Unit 6 - Lecture 13

Beam loading

William A. Barletta
Director, United States Particle Accelerator School
Dept. of Physics, MIT

Source: Wake field slides are based on Sannibale lecture 9

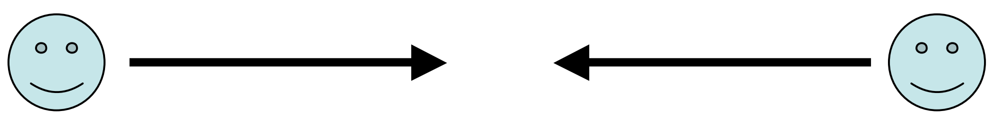
Figure of merit 1: Beam energy

- Two particles have equal rest mass m_0.

Laboratory Frame (LF): one particle at rest, total energy is E_{lab}.

$$\mathbf{P_1} = (E_1/c, \mathbf{p_1}) \qquad \mathbf{P_2} = (m_0 c, \mathbf{0})$$

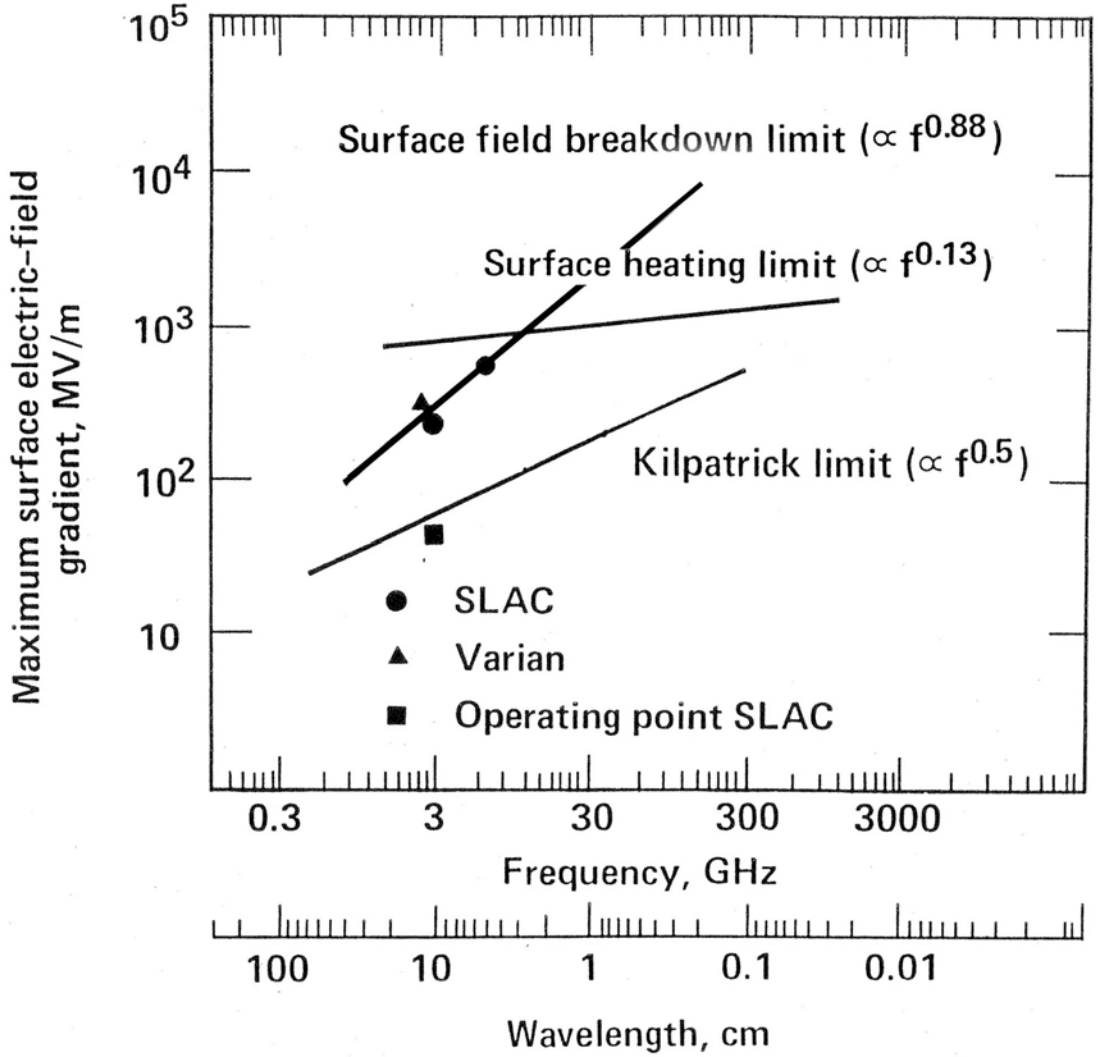
Centre of Momentum Frame (CMF): Velocities are equal & opposite, total energy is E_{cm}.



$$\mathbf{P_1} = (E_{\rm cm}/(2c), \mathbf{p}) \qquad \mathbf{P_2} = (E_{\rm cm}/(2c), -\mathbf{p})$$

$$\boxed{E_{cm} \approx \sqrt{2 m_o c^2 E_{lab}} \quad \text{for } E_{lab} >> m_o c^2}$$

Surface field breakdown behavior



Beam loading

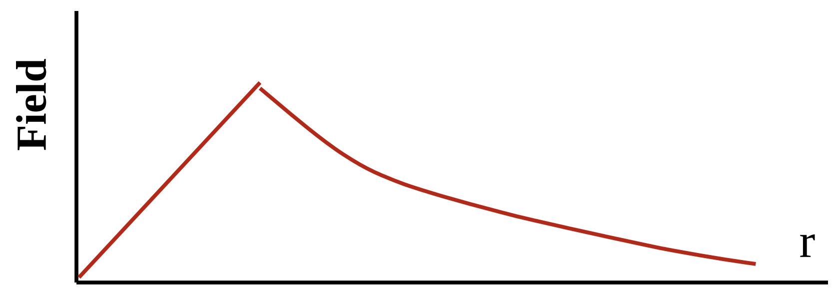
Assumptions in our discussion

1. Particle trajectories are parallel to z-axis in the region of interest
2. The particles are highly relativistic
3. (1) + (2) ==> The beam is rigid,
   - ➞ Particle trajectories are not changed in the region of interest
4. Linearity of the particle motion
   - ➞ Particle dynamics are independent of presence of other particles
5. Linearity of the electromagnetic fields in the structure
   - ➞ The beam does not detune the structure
6. The power source is unaffected by the beam
7. The interaction between beam and structure is linear

Recall our discussion of space charge fields

- Coulomb interaction ==> space charge effect
  - A generic particle in the bunch experiences the *collective* Coulomb force due to fields generated by all the other particles in the bunch
- Such self-fields are usually nonlinear
  - Their evaluation usually requires numerical techniques
  - Special cases can be evaluated analytically

We've already written the expressions for an axisymmetric beam with uniform charge density



Lee Teng’s solution for fields inside the beam

- Conditions:
  - Continuous beam with constant linear charge density l
  - Stationary uniform elliptical distribution in the transverse plane
  - *a* and *b* the ellipse half-axes,
  - the beam moves along *z* with velocity βc.

$$E_x = \frac{1}{\pi\varepsilon_0}\frac{\lambda x}{a(a+b)} \qquad E_y = \frac{1}{\pi\varepsilon_0}\frac{\lambda y}{b(a+b)}$$

$$B_x = -\frac{\mu_0}{\pi}\frac{\lambda\beta c y}{b(a+b)} \qquad B_y = \frac{\mu_0}{\pi}\frac{\lambda\beta c x}{a(a+b)}$$

$$B_x = -\frac{\beta}{c}E_y, \qquad B_y = \frac{\beta}{c}E_x,$$

Space charge for Gaussian distribution

- Conditions
  - Charge density is gaussian in the transverse plane
  - $x << \sigma_x$ and $y << \sigma_y$:

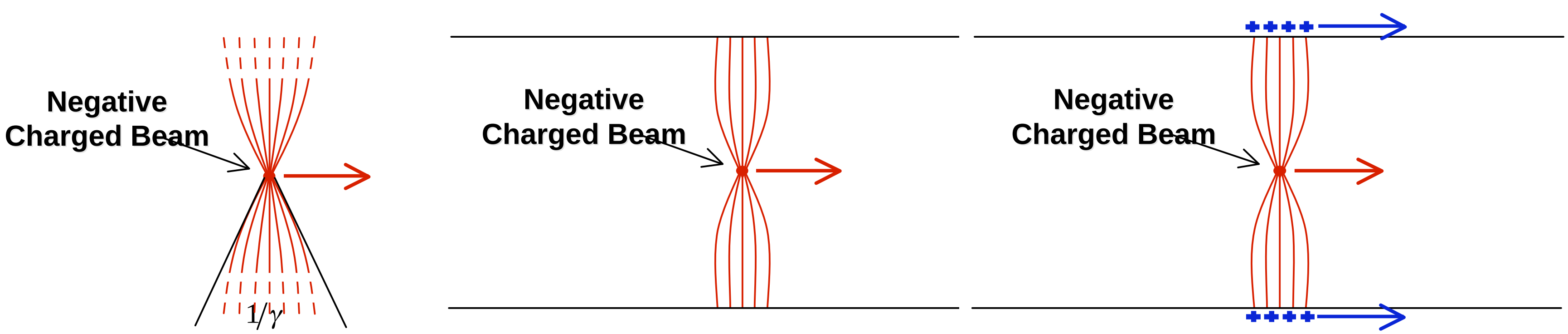
$$E_x = \frac{1}{2\pi\varepsilon_0} \frac{\lambda x}{\sigma_x\left(\sigma_x + \sigma_y\right)} \qquad E_y = \frac{1}{2\pi\varepsilon_0} \frac{\lambda y}{\sigma_y\left(\sigma_x + \sigma_y\right)}$$

$$B_x = -\frac{\mu_0}{2\pi} \frac{\lambda \beta c y}{\sigma_y\left(\sigma_x + \sigma_y\right)} \qquad B_y = \frac{\mu_0}{2\pi} \frac{\lambda \beta c x}{\sigma_x\left(\sigma_x + \sigma_y\right)}$$

$$B_x = -\frac{\beta}{c} E_y, \qquad B_y = \frac{\beta}{c} E_x,$$

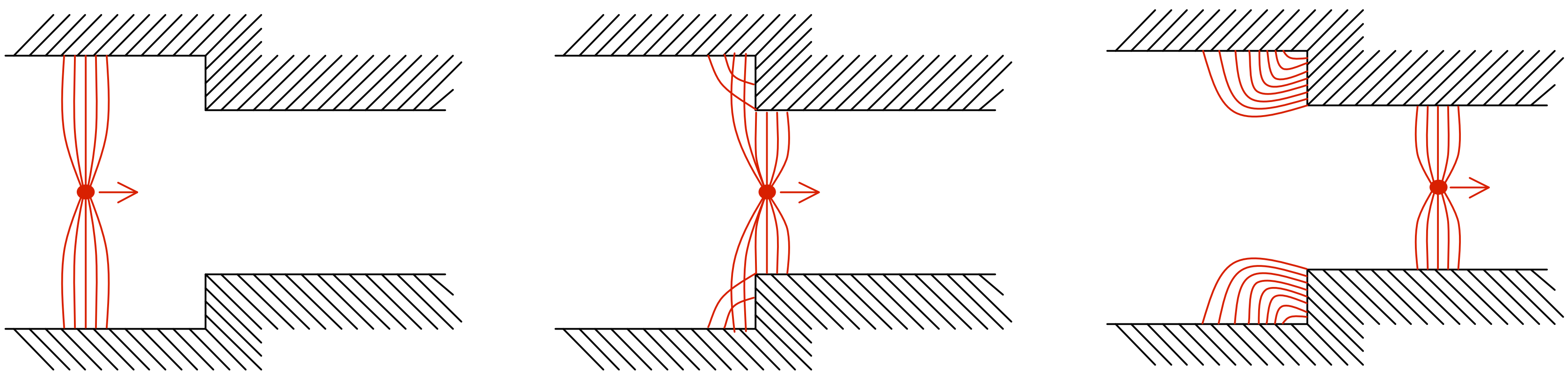
Vacuum Chamber Effects:Image Charge

- In the lab frame, the EM field of a relativistic particle is transversely confined within a cone of aperture of ~ $1/\gamma$
- Particle accelerators operate in an ultra high vacuum environment provided by a metal *vacuum chamber*
- By Maxwell equations, the beam's E field terminates perpendicular to the chamber (conductive) walls
- An equal image charge, but with opposite sign, travels on the vacuum chamber walls following the beam



Vacuum Chamber Wake Fields

- Any variation in chamber profile, chamber material, or material properties perturbs this configuration.
- The beam loses part of its energy to establish EM (wake) fields that remain after the passage of the beam.

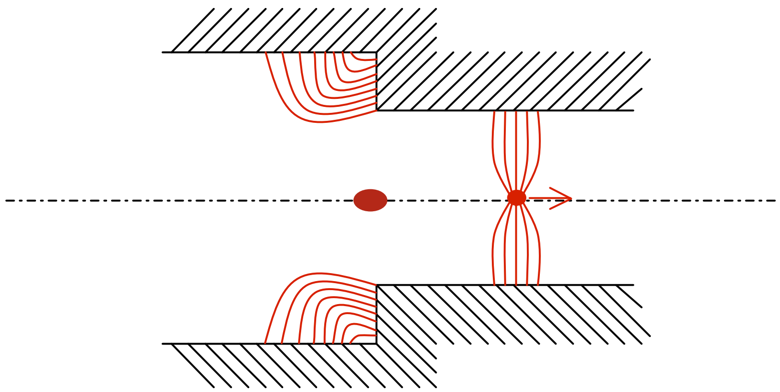


- By causality in the case of ultra-relativistic beams, chamber wakes can <u>only</u> affect trailing particles

The accelerator cavity is, by design, such a variation

Longitudinal wakes & beam loading

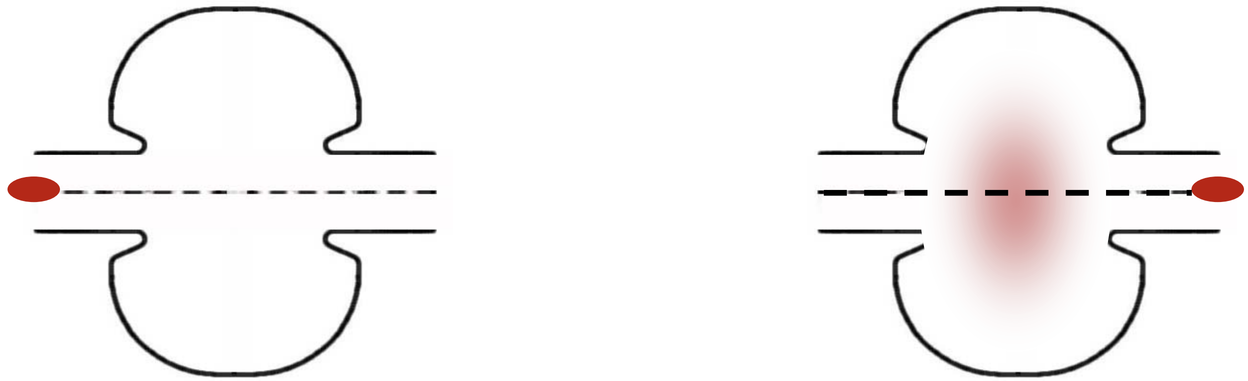
- If the structure is axisymmetric & if the beam passes on the axis of symmetry…



- … the force on axis can only be longitudinal

In a cavity the longitudinal wake (HOMs)
is closely related to beam loading via the cavity impedance

Fundamental theorem of beam loading



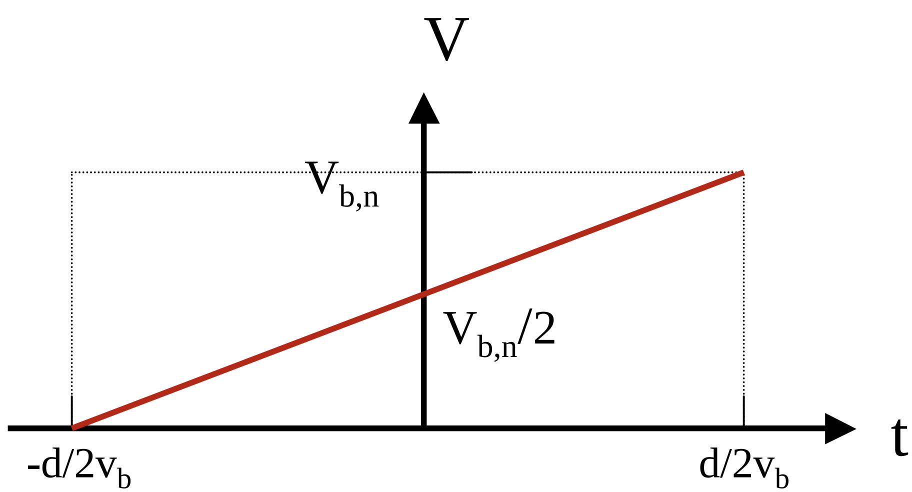
A point charge crosses a cavity initially empty of energy.

After the charge leaves the cavity, a beam-induced voltage $V_{b,n}$ remains in each mode.

By energy conservation the particle must have lost energy equal to the work done by the induced voltage on the charge

What fraction (f) of $V_{b,n}$ does the charge itself see?

The naïve guess is correct for any cavity



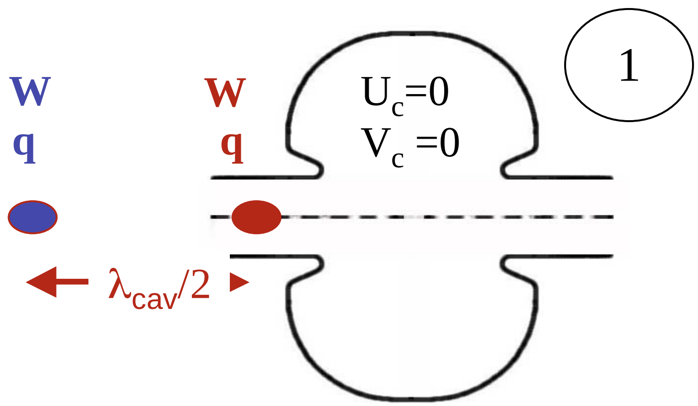
This theorem relates the energy loss by a charge passing through a structure to the electromagnetic properties of modes of that structure.

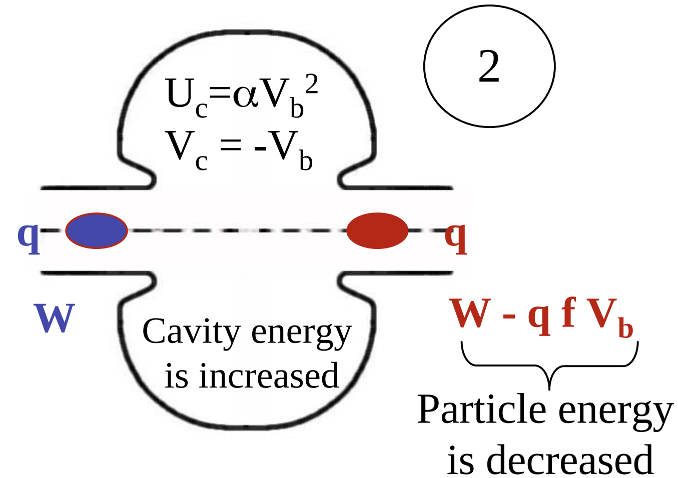
By superposition,
$V_{b,n}$ in a cavity is the same whether or not a generator voltage is present.

A simple proof

W's are the particle energies
U is the cavity energy

Half an rf period later, the voltage has changed in phase by π





For simplicity:

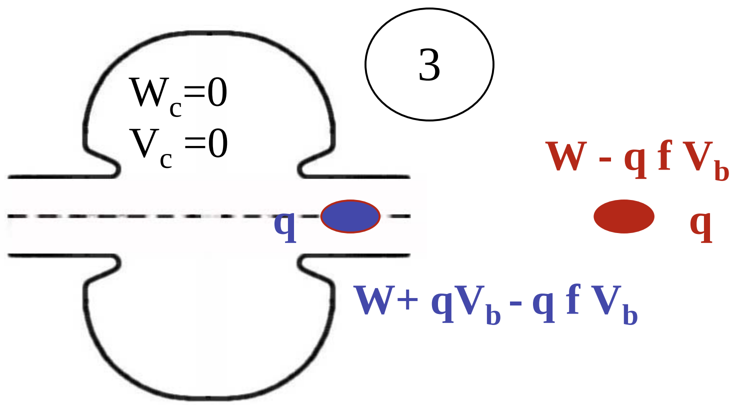
Assume that the change in energy of the particles does not appreciably change their velocity

Increase in U = decrease in W

$$\alpha V_b^2 = q\, f\, V_b \implies V_b = q\, f / \alpha$$

V_b is proportional to q

The simplest wakefield accelerator: q sees an accelerating voltage



Half an rf period later, the voltage has changed in phase by π

Note that **the second charge** has gained energy

$$\Delta W = 1/2\ qV_b$$

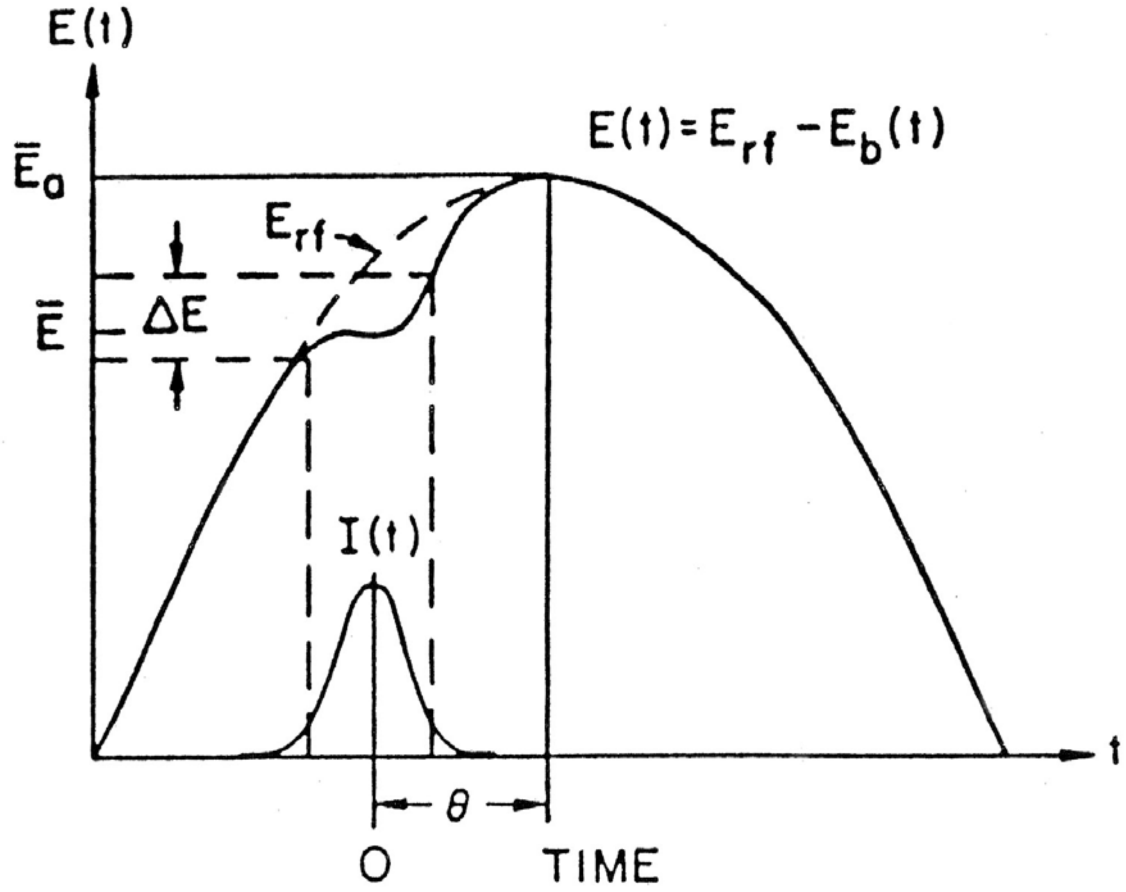
from longitudinal wake field of **the first charge**

By energy conservation:

$$W + qV_b - q\,fV_b + W - q\,fV_b = W + W$$

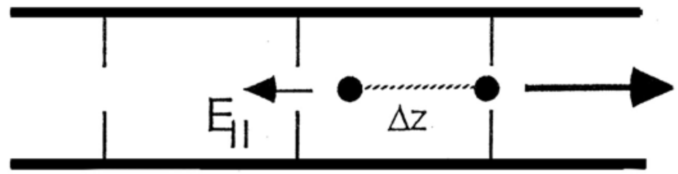
$$\Longrightarrow f = 1/2$$

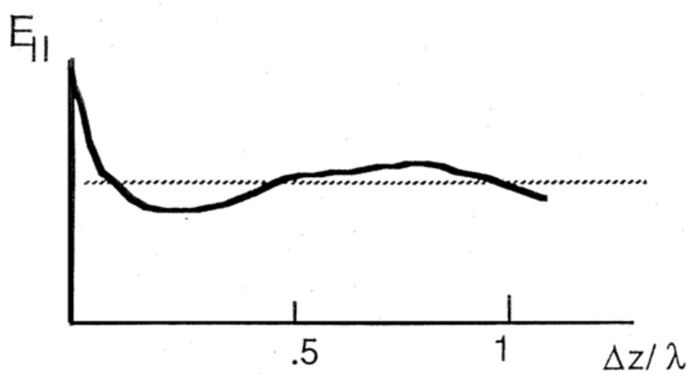
Beam loading lowers accelerating gradient

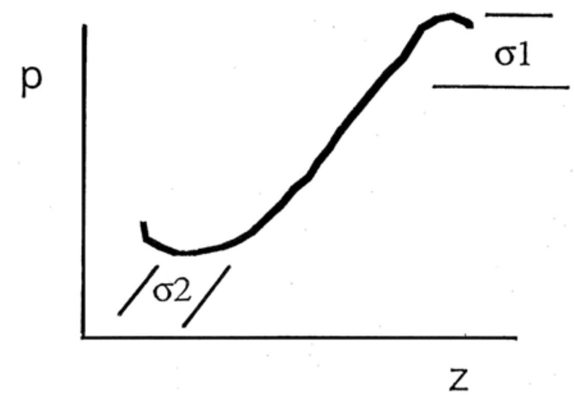


Locating the bunch at the best rf-phase minimizes energy spread

Longitudinal wake field determines the (minimum) energy spread







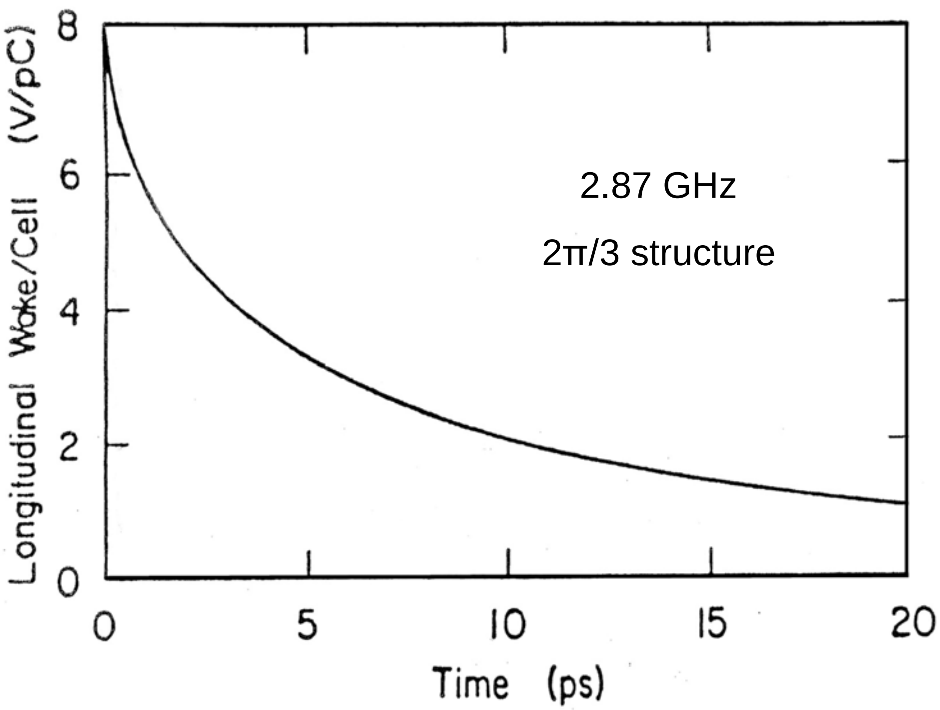
The wake potential, $W_{||}$ varies roughly linearly with distance, s, back from the head

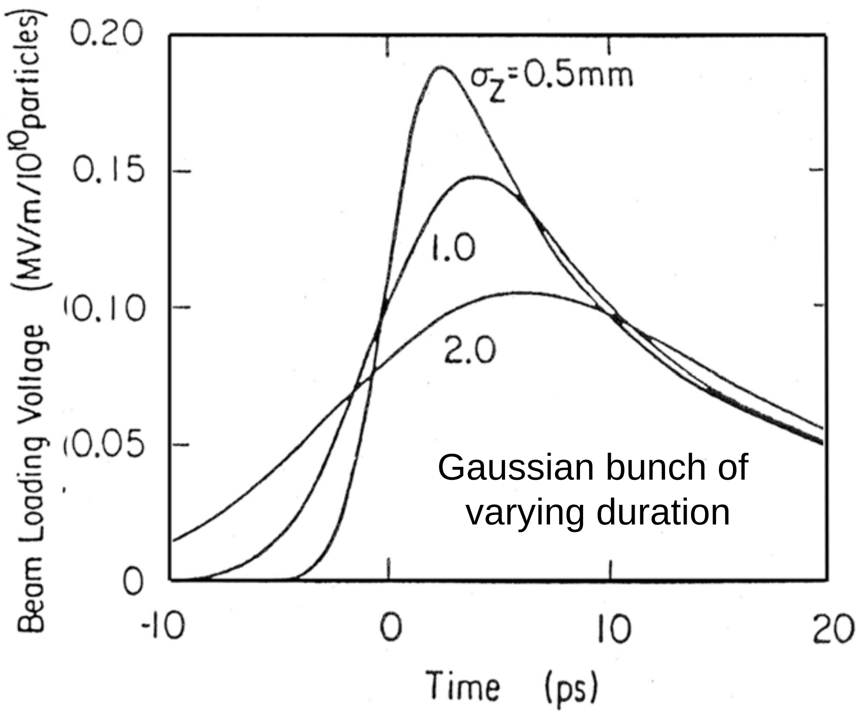
$$W_{ll}(s) \approx W'_{ll} s$$

The energy spread per cell of length d for an electron bunch with charge q is

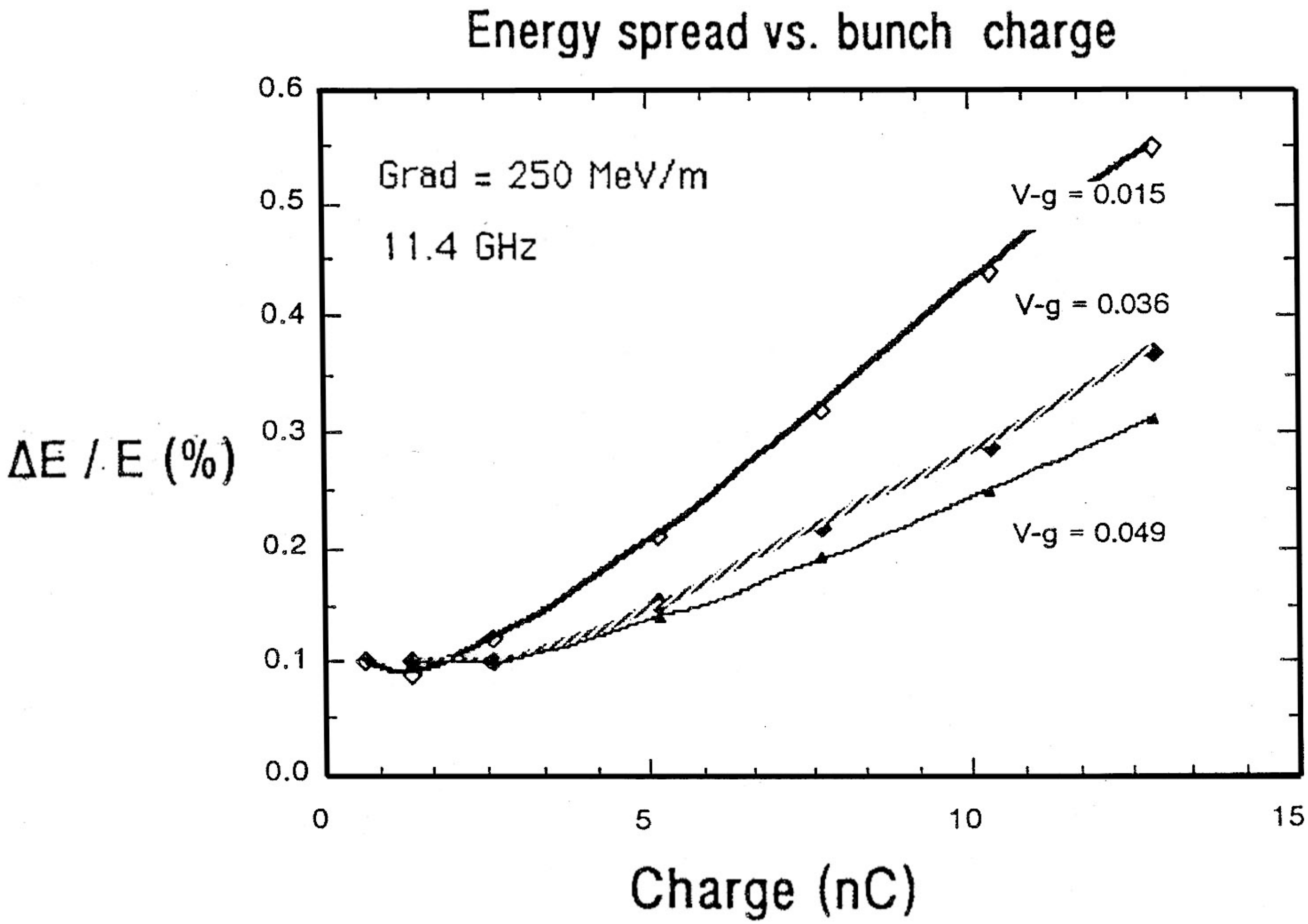
$$\Delta W_{ll}(s) \approx -qeW'_{ll} s_{tail}$$

Beam loading effects for the SLAC linac





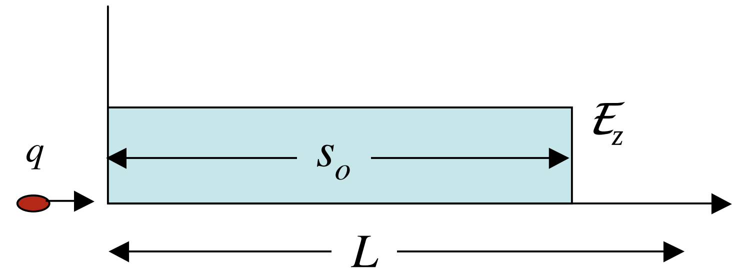
My calculation for a CLIC-like structure



Energy gain in a partially filled structure

- For a test charge at the beginning of the bunch

$$\text{Energy gain} \equiv \Delta E \equiv \int_0^L \mathcal{E}_z(s)\,ds$$



$\mathcal{E}_z$ is the rf - field

$\mathcal{E}_z(s) = \mathcal{E}_o e^{-s/l}$ where l is the attenuation length

$$l = L\frac{T_o}{T_f} \quad \text{with } T_o = {2Q}/{\omega} \quad \text{and} \quad T_f = {L}/{v_g}$$

$$\therefore \Delta E \approx \mathcal{E}_o\left(1 - e^{-s_o/l}\right) \approx \mathcal{E}_o s_o + \ldots$$

In terms of the longitudinal wakefield…

- Bunch induces a wake in the fundamental accelerating mode

$$\mathcal{E}_{z,w} = -\, 2kq$$

- The efficiency of energy extraction is

$$\eta = 1 - \frac{\text{Remaining stored energy}}{\text{Initial stored energy}}$$

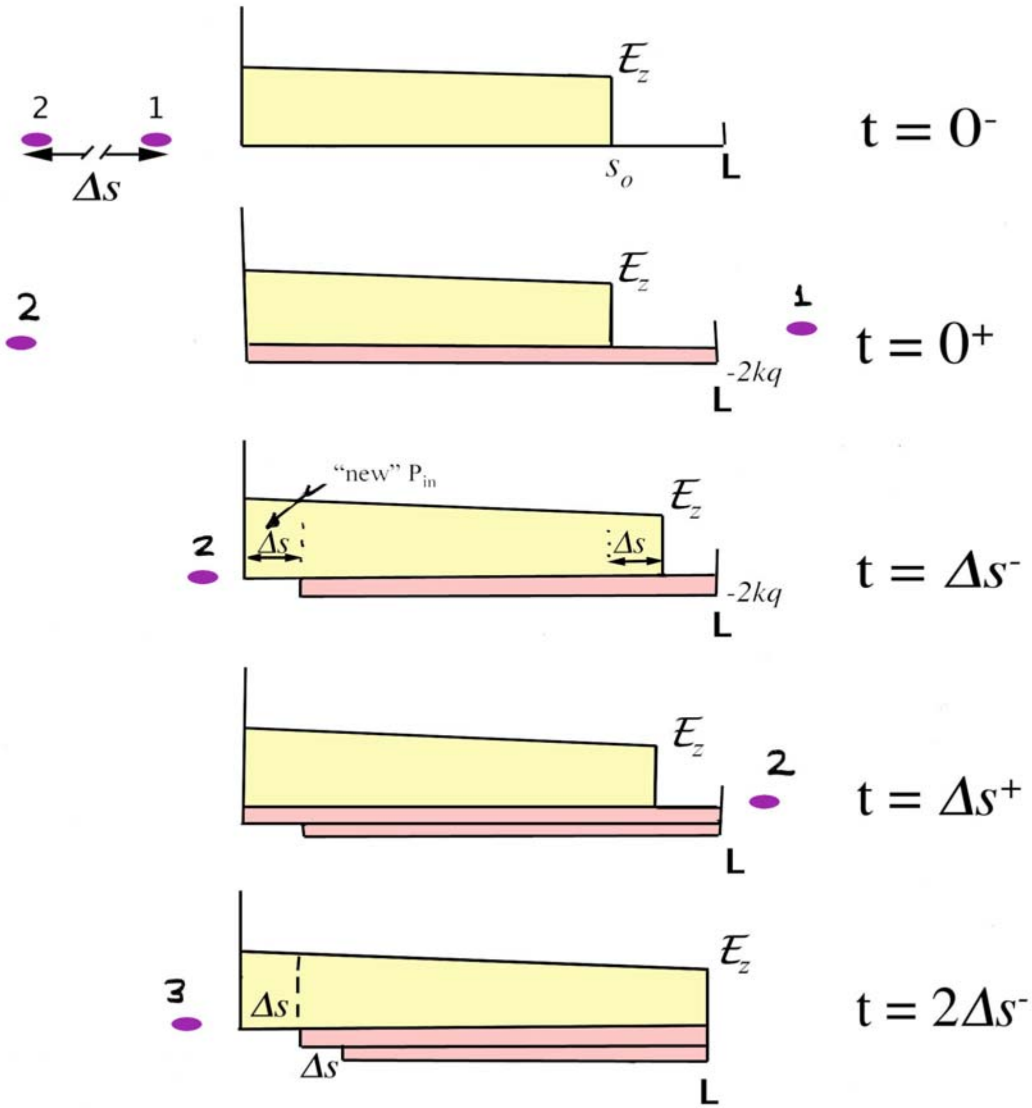
- Stored energy ~ $\mathcal{E}_z^2$

$$\eta = 1 - \frac{\left(\mathcal{E}_z + \mathcal{E}_{z,w}\right)^2}{\mathcal{E}_z^2} = \frac{4kq}{\mathcal{E}_z} - \frac{4k^2q^2}{\mathcal{E}_z^2} \approx \frac{4kq}{\mathcal{E}_z}$$

- The particle at the end of the bunch sees $\mathcal{E}_z = \mathcal{E}_{z,o} - 2kq$

$$\therefore \text{ Average } \Delta E \equiv \langle \Delta E \rangle = \left(\mathcal{E}_z - kq\right)s_o - (kq)\left(l - s_o\right)$$

Now look at the second bunch setting $\Delta s = \Delta t/v_g$



$$\Delta E_3 = \int_0^{\Delta s} \mathcal{E}_z(s)ds + \int_{\Delta s}^{2\Delta s} \left(\mathcal{E}_z(s) - 2kq\right)ds$$

$$+ \int_{2\Delta s}^{s_o + 2\Delta s} \left(\mathcal{E}_z(s) - 4kq\right)ds$$

$$-4kq\left(L - s_o - 2\Delta s\right)$$

We can extend this idea to N bunches

- By analogy

$$\Delta E_n = \mathcal{E}_o l\left(1 - e^{-\left(s_o + (n+1)\Delta s\right)/l}\right) - (n-1)2kql + n(n+1)kq\Delta s$$

- Assume a small attenuation parameter ($T_f/T_o << 1$)

$$\Delta E_n \approx \mathcal{E}_o s_o + (n-1)\left(\mathcal{E}_o \Delta s - 2kqL\right) + n(n-1)kq\Delta s$$

- The quadratic tern prevents all ΔE_n from being equal
- We can choose Δs so $\Delta E_1 = \Delta E_N$; i.e., such that

$$(n-1)\left(\mathcal{E}_o \Delta s - 2kqL\right) + n(n-1)kq\Delta s = 0 \quad \text{for } n = N$$

… finally…

- That is $$\frac{\Delta s}{L} = \frac{2kq}{E_o + Nkq}$$
- Then the maximum energy spread between the bunches is

$$\delta E_{\max} = Max\left(\Delta E_i, \Delta E_j\right) = -\frac{N(N-2)}{4} kq\Delta s$$

- In terms of the single bunch beam loading $\eta_o = 4kq/\mathcal{E}_o$

$$\frac{\Delta s}{L} = \frac{\Delta t}{T_f} = \frac{\eta_o}{2}\frac{1}{1+\eta_o N/4}$$

- Where the maximum δE_{max} is set by the application

$$\eta_o \approx \left[\frac{32\ \delta E_{\max}}{N(N-2)}\right]^{1/2}$$

Costs of making a multi-bunch train

- Decreased gradient:

$$E_{actual}/E_{max} \approx 1\text{-}N\eta_o/2$$

- Decreased efficiency

$$\eta_N \approx N\eta_o(\sim 1\text{-}N\eta_o/2)$$

- Example: Say $(\Delta E/E)_{max} = 10^{-3}$ & N = 10 bunches

$$\eta_o \approx 2\%$$

- The bunch separation $(\Delta t/T_f) \approx 9.95 \times 10^{-3}$

- For f_{rf}= 17 GHz & T_f = 70 ns
  - ➞ 21 cm spacing ==> 12 rf periods between bunches

$$\eta_{10} \approx 18\%$$

Consider a 17 GHz MIT structure

- For f_{rf}= 17 GHz & T_f = 70 ns
- 21 cm spacing ==> 12 rf periods between bunches

$$\eta_{10} \approx 18\%$$

- Since the rf is making up for the wakefields
  - ➞ tight tolerance on N
- In this case,

$$\langle \Delta N/N \rangle < 1\ \%$$

Scaling of wakefields with geometry & frequency in axisymmetric structures

For the disk-loaded waveguide structure (and typically)

- Longitudinal wake field scales as $\boxed{a^{-2} \sim \lambda_{rf}^{-2}}$
- Transverse wakes scale as $\boxed{a^{-3} \sim \lambda_{rf}^{-3}}$