Lecture 5:
Beam Dynamics with Space Charge

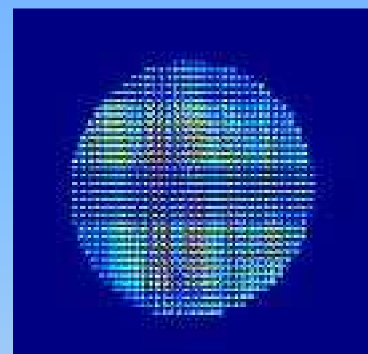
•In this lecture the concept of emittance compensation is explained for the RF gun using the concept of coordinated plasma oscillations of the slices.
•Beam matching into the booster after the gun following the Ferrario condition is described.
•The method of RF compression is explained in conjunction with the tapering of the confining solenoid field to control emittance growth
•The concept of generalized dispersion is defined and used to discuss the emittance growth in beam transport due to space charge or CSR.
•A simple analytic model is described to compute the emittance growth resulting from various laser produced beam shapes.

Analysis of Space Charge Emittance Growth Due to Shaped Beams

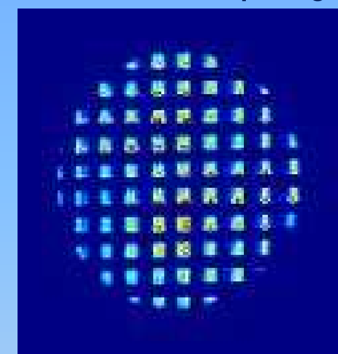
The Seven Laser Shapes

Nominal
1.2 mm dia.

180 mesh
32 cycles per 1.2 mm
38 micron line spacing

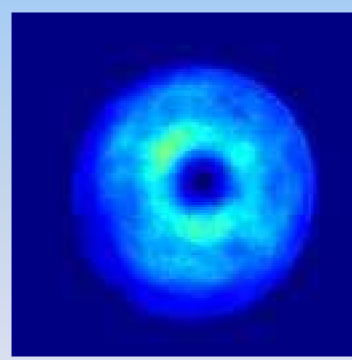


50 mesh
9 cycles per 1.2 mm dia.
133 micron line spacing



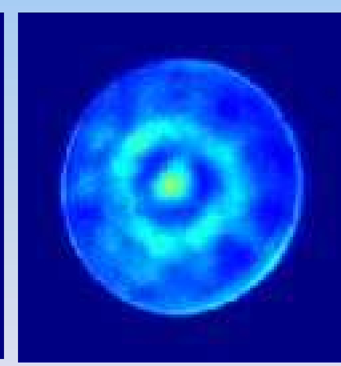
Gaussian

Bagel



Donut

Airy Diffraction

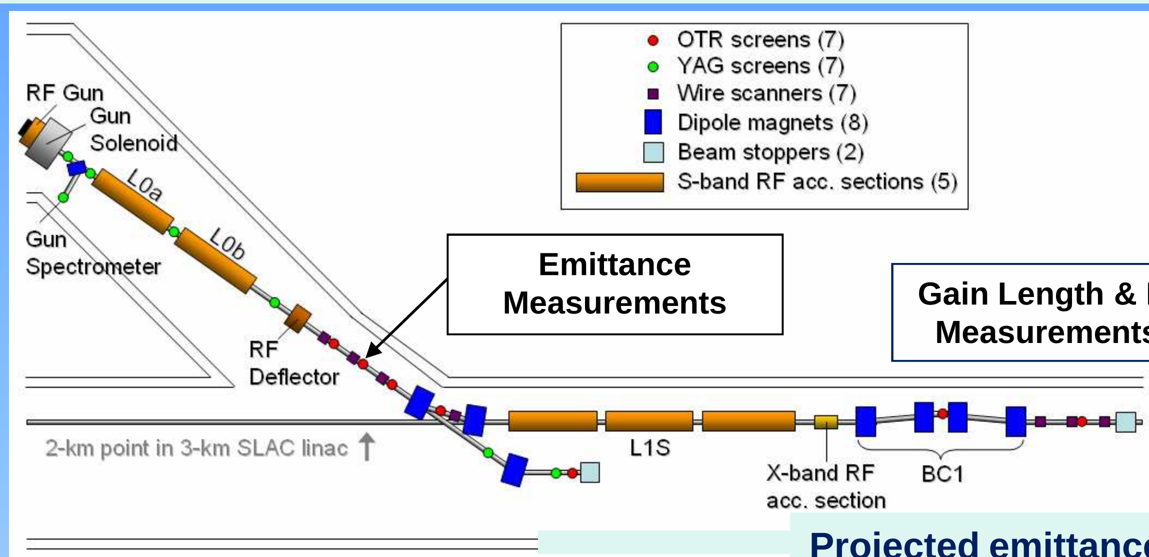


Measure for Each Shape:

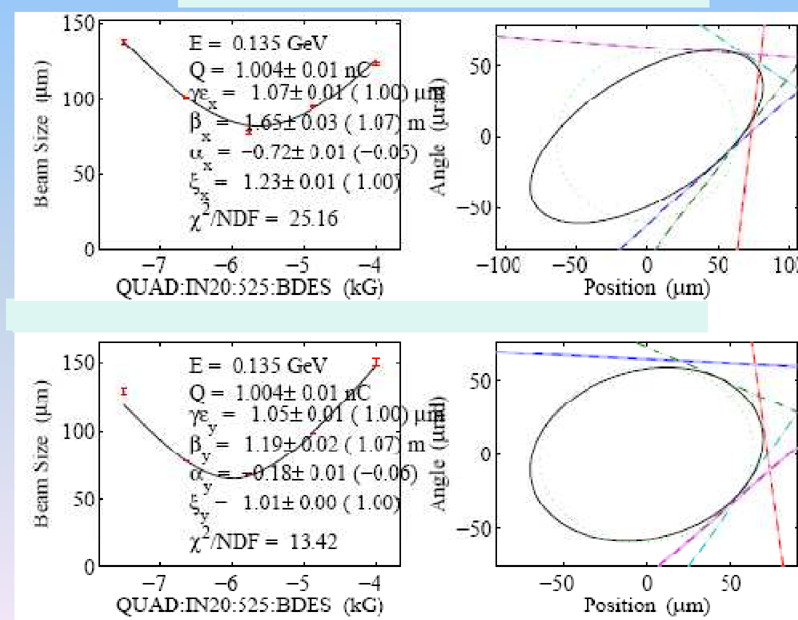
- **Projected & Slice Emittance**
- **Gain Length**
- **FEL Extraction**

The LCLS Injector with Diagnostics

Location of Injector Diagnostics



Projected emittance at 1 nC



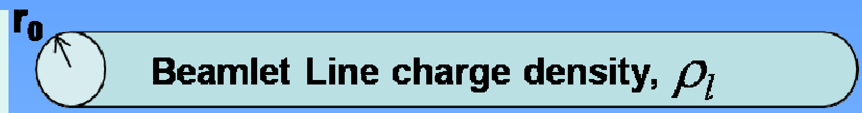
Nominal Operating Parameters

Parameter	Measured Value
Bunch Charge	250 pC
Projected Emittance (x-plane)	0.44 microns (rms)
Projected Emittance (y-plane)	0.46 microns (rms)
Slice Emittance (x-plane)	0.39 microns (rms)
Bunch Length	697.6 microns (rms)
Gain Length	3.7 meters
Electron Energy Loss	6.4 MeV

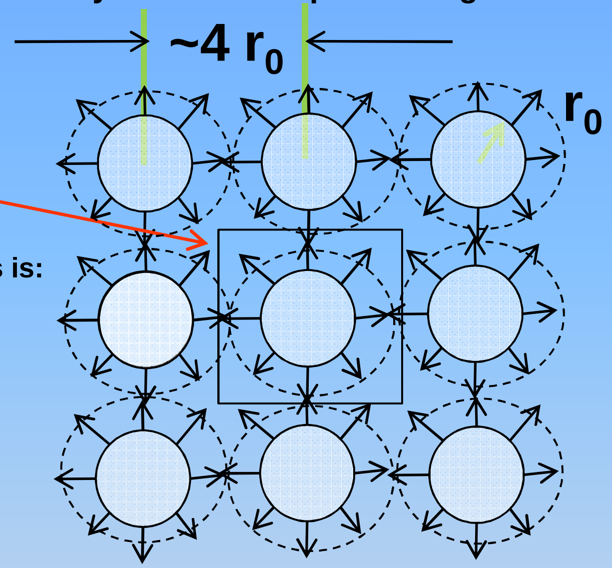
A Simple Model for the Emittance Growth

Basic Assumptions of Model

1. **Charge is distributed in a regular array of tubes, beamlets.**
2. **Beamlets see radial space charge force until they overlap.**
3. **After overlapping the sc-force becomes small, the electrons are left with radial velocity which becomes emittance.**



Beam consists of a rectangular array of beamlets, each driven outward by their radial space charge force



Radial electric field outside the tube/beamlet of charge:

$$E_r = \frac{\rho_l}{2\pi\varepsilon_0 r} \qquad \rho_l = \frac{Q}{N_b l_b} = \frac{16 r_0^2 Q}{\pi R^2 l_b}$$

Area (cell) occupied by each beamlet is: $(4r_0)^2$

∴ Number of beamlets is:

$$N_b = \frac{\pi R^2}{16 r_0^2}$$

$$E_r = \frac{8}{\pi^2}\frac{Q}{\varepsilon_0}\frac{r_0^2}{R^2 l_b r}$$

Integrate to get energy gain of an electron at radial edge of beamlet:

$$\frac{p_r^2}{2m} = \frac{8eQr_0^2}{\pi^2\varepsilon_0 R^2 l_b}\int_{r_0}^{ar_0}\frac{dr}{r} = \frac{8eQr_0^2}{\pi^2\varepsilon_0 R^2 l_b}\ln a$$

Leap of faith: Assume $\left\langle p_x^2 \right\rangle \approx \frac{p_r^2}{4}$

Then the emittance definition:

$$\varepsilon_n = \sigma_x \frac{\sqrt{\left\langle p_x^2 \right\rangle}}{mc}$$

Gives the emittance due to the rectangular array of beamlets:

$$\Delta\varepsilon_n \propto \sigma_x \frac{2r_0}{\pi R}\sqrt{\frac{eQ\ln a}{\varepsilon_0 mc^2 l_b}}$$

Simple Model Compared to the Expt.

General parameters:

Laser Radius: $R = 0.6mm \Rightarrow \sigma_x = 0.3mm$

Bunch Charge: $Q = 250\,pC$

Bunch Length: $l_b = 6.6\,ps = 2mm$

Nominal Emittance: $\varepsilon_{nominal} = 0.45 microns$

$$\Delta\varepsilon_n \approx \sigma_x \frac{2r_0}{\pi R}\sqrt{\frac{eQ\ln a}{\varepsilon_0 mc^2 l_b}}$$

$$\varepsilon_n = \sqrt{\Delta\varepsilon_n^2 + \varepsilon_{nominal}^2}$$

For 50 mesh pattern: $r_0 = 33\mu m$

$$\left.\frac{\Delta\varepsilon_n}{\sigma_x}\right|_{50mesh} = 5.8 microns/mm(rms)$$

$$\left.\Delta\varepsilon_n\right|_{50mesh} = 1.7 microns$$

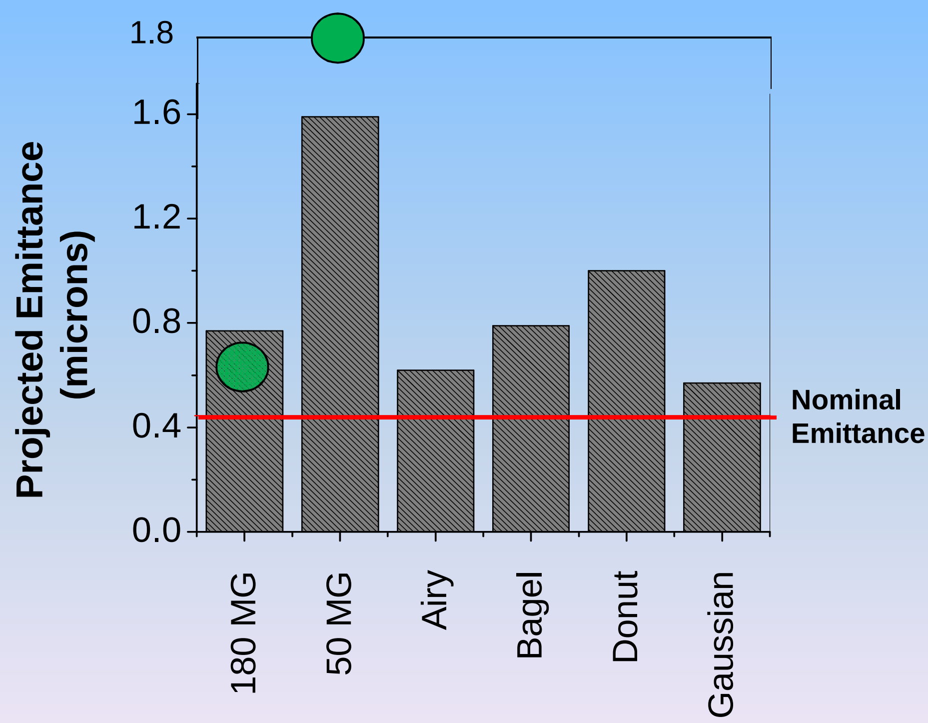
$$\left.\varepsilon_n\right|_{50mesh} = \sqrt{1.7^2 + 0.45^2} = 1.8 microns$$

For 180 mesh, the emittance will be 180/50= 3.6 times smaller: $r_0 = 9\mu m$

$$\left.\frac{\Delta\varepsilon_n}{\sigma_x}\right|_{180mesh} = 1.6 microns/mm(rms)$$

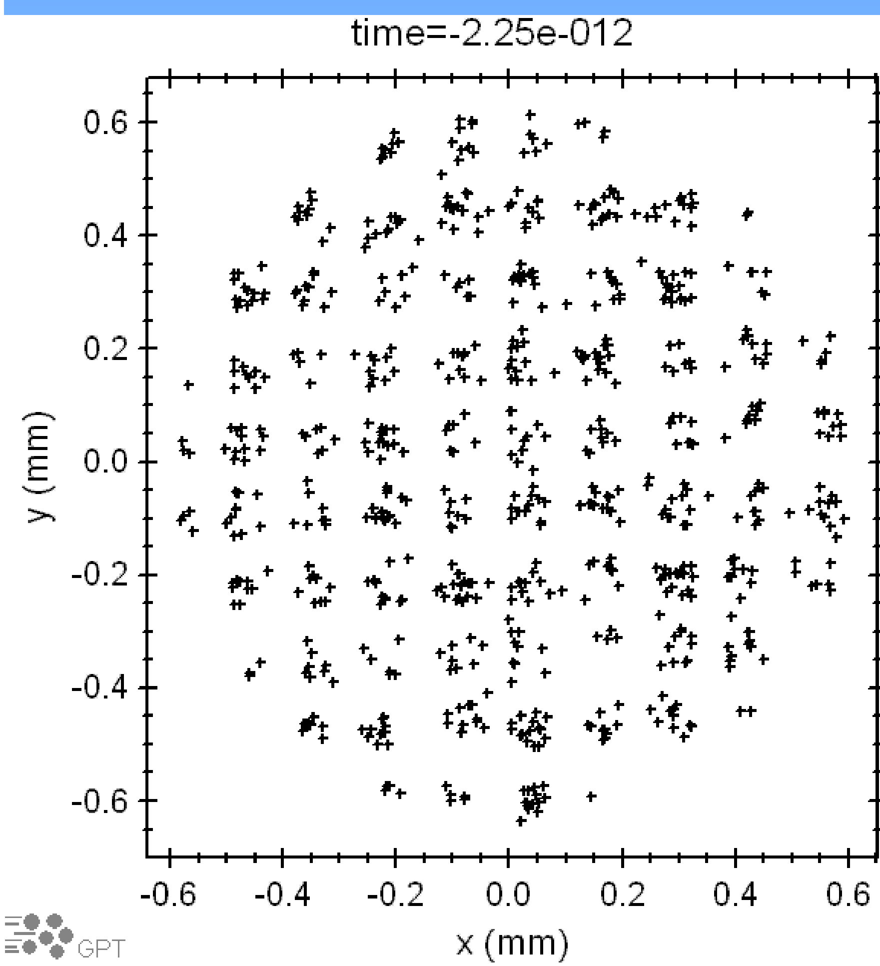
$$\left.\Delta\varepsilon_n\right|_{180mesh} = 0.48 microns$$

$$\left.\varepsilon\right|_{180mesh} = \sqrt{0.48^2 + 0.45^2} = 0.65 microns$$

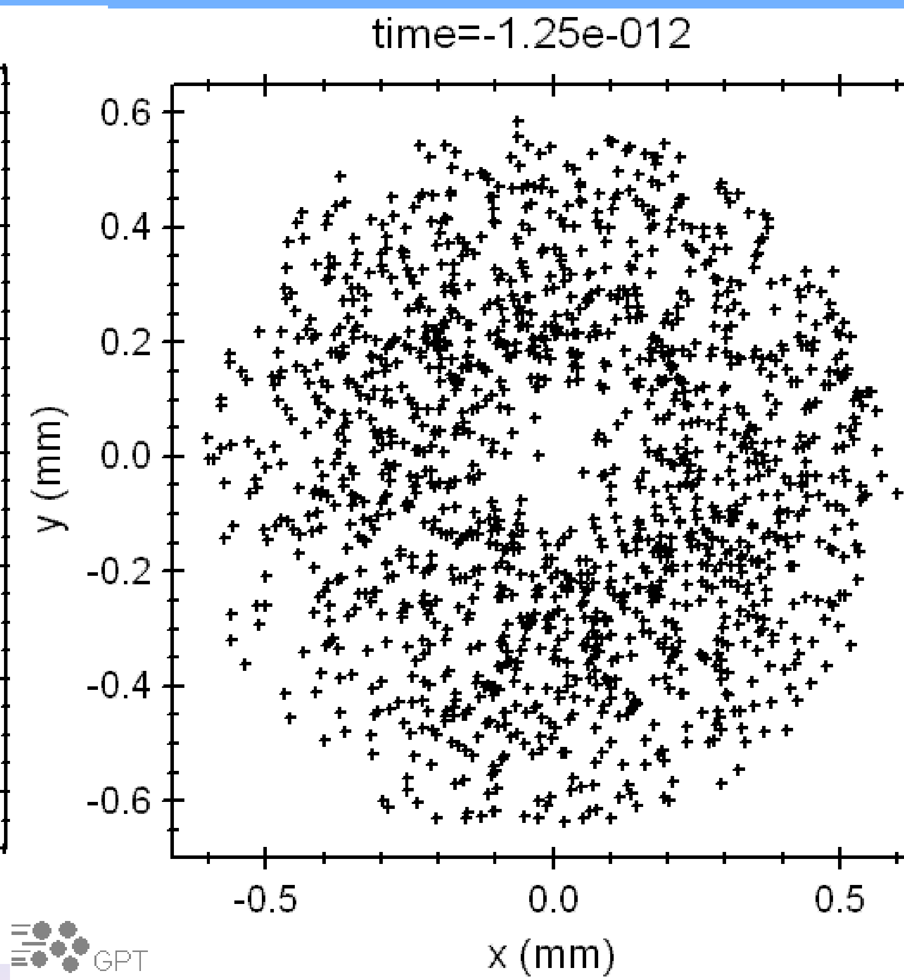


GPT Simulation Shows Beamlet Expansion in Early Life of the Beam

50 Mesh Laser Shape



Bagel Laser Shape



GPT: General Particle Tracer, Pulsar Physics, www.pulsar.nl