



Pulsed Power Engineering Homework Solution Set

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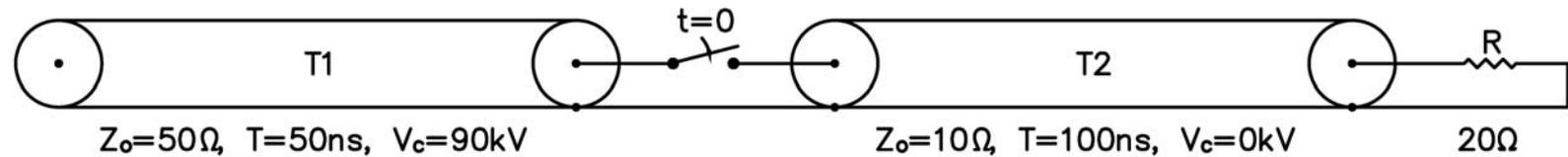
Power Conversion Department

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Homework #1: Transmission Line Bounce



- The switch is closed at $t=0$ (and remains closed), calculate the voltage at the mid-point of line T2 for the following delays:
 - 45 ns
 - 135 ns
 - 195 ns
 - 310 ns
 - 470 ns



Solution Approach

- The traveling wave model for transmission line behavior, translates the 90 kV charge voltage on T1 to 2 recirculation 45 kV waves.
- On closure of the switch, the wave incident on T2 will be partially transmitted and partially reflected, replacing the existing wave with 2 new ones.
- When each new wave encounters an impedance discontinuity, some portion will be transmitted and the rest reflected, again, replacing the existing wave with 2 new ones.
- Waves transmitted to the resistive load are dissipated.
- At any location and time, the voltage is the sum of the wave voltages.
- Calculate transmission and reflection coefficients.
- Track waves
 - Tree structure (as done in class)
 - Bounce diagram (used in the TTU notes)

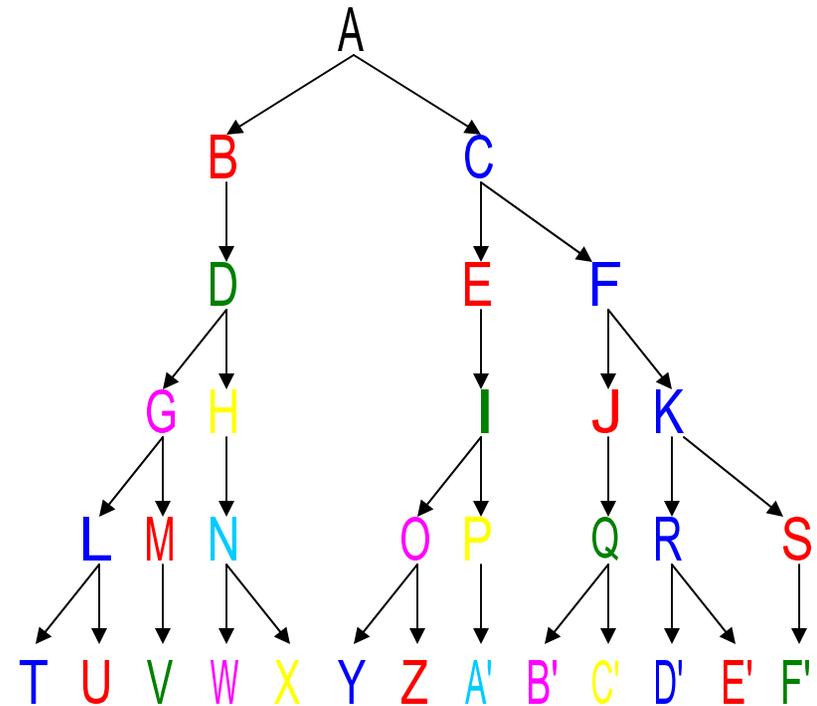
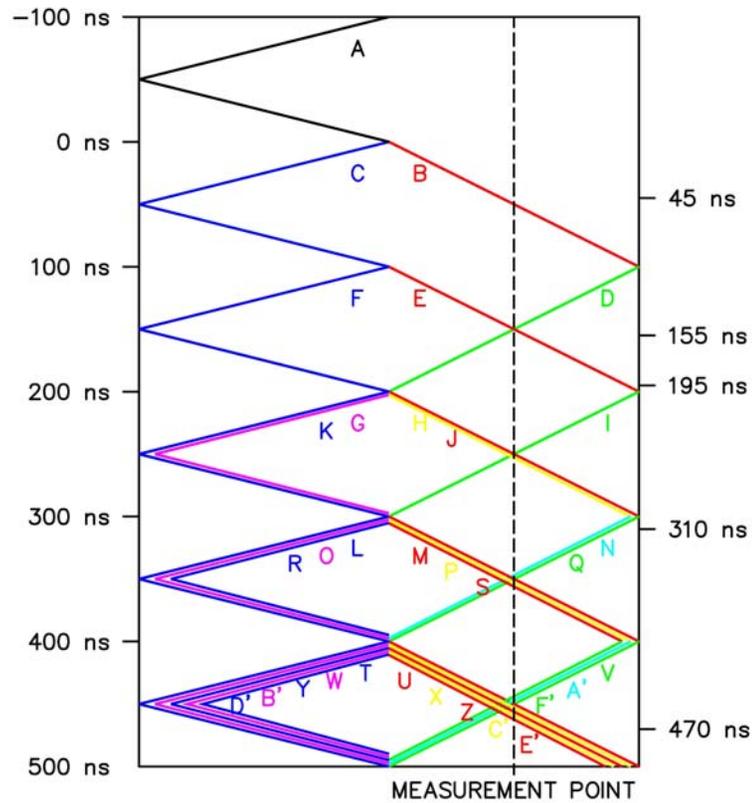
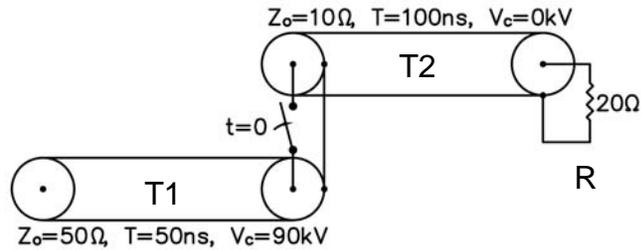


Transmission and Reflection Coefficients

- At each transition from impedance Z_0 to R :
 - Transmission coefficient $= V_T/V_I = (2 R) / (R + Z_0)$
 - Reflection coefficient $V_R/V_I = (R - Z_0) / (R + Z_0)$
- T1 to T2
 - $V_T/V_I = (2 \cdot 10) / (10 + 50) = 1/3$
 - $V_R/V_I = (10 - 50) / (10 + 50) = -2/3$
- T2 to R
 - $V_T/V_I = (2 \cdot 20) / (20 + 10) = 4/3$ (dissipated in load, removed from system)
 - $V_R/V_I = (20 - 10) / (20 + 10) = 1/3$ (shown in cyan for incident yellow)
- T2 to T1
 - $V_T/V_I = (2 \cdot 50) / (50 + 10) = 5/3$
 - $V_R/V_I = (50 - 10) / (50 + 10) = 2/3$



Bounce Diagram





Wave Amplitude: $A = 45 \text{ kV}$

- $B = 1/3 A = 15 \text{ kV}$
 - $D = 1/3 B = 5 \text{ kV}$
 - $G = 5/3 D = 25/3 \text{ kV}$
 - $L = -2/3 G = -50/9 \text{ kV}$
 - » $T = -2/3 L = 100/27$
 - » $U = 1/3 L = -50/27$
 - $M = 1/3 G = 25/9 \text{ kV}$
 - » $V = 1/3 M = 25/27 \text{ kV}$
 - $H = 2/3 D = 10/3 \text{ kV}$
 - $N = 1/3 H = 10/9 \text{ kV}$
 - » $W = 5/3 N = 50/27 \text{ kV}$
 - » $X = 2/3 N = 20/27 \text{ kV}$



Wave Amplitude: $A = 45 \text{ kV}$

- $C = -2/3 A = -30 \text{ kV}$
 - $E = 1/3 C = -10 \text{ kV}$
 - $I = 1/3 E = -10/3 \text{ kV}$
 - $O = 5/3 I = -50/9 \text{ kV}$
 - » $Y = -2/3 O = 100/27 \text{ kV}$
 - » $Z = 1/3 O = -50/27 \text{ kV}$
 - $P = 2/3 I = -20/9 \text{ kV}$
 - » $A' = 1/3 P = -20/27 \text{ kV}$
 - $F = -2/3 C = 20 \text{ kV}$
 - $J = 1/3 F = 20/3 \text{ kV}$
 - $Q = 1/3 J = 20/9 \text{ kV}$
 - » $B' = 5/3 Q = 100/27 \text{ kV}$
 - » $C' = 2/3 Q = 40/27 \text{ kV}$
 - $K = -2/3 F = -40/3 \text{ kV}$
 - $R = -2/3 K = 80/9 \text{ kV}$
 - » $D' = -2/3 R = -160/27 \text{ kV}$
 - » $E' = 1/3 R = 80/27 \text{ kV}$
 - $S = 1/3 K = -40/9 \text{ kV}$
 - » $F' = 1/3 S = -40/27 \text{ kV}$

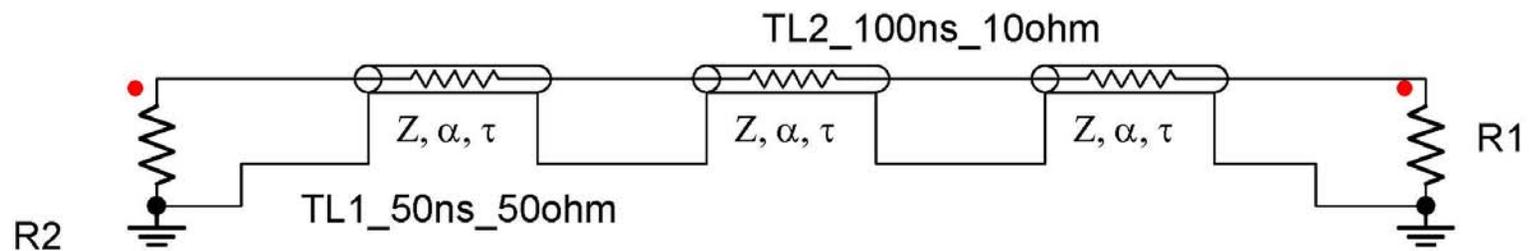
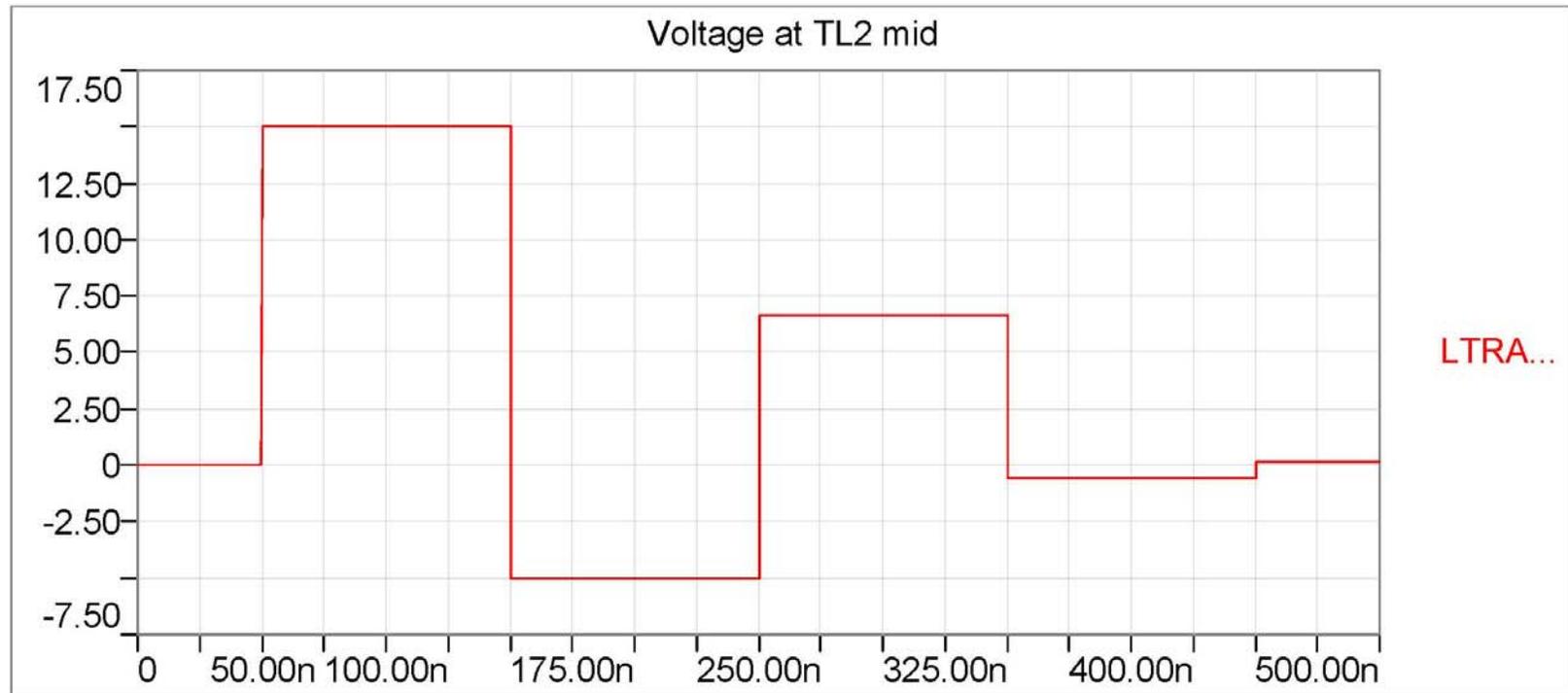


Solutions

- $t = 45 \text{ ns}$
 - Wave B is in T2, but has not yet reached the observation point
 - $V = 0$
- $T = 155 \text{ ns}$
 - Waves D and E are in T2 at the observation point
 - $V = 5 + (-10) = -5 \text{ kV}$
- $T = 195 \text{ ns}$
 - Same waves in at the observation point in T2 as at 155 ns
 - $V = 5 + (-10) = -5 \text{ kV}$
- $T = 310 \text{ ns}$
 - Waves H, I and J are in T2 at the observation point
 - $V = 10/3 - 10/3 + 20/3 = 20/3 \text{ kV}$
- $T = 470 \text{ ns}$
 - Waves U, V, X, Z, A', C', E' and F' are in T2 at the observation point
 - $V = (-50 + 25 + 20 - 50 - 20 + 40 + 80 - 40)/27 = 5/27 \text{ kV}$



Simplorer Simulation of Circuit





Homework #2: Pulse Forming Network Design

- Design a PFN to the following specifications:
 - Impedance: $Z = 10 \Omega$
 - Pulse duration: $\tau = 5 \mu\text{s}$
 - Number of stages: $N = 5$
 - 50 kV charge voltage
 - We discussed several approaches for PFN design. For this exercise, please use the “practical” implementation of the E-type Guillemin topology (page 51 of PPE Basic Topologies).
 - Include a design for the inductor(s)



Solution: PFN

- $L_T = (\tau/2) Z = (2.5 \mu\text{s}) (10 \Omega) = 25 \mu\text{H}$
 - $\sum L_n = L_T$, all the same except 30% higher on both ends
 - $L_n = L1$ for $n = 2, 3, 4$
 - $= 4.5 \mu\text{H}$
 - $L_n = 1.3 \cdot L1$ for $n = 1, 5$
 - $= 5.8 \mu\text{H}$
- $C_T = (\tau/2)/Z = (2.5 \mu\text{s})/(10 \Omega) = 0.25 \mu\text{F}$
 - $\sum C_n = C_T$, all the same
 - $C_n = C_T / N$
 - $= 50 \text{ nF}$



A Solution (E Pluribus Unum): Inductor

- Design considerations
 - High voltage:
 - Charge to 50 kV, avoid field enhancements that could arc or corona
 - Transient voltage as high as 50 kV across inductor, avoid turn-turn breakdown
 - Current
 - $I_{\text{matched}} = 2.5 \text{ kA}$
 - Modest magnetic forces on coils
 - Average current
 - Don't know PRF, if we assume 100 Hz
 - $\langle I \rangle = 100 \text{ Hz} \cdot 5 \mu\text{s} \cdot 2.5 \text{ kA} = 1.25 \text{ A}$, therefore don't need much copper
 - Skin depth, $\delta = (2.6 \cdot \tau^{1/2}) (\rho/\rho_c)^{1/2} = (2.6 \cdot 5 \mu\text{s}^{1/2}) (2.6/1.7)^{1/2} = 7 \text{ mm}$
[see page 4 of the “Materials” slides, above is simplified form for pulse length]
 - Size
 - Want something that will “mate” to capacitors with short leads (stray L)
 - Caps will be similar to Maxwell 37612, except at 2X voltage (4X volume), so expect caps to be ~4” X 6” X 10” long, so caps all side-by-side will be ~0.6 m, so that would be a good length for inductor
 - Mutual inductance
 - Need ~15% mutual inductance between adjacent inductors E-type design
 - Can be achieved with a coil radius of 3 cm (for the rest of the final coil parameters)
 - Estimate the mutual inductance by calculating inductance for the length of a single coil ($L = L_n$) then repeat calculation for a coil of twice the length ($L = 2L_n + M$), “extra” inductance is M



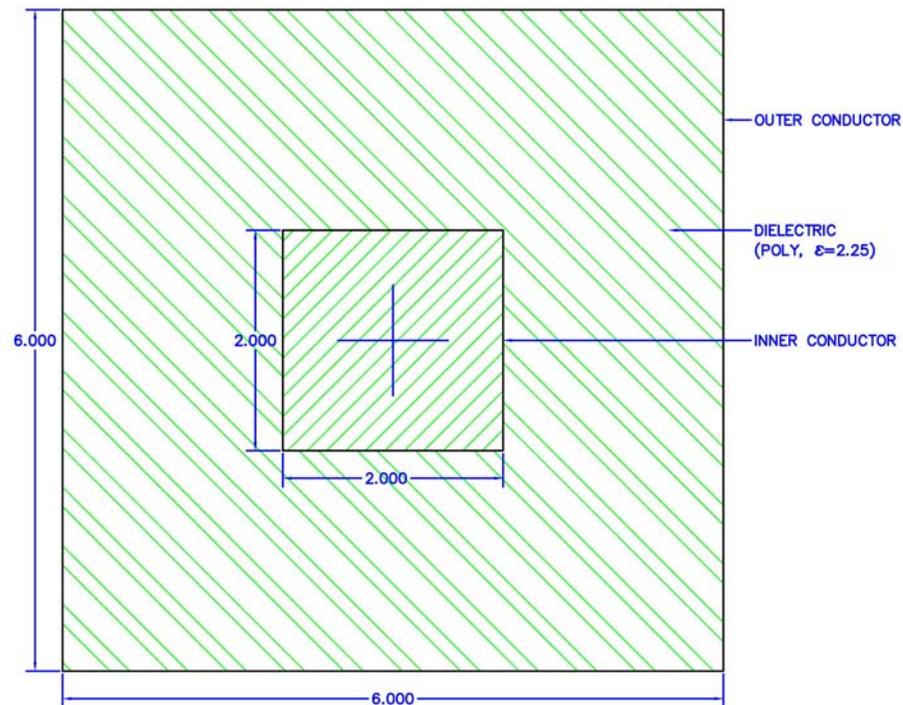
Coil design

- $L = N^2 \mu \pi r^2 / \ell$ (will be a pretty good approximation, for a long coil)
 $25 \mu\text{H} = N^2 \cdot 1.26 \times 10^{-6} \cdot \pi \cdot (3 \times 10^{-2})^2 / 0.6$
→ $N = 66$ turns: 15 turns for coils 1 and 5, 12 turns for coils 2, 3 and 4
- Construction
 - Conductor: 1/4" copper refrigeration tubing
 - Flexible enough to wind
 - Rigid enough to be self-supporting
 - Large enough diameter to minimize electrical field enhancement
 - Mandrel: 2-1/8" diameter heavy wall PVC tubing
 - Puts conductor centers at 3-cm-radius
 - Good mechanical material
 - Inexpensive

Homework #3: Application of Numerical Electrostatic Field Calculation



- Calculate the impedance of a transmission line with the following cross-section





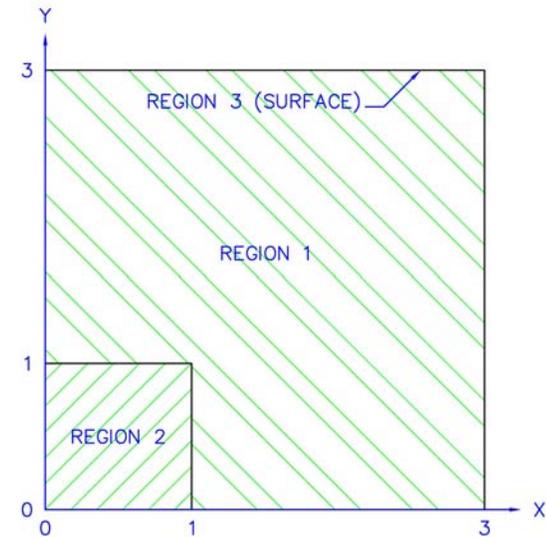
Solution Technique

- Transmission line impedance can be calculated from
 - $Z = \tau/C$, or
 - $= 1/[(C/\ell) \cdot v]$
 - Where C/ℓ is capacitance per unit length
 - And v is the propagation velocity for EM energy in the transmission line
 - $v = c/\epsilon^{0.5}$
- Capacitance per unit length can be calculated from electric field energy per unit length (E/ℓ) and voltage difference between conductors (V)
 - $C/\ell = (2 \cdot E/\ell)/V^2$
- The electrostatic field solver can calculate the electric field energy



Solution

- Identify symmetry planes to reduce extent of solution volume to increase computational efficiency (optional)
 - Most efficient configuration is
- Create a script to generate the solution mesh
 - Region 1 (solution volume): dielectric
 - Region 2: conductor
 - Region 3: conductor
- Create a script for the electrostatic solver
 - Region 1: $\epsilon = 2.25$
 - Region 2: $V = 1$
 - Region 3: $V = 0$
- Perform a volume integral of the electrostatic energy in the solution volume: $E/\ell = 15.5 \text{ pJ/m}$, but only $1/4$ of line, hence $E/\ell = 62 \text{ pJ/m}$
- $V = 1$, so $C/\ell = 2 \cdot E/\ell = 124 \text{ pF/m}$
- $v = 2 \times 10^8 \text{ m/s}$
- $Z = 40.3 \text{ } \Omega$



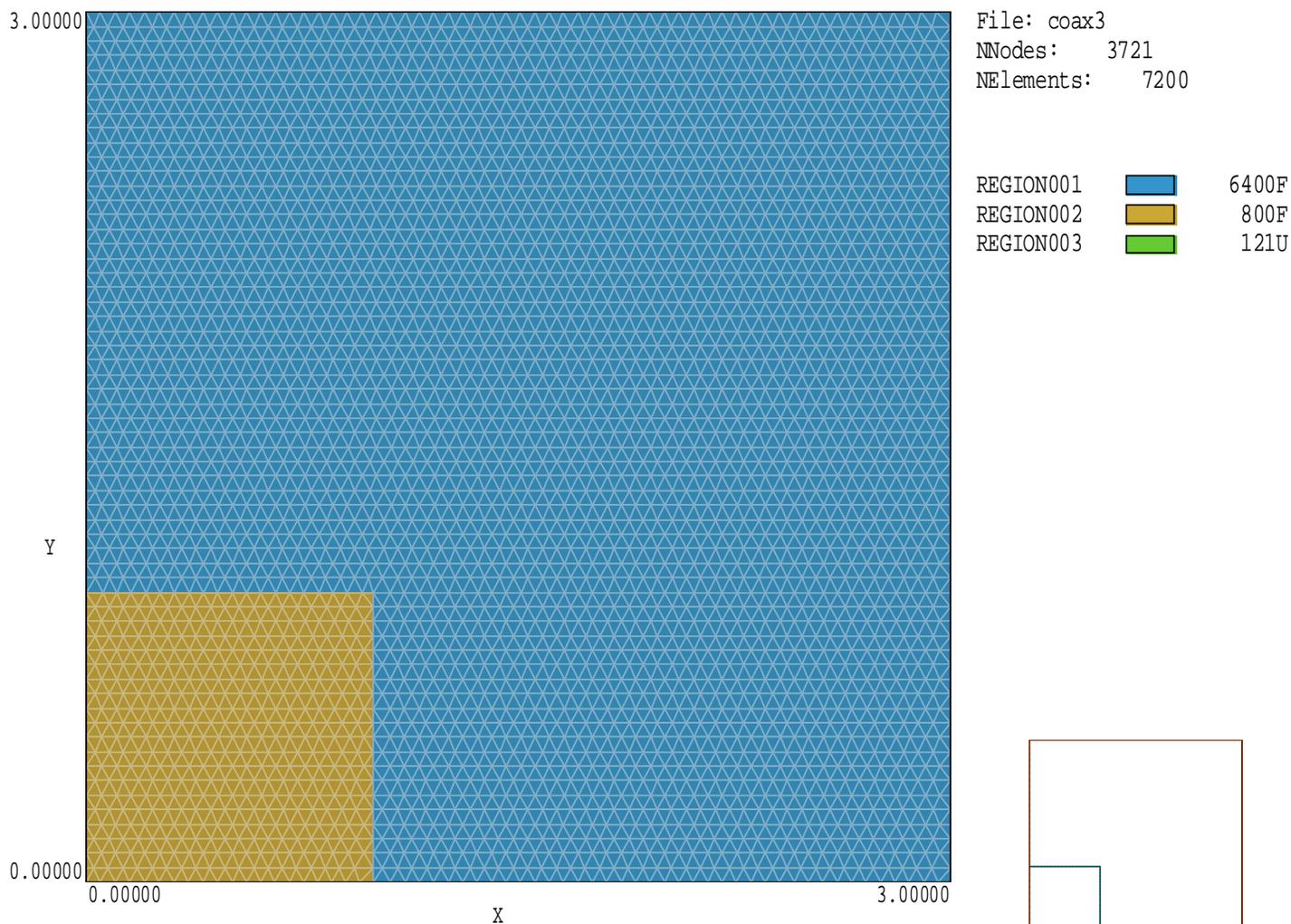
Mesh Script



```
coax3.MIN
* File: coax3.MIN
* MESH 6.1 script (Field Precision)
* Date: 01/13/09
* Time: 17:36:33
* -----
GLOBAL
  XMESH 0.00000 3.000 0.05000
  END
  YMESH 0.00000 3.000 0.05000
  END
END
* -----
REGION  FILL REGION001
  L 0.00000 0.00000 3.000 0.00000
  L 3.000 0.00000 3.000 3.000
  L 3.000 3.000 0.00000 3.000
  L 0.00000 3.000 0.00000 0.00000
END
* -----
REGION  FILL REGION002
  L 0.00000 0.00000 1.00000 0.00000
  L 1.00000 0.00000 1.00000 1.00000
  L 1.00000 1.00000 0.00000 1.00000
  L 0.00000 1.00000 0.00000 0.00000
END
* -----
REGION  REGION003
  L 3.000 0.00000 3.000 3.000
  L 3.000 3.000 0.00000 3.000
END
* -----
ENDFILE
```



Computational Mesh





Electrostatic Solver Script

```
coax3.EIN
* EStat_Edu 3.0 Script (Field Precision)
* File: coax3.EIN
* Date: 01/13/2009
* Time: 18:44:43

Mesh = coax3
Geometry = Rect
DUnit = 1.0000E+02
ResTarget = 5.0000E-08
MaxCycle = 5000

* Region 1: REGION001
Epsi(1) = 2.2500E+00

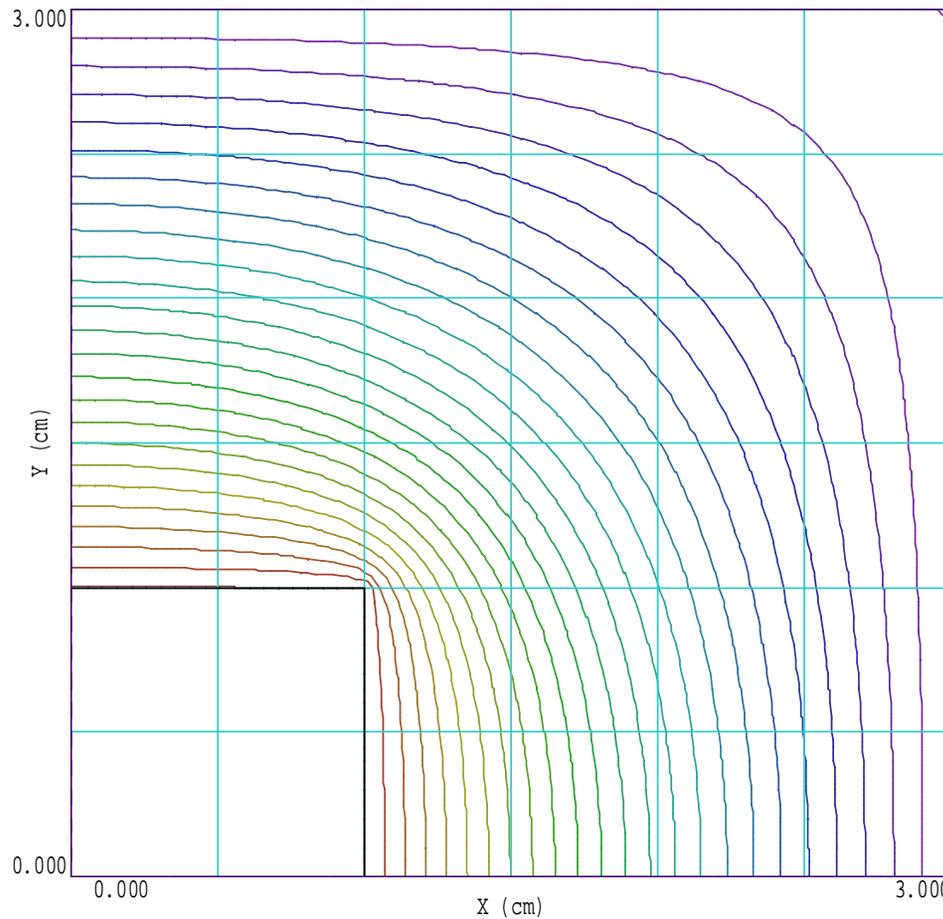
* Region 2: REGION002
Potential(2) = 1.0000E+00

* Region 3: REGION003
Potential(3) = 0.0000E+00

EndFile
```

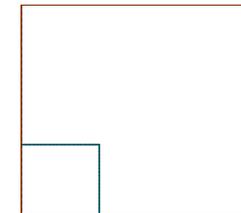
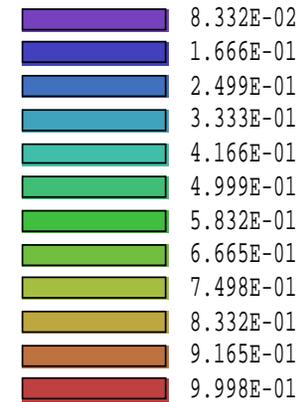


Electrostatic Solution



File prefix: coax3.EOU
Plot type: Contour
Quantity: Potential (V)

XGrid: 0.500
YGrid: 0.500
Minimum value: 0.000E+00
Maximum value: 9.998E-01



Volume Integral



coax3.DAT
TriComp Data Analysis File
Field Precision, Albuquerque, NM 87111 USA
Telephone: 505-296-6689 FAX: 505-294-0222
E Mail: techinfo@fieldp.com Internet: www.fieldp.com
=====

Analyzing solution file coax3.EOU

--- Volume Integrals ---
Energy: 1.550E-11 J/m
Power: 3.501E+00 W/m
Epeak: 1.866E+02 V/m
XPeak: 1.029E+00
YPeak: 1.017E+00

Integrals by region

NReg	Area (m2)	Energy (J/m)	Power (W/m)	PeakE (V/m)	PeakX	PeakY
1	8.000E-04	1.550E-11	3.501E+00	1.866E+02	1.029E+00	1.017E+00
2	1.000E-04	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00



Homework 4: Simulation of a PFN with Klystron load

- Design a PFN for the following system
 - Klystron load
 - 250 kV
 - 2 μ P
 - Transformer
 - Step-up ratio: $N = 1:10$
 - Secondary leakage inductance: 200 μ H
 - Primary inductance, L_p , such that the peak magnetization current, I_p , is 0.5% of the PFN output current
 - PFN
 - 5 μ s pulse length
 - “Practical” Type-E design
 - 5-stage
- Simulate the system behavior using Simplorer



Solution: Equivalent PFN Load

- $I_{\text{klystron}} = \mu V^{1.5} = 2 \times 10^{-6} \cdot (2.5 \times 10^5)^{1.5} = 250 \text{ A}$
- $Z_{\text{klystron}} = V/I = 250 \text{ kV}/250 \text{ A} = 1 \text{ k}\Omega$
- $Z_{\text{PFN}} = Z_{\text{primary}} = Z_{\text{secondary}}/N^2 = 1000/10^2 = 10 \Omega$



Solution: PFN

- $L_T = (\tau/2) Z = (2.5 \mu\text{s}) (10 \Omega) = 25 \mu\text{H}$
 - $\sum L_n = L_T$, all the same except 30% higher on both ends
 - $L_n = L1$ for $n = 2, 3, 4$
 - $= 4.5 \mu\text{H}$
 - $L_n = 1.3 \cdot L1$ for $n = 1, 5$
 - $= 5.8 \mu\text{H}$
- $C_T = (\tau/2)/Z = (2.5 \mu\text{s})/(10 \Omega) = 0.25 \mu\text{F}$
 - $\sum C_n = C_T$, all the same
 - $C_n = C_T / N$
 - $= 50 \text{ nF}$

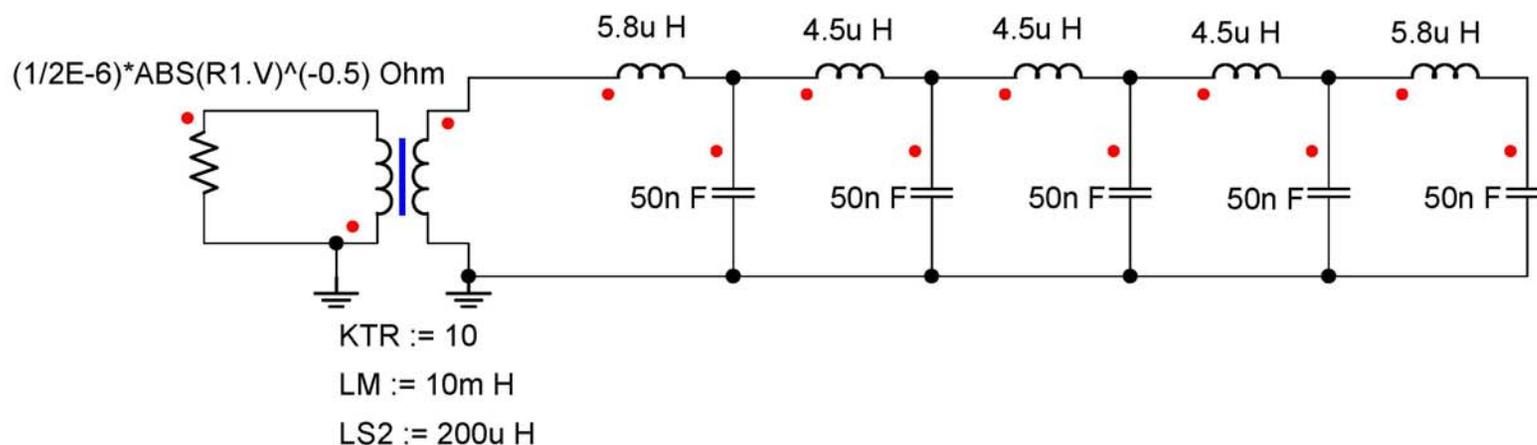
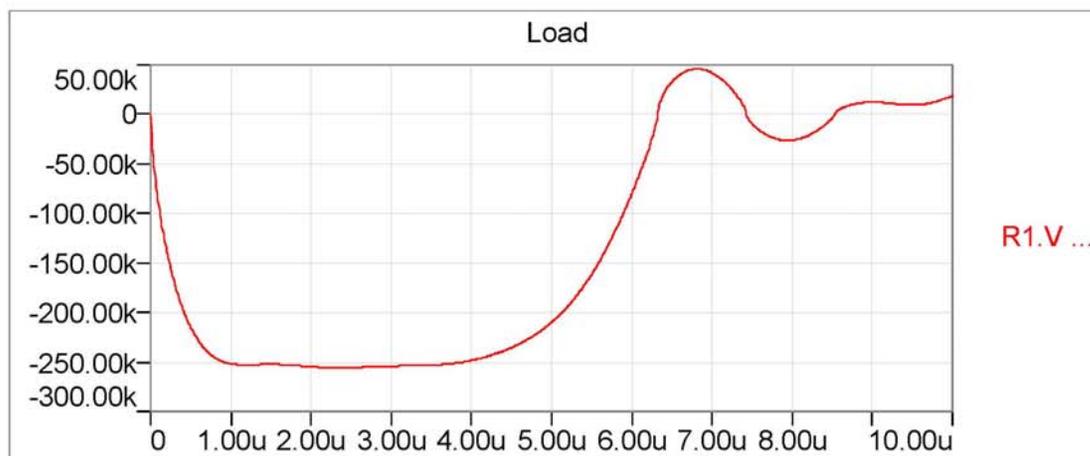


Solution: Primary Magnetization Current

- Current through an inductor
 - $I(t) = (1/L) \cdot \int V dt$
 - For a “square” pulse, $\int V dt = V\tau$
 - $I_M = (1/L_M) \cdot V\tau = (1/L_M) \cdot (25 \text{ kV})(5 \mu\text{s})$
- PFN current
 - $I_{\text{PFN}} = 25 \text{ kV}/10 \Omega = 2.5 \text{ kA}$
- Primary inductance
 - $L_M = (25 \text{ kV})(5 \mu\text{s}) / (0.005)(2.5 \text{ kA}) = 0.125 \text{ Vs} / 12.5 \text{ A} = 10 \text{ mH}$



Circuit Simulation: Model and Results





Final Exam

- 1) Calculate the perveance of a 500 kV, 250 A klystron

$$\begin{aligned}\mu &= I / V^{1.5} \\ &= 250 / (5 \times 10^5)^{1.5} \\ &= 0.707 \mu\text{P}\end{aligned}$$

- 2) Calculate the inductance of 1 m of 50 Ω poly dielectric ($\epsilon_r = 2.25$) coax

$$L = \tau \cdot Z$$

$$\tau = \ell/v$$

$$\begin{aligned}v &= c/\epsilon^{0.5} \\ &= 3 \times 10^8/2.25^{0.5} \\ &= 2 \times 10^8 \text{ m/s}\end{aligned}$$

$$= 1 \text{ m} / 2 \times 10^8 \text{ m/s}$$

$$= 5 \text{ ns}$$

$$= 5 \text{ ns} \cdot 50 \Omega$$

$$= 0.25 \mu\text{H}$$



Final Exam (cont.)

- 3) If a PFN charged to 50 kV is coupled to the klystron in problem (1) through a step-up transformer,
- A) What is the “matched” impedance of the PFN be?
 - B) What is the turns ratio of the transformer be?
 - C) If the primary leakage inductance is negligible but the secondary leakage inductance is 0.5 mH, what is the minimum risetime of the klystron current ($\tau_R = 2.2 L/R$)

B) Klystron: $Z = V/I = 500 \text{ kV} / 250 \text{ A} = 2 \text{ k}\Omega$

Matched PFN: $V_T = 0.5 V_{\text{charge}} = (0.5)(50 \text{ kV}) = 25 \text{ kV}$

$$N = V_S / V_P = 500 \text{ kV} / 25 \text{ kV} \\ = 20$$

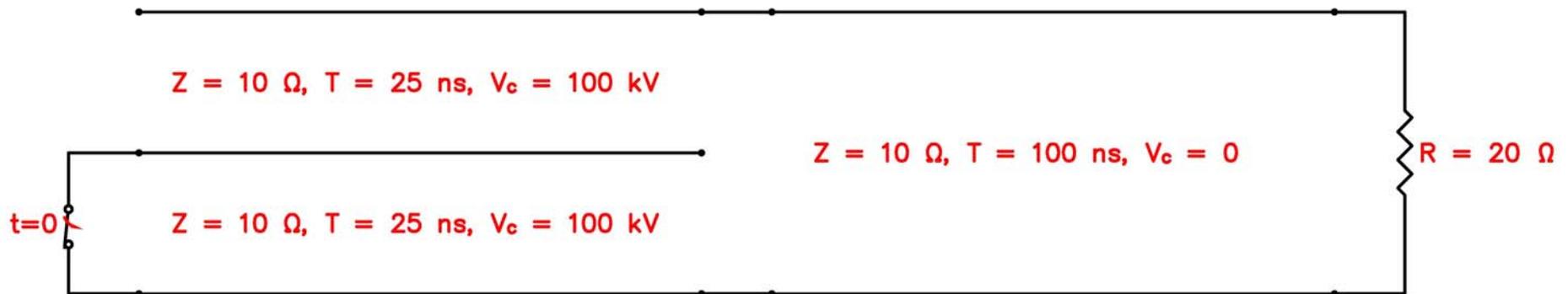
A) $Z_{\text{PFN}} = Z_{\text{primary}} = Z_{\text{secondary}} / N^2 = 2000 / 20^2 = 5 \Omega$

C) $\tau_R = 2.2 L/R = (2.2)(0.5 \text{ mH}) / (2 \text{ k}\Omega) = 0.55 \mu\text{s}$



Final Exam (cont.)

- 4) Given the transmission line system shown below, a $20\ \Omega$ Blumlein connected to a $10\ \Omega$ transmission line connected to a $20\ \Omega$ load (T is the 1-way transit time), plot the load voltage for $0 \leq t \leq 200\ \text{ns}$.



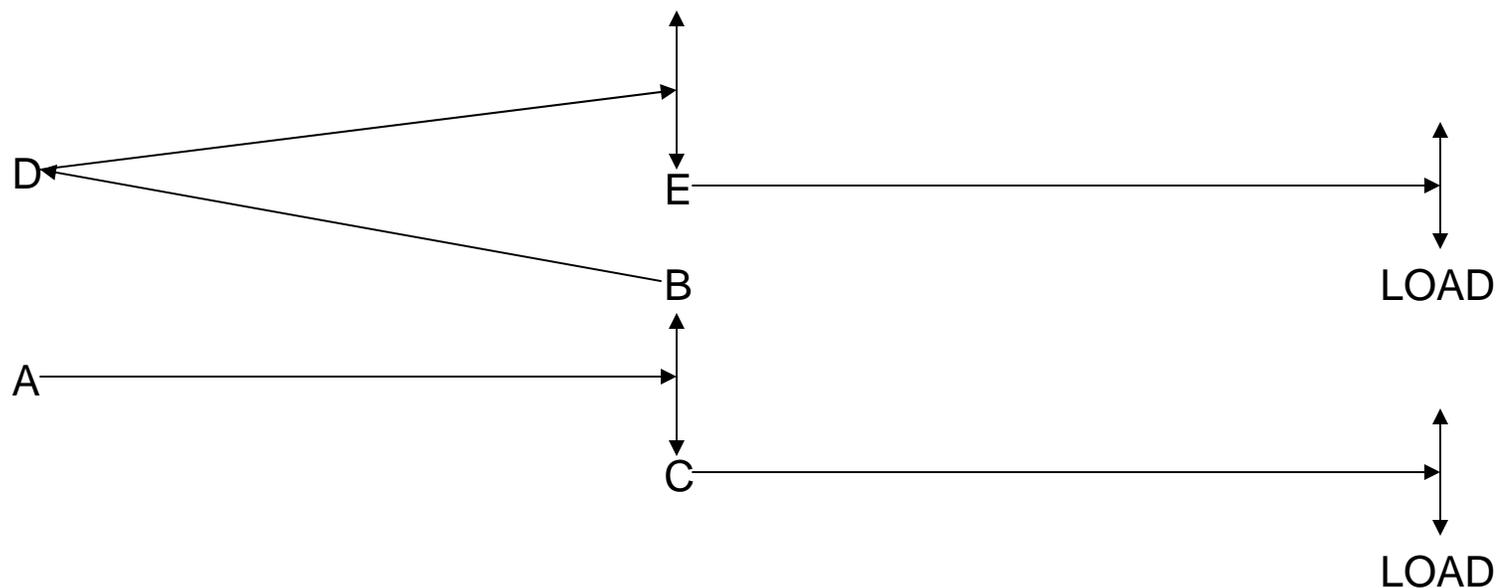
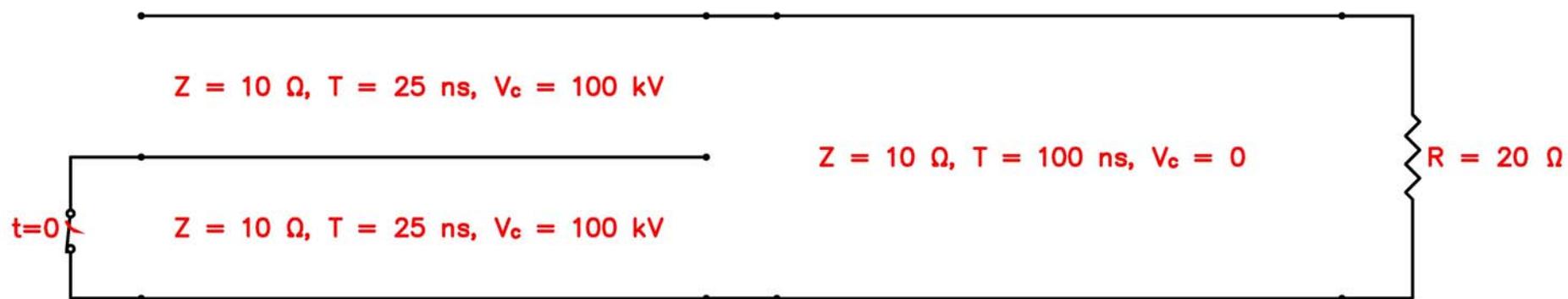


Final Exam: Solution to #4

- This was intended to be a variation on the transmission line reflection and transmission problems we have examined throughout the class, with the addition of the Blumlein.
- Full credit was awarded for recognizing
 - There is a transit-time delay between closing the Blumlein switch and the delivery of a pulse to the output (25 ns).
 - The amplitude of the incident wave (same as output into a matched load) for the Blumlein (T1) is the same as the charge voltage (100 kV)
 - The Blumlein impedance twice the impedance of the individual lines and hence there is a mismatch at the 100 ns transmission line (T2)
 - $V_T/V_I = (2 R) / (R + Z_O) = (2)(10) / (10 + 20) = 2/3$
 - $V_R/V_I = (R - Z_O) / (R + Z_O) = (10 - 20) / (10 + 20) = -1/3$
 - There is a 100 ns propagation delay down the transmission line
 - There is a mismatch at the load
 - $V_T/V_I = (2 R) / (R + Z_O) = (2)(20) / (20 + 10) = 4/3$



Final Exam: Solution to #4 (cont.)





Final Exam: Solution to #4 (cont.)

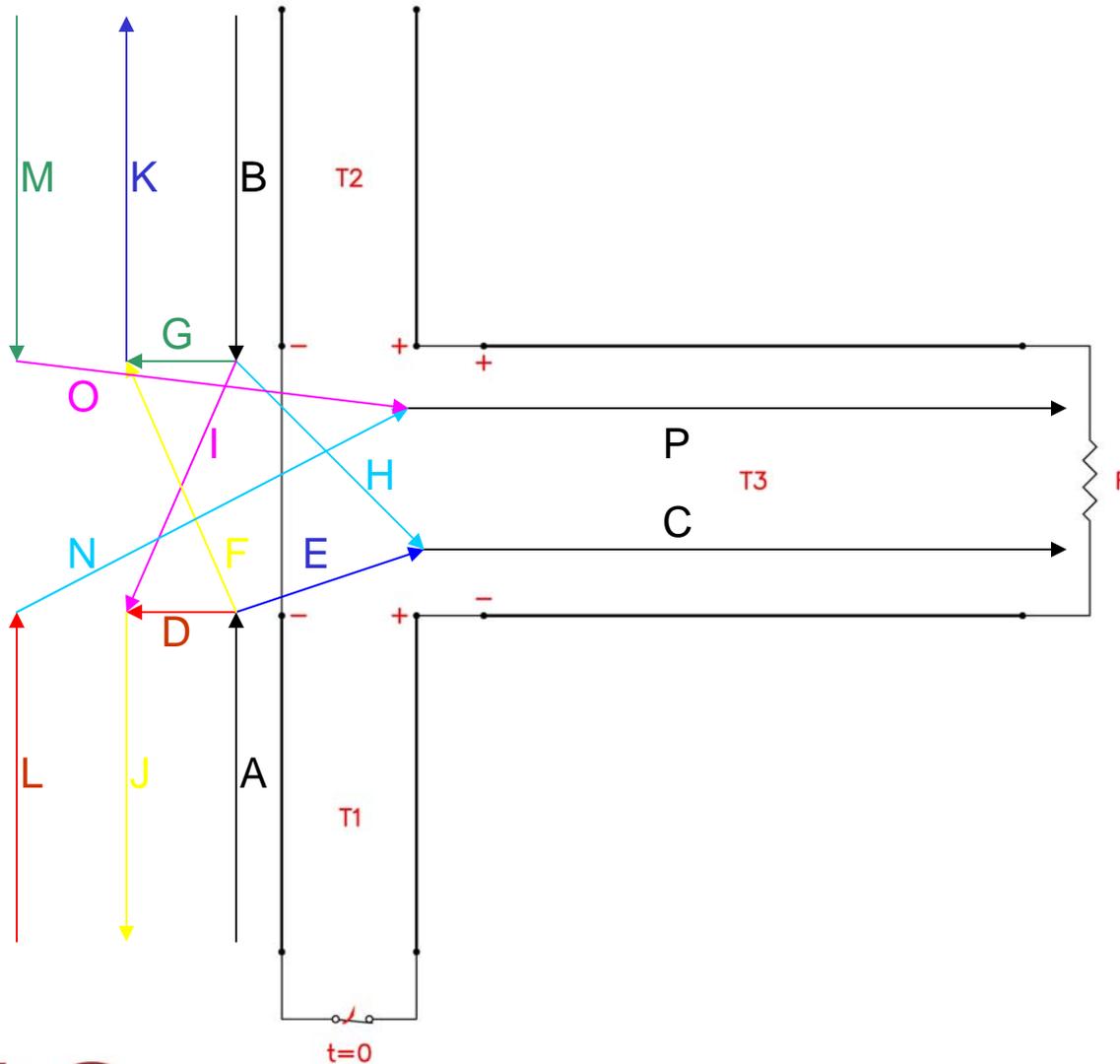
- A is the incident wave from the Blumlein
 - 100 kV amplitude
 - Arrives at T1-T2 junction at $t = 25$ ns
 - 50 ns duration
- B is the reflection of A back into T1
 - $(-1/3)(100$ kV) amplitude
 - Arrives at the “upstream end” of T1 at $t = 50$ ns
 - 50 ns duration
- C is the transmission of A into T2
 - $(2/3)(100$ kV) amplitude
 - Arrives at the load at $t = 125$ ns
 - 50 ns duration
 - Delivers a voltage of $(4/3)(2/3)(100$ kV) to the load, 125 ns $\leq t \leq 175$ ns
 - Reflection from the load won't return to load until 325 ns, after solution period



Final Exam: Solution to #4 (cont.)

- D is the reflection of B at the “upstream end” T1
 - Amplitude
 - If the upstream end of the Blumlein is assumed to be open: $(-1/3)(100 \text{ kV})$
 - If the upstream end of the Blumlein is assumed to be shorted: $(1/3)(100 \text{ kV})$
 - Arrives at T1-T2 at $t = 75 \text{ ns}$
 - 50 ns duration
- E is the transmission of D into T2
 - $(2/3)(\pm 1/3)(100 \text{ kV})$ amplitude
 - Arrives at the load at $t = 175 \text{ ns}$
 - 50 ns duration
 - Load voltage of $(4/3)(\pm 1/3)(2/3)(100 \text{ kV})$, $175 \text{ ns} \leq t \leq 225 \text{ ns}$
 - Reflection from the load won't return to load until after solution period
- The apparent ambiguity in translating B to D indicates the deficiency in this simple model

Final Exam: Solution to #4 (in all its bloody glory)





Final Exam: #4 Transmission & Reflection

- T1
 - $V_T/V_I = (2R) / (R + Z_O) = (2)(10 + 10) / (10 + 10 + 10) = 4/3$
 - $V_T/V_I(T2) = (1/2) V_T/V_I = 2/3$
 - $V_T/V_I(T3) = (-1/2) V_T/V_I = -2/3$ (“sense” of 2 lines reversed)
 - $V_R/V_I = (R - Z_O) / (R + Z_O) = (10 + 10 - 10) / (10 + 10 + 10) = 1/3$
- T2
 - $V_T/V_I = (2)(10 + 10) / (10 + 10 + 10) = 4/3$
 - $V_T/V_I(T3) = (1/2) V_T/V_I = 2/3$
 - $V_T/V_I(T1) = (1/2) V_T/V_I = 2/3$
 - $V_R/V_I = (10 + 10 - 10) / (10 + 10 + 10) = 1/3$
- T3
 - $V_T/V_I = (2)(10 + 10) / (10 + 10 + 10) = 4/3$
 - $V_T/V_I(T1) = (-1/2) V_T/V_I = -2/3$
 - $V_T/V_I(T2) = (1/2) V_T/V_I = 2/3$
 - $V_R/V_I = (10 + 10 - 10) / (10 + 10 + 10) = 1/3$
- Load: as before



Final Exam: #4 Wave Amplitude

- $t < 0$
 - A
 - $V_I = 50$ kV
 - Open at bottom end of T1: $V_R/V_I = 1$, $A = J = 50$ kV, ($V_{\text{charge}} = V_R + V_I = 100$ kV)
 - Junction with T2 & T3
 - $V_T/V_I(T2) = 2/3$: $V_T(T2) = 100/3$ kV = F
 - $V_T/V_I(T3) = -2/3$: $V_T(T3) = -100/3$ kV = E
 - $V_R/V_I = 1/3$: $V_R(T1) = 50/3$ kV = D
 - B
 - $V_I = 50$ kV
 - Open at top end of T2: $V_R/V_I = 1$, $B = K = 50$ kV, ($V_{\text{charge}} = V_R + V_I = 100$ kV)
 - Junction with T3 & T1
 - $V_T/V_I(T3) = 2/3$: $V_T(T3) = 100/3$ kV = H
 - $V_T/V_I(T1) = 2/3$: $V_T(T1) = 100/3$ kV = I
 - $V_R/V_I = 1/3$: $V_R(T2) = 50/3$ kV = G
 - C
 - $V_R = 0$ kV (T3 not charged)
 - Waves launched down lines from T1-T2-T3 Junction
 - $J = D + I = 50/3 + 100/3 = 50$ kV (as per our “simple” Blumlein model)
 - $K = G + F = 50/3 + 100/3 = 50$ kV (as per our “simple” Blumlein model)
 - $C = E + H = -100/3 + 100/3 = 0$ kV (as per our “simple” Blumlein model)



Final Exam: #4 Wave Amplitude

- $0 \leq t < 25$ ns
 - J
 - $V_I = 50$ kV
 - Switch closure shorts bottom end of T1: $V_R/V_I = -1$
 - $A = -J = -50$ kV, ($V_{T1} = A + J = 0$ kV, shorted)
 - All else unchanged: knowledge of closure of switch cannot propagate faster than wave A



Final Exam: #4 Wave Amplitude

- $25 \text{ ns} \leq t < 75 \text{ ns}$
 - A
 - $V_I = -50 \text{ kV}$
 - Junction with T2 & T3
 - $V_T/V_I(T2) = 2/3: V_T(T2) = -100/3 \text{ kV} = F$
 - $V_T/V_I(T3) = -2/3: V_T(T3) = +100/3 \text{ kV} = E$
 - $V_R/V_I = 1/3: V_R(T1) = -50/3 \text{ kV} = D$
 - B
 - $V_I = 50 \text{ kV}$
 - Junction with T3 & T1
 - $V_T/V_I(T3) = 2/3: V_T(T3) = 100/3 \text{ kV} = H$
 - $V_T/V_I(T1) = 2/3: V_T(T1) = 100/3 \text{ kV} = I$
 - $V_R/V_I = 1/3: V_R(T2) = 50/3 \text{ kV} = G$
 - Waves launched down lines from T1-T2-T3 Junction
 - $J = D + I = -50/3 + 100/3 = 50/3 \text{ kV}$
 - $K = G + F = 50/3 - 100/3 = -50/3 \text{ kV}$
 - $C = E + H = 100/3 + 100/3 = 200/3 \text{ kV}$



Final Exam: #4 Wave Amplitude

- $50 \leq t < 100$ ns
 - J
 - $V_I = 50/3$ kV
 - $V_R/V_I = -1$
 - $L = -J = -50/3$ kV
 - K
 - $V_I = -50/3$ kV
 - $V_R/V_I = 1$
 - $M = K = -50/3$ kV



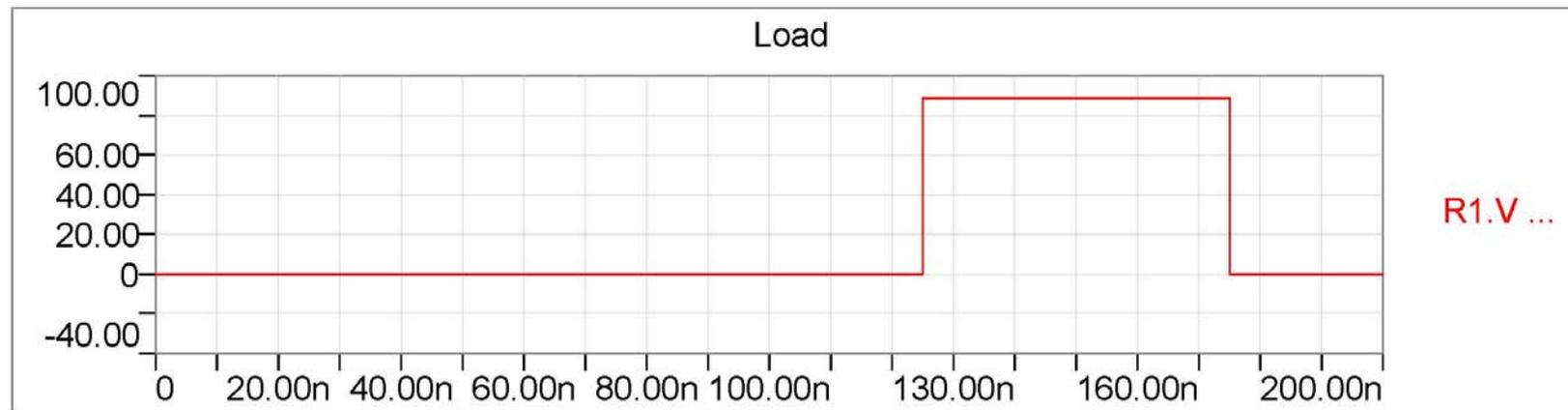
Final Exam: #4 Wave Amplitude

- $75 \text{ ns} \leq t < 125 \text{ ns}$
 - L
 - $V_I = -50/3 \text{ kV}$
 - Junction with T2 & T3
 - $V_T/V_I(T2) = 2/3: V_T(T2) = -100/9 \text{ kV}$
 - $V_T/V_I(T3) = -2/3: V_T(T3) = 100/9 \text{ kV} = N$
 - $V_R/V_I = 1/3: V_R(T1) = -50/9 \text{ kV}$
 - M
 - $V_I = -50/3 \text{ kV}$
 - Junction with T3 & T1
 - $V_T/V_I(T3) = 2/3: V_T(T3) = -100/9 \text{ kV} = O$
 - $V_T/V_I(T1) = 2/3: V_T(T1) = -100/9 \text{ kV}$
 - $V_R/V_I = 1/3: V_R(T2) = -50/9 \text{ kV}$
 - Waves launched down lines from T1-T2-T3 Junction (that can reach the load prior to 200 ns)
 - $P = N + O = 100/9 - 100/3 = 0 \text{ kV}$



Final Exam: #4 Solution

- Have wave C = 200/3 kV launched down T3 $25 \text{ ns} \leq t < 75 \text{ ns}$ and wave P = 0 kV launched down T3 $75 \text{ ns} \leq t < 125 \text{ ns}$ to apply voltage to load.
- Propagation delay down T3 is 100 ns
- Transmission coefficient from T3 to R is 4/3
- V_{load}
 - 0 kV: $0 \leq t < 125 \text{ ns}$
 - $(4/3)(200/3) = 800/9 \text{ kV}$: $125 \leq t < 175 \text{ ns}$
 - 0 kV: $175 \text{ ns} \leq t < 200 \text{ ns}$





Final Exam: #4 Extended solution

- Even though the load voltage is at zero at the end of the 200 ns solution period, there are still waves bouncing through the system that will deposit more energy to the load
- Solution plotted to 400 ns for load and open end of T2 (shown below T1). Open on T2 represented by 1 GΩ and short on T1 by 1 mΩ.

