C during chemical etching.

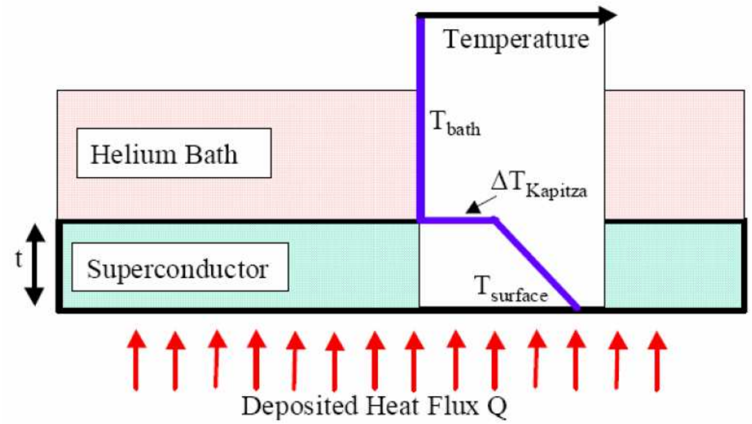


Thermal breakdown

- If there is a localized heating, the hot area will increase with field. At a certain field there is a thermal runaway and the filed collapses (loss of superconductivity or quench).
- Thermal breakdown occurs when the heat generated at the hot spot is larger than that can be evacuated to the helium bath.

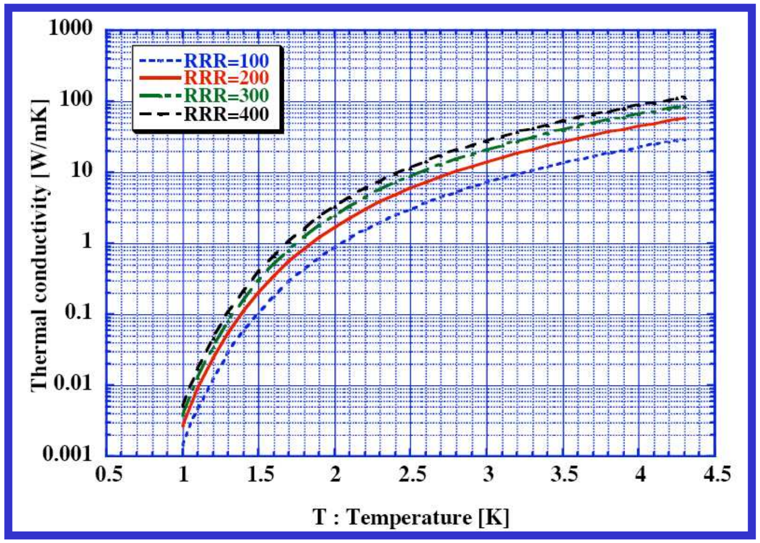


► Both the thermal conductivity and the surface resistance of Nb are highly temperature dependent between 2 and 9 K.



RRR

- **Residual Resistivity Ratio (RRR) is a measure of material purity and is defined as the ratio of the resistivity at 273 K (or at 300 K) to that at 4.2 K in normal state.**
- **High purity materials have better thermal conductivity, hence better handling of RF losses.**
- **The ideal RRR of niobium due to phonon scattering is 35,000. Typical “reactor grade” Nb has RRR ≈ 30. Nb sheets used in cavity fabrication have RRR ≥ 200.**



$$\lambda(4.2K) \approx 0.25 \cdot RRR \quad [W/(m \cdot K)]$$

$$RRR = \left(\sum_i f_i / r_i \right)^{-1}$$

where the f_i denote the fractional contents of impurity i (measured in weight ppm) and the r_i the corresponding resistivity coefficients which are listed in the following table.

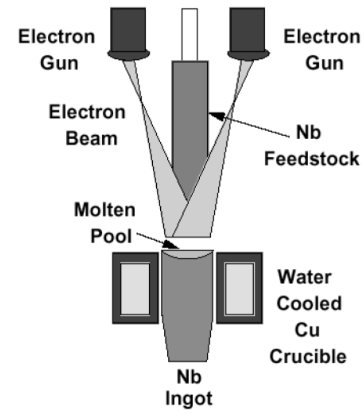
Table II Weight factor r_i of some impurities (see equation (4))

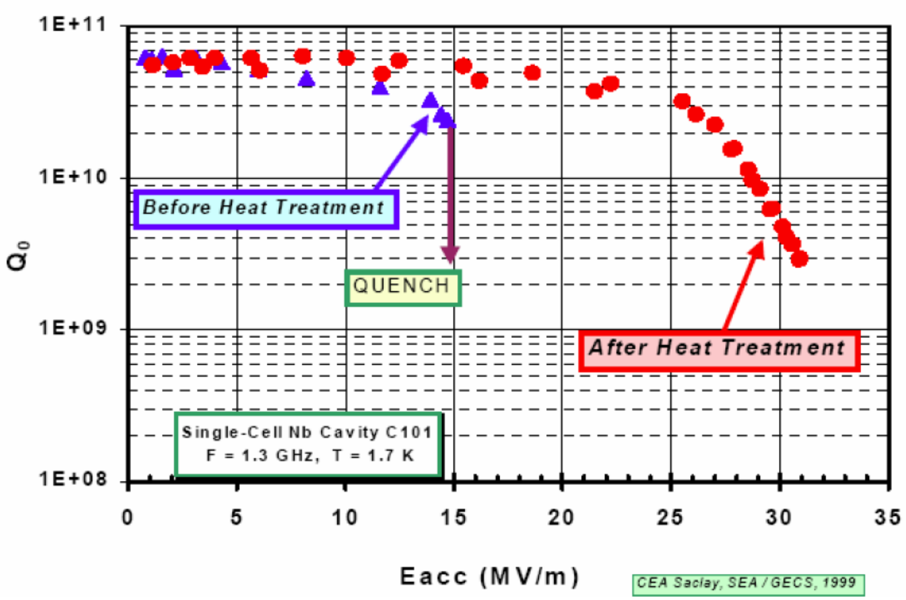
Impurity atom i	N	O	C	H	Ta
r_i in 10^4 wt. ppm	0.44	0.58	0.47	0.36	111

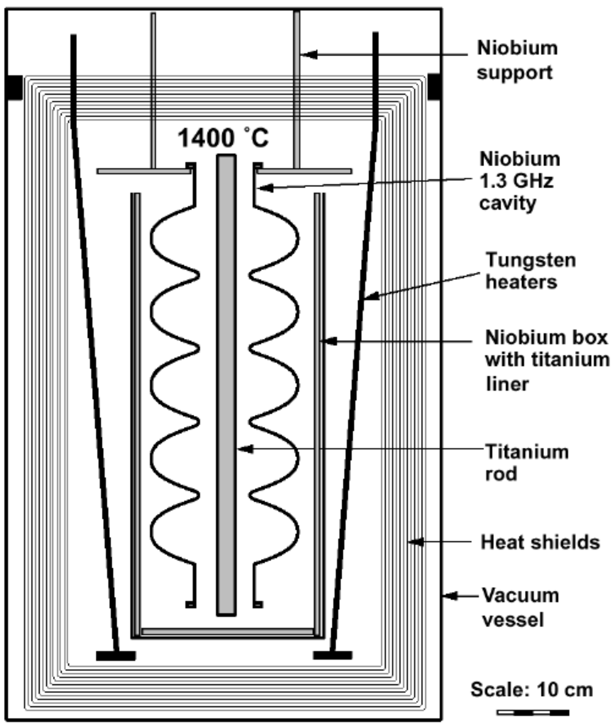
Improving thermal conductivity

There are several ways to improve material purity of Nb:

- Industry can produce high Nb purity by e-beam melting in a vacuum furnace. RRR = 300 – 400 Nb is available.
- Cavities can be post-purified in a vacuum furnace by heating to ~ 1400°C, evaporating Ti on cavity surface (use titanium as getter to capture impurities), upon cooldown etching away the titanium. This doubles the purity (RRR ~ 600 if originally RRR = 300).

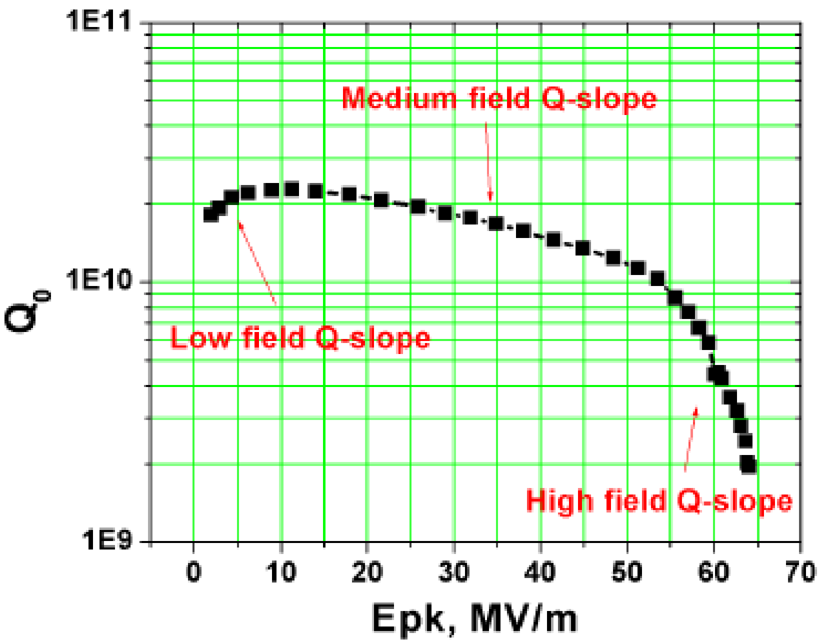






Q-slopes

- The observed *Q* of a niobium cavity shows several interesting features with increasing field. As there is still no commonly accepted explanation of physics behind each of the *Q*-slopes, we are mentioning those just to have a complete picture and refer for further details to second volume of H. Padamsee's book.
- In the low-field region *Q* surprisingly increases. This does not present any limitation of cavity performance. Mild baking generally enhances the low-field *Q*-slope.
- At medium fields Q gradually decreases, a common feature of all Nb cavities. This is generally attributed to a combination of surface heating and "nonlinear" BCS resistance. Mild baking (100 – 120°C for 48 hrs) usually decreases this *Q*-slope.
- Finally, there is a strong *Q*-drop at the highest fields. This is still highly active area of basic SRF research. Mild baking helps under certain conditions.
- Eventually superconductivity quenches due to a thermal instability at defects.



What have we learned?

- The superconducting state is characterized by the critical temperature and magnetic field.
- There are Type I and Type II superconductors.
- Two-fluid model and BCS theory explain surface resistivity of superconductors.
- Nb is a material of choice in either bulk form or as a film on a copper substrate.
- Other materials are being investigated.
- At low temperatures residual resistivity limits performance of superconducting cavities.
- There are several phenomena responsible for the deviation of “real world” losses from theoretical predictions.
- Material quality (impurities, mechanical damage) plays important role.
- Performance of SC cavities is dependant on the quality of a thin surface layer.

✪ We will discuss other mechanisms limiting performance of superconducting cavities in the next lecture. Also, we will assume that cavities are made of bulk Nb (unless other materials are specifically mentioned.)