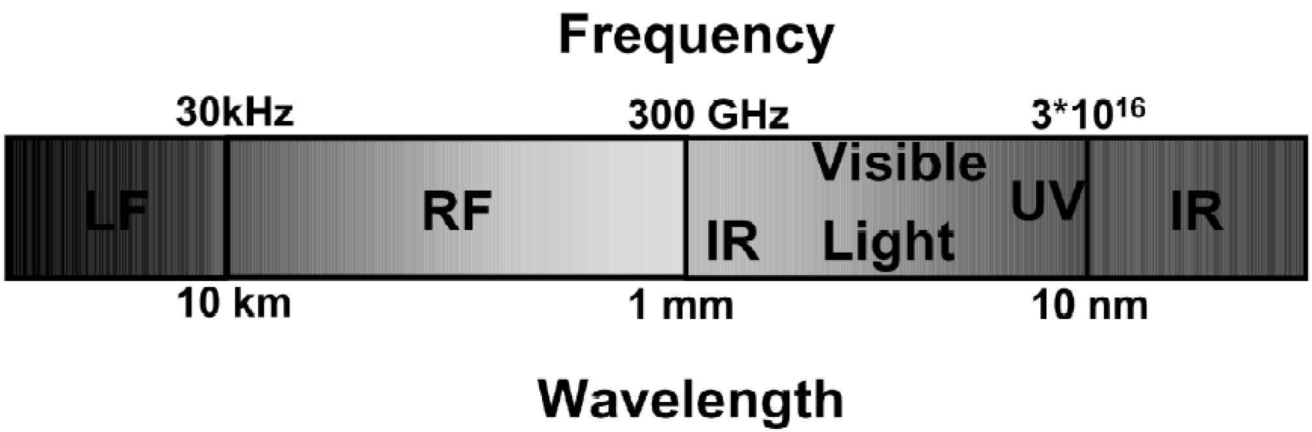
S-Parameters

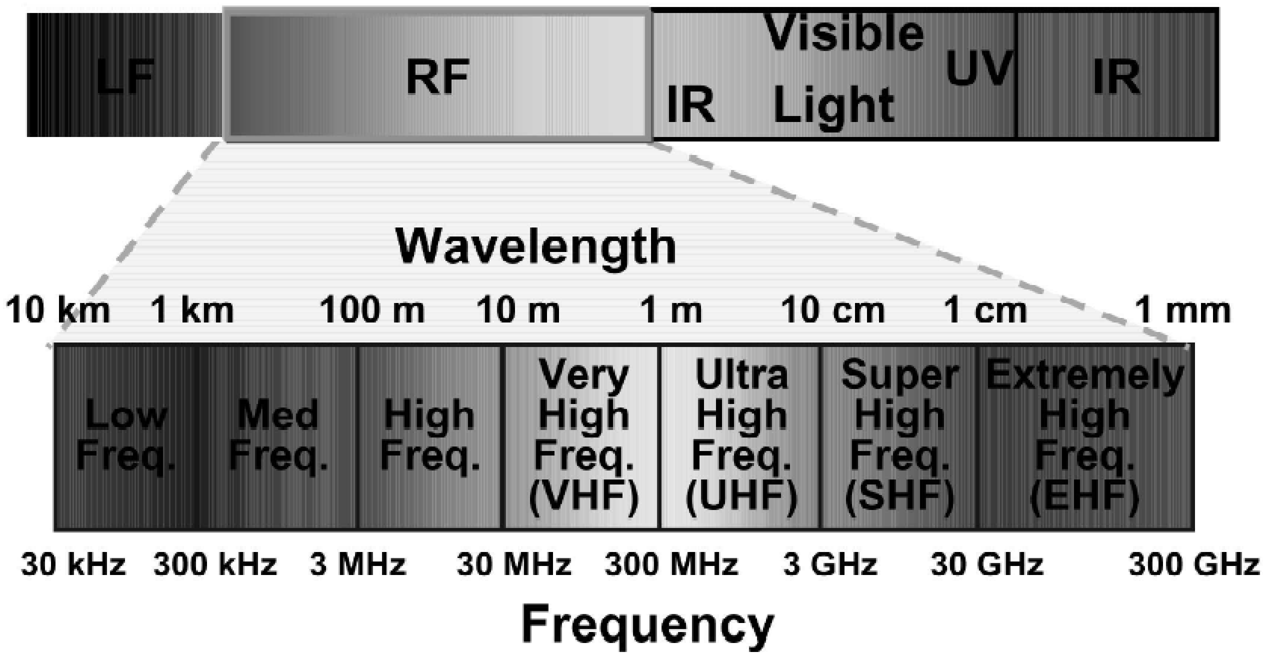
A. Nassiri-ANL

What is RF? (1)

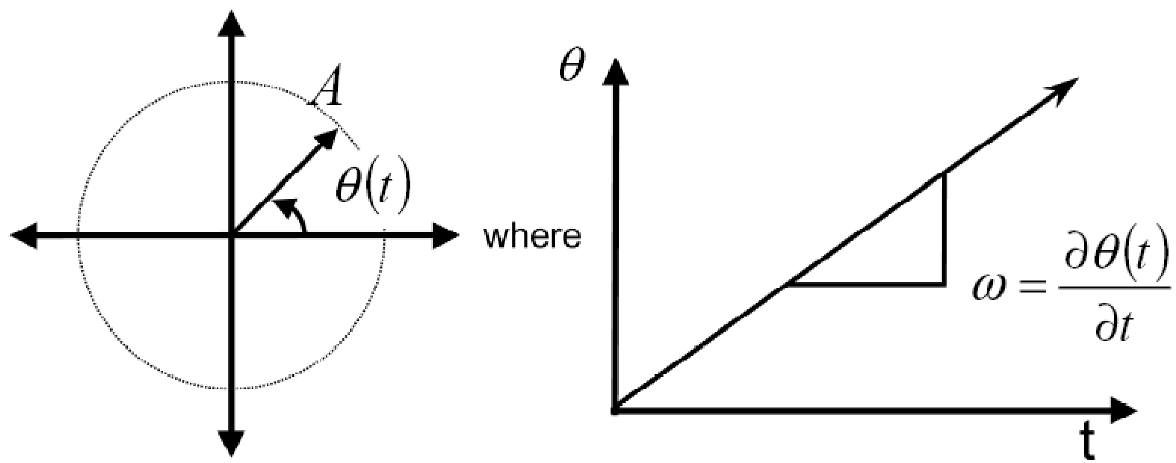
Radio Frequency (RF) ranges from 30KHz to 300GHz



What is RF? (2)


LF
RF
IR
Visible
Light
UV
IR
Wavelength
10 km
1 km
100 m
10 m
1 m
10 cm
1 cm
1 mm
Low Freq.
Med Freq.
High Freq.
Very High Freq. (VHF)
Ultra High Freq. (UHF)
Super High Freq. (SHF)
Extremely High Freq. (EHF)
30 kHz
300 kHz
3 MHz
30 MHz
300 MHz
3 GHz
30 GHz
300 GHz
Frequency

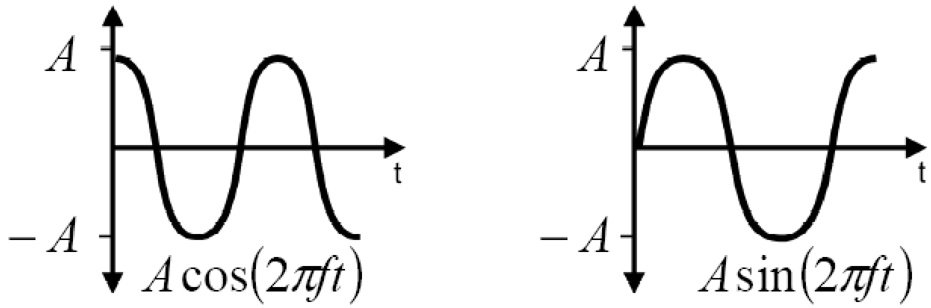
Fundamentals of a Wave (1)



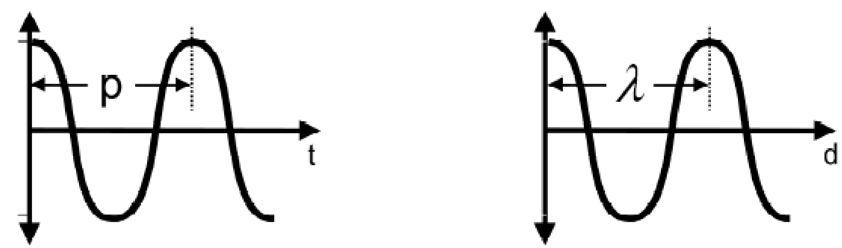
$$A\sin(\omega t+\phi(t)) = A\sin(\theta(t))$$

- In polar display (magnitude & phase represented together) a wave can be represented like a rotating vector (phasor)
- A wave's angular frequency ω is the derivative of phase with respect to time
- A wave's frequency is $f = \omega / 2\pi$

Fundamentals of a Wave (2)



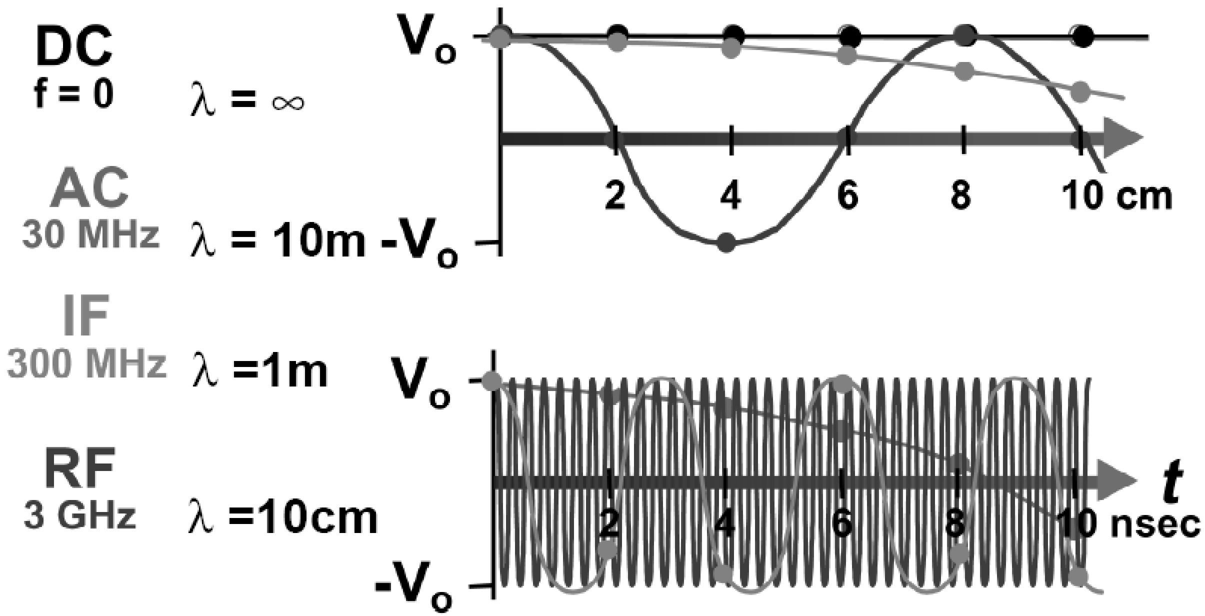
The phasor's x and y axis projections, as a function of time, map out cosine and sine waves, respectively



The speed of light defines the relationship between time and length for a wave

$$\lambda = \frac{c}{f} = c \bullet p$$

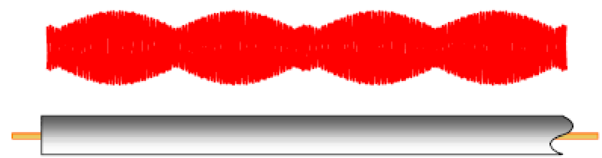
Fundamentals of a Wave (3)



Why Measure Signal Power at RF?

Low frequencies

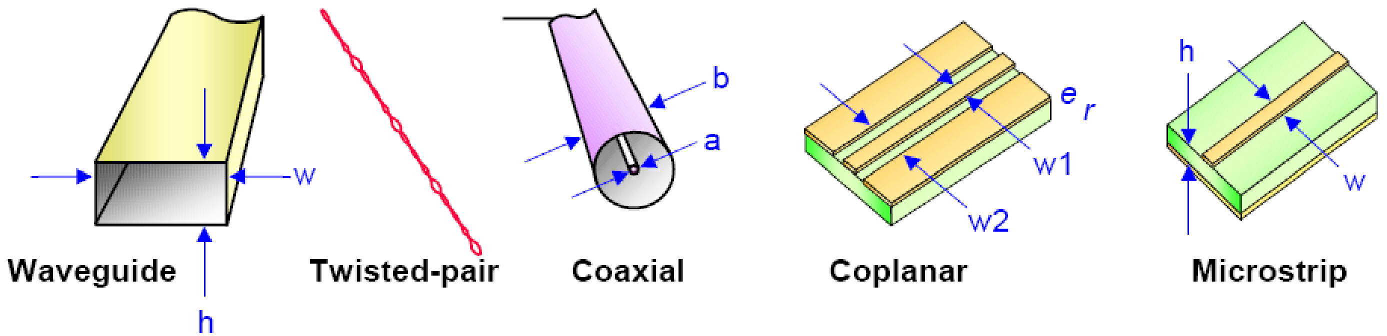
- wavelengths >> wire length
- current (I) travels down wires easily for efficient power transmission
- measured **voltage and current not dependent on position along wire**



High frequencies

- wavelength ~ or << length of conductor ⇒ **traveling waves**
- measured **envelope voltage dependent on position along line**
- **power** is a **reliable signal attribute** and is **constant along a lossless line**
- need **transmission lines** for efficient power transmission
- RMS voltage & current can be extracted directly from RF signal power

Transmission Line Basics

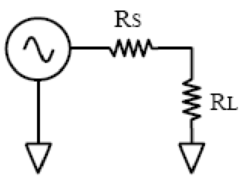


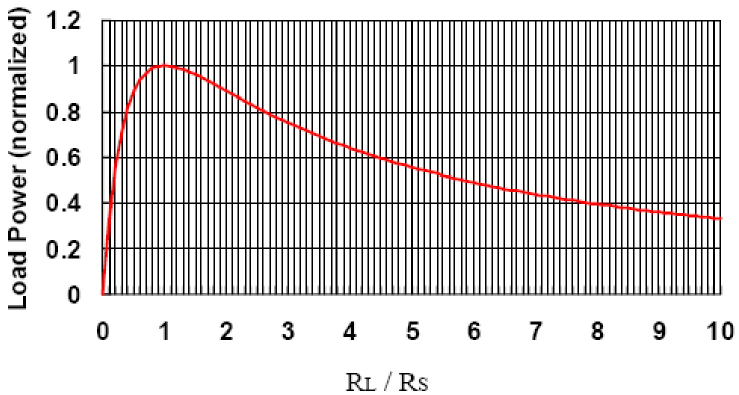
- The fundamental parameter of a transmission line is its characteristic impedance Z_0
- Z_0 describes the relationship between the voltage and current traveling waves and is a function of physical dimensions and the dielectric constant ε_r
- Z_0 is usually defined a real impedance (e.g. 50 or 75 ohms)
- For a loss-less transmission line:

$$Z_0 = \sqrt{\frac{L}{C}}$$

L - the distributed inductance of the line
C - the distributed capacitance of the line

Power Transfer Efficiency (1)

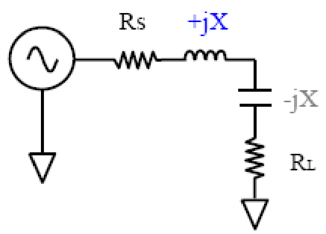


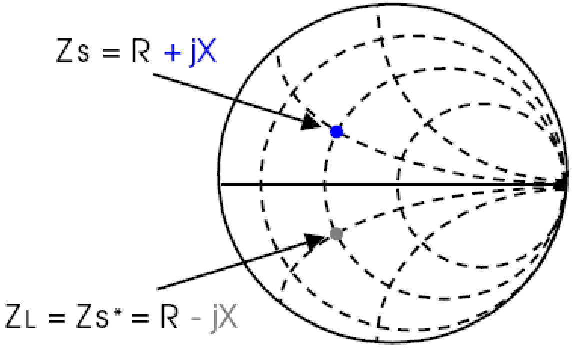


Maximum power is transferred when $R_L = R_S$

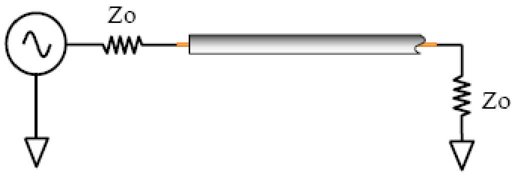
Power Transfer Efficiency (2)

For complex impedances, maximum power transfer occurs when $Z_L = Z_S^*$ (conjugate match)

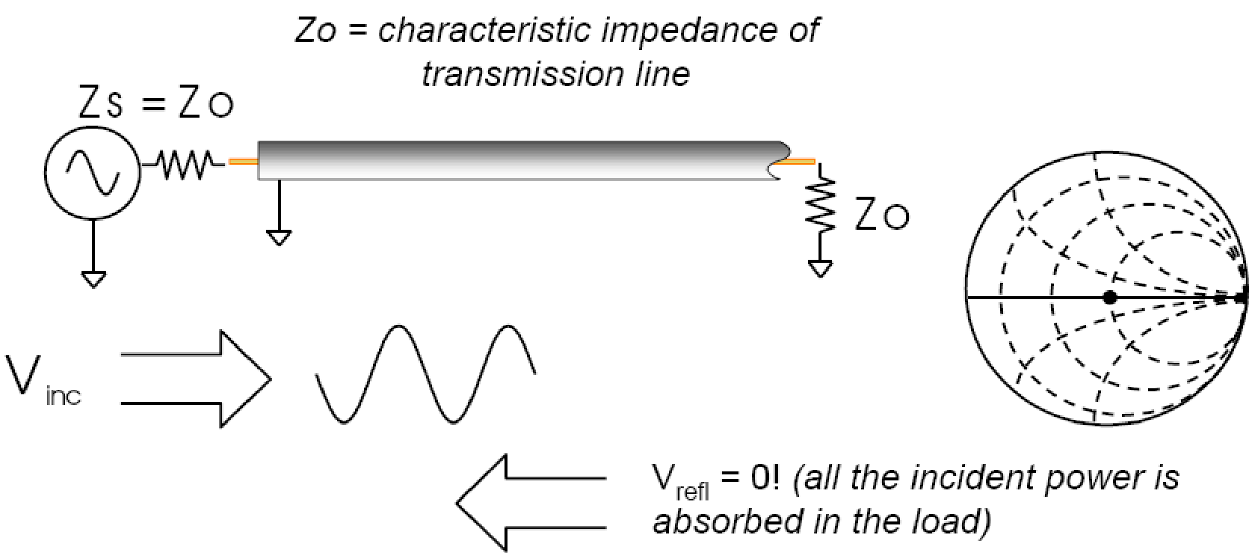




At high frequencies, maximum power transfer occurs when $R_S = R_L = Z_0$

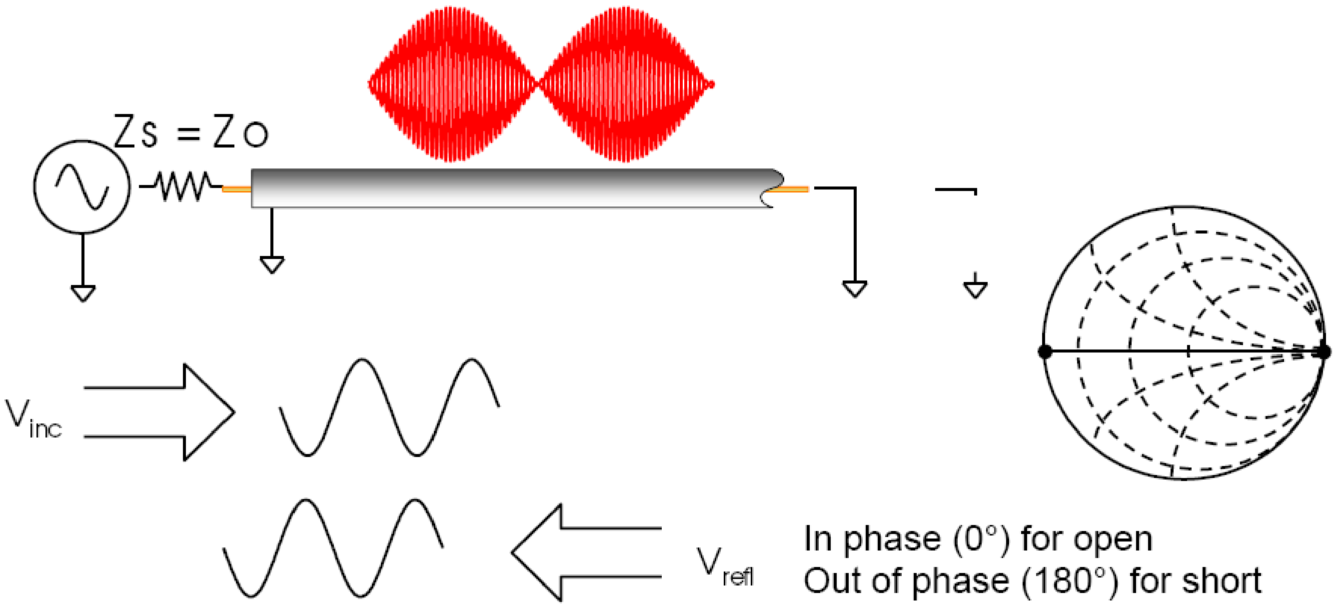


Transmission Line Terminated with Z_0



For reflection, a transmission line terminated in Zo behaves like an infinitely long transmission line

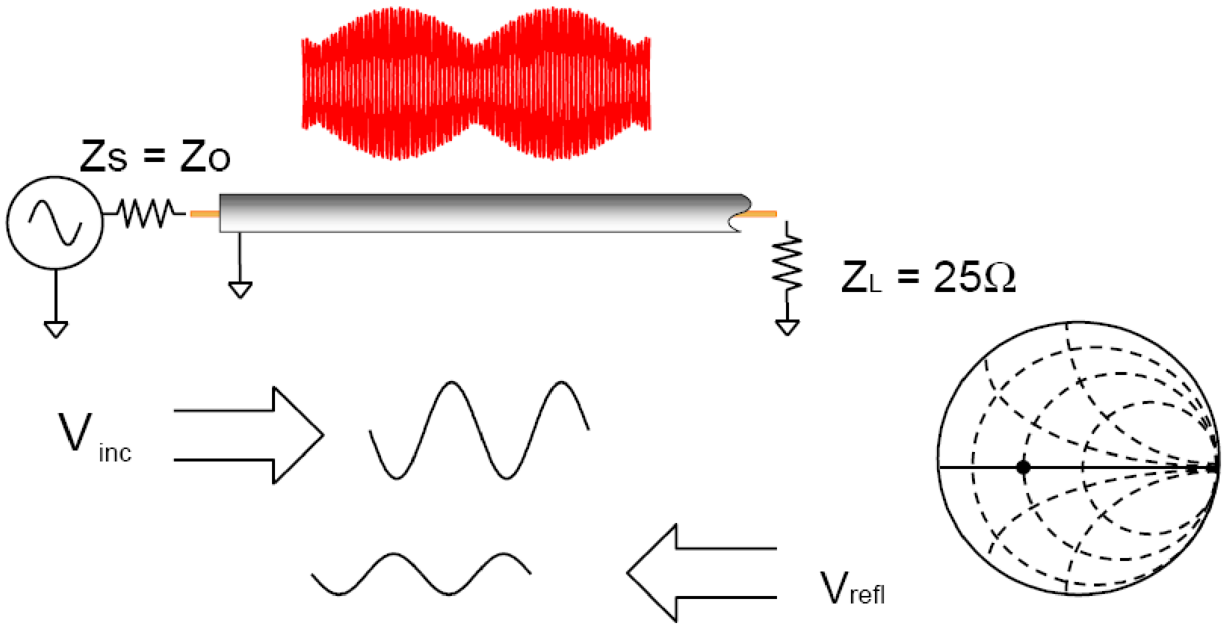
Transmission Line Terminated with Short, Open



In phase (0°) for open
Out of phase (180°) for short

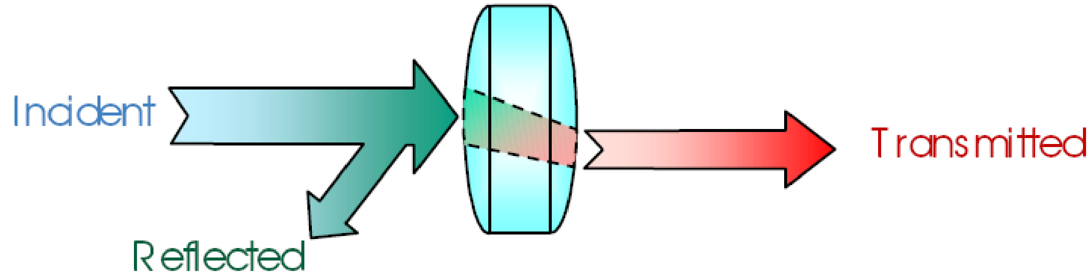
For reflection, a transmission line terminated in a short or open reflects all power back to source

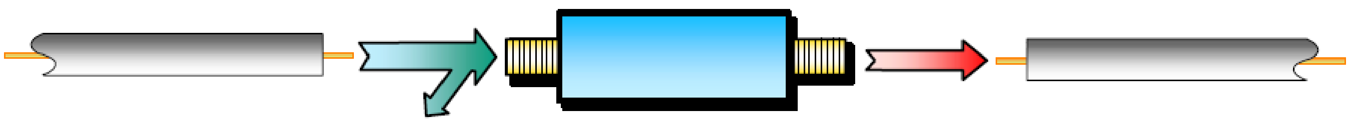
Transmission Line Terminated with 25Ω



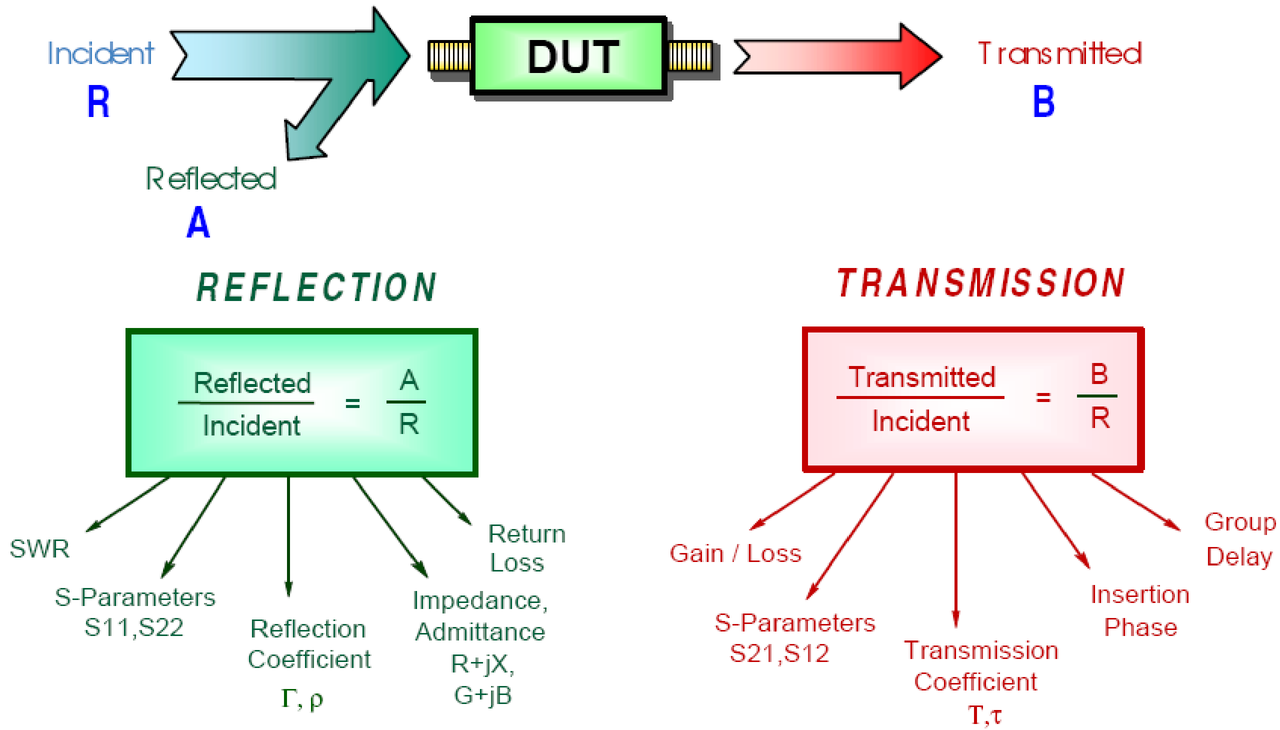
Standing wave pattern does not go to zero as with short or open

Light Wave Analogy to RF Energy





High-Frequency Device Characterization



Reflection Parameters

Reflection Coefficient (Γ)

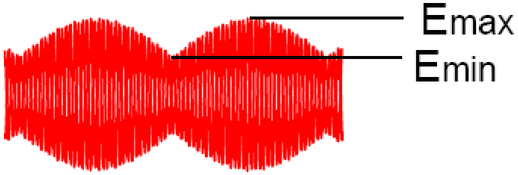
$$\Gamma = \frac{V_{reflected}}{V_{incident}} = \rho \angle \Phi = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Return Loss (RL)

$$RL = -20\log(\rho), \quad \rho = |\Gamma|$$

Voltage Standing Wave Ratio (VSWR)

$$VSWR = \frac{E_{max}}{E_{min}} = \frac{1+\rho}{1-\rho}$$



No reflection $(Z_L = Z_o)$		*Full reflection* $(Z_L = open, short)$
0	ρ	1
∞ dB	RL	0 dB
1	VSWR	∞

Transmission Parameters

Transmission Coefficient (T) $$T = \frac{V_{transmitted}}{V_{incident}} = \tau\angle\Phi$$

Insertion Loss (IL) $$IL\ (dB) = -20\log(\tau)$$

Gain $$Gain\ (dB) = 20\log(\tau)$$

Insertion Phase $$Insertion\ Phase\ (deg) = \angle\frac{V_{transmitted}}{V_{incident}} = \phi$$

Characterizing Unknown Devices

Using parameters (H, Y, Z, S) to characterize devices:

- gives us a linear behavioral model of our device
- measure parameters (e.g. voltage and current) versus frequency under various source and load conditions (e.g. short and open circuits)
- compute device parameters from measured data
- now we can predict circuit performance under any source and load conditions

H-parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$

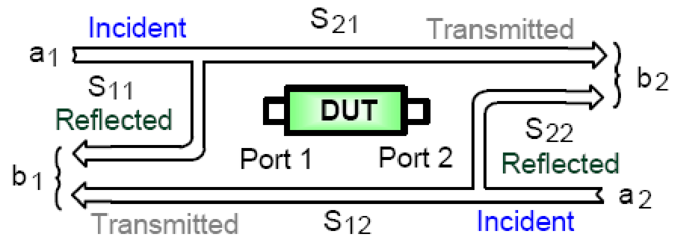
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0}$$ *(requires **short circuit**)*

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0}$$ *(requires **open circuit**)*

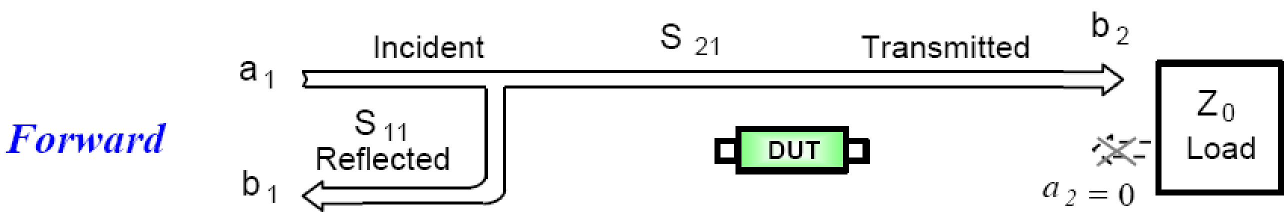
Why Use S-Parameters?

- relatively easy to **obtain** at high frequencies
  - measure voltage traveling waves with a vector network analyzer
  - don't need shorts/opens which can cause active devices to oscillate or self-destruct
- relate to **familiar** measurements (gain, loss, reflection coefficient ...)
- can **cascade** S-parameters of multiple devices to predict system performance
- can **compute** H, Y, or Z parameters from S-parameters if desired
- can import and use S-parameter files in **electronic-simulation** tools



$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

Measuring S-Parameters

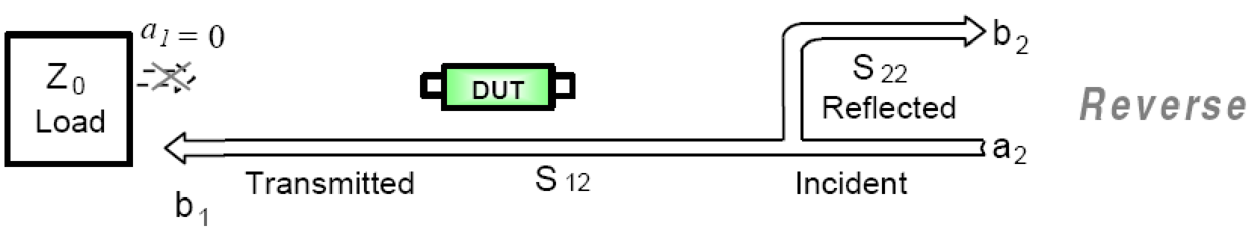


$$S_{11} = \frac{Reflected}{Incident} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$

$$S_{21} = \frac{Transmitted}{Incident} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

$$S_{22} = \frac{Reflected}{Incident} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

$$S_{12} = \frac{Transmitted}{Incident} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

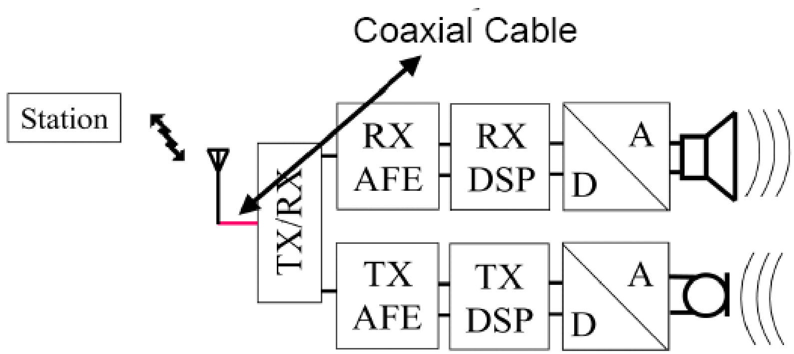


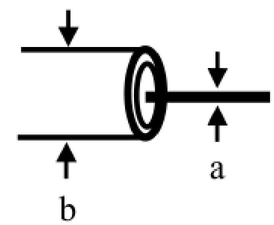
Equating S-Parameters with Common Measurement Terms

- S11 = forward reflection coefficient *(input match)*
- S22 = reverse reflection coefficient *(output match)*
- S21 = forward transmission coefficient *(gain or loss)*
- S12 = reverse transmission coefficient *(isolation)*

Remember, S-parameters are inherently linear quantities -- however, we often express them in a log-magnitude format

Why 50Ω? (1)





Use of coaxial cable: Antenna / PCB interface

Considering the dielectric **air** (μ_R =1, ε_R = 1) the characteristic impedance is:

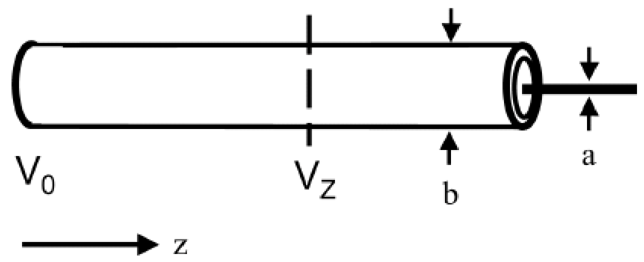
$$Z_0 = \sqrt{\frac{\mu_0 \mu_R}{\varepsilon_0 \varepsilon_R}} \frac{\ln \frac{b}{a}}{2\pi} \approx 60 \ln \frac{b}{a}$$

Receiver: **attenuation** of the coaxial cable as low as possible - Z_0 = ?

Transmitter: **power efficiency** of the coaxial cable as high as possible - Z_0 = ?

Why 50Ω? (2)

Cable attenuation



Considering only resistive loss: $$V_z = V_0 e^{-\alpha z}$$

$$\alpha \approx \frac{2R^*}{Z_0}$$ R* - resistance / meter

$$R^* \approx \frac{1}{2\pi\delta\sigma}\left(\frac{1}{a}+\frac{1}{b}\right)$$ σ - wire conductivity [1/Ωm]

$$\delta = \sqrt{\frac{1}{\pi f \mu_0 \mu_R \sigma}}$$

Why 50Ω? (3)

$$\alpha \approx \frac{R^*}{2Z_0} \approx \frac{\frac{1}{2\pi\delta\sigma}\left(\frac{1}{a}+\frac{1}{b}\right)}{2\left[60\ \ln\left(\frac{b}{a}\right)\right]}$$

Minimum of α:

$$\frac{\partial\alpha}{\partial a} = 0 \Rightarrow \frac{\partial}{\partial a}\frac{\left(\frac{1}{a}+\frac{1}{b}\right)}{\ln\left(\frac{b}{a}\right)} = 0$$

$$\ln\left(\frac{b}{a}\right) = 1 + \frac{a}{b}$$

and after few iterations

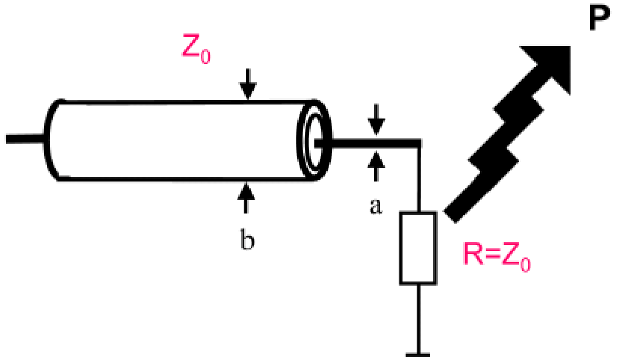
$$\frac{b}{a} = 3.6$$

$$Z_0 \approx 60\ \ln(3.6) \approx 77\ \Omega$$

75Ω - is used for Radio/TV cables

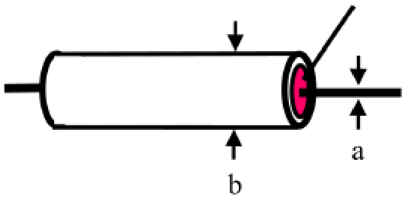
Why 50Ω? (4)

Power transfer efficiency



$$P \propto \frac{V^2}{Z_0} = \frac{V^2}{60 \ln\left(\frac{b}{a}\right)}$$

Power efficiency will be limited by the electric field between the 2 conductors



$$E_{\max} \propto \frac{V}{a \ln\left(\frac{b}{a}\right)}$$

Why 50Ω? (5)

$$P = \frac{E^2_{\max} a^2 \ln\left(\frac{b}{a}\right)}{60}$$

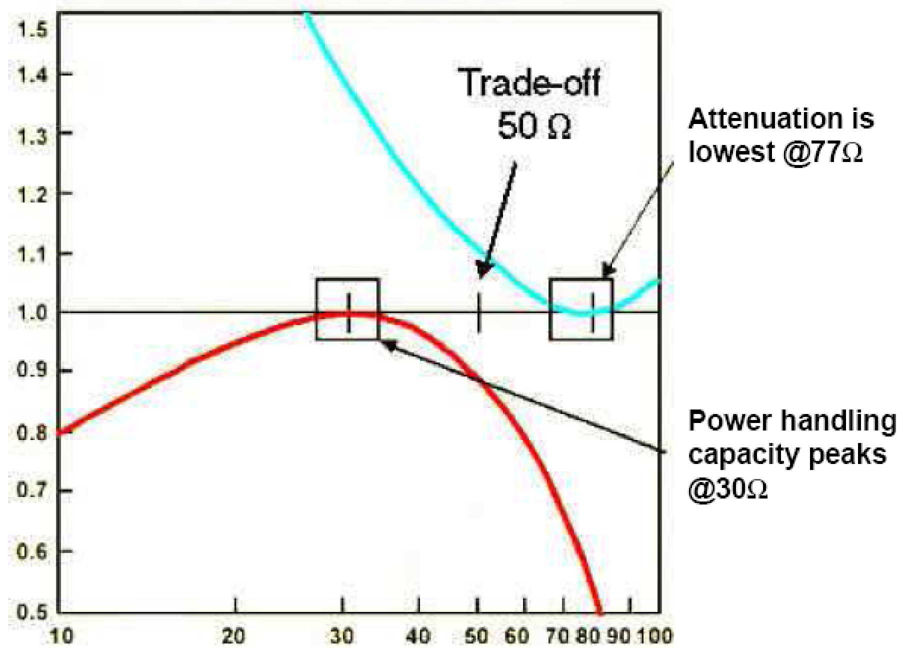
Maximum power efficiency: $$\frac{\partial P}{\partial a} = 0 \Rightarrow \frac{\partial}{\partial a}\left[a^2 \ln\left(\frac{b}{a}\right)\right] = 0$$

$$\frac{b}{a} = \sqrt{e}$$

$$Z_0 \approx 60 \ln\left(\sqrt{e}\right) \approx 30\,\Omega$$

Why 50Ω? (6)

Attenuation / Power efficiency (normalized)



Characteristic impedance for coaxial airlines (Ω)

Why use dB? (1)

In applications where analog signals levels span multiple decades, logarithmic scales are employed

$$x(Bel) = \log_{10}\frac{P}{P_{ref}} = \log_{10} a$$

Example: P/P_{ref} = 100:1 = 2*Bel*

$$x(Deci-Bel) = x(dB) = 10\log_{10}\frac{P}{P_{ref}} = 10\log_{10} a$$

Linear to logarithmic transformation $10\log_{10} a = xdB$

Logarithmic to linear transformation $10^{\frac{xdB}{10}} = a$

Why use dB? (2)

„dB“ properties

1) Linear multiplication ⇒ dB addition $$10\log_{10}(a \bullet b) = 10\log_{10} a + 10\log_{10} b$$

2) Linear division ⇒ dB subtraction $$10\log_{10}(a / b) = 10\log_{10} a - 10\log_{10} b$$

3) Linear power ⇒ dB multiplication $$10\log_{10}(a^x) = x \bullet 10\log_{10} a$$

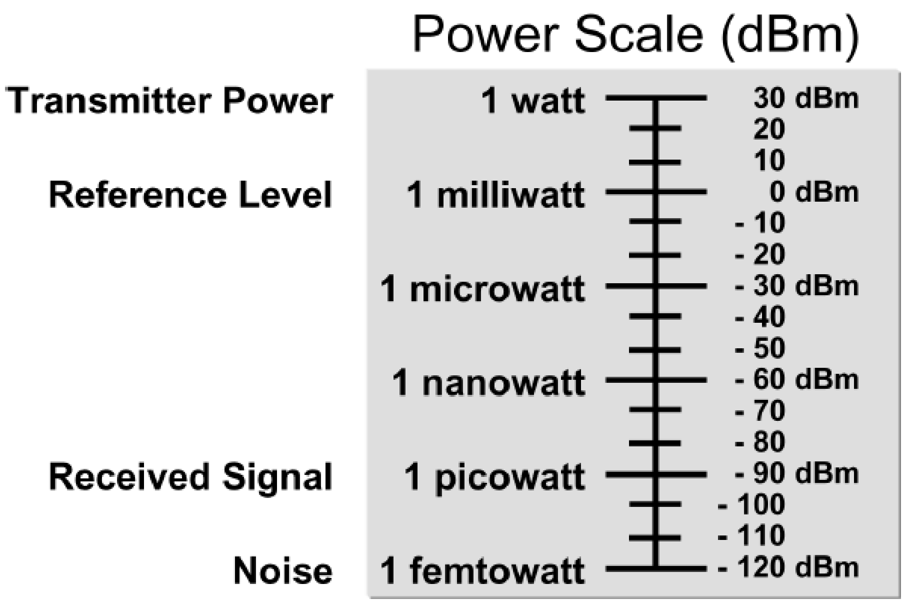
Power can be expressed in dBW, dBm:

$$PdB(W) = 10\log_{10}\left(\frac{\mathrm{P}}{1\mathrm{W}}\right)$$

$$PdBm(W) = 10\log_{10}\frac{P}{1mW}$$

dBm – logarithmic power measurements employing **1mW** as the reference level

Why use dB? (3)


Power Scale (dBm)
Transmitter Power
Reference Level
Received Signal
Noise
1 watt
1 milliwatt
1 microwatt
1 nanowatt
1 picowatt
1 femtowatt
30 dBm
20
10
0 dBm
- 10
- 20
- 30 dBm
- 40
- 50
- 60 dBm
- 70
- 80
- 90 dBm
- 100
- 110
- 120 dBm

Why use dB? (4)

Basic Relationships

1/10	$10^{-1.0}$	-10 dB
1/5	$10^{-0.7}$	-7 dB
1/2	$10^{-0.3}$	-3 dB
1	$10^{0.0}$	0 dB
2	$10^{0.3}$	3 dB
5	$10^{0.7}$	7 dB
10	$10^{1.0}$	10 dB
20	$10^{1.3}$	13 dB
50	$10^{1.7}$	17 dB
100	$10^{2.0}$	20 dB

Basic RF dB Arithmetic

2 x 2 = 4	•	3 + 3 = 6 dB
5 x 5 = 25	•	7 + 7 = 14 dB
10 / 2 = 5	•	10 - 3 = 7 dB
2 x 2 x 2 x 10 = 80	•	3 + 3 + 3 + 10 = 19 dB
5 x 100 = 500	•	7 + 20 = 27 dB
1000 / 2 = 500	•	30 - 3 = 27 dB
2 / 100 =0.02	•	3- 20 = - 17 dB

Why use dB? (5)

A voltage or a current can be also expressed in dB: $VdB = 20\log_{10}\left(\frac{V}{V_{ref}}\right)$

$$\mathrm{V}dB(V) = 20\log_{10}\left(\frac{\mathrm{V}}{1V}\right) \qquad VdBm(V) = 20\log_{10}\left(\frac{V}{1mV}\right)$$

The factor 20 appears because the power is proportional to the squared voltage
Attention : the factor values for powers expressed in (dB) should be multiplied by 2 when referred like voltages (dB)!

Example:

a factor of 2 represents in “power” 3dB
a factor of 2 represents in “voltage / current” 6dB

Why use dB? (6)

Examples

1. For a GSM mobile phone the max signal power is 1W while the weakest signal is only 10μW. Determine the dynamic range.

- for the linear scale: 1W/10μW = 10^5
- in dB: $$10\log_{10} 10^5 = 50dB$$

The range from 1W to 10μW is 50dB

2. An amplifier has a 26dB **voltage gain**. What is the output voltage for an input voltage of 100mV.

26dB = 20dB + 6dB $\Rightarrow$ the multiplication factor is 10x2 = 20

V_{out} = V_{in}x20 = 100mVx20 = 2V

Why use dB? (7)

3. Calculate the voltage corresponding to a power of 0dBm and -20dBm in a 50Ω system.

P_1 = 0dBm ⇒ P_1 = 1mW = 10^{-3} W,

$P_1 = V_1^2/R \Rightarrow V_1^2 = P_1 x R = 0.05V^2 \Rightarrow V_{rms} = 0.224V = 224mV$

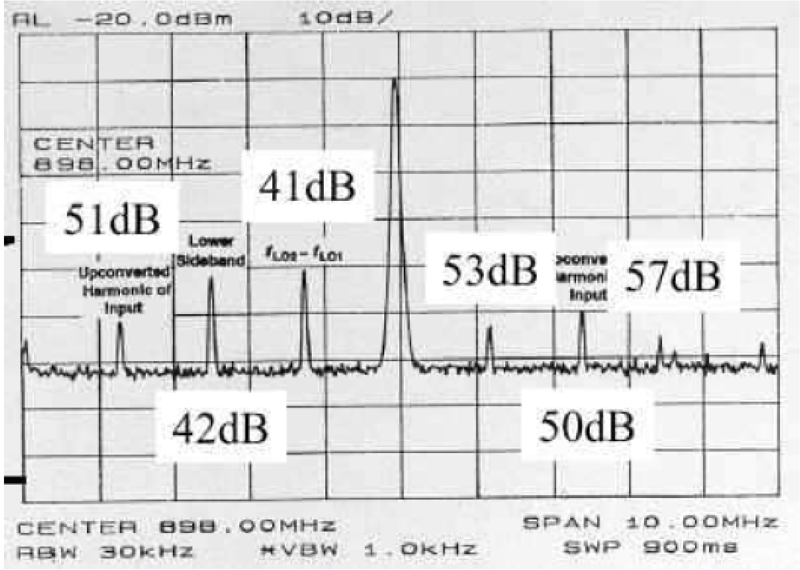
P_2 = -20dBm ⇒ $P_2 = 10^{-20/10}$ mW = 10^{-5} W,

$P_2 = V_2^2/R \Rightarrow V_2^2 = P_2 x R = 0.0005V^2 \Rightarrow V_{rms} = 0.0224V = 22.4mV$

Why use dB? (8)

dBc (c = carrier) ⇒ difference in dB between an interfering signal / noise (interferer) and the desired signal (carrier)

$$dBc = 10\log_{10}\left(\frac{P_{carrier}}{P_{interferer}}\right)$$
$$= P_{carrier}(dB) - P_{interferer}(dB)$$



For 50Ω System

dBm	mW	dBmV	mV_{RMS}	mV_P	mV_{PP}
-50	0.000	-3.0	0.7	1.0	2.0
-45	0.000	2.0	1.3	1.8	3.6
-40	0.000	7.0	2.2	3.2	6.3
-35	0.000	12.0	4.0	5.6	11.2
-30	0.001	17.0	7.1	10.0	20.0
-25	0.003	22.0	12.6	17.8	35.6
-20	0.010	27.0	22.4	31.6	63.2
-15	0.032	32.0	39.8	56.2	112.5
-10	0.100	37.0	70.7	100.0	200.0
-5	0.316	42.0	125.7	177.8	355.7
0	1.000	47.0	223.6	316.2	632.5
1	1.259	48.0	250.9	354.8	709.6
2	1.585	49.0	281.5	398.1	796.2
3	1.995	50.0	315.9	446.7	893.4
4	2.512	51.0	354.4	501.2	1002.4
5	3.162	52.0	397.6	562.3	1124.7
6	3.981	53.0	446.2	631.0	1261.9
7	5.012	54.0	500.6	707.9	1415.9
8	6.310	55.0	561.7	794.3	1588.7
9	7.943	56.0	630.2	891.3	1782.5
10	10.000	57.0	707.1	1000.0	2000.0
11	12.589	58.0	793.4	1122.0	2244.0
12	15.849	59.0	890.2	1258.9	2517.9
13	19.953	60.0	998.8	1412.5	2825.1
14	25.119	61.0	1120.7	1584.9	3169.8
15	31.623	62.0	1257.4	1778.3	3556.6
16	39.811	63.0	1410.9	1995.3	3990.5
17	50.119	64.0	1583.0	2238.7	4477.4
18	63.096	65.0	1776.2	2511.9	5023.8
19	79.433	66.0	1992.9	2818.4	5636.8
20	100.000	67.0	2236.1	3162.3	6324.6
21	125.893	68.0	2508.9	3548.1	7096.3
22	158.489	69.0	2815.0	3981.1	7962.1
23	199.526	70.0	3158.5	4466.8	8933.7
24	251.189	71.0	3543.9	5011.9	10023.7
25	316.228	72.0	3976.4	5623.4	11246.8
26	398.107	73.0	4461.5	6309.6	12619.1
27	501.187	74.0	5005.9	7079.5	14158.9
28	630.957	75.0	5616.7	7943.3	15886.6
29	794.328	76.0	6302.1	8912.5	17825.0
30	1000.000	77.0	7071.1	10000.0	20000.0

Conversion table for 50Ohm systems