Interferometers and Spectrometers

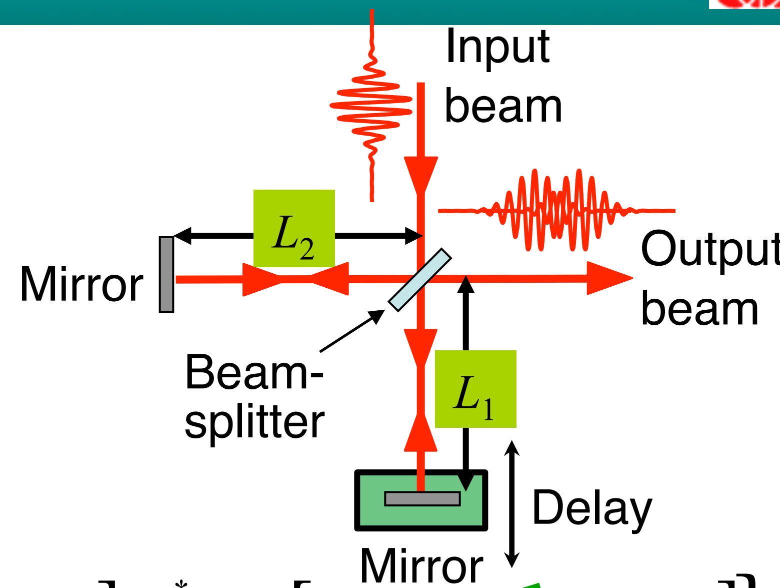
John Byrd

Overview

- The Michelson Interferometer
- Martin Puplett configuration
- Grating spectrometers

The Michelson Interferometer

The Michelson Interferometer splits a beam into two and then recombines them at the same beam splitter.



Suppose the input beam is a plane wave:

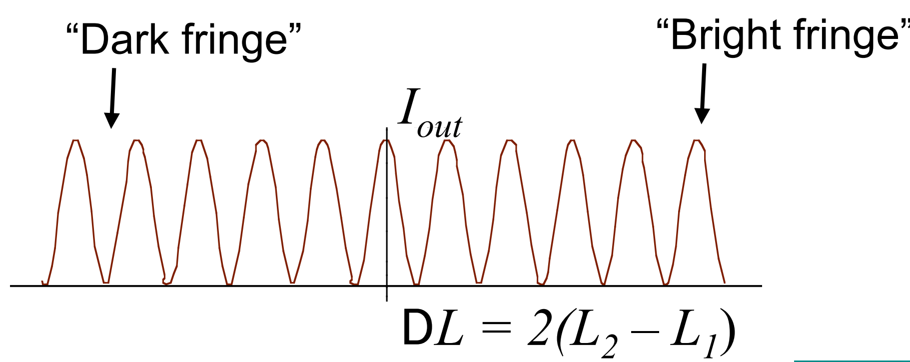
$$I_{out} = I_1 + I_2 + c\varepsilon \,\mathrm{Re}\left\{E_0 \exp\left[i(\cancel{\omega t - kz} - 2kL_1)\right] E_0^* \exp\left[-i(\cancel{\omega t - kz} - 2kL_2)\right]\right\}$$

$$= I + I + 2I\,\mathrm{Re}\left\{\exp\left[2ik(L_2 - L_1)\right]\right\}$$ since $I \equiv I_1 = I_2 = (c\varepsilon_0 / 2)\left|E_0\right|^2$

$$= 2I\left\{1 + \cos(k\Delta L)\right\}$$

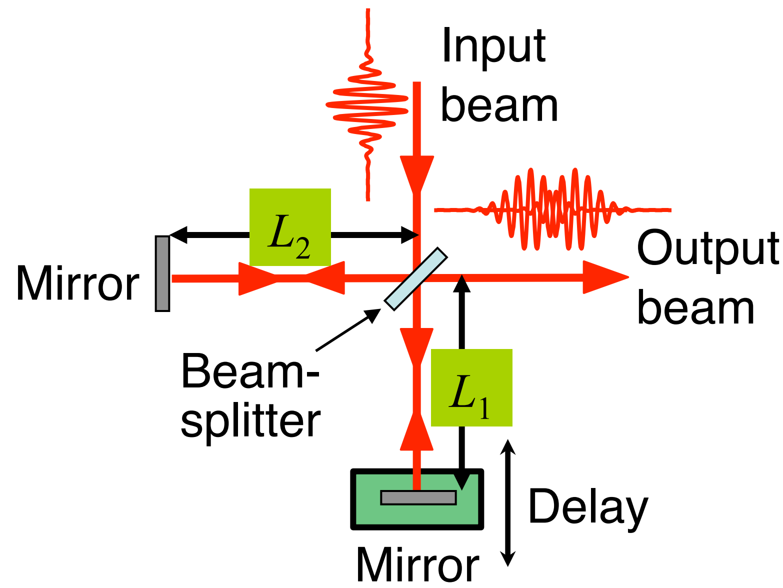
where: D$L = 2(L_2 - L_1)$

Fringes (in delay):

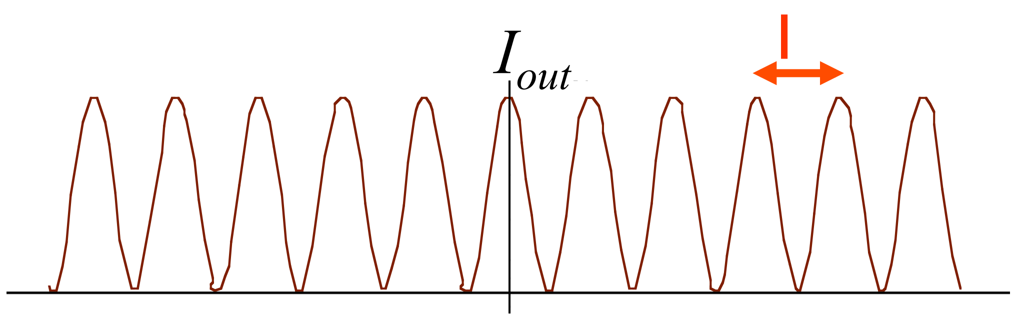


The Michelson Interferometer

The most obvious application of the Michelson Interferometer is to measure the wavelength of monochromatic light.

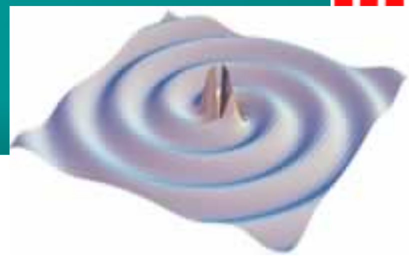


$$I_{out} = 2I\left\{1+\cos(k\Delta L)\right\} = 2I\left\{1+\cos(2\pi\, \Delta L/\lambda)\right\}$$



D$L = 2(L_2 - L_1)$

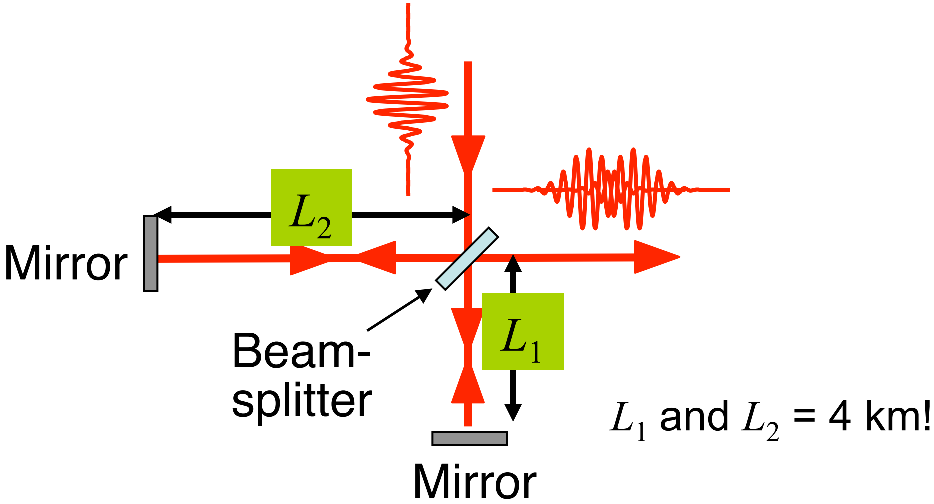
Huge Michelson Interferometers may someday detect gravity waves.



Gravity waves (emitted by all massive objects) ever so slightly warp space-time. Relativity predicts them, but they've never been detected.

Supernovae and colliding black holes emit gravity waves that may be detectable.

Gravity waves are "quadrupole" waves, which stretch space in one direction and shrink it in another. They should cause one arm of a Michelson interferometer to stretch and the other to shrink.



Unfortunately, the relative distance (L_1-$L_2 \sim 10^{-16}$ cm) is less than the width of a nucleus! So such measurements are very very difficult!

The LIGO project

The building
containing an arm

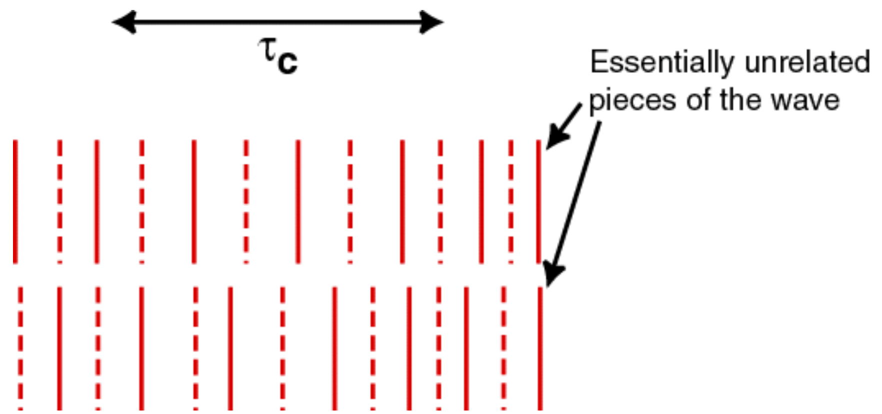
A small fraction of
one arm of the
CalTech LIGO
interferometer…

Hanford LIGO

The control center

Interference of white beams

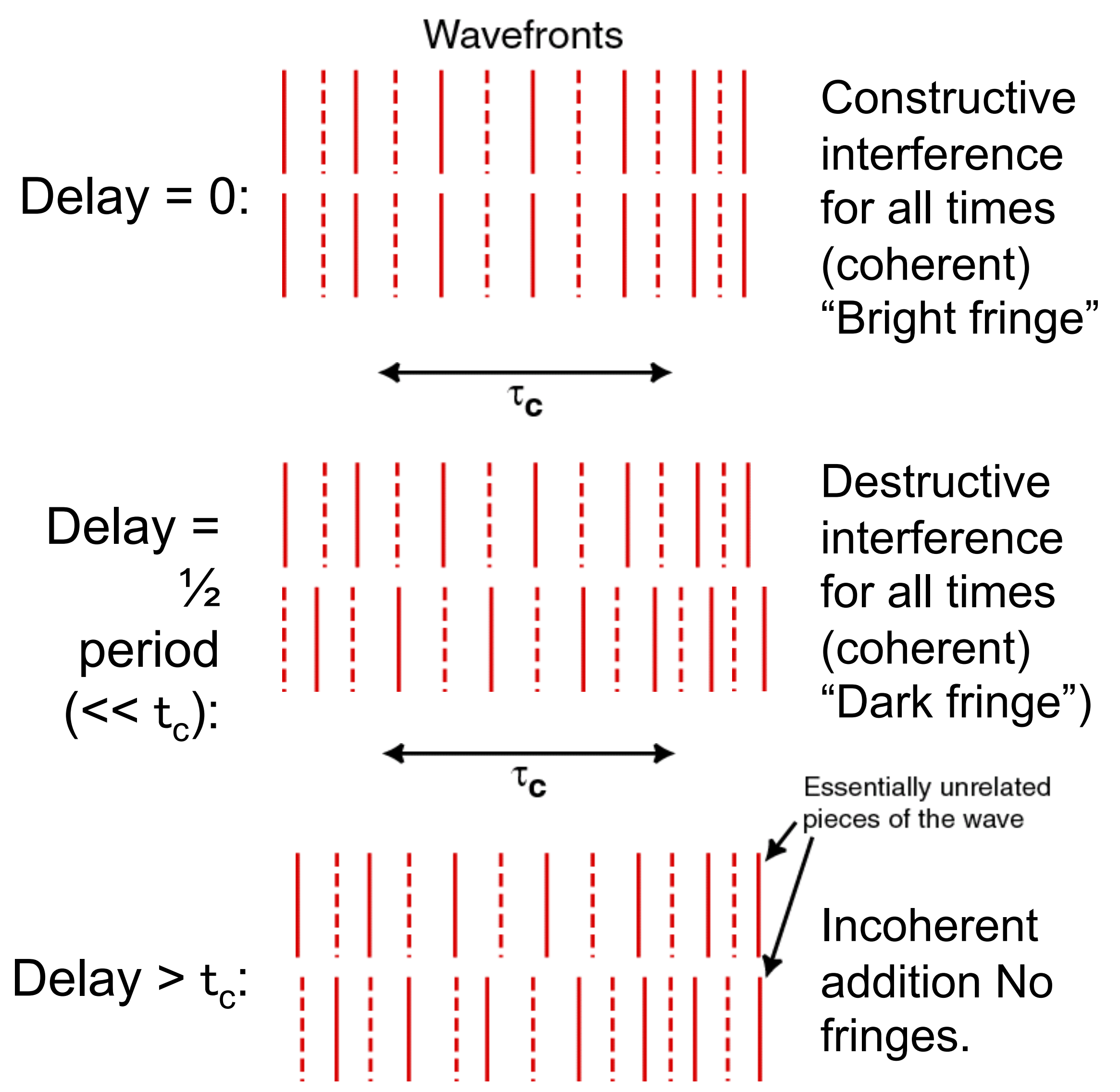
For perfect sine waves, the two beams are either in phase or they're not. What about a beam with a short coherence time????



The beams could be in phase some of the time and out of phase at other times, varying rapidly.

Remember that most optical measurements take a long time, so these variations will get averaged.

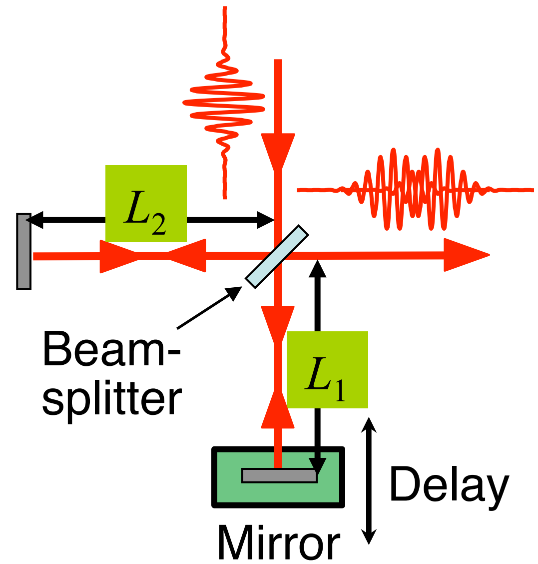
Adding a non-monochro-matic wave to a delayed replica of itself



The Michelson Interferometer is a Fourier Transform Spectrometer

Suppose the input beam is **not monochromatic** (but is perfectly spatially coherent):

$$\Rightarrow \quad I_{out} = 2I + c\,\varepsilon\,\mathrm{Re}\{E(t+2L_1/c)\,E^*(t+2L_2/c)\}$$



Now, I_{out} will vary rapidly in time, and most detectors will simply integrate over a relatively long time, T:

$$U \propto \int_{-T/2}^{T/2} I_{out}(t)\,dt \quad \Rightarrow \quad U \propto 2IT + c\varepsilon\,\mathrm{Re}\int_{-T/2}^{T/2} E(t+2L_1/c)E^*(t+2L_2/c)\,dt$$

Changing variables: $t' = t + 2L_1/c$ and letting $\tau = 2(L_2 - L_1)/c$ and $T \to \infty$

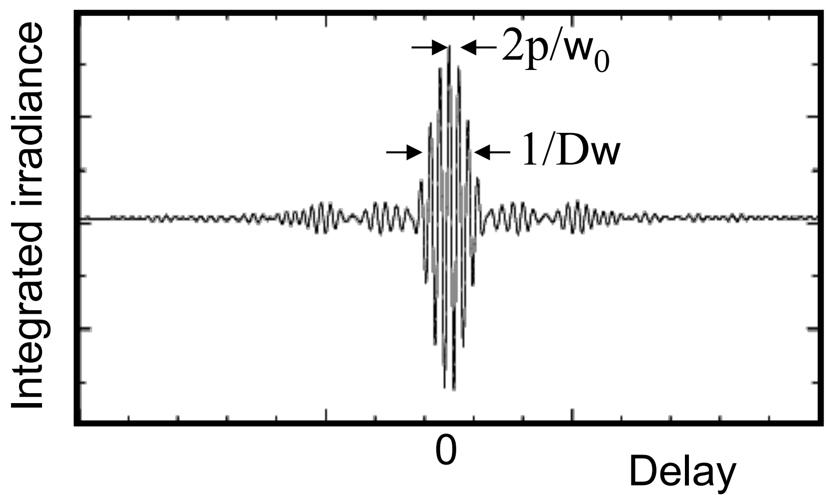
$$U \propto 2IT + c\varepsilon\,\mathrm{Re}\int_{-\infty}^{\infty} E(t')E^*(t'-\tau)\,dt'$$ **The Field Autocorrelation!**

The Fourier Transform of the Field Autocorrelation is the spectrum!!

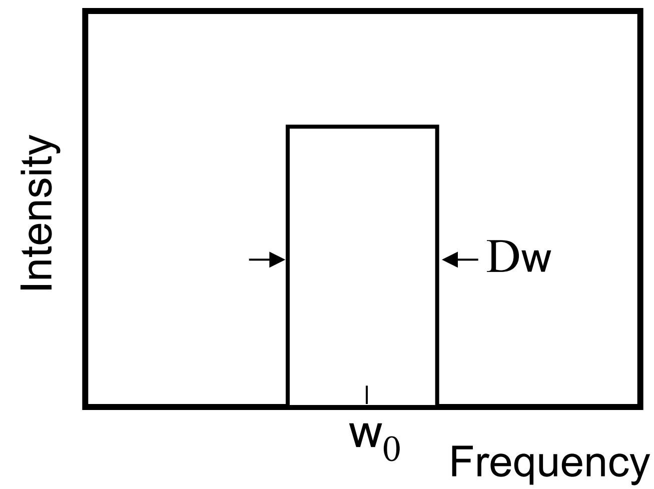
Fourier Transform Spectrometer Interferogram

A Fourier Transform Spectrometer's detected light energy vs. delay is called an interferogram.

Michelson interferometer integrated irradiance



Spectrum



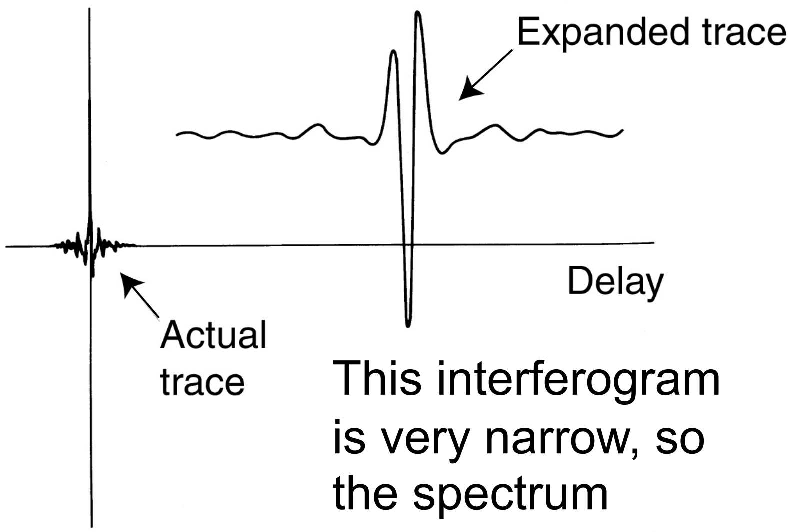
The Michelson interferometer output—the interferogram—Fourier transforms to the spectrum.

The spectral phase plays no role! (The temporal phase does, however.)

Fourier Transform Spectrometer Data

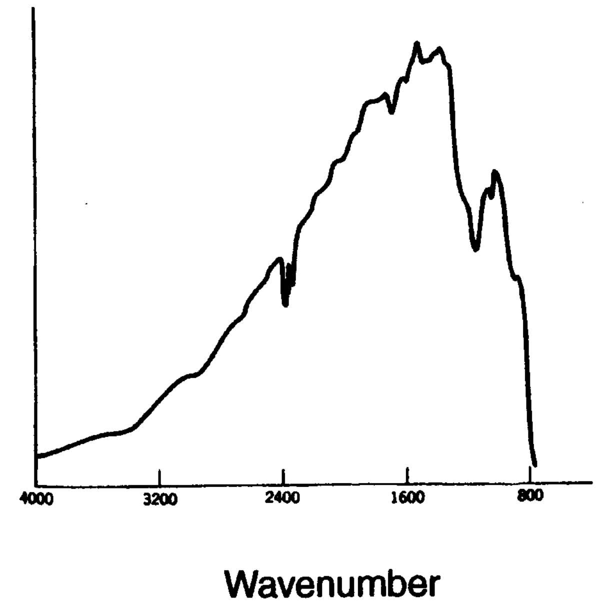
Actual interferogram from a Fourier Transform Spectrometer

Interferogram



This interferogram is very narrow, so the spectrum is very broad.

Spectrum



Fourier Transform Spectrometers are most commonly used in the infrared where the fringes in delay are most easily generated. As a result, they are often called FTIR's.

Fourier Transform Spectrometers

Maximum path difference: 1 m

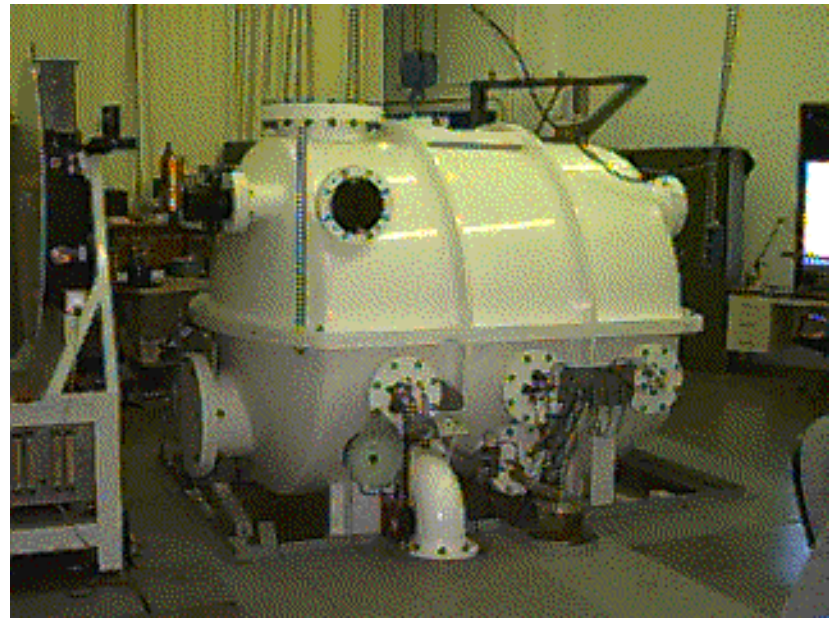
Minimum resolution: 0.005 /cm

Spectral range: 2.2 to 18 mm

Accuracy: 10^{-3} /cm to 10^{-4} /cm

Dynamic range: 19 bits (5 x 10^{5})

National Solar Observatory



A compact commercial FT spectrometer from Nicolet

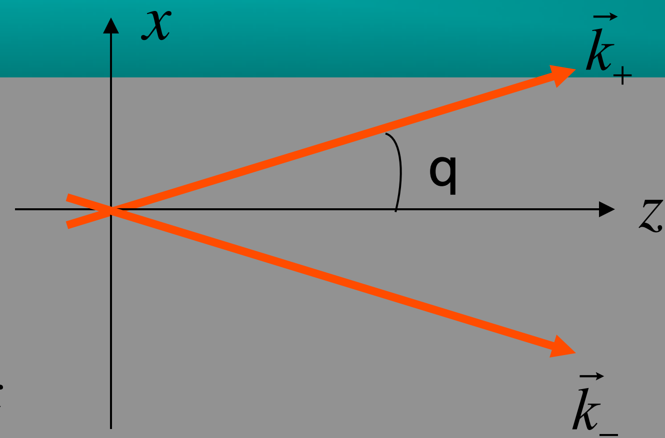
Fourier-transform spectrometers are now available for wave-lengths even in the UV! Strangely, they're still called FTIR's.

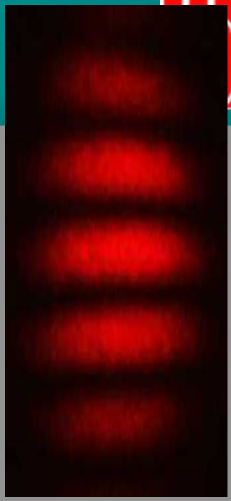
Crossed Beams

$$\vec{k}_+ = k\cos\theta\ \hat{z} + k\sin\theta\ \hat{x}$$
$$\vec{k}_- = k\cos\theta\ \hat{z} - k\sin\theta\ \hat{x}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\Rightarrow \vec{k}_+ \cdot \vec{r} = k\cos\theta\ z + k\sin\theta\ x$$
$$\vec{k}_- \cdot \vec{r} = k\cos\theta\ z - k\sin\theta\ x$$





$$I = 2I_0 + c\varepsilon\,\mathrm{Re}\left\{E_0 \exp[i(\omega t - \vec{k}_+ \cdot \vec{r})]E_0^* \exp[-i(\omega t - \vec{k}_- \cdot \vec{r})]\right\}$$

Cross term is proportional to:

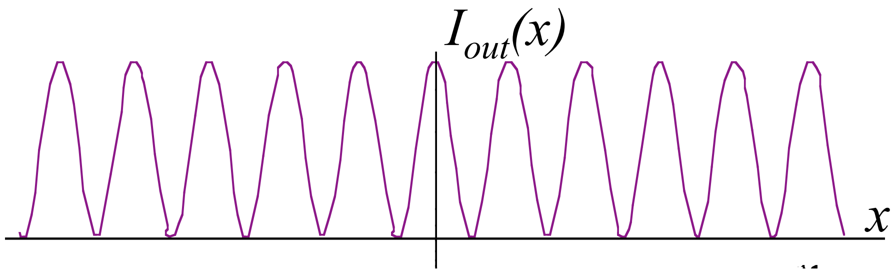
$$\mathrm{Re}\left\{E_0 \exp\left[i(\omega t - kz\cos\theta - kx\sin\theta\right]E_0^* \exp\left[-i(\omega t - kz\cos\theta + kx\sin\theta\right]\right\}$$

$$\propto \mathrm{Re}\left\{\left|E_0\right|^2 \exp\left[-2ikx\sin\theta\right]\right\}$$

$$\propto \left|E_0\right|^2 \cos(2kx\sin\theta)$$

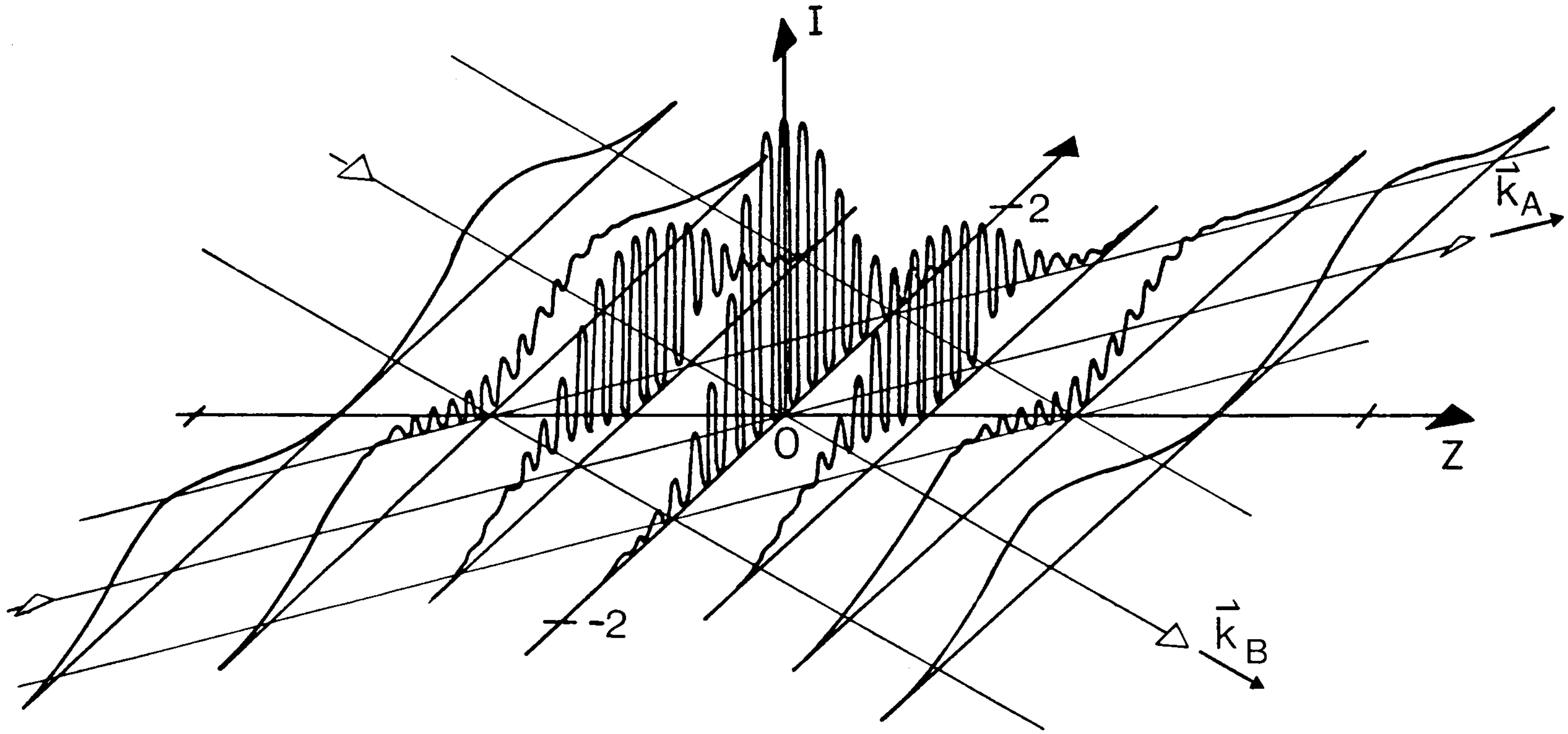
Fringe spacing: $\Lambda = 2\pi/(2k\sin\theta)$

Fringes (in position)



Irradiance vs. position for crossed beams

Irradiance fringes occur where the beams overlap in space and time.

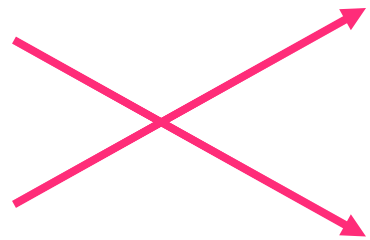


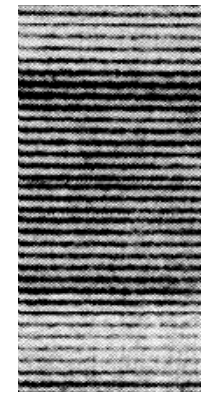
Big angle: small fringes.
Small angle: big fringes.

The fringe spacing, L:

$$\Lambda = 2\pi/(2k\sin\theta)$$
$$= \lambda/(2\sin\theta)$$

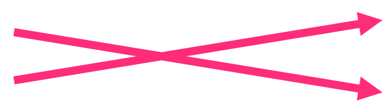
Large angle:

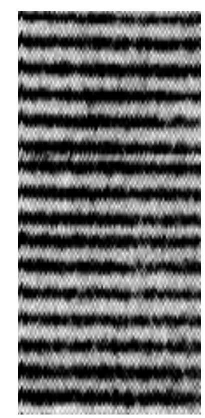




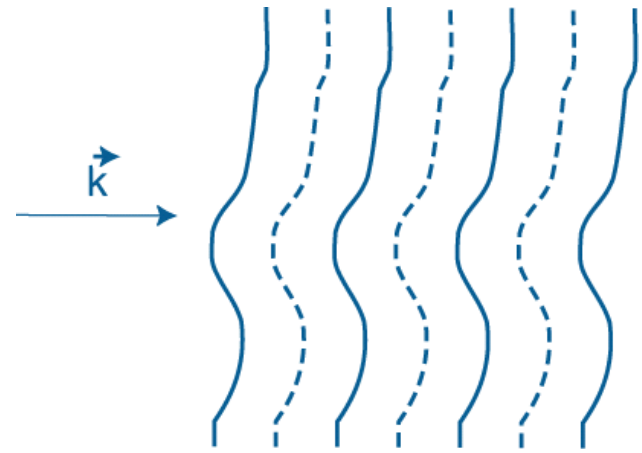
As the angle decreases to zero, the fringes become larger and larger, until finally, at q = 0, the intensity pattern becomes constant.

Small angle:

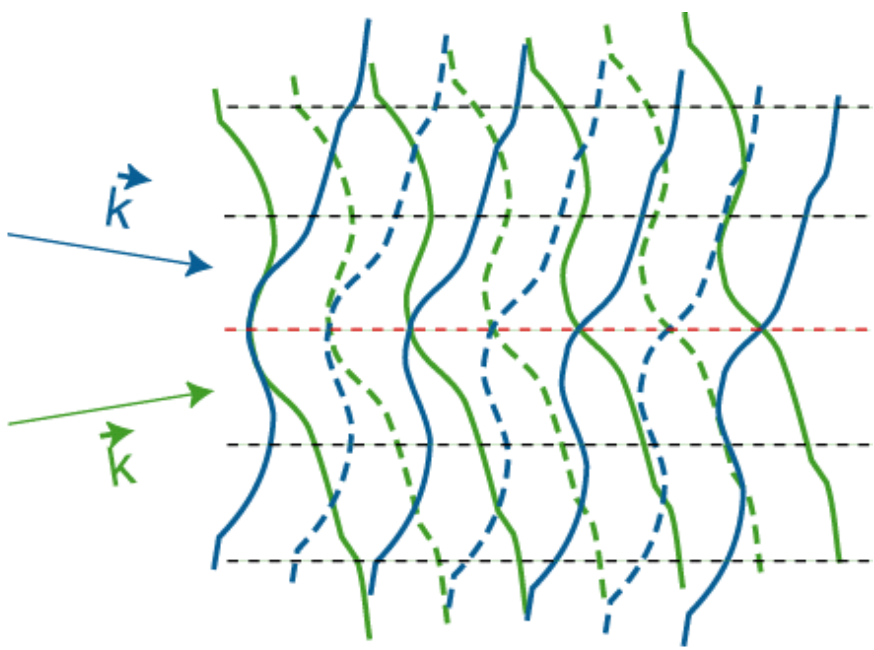




Spatial fringes and spatial coherence

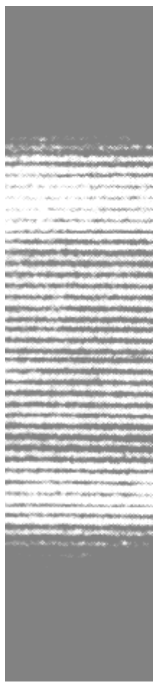


Suppose that a beam is temporally, but not spatially, coherent.

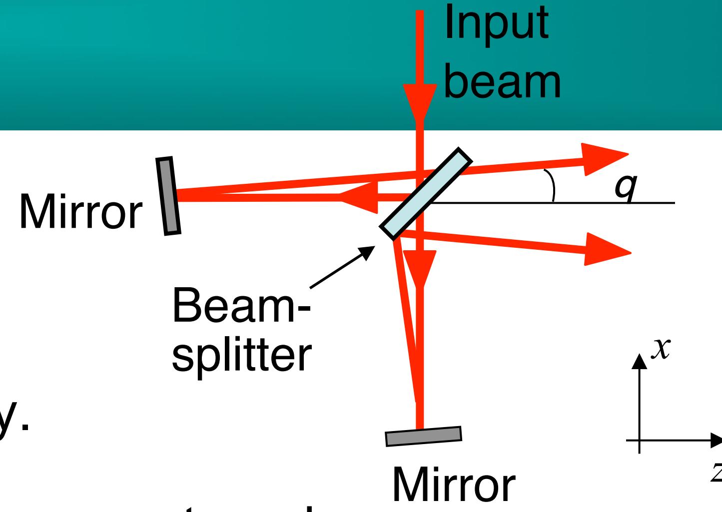


Interference is incoherent (no fringes) far off the axis, where very different regions of the wave interfere.

Interference is coherent (sharp fringes) along the center line, where same regions of the wave interfere.



The Michelson Interferometer and Spatial Fringes



Suppose we misalign the mirrors so the beams cross at an angle when they recombine at the beam splitter. And we won't scan the delay.

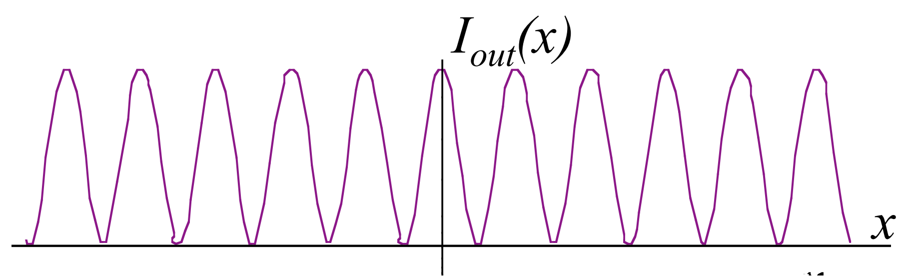
If the input beam is a plane wave, the cross term becomes:

$$\text{Re}\left\{E_0 \exp\left[i(\cancel{\omega t - kz\cos\theta} - kx\sin\theta\right] E_0^* \exp\left[-i(\cancel{\omega t - kz\cos\theta} + kx\sin\theta\right]\right\}$$

$$\propto \text{Re}\left\{\exp\left[-2ikx\sin\theta\right]\right\}$$

$$\propto \cos(2kx\sin\theta)$$

Fringes (in position)



Crossing beams maps delay onto position.

Effect of intensity variations

- Variations in the detector signal are assumed to result from interference.
- Periodic variations in the input intensity can give an apparent additional interference term.
- Beam signals are susceptible to intensity variations from changes in
  - bunch charge
  - beam motion via an aperture
  - bunch length (with CSR)

Effect of beam noise

ALS IR beamline observed unwanted peaks in FTIR spectra.
Effective spectrogram frequency changes with mirror speed.

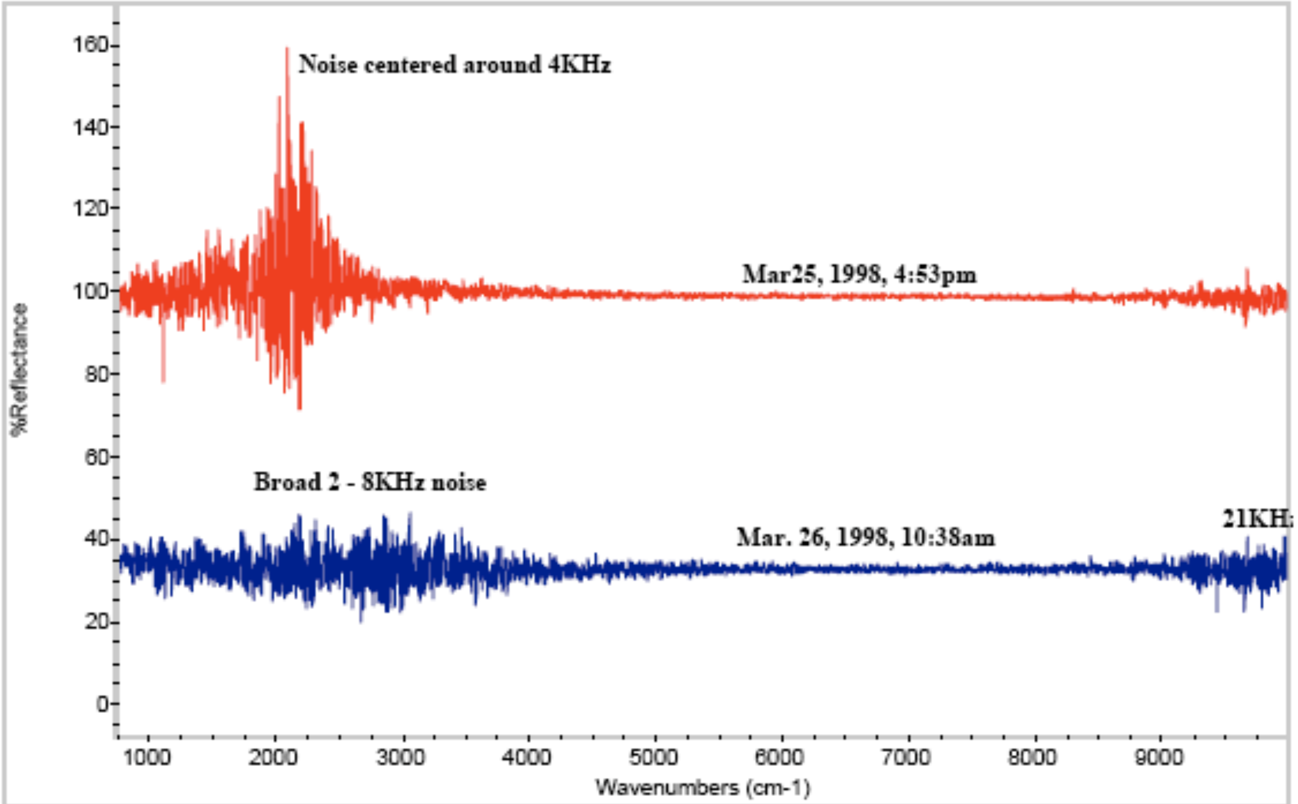
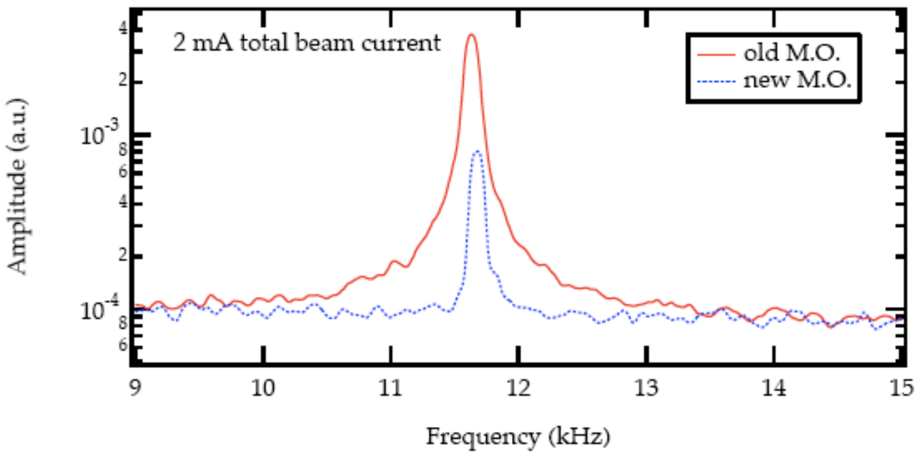
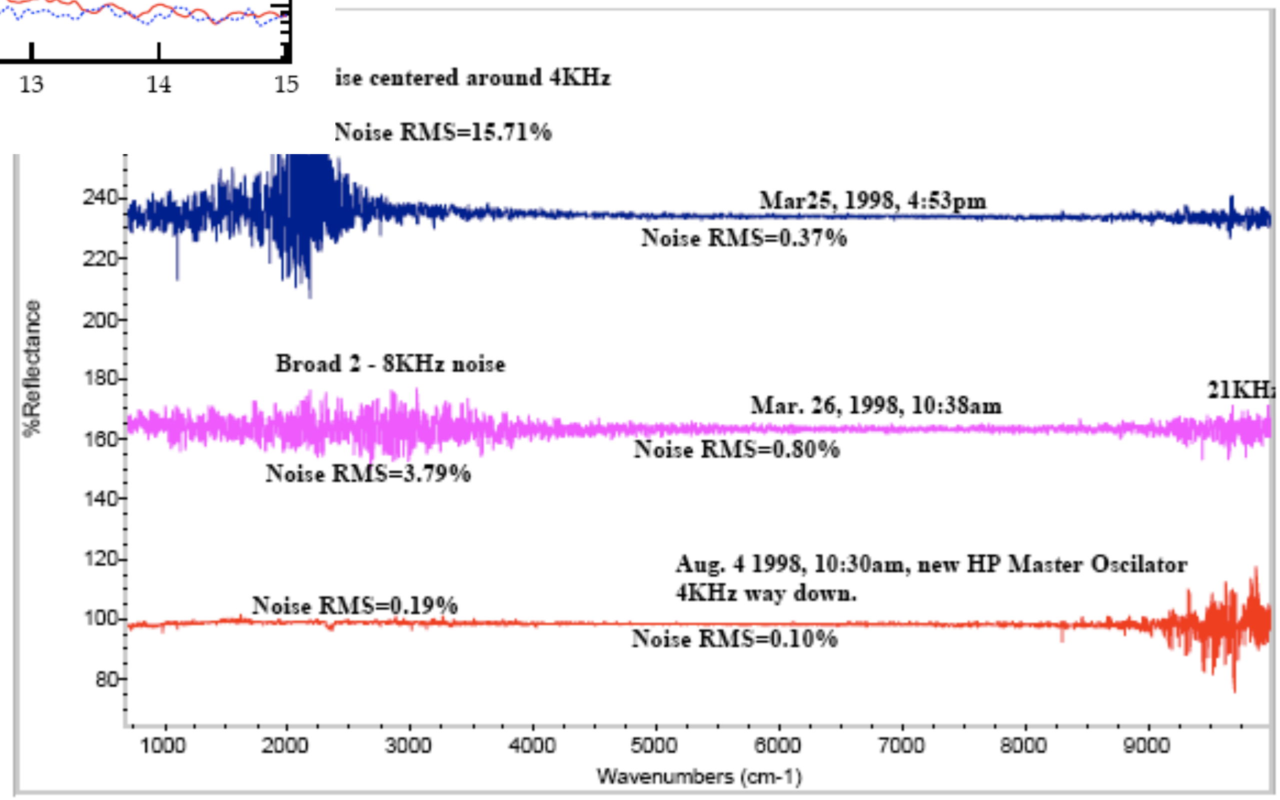


Figure 1. FTIR Spectra showing extra noise in the 2 – 8 kHz frequency regime.

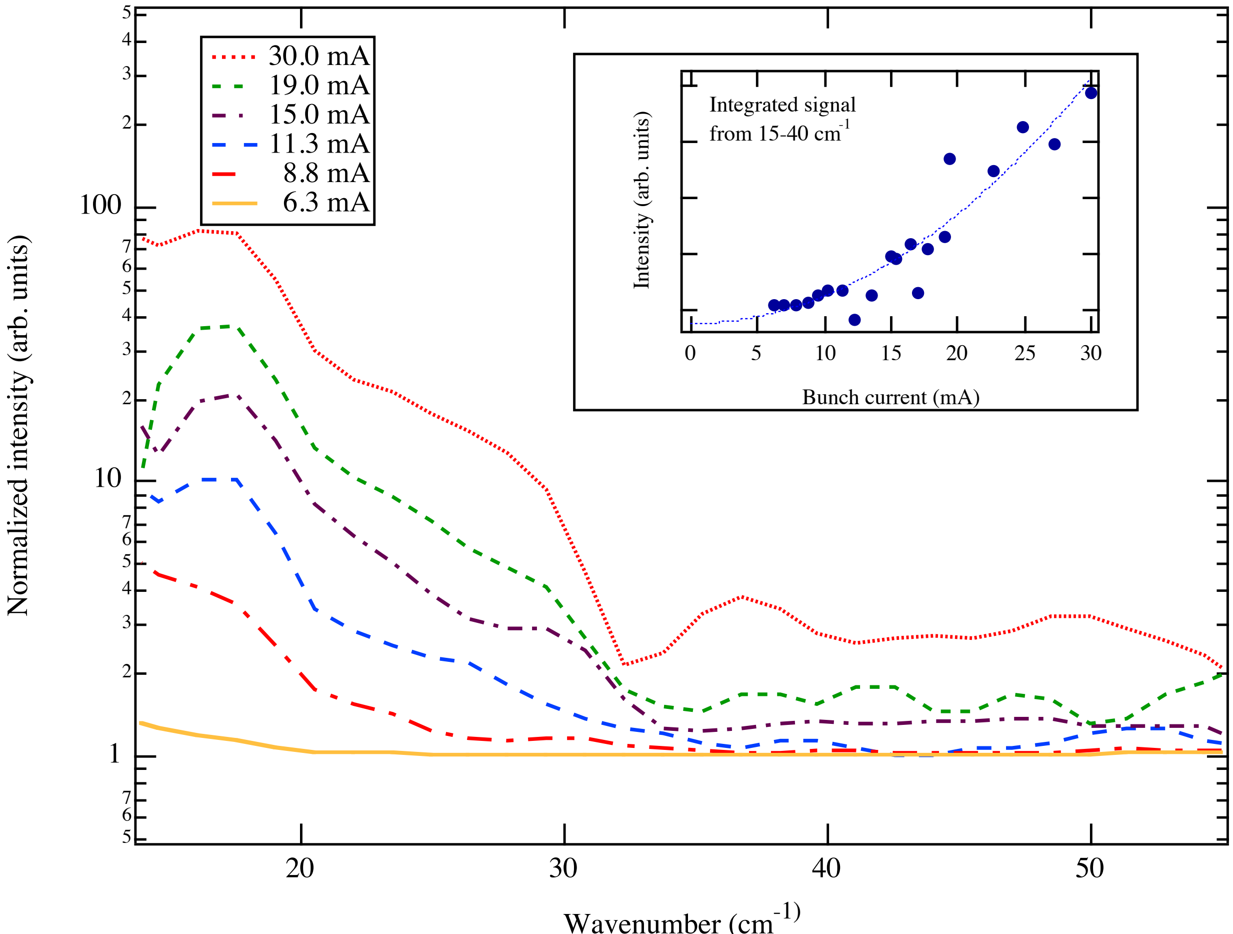
Reduced beam noise



Noise identified as synchrotron motion (Robinson mode). New master oscillator reduces motion and solves problem.



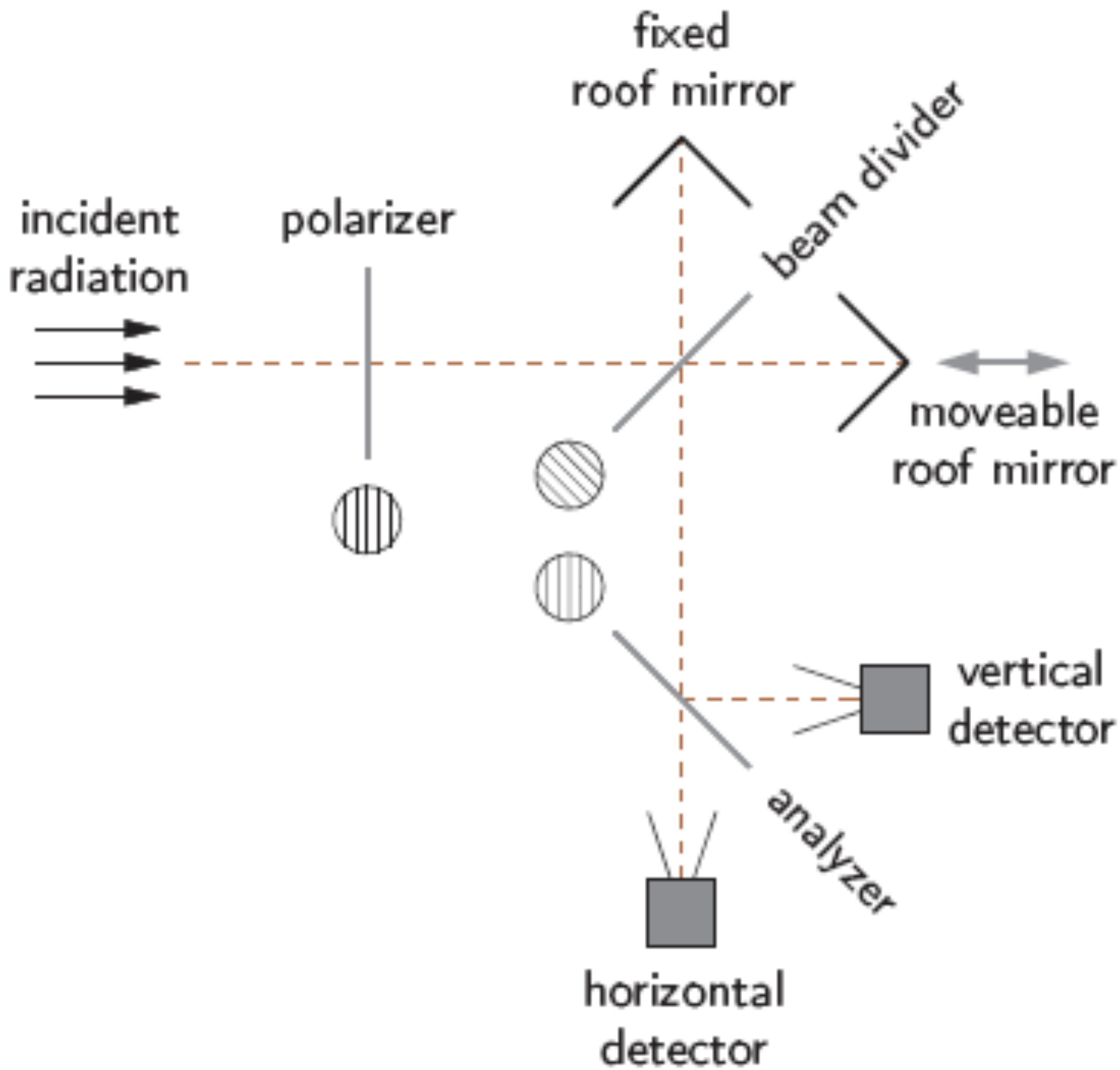
Example ALS FTIR Measurements



Signal during microbunching “bursting” instability in the ALS.

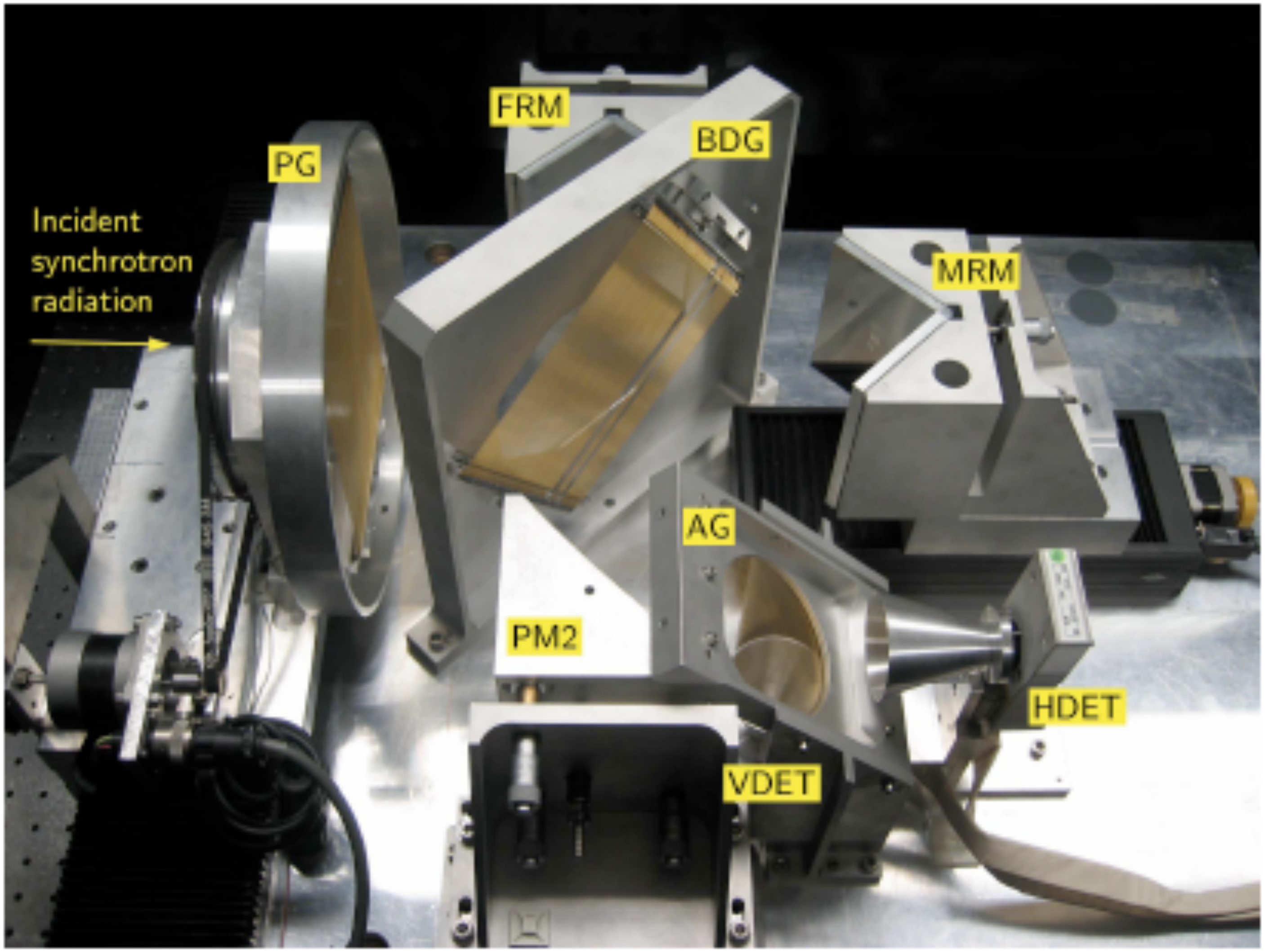
Bruker FTIR spectometer used with a Si bolometer.

Martin Puplett Configuration

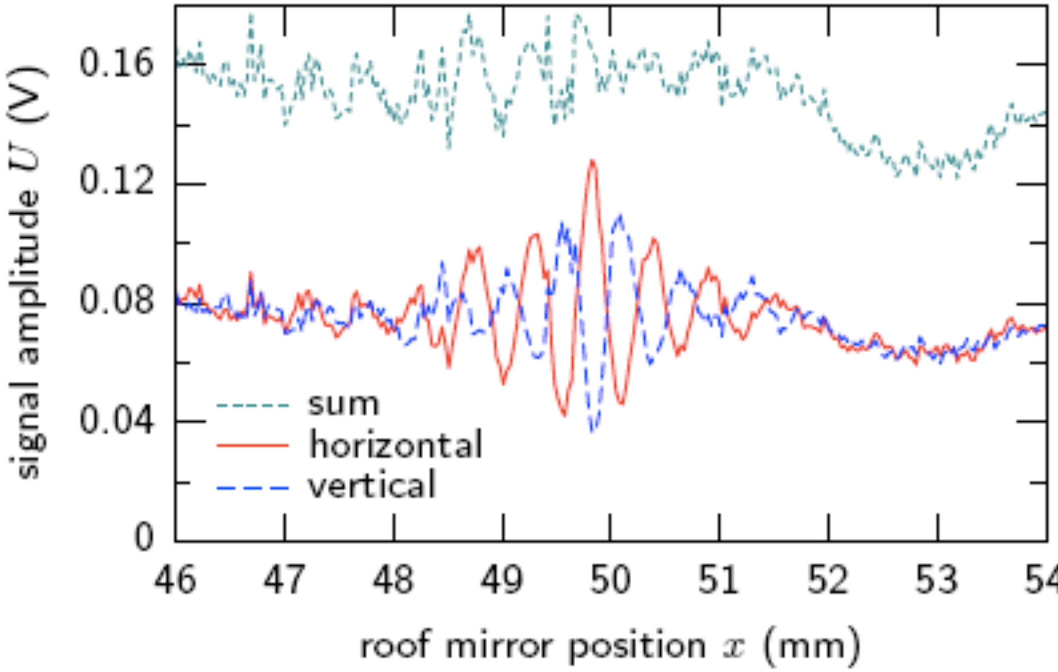


- Operates much like the M-I
- Horizontally polarized input light is split into two orthogonal 45 deg polarizations.
- Polarization is mirrored by roof mirrors, allowing transmission/reflection through splitter.
- Analyzer recombines horz and vert polarizations and detects each signal.

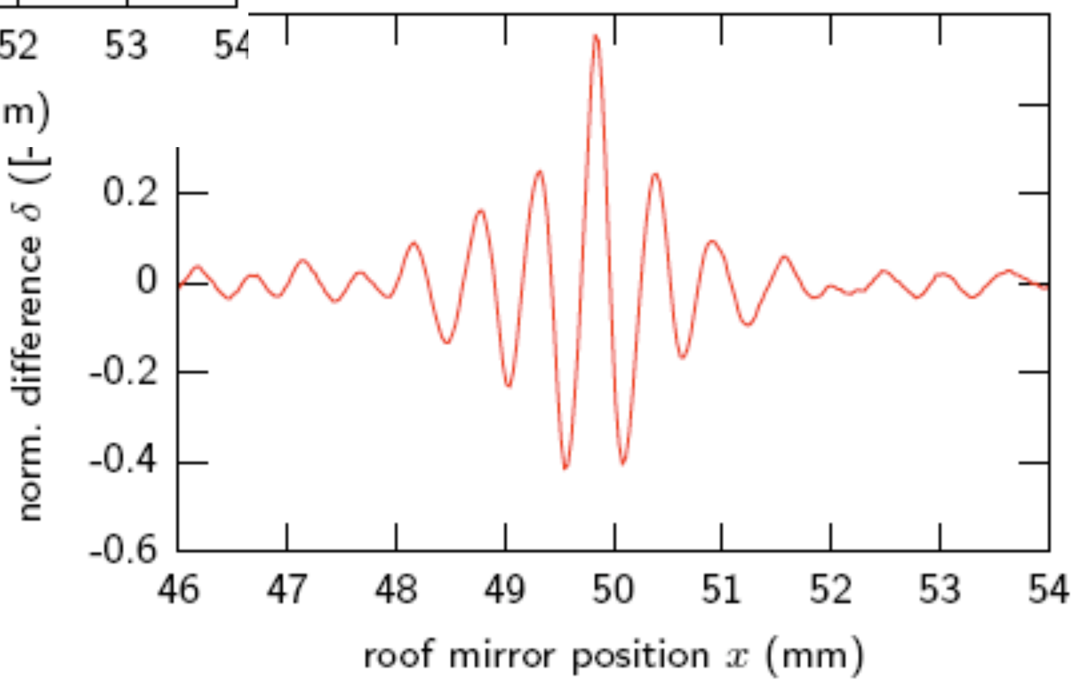
FLASH Configuration



Example measurement



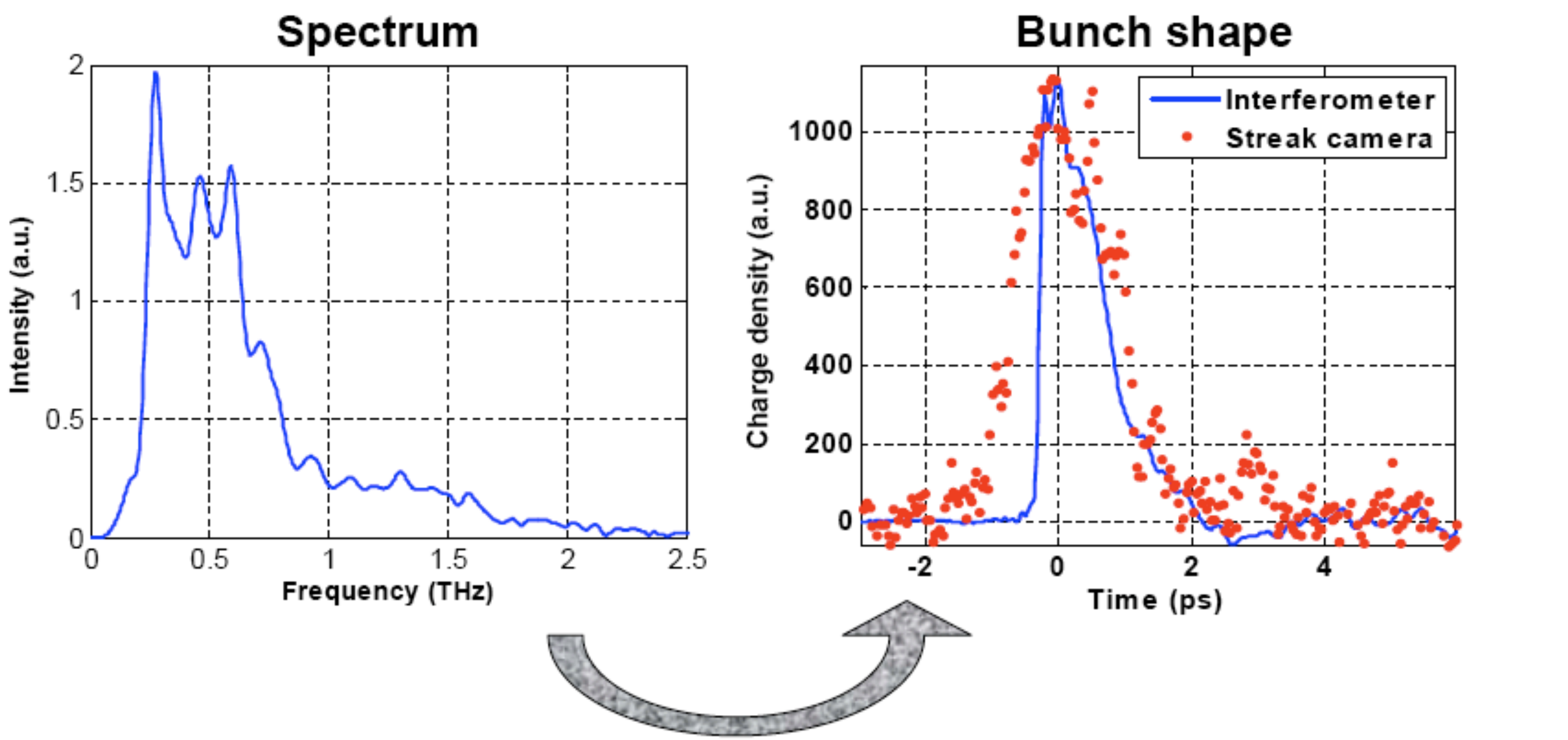
$$\delta(x) = \frac{U_h(x) - U_v(x)}{U_h(x) + U_v(x)}$$



Comparison of M-P-I and M-I

- Differential detection of two polarizations gives better S/N
- Detected intensity is equal to input intensity (past first polarizer) and provides good input signal normalization

Bunch Reconstruction



through inversion of $$\frac{dU}{d\upsilon} = \left(\frac{dU}{d\upsilon}\right)_1 \left(N + N(N-1)\left|F(\upsilon)\right|^2\right) \quad F(\upsilon) = \int S(t) e^{2\pi i \upsilon t} dt$$

Time and Frequency domain

$$\left(\frac{\mathrm{d}U}{\mathrm{d}\upsilon}\right)_{\text{measured}} \Rightarrow \frac{\mathrm{d}U}{\mathrm{d}\upsilon} \Rightarrow \left|F(\upsilon)\right| \Rightarrow F(\upsilon) \Rightarrow S(z)$$

Beam line transmission, instrument response

Single-electron spectrum, bunch charge

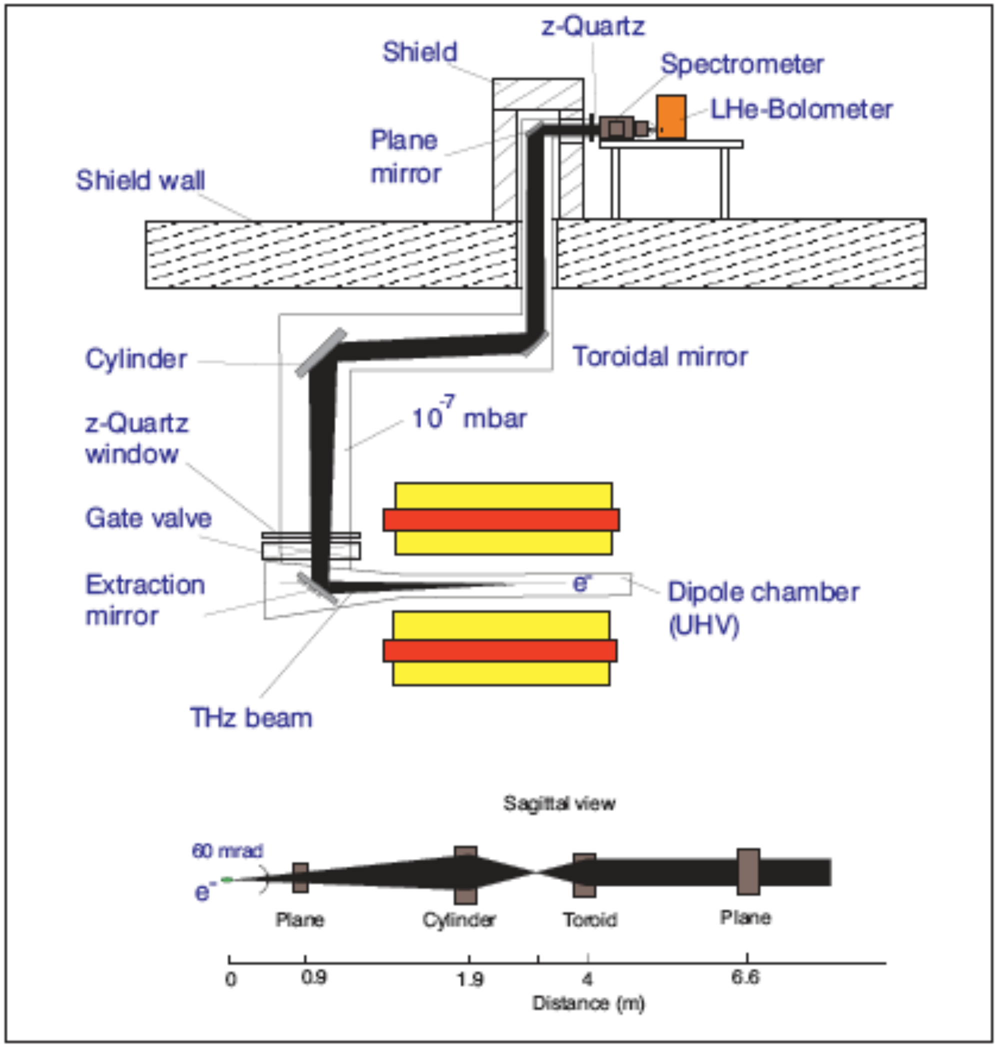
Phase reconstruction

Inverse Fourier transform

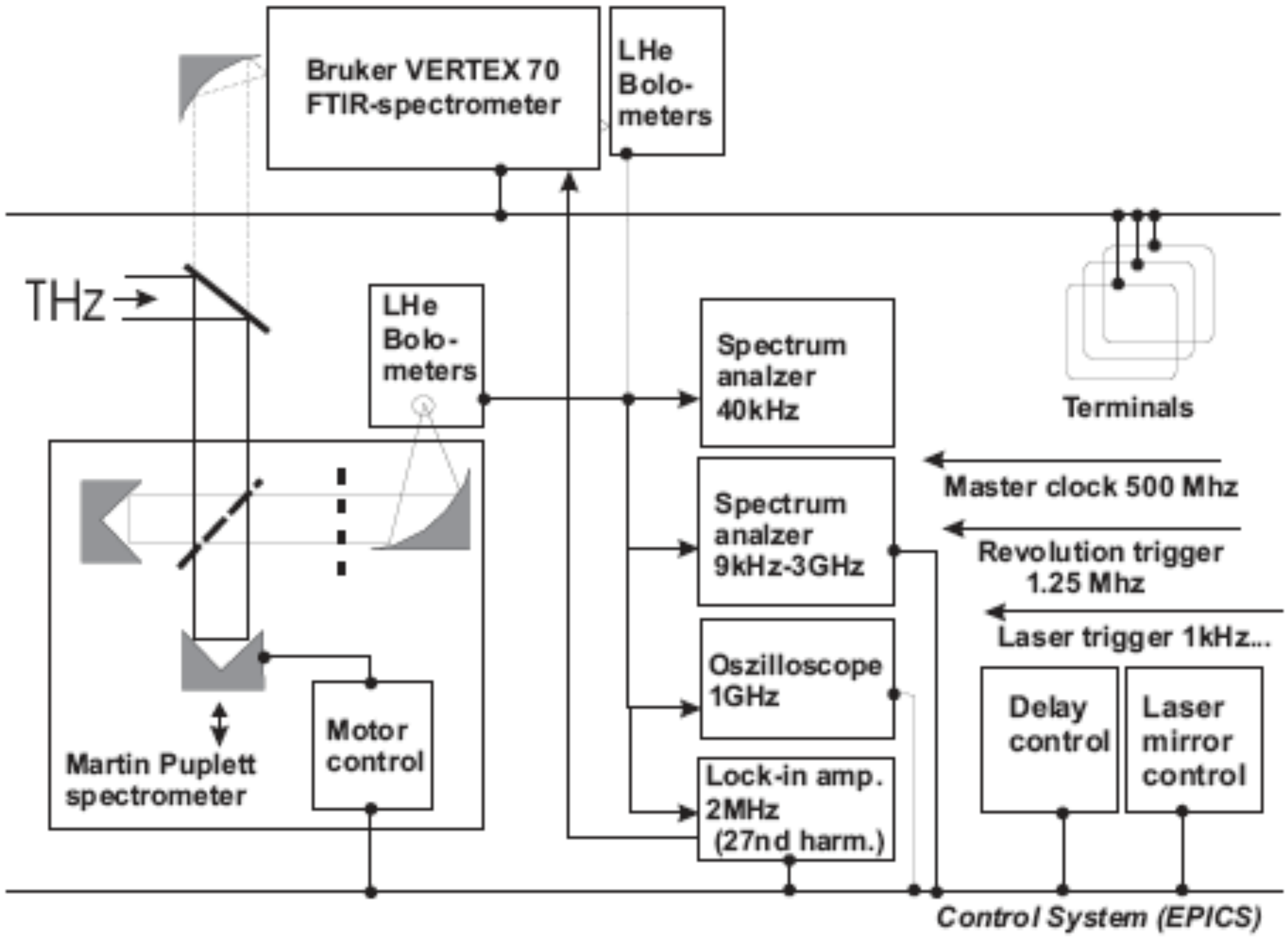
Complex form factor $F(\upsilon) = \left|F(\upsilon)\right| e^{i\Theta(\upsilon)}$

Kramers - Kronig relation (phase retrieval) $\Theta(\upsilon) \geq \frac{2\nu}{\pi} \int_0^{\infty} \frac{\ln \frac{\left|F(\upsilon')\right|}{\left|F(\upsilon)\right|}}{\upsilon^2 - \upsilon'^2} \mathrm{d}\upsilon'$

A THz beamline: BESSY



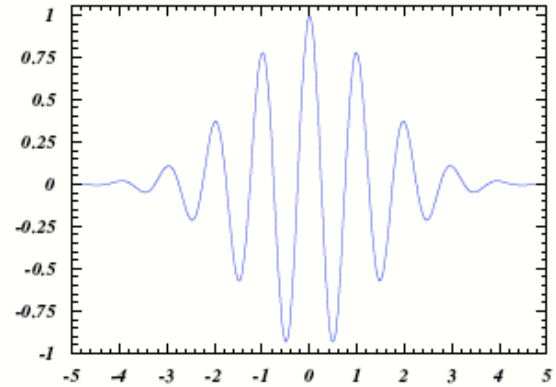
Martin-Puplett: Bessy



Bunch length via interferometer

- Simplified view: wave train of frequency ω emitted by charge distribution of RMS length σ:

$$A(t) = e^{-\left(\frac{1}{2}\frac{t^2}{\sigma^2}+i\omega t\right)}$$



- The pulse overlayed with itself shifted by a time Δ due to the Michelson interferometer is

$$A(t,\Delta) = e^{-\left(\frac{1}{2}\frac{t^2}{\sigma^2}+i\omega t\right)} + e^{-\left(\frac{1}{2}\frac{(t+\Delta)^2}{\sigma^2}+i\omega(t+\Delta)\right)}$$

- The time integrated intensity observed by the detector is therefore

$$\tilde{I}(\Delta) = \int I(\Delta)dt = \int |A(\Delta)|^2 dt \propto \cos(\omega\Delta)\, e^{-\frac{\Delta^2}{4\sigma^2}}$$

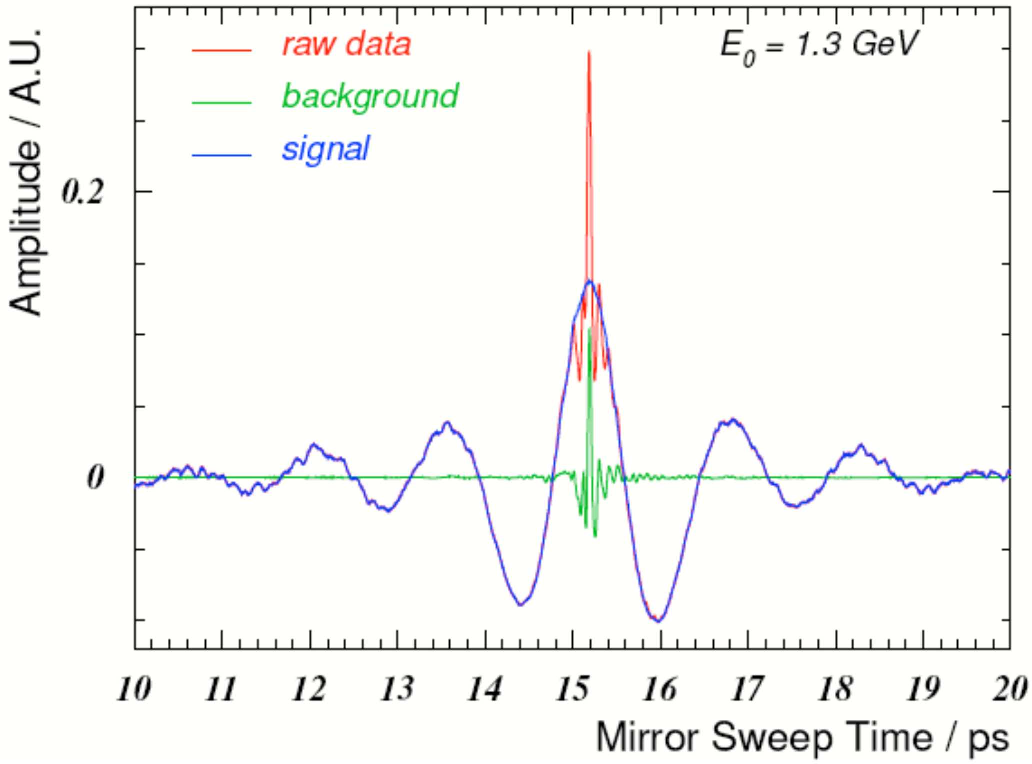
- Assumption: Since the shortest wave length emitted coherently is equal to full bunch length $\lambda_{min} = 2\sigma_w$, the max. frequency is $\omega_{max} = 2\pi c/\lambda_{min} = \pi c/\sigma_w$. It follows that

$$\tilde{I}(\Delta) \propto \cos(\omega_{max}\Delta)\, e^{-\frac{\Delta^2}{4\sigma^2}} = \cos\left(\frac{\pi}{\sigma}\Delta\right) e^{-\frac{\Delta^2}{4\sigma^2}}$$

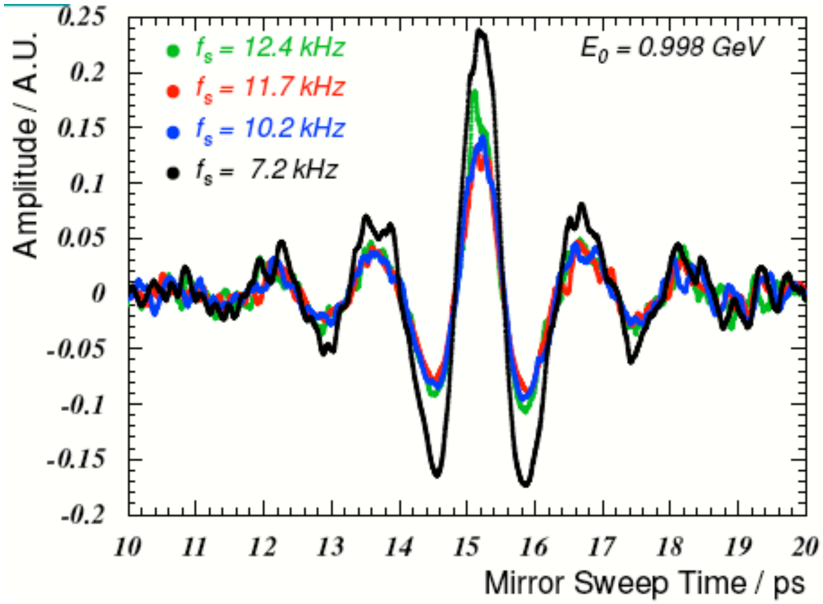
➜ Exponential doesn't change peak width: FWHM yields σ

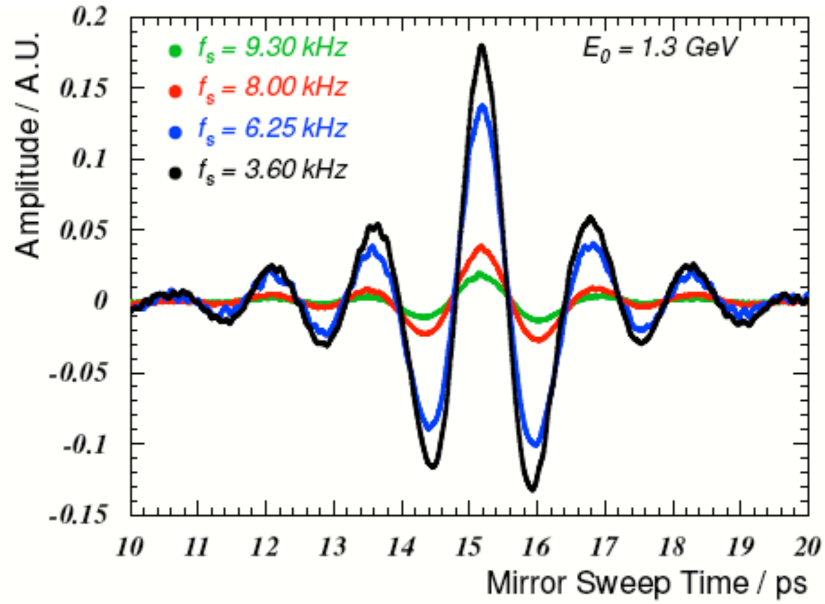
Background subtraction

- The coherent signal in the interferogram is superimposed by the incoherent and the thermal contributions
  - ➜ The width of the central peak (i.e. the cos term) must be determined only after background subtraction
  - ➜ FWHM is very sensitive to noise ⇒ determine zero-crossings

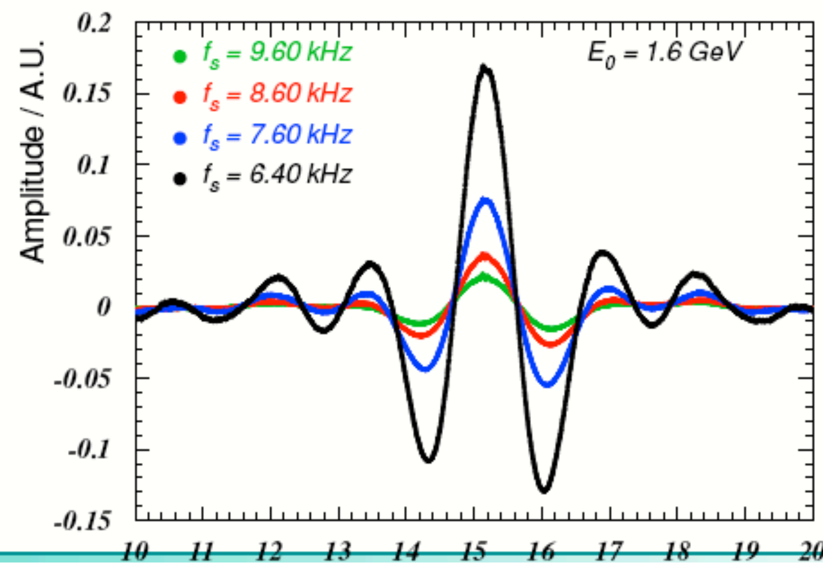


Anke Examples

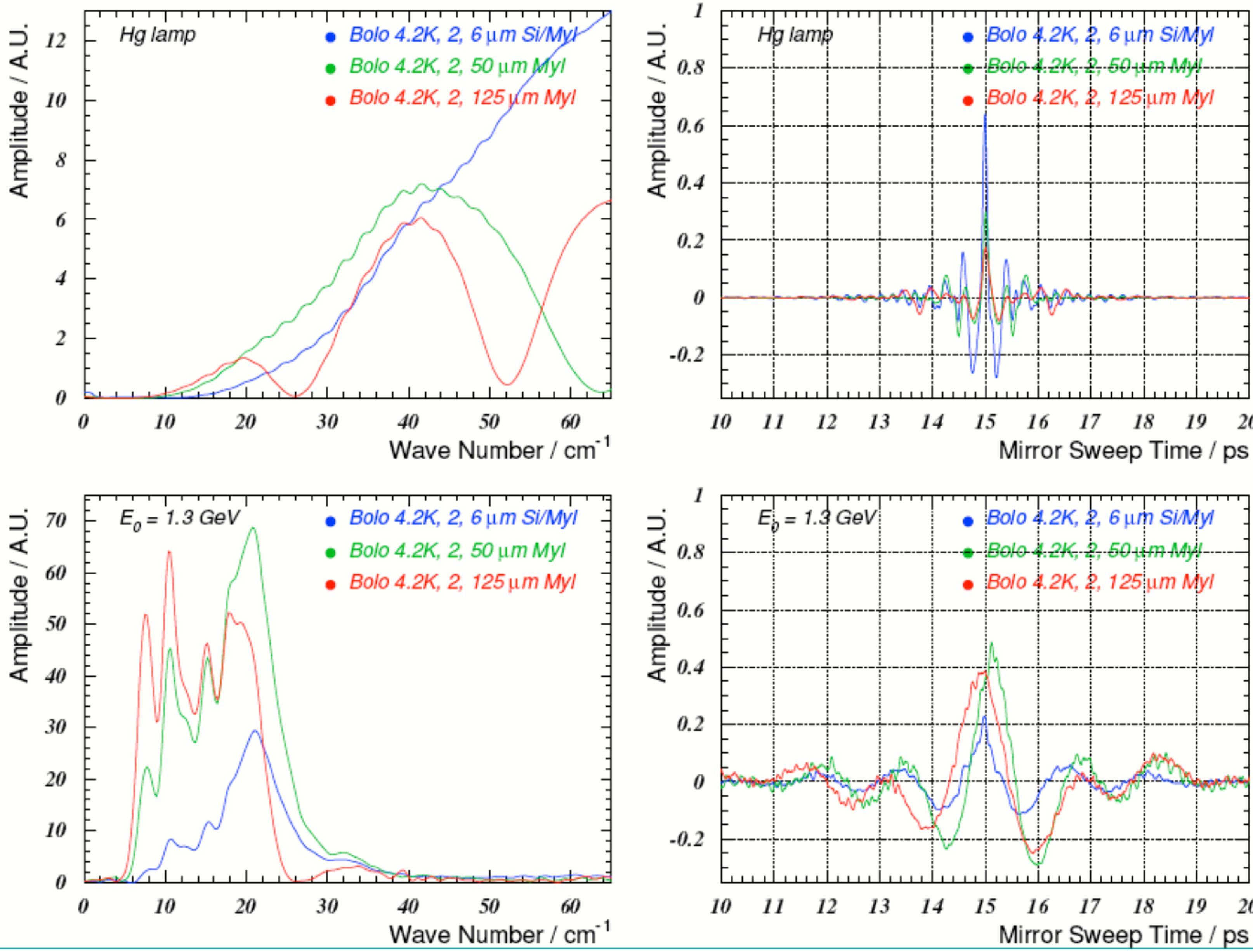




- Background subtracted data for different beam energies and "squeeze states":
  - ➜ the higher the energy, the lower the noise
  - ➜ cavity mode?

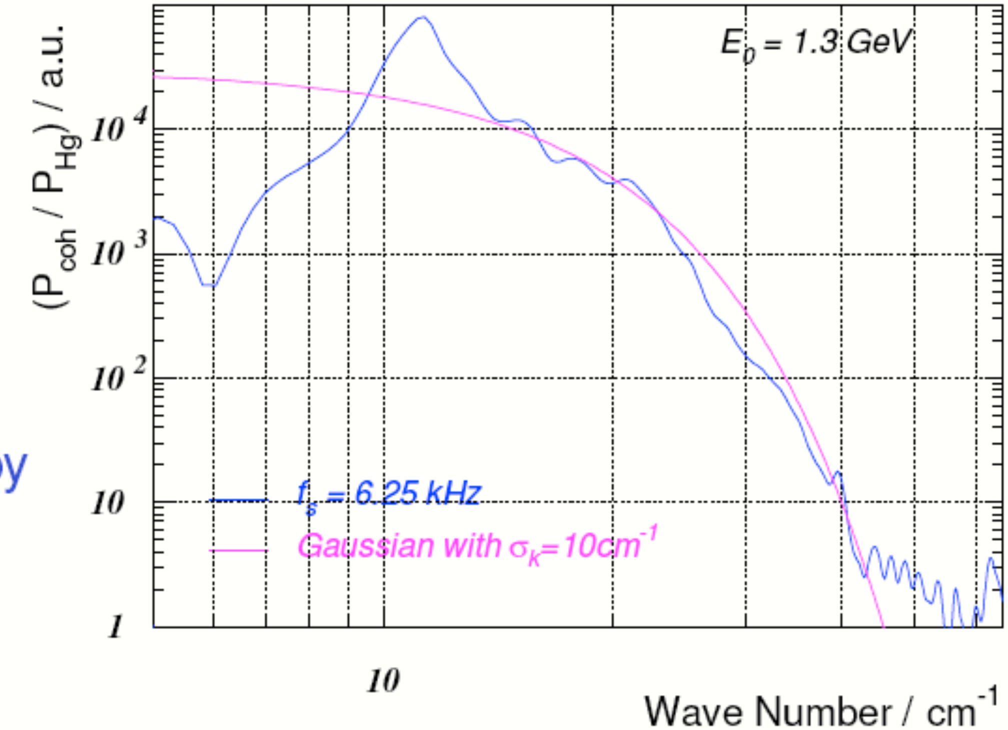


Effect of beam splitter



Bunch length from Spectrum

- Determination of σ from normalised spectrum
- Ideally normalisation by incoherent spectrum
  - ➜ Problem: low intensity
  - ➜ Alternative: Normalise by spectrum of Hg lamp



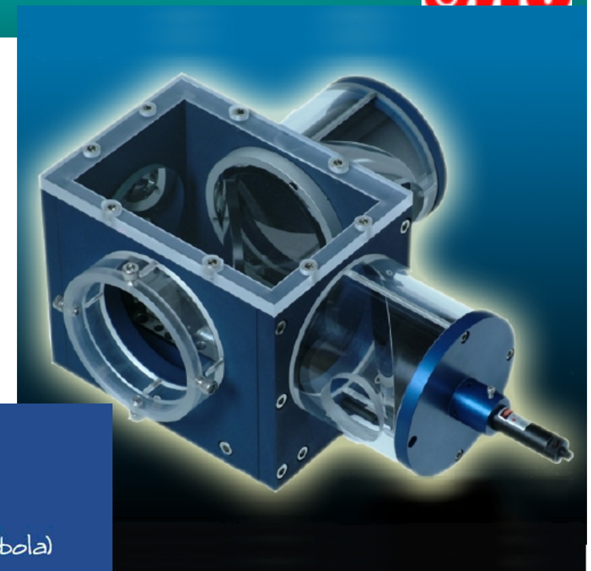
- The bunch length is related to the spectral bandwidth σ_k by (G. Wüstefeld, SBSR05):

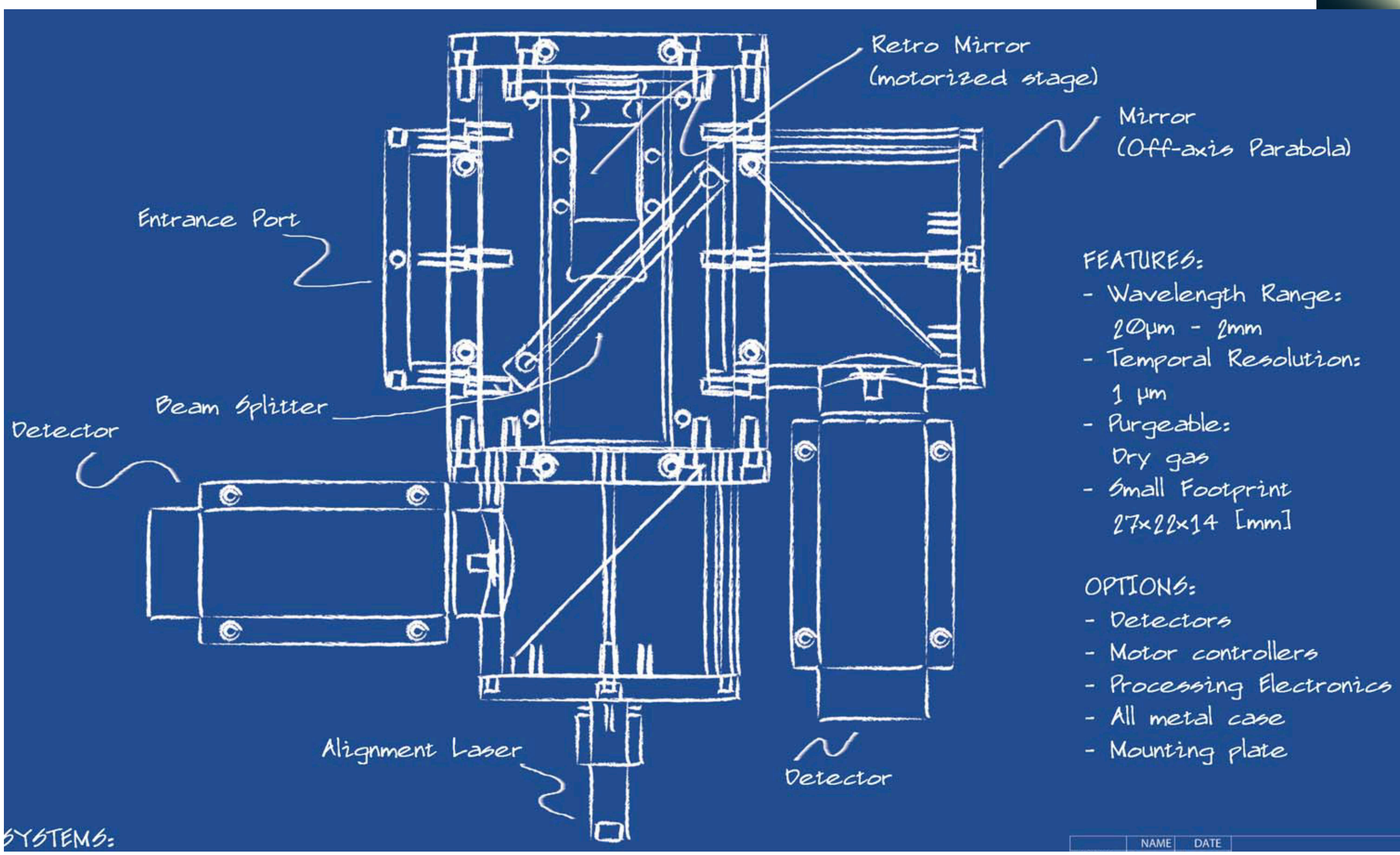
$$\sigma_s = \frac{1}{2\,\pi\,\sqrt{2}\,\sigma_k}$$

- The bunch length determined thus is 0.375 ps.

Commercial bunch length monitors

Radiabeam

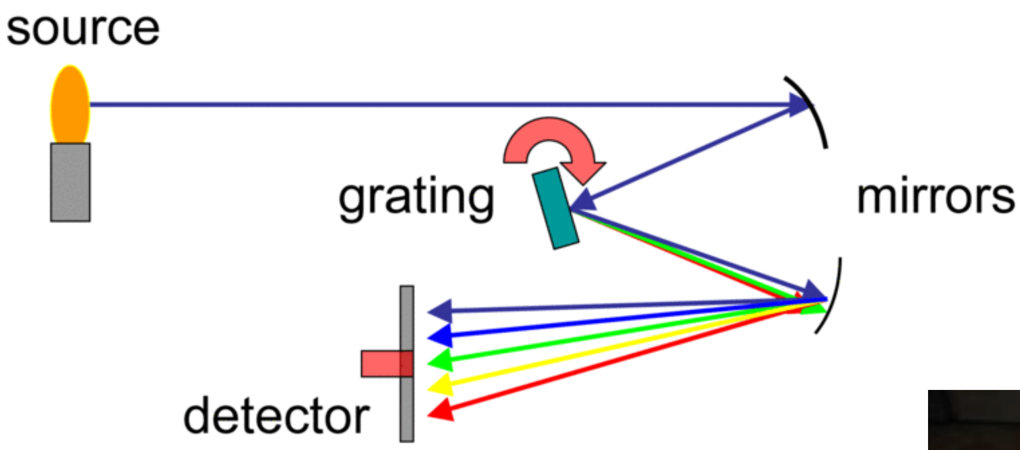




Laboratory models

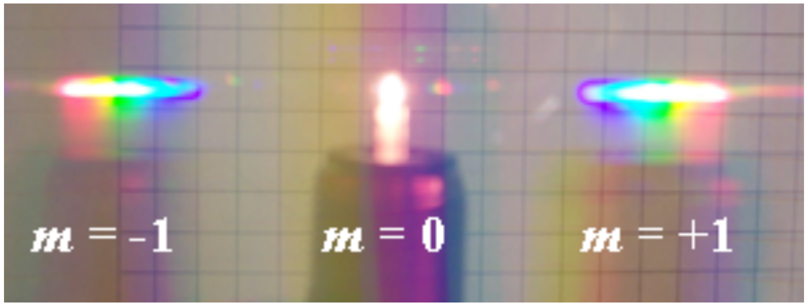
Sciencetech Model SPS-300
Far Infrared THz FTIR

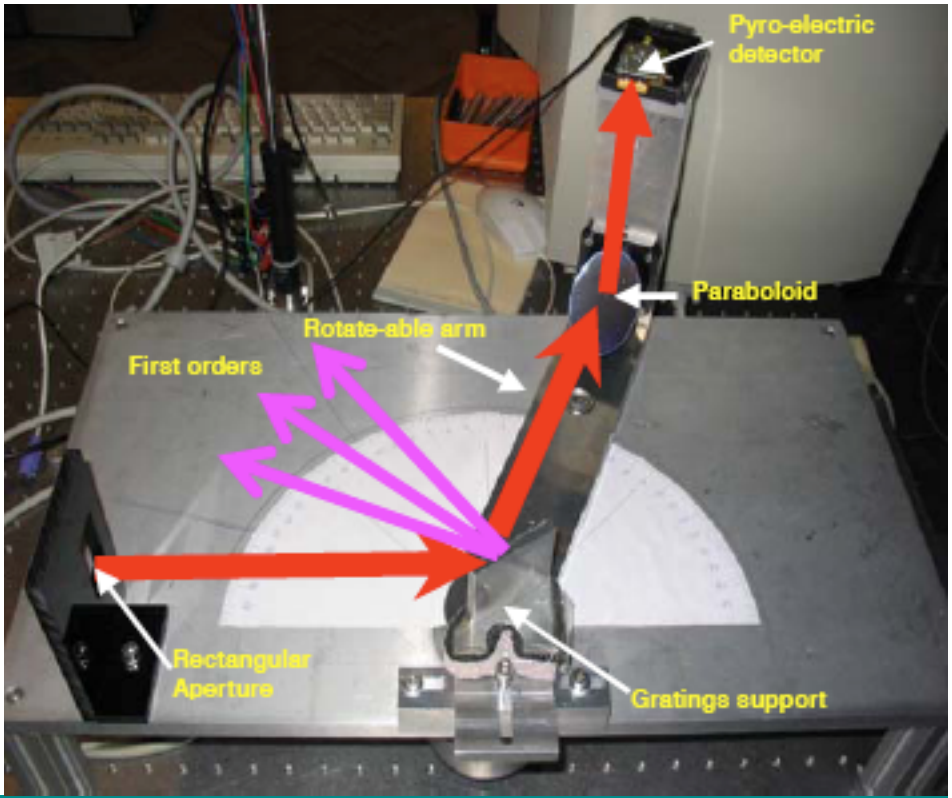
Grating spectrometers



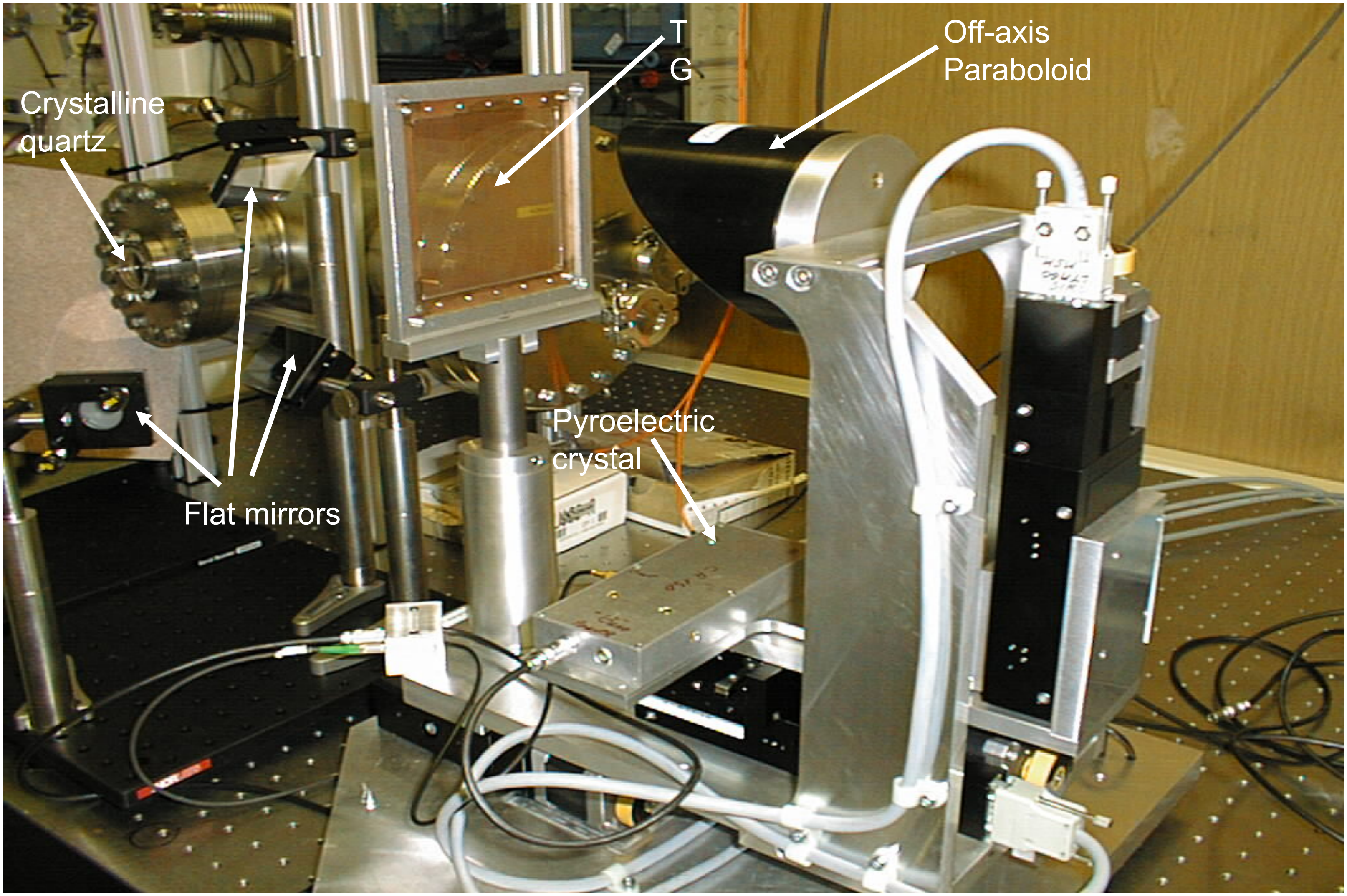
Use dispersion to separate wavelength components

$$d\,(\sin\theta_m(\lambda) + \sin\theta_i) = m\lambda$$

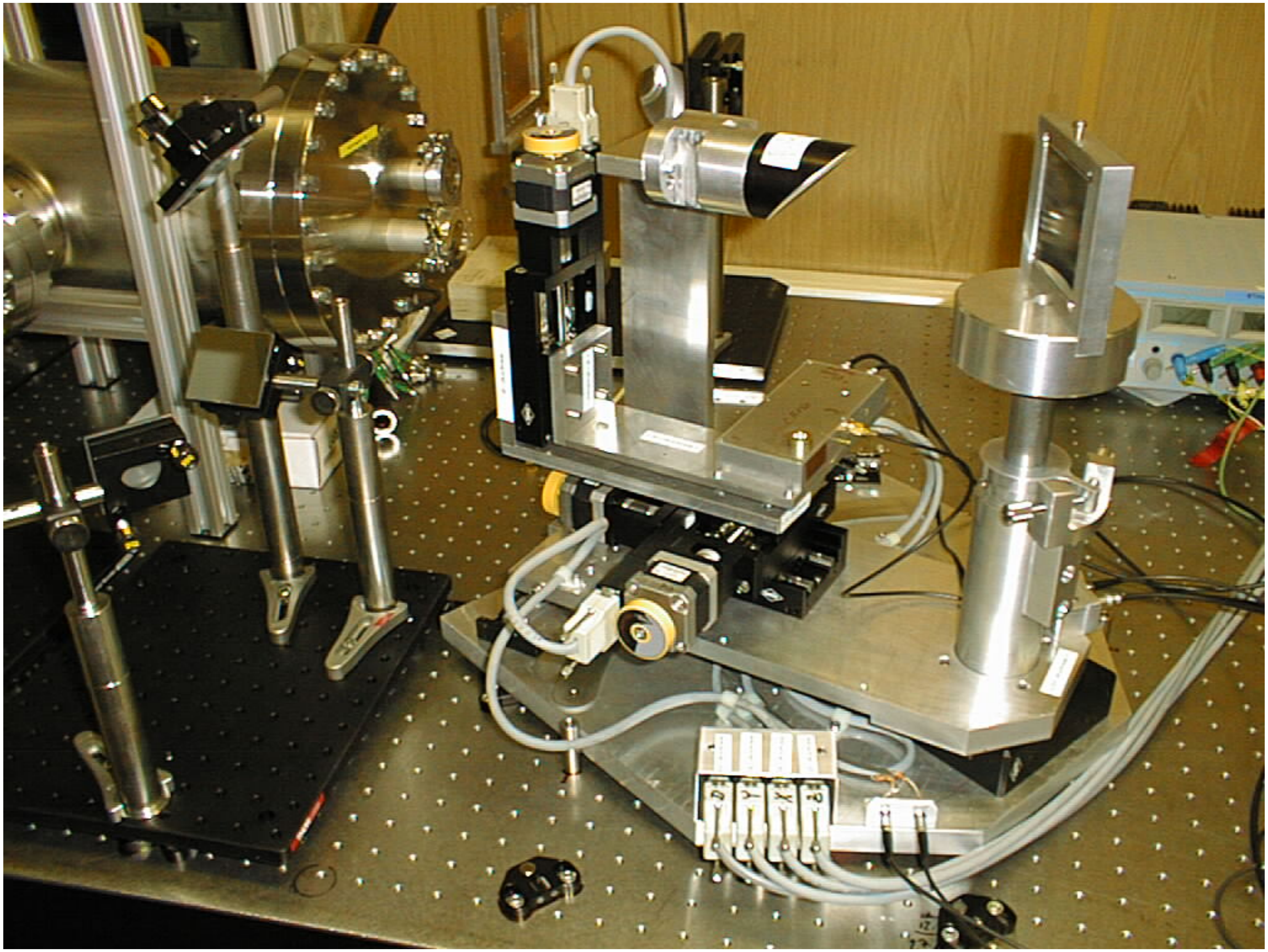




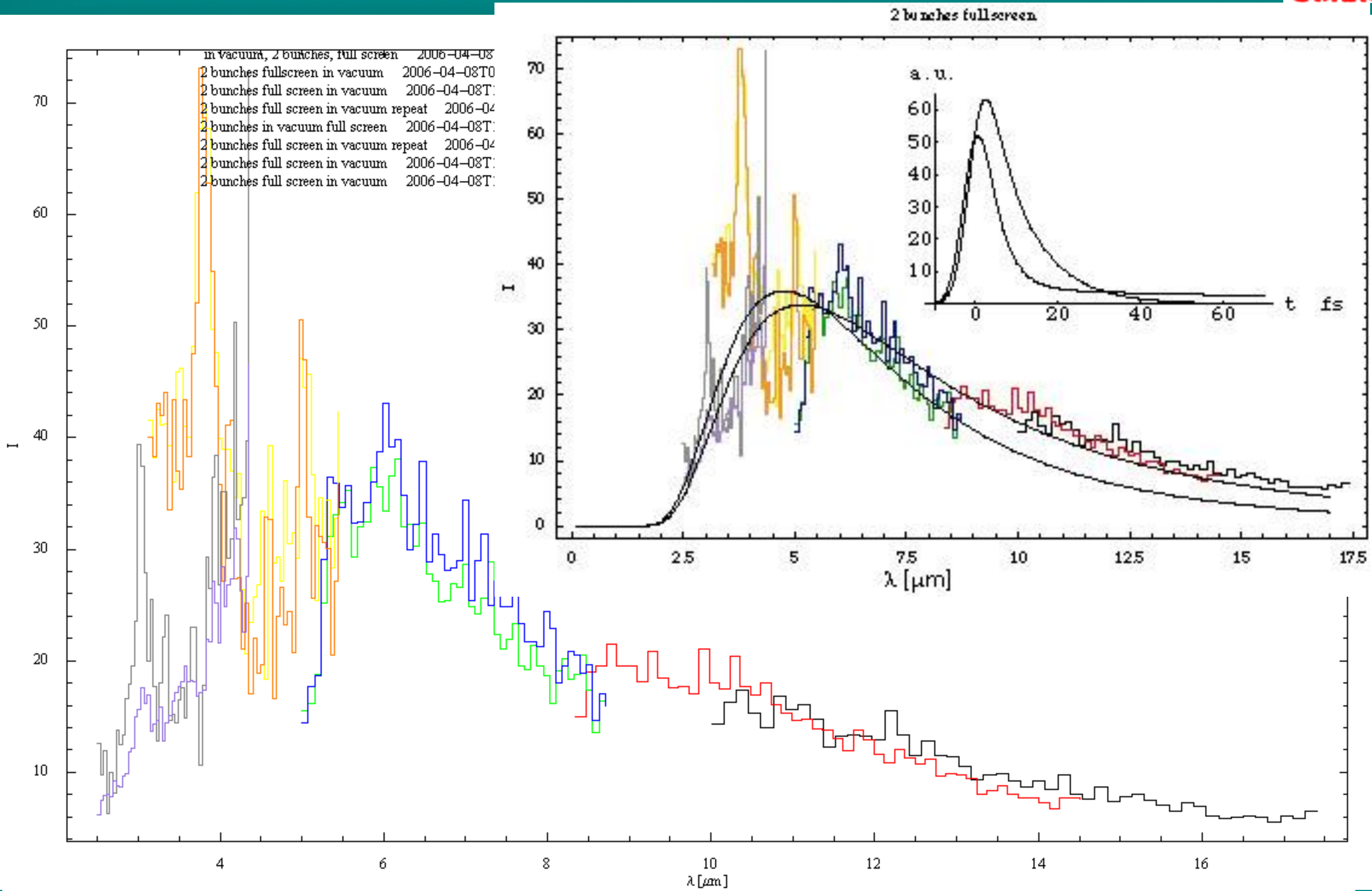
Rotating grating spectrometer



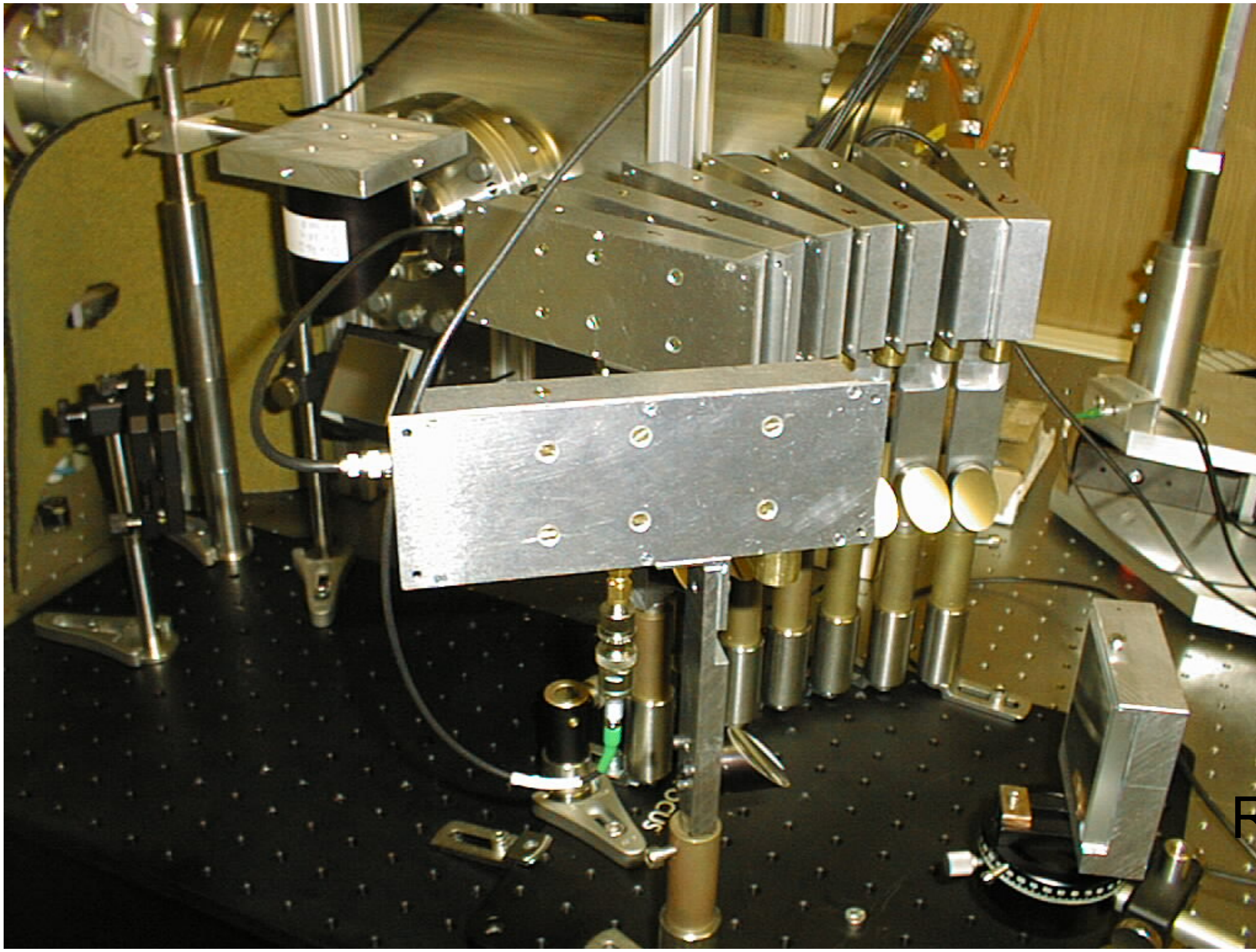
Rotation reflection grating



Some results (short wavelengths in vacuum)



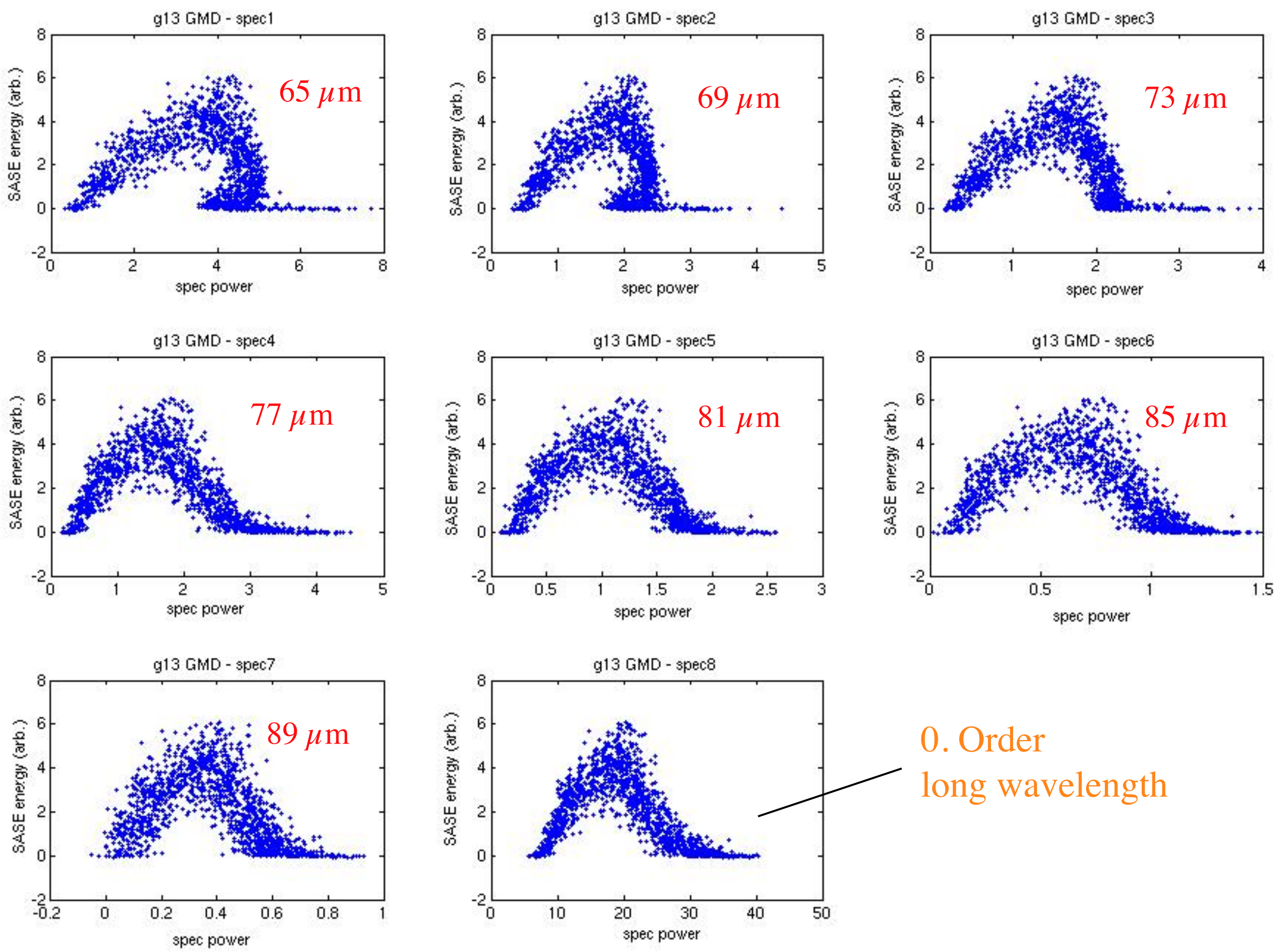
Single shot spectrograph



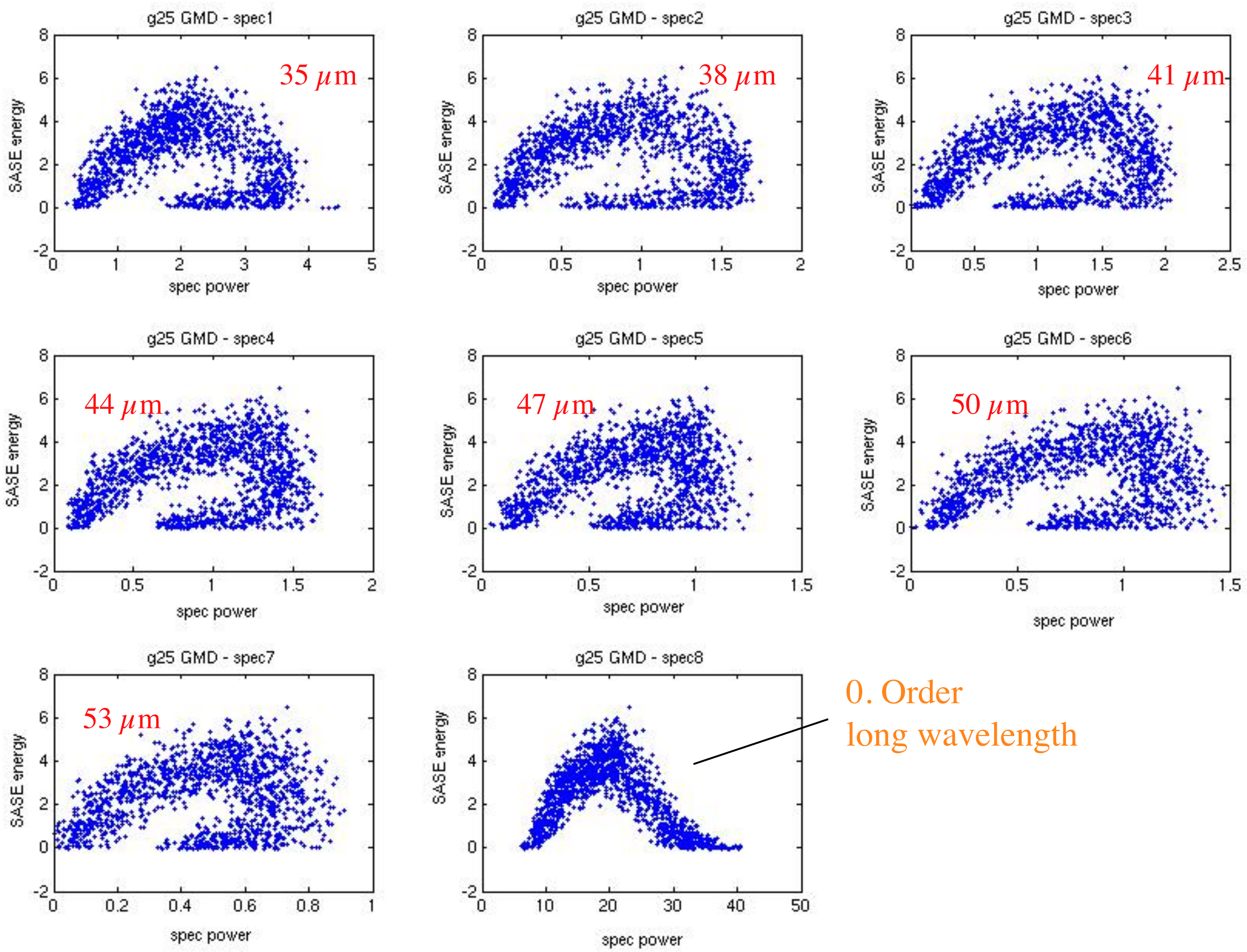
PD array

Reflection grating

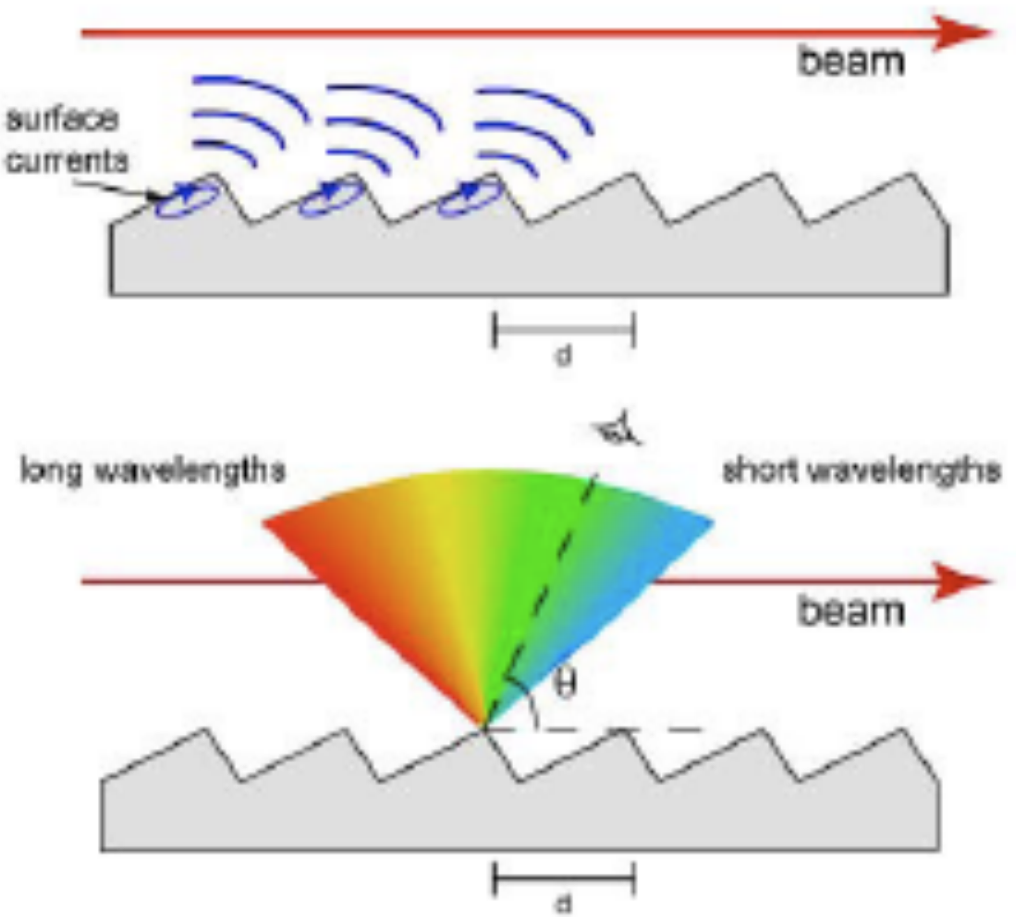
Correlations SASE - short wavelengths - I



Correlations SASE - short wavelengths - II

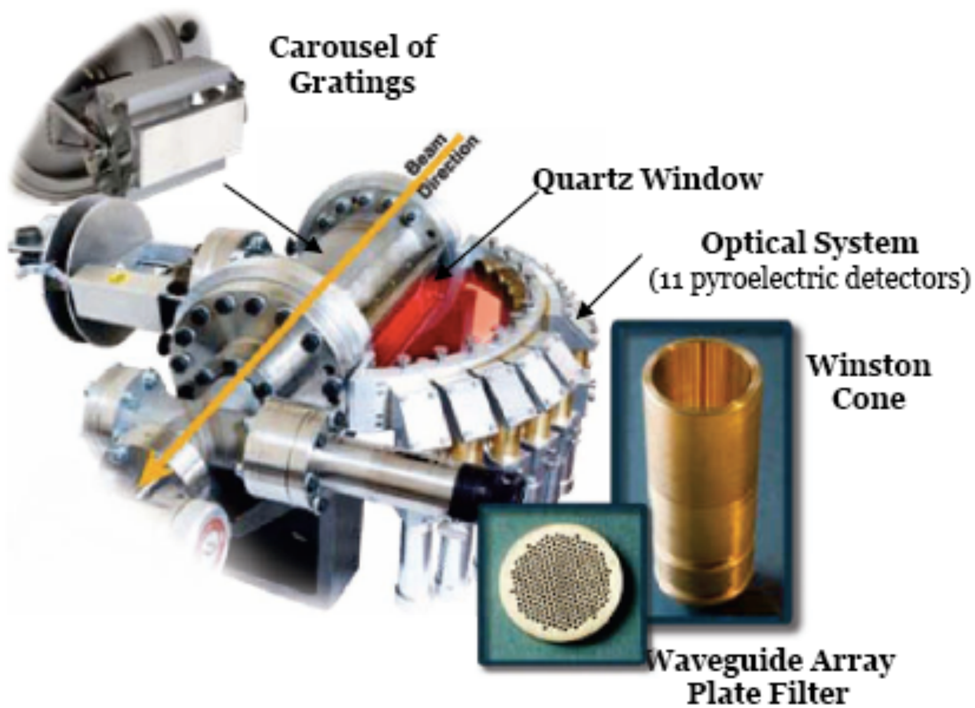


Direct grating spectrometer; Smith-Purcell radiation



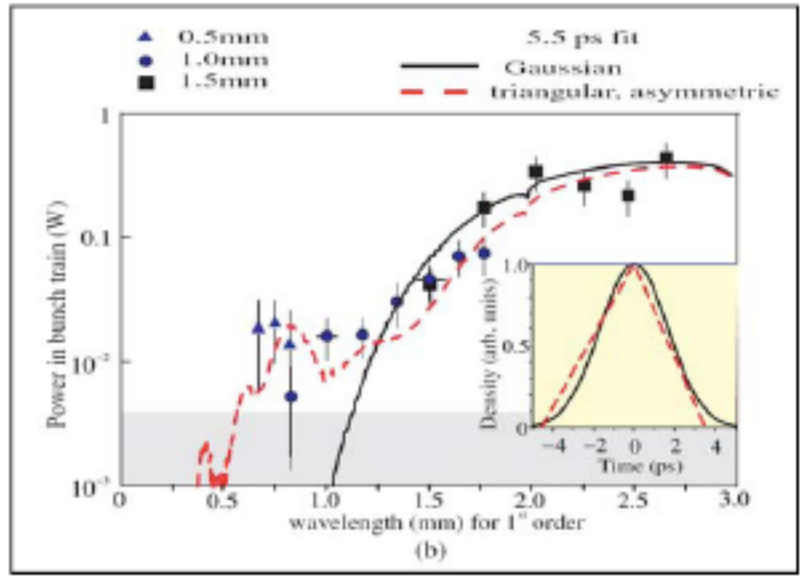
Smith-Purcell radiation is interference of multiple diffraction radiators

Smith-Purcell spectrometer



Results of a run at 28.5 GeV from SLAC are currently being analyzed.

Measurement at 45 MeV, FELIX



see PRST 9,092801 (2006)