Introduction to Accelerators

Lecture 4

Basic Properties of Particle Beams

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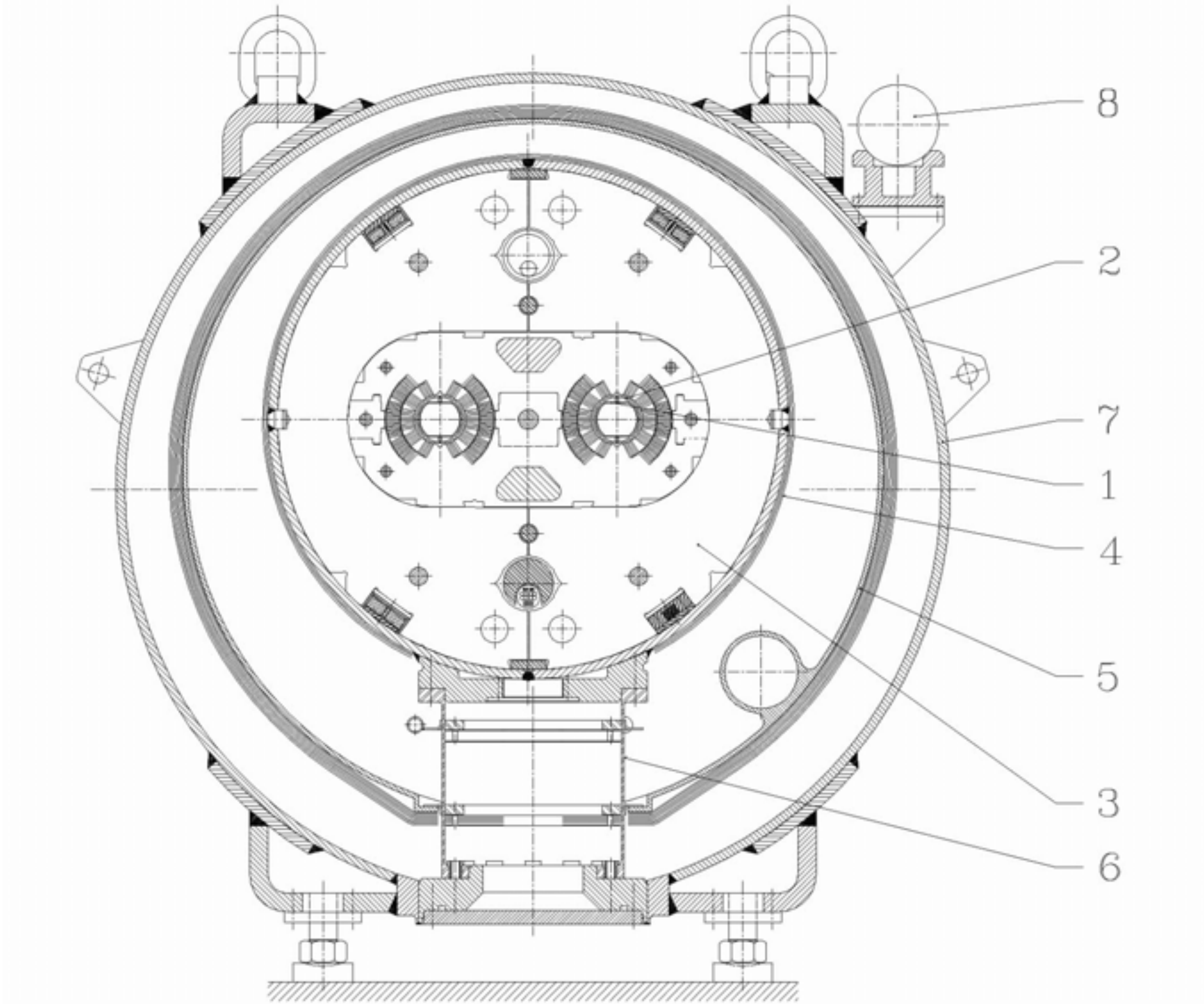
Homework item



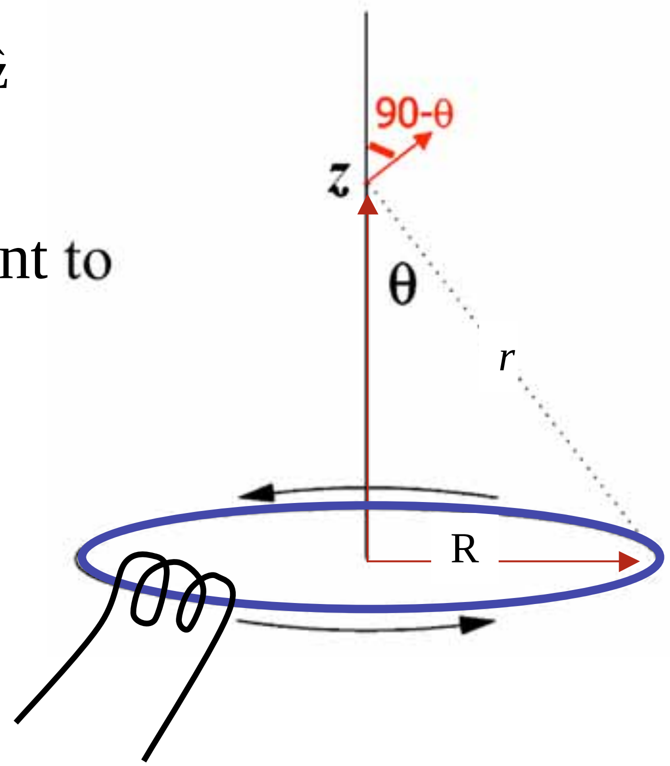
From the last lecture

We computed the B-field from current loop with I = constant

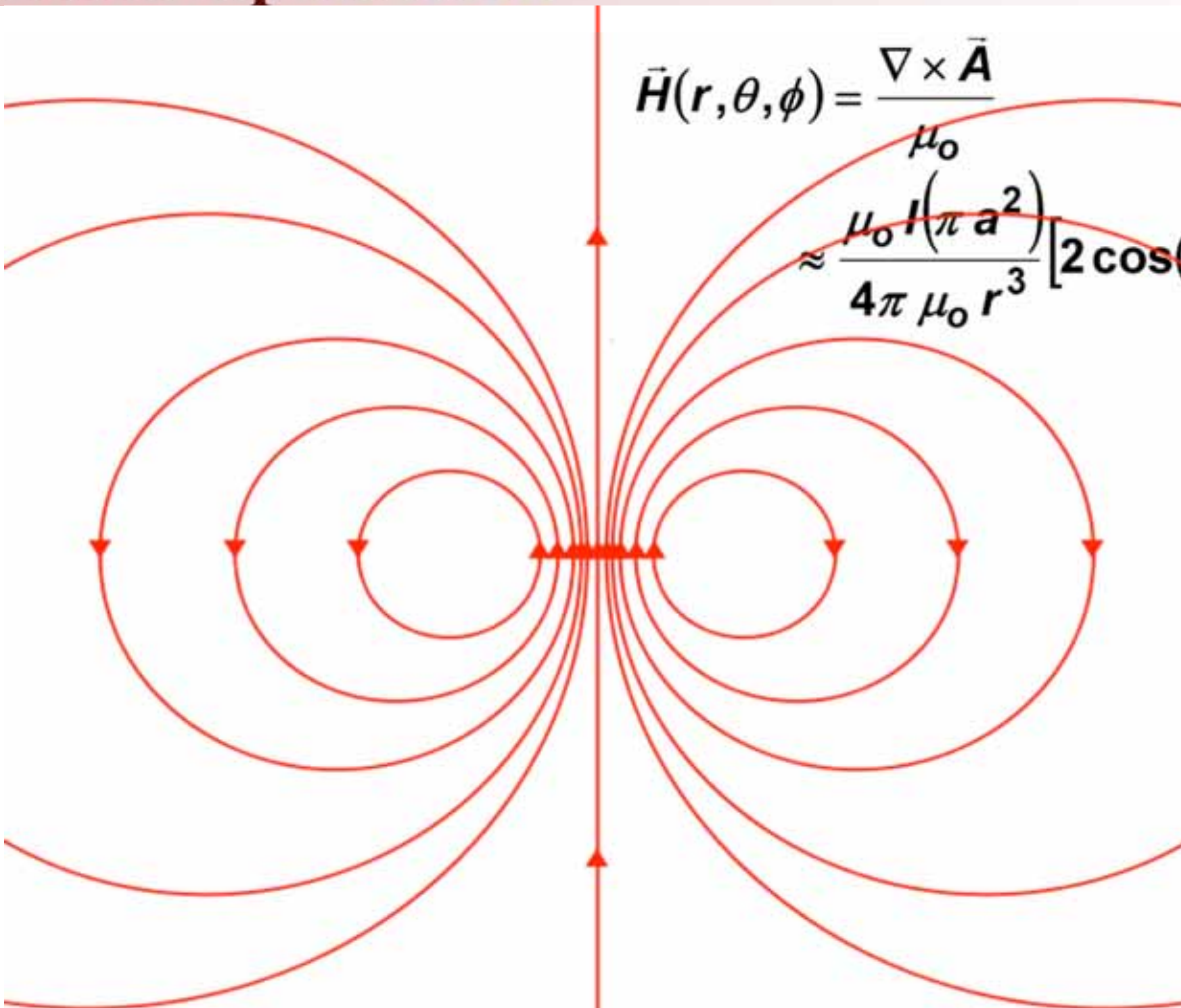
- By the Biot-Savart law we found that on the z-axis

$$\mathbf{B} = \frac{I}{cr^2} R \sin\theta \int_0^{2\pi} d\varphi \,\hat{\mathbf{z}} = \frac{2\pi I R^2}{c\left(R^2 + z^2\right)^{3/2}} \hat{\mathbf{z}}$$

- What happens if we drive the current to have a time variation?



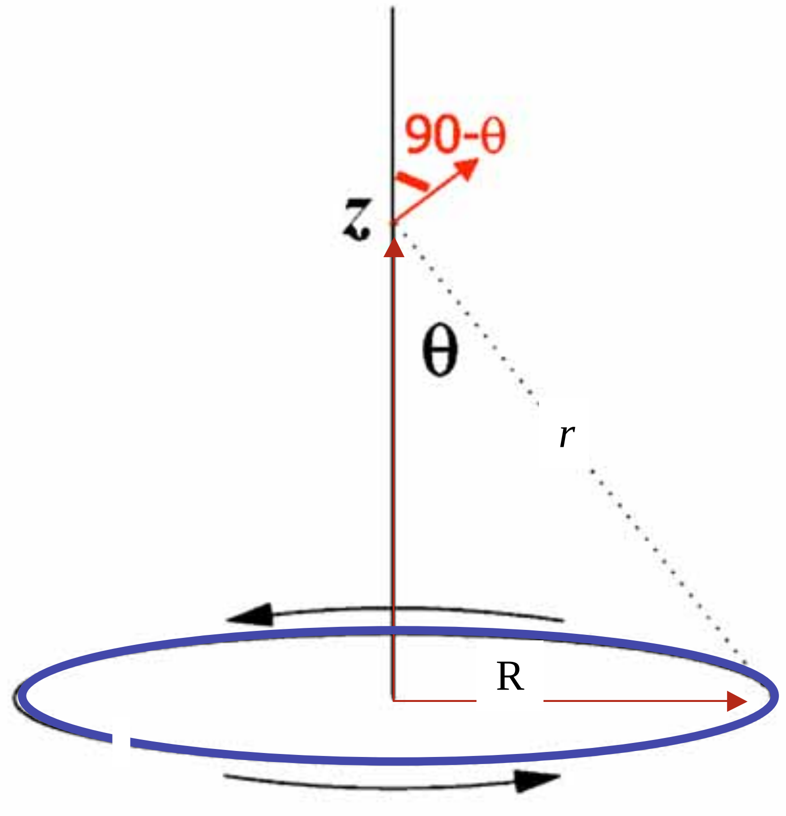
The far field B-field has a static dipole form



$$\vec{H}(r,\theta,\phi) = \frac{\nabla \times \vec{A}}{\mu_o} \approx \frac{\mu_o I(\pi a^2)}{4\pi\, \mu_o\, r^3}\left[2\cos(\theta)\hat{r} + \sin(\theta)\hat{\theta}\right]$$

Importantly the ring of current does not radiate

Question to ponder:
What is the field from this situation?



We'll return to this question in the second half of the course

Is this really paradoxical?

- Let's look at Maxwell's equations

$$\nabla . \mu_o \vec{H}(\vec{r},t) = 0 \qquad \nabla \times \vec{E}(\vec{r},t) = -\frac{\partial \mu_o \vec{H}(\vec{r},t)}{\partial t}$$

$$\nabla . \varepsilon_o \vec{E}(\vec{r},t) = \rho(\vec{r},t) \qquad \nabla \times \vec{H}(\vec{r},t) = \vec{J}(\vec{r},t) + \frac{\partial \varepsilon_o \vec{E}(\vec{r},t)}{\partial t}$$

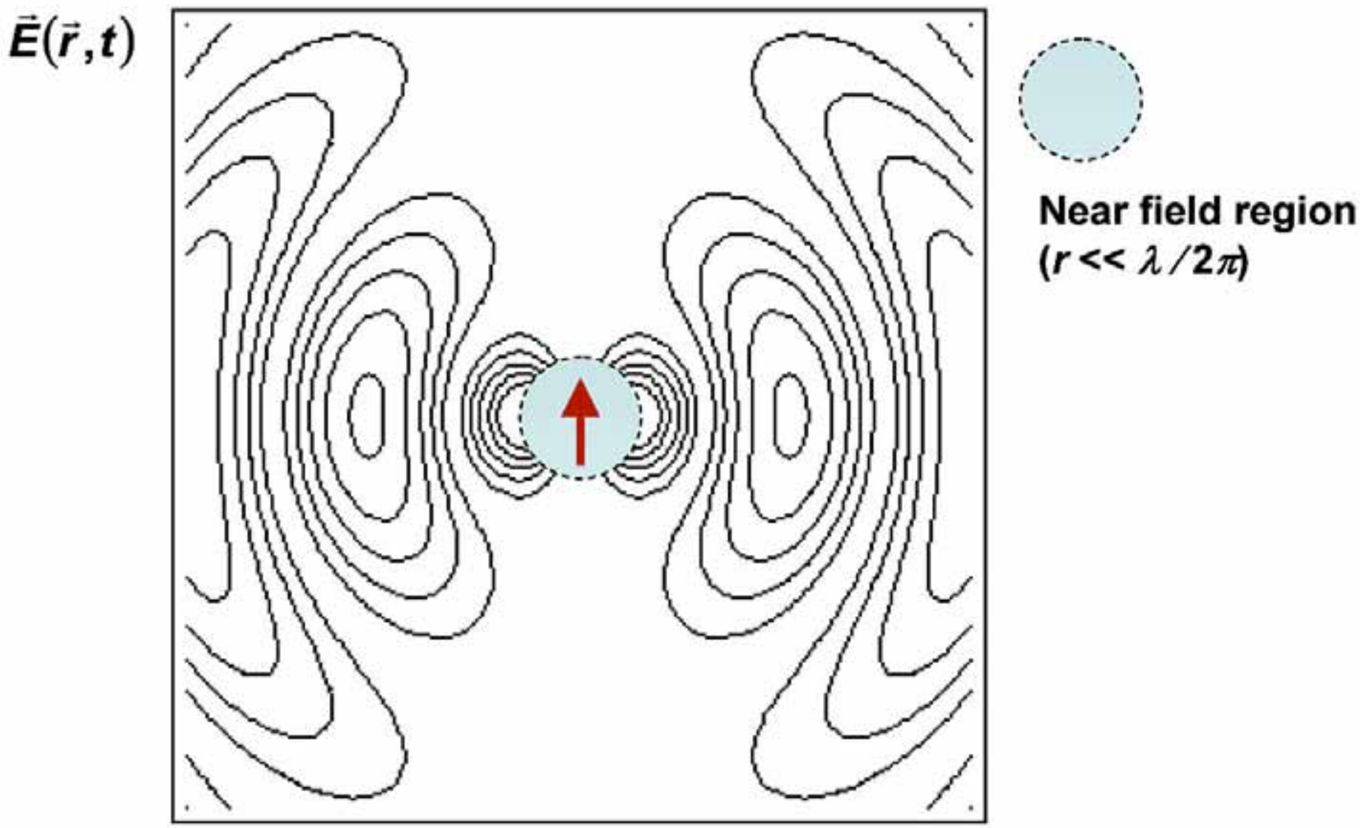
- Take the curl of ∇x**E**

$$\nabla \times \nabla \times \vec{E}(\vec{r},t) = -\frac{\partial \mu_o \nabla \times \vec{H}(\vec{r},t)}{\partial t} = -\frac{\partial \mu_o \vec{J}(\vec{r},t)}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2}$$

- Hence

$$\Rightarrow \boxed{\nabla \times \nabla \times \vec{E}(\vec{r},t) + \frac{1}{c^2}\frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2} = -\frac{\partial \mu_o \vec{J}(\vec{r},t)}{\partial t}}$$

The dipole radiation field: note the similarity to the static dipole

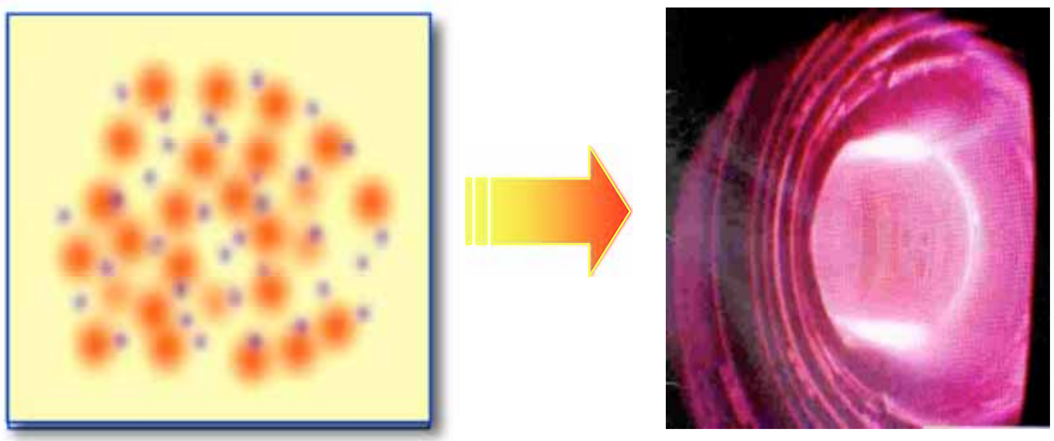


$$\vec{E}(\vec{r}) = \eta_o \frac{j\,k\,Id}{4\pi\,r} e^{-j\,k\,r} \left\{ \hat{r} \left[\frac{1}{j\,k\,r} + \left(\frac{1}{j\,k\,r} \right)^2 \right] 2\cos(\theta) + \hat{\theta} \left[1 + \frac{1}{j\,k\,r} + \left(\frac{1}{j\,k\,r} \right)^2 \right] \sin(\theta) \right\}$$

Now on to beams

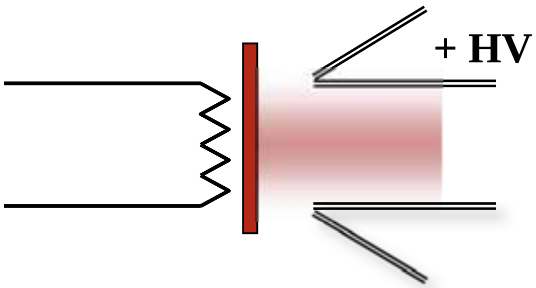
Beams: particle bunches with directed velocity

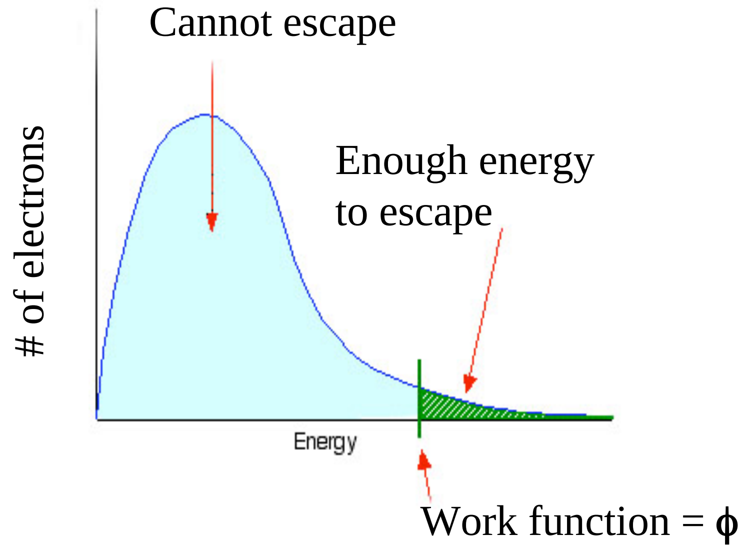
- Ions - either missing electrons (+) or with extra electrons (-)
- Electrons or positrons
- Plasma - ions plus electrons
- Source techniques depend on type of beam & on application



Electron sources - thermionic

- Heated metals
  - Some electrons have energies above potential barrier





Electrons in a metal obey Fermi statistics

$$\frac{dn(E)}{dE} = A\sqrt{E}\,\frac{1}{\left[e^{(E-E_F)/kT}+1\right]}$$

Electrons with enough momentum can escape the metal

- Integrating over electrons going in the z direction with

$$p_z^2/2m > E_F + \phi$$

yields

$$J_e = \int_{-\infty}^{\infty} dp_x \int_{-\infty}^{\infty} dp_y \int_{p_{z,\,free}}^{\infty} dp_x (2/h^3) f(E) v_z$$

some considerable manipulation yields the Richardson-Dushman equation

$$I \propto AT^2 \exp\left(\frac{-q\phi}{k_B T}\right)$$

$A=1202\ mA/mm^2K^2$

Brightness of a beam source

- A figure of merit for the performance of a beam source is the brightness

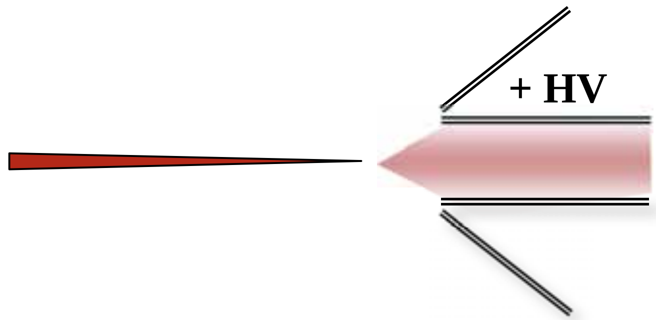
$$B = \frac{\text{Beam current}}{\text{Beam area} \cdot \text{Beam Divergence}} = \frac{\text{Emissivity (J)}}{\sqrt{\text{Temperature/mass}}}$$

$$= \frac{J_e}{\left(\sqrt{\frac{kT}{\gamma m_o c^2}}\right)^2} = \frac{J_e \gamma}{\left(kT / m_o c^2\right)}$$

Typically the normalized brightness is quoted for $\gamma = 1$

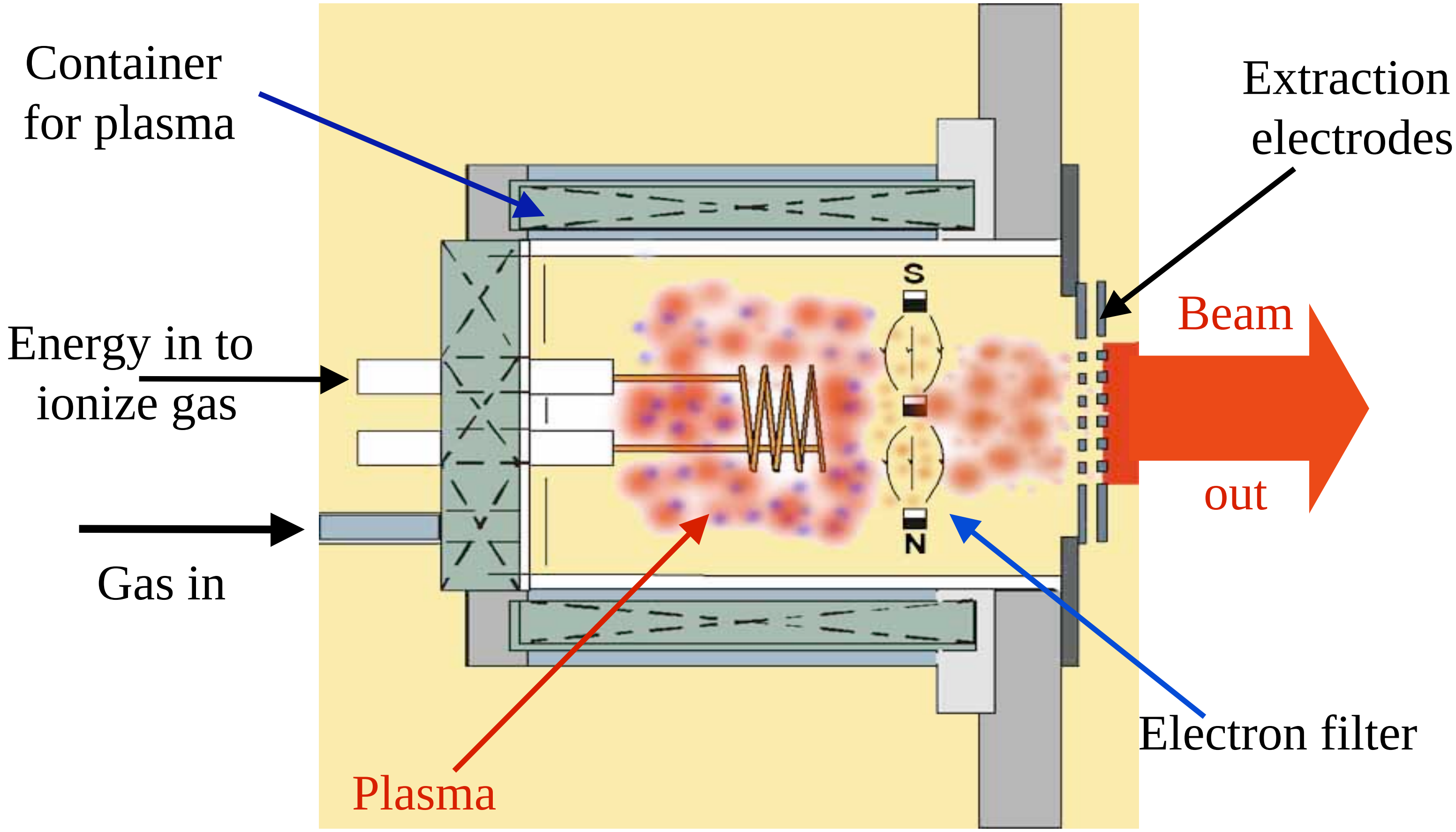
Other ways to get electrons over the potential barrier

- Field emission
  - Sharp needle enhances electric field



- Photoemission from metals & semi-conductors
  - Photon energy exceeds the work function
  - These sources produce beams with high current densities and low thermal energy
  - This is a topic for a separate lecture

Anatomy of an ion source



Electron beams can also be used to ionize the gas or sputter ions from a solid

What properties characterize particle beams?

Beams have directed energy

- The beam momentum refers to the average value of p_z of the particles

$$p_{beam} = \langle p_z \rangle$$

- The beam energy refers to the mean value of

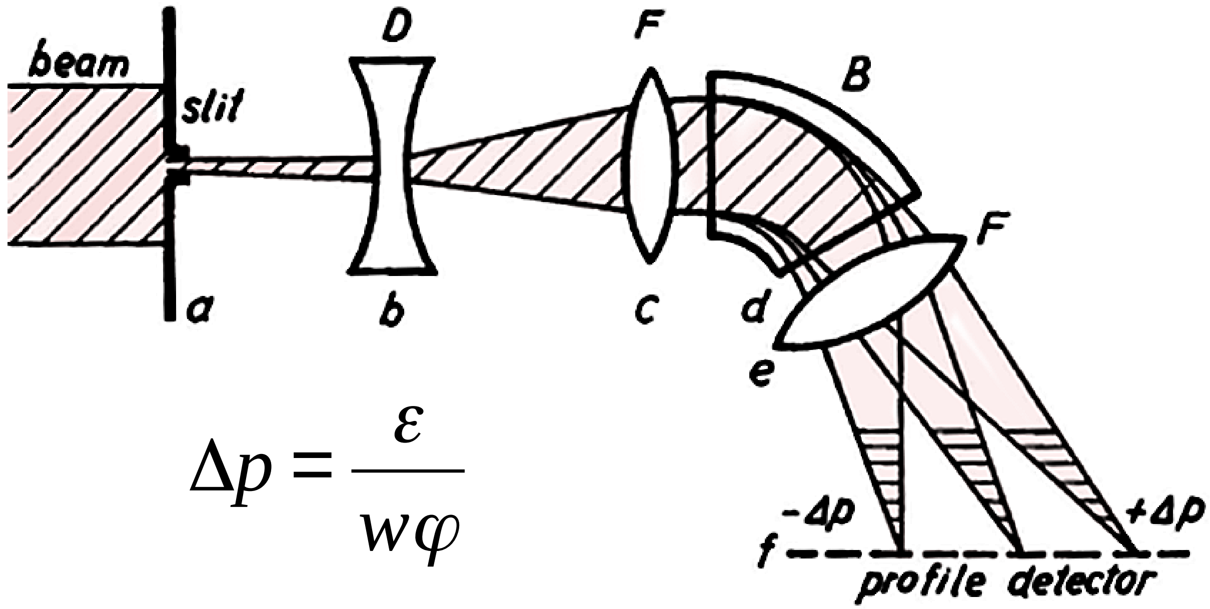
$$E_{beam} = \left[\langle p_z \rangle^2 c^2 + m^2 c^4 \right]^{1/2}$$

- For highly relativistic beams $pc \gg mc^2$, therefore

$$E_{beam} = \langle p_z \rangle c$$

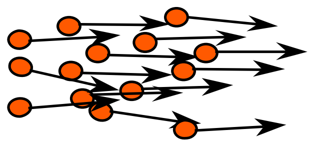
Measuring beam energy & energy spread

- Magnetic spectrometer - for good resolution, Δp one needs
  - ➙ small sample emittance ε, (parallel particle velocities)
  - ➙ a large beamwidth w in the bending magnet
  - ➙ a large angle φ



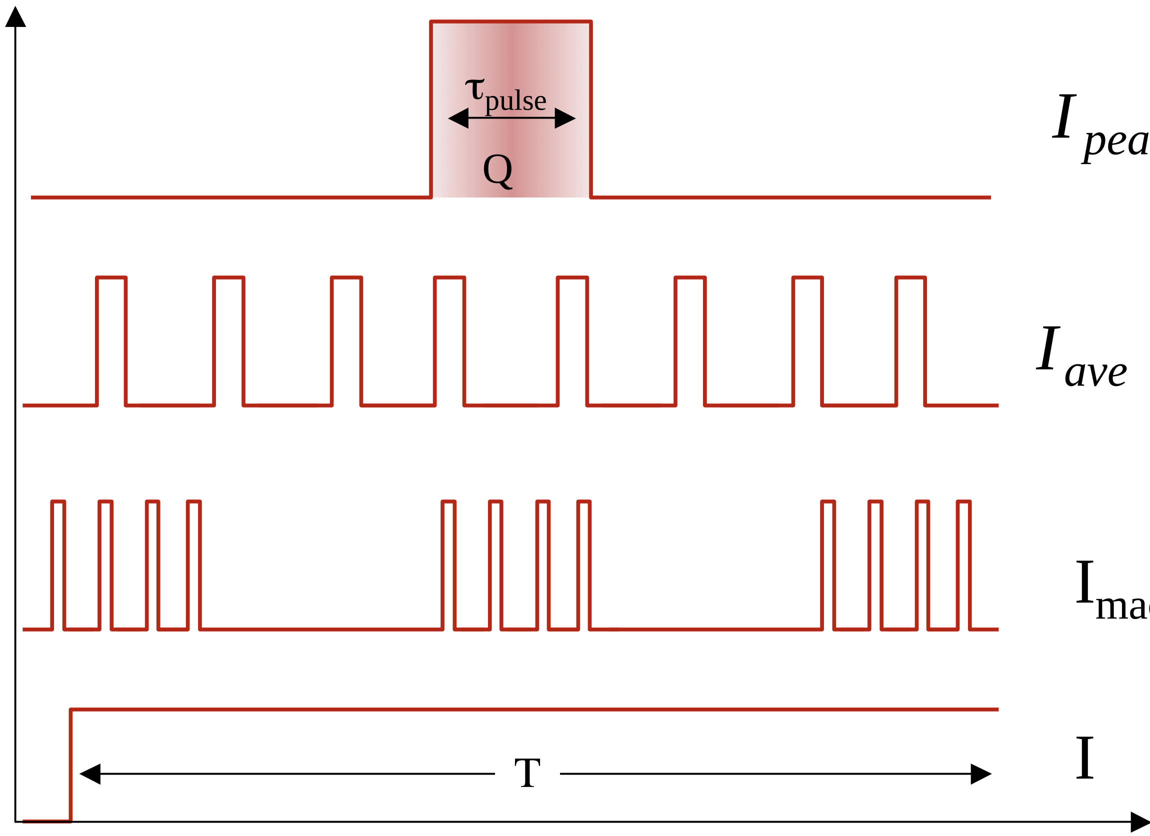
$$\Delta p = \frac{\varepsilon}{w\varphi}$$

Beam carry a current



$$I \sim ne\langle v_z \rangle$$

$$Duty\ factor = \frac{\sum \tau_{pulse}}{T}$$



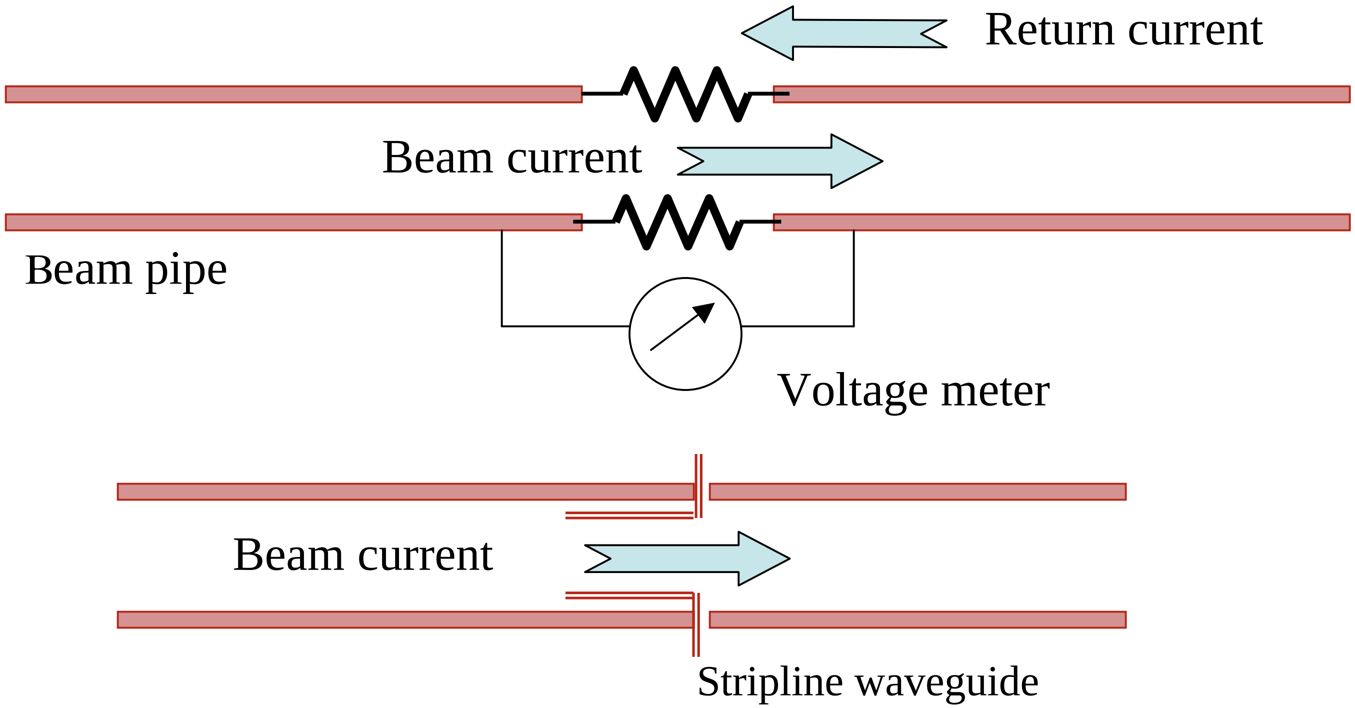
$$I_{peak} = Q / \tau_{pulse}$$

$$I_{ave} = Q_{tot} / T$$

I_{macro}

I

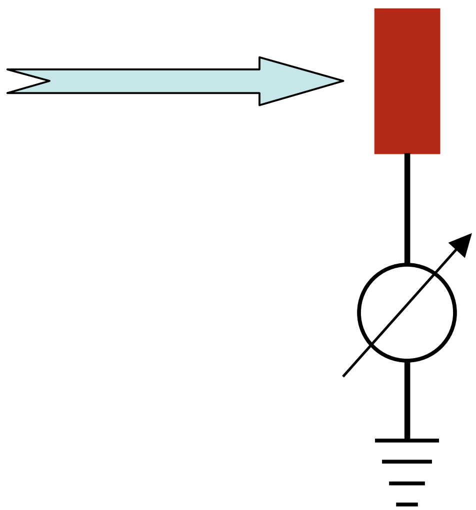
Measuring the beam current



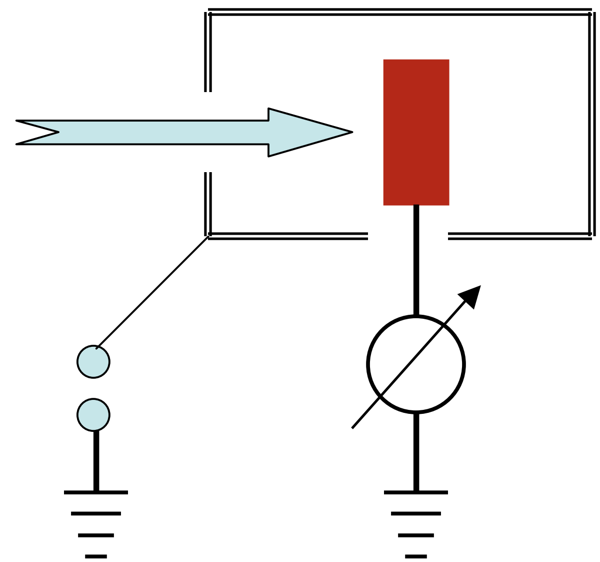
- Examples:
  - ➙ Non-intercepting: Wall current monitors, waveguide pick-ups
  - ➙ Intercepting: Collect the charge; let it drain through a current meter
    - Faraday Cup

Collecting the charge: Right & wrong ways

The Faraday cup

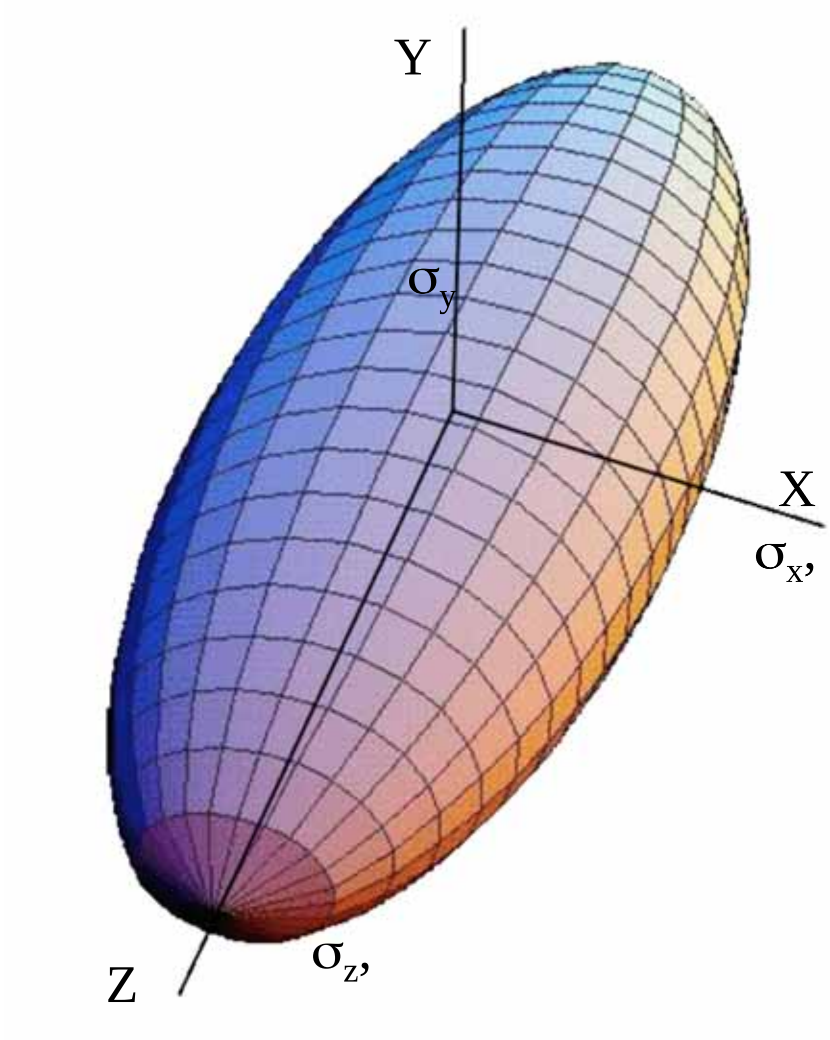


Simple collector



Proper Faraday cup

Bunch dimensions



For uniform charge distributions

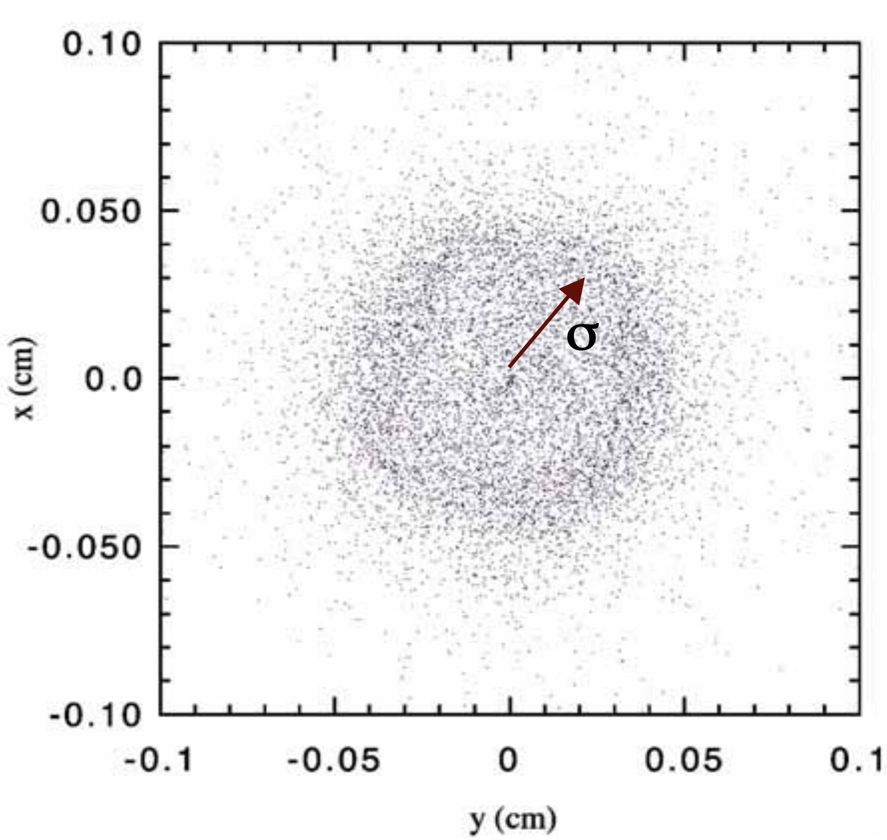
We may use "hard edge values

For gaussian charge distributions

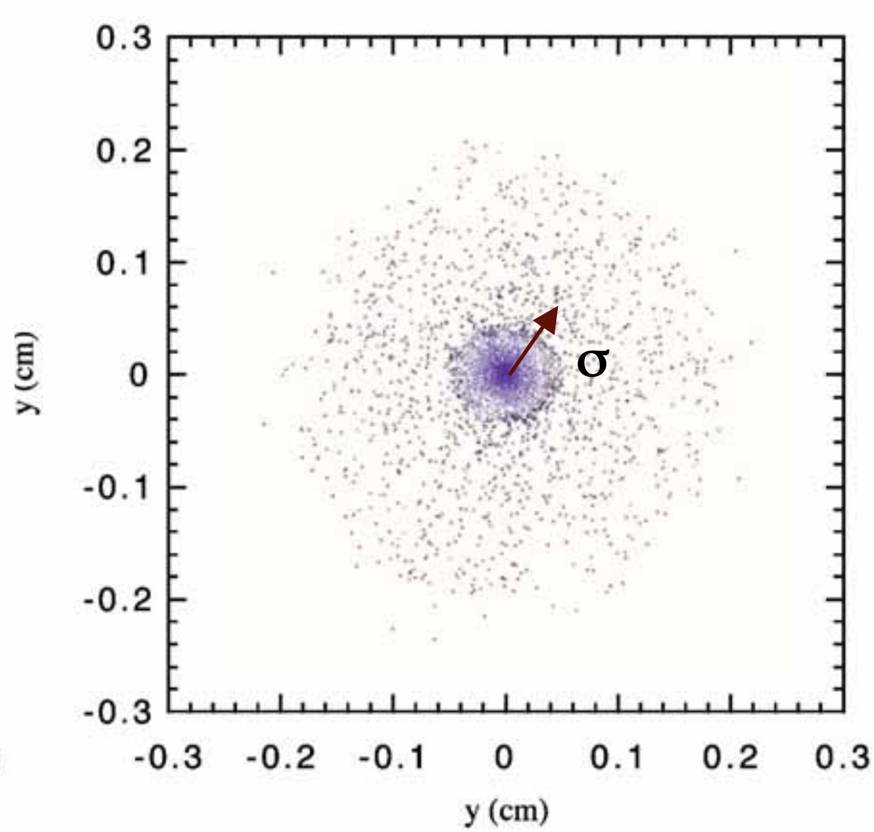
Use rms values σ_x, σ_y, σ_z

We will discuss measurements of bunch size and charge distribution later

But rms values can be misleading



Gaussian beam



Beam with halo

We need to measure the particle distribution

Measuring beam size & distribution

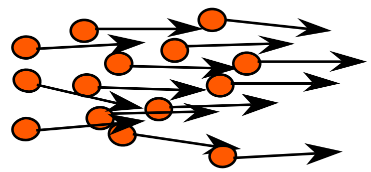
PROPERTY MEASURED →	Intensity/charge	transverse			longit.		Q-value + ΔQ	Energy + ΔE	Polarization	Effect on beam			
		Position	Size/shape	Emittance	Size/shape	Emittance				N	-	+	D
Secondary emission monitors	●	●	●	●				•			x	x	
Wire scanners		●	●	●							x		
Wire chambers		●	●								x	x	
Gas curtain		●	●	●							x		
Residual-gas profile monitors		●	●	●						x			
Scintillator screens		●	●								x	x	x
Optical transition radiation		●	●	●							x		
Synchrotron radiation		●	●	●	●	●				x			
Scrapers and measurement targets		●	●	●									x
Beamscope		●	●	●									x

Effect on beam: N none
- slight, negligible
+ perturbing
D destructive

● primary purpose
• indirect use

Some other characteristics of beams

- Beams particles have random (thermal) ⊥ motion



$$\vartheta_x = \left\langle \frac{p_x^x}{p_z^2} \right\rangle^{1/2} > 0$$

- Beams must be confined against thermal expansion during transport

Beams have internal (self-forces)

- Space charge forces
  - Like charges repel
  - Like currents attract
- For a long thin beam

$$E_{sp}(V/cm) = \frac{60\ I_{beam}(A)}{R_{beam}(cm)}$$

$$B_{\theta}(gauss) = \frac{I_{beam}(A)}{5\ R_{beam}(cm)}$$

Net force due to transverse self-fields

In vacuum:

Beam's transverse self-force scale as $1/\gamma^2$

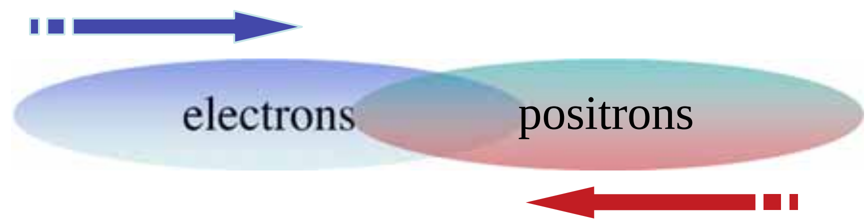
➞ Space charge repulsion: $E_{sp,\perp} \sim N_{beam}$

➞ Pinch field: $B_\theta \sim I_{beam} \sim v_z N_{beam} \sim v_z E_{sp}$

$$\therefore F_{sp,\perp} = q\,(E_{sp,\perp} + v_z \times B_\theta) \sim (1-v^2)\, N_{beam} \sim N_{beam}/\gamma^2$$

Beams in collision are ***not*** in vacuum (beam-beam effects)

Example: Megagauss fields in linear collider



At Interaction Point space charge cancels; currents add

==> strong beam-beam focus

--> Luminosity enhancement

--> Strong synchrotron radiation

Consider 250 GeV beams with 1 kA focused to 100 nm

$$B_{peak} \sim 40 \text{ Mgauss}$$

Applications determine the desired beam characteristics

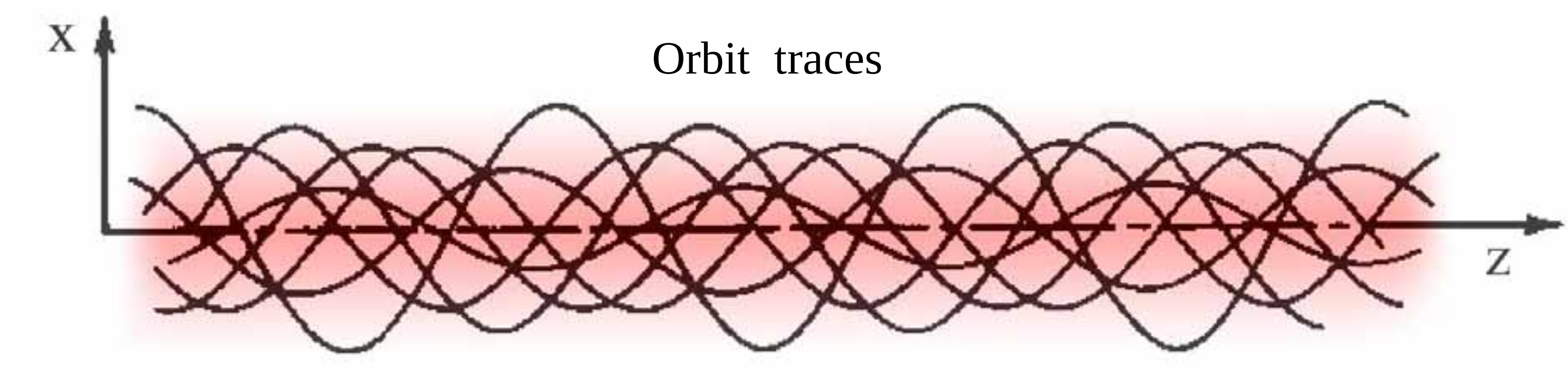
Energy	$E = \gamma mc^2$	MeV to TeV
Energy Spread (rms) Momentum spread	$\sigma = \Delta E/E$, $\Delta\gamma/\gamma$ $\Delta p/p$	~0.1%
Beam current (peak)	I	$10 - 10^4$ A
Pulse duration (FWHM)	T_p	50 fs - 50 ps
Pulse length (Standard deviation)	σ_z	mm - cm
Charge per pulse # of Particles number	Q_b N_b	1 nC
Emittance (rms) Normalized emittance	ε $\varepsilon_n = \gamma\beta\varepsilon$	1 π mm-mrad / γ
Bunches per macropulse	M_b	1- 100
Pulse repetition rate	f	$1 - 10^7$
Effective bunch rate	f M_b	$1 - 10^9$

Emittance is a measure of beam quality

What is this thing called beam quality?
or
How can one describe the dynamics of a bunch of particles?

Coordinate space

Each of N_b particles is tracked in ordinary 3-D space

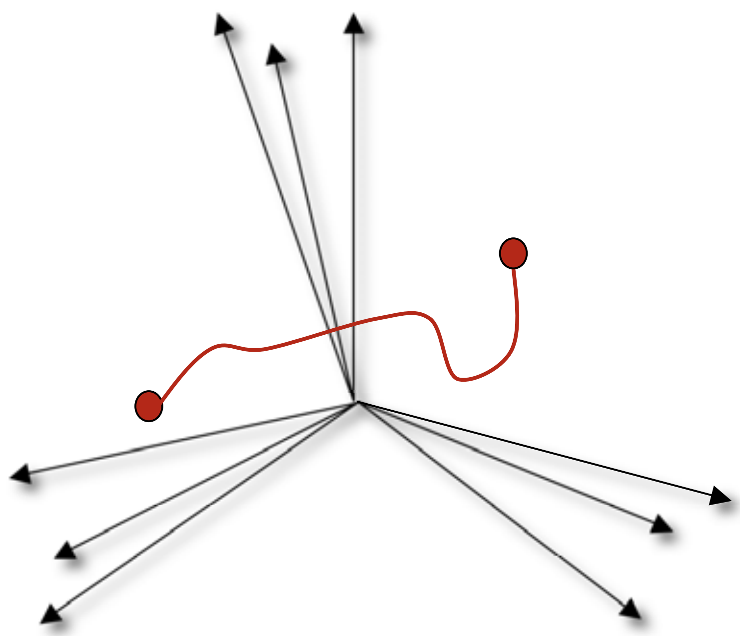


Not too helpful

Configuration space:

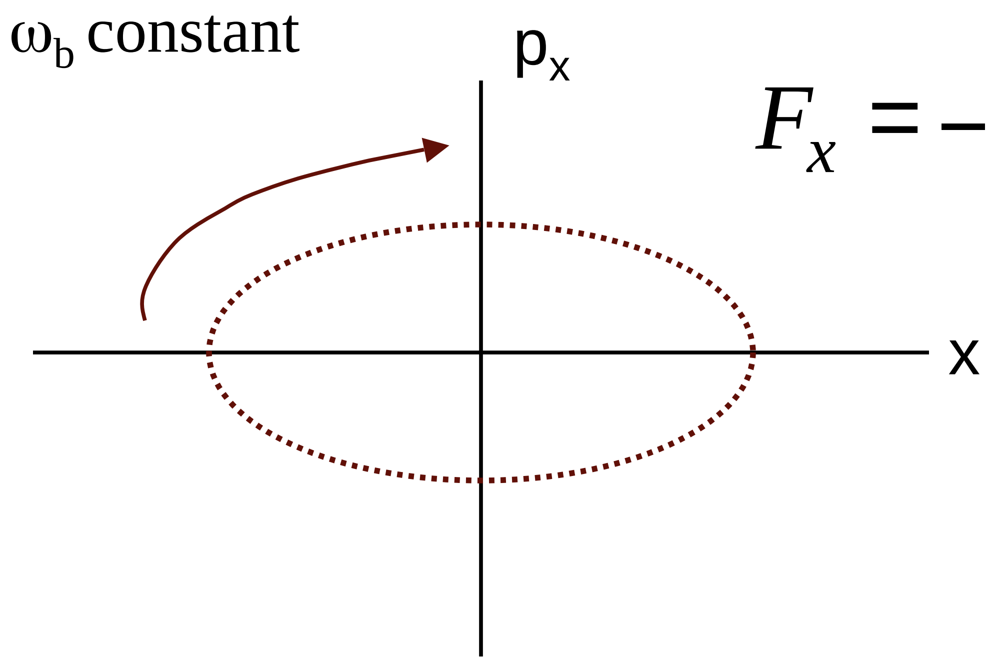
$6N_b$-dimensional space for N_b particles; coordinates (x_i, p_i), $i = 1,\ldots, N_b$

The bunch is represented by a single point that moves in time



Useful for Hamiltonian dynamics

Configuration space example: 1 particle in an harmonic potential



$$F_x = -kx = m\ddot{x}$$

But for many problems this description carries much more information than needed :

We don’t care about each of 10^{10} individual particles

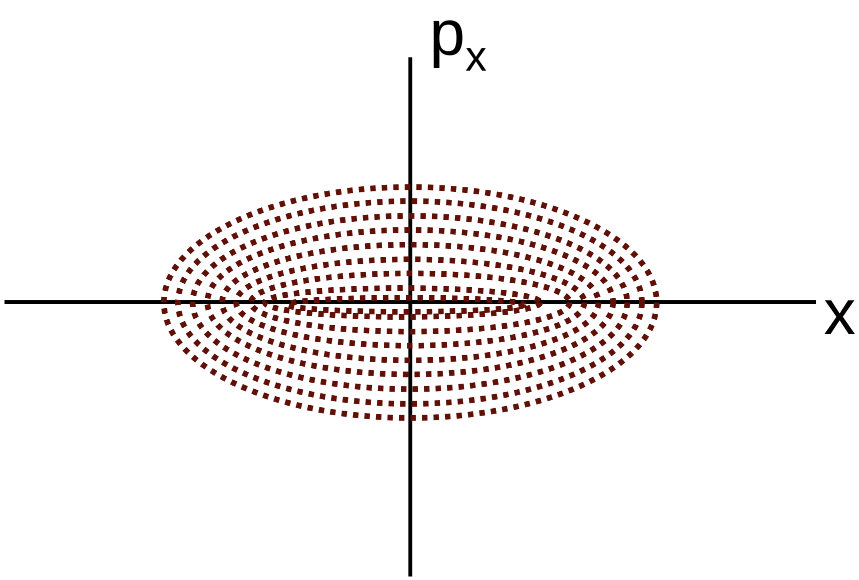
But seeing both the x & p_x looks useful

Option 3: Phase space (gas space in statistical mechanics)

6-dimensional space for N_b particles

The i^{th} particle has coordinates (x_i, p_i), i = x, y, z

The bunch is represented by N_b points that move in time



In most cases, the three planes are to very good approximation decoupled

==> One can study the particle evolution independently in each planes:

Particles Systems & Ensembles

- The set of possible states for a system of N particles is referred as an *ensemble* in statistical mechanics.
- In the statistical approach, particles lose their individuality.
- Properties of the whole system are fully represented by particle density functions f_{6D} and f_{2D} :

$$f_{6D}\left(x, p_x, y, p_y, z, p_z\right)\, dx\, dp_x\, dy\, dp_y\, dz\, dp_z \qquad f_{2D}\left(x_i, p_i\right) dx_i\, dp_i \quad i = 1,2,3$$

where

$$\int f_{6D}\, dx\, dp_x\, dy\, dp_y\, dz\, dp_z = N$$

From: Sannibale USPAS lectures

Longitudinal phase space

- In most accelerators the phase space planes are only weakly coupled.
  - ➙ Treat the longitudinal plane independently from the transverse one
  - ➙ Effects of weak coupling can be treated as a perturbation of the uncoupled solution
- In the longitudinal plane, electric fields accelerate the particles
  - ➙ Use *energy* as longitudinal variable together with its canonical conjugate *time*
- Frequently, we use *relative energy variation* δ & *relative time* τ with respect to a reference particle

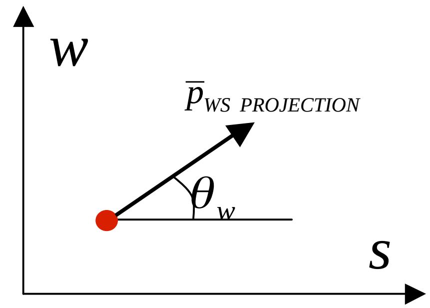
$$\delta = \frac{E - E_0}{E_0} \qquad \tau = t - t_0$$

- According to Liouville, in the presence of Hamiltonian forces, the area occupied by the beam in the longitudinal phase space is conserved

From: Sannibale USPAS lectures

Transverse phase space

- For transverse planes $\{x, p_x\}$ and $\{y, p_y\}$, use a modified phase space where the momentum components are replaced by:



$$p_{xi} \rightarrow x' = \frac{dx}{ds} \qquad\qquad p_{yi} \rightarrow y' = \frac{dy}{ds}$$

where s is the direction of motion

- We can relate the old and new variables (for Bz $\neq 0$) by

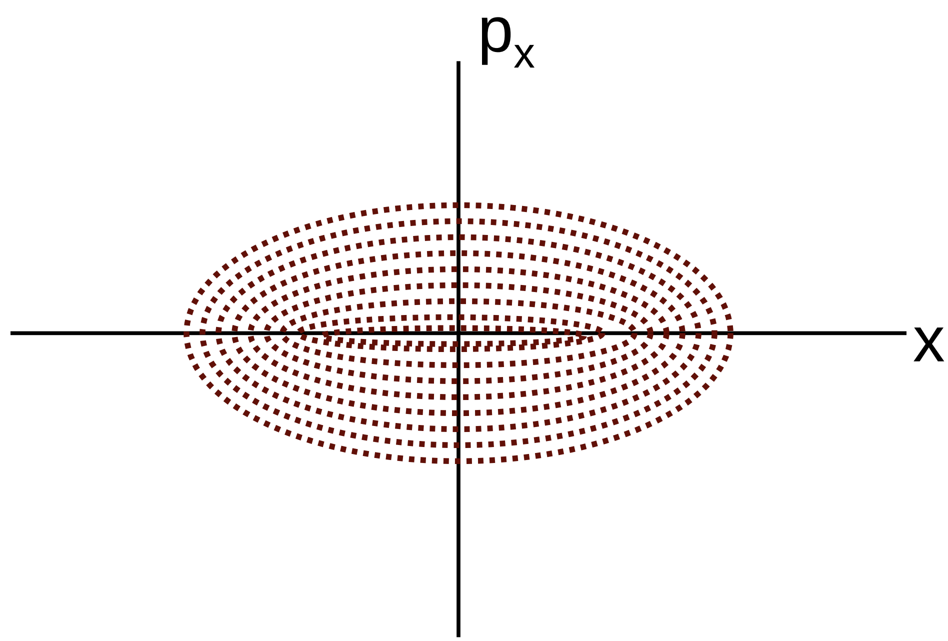
$$p_i = \gamma\, m_0 \frac{dx_i}{dt} = \gamma\, m_0\, v_s \frac{dx_i}{ds} = \gamma\, \beta\, m_0\, c\, x_i' \qquad \text{i = x,y}$$

$$where \quad \beta = \frac{v_s}{c} \quad and \quad \gamma = \left(1 - \beta^2\right)^{-1/2}$$

Note: x_i and p_i are canonical conjugate variables while x and x_i' are not, unless there is no acceleration (γ and β constant)

From: Sannibale USPAS lectures

Look again at our ensemble of harmonic oscillators

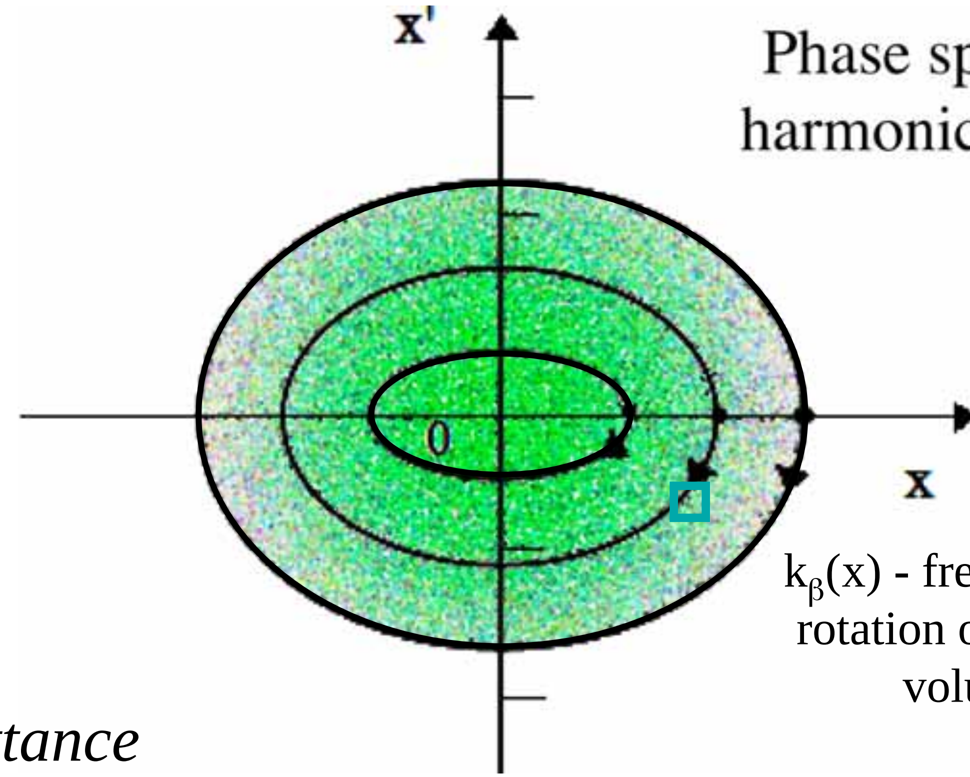


Particles stay on their energy contour.

Again the phase area of the ensemble is conserved

Emittance describes the area in phase space of the ensemble of beam particles

Emittance - Phase space volume of beam



Phase space of an harmonic oscillator

$k_\beta(x)$ - frequency of rotation of a phase volume

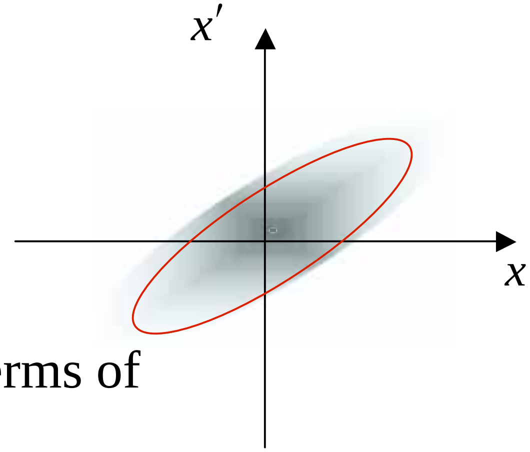
RMS emittance

$$\varepsilon^2 \equiv R^2 (V^2 - (R')^2)/c^2$$

Twiss representation of the emittance

- A beam with arbitrary phase space distribution can be represented by an equivalent ellipse with area equal to the rms emittance divided by π.

- The equation for such an ellipse can be written as



$$\frac{\langle w'^2 \rangle}{\varepsilon_{w,rms}} w^2 + \frac{\langle w^2 \rangle}{\varepsilon_{w,rms}} w'^2 - 2\frac{\langle w\, w' \rangle}{\varepsilon_{w,rms}} w\, w' = \varepsilon_{w,rms} \qquad w = x, y$$

- Accelerator physicists often write this equation in terms of the so-called *Twiss Parameters* β_T, γ_T and α_T

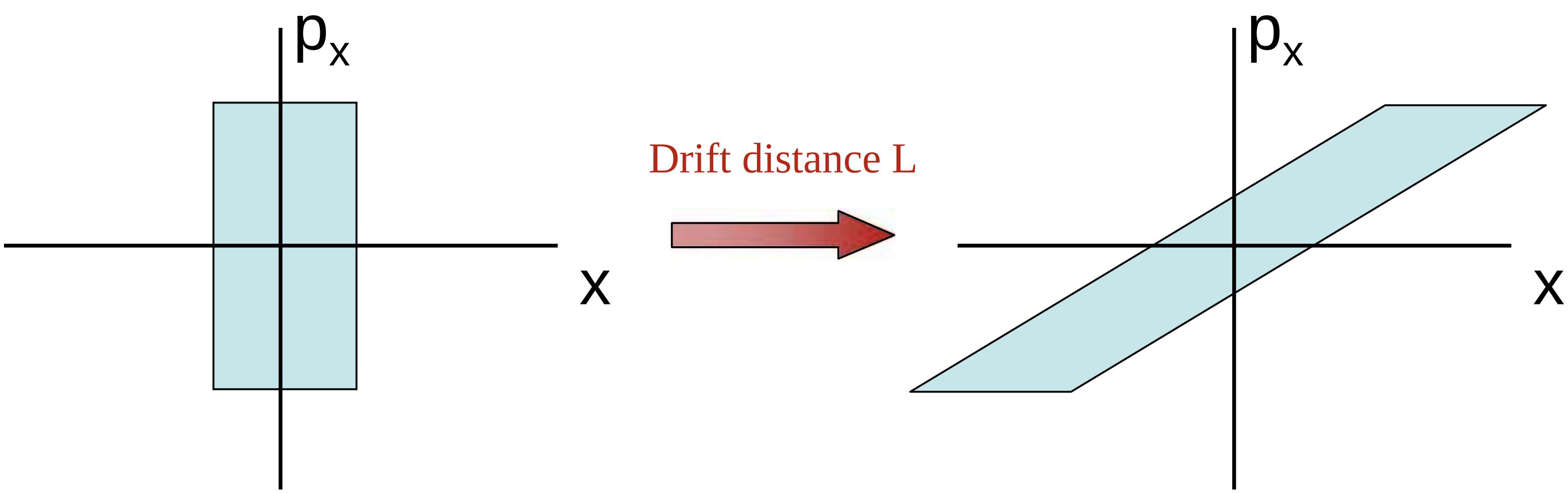
$$\beta_{Tw} w'^2 + \gamma_{Tw} w^2 + 2\alpha_{Tw} w w' = \varepsilon_w \qquad w = x, y$$

where

$$\langle w^2 \rangle = \beta_{Tw}\varepsilon_w \qquad \langle w'^2 \rangle = \gamma_{Tw}\varepsilon_w \qquad \langle w\, w' \rangle = -\alpha_{Tw}\varepsilon_w \qquad w = x, y$$

From: Sannibale USPAS lectures

Force-free expansion of a beam



Notice: The phase space area is conserved

Matrix representation of a drift

- From the diagram we can write by inspection

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \Rightarrow \begin{array}{c} x = x_0 + Lx'_0 \\ x' = x'_0 \end{array}$$

$$\Rightarrow \begin{array}{c} \langle x^2 \rangle = \langle (x_0 + Lx'_0)^2 \rangle = \langle x_0^2 \rangle + L^2 \langle x_0'^2 \rangle + 2L \langle x_0 x'_0 \rangle \\ \langle x'^2 \rangle = \langle x_0'^2 \rangle \\ \langle xx' \rangle = \langle (x_0 + Lx'_0) x'_0 \rangle = L \langle x_0'^2 \rangle + \langle x_0 x'_0 \rangle \end{array}$$

- Now write these last equations in terms of β_T, γ_T and α_T

Recalling the definition of the Twiss parameters

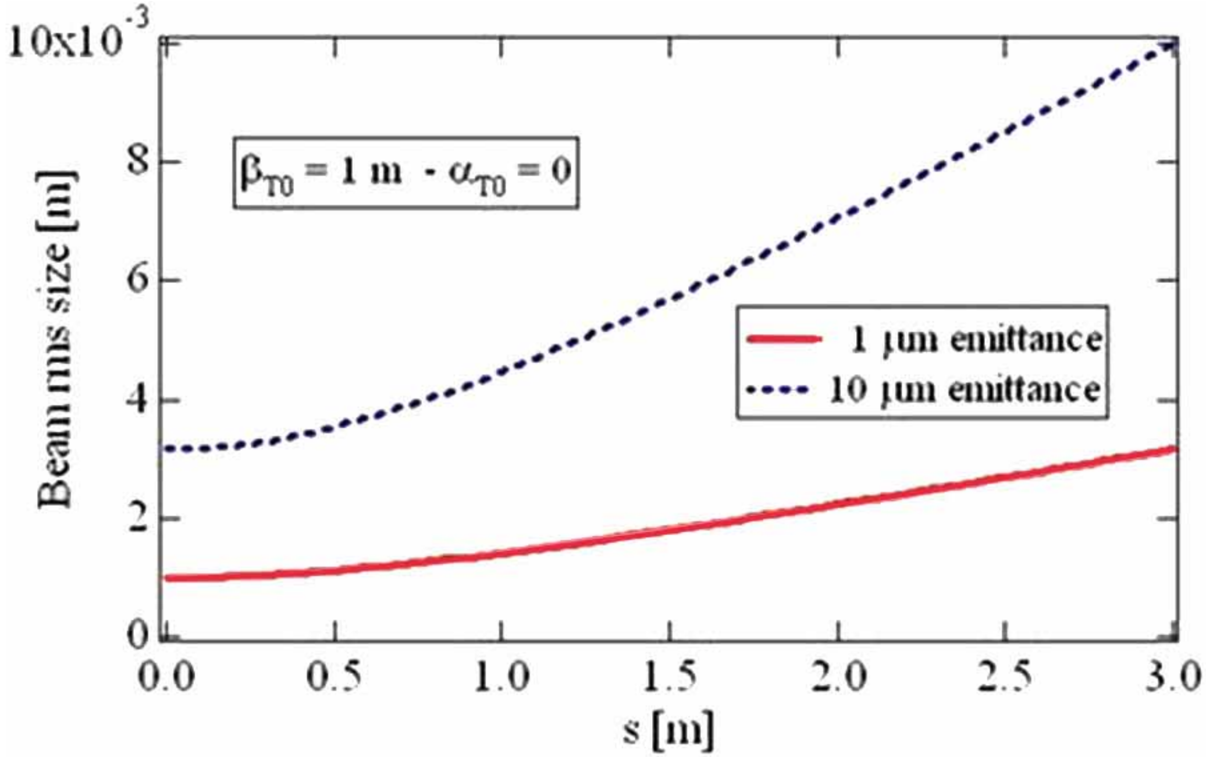
$$\beta_T\varepsilon = \beta_{T0}\varepsilon + L^2\gamma_{T0}\varepsilon - 2L\alpha_{T0}\varepsilon$$

$$\gamma_T\varepsilon = \gamma_{T0}\varepsilon$$

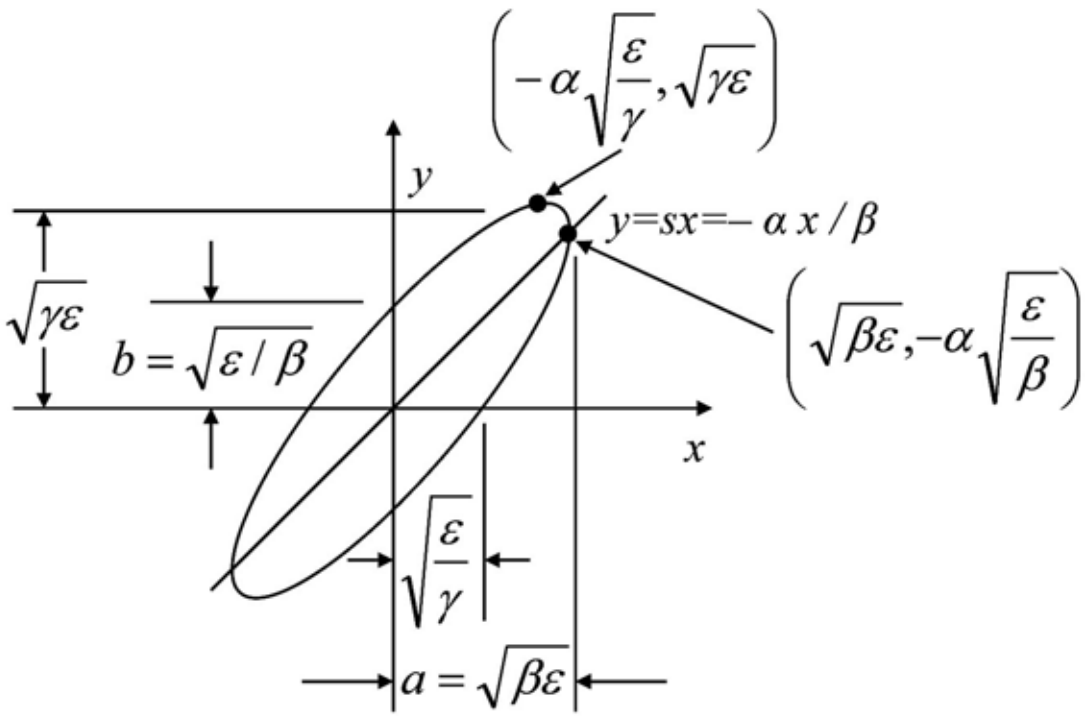
$$-\alpha_T\varepsilon = L\gamma_{T0}\varepsilon - \alpha_{T0}\varepsilon$$

$$\Rightarrow \begin{pmatrix} \beta_T \\ \gamma_T \\ \alpha_T \end{pmatrix} = \begin{pmatrix} 1 & L^2 & -2L \\ 0 & 1 & 0 \\ 0 & -L & 1 \end{pmatrix} \begin{pmatrix} \beta_{T0} \\ \gamma_{T0} \\ \alpha_{T0} \end{pmatrix}$$

A numerical example



For your notes, as shown in many books:

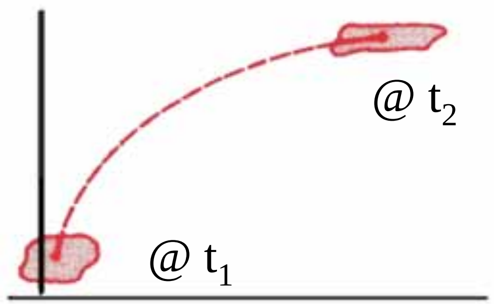


As for the upright ellipse $x_{max} = \sqrt{\beta\varepsilon}$, $y_{max} = \sqrt{\gamma\varepsilon}$

This emittance is the phase space area occupied by the system of particles, divided by π

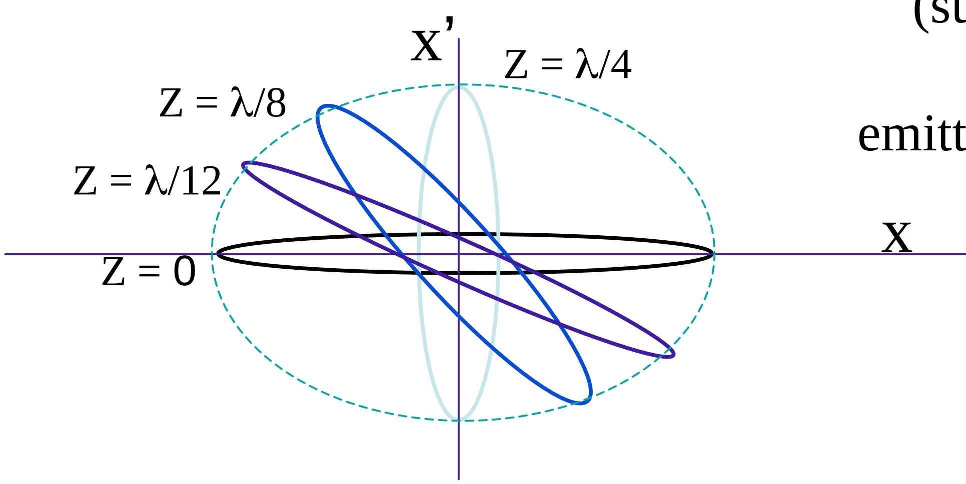
The rms emittance is a measure of the mean non-directed (thermal) energy of the beam

Why is emittance an important concept



1) Liouville: Under conservative forces phase space evolves like an incompressible fluid ==>

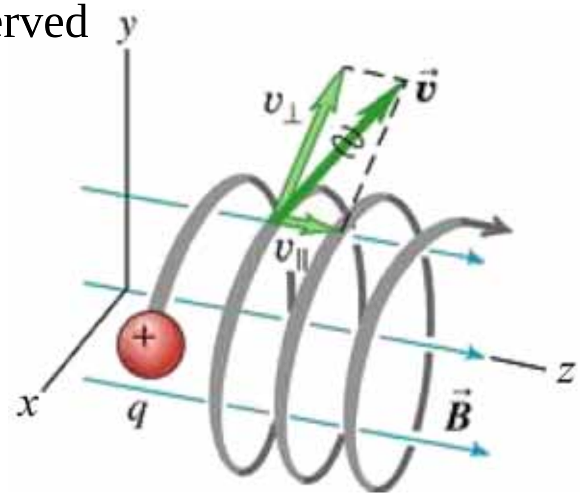
2) Under linear forces macroscopic (such as focusing magnets) &
γ =constant
emittance is an invariant of motion



3) Under acceleration
$\gamma\varepsilon = \varepsilon_n$
is an adiabatic invariant

Emittance conservation with B_z

- An axial B_z field, (e.g.,solenoidal lenses) couples transverse planes
  - ➙ The 2-D Phase space area occupied by the system in each transverse plane is no longer conserved

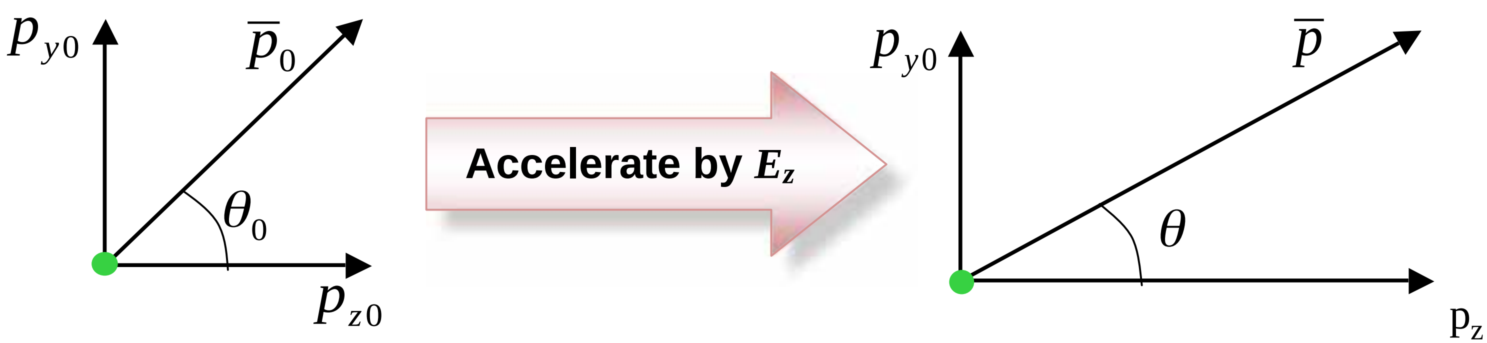


- Liouville's theorem still applies to the 4D transverse phase space
  - ➙ the 4-D hypervolume is an invariant of the motion

- In a frame rotating around the *z* axis by the *Larmor frequency* $\omega_L = qB_z / 2g\, m_0$, the transverse planes decouple
  - ➙ The phase space area in each of the planes is conserved again

Emittance during acceleration

- When the beam is accelerated, β & γ change
  - → x and x' are no longer canonical
  - → Liouville theorem does not apply & emittance is not invariant



$$p_z = \sqrt{\frac{T^2 + 2Tm_0c^2}{T_0^2 + 2T_0m_0c^2}}\, p_{z0}$$

$$T \equiv kinetic\ energy$$

From: Sannibale USPAS lectures

Then…

$$y_0' = \tan\theta_0 = \frac{p_{y0}}{p_{z0}} = \frac{p_{y0}}{\beta_0 \gamma_0 m_0 c} \qquad y' = \tan\theta = \frac{p_y}{p_z} = \frac{p_{y0}}{\beta \gamma m_0 c} \qquad \frac{y'}{y_0'} = \frac{\beta_0 \gamma_0}{\beta \gamma}$$

In this case $\frac{\varepsilon_y}{\varepsilon_{y0}} = \frac{y'}{y_0'}$ ==> $\boxed{\beta \gamma \varepsilon_y = \beta_0 \gamma_0 \varepsilon_{y0}}$

- Therefore, the quantity $\beta \gamma \varepsilon$ is invariant during acceleration.
- Define a conserved *normalized emittance*

$$\boxed{\varepsilon_{n\,i} = \beta \gamma \varepsilon_i \qquad i = x, y}$$

Acceleration couples the longitudinal plane with the transverse planes
The 6D emittance is still conserved but the transverse ones are not

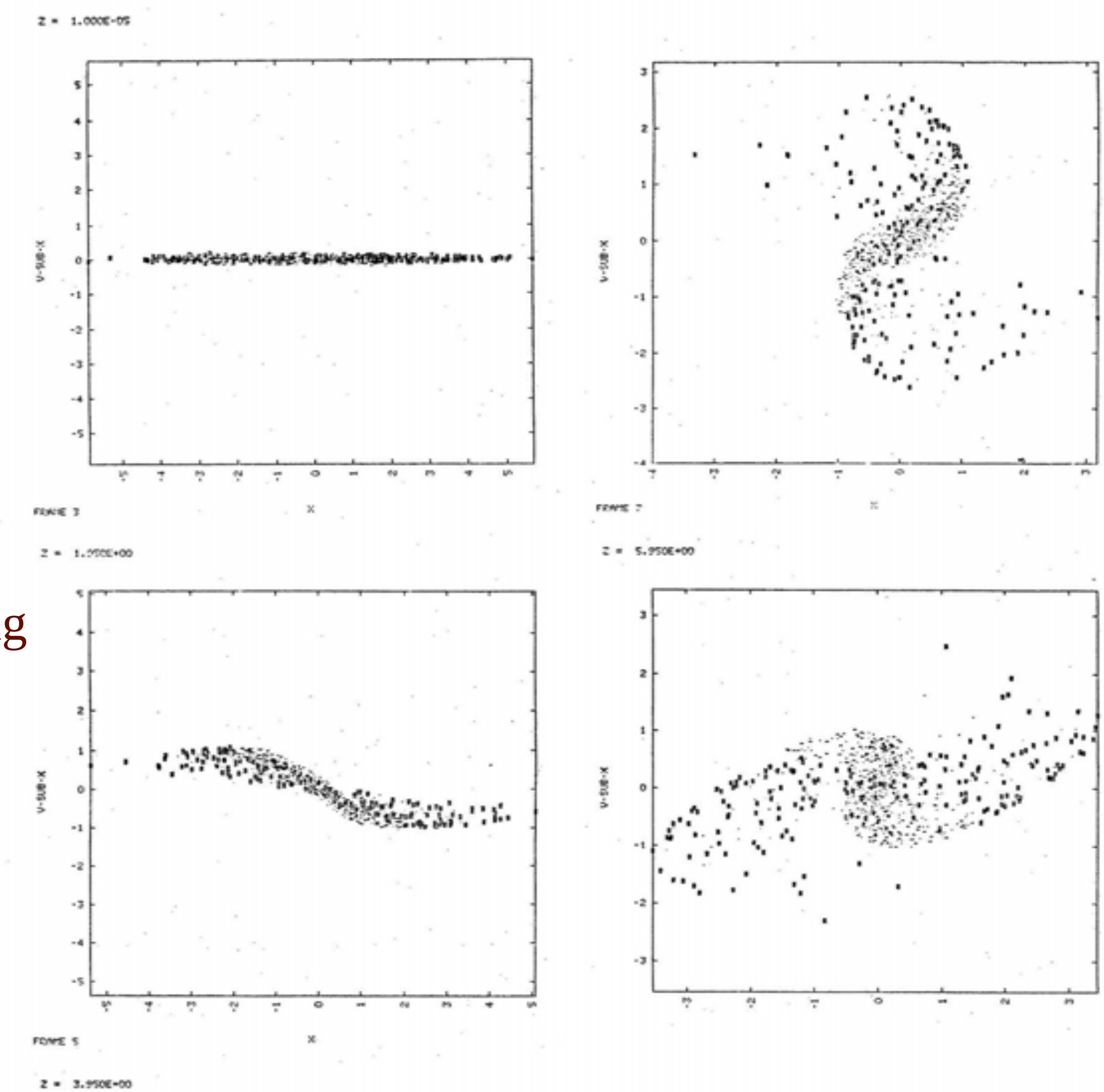
From: Sannibale USPAS lectures

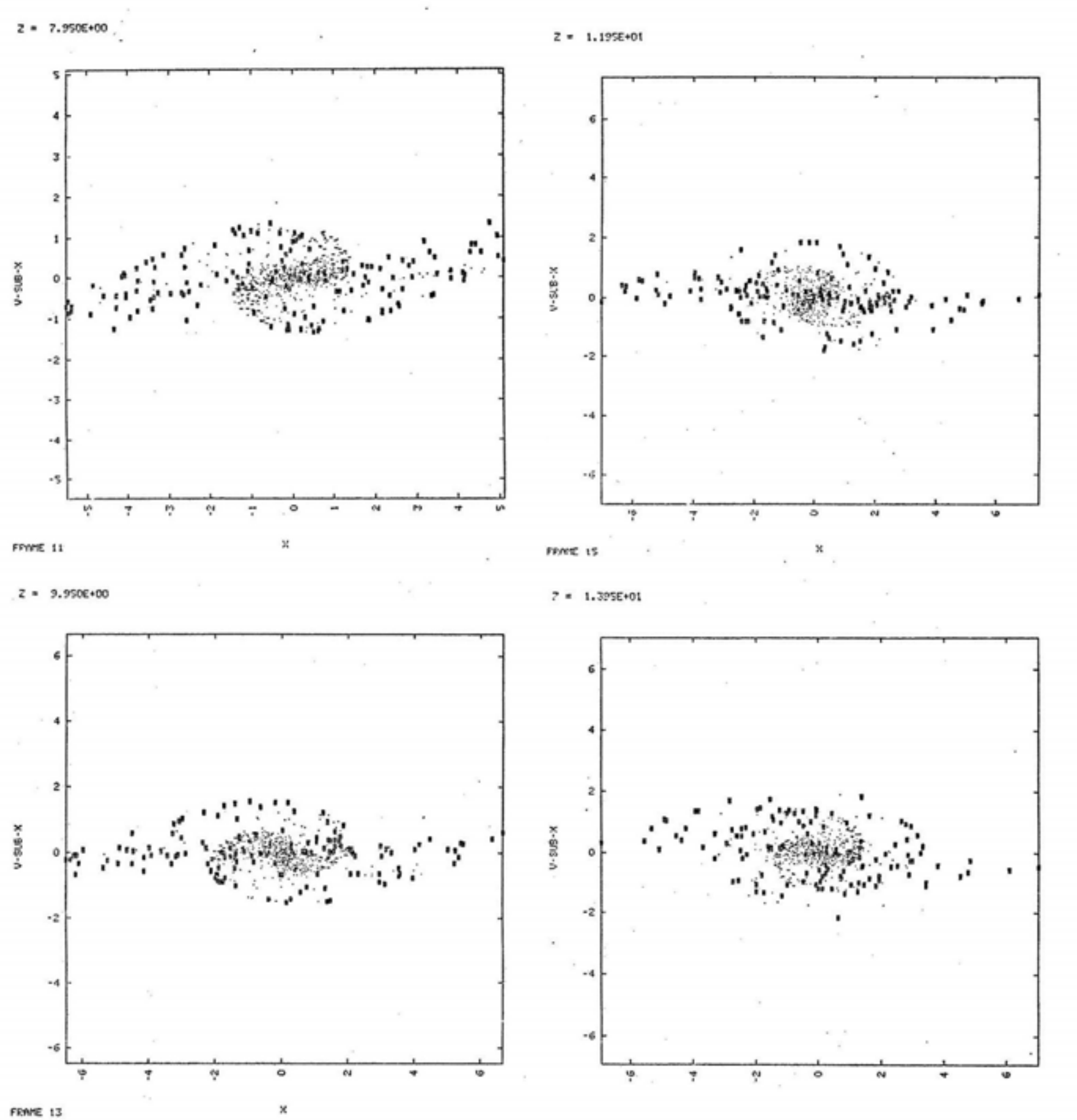
Nonlinear space-charge fields filament phase space

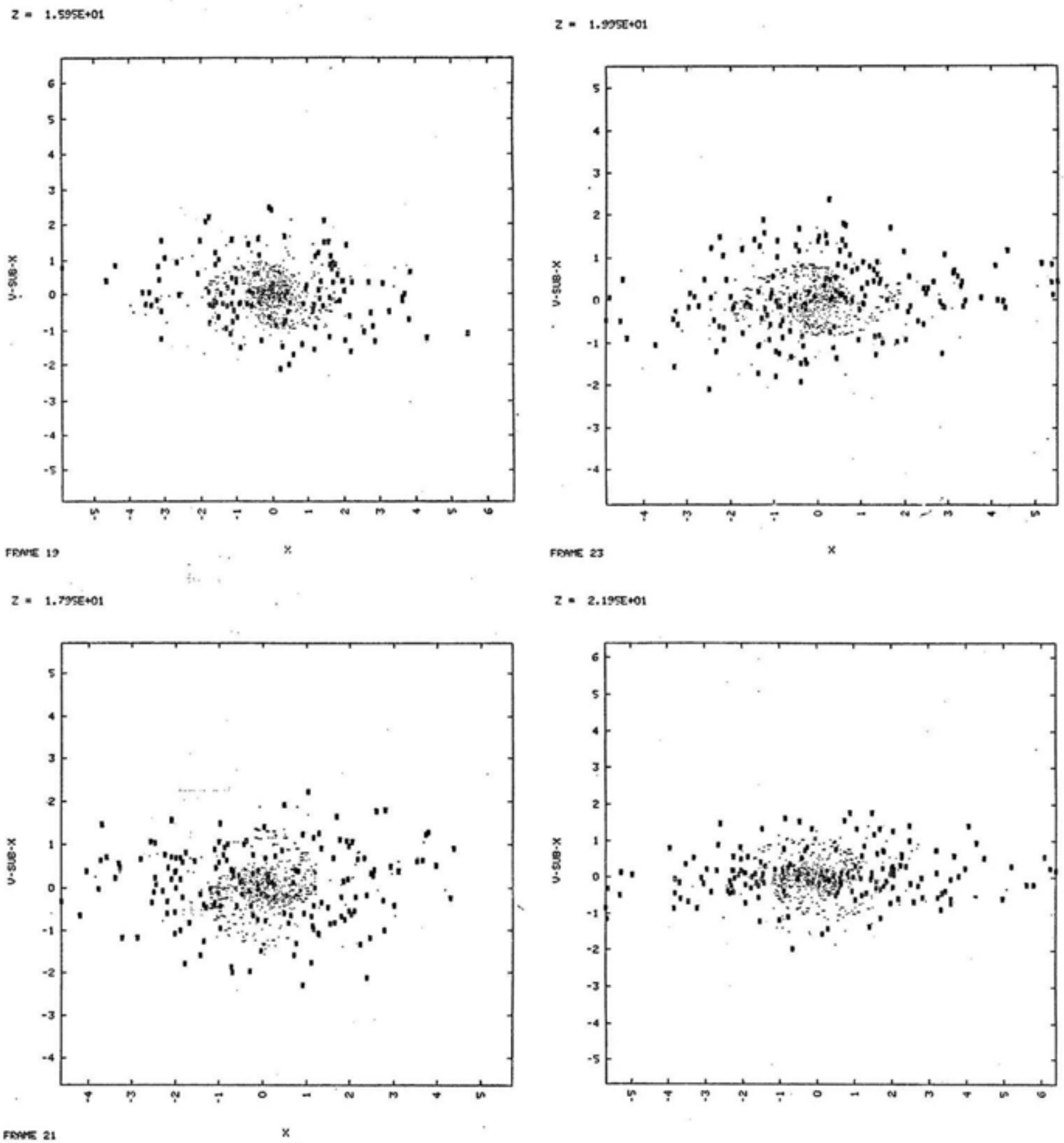
Consider a cold beam with a Gaussian charge distribution entering a dense plasma

At the beam head the plasma shorts out the E_r leaving only the azimuthal B-field

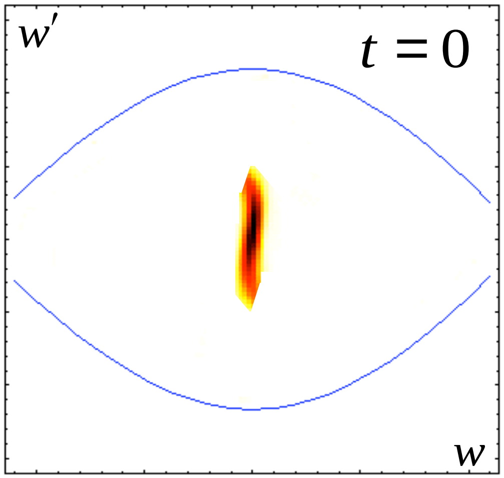
The beam begins to pinch trying to find an equilibrium radius

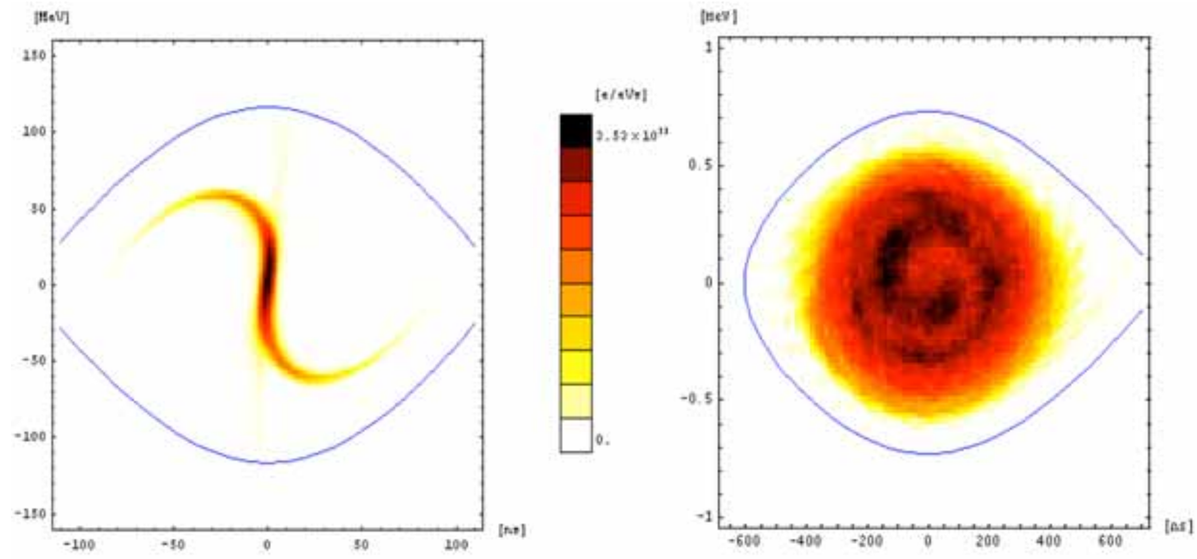



Z = 7.950E+00
V-SUB-X
X
FRAME 11
Z = 1.195E+01
V-SUB-X
X
FPAME 15
Z = 9.950E+00
V-SUB-X
X
FRAME 13
? = 1.395E+01
V-SUB-X


Z = 1.595E+01
V-SUB-X
X
FRAME 19
Z = 1.995E+01
V-SUB-X
X
FRAME 23
Z = 1.795E+01
V-SUB-X
X
FRAME 21
Z = 2.195E+01
V-SUB-X

Example 2: Filamentation of longitudinal phase space



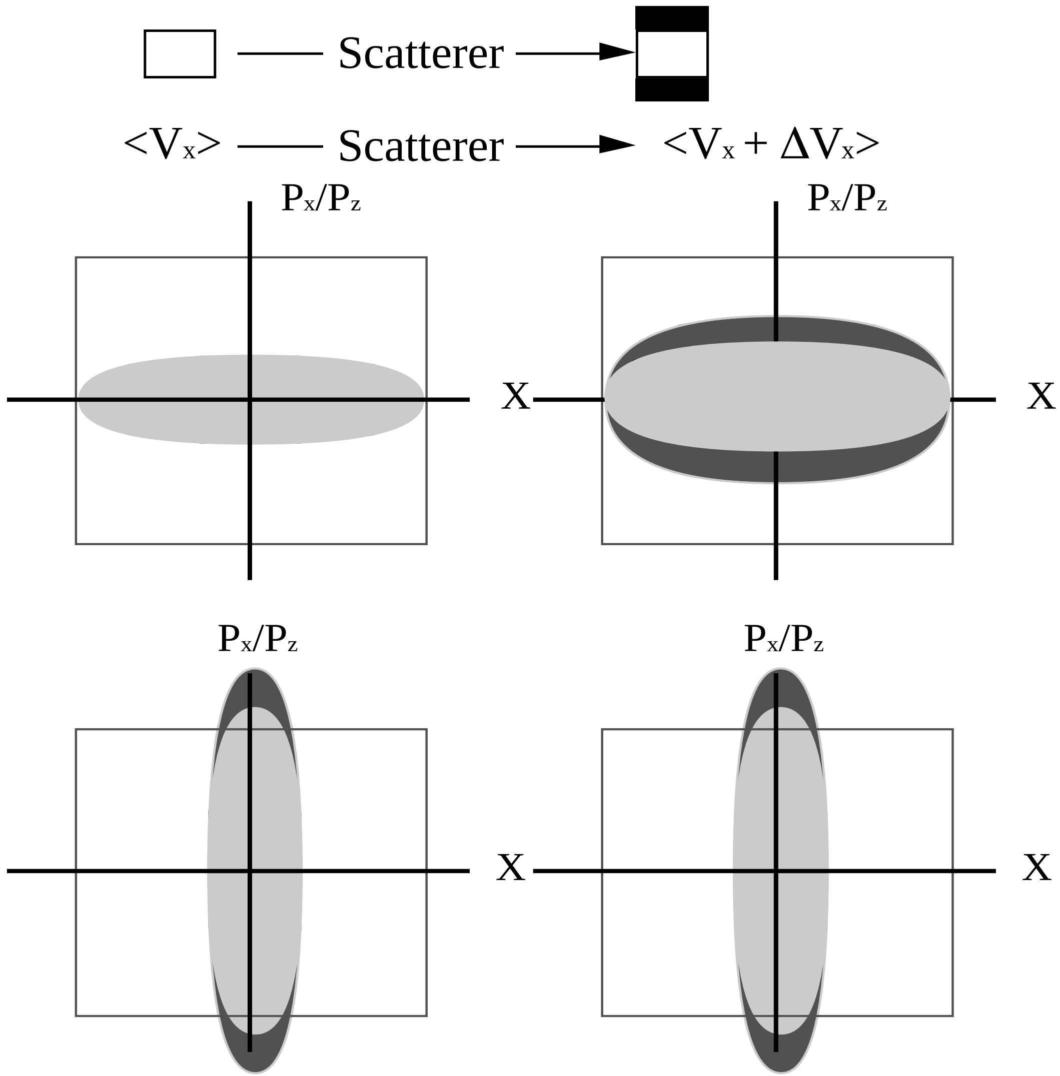


Data from CERN PS

The emittance according to Liouville is still conserved

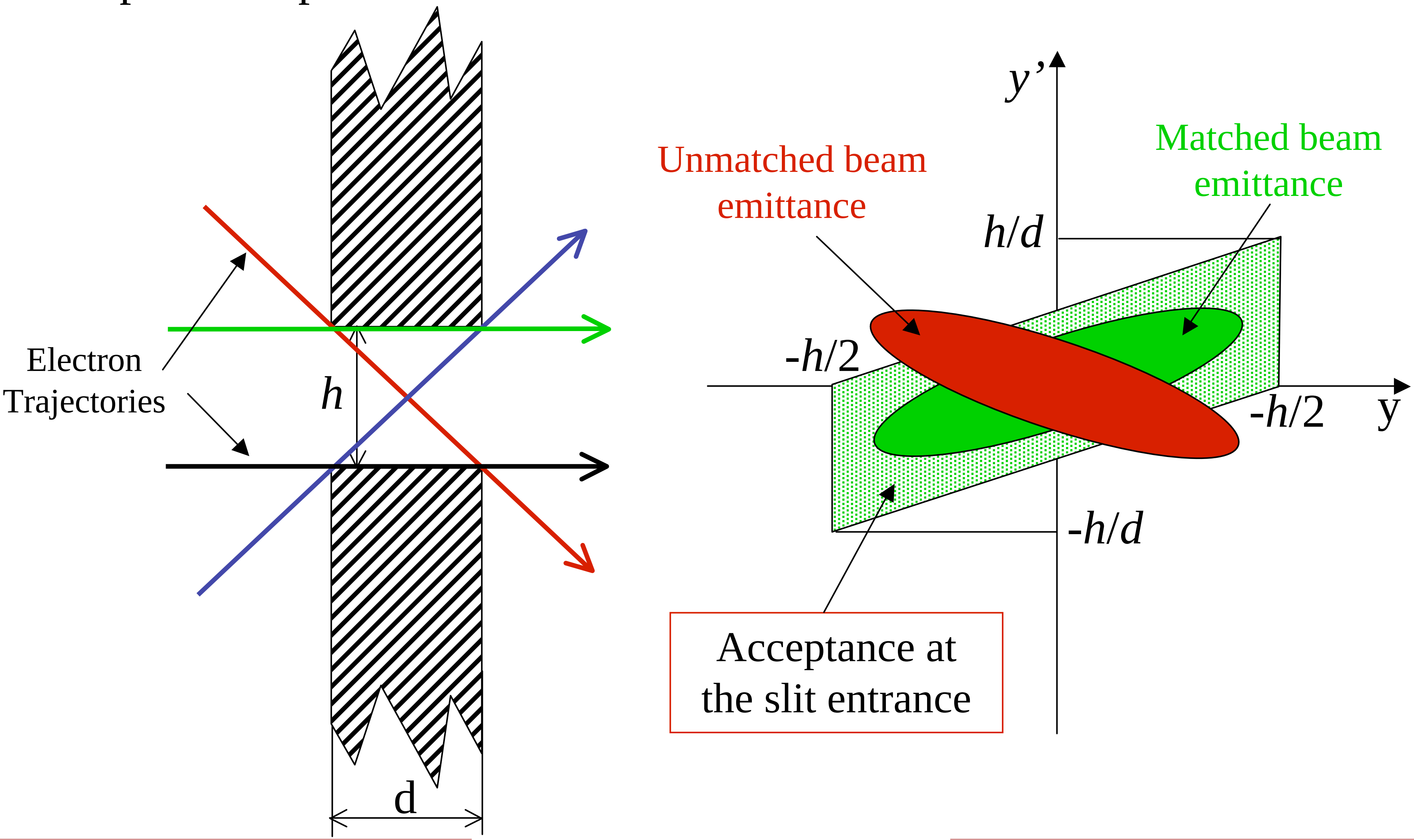
Macroscopic (rms) emittance is not conserved

Non-conservative forces (scattering) increases emittance



The Concept of Acceptance

Example: Acceptance of a slit



From: Sannibale USPAS lectures

Is there any way to decrease the emittance?

This means taking away mean transverse momentum,

but

keeping mean longitudinal momentum

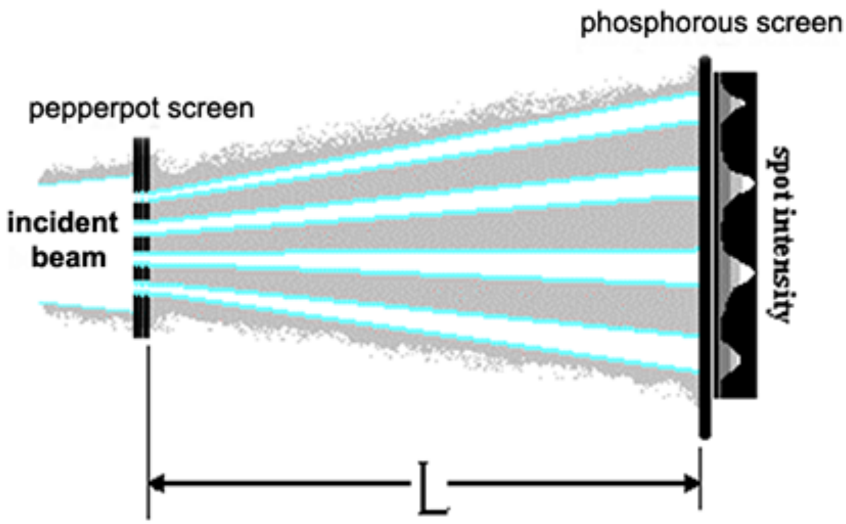
We'll leave the details for later in the course.

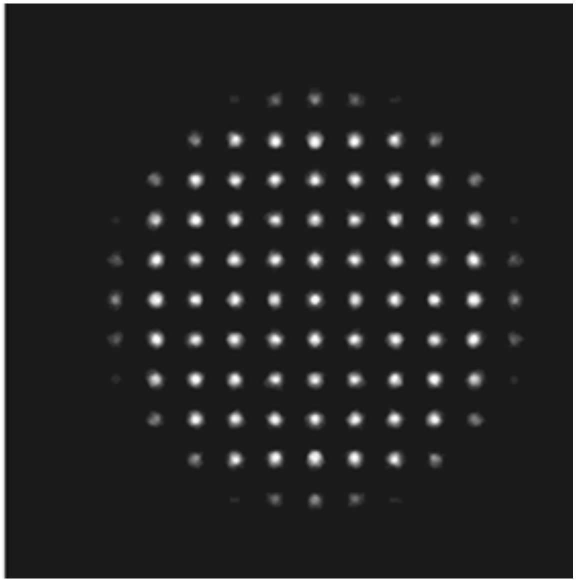
Measuring the emittance of the beam

$$\varepsilon^2 = R^2 (V^2 - (R')^2)/c^2$$

- RMS emittance
  - Determine rms values of velocity & spatial distribution
- Ideally determine distribution functions & compute rms values
- Destructive and non-destructive diagnostics

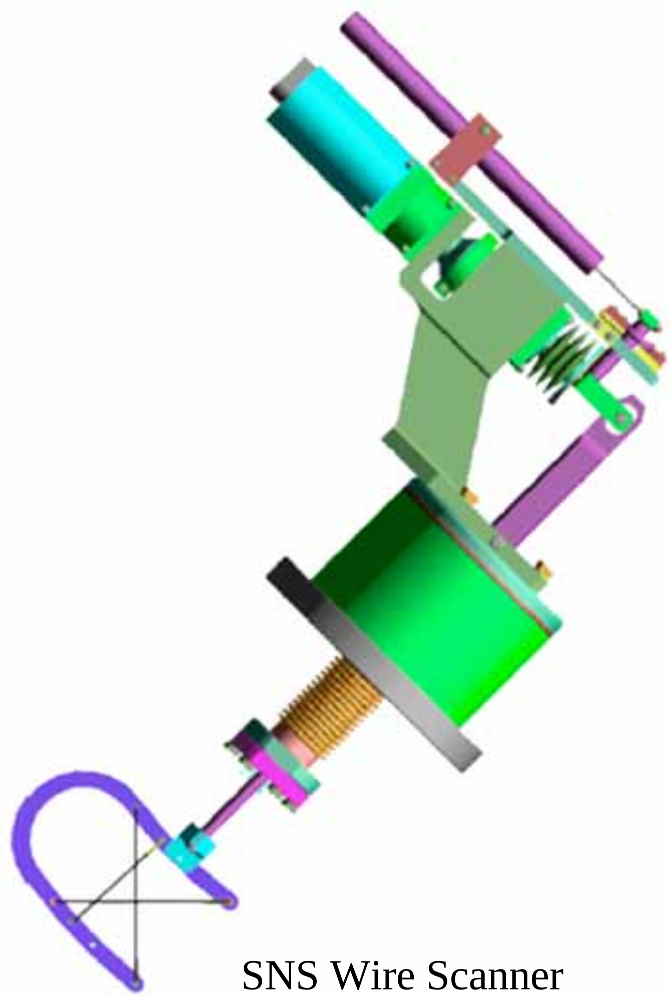
Example of pepperpot diagnostic





- Size of image ==> R
- Spread in overall image ==> R´
- Spread in beamlets ==> V
- Intensity of beamlets ==> current density

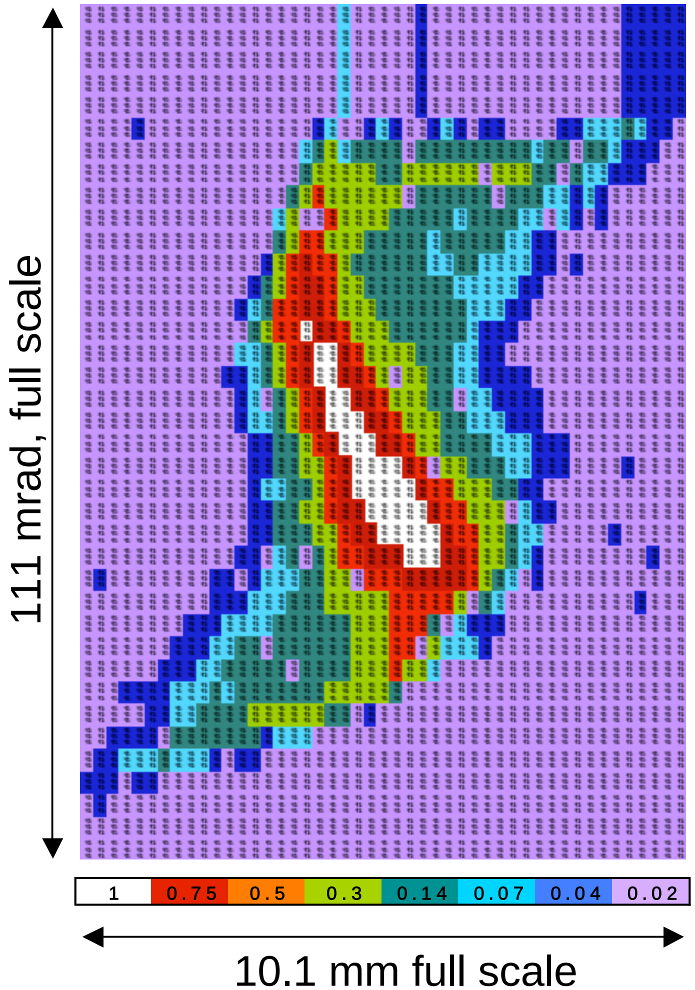
Wire scanning to measure R and ε



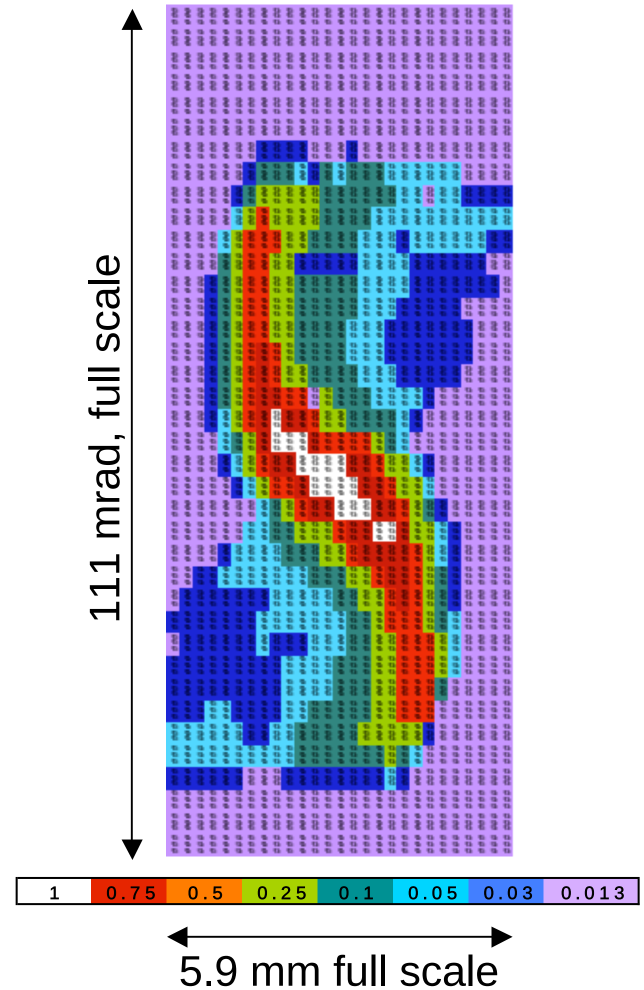
SNS Wire Scanner

- Measure x-ray signal from beam scattering from thin tungsten wires
- Requires at least 3 measurements along the beamline

Measured 33-mA Beam RMS Emittances



Horizontal, 0.22 pi mm mrad



Vertical, 0.15 pi mm mrad

Matching beams & accelerators to the task

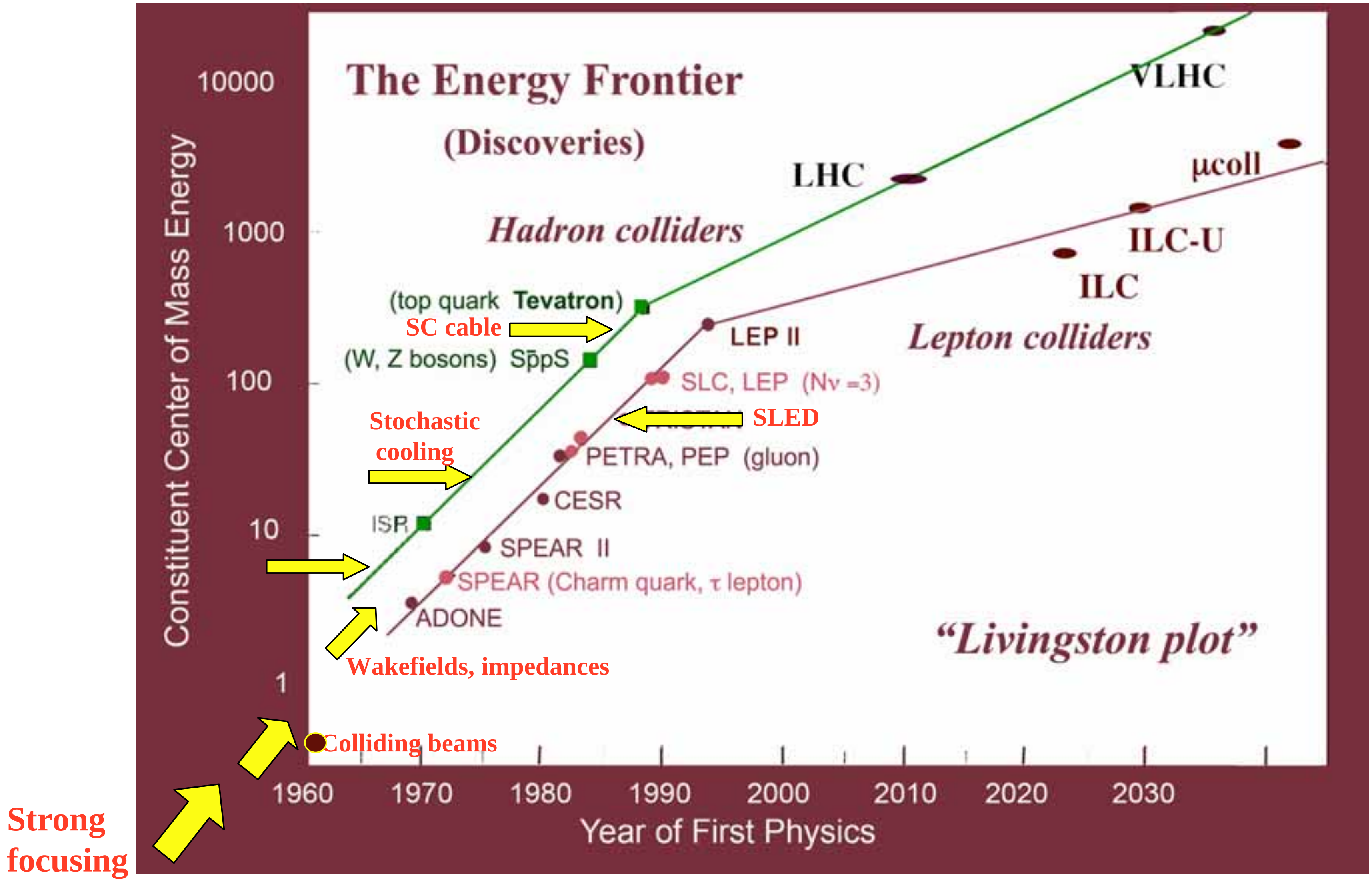
What are the design constraints?

- Beam particle
- Beam format
- Type of accelerator
- Machine parameters

Accelerator designer needs figures of merit to compare machine alternatives

- Physics based
  - Colliders
    - Energy reach, Collision rate (Luminosity), Energy resolution
  - Light sources
    - Spectral range, Spectral brilliance
- Economics based
  - Total cost, $/Watt, €/Joule, operating cost
  - Lifetime, Reliability, Availability
- Facility based
  - Size, Weight, power consumption
    ==> Accelerating gradient, efficiency
- Technology based
  - Technical risk, expansion potential

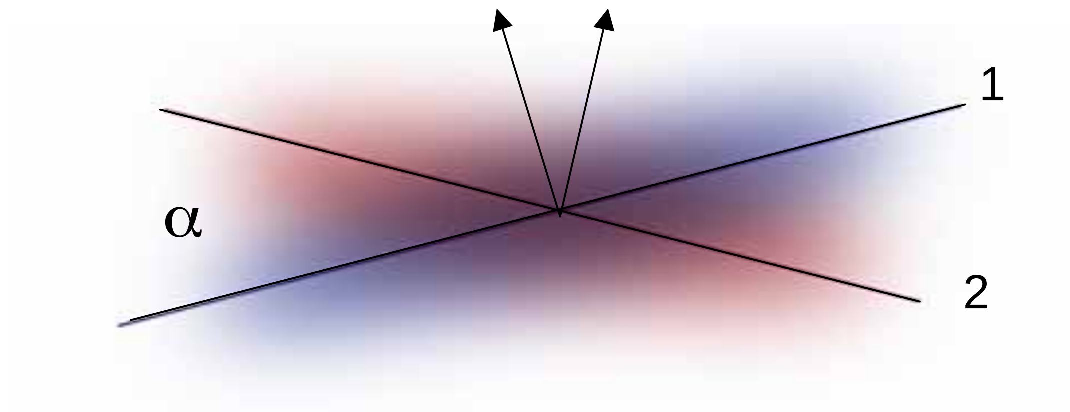
Figure of Merit 1: Beam Energy ==> Energy frontier of discovery



High Energy Physics Figure of Merit 2: Number of events

$$Events = Cross\text{-}section \times \langle Collision\ Rate \rangle \times Time$$

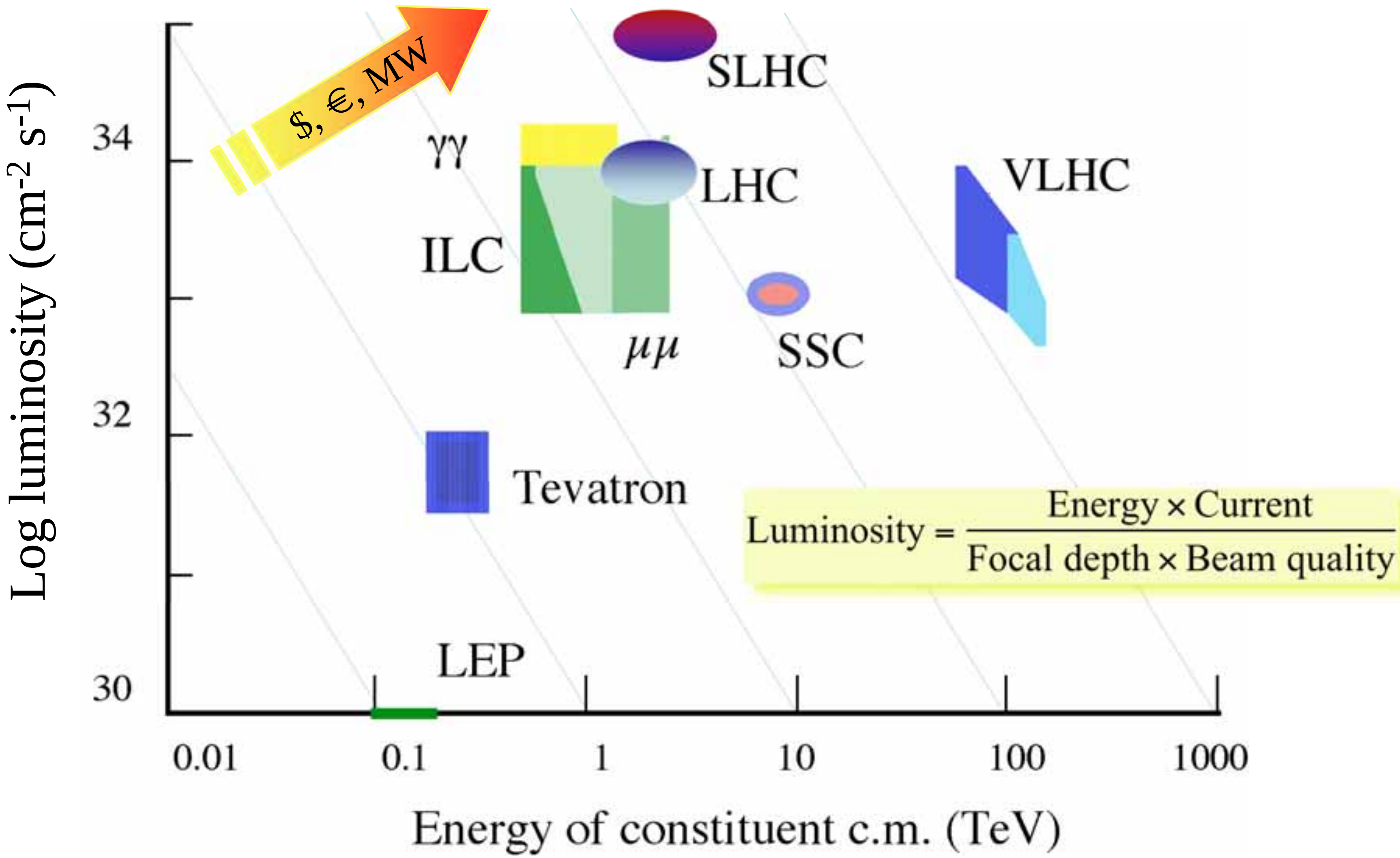
Beam energy: sets scale of physics accessible



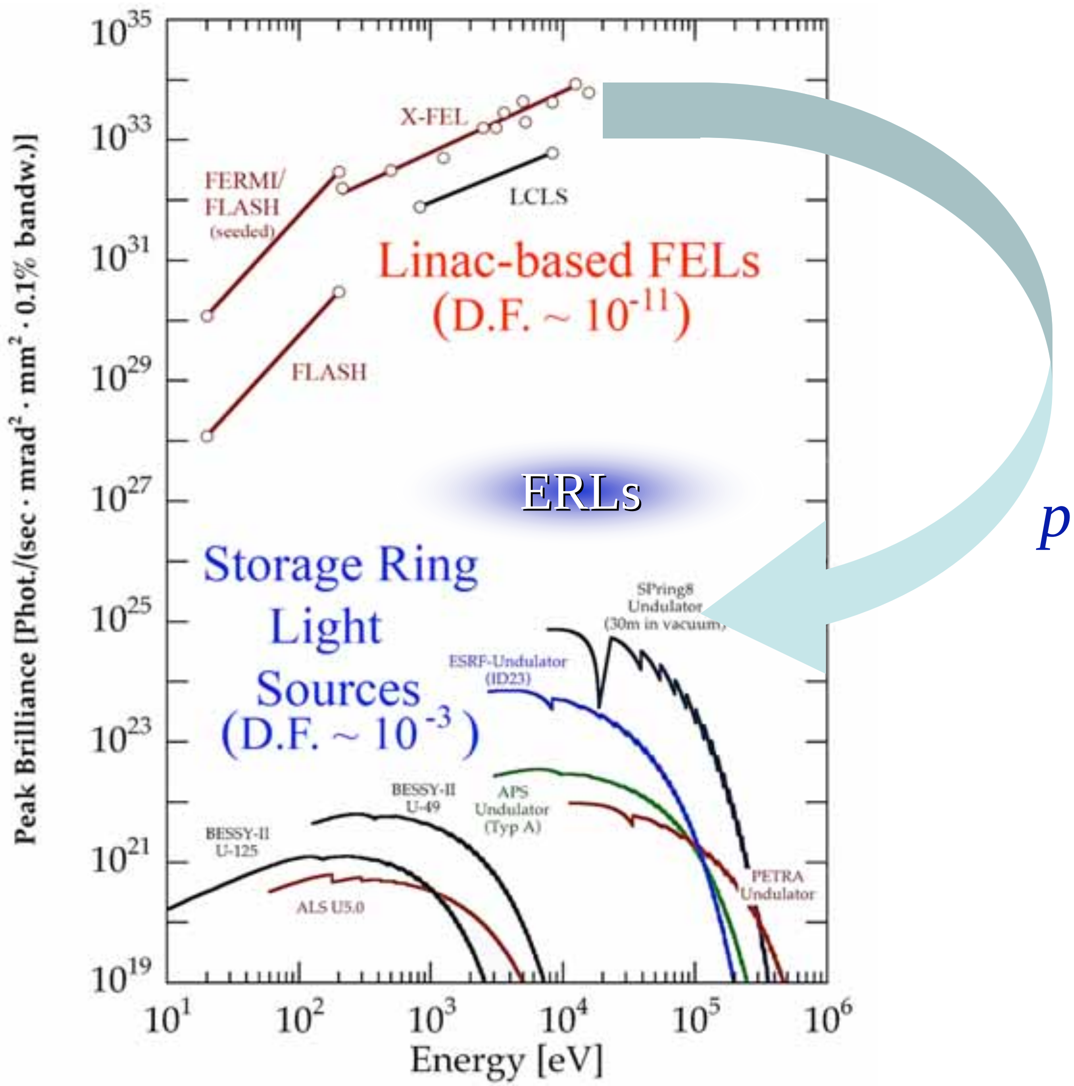
$$\text{Luminosity} = \frac{N_1 \times N_2 \times frequency}{Overlap\ Area} = \frac{N_1 \times N_2 \times f}{4\pi\sigma_x\sigma_y} \times Correction\ factors$$

We want large charge/bunch, high collision frequency & small spot size

Example from high energy physics: Discovery space for future accelerators

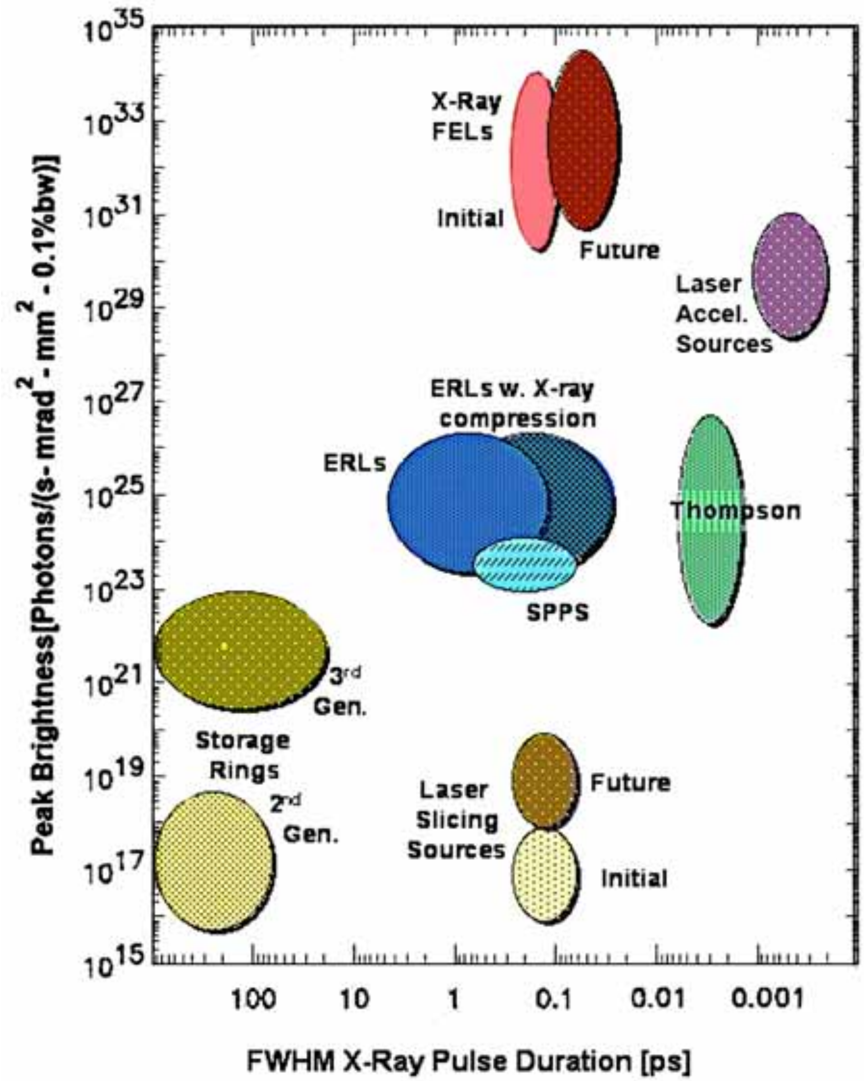


FOM 1 from condensed matter studies: Light source brilliance v. photon energy



Duty factor correction for pulsed linacs

FOM 2 from condensed matter studies: Ultra-fast light sources



Primary, secondary, & tertiary design constraints

	Particle type	Energy	Current	Quality	ΔE/E	Pulse rate	Pulse length	Micro-bunch	Polarize
HEP. NP	p, i, e	1	2	2	1	2	2	1	1 / 3
Light sources	e	2	1	1	1	2	3	1	–
FELs	e	2	1	1	1	2	1	3	–
Spallation sources	p, i	3	1	2	1	2	2	3	–
Radiography	e, p	2	1	1	2	3	1	2	–
Therapy	e, p	1	1	2	2	3	3	3	–
Cargo screening	i	2	1	3	3	2	3	3	–