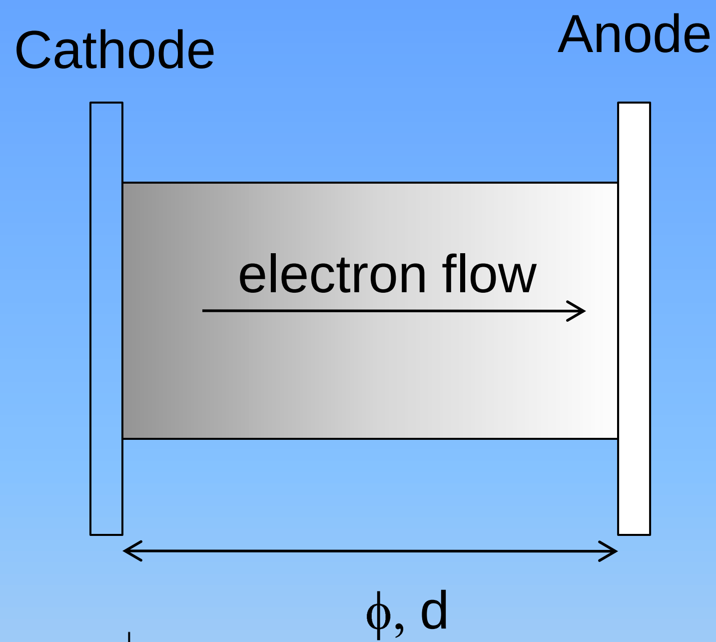
Lecture 3: Space Charge Limited Emission

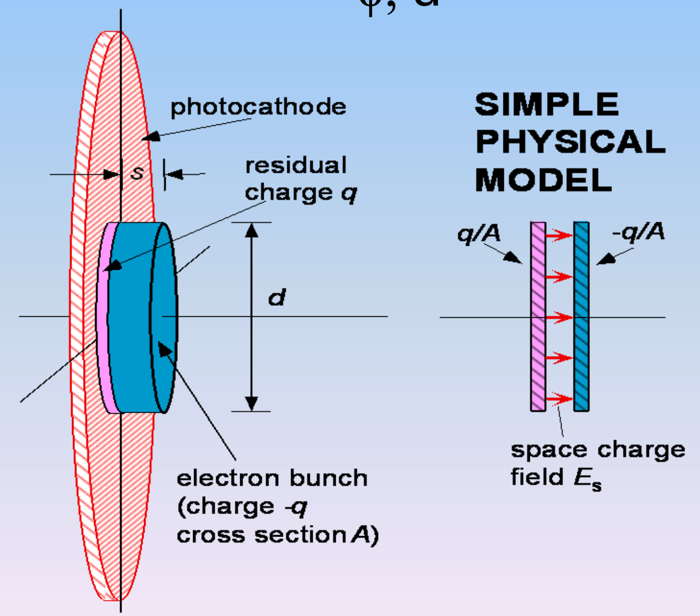
- The object of this lecture is to define the concept of space charge limited emission. This is done by deriving the Child-Langmuir law for a DC diode and the space charge limit for the short pulse of a photocathode gun. The differences of these two cases are discussed.
- The space charge limit is applied to the observed operation of RF photocathode guns.
- The connection between the space charge limit and the lowest achievable emittance is derived and discussed.

Difference between SCL for DC diode and short pulse photo-emission

Space Charge Field Across a Diode, Child-Langmuir law:



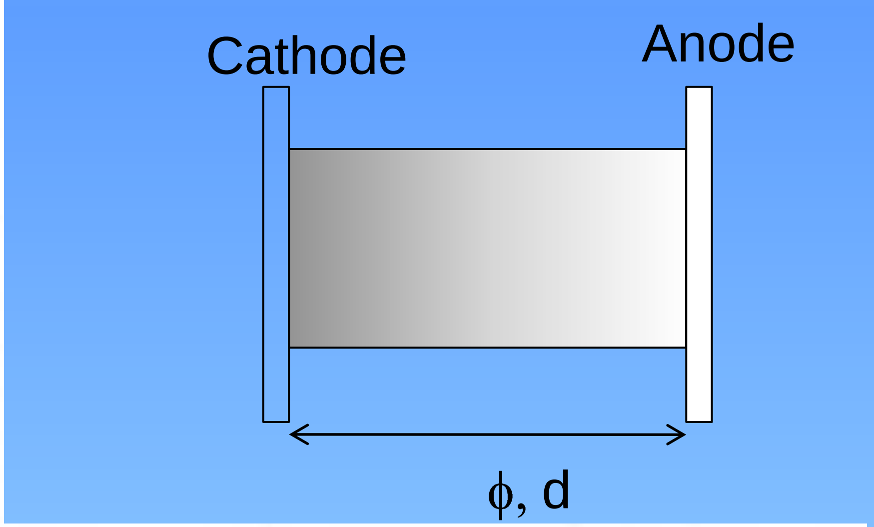
Space Charge Field from a Short Electron Bunch, Laser-driven Photocathodes:



Drawing compliments of A. Vetter

Child's Law Lecture

See Phys. Rev. 32, 492 (1911)



Space Charge in a Diode

$$\frac{1}{2}mv^2 = eV \qquad \text{Conservation of energy}$$

$$\nabla^2 V = \frac{\rho}{\epsilon_0} \qquad \text{Poisson's Eqn.}$$

$$\vec{E} = \vec{\nabla}V, \quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

Assume a steady-state, one-dimensional flow of the current,

$$\frac{d^2V}{dx^2} = \frac{\rho_a}{\epsilon_0} \quad \text{and} \quad E = \frac{dV}{dx}$$

The current surface density, amps/m², is

$$I_a = \rho_a v \; ; \quad v = \sqrt{\frac{2eV}{m}}$$

$$\rho_a = \frac{I_a}{v} = I_a\sqrt{\frac{m}{2eV}}$$

$$\frac{d^2V}{dx^2} = \frac{I_a}{\epsilon_0}\sqrt{\frac{m}{2eV}}$$

Realize/Common Trick:

$$d\left(\frac{dV}{dx}\right)^2 = 2\frac{dV}{dx}\frac{d^2V}{dx^2}dx = 2\frac{d^2V}{dx^2}dV$$

$$\left(\frac{dV}{dx}\right)^2 = E^2 = 2\frac{I_a}{\epsilon_0\sqrt{2\frac{e}{m}}}\int_0^V \frac{dV}{V^{1/2}} = \frac{4I_a}{\epsilon_0\sqrt{2\frac{e}{m}}}V^{\frac{1}{2}}$$

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$$\frac{dV}{V^{1/4}} = \left[\frac{4I_a}{\epsilon_0\sqrt{2\frac{e}{m}}}\right]^{1/2}dx$$

$$\frac{4}{3}V^{3/4} = \left[\frac{4I_a}{\epsilon_0\sqrt{2\frac{e}{m}}}\right]^{1/2}x$$

Solve for the steady-state current to get Child's Law, (Child-Langmuir law)

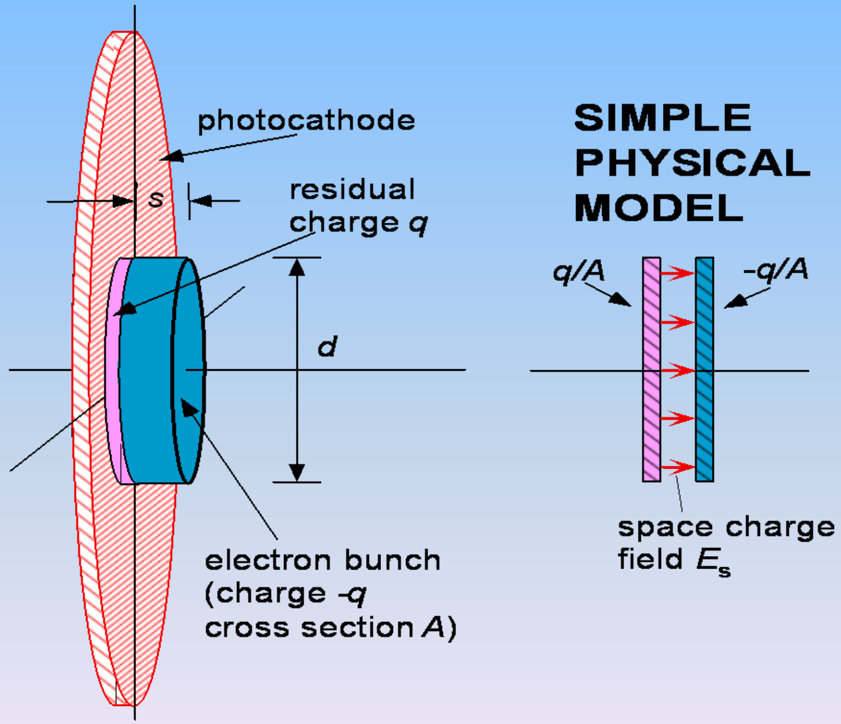
$$I_a = \frac{4}{9}\epsilon_0\sqrt{2\frac{e}{m}}\,\frac{V^{3/2}}{d^2} = J \quad \text{current surface density}$$

d is the gap length of the diode.

$$J = 2.33\times10^{-6}\frac{V^{3/2}}{d^2} \quad [A/m^2]$$

with V in volts and d in meters.

Electron emission is strongly affected by self-fields produced by the electron bunch itself. Immediately at the cathode surface, the electrons experience their own image charge which for metal cathodes produces a field which opposes the applied electric field. The magnitude of this field is easily estimated by considering the electron bunch as a very thin charge sheet very close to the cathode surface. Then the space charge field is similar to that between the plates of a capacitor as shown



Space charge electric field:

$$E_{sc} = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

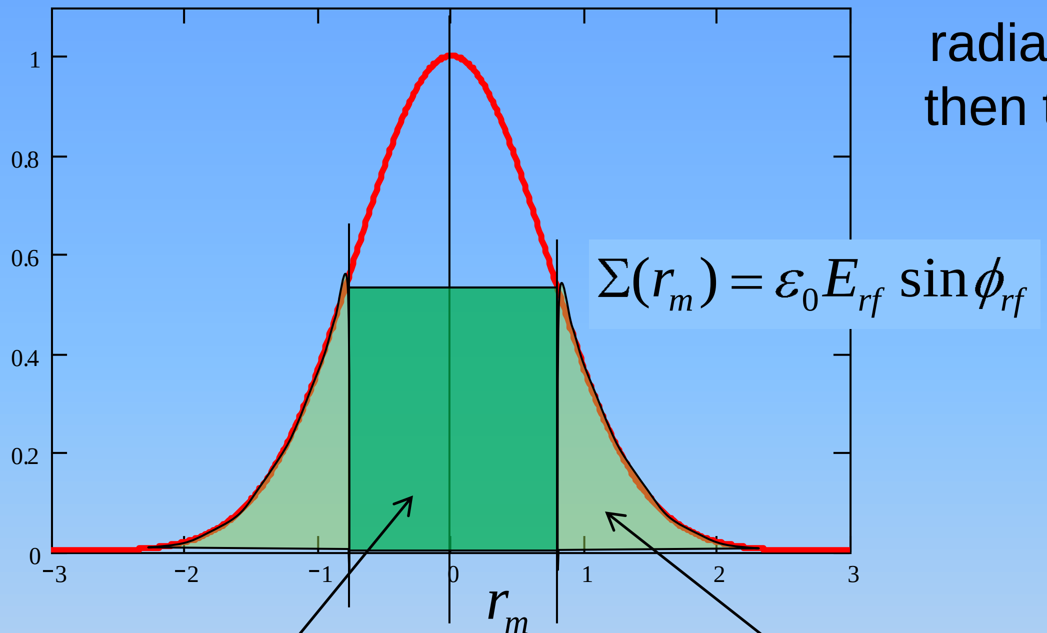
SCL occurs when
space charge field = applied field

$$E_{sc} = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = E_{applied}$$

Limiting surface charge density:

$$\sigma_{sc-\mathrm{limit}} = \varepsilon_0 E_{applied}$$

Space charge limited (SCL) emission in a RF gun



Assume a Gaussian radial distribution on the cathode, then the surface charge density is

$$\Sigma(r) = \frac{Q_{bunch}}{2\pi\sigma_r^2} e^{-\frac{r^2}{2\sigma_r^2}}$$

$$Q_{bunch} = \frac{eE_{laser}}{\hbar\omega} QE$$

SCL occurs when

$$\Sigma(r_m) = \varepsilon_0 E_{rf} \sin\phi_{rf}$$

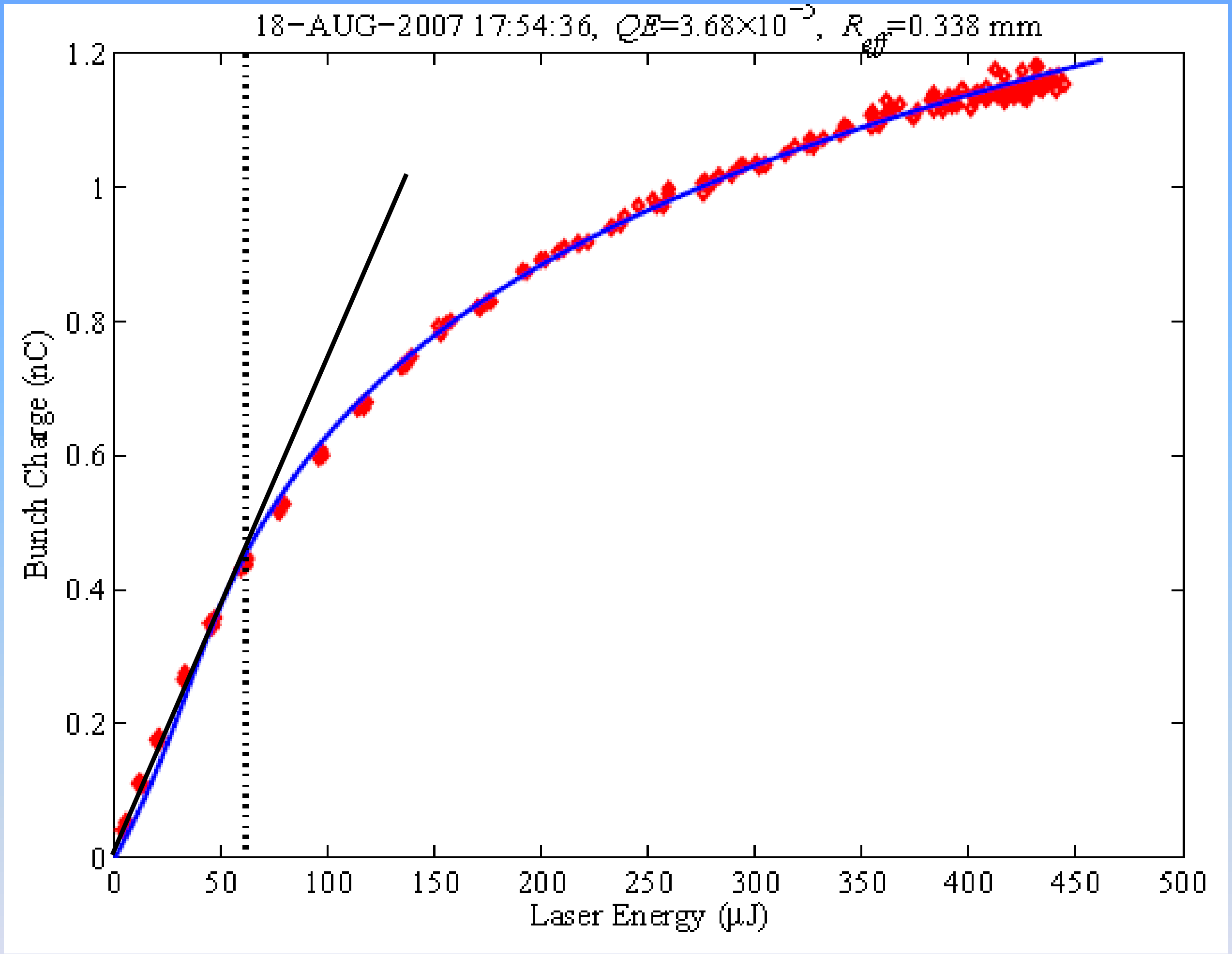
$$Q_{core} = \pi r_m^2 \varepsilon_0 E_{rf} \sin\phi_{rf}$$

$$Q_{tail} = 2\pi \int_{r_m}^{\infty} \Sigma(r) r dr = Q_{bunch} e^{-\frac{r_m^2}{2\sigma_r^2}}$$

$$Q_{emitted} = Q_{core} + Q_{tail} = \pi r_m^2 \varepsilon_0 E_{rf} \sin\phi_{rf} + QE \frac{eE_{laser}}{\hbar\omega} e^{-\frac{r_m^2}{2\sigma_r^2}}$$

See Rosenzweig et al., NIM A341(1994)379-385

Theory is used at LCLS to obtain QE and effective emission size



The measured bunch charge vs. laser energy fit with an analysis for the *QE* and the space charge limit. The *QE* in this case was 3.7×10^{-5} and the effective transverse radius is 0.34 mm rms.

SCL for Different QE's

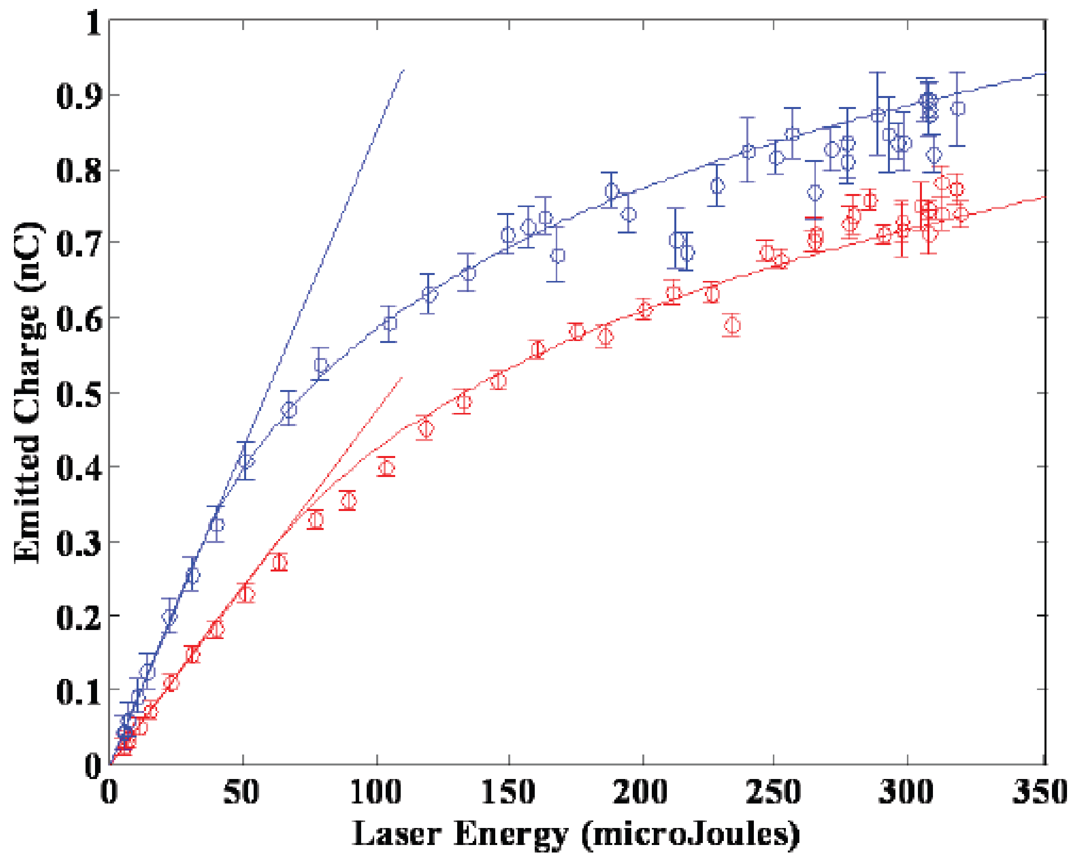


FIGURE 22. Measurements of the charge vs. the laser energy for the LCLS-gun operating at a peak cathode field of 115 MV/m. The data were taken with the same beam size (1.2 mm diameter) but for different QE.

13. The Envelope Equation and Beam Perveance

The envelope equation without external focusing is,

$$r''_m = \frac{\epsilon^2}{r_m^3} + \frac{K}{r_m} \tag{66}$$

where K is the generalized perveance (Lawson, p117) and ϵ is the geometric emittance,

$$K_{general} = \frac{I}{I_0}\frac{2}{\beta^3\gamma^3} = \frac{\omega_p^2 a^2}{2\beta^2 c^2}. \tag{67}$$

Otherwise the perveance is defined as (Reiser, Eqn. 4.28, p.197)

$$K = \frac{I}{4\pi\epsilon_0 V^{3/2}\left(\frac{2e}{m}\right)^{1/2}} = \frac{I}{d^2 J_{SCL}^{CL}} \tag{68}$$

Where the denominator is identified as the Child-Langmiur current from Eqn. 60. Therefore the perveance is defined as the beam current divided by the space charge limited current, $d^2 J_{SCL}^{CL}$.

When the beam is in radial equilibrium, $r''_e = 0$, the geometric emittance can be related to the perveance,

$$\frac{\epsilon^2}{r_e^3} = \frac{K}{r_e} \tag{69}$$

$$\epsilon = r_e\sqrt{K} \tag{70}$$

$$\epsilon = r_e\sqrt{\frac{I}{V^{3/2}}\left[\frac{1}{4\pi\epsilon_0\left(\frac{2e}{m}\right)^{1/2}}\right]} = r_e\sqrt{\frac{I}{d^2 J_{SCL}^{CL}}} = r_e\sqrt{K} \tag{71}$$

Thus the square root of the perveance is the space charge limited *geometric* divergence.

The Space Charge Limited Emittance: The Lowest Possible Emittance

The space charge limited emittance is found by combining the beam size for a the short bunch at the SCL with the normalized divergence derived for photo-electric emission. If we assume the beam is transversely unform with radius a then

$$a = \sqrt{\frac{Q_{bunch}}{\epsilon_0 \pi E_a}} \tag{61}$$

Which has the root-mean-square size of

$$\sigma_x = \frac{a}{2} = \sqrt{\frac{Q_{bunch}}{4\pi\epsilon_0 E_a}} \tag{62}$$

The normalized cathode emittance is given by

$$\varepsilon = \beta\gamma\sigma_x\sigma_{x'} \tag{63}$$

And substituting the normalized divergence for photo-electric emission results in the SCL photo-electric emittance,

$$\varepsilon^{SCL}_{photo} = \sqrt{\frac{Q_{bunch}(\hbar\omega - \phi_{eff})}{4\pi\epsilon_0 mc^2 E_a}} \tag{64}$$

Here ϕ_{eff} is the effective cathode work function which includes the Schottky effect and $\hbar\omega$ is the laser photon energy. For a bunch charge of 1 nC, an applied field of 50 MV/m, a laser energy of 4.86 eV and a copper cathode ($\phi_{eff} = 4.5eV$), the space charge limited thermal emittance is

$$\varepsilon^{SCL}_{photo} = 0.34\ microns \tag{65}$$

Lecture 3 Summary

•This lecture defines the space charge limit of electron emission for cathodes and describes the difference between the Child-Langmuir law for long pulse thermionic-like emission and short pulse photoemission. Formulas for the SCL for each case are derived.

•For the photo-emission SCL the function for the emitted charge vs the drive laser energy is derived for a Gaussian laser profile which is useful for obtaining the QE from laser-limited emission and for obtaining the rms of the effective (laser+QE) emission profile.

•The perveance was related to the geometric divergence at the SCL and the ultimate emittance derived in terms of the SCL and the intrinsic cathode emittance.

Homework problem