Lecture 4: Beam Dynamics w/o Space Charge

- This lecture describes electron beam dynamics in the absence of space charge forces. It begins by showing the bunch in a photocathode gun is pancake-like, justifying the SCL formulas given in the previous lecture.
- Longitudinal beam cooling longitudinally and conservation of longitudinal emittance are demonstrated.
- The emittance due to RF field of the gun is presented.
- The formula for the “Schottky scan”, commonly used to establish the laser launch phase, is derived w/o space charge.

Formation Length of the Bunch at the Cathode

The oscillation of space charge waves determine the amount of and the ratio of energy to charge density modulation. The initial modulation, which is dominantly in charge density, results from the drive laser temporal variations. The variations in the laser intensity during the 6 to 10 ps long pulse imprints the same variation onto the electron bunch, modulating it in longitudinal charge density. To a good approximation we assume the electrons have just escaped and are less than a few 100 microns away from the cathode. This can be shown by integrating the relativistic force equation,

$$F_{rf} = eE_0(1 - \beta^2) = m\ddot{z} \tag{1}$$

As justified by the result, since the electrons are being accelerated from rest, β is no more than 10% of the speed of light and therefore the β^2 term can be ignored. Double integration gives

$$z(t) = \frac{eE_0}{mc^2}\frac{(ct)^2}{2} \tag{2}$$

As noted earlier, this is valid for $\beta = \dot{z}/c << 1$ or when

$$ct << \frac{mc^2}{eE_0} \tag{3}$$

This relation is valid for $t <\sim 30ps$ when the launch field is $E_0 = 50MV/m$.

For an rf gun the field varies in both time and space, giving the complete equation of motion,

$$m\ddot{z} = e(1 - \beta^2)E_{peak}\sin(\omega_{rf}t + \phi_{rf})\cos(k_{rf}z) \tag{4}$$

It can be shown that the $\cos(k_{rf}z)$ space term is close to unity (0.9), and to second order the simple formula given in Equation 1 is reasonably valid.

Using Equation 2 the head of the bunch at the end of the laser pulse is at

$$z_{formation} = \frac{eE_0}{2mc^2}(c\tau_{laser})^2 = \frac{eE_0}{2mc^2}z_{laser}^2 \tag{5}$$

This length is defined as the formation length of the electron bunch and is bunch length when the last electron has just escaped the cathode. At this instant of time the bunch is a continuous stream of electrons from the cathode surface to the bunch head. Using LCLS parameters of 50 MV/m, $\tau_{laser} = 6$ ps and $z_{laser} = 1.8$ mm gives $z_{formation}$ of 160 microns. This is approximately 10% of the laser's length. At the formation time, the bunch head has an energy of $159 microns * 50 MV/m = 7.95 keV$ and tail energy is just the laser photon energy and the work function or $4.86 - 4.6$ 0.3 eV. The velocity of the head is approximately,

$$\beta_{head} = \sqrt{\frac{2 * 7.95 keV}{mc^2}} = 0.12 \quad (6)$$

Which validates the starting assumption that $\beta \ll 1$.

Therefore the electrons are essentially at rest (10 per cent the speed of light), and we can assume the reference frame of the bunch is the same as the lab frame. The result is a slug of charge which is modulated according to the drive laser variation. The rate the laser can vary is limited by its bandwidth, which for the LCLS drive laser is 1 ps or slower. In terms of the bunch z-coordinate, the electron bunch length when the last electron escapes is approximately 1/10 the laser pulse length or for a 6 ps, 1.8 mm long laser pulse, the bunch length is 159 microns. The energy of the head electrons is 7950 KeV and the energy of the tail is 0.2 eV. In this case, a 1 ps variation corresponds to $159 \times 1ps/6ps = 26 microns$. However it is important to note that during acceleration, the bunch expands to a length near that of the laser pulse length, thus the fastest longitudinal charge modulation is 260 microns.

Longitudinal Beam Cooling During Acceleration

Beam Cooling During Acceleration

(Reiser p399-400)

Two particles (electrons), one with velocity v_1 and another with velocity $v_1+\Delta v_1$, transverse and an acceleration gap with energy gain $e\phi$. After traversing the gap the particle's energies are (classically) given by

$$\frac{m}{2}v_2^2 = \frac{mv_1^2}{2} + e\phi$$

$$\frac{m}{2}(v_2+\Delta v_2)^2 = \frac{m(v_1+\Delta v_1)^2}{2} + e\phi$$

$$e\phi = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

$$\frac{m}{2}\left(v_2^2+2v_2\Delta v_2+(\Delta v_2)^2\right) = \frac{m}{2}\left(v_1^2+2v_1\Delta v_1+\Delta v_1^2\right)+e\phi$$

Since $\Delta v_1, \Delta v_2 \ll v_1, v_2$; In general, $\Delta v \ll v$ so the $(\Delta v)^2$ term can be ignored.

$$v_2^2+2v_2\Delta v_2 = v_1^2+2v_1\Delta v_1 + v_2^2 - v_1^2$$

$$v_2\Delta v_2 = v_1\Delta v_1 \Rightarrow \Delta v_2 = \frac{v_1}{v_2}\Delta v_1$$

Thus the velocity spread decreases as $\frac{v_1}{v_2}$

or if $v_2 > v_1$ then $e\phi = \frac{mv_2^2}{2} \Rightarrow v_2 = \sqrt{\frac{2e\phi}{m}}$

or $\Delta v_2 = v_1\sqrt{\frac{m}{2e\phi}}\,\Delta v_1 = \sqrt{\frac{m}{2e\phi}}\,v_1\Delta v_1 = \sqrt{\frac{m}{2eE_z z}}\,v_1\Delta v_1$

Since the beam temperature is due to the random, uncorrelated energy spread corresponding to a rms velocity spread of σ_{v_1},

$\sigma_{v_2} = \frac{v_1}{v_2}\sigma_{v_1}$ is also reduced by the acceleration cooling.

Longitudinal Cooling (cont'd)

Longitudinally the bunch expands:

(Reiser p.378)

Particle A velocity is v_1 and particle B velocity is $v_1+\Delta v_1$
assume A is behind B Δz_1, then A arrives at the gap

$$\Delta t = \frac{\Delta z_1}{v_1} \text{ later than B.}$$

During Δt B has traveled $\Delta z_2 = (v_2+\Delta v_2)\Delta t$

for $\Delta v_2 < v_2$ $\rightarrow$ $\Delta z_2 = v_2 \Delta t \Rightarrow \Delta t = \frac{\Delta z_2}{v_2}$

$$\frac{\Delta z_1}{v_1} = \frac{\Delta z_2}{v_2} \Rightarrow \Delta z_2 = \frac{v_2}{v_1}\Delta z_1$$

Thus a bunch will be elongated after acceleration, in the absence of any other longitudinal forces.

By ~~noting~~ using $\frac{v_1}{v_2} = \frac{\Delta z_1}{\Delta z_2}$ in the relation for

the energy spread: $\Delta v_2 = \frac{v_1}{v_2}\Delta v_1 \Rightarrow \Delta v_2 = \frac{\Delta z_1}{\Delta z_2}\Delta v_1$

$$\Delta z_2 \Delta v_2 = \Delta z_1 \Delta v_1$$

↑ Invariance of longitudinal Emittance.

RF emittance in Photocathode Guns

For a high field gun operating at 100 MV/m the defocusing at the gun exit is quite strong:

$$f_{rf} = -\frac{2\gamma mc^2}{eE_0 \sin\phi_e}$$

$$f_{rf} = -\frac{2 \times 6MeV}{100MeV/m \times 1} = -12cm$$

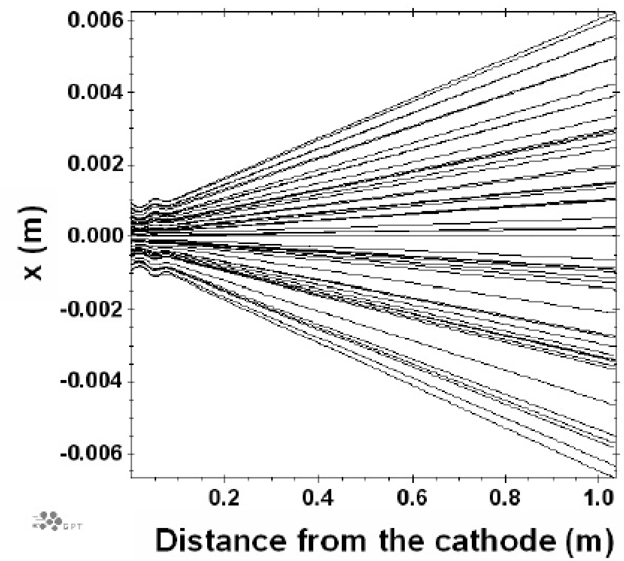
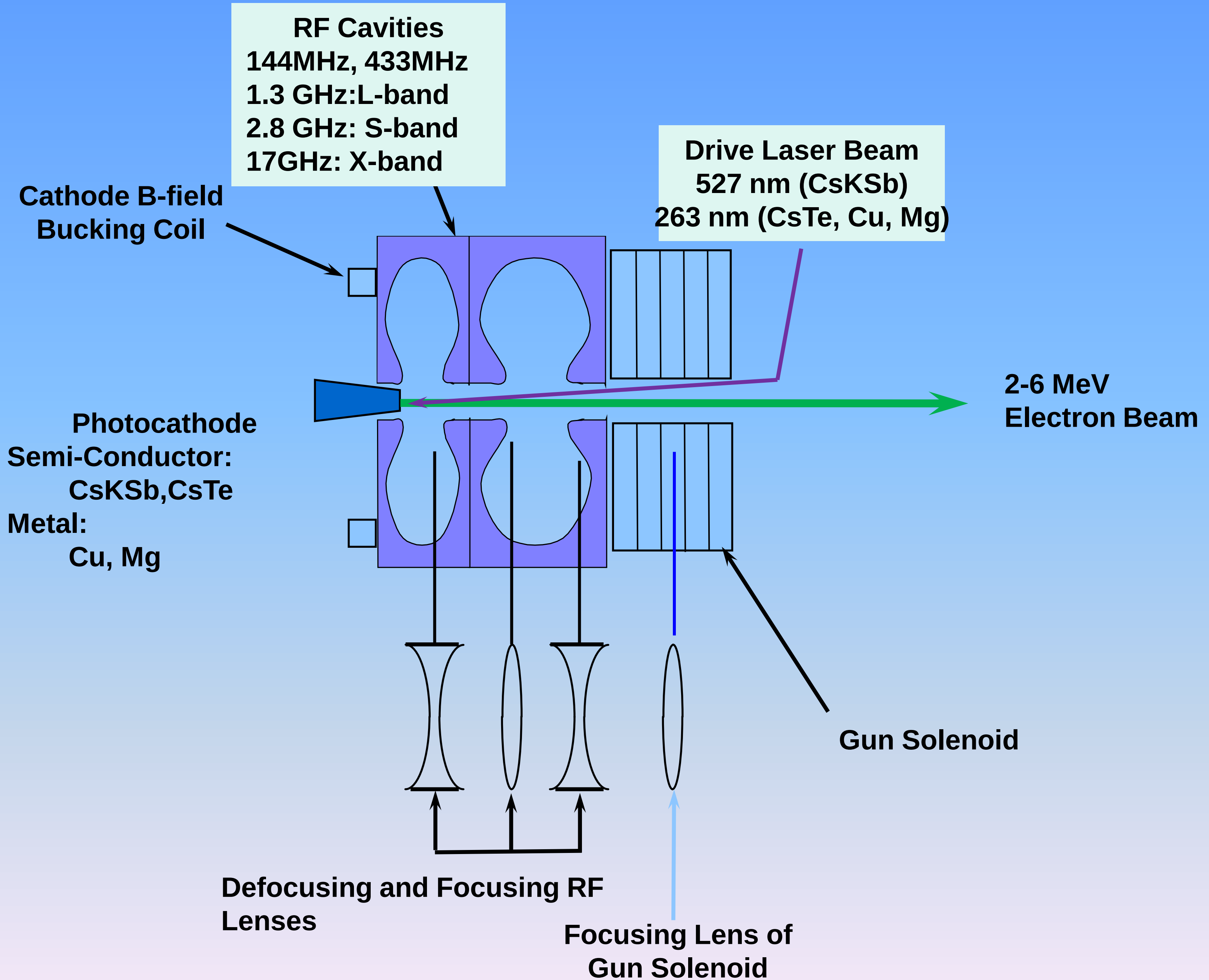
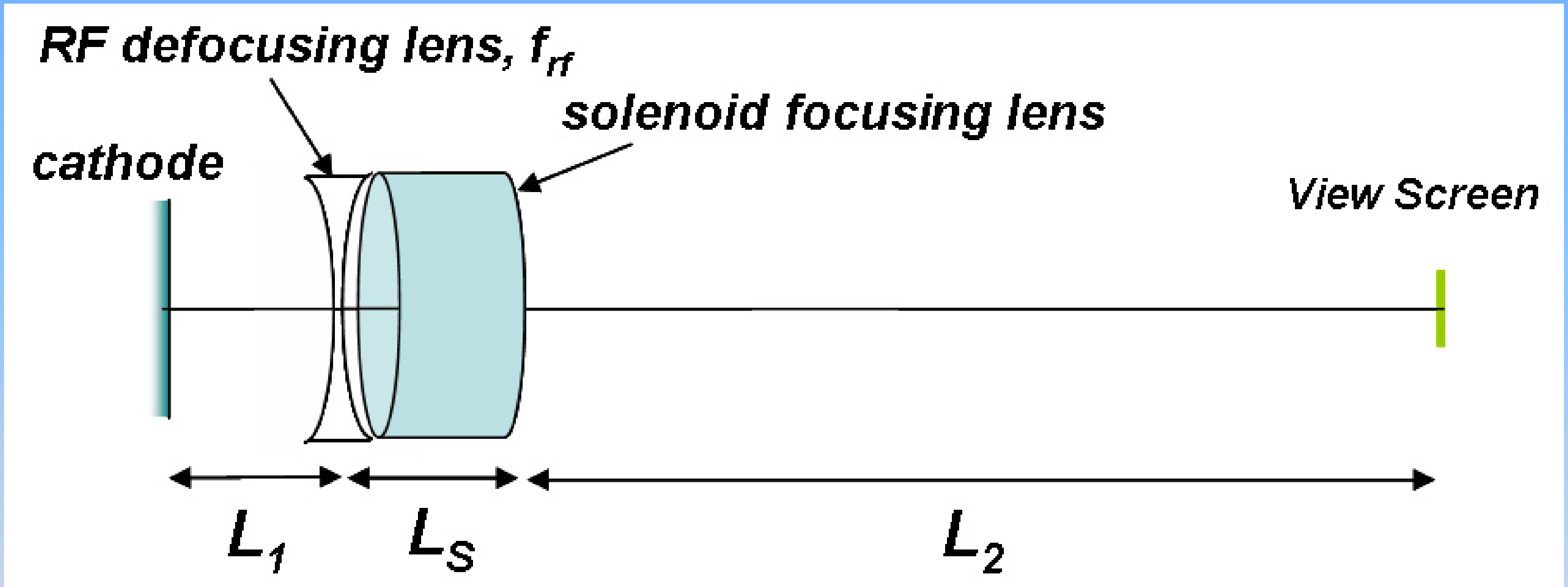
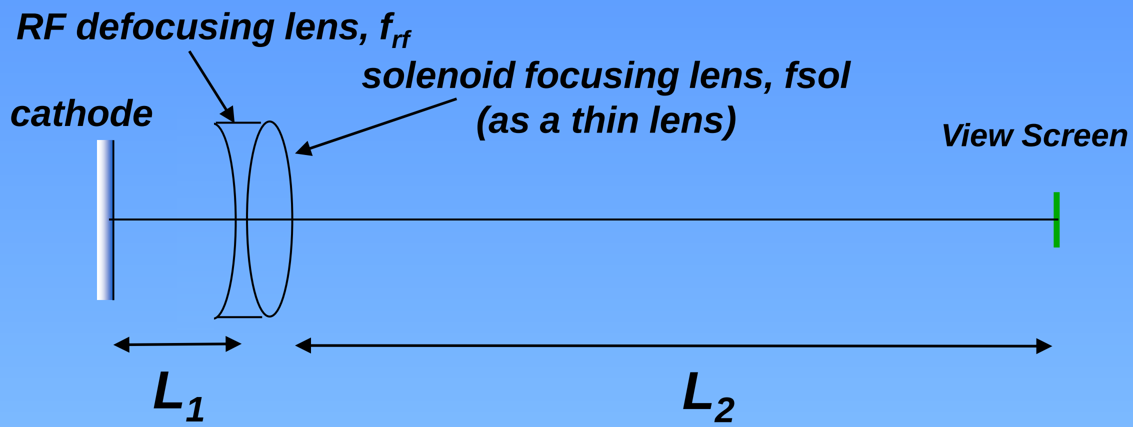


FIGURE 3. Electron trajectories for a rf gun with a cathode field on 115 MV/m as computed by General Particle Tracker Program,


RF Cavities
144MHz, 433MHz
1.3 GHz:L-band
2.8 GHz: S-band
17GHz: X-band
Drive Laser Beam
527 nm (CsKSb)
263 nm (CsTe, Cu, Mg)
Cathode B-field
Bucking Coil
2-6 MeV
Electron Beam
Photocathode
Semi-Conductor:
CsKSb,CsTe
Metal:
Cu, Mg
Gun Solenoid
Defocusing and Focusing RF
Lenses
Focusing Lens of
Gun Solenoid


RF defocusing lens, f_rf
solenoid focusing lens
cathode
View Screen
L_1
L_S
L_2



$$\begin{pmatrix} 1 & L_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_{sol}} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_{rf}} & 1 \end{pmatrix} \begin{pmatrix} 1 & L_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & L_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_{eff}} & 1 \end{pmatrix} \begin{pmatrix} 1 & L_1 \\ 0 & 1 \end{pmatrix}$$

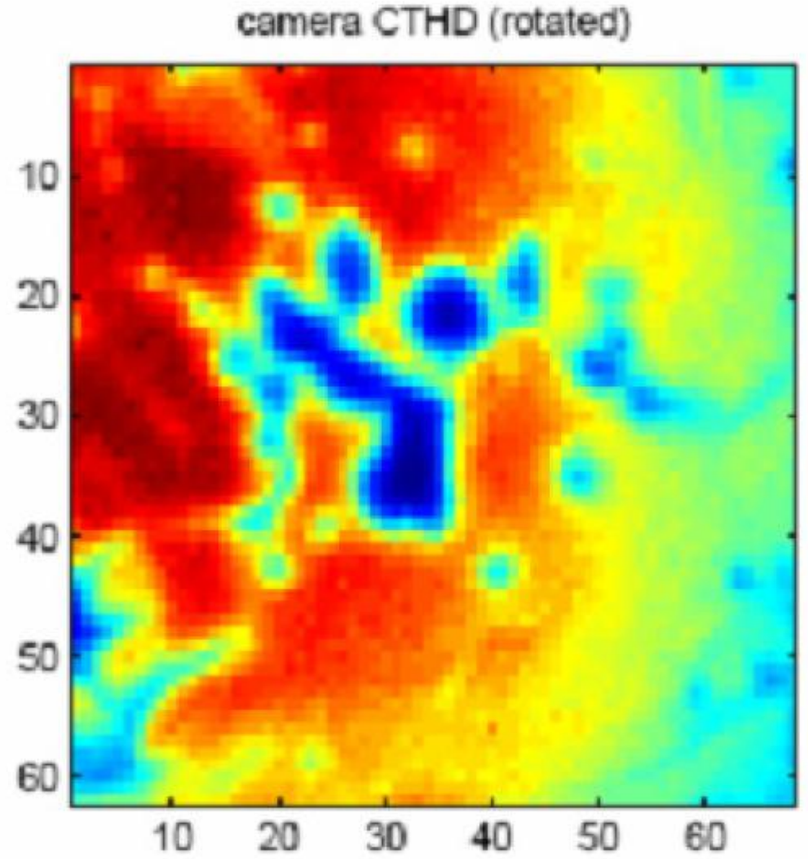
$$\begin{pmatrix} 1-\frac{L_2}{f_{eff}} & L_1 + L_2 - \frac{L_1 L_2}{f_{eff}} \\ -\frac{1}{f_{eff}} & 1-\frac{L_1}{f_{eff}} \end{pmatrix}$$

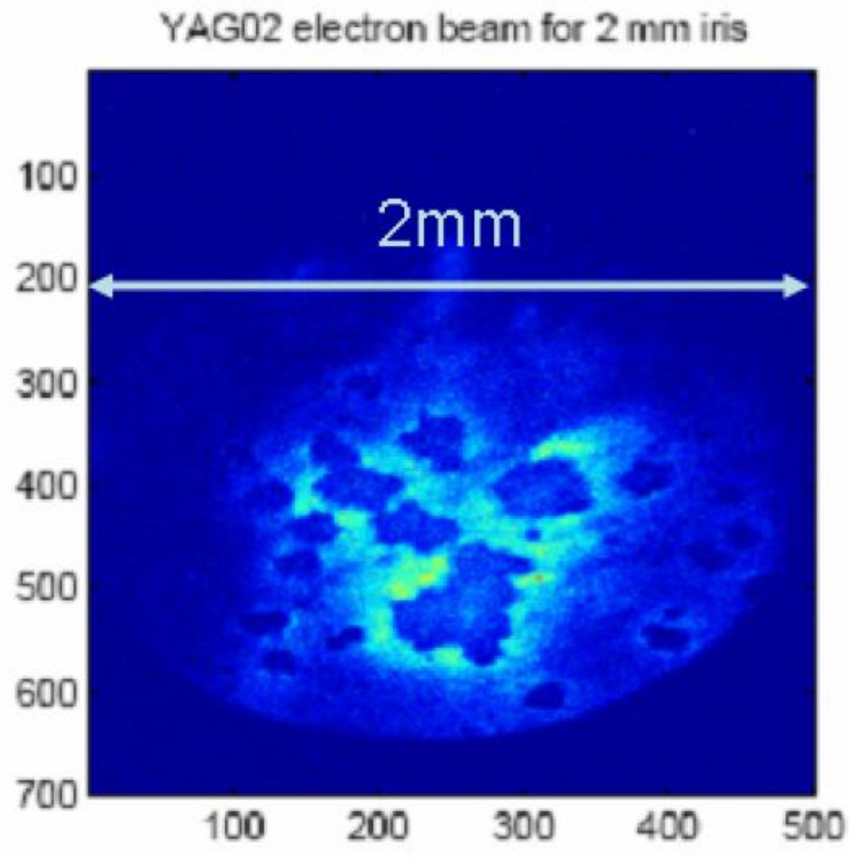
to image cathode on view screen:

$$L_1 + L_2 - \frac{L_1 L_2}{f_{eff}} = 0$$

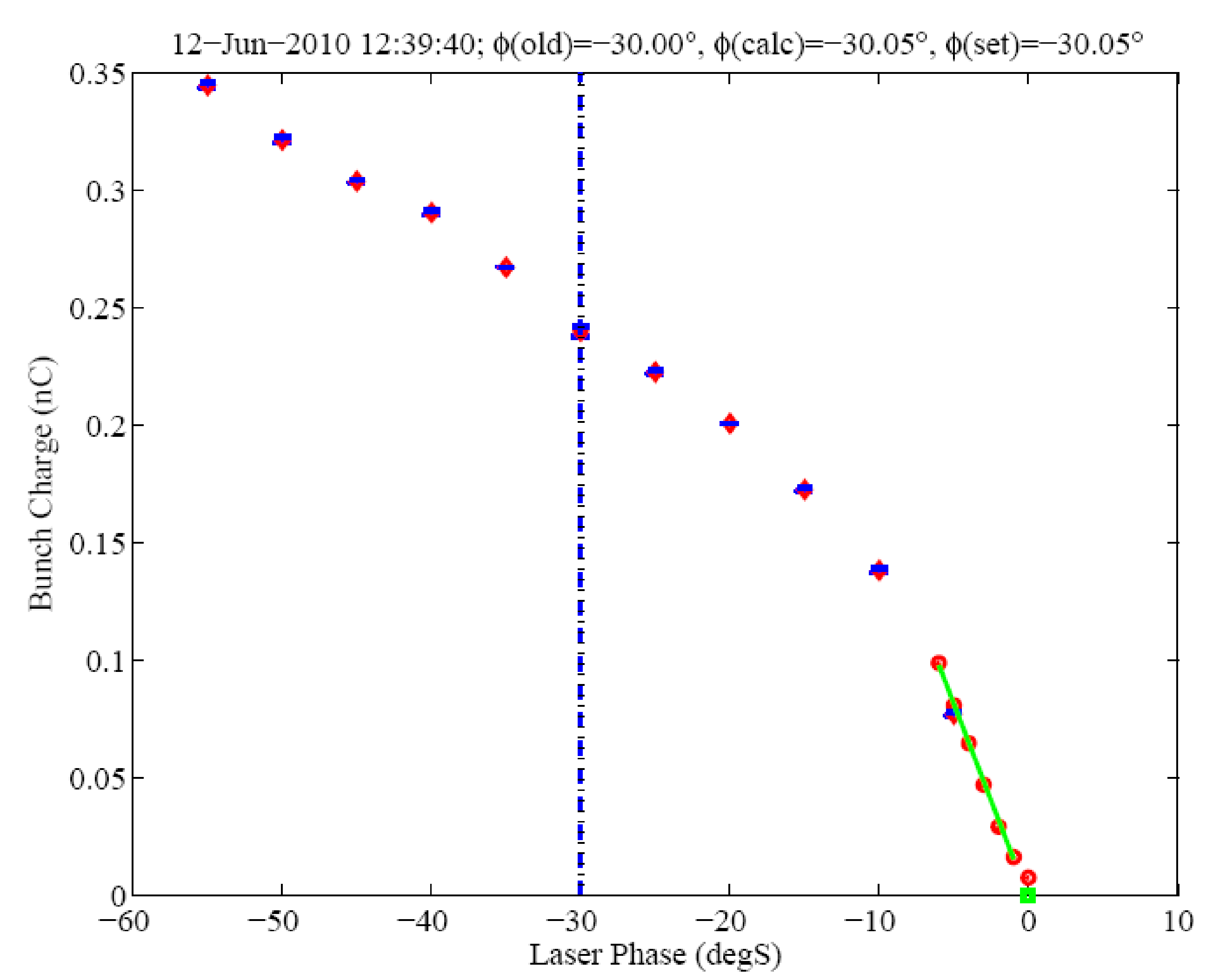
$$\frac{1}{f_{eff}} = \frac{1}{f_{rf}} + \frac{1}{f_{sol}} = \frac{L_1 L_2}{L_1 + L_2}$$

$$\frac{1}{f_{sol}} = \frac{L_1 + L_2}{L_1 L_2} - \frac{1}{f_{rf}} \qquad f_{rf} = -\frac{2\gamma mc^2}{eE_0 \sin\phi_e} \qquad \frac{1}{f_{sol}} = \frac{L_1 + L_2}{L_1 L_2} + \frac{eE_0 \sin\phi_e}{2\gamma mc^2}$$





The Schottky Scan



Derivation of Schottky Scan Function

In Lecture 2 we derived the following formula for the QE of a metal

$$QE = \frac{1-R}{1+\frac{\lambda_{opt}}{\overline{\lambda}_{e-e}}} \frac{E_F + \hbar\omega}{2\hbar\omega}\left(1-\sqrt{\frac{E_F + \phi_{eff}}{E_F + \hbar\omega}}\right)^2$$

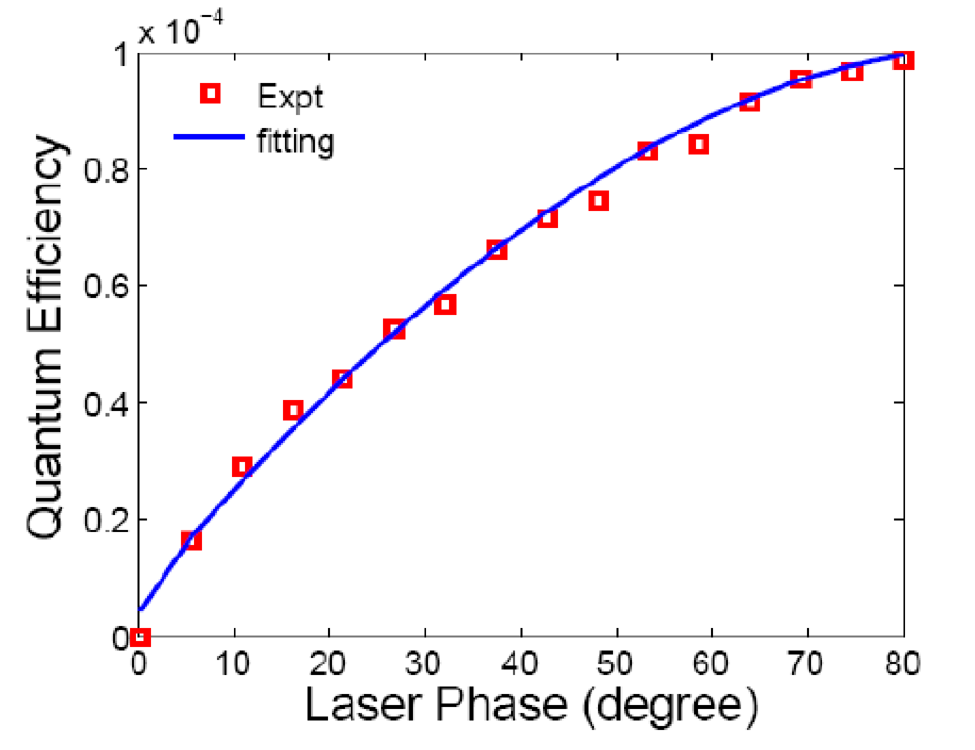
where the effective work function is

$$\phi_{eff} = \phi_W - e\sqrt{\frac{e\beta E_{rf}\sin\phi_{rf}}{4\pi\varepsilon_0}}$$

Putting this into the QE formula gives,

$$QE = \frac{1-R}{1+\frac{\lambda_{opt}}{\overline{\lambda}_{e-e}}} \frac{E_F + \hbar\omega}{2\hbar\omega}\left(1-\sqrt{\frac{E_F + \phi_W - e\sqrt{e\beta E_{rf}\sin\phi_{rf}/(4\pi\varepsilon_0)}}{E_F + \hbar\omega}}\right)^2$$

Fit of the Schottky Function to LCLS Data



Find: $\phi_W = 4.83eV \quad \beta = 1.02$

Fit to LCLS data by Dao Xiang