

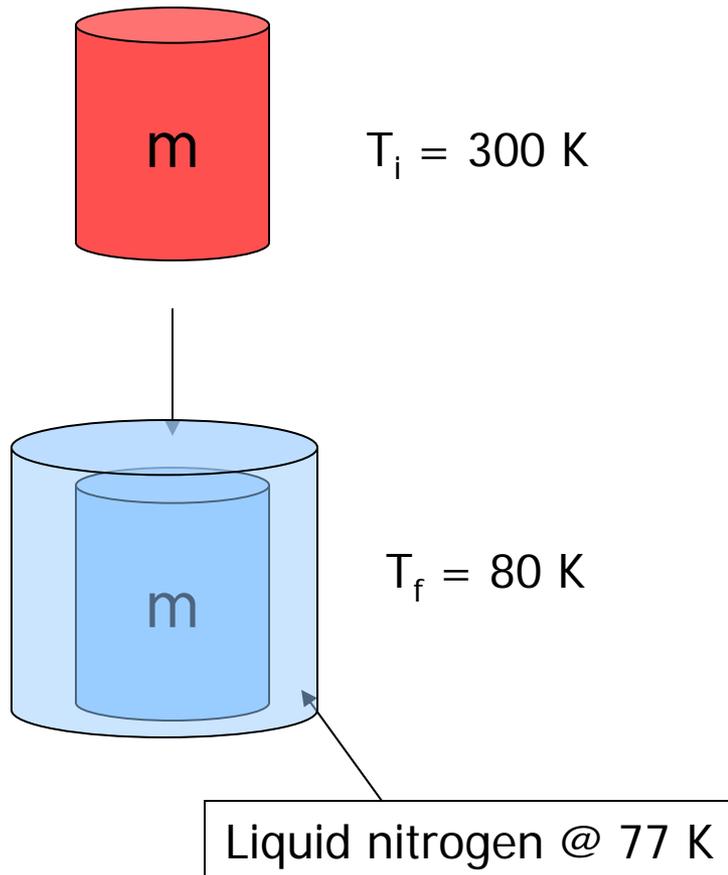
# 1.2 Low Temperature Properties of Materials

- Materials properties affect the performance of cryogenic systems.
- Properties of materials vary considerably with temperature
  - Thermal Properties: Heat Capacity (internal energy), Thermal Expansion
  - Transport Properties: Thermal conductivity, Electrical conductivity
  - Mechanical Properties: Strength, modulus or compressibility, ductility, toughness
  - Superconductivity
- Many of the materials properties have been recorded and models exist to understand and characterize their behavior
  - Physical models
  - Property data bases (CryoComp®)
  - NIST: [www.cryogenics.nist.gov/MPropsMAY/material%20properties.htm](http://www.cryogenics.nist.gov/MPropsMAY/material%20properties.htm)

What are the cryogenic engineering problems that involve materials?

# Cooldown of a solid component

Cryogenics involves cooling things to low temperature. Therefore one needs to understand the process.

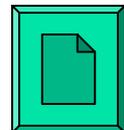


- If the mass and type of the object and its material are known, then the heat content at the designated temperatures can be calculated by integrating 1<sup>st</sup> Law.

$$dQ = Tds = dE + p\overset{\sim 0}{dv}$$

- The heat removed from the component is equal to its change of internal energy,

$$\Delta E = m \left( \int_{T_f}^{T_i} C dT \right)$$



# Heat Capacity of Solids

- General characteristics:
- The heat capacity is defined as the change in the heat content with temperature. The heat capacity at constant volume is,

$$C_v = \left. \frac{\partial E}{\partial T} \right|_v \quad \text{and at constant pressure, } C_p = T \left. \frac{\partial s}{\partial T} \right|_p$$

- These two forms of the heat capacity are related through the following thermodynamic relation,

$$C_p - C_v = -T \left. \left( \frac{\partial v}{\partial T} \right)_p^2 \frac{\partial p}{\partial v} \right|_T = \frac{T v \beta^2}{\kappa}$$

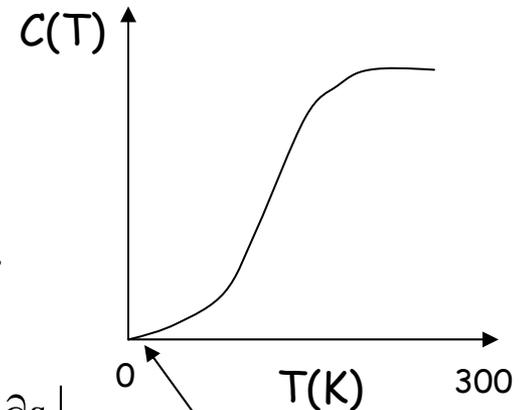
Note:  $C_p - C_v$  is small except for gases, where  $\sim R = 8.31 \text{ J/mole K}$ .

$$\kappa = - \left. \frac{1}{v} \frac{\partial v}{\partial p} \right|_T$$

Isothermal  
compressibility

$$\beta = - \left. \frac{1}{v} \frac{\partial v}{\partial T} \right|_p$$

Volume  
expansivity



3<sup>rd</sup> Law:  $C \rightarrow 0$  as  $T \rightarrow 0$

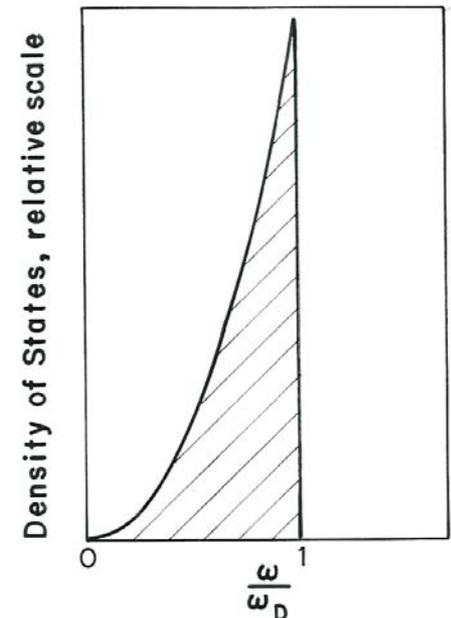
# Heat Capacity of Solids (Lattice Contribution)

- Lattice vibration (Phonon) excitations are the main contribution to the heat capacity of solids at all except the lowest temperatures.
- Internal energy of a phonon gas is given by  $E_{ph} = \frac{h}{2\pi} \int \omega d\omega D(\omega) n(\omega)$ 
  - $D(\omega)$  is the density of states and depends on the choice of model
  - $n(\omega)$  is the statistical distribution function

$$n(\omega) = \frac{1}{e^{h\omega/2\pi k_B T} - 1}$$

$h$  = Planck's constant =  $6.63 \times 10^{-34}$  J.s  
 $k_B$  = Boltzmann's constant =  $1.38 \times 10^{-23}$  J/K

- Debye Model for density of states
  - Constant phonon velocity
  - Maximum frequency =  $\omega_D$
  - Debye temperature:  $\Theta_D = h\omega_D/2\pi k_B$



# Debye Internal Energy & Heat Capacity

- In Debye model the internal energy and heat capacity have simple forms

$$E_{ph} = 9RT \left( \frac{T}{\theta_D} \right)^3 \int_0^{x_D} dx \frac{x^3}{e^x - 1}$$

$$C_{ph} = 9R \left( \frac{T}{\theta_D} \right)^3 \int_0^{x_D} dx \frac{x^4 e^x}{(e^x - 1)^2}$$

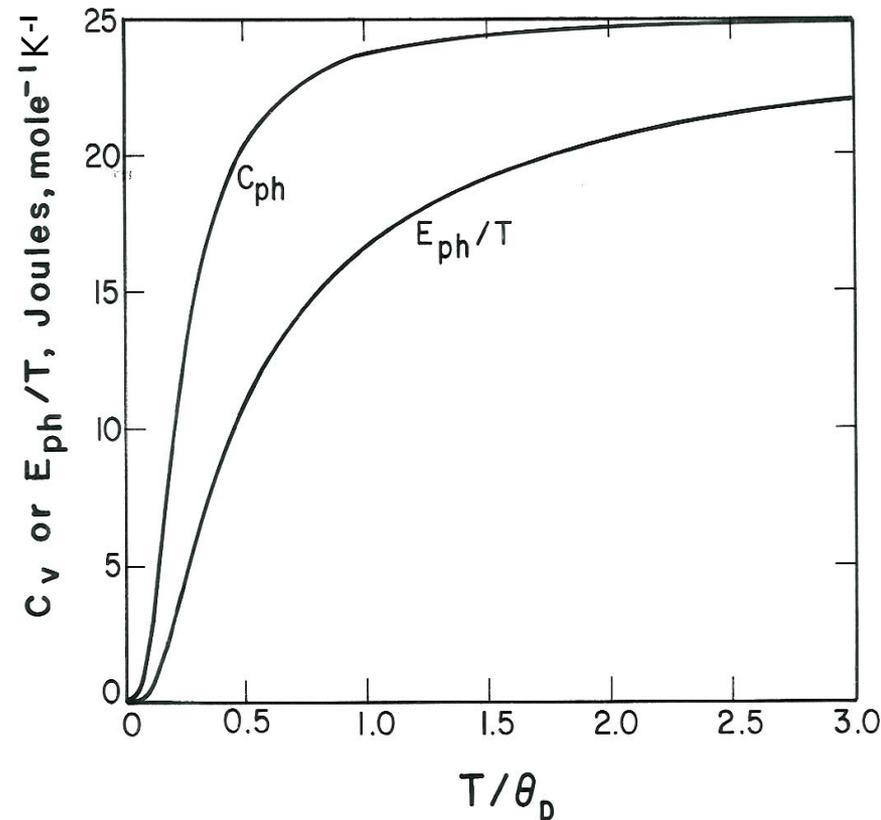
- where  $x = h\omega/2\pi k_B T$  and  $x_D = \Theta_D/T$

- Limits:

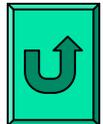
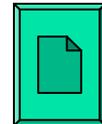
- $T > \Theta_D$ ,  $C_{ph} \approx 3R$  (Dulong-Petit value)

- $T \ll \Theta_D$ ,  $C_{ph} \approx 234R \left( \frac{T}{\theta_D} \right)^3$

$$R = N_0 k_B = 8.31 \text{ J/mole K (gas constant)}$$



Example



# Values of Debye Temperature (K)

Metals <sup>1</sup>	$\Theta_D$
Ag	225
Al	428
Au	165
Cd	209
Cr	630
Cu	343
Fe	470
Ga	320
Hf	252
Hg	71.9
In	108
Nb	275
Ni	450
Pb	105
Sn	200
Ti	420
V	380
Zn	327
Zr	291

Non-metals & Compounds <sup>2</sup>	$\Theta_D$
C (graphite)	2700
C (diamond)	2028
H <sub>2</sub> (solid)	105-115
He (solid)	30
N <sub>2</sub> (solid)	70
O <sub>2</sub> (solid)	90
Si	630
SiO <sub>2</sub> (quartz)	255
TiO <sub>2</sub>	450

The Debye temperature is normally determined by measurements of the specific heat at low temperature. For  $T \ll \Theta_D$ ,

$$C_{ph} \approx 234R \left( \frac{T}{\theta_D} \right)^3$$

<sup>1</sup> Kittel, Introduction to Solid State Physics

<sup>2</sup> Timmerhaus and Flynn, Cryogenic Process Engineering

# Electronic Heat Capacity (Metals)

- The free electron model treats electrons as a gas of particles obeying Fermi-Dirac statistics

$$E_e = \int \varepsilon d\varepsilon D(\varepsilon) f(\varepsilon)$$

- Where

$$f(\varepsilon) = \frac{1}{e^{\varepsilon - \varepsilon_f / k_B T} + 1} \quad \varepsilon_f \equiv \frac{h^2}{8\pi^2 m} \left( 3\pi^2 \frac{N}{V} \right)^{2/3}$$

- and  $D(\varepsilon) = \frac{V}{2\pi^2} \left( \frac{8\pi^2 m}{h^2} \right)^{3/2} \varepsilon^{1/2}$

- At low temperatures,  $T \ll \varepsilon_f / k_B \sim 10^4$  K

$$C_e \approx \frac{1}{3} \pi^2 D(\varepsilon_f) k_B^2 T = \gamma T$$

The electronic and phonon contributions to the heat capacity of copper are approximately equal at 4 K

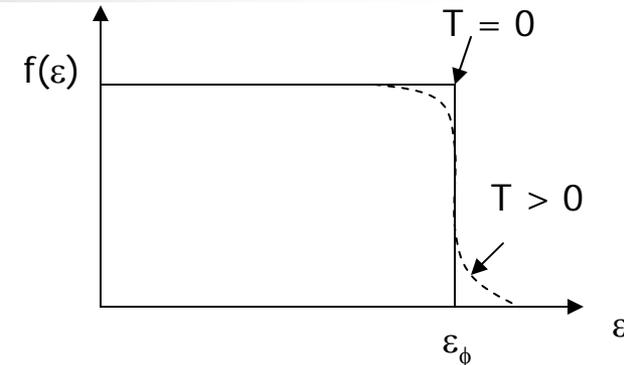


Table 2.2. Coefficient of the Electronic Specific Heat for Various Metallic Elements of Technical Interest<sup>a</sup>

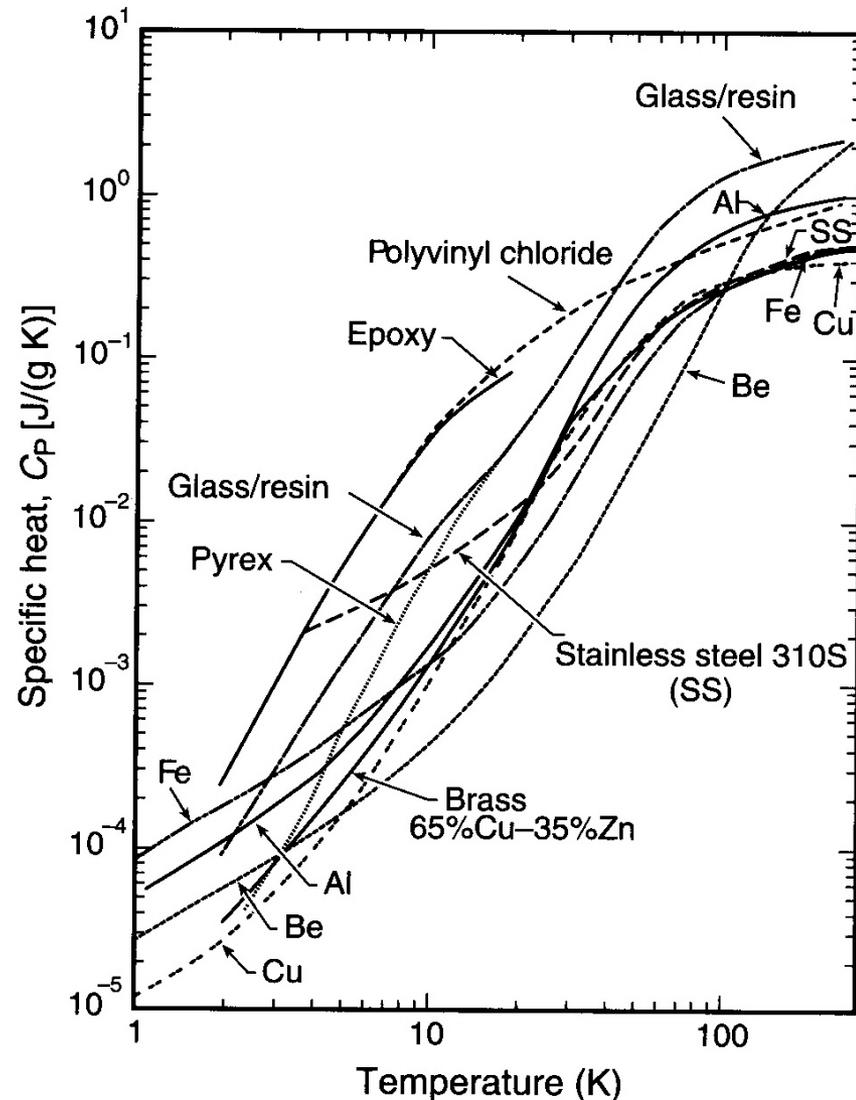
Element	$\gamma$ (mJ/mol · K <sup>2</sup> )
Ag	0.646
Al	1.35
Au	0.729
Cd	0.688
Cr	1.40
Cu	0.695
Fe	4.98
Ga	0.596
Hf	2.16
Hg	1.79
In	1.69
Nb	7.79
Ni	7.02
Pb	2.98
Sn	1.78
Ti	3.35
V	9.26
Zn	0.64
Zr	2.80

<sup>a</sup> From Kittel.<sup>1</sup>

# Summary: Specific Heat of Materials

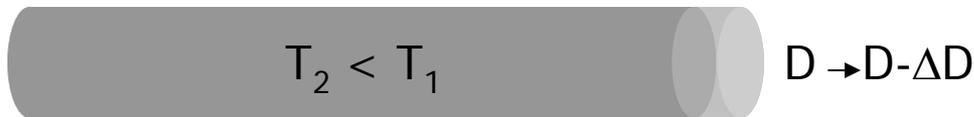
## General characteristics:

- Specific heat decreases by  $\sim 10\times$  between 300 K and LN<sub>2</sub> temperature (77 K)
- Decreases by factor of  $\sim 1000\times$  between RT and 4 K
- Temperature dependence
  - $C \sim \text{constant}$  near RT
  - $C \sim T^n$ ,  $n \sim 3$  for  $T < 100$  K
  - $C \sim T$  for metals at  $T < 1$  K



# Thermal Contraction

- All materials change dimension with temperature. The expansion coefficient is a measure of this effect. For most materials, the expansion coefficient  $> 0$

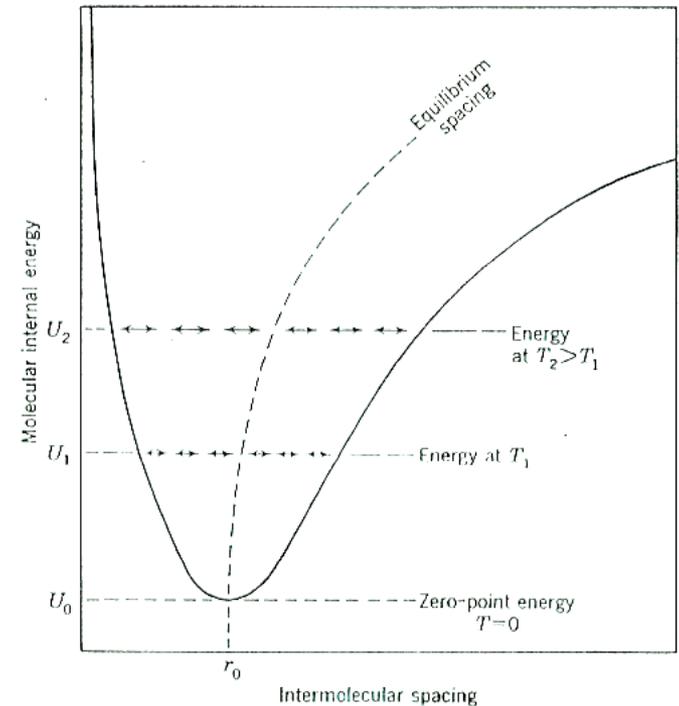


Linear expansion coefficient  $\rightarrow \Delta L \leftarrow \uparrow$

$$\alpha = \left. \frac{1}{L} \frac{\partial L}{\partial T} \right)_p = \frac{1}{3} \beta \quad \text{For isotropic materials}$$

Bulk expansivity (volume change):

$$\beta = \left. \frac{1}{v} \frac{\partial v}{\partial T} \right)_p = - \left. \frac{1}{\rho} \frac{\partial \rho}{\partial T} \right)_p$$



Expansivity caused by anharmonic terms in the lattice potential

# Temperature dependence to $\alpha$ and $\beta$

- Most materials contract when cooled
  - The magnitude of the effect depends on materials
  - Plastics > metals > glasses
- Coefficient ( $\alpha$ ) decreases with temperature
  - Thermodynamics:

$$C_p - C_v = \frac{Tv\beta^2}{\kappa} \xrightarrow{T \rightarrow 0} 0$$

$$\left. \frac{\partial v}{\partial T} \right)_p = - \left. \frac{\partial s}{\partial p} \right)_T \xrightarrow{T \rightarrow 0} 0$$

(Third law:  $s \rightarrow 0$ )

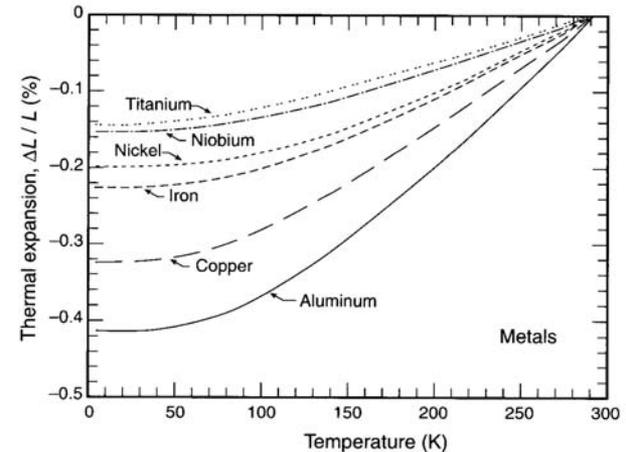


Fig. 6.6 Thermal linear expansion  $\Delta L/L \equiv (L_T - L_{293})/L_{293}$  of common metals. (Compiled by Clark 1983 from data by Corruccini and Gniewek 1961, and Hahn 1970.) Tabulated values for these and other materials are given in Appendix A6.4.

From Ekin (2006)

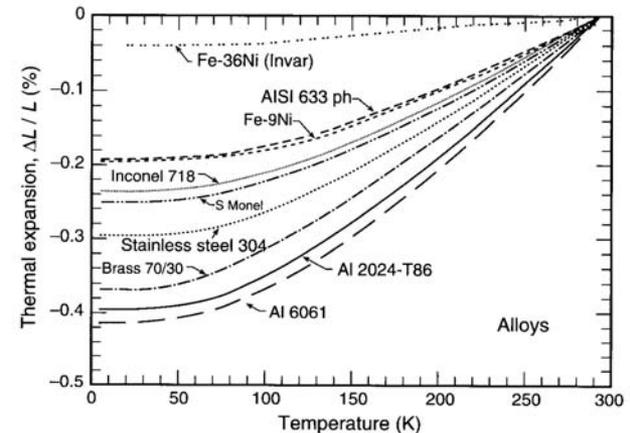


Fig. 6.7 Thermal linear expansion  $\Delta L/L \equiv (L_T - L_{293})/L_{293}$  of common alloys. (Compiled by Clark 1983 from data by Clark 1968 and Arp et al. 1962.) Tabulated values for these and other materials are given in Appendix A6.4.

# Expansion coefficient for materials

Table 2.5. Linear Thermal Contractions Relative to 293 K<sup>a</sup>

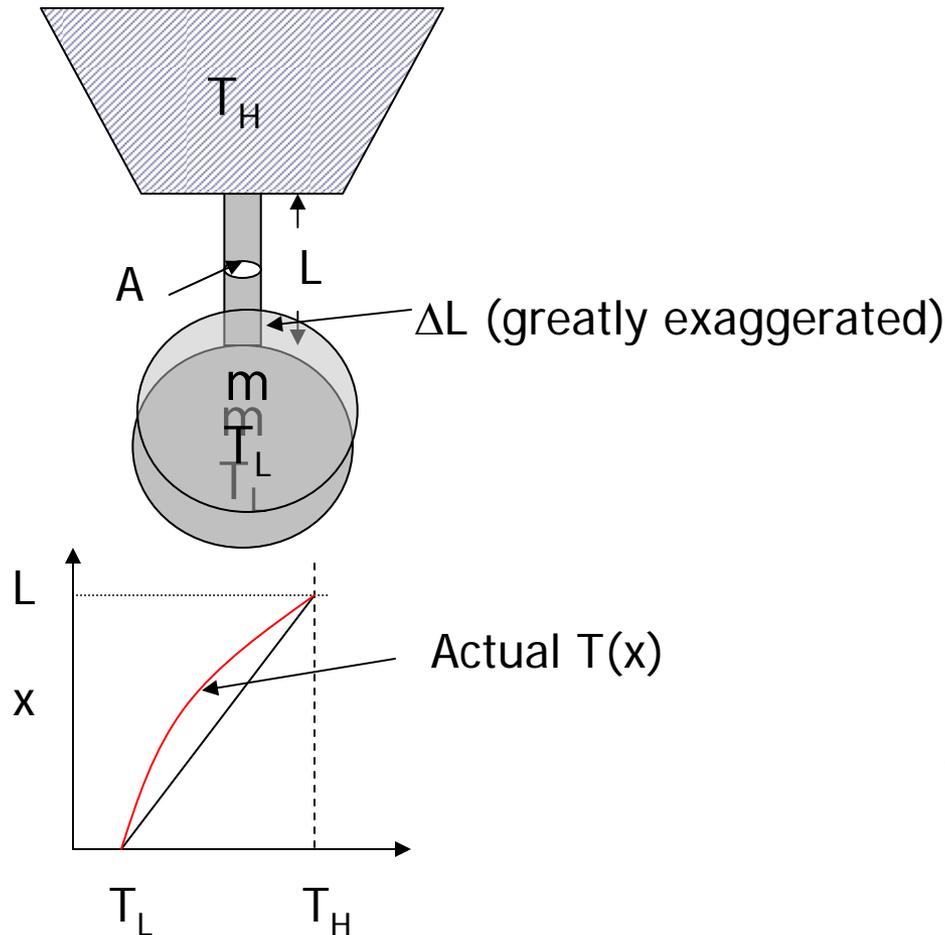
Substance	T(K) :	0	20	40	60	80	100	150	200	250
Aluminum		41.4	41.4	41.2	40.5	39.0	36.9	29.4	20.1	9.6
Copper		32.6	32.6	32.3	31.6	30.2	28.3	22.1	14.9	7.1
Germanium		9.3	9.3	9.3	9.4	9.3	8.9	7.3	5.0	2.4
Iron		20.4	20.4	20.3	19.9	19.5	18.4	14.9	10.2	4.9
Lead		70.8	70.0	66.7	62.4	57.7	52.8	39.9	26.3	12.4
Nickel		23.1	23.0	22.9	22.6	21.8	20.8	16.5	11.4	5.4
Silicon		2.16	2.16	2.17	2.23	2.32	2.40	2.38	1.90	1.01
Silver		41.0	41.0	40.3	38.7	36.5	33.7	25.9	17.2	8.2
Titanium		15.1	15.1	15.0	14.8	14.2	13.4	10.7	7.3	3.5
Tungsten		8.6	8.6	8.5	8.4	8.1	7.6	5.9	4.0	1.9
Brass (65% Cu, 35% Zn)		38.4	38.3	38.0	36.8	35.0	32.6	25.3	16.9	8.0
Cu + 2 Be		32.4	32.4	31.9	31.6	30.0	28.3	22.0	16.0	7.0
Constantan		—	—	26.4	25.8	24.7	23.2	18.3	12.4	5.85
Invar <sup>b</sup>		4.5	4.6	4.8	4.9	4.8	4.5	3.0	2.0	1.0
304, 316 Stainless steel		—	29.7	29.6	29.0	27.8	26.0	20.3	13.8	6.6
Pyrex		5.6	5.6	5.7	5.6	5.4	5.0	3.95	2.7	0.8
Silica (1000° C) <sup>c</sup>		-0.1	-0.05	0.05	0.2	0.3	0.4	0.5	0.4	0.2
Silica (1400° C) <sup>c</sup>		-0.7	-0.65	-0.5	-0.3	-0.2	-0.05	0.2	0.2	0.1
Araldite		106	105	102	98	94	88	71	50	25
Nylon		139	138	135	131	125	117	95	67	34
Polystyrene		155	152	147	139	131	121	93	63	30
Teflon		214	211	206	200	193	185	160	124	75

<sup>a</sup> Units are  $10^4 \times (L_{293} - L_T)/L_{293}$ . Sources of data include *Thermophysical Properties of Matter* (1977), Corruccini and Gniewek (1961), and *American Institute of Physics Handbook* (1972). Compiled by White.<sup>2</sup>

<sup>b</sup> The expansion of Invar NiFe alloys containing ~36% Ni is very sensitive to composition and heat treatment.

<sup>c</sup> These silicas were aged at 1000° C and 1400° C.

# Thermal contraction of supports



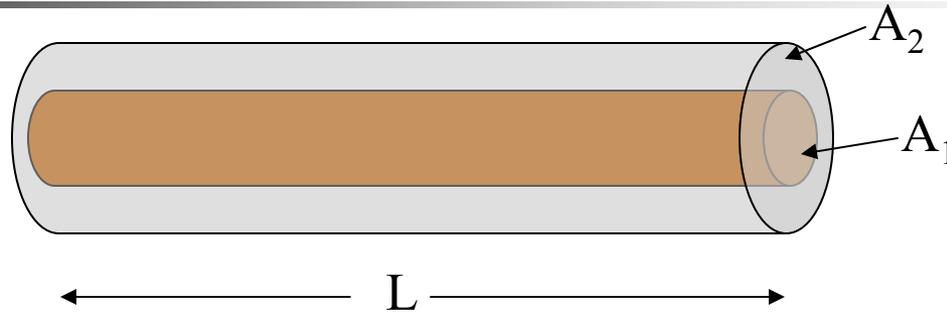
- The change in length of a support is determined by the temperature distribution along the support
- Tabulated  $\Delta L/L$  values are for uniform temperature of support
- For non-uniform temperature

$$\Delta L = \int_0^L dx \int_{T(x)}^{T_H} \alpha(T) dT$$

Where  $T(x)$  is defined according to,

$$\int_{T_x}^{T_H} k(T) dT = \left( \frac{x}{L} \right) \int_{T_L}^{T_H} k(T) dT$$

# Thermal stress in a composite



## Assumptions:

- Composite is stress free at  $T_0$
- Materials remain elastic:  $\sigma = E_y \varepsilon$
- No slippage at boundary
- Ends are free

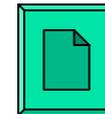
## Force balance

$$F_1 = \sigma_1 A_1 = F_2 = \sigma_2 A_2$$

$$\varepsilon_1 = \frac{\Delta L}{L} \Big|_1 - \frac{\Delta L}{L} \Big|_{\text{composite}}$$

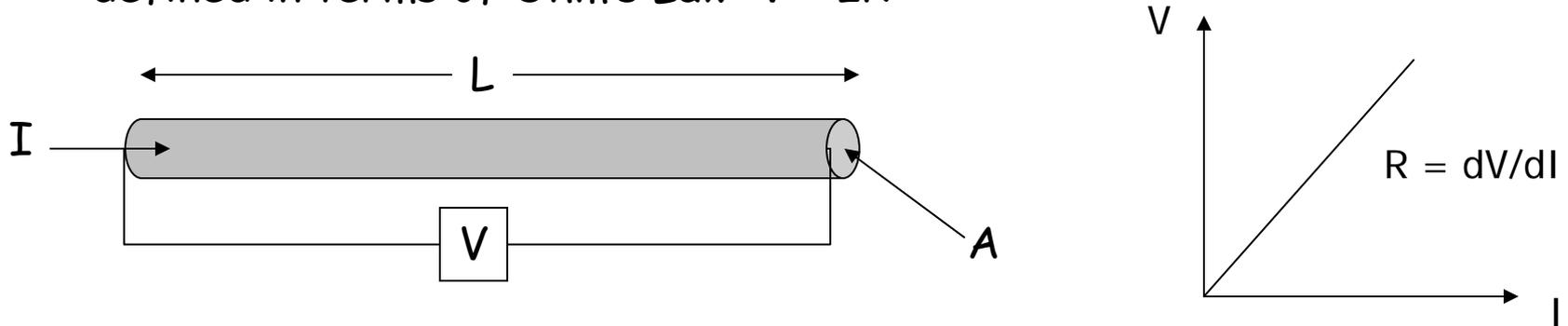
$$\varepsilon_2 = \frac{\Delta L}{L} \Big|_{\text{composite}} - \frac{\Delta L}{L} \Big|_2$$

$$\sigma_1 = E_{y1} \varepsilon_1 = E_{y1} \left[ \frac{\frac{\Delta L}{L} \Big|_1 - \frac{\Delta L}{L} \Big|_2}{\left( 1 + \frac{E_{y1} A_1}{E_{y2} A_2} \right)} \right]$$



# Electrical conductivity of materials

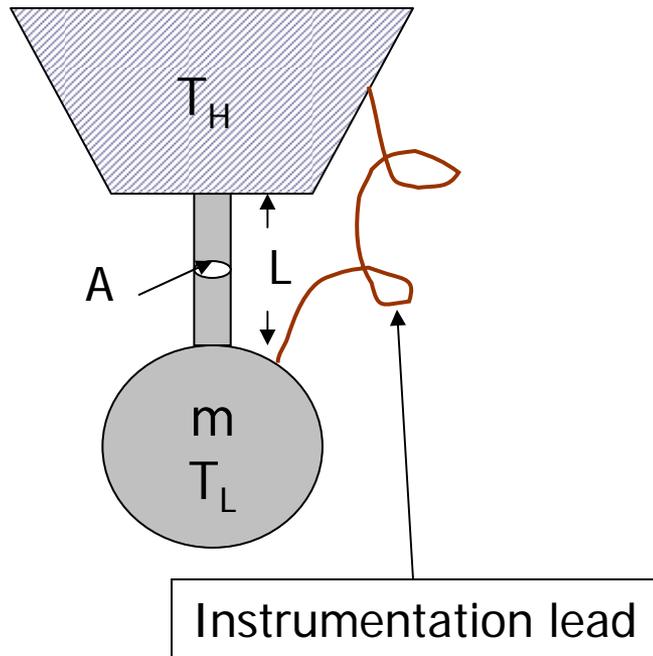
- The electrical conductivity or resistivity of a material is defined in terms of Ohm's Law:  $V = IR$



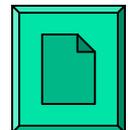
- Heat generation by electrical conduction  $Q = I^2R$
- Conductivity ( $\sigma$ ) or resistivity ( $\rho$ ) are material properties that depend on extrinsic variables:  $T$ ,  $p$ ,  $B$  (magnetic field)

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

# Instrumentation leads



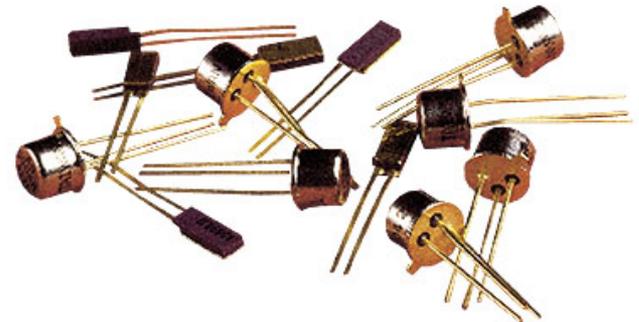
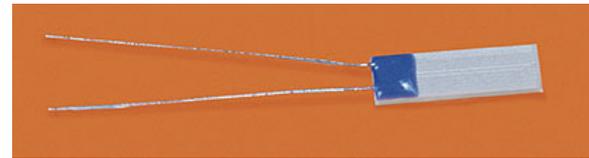
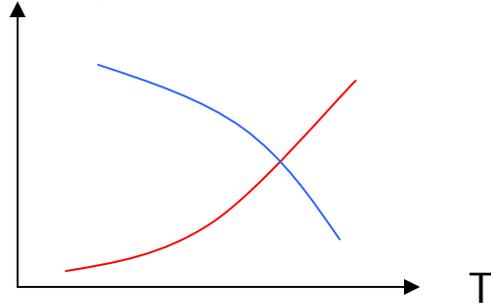
- An instrumentation lead usually carries current between room temperature and low temperature
  - Material selection (pure metals vs. alloys)
  - Lead length determined by application
  - Optimizing the design
- Leads are an important component and should be carefully designed
  - Often one of the main heat loads to the system
  - Poor design can also affect one's ability to make a measurement.



# Cryogenic temperature sensors

- A temperature sensor is a device that has a measurable property that is sensitive to temperature.
- Measurement and control of temperature is an important component of cryogenic systems.
- Resistive sensors are frequently used for cryogenic temperature measurement

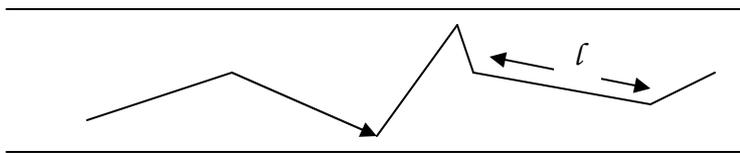
$$R(T) = \frac{V}{I}$$



- Knowing what temperature is being measured is often a challenge in cryogenics (more later)

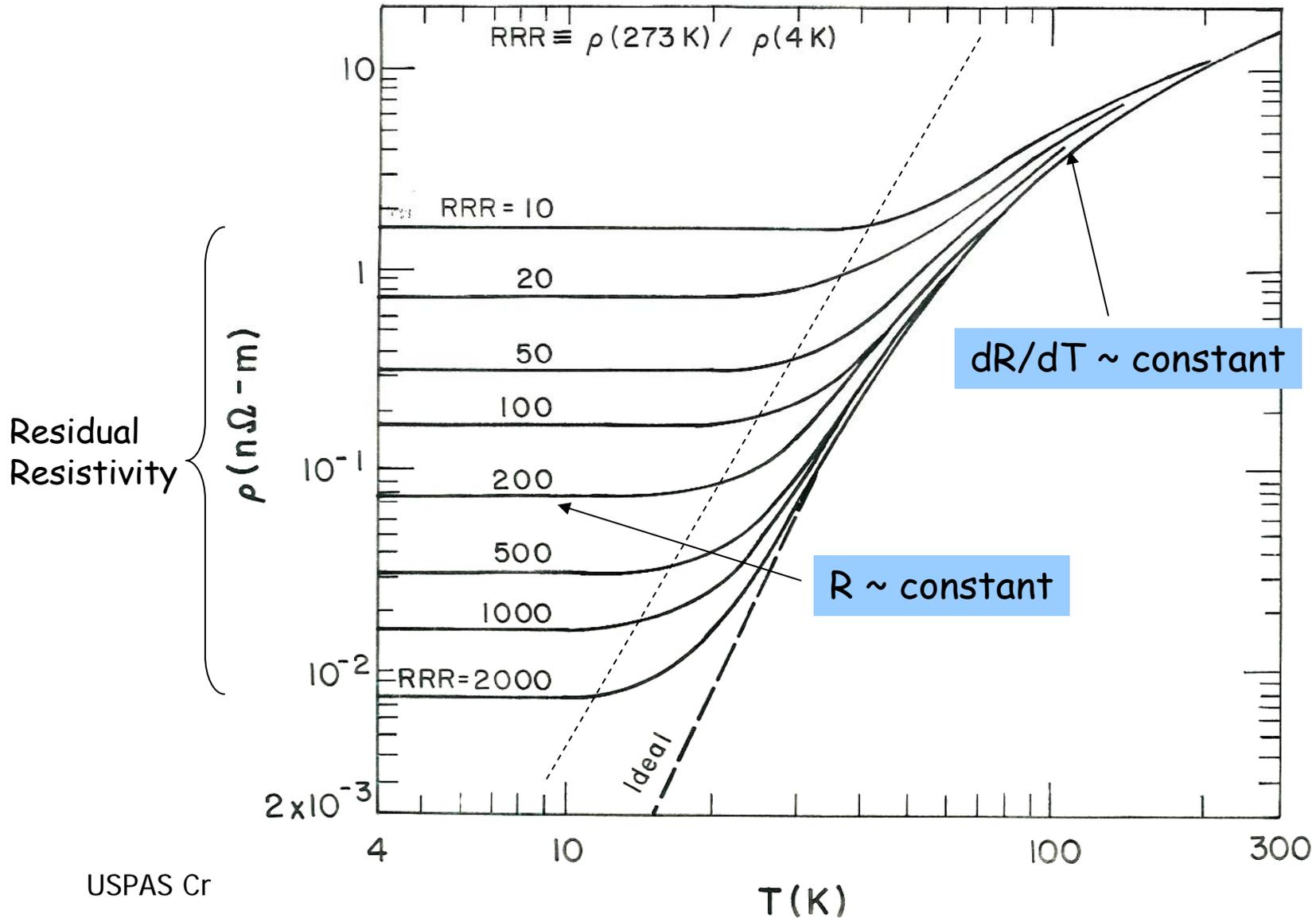
# Electrical Conductivity in Metals

- Resistivity in metals develops from two electron scattering mechanisms (Matthiessen's rule)
  - Electron-phonon scattering,  $T > \Theta_D$   
Scattering probability  $\sim$  mean square displacement due to thermal motion of the lattice:  $\langle x^2 \rangle \sim k_B T$  and  $\rho \sim T$
  - Electron-defect scattering (temperature independent),  $T \ll \Theta_D$   
Scattering depends on concentration of defects  
 $\rho \sim \rho_0 \sim \text{constant}$ ; Residual Resistivity Ratio (RRR =  $\rho(273 \text{ K})/\rho_0$ )  
indication of the purity of a metal
  - Intermediate region,  $T \sim \Theta_D/3$   
 $\rho \sim (\text{phonon density}) \times (\text{scattering probability}) \sim T^5$
  - Scattering process:



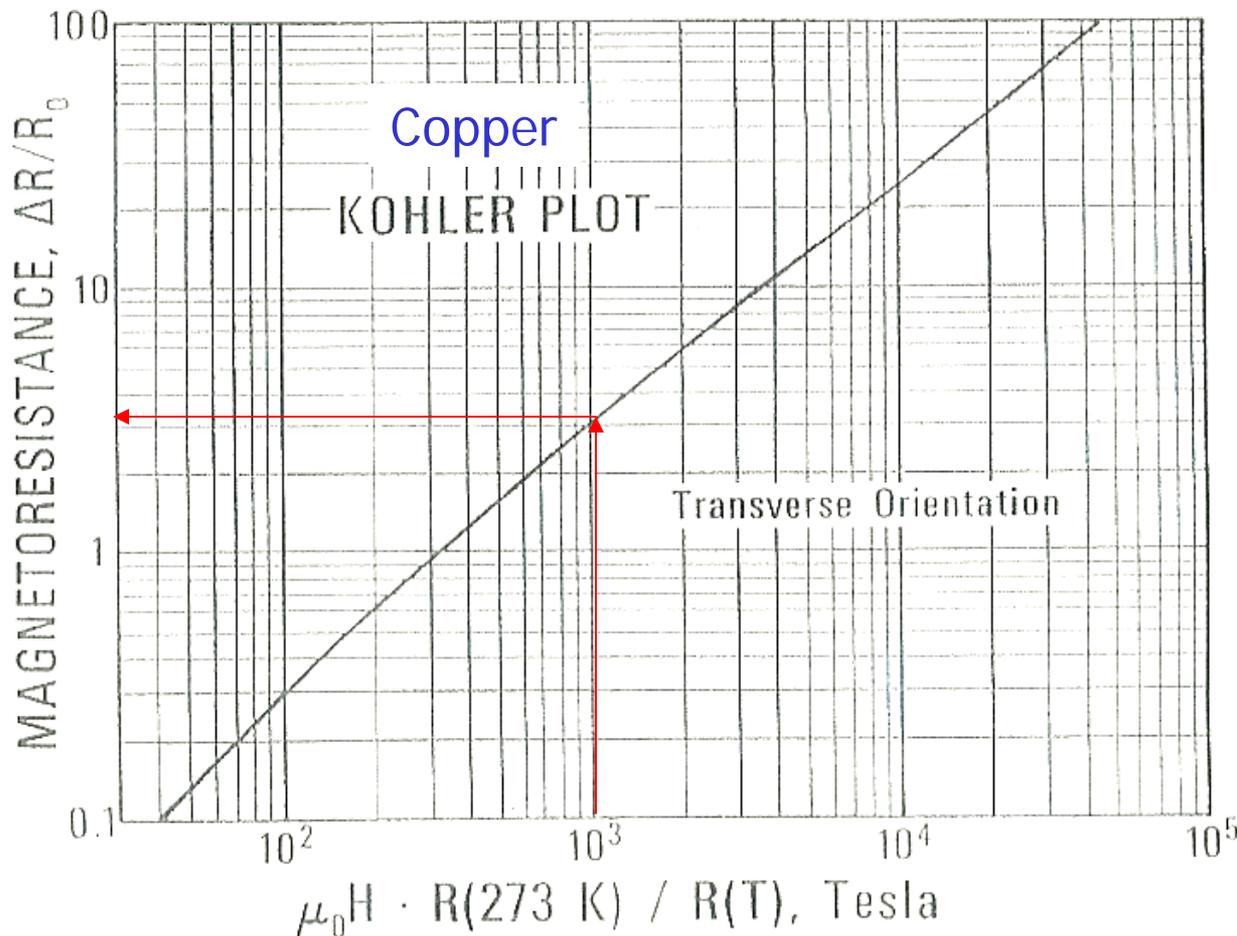
“ $l$ ” is the mean free path  
 $\tau$  is the mean scattering  
time =  $l/v_f$

# Resistivity of Pure Metals (e.g. Copper)



# Kohler plot for Magneto-resistance

The resistance of pure metals increases in a magnetic field due to more complex electron path and scattering (Why?)



How to use a Kohler plot?

1. Determine RRR of metal
2. Compute product: RRR X B
3. Use graph to estimate incremental increase in R (or  $\rho$ ).
4. Add to base value

Example:

$$\left. \begin{array}{l} \text{RRR} = 100 \\ B = 10 \text{ T} \end{array} \right\} ( ) = 1000$$

$$\begin{array}{l} \Delta R/R = 3 \\ R(10 \text{ T})/R(0 \text{ T}) = 4 \\ \text{RRR (equival.)} = 25 \end{array}$$

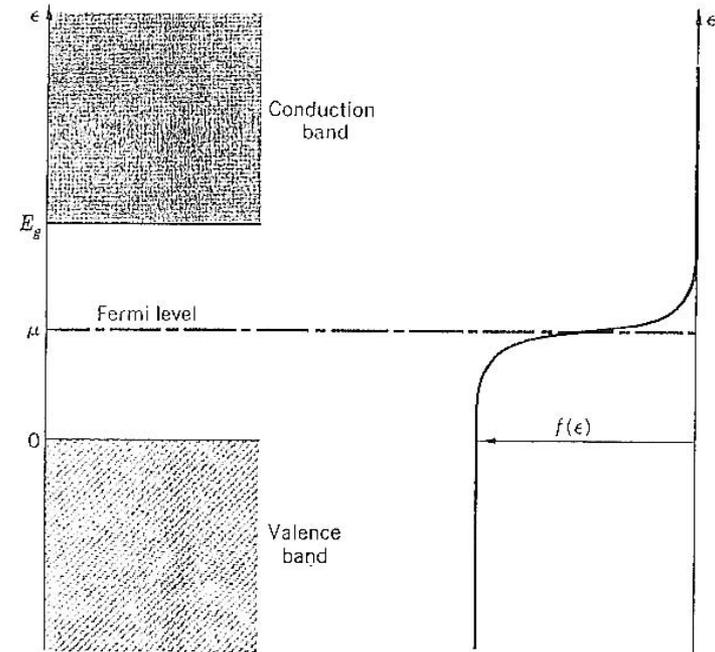
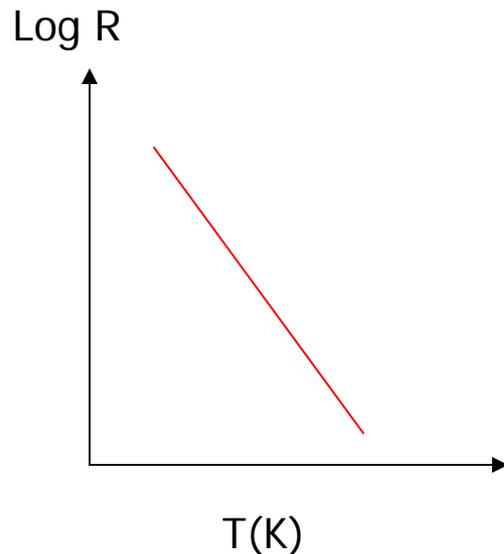
# Electrical conductivity of alloys

Electrical resistivity of various alloys ( $\times 10^{-9} \Omega\text{-m}$ ) (from Cryocomp)

Alloy	10 K	20 K	50 K	100 K	200 K	300 K	RRR
AL 5083	30.3	30.3	31.3	35.5	47.9	59.2	1.95
AL 6061-T6	13.8	13.9	14.8	18.8	30.9	41.9	3
304 SUS	495	494	500	533	638	723	1.46
BeCu	56.2	57	58.9	63	72	83	1.48
Manganin	419	425	437	451	469	476	1.13
Constantan	461	461	461	467	480	491	1.07
Ti-6%Al- 4%V	1470	1470	1480	1520	1620	1690	1.15
PbSn (56-44)	4.0	5.2	16.8	43.1	95.5	148	37

# Electrical conductivity of semiconductors

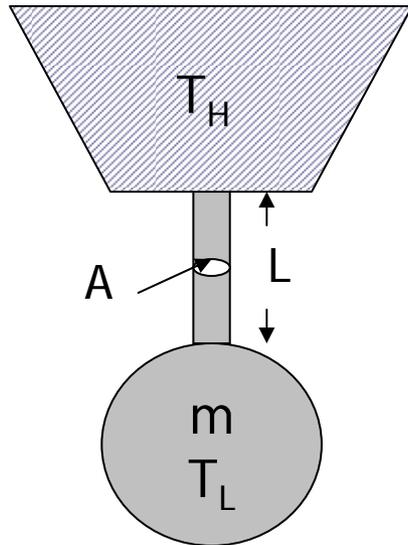
- Carrier concentration
- Energy gap
- Impurity concentration
- $\rho \sim 10^{-4}$  to  $10^7 \Omega\text{-m}$
- Basis for low temperature thermometry



Conductivity depends on the number of carriers in the conduction band.

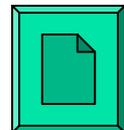
$$N_c \approx e^{-E_g/k_B T}$$

# Steady heat leak of a support



- The cross section and length of the support is determined by the requirements of the application
  - Gravitational loading, vibration, etc.
  - Strength of material used for support
- Heat leak is determined by the temperature dependent thermal conductivity of the material,  $k(T)$  and physical dimensions ( $A, L$ )

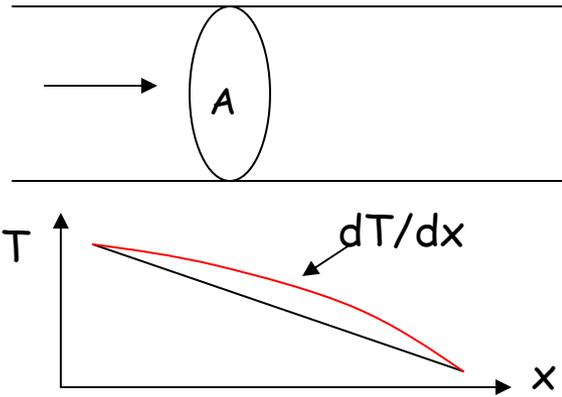
$$Q = \left( \frac{A}{L} \right) \int_{T_L}^{T_H} k(T) dT$$



# Thermal Conductivity of Materials

- The thermal conductivity is defined in terms of the relationship between the temperature gradient and heat flux (Fourier's Law):

$$\dot{Q} = -k(T)A \frac{dT}{dx}$$



Note that in general  $k(T)$  and may vary significantly over the temperature range of interest

- Two contributions to the thermal conductivity
  - Electronic contribution dominates in pure metals
  - Lattice (Phonon) contribution mostly in insulating materials
- Kinetic theory

$$k = \frac{1}{3} \rho C v l$$

$\rho C$  = volumetric heat capacity

$v$  = characteristic velocity

$l$  = mean free path

# Electronic Thermal Conductivity

- Free electron model:

$$k = \frac{\pi^2 n k_B^2 T \tau}{3 m_e} \quad \text{Recall: } \tau \text{ is scattering time} = \ell/v$$

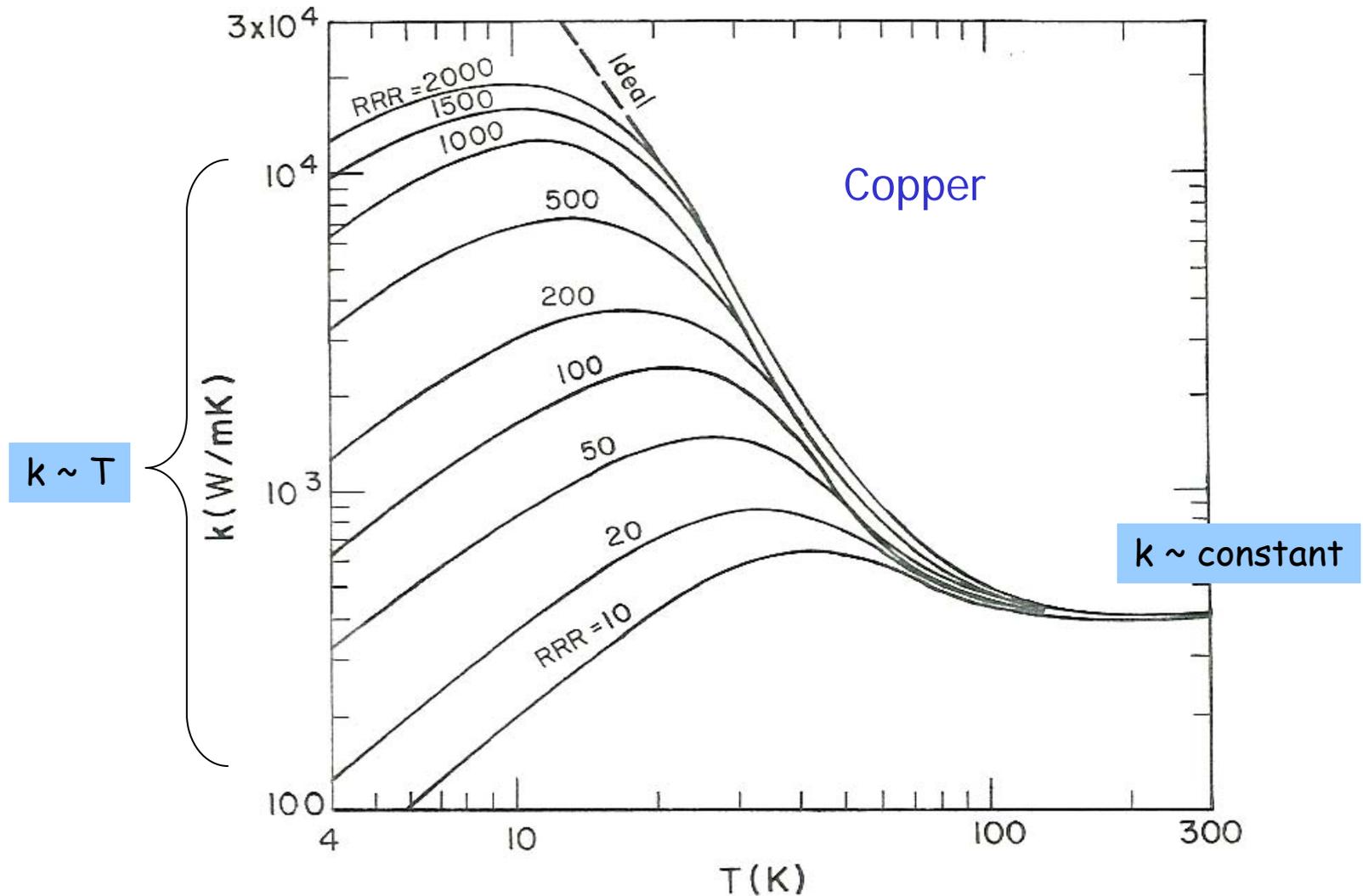
~ constant at high temperature  
~ T at low temperature

- Weidemann-Franz law (for free electron model)

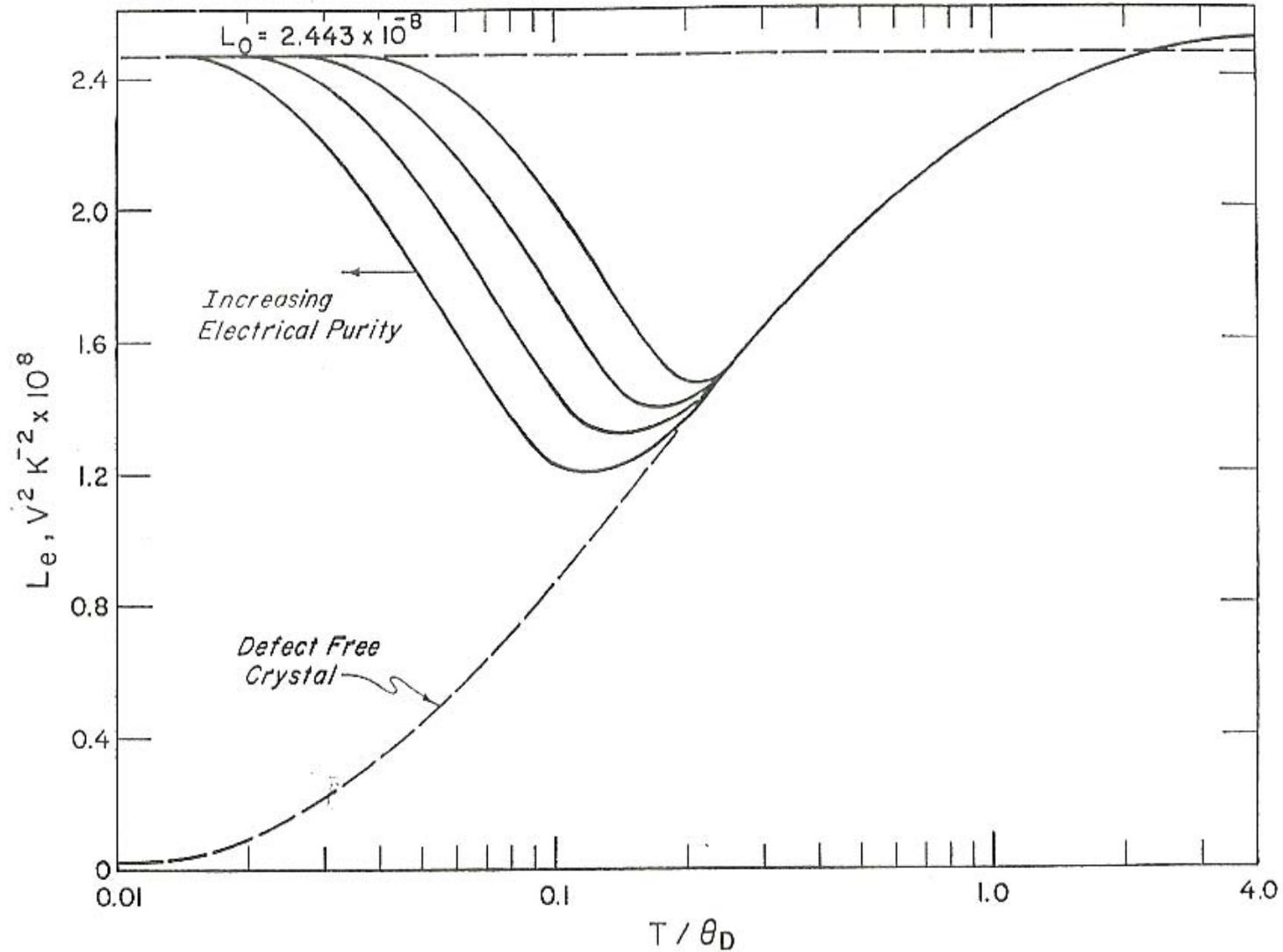
$$\frac{k}{\sigma} = f(T) = \frac{\pi^2 k_B^2}{3 e^2} T \equiv L_0 T \quad L_0 = \text{Lorentz number} = 2.443 \times 10^{-8} \text{ W}\Omega/\text{K}^2$$

- Note that no real material obeys the W-F law, although it is a good approximation at low T and near and above RT.

# Thermal Conductivity of Pure Metals



# Lorentz Number for Pure Metals



# Thermal Conductivity (continued)

- Thermal conduction by lattice vibrations (phonons) is a significant contributor to overall heat conduction particularly in non-metals and alloys.
- Metals (Alloys):  $k_{\text{total}} = k_{\text{electrons}} + k_{\text{phonons}}$  (Typ. 1 to 3 orders less than that of pure metals)
- Insulating crystals only have lattice contribution, which can be large for single crystals (e.g.  $\text{Al}_2\text{O}_3$ , Sapphire)
- Insulating polymers have very low thermal conductivity (Nylon, Teflon, Mylar, Kapton)
- Insulating composites have complex behavior depending on components

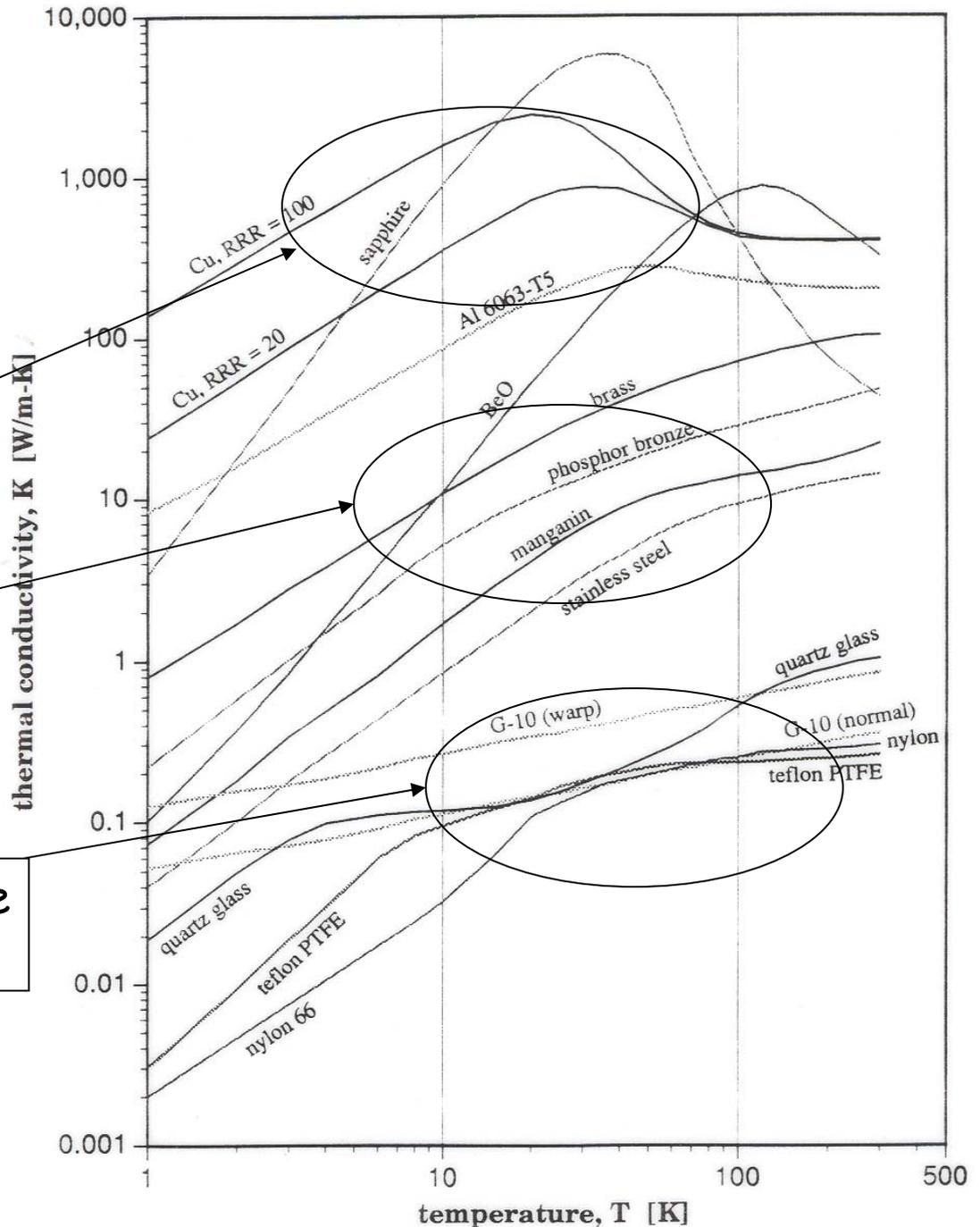
# Examples

- Low temp. range,  $k \sim T^n$  with  $1 < n < 3$

Pure metals and crystalline insulators

Alloys

Non-crystalline  
Non-metallics



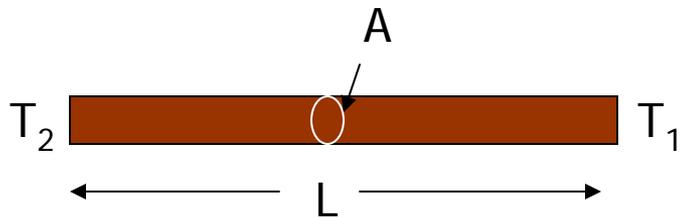
# Thermal Conductivity Integrals

$$\bar{k}(T_1, T_2) = \int_{T_1}^{T_2} k(T) dT \quad , \quad [\text{W/m}]$$

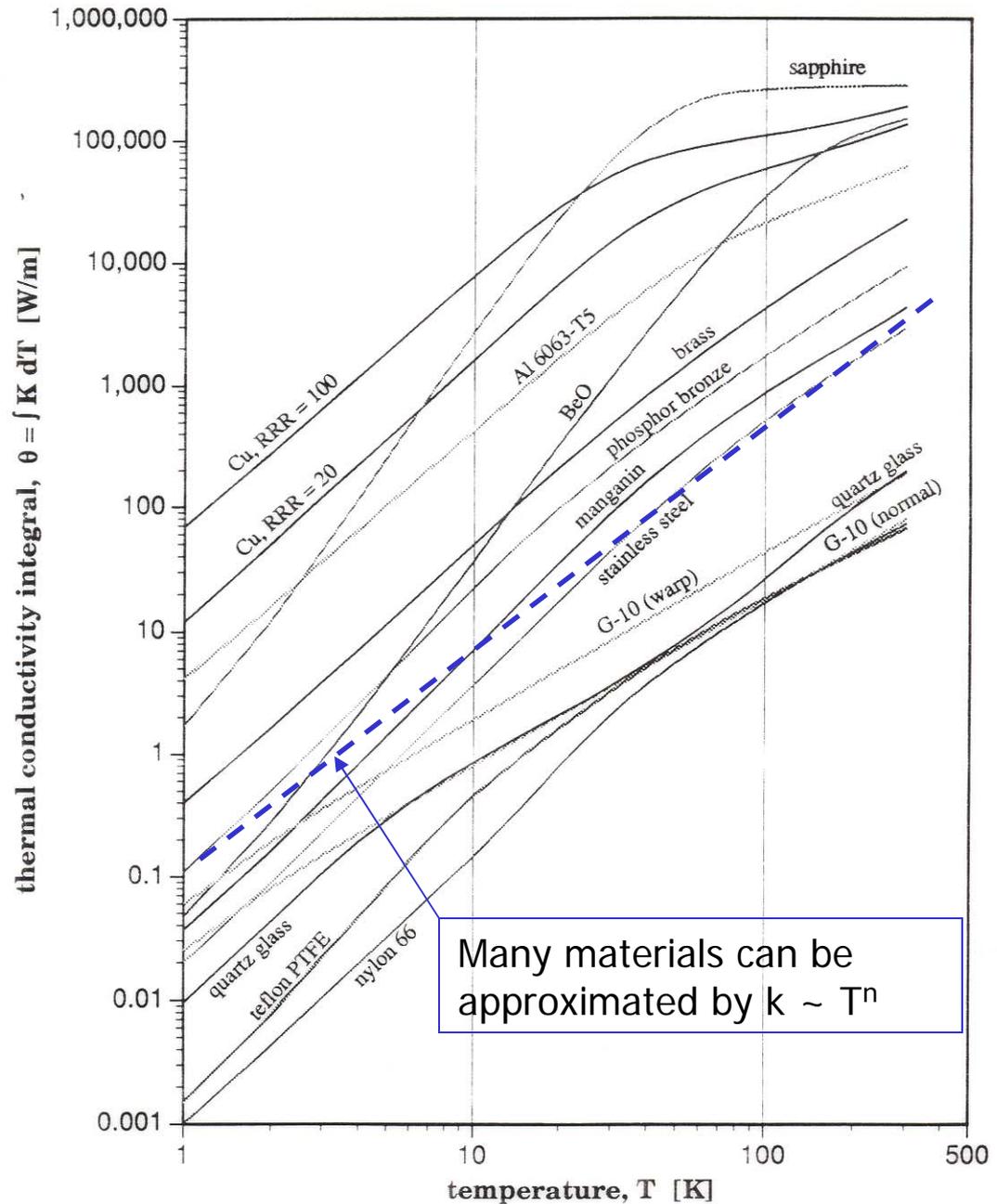
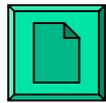
To use the graph

$$\bar{k}(T_1, T_2) = \bar{k}(0, T_2) - \bar{k}(0, T_1)$$

Heat conduction along a rod

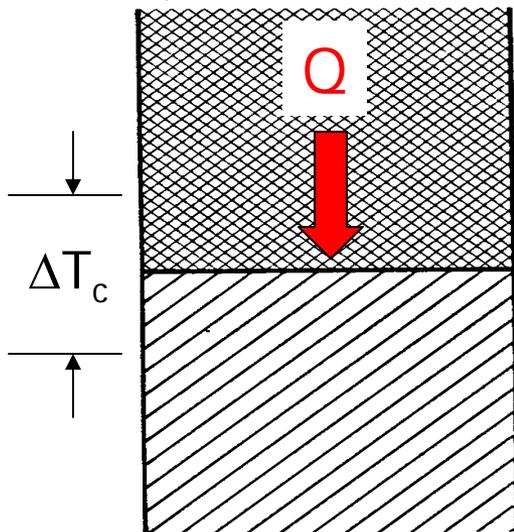


$$Q = \bar{k}(T_1, T_2) \frac{A}{L}$$

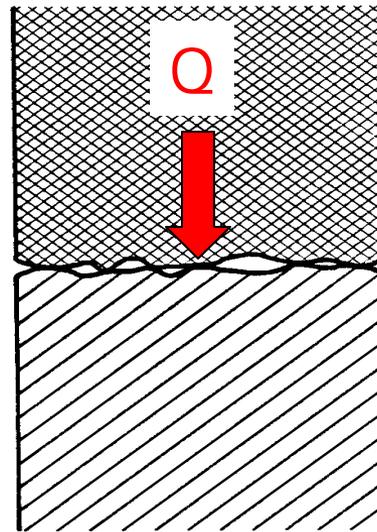


# Contact Resistance (conductance)

- Joints or contacts can lead to considerable resistance in a thermal (electrical) circuit.
- Contact resistance can vary considerably depending on a number of factors
  - Bulk material properties (insulators, metals)
  - Surface condition (pressure, bonding agents)



a) ideal



b) real

Heat transfer coefficient

$$Q = h_c A \Delta T$$

# Thermal contact conductance (conductive)

Contact point between two materials can produce significant thermal resistance

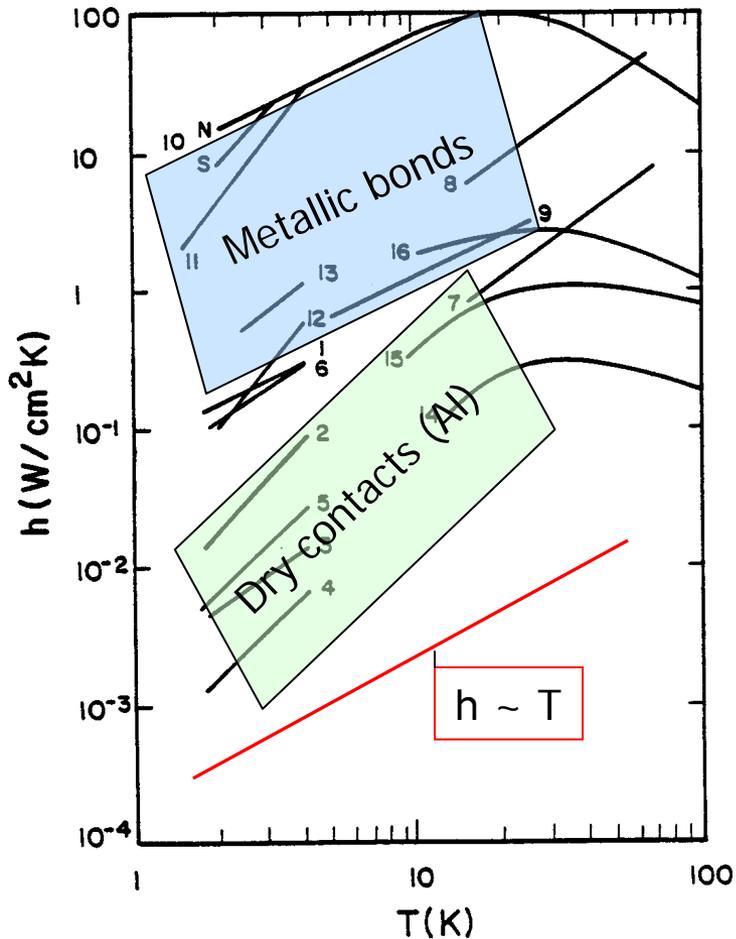
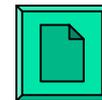


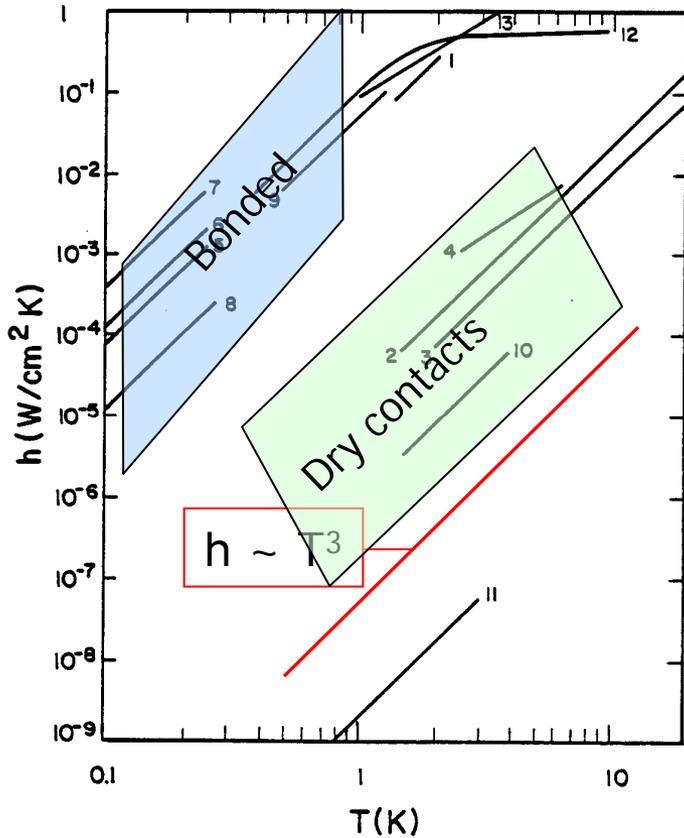
Table II. Thermal Conductance of Metallic Contacts

Material	Contact	Pressure (MPa)	$h$ (W/cm <sup>2</sup> K)	Temp. Range (K)	Ref
1. Al-Al (alloy)	machined	torque=20 Nm	0.075 T	1.8-4.2	Wanner /13/
2. Al-Al alloy	electropolished	torque=20 Nm	$3.6 \times 10^{-3} T^{2.3}$	1.8-4.2	
3. Al-Al alloy	Au plated	torque=20 Nm	$1.9 \times 10^{-3} T^{1.4}$	1.8-4.2	
4. Cu-Cu	machined	2.8	$4 \times 10^{-4} T^2$	1.8-4.2	Berman /8/
5. Cu-Cu	machined	14	$1.67 \times 10^{-3} T^2$	1.8-4.2	
6. Au-Au	--	5.6	$0.05 T^{1.3}$	2-4	Berman & Mate /7/
7. SS-SS (302)	polished	21	$0.014 T^{1.5*}$	15-300	Lyon & Parrish /14/
8. SS-SS (302)	polished	390	$0.10 T^{1.5*}$	15-300	
9. Cu-Cu	machined	7	0.13 T	5-25	Nilles & Van Sciver /15/
10. Cu-Cu	in solder	--	$7.5 T^*$	2-150	Radebaugh /16/
11. Cu-Cu	Pb solder	--	$0.64 T^{2.8}$	1.5-4	Challis & Cheeke /17/
12. Cu-Cu	woods metal	--	$0.018 T^{2.5}$	2-4	
13. Cu-Cu	PbSn solder	--	$0.13 T^{1.6}$	2.5-4	Foster /18/
14. Al-Al	SnPb foil	26	$0.02 T^{0.8*}$	10-300	Friedman & Gasser /19/
15. Cu-Al	SnPb foil	9	$0.04 T^*$	10-300	
16. Cu-Cu	SnPb Foil	7	$0.17 T^*$	10-300	

Example:

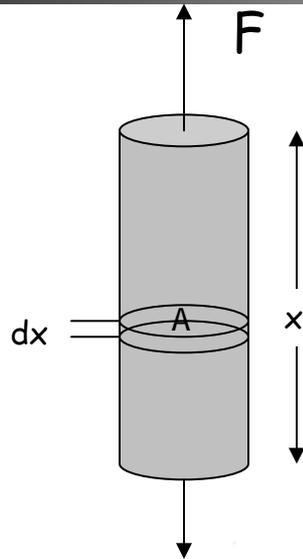


# Contact conductance (Insulating)



Material	Contact	Pressure (MPa)	$h$ (W/cm <sup>2</sup> K)	Temp range (K)	Ref.
1. In-sapphire	bonded	--	$0.03 T^3$	1.4-2.1	Neeper & Dillinger /6/
2. sapphire-sapphire	dry	4	$9 \times 10^{-6} T^3$	2-20	Berman & Mate /7/
3. Cu-diamond	dry	4	$2 \times 10^{-5} T^3$	1.5-20	
4. Cu-Teflon-Cu	12 mil foil	4	$1.8 \times 10^{-4} T^2$	2-5	Berman /8/
5. Cu-epoxy-Cu	bonded	--	$0.09 T^3$	0.05-0.25	Peterson & Anderson /2/
6. Al-epoxy-Al	bonded	--	$0.13 T^3$	0.05-0.25	
7. Pb-epoxy-Pb	bonded	--	$0.40 T^3$	0.05-0.25	
8. Be-epoxy-Be	bonded	--	$0.013 T^3$	0.05-0.25	
9. Cu-LiF	Ge-7031	--	$0.05 T^3$	0.4-1.3	Ackerman & Anderson /9/
10. Cu-sapphire-Cu	dry	0.1	$1 \times 10^{-6} T^3$	1.5-4	Yoo & Anderson /10/
11. Cu-sapphire-Cu	Al <sub>2</sub> O <sub>3</sub>	0.1	$2 \times 10^{-9} T^3$	0.8-3	
12. Cu-epoxy-Cu	bonded	--	$0.16 T^{0.5}$	2-8	Matsumoto et al. /11/
13. Cu-epoxy-Cu	bonded	--	$0.089 T^{1.9}$	1-4	Schmidt /12/

# Mechanical Properties

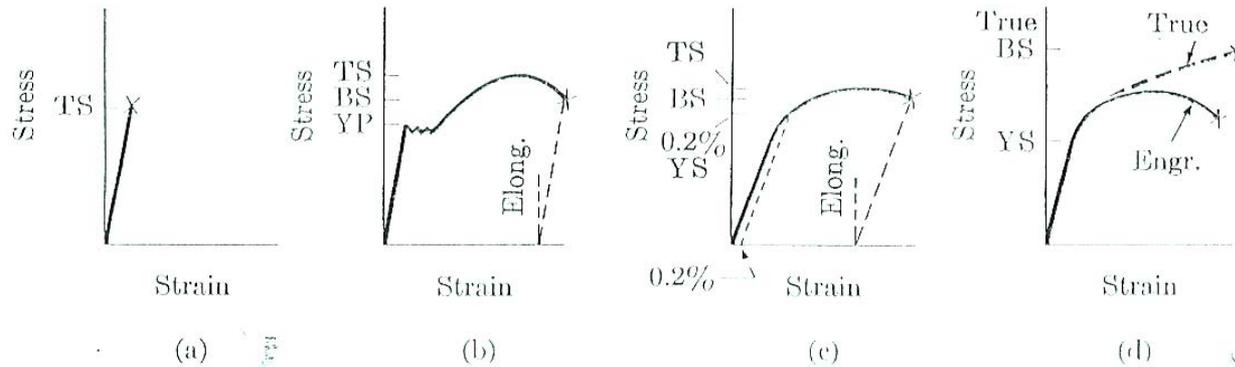


## Definitions

Stress:  $\sigma = \frac{F}{A}$  [N] ;  $\sigma_u$  &  $\sigma_y$

Strain:  $\epsilon = \frac{dx}{x}$

Modulus:  $E_y = \frac{\sigma}{\epsilon} = \frac{F/A}{dx/x}$  [Pa]



Stress-strain diagrams. (a) Nonductile material with no plastic deformation (example: cast iron). (b) Ductile material with yield point (example: low-carbon steel). (c) Ductile material without marked yield point (example: aluminum). (d) True stress-strain curve versus engineering stress-strain curve. BS = breaking strength, TS = tensile strength, YS = yield strength, Elong. = elongation, X = rupture, YP = yield point.

# $E_y$ and $\sigma_y$ at Low Temperatures

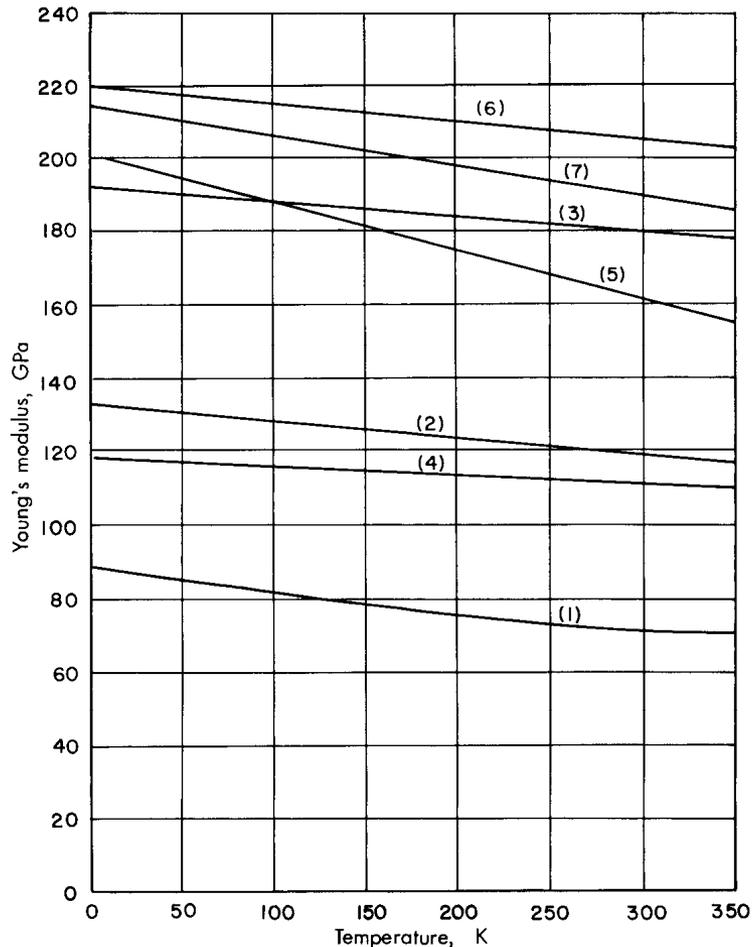


Fig. 2.6. Young's modulus at low temperatures: (1) 2024-T4 aluminum; (2) beryllium copper; (3) K Monel; (4) titanium; (5) 304 stainless steel; (6) C1020 carbon steel; (7) 9 percent Ni steel (Durham et al. 1962).

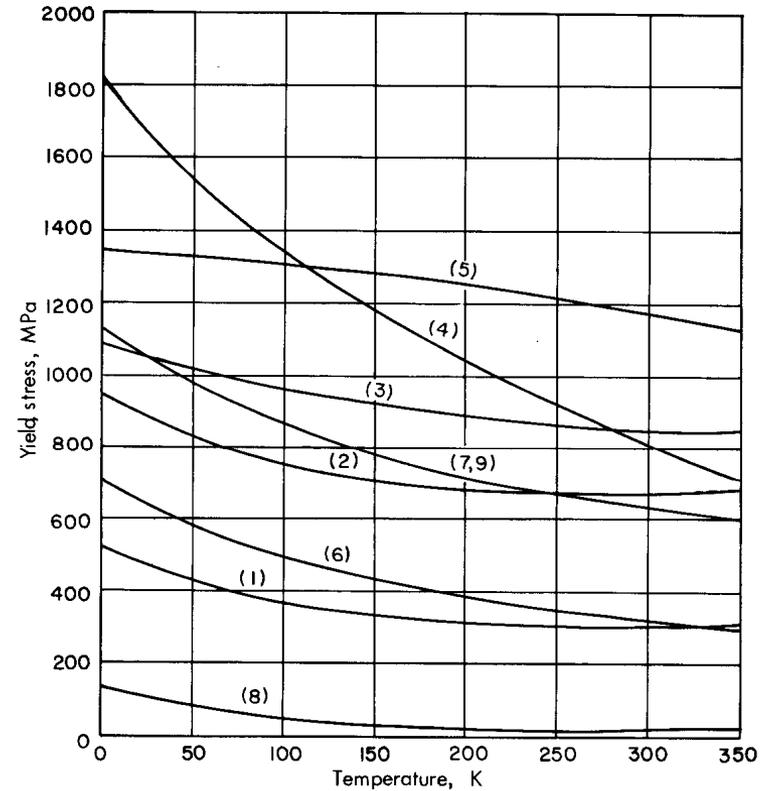


Fig. 2.2. Yield strength for several engineering materials: (1) 2024-T4 aluminum; (2) beryllium copper; (3) K Monel; (4) titanium; (5) 304 stainless steel; (6) C1020 carbon steel; (7) 9 percent Ni steel; (8) Teflon; (9) Invar-36 (Durham et al. 1962).

# $\sigma_y$ and Elongation at Low Temp

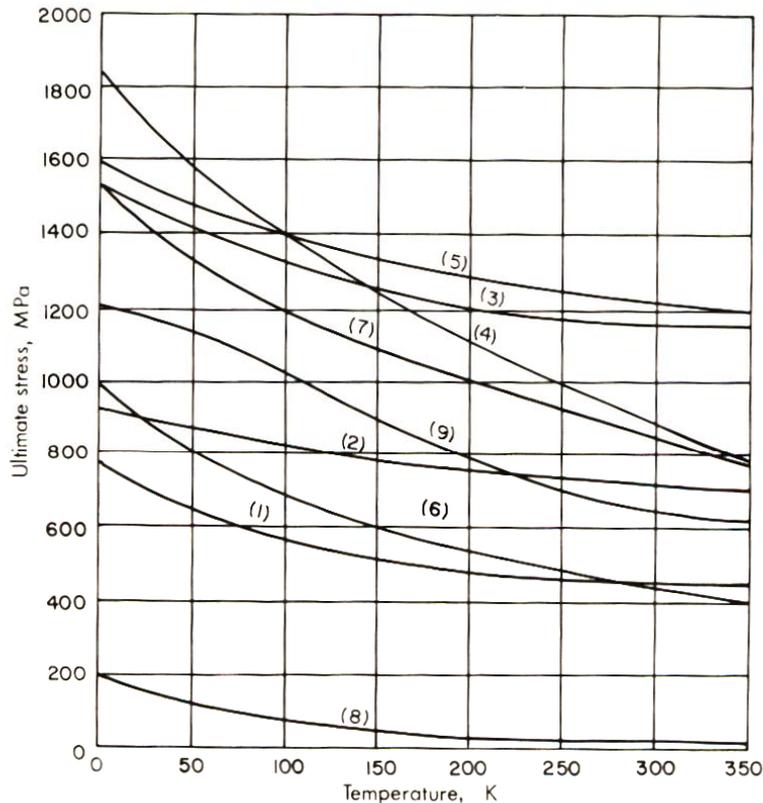


Fig. 2.1. Ultimate strength for several engineering materials: (1) 2024-T4 aluminum; (2) beryllium copper; (3) K Monel; (4) titanium; (5) 304 stainless steel; (6) C1020 carbon steel, (7) 9 percent Ni steel; (8) Teflon; (9) Invar-36 (Durham et al. 1962).

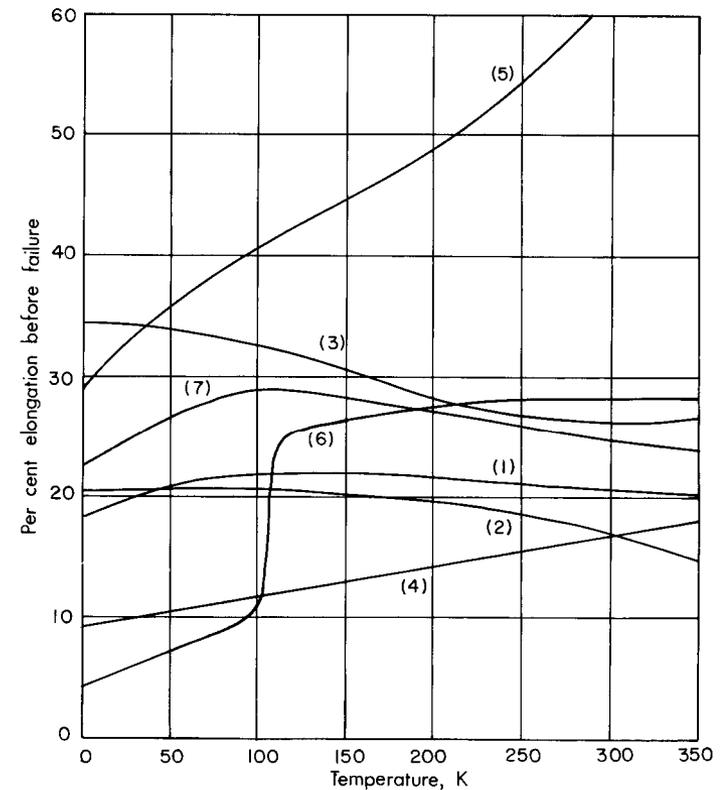
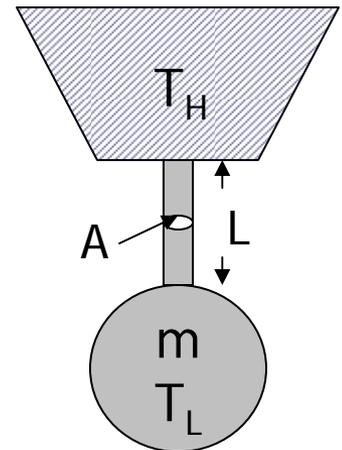


Fig. 2.5. Percent elongation for various materials: (1) 2024-T4 aluminum; (2) beryllium copper; (3) K Monel; (4) titanium; (5) 304 stainless steel; (6) C1020 carbon steel; (7) 9 percent Ni steel (Durham et al. 1962).

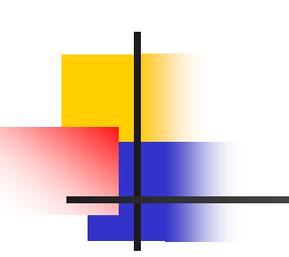
# Figure of Merit (FOM) for structural support materials

- Simple structural supports in cryogenic systems should be designed to minimize conductive heat leak,  $Q$ .
- Note that  $Q = -kA\Delta T/L$  and  $A = F/\sigma$ , force/allowable stress in the material.
- Thus, for a constant load, the best material for supports has a minimum value of  $k/\sigma$ .

Material	$k/\sigma$ (4 K), W/Pa*m*K	$k/\sigma$ (80 K) W/Pa*m*K	$k/\sigma$ (300 K) W/Pa*m*K
304 ss	0.042	1.8	3.7
6061 T6 AL	2.8	36	57
G-10	0.008	0.057	0.19
brass	1.3	21	46
copper	345	566	523
Note all values x $10^{-8}$			



Note that for a simple structural support, the cross sectional area is determined by the room temperature properties.



# Superconducting Materials

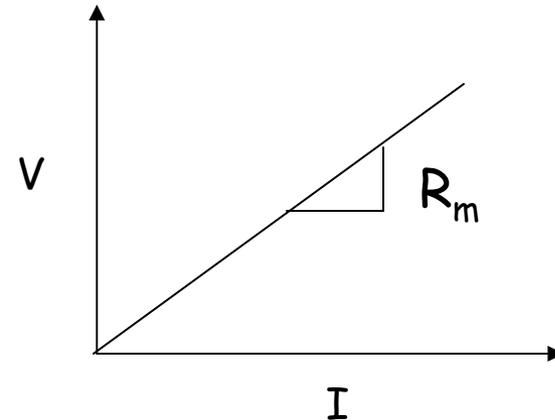
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- Superconductors are special materials that have (near) zero electrical resistance at low temperature
- Main application for superconductors is magnet technology
- LTS (low temperature superconductors) are used in most magnets and consist of niobium alloys (NbTi & Nb<sub>3</sub>Sn) co-processed with copper or copper alloys
- HTS are ceramics and have seen some use in magnet systems. Main materials are BSCCO and YBCO
- Both LTS and HTS are fabricated with normal metal (copper or silver) to provide strength and electrical protection
- Additional normal materials (copper stabilizer or structural support) to optimize conductor for final application.
- Other applications (Electronics, sensors)

# Superconducting vs. Resistive Materials

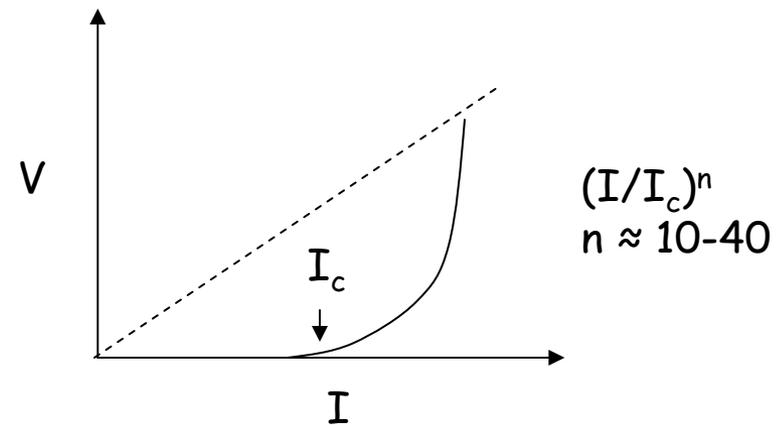
## ■ Resistive Material

- $V = IR$ ;  $P = I^2R$
- Copper and copper alloys
- Aluminum and Al alloys
- Stainless steel



## ■ Superconducting Materials

- $V = 0$  below  $I_c$
- $V \sim I^n$  above  $I_c$
- Transport properties depend on
  - Temperature
  - Magnetic field
  - Metallurgical processes
  - Strain in conductor



# Elemental superconductors

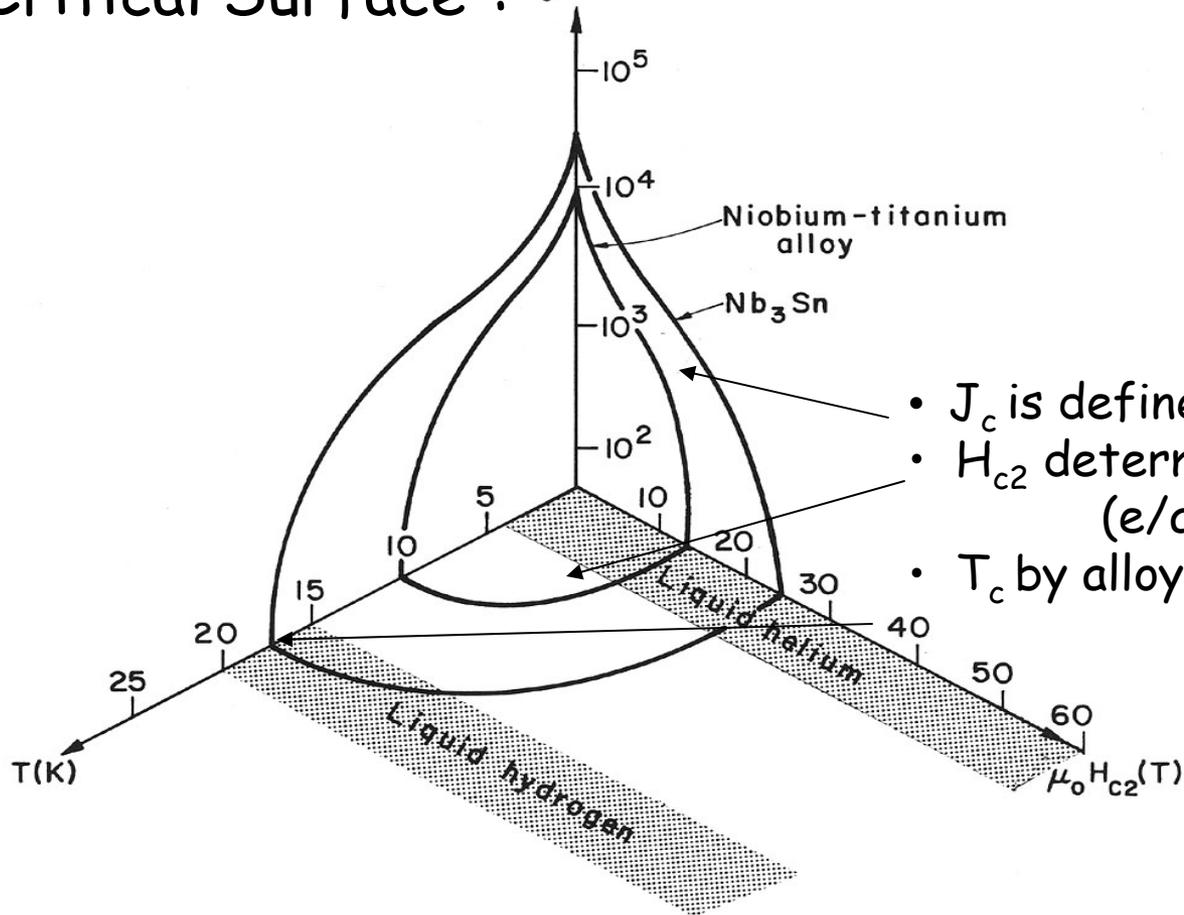
- ~ 1/3 elements are Type I superconductors
  - Not suitable for high field, current applications
- $T_c$  (transition from superconductor to normal state)
  - 7.2 K for Pb
  - 1.2 K for Al
- $\mu_0 H_c$  (critical field)
  - 80 mT for Pb
  - 10 mT for Al

Critical Temperature and Critical Field of Type I Superconductors

Material	$T_c$ (K)	$\mu_0 H_c$ (mT)
Aluminum	1.2	9.9
Cadmium	0.52	3.0
Gallium	1.1	5.1
Indium	3.4	27.6
Iridium	0.11	1.6
Lead	7.2	80.3
Mercury $\alpha$	4.2	41.3
Mercury $\beta$	4.0	34.0
Osmium	0.7	6.3
Rhenium	1.7	20.1
Rhodium	0.0003	4.9
Ruthenium	0.5	6.6
Tantalum	4.5	83.0
Thalium	2.4	17.1
Thorium	1.4	16.2
Tin	3.7	30.6
Tungsten	0.016	0.12
Zinc	0.9	5.3
Zirconium	0.8	4.7

# Practical LTS Superconductors

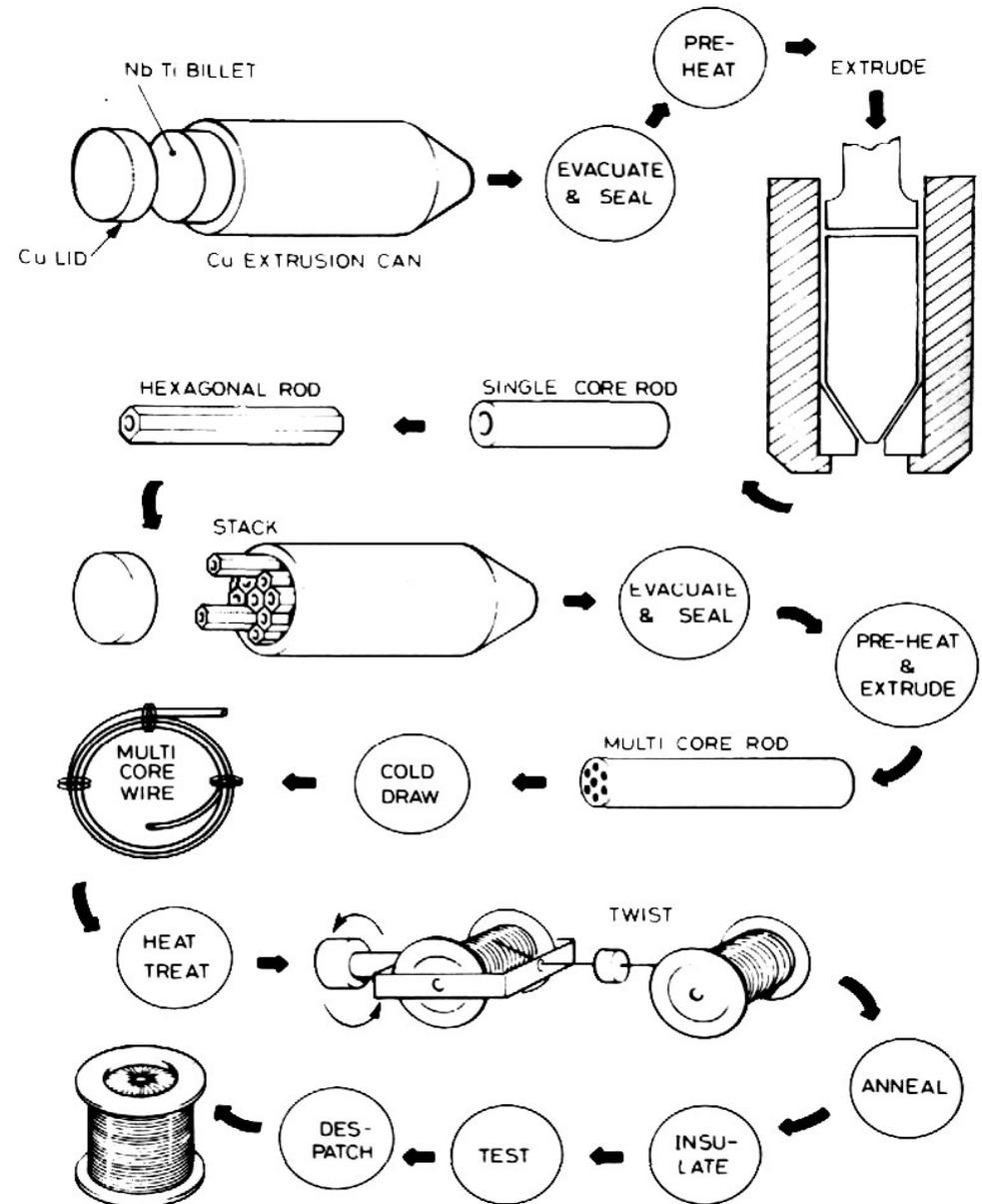
"Critical Surface":  $J_c$  (A/mm<sup>2</sup>)



- $J_c$  is defined by flux pinning
- $H_{c2}$  determined by alloy composition (e/a, crystal structure)
- $T_c$  by alloy composition

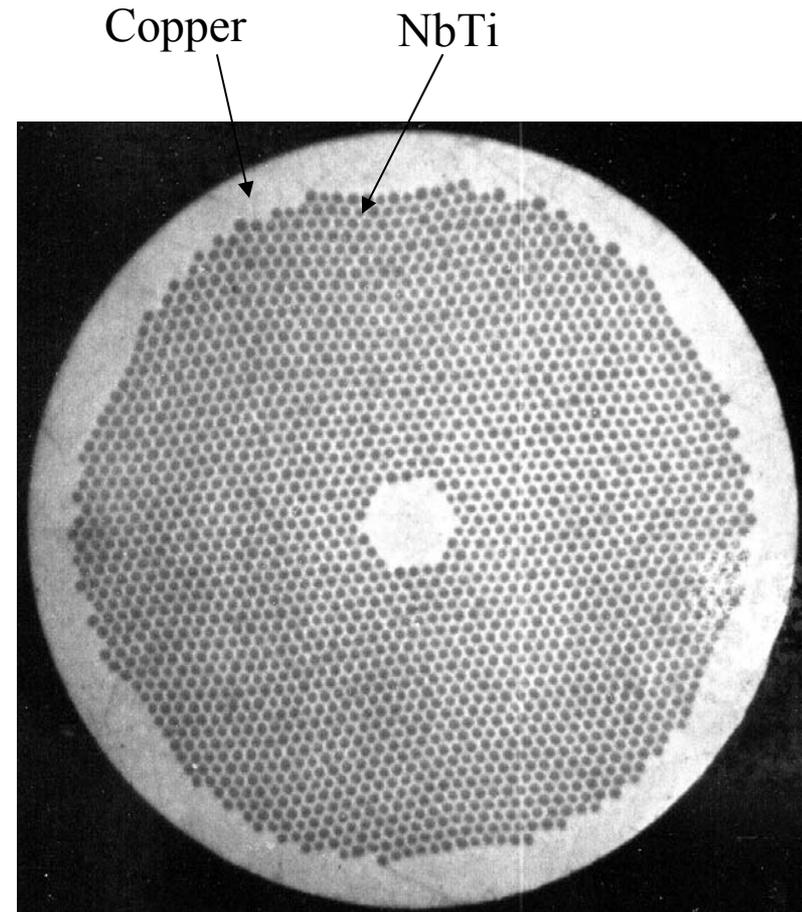
# Processing NbTi into Wire Form:

- Billet diameter  $\approx 10''$
- Extrusion ratio  $\approx (20)^2$
- Draw ratio  $\approx (15)^2$
- Final diameter  $\approx 1\text{ mm}$
- Heat Treatment @ 375 C  
anneal copper  
create grain structure
- Final product (1 mm wire)  
Cu:SC ratio (1:1-4:1)  
 $I_c \approx 500\text{ A @ } 5\text{ T}$
- Final processing:  
Cabling, insulation, etc.



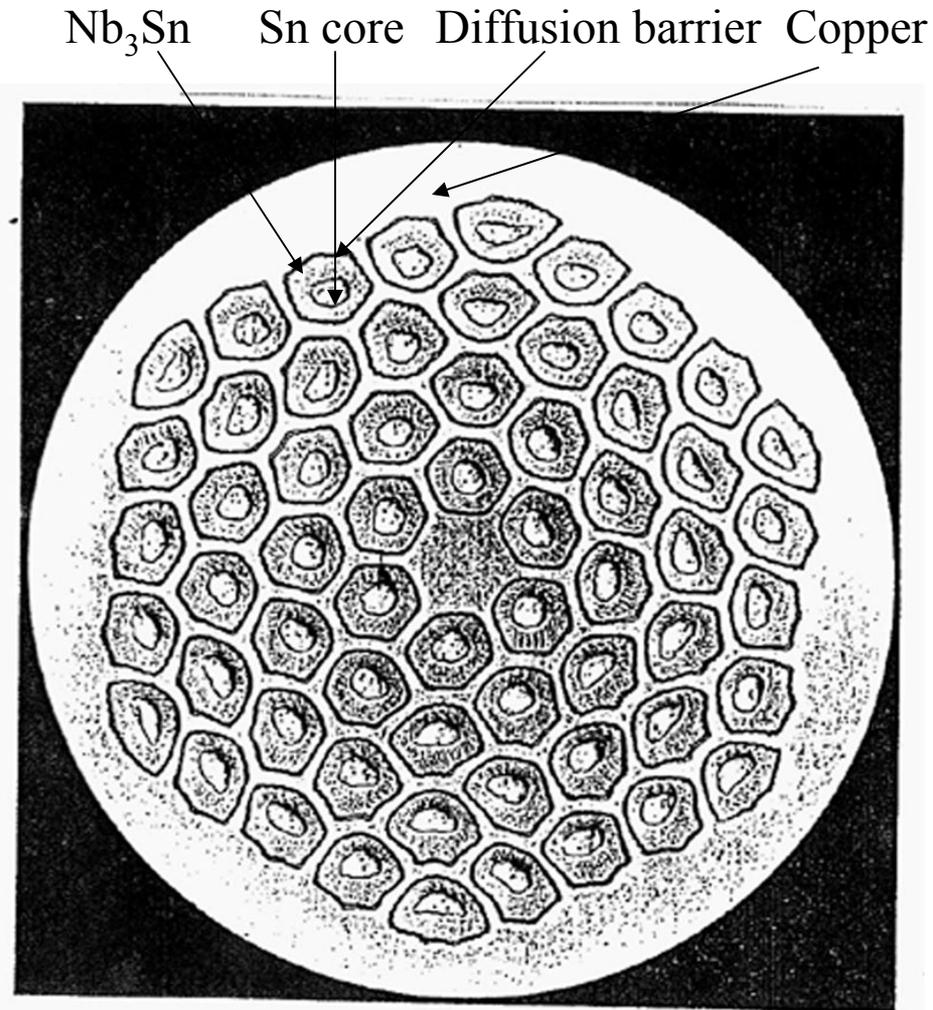
# NbTi/Cu Wire:

- Final wire is a composite of copper and  $\approx 1000$  NbTi filaments.
- Filament diameter is  $\approx 10 \mu\text{m}$ .
- Good metallurgical properties allow the wire to be processed to optimum properties, insulated and then wound into coil.
- Fine filaments are required for stability of the SC strand and to reduce AC loss
- Copper "stabilizer" provides alternate current path if superconductor becomes "normal"



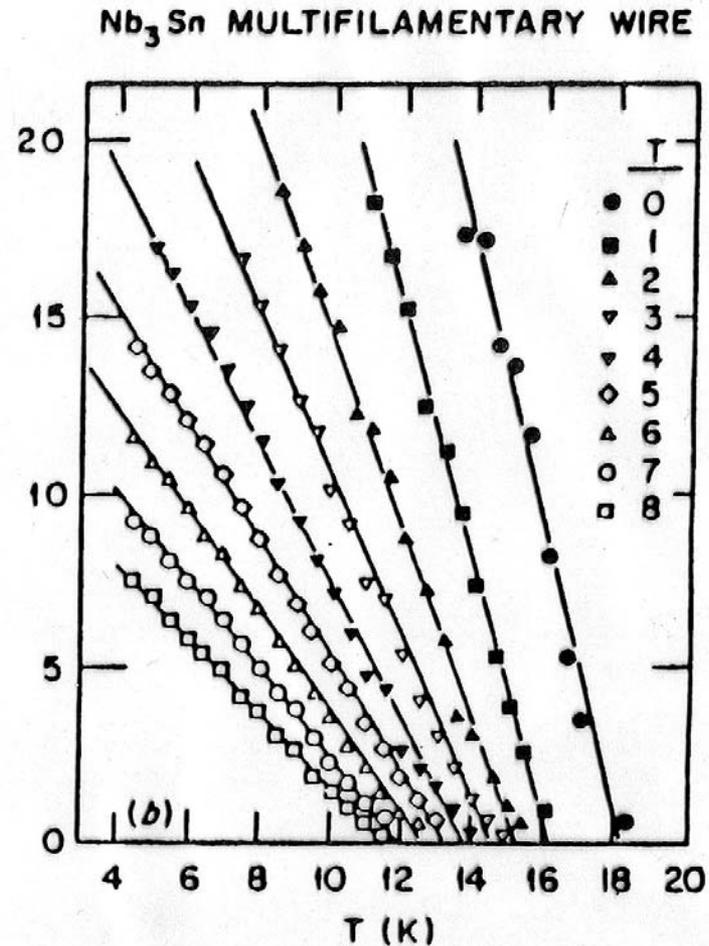
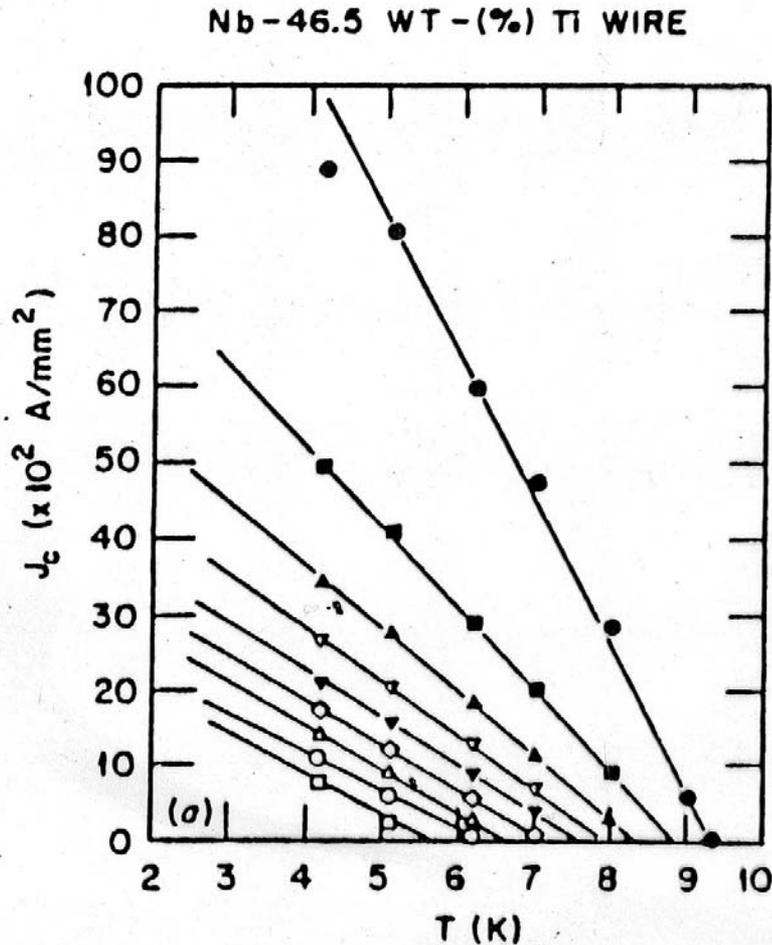
# Nb<sub>3</sub>Sn/Cu wire

- Nb<sub>3</sub>Sn is a metallurgical compound produced by solid state reaction at  $\approx 700\text{ C}$  for 100 hours
- Grown from pure Nb + Sn or from Nb + bronze (15% Sn in copper)
- Diffusion barrier (Ta) prevents Sn from mixing with Cu stabilizer
- Final wire (after reaction) is brittle
- In coil applications, Nb<sub>3</sub>Sn conductor is often wound first then reacted. This requires a high temperature insulation system



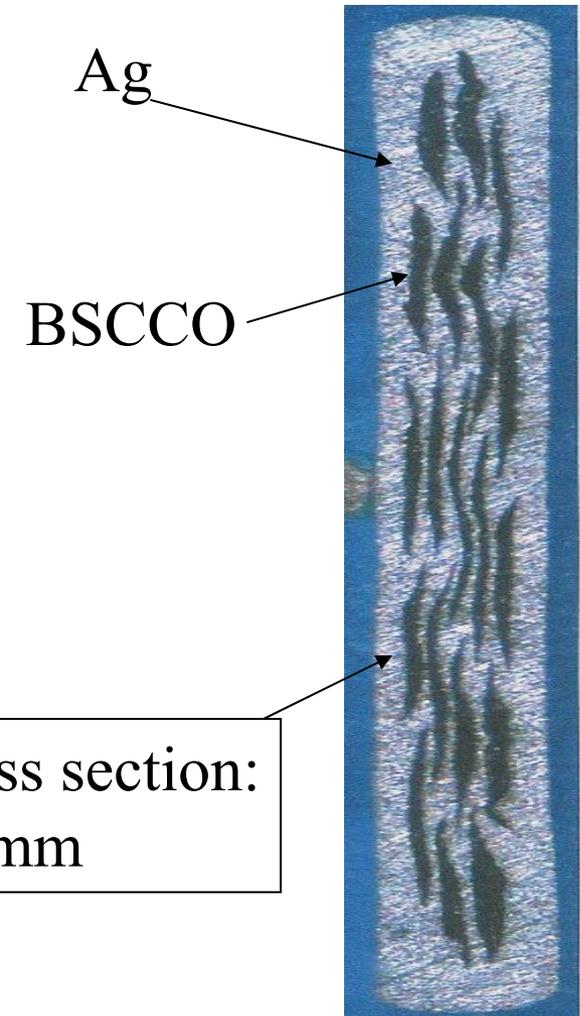
# Comparison of $J_c$ between $Nb_3Sn$ & $NbTi$

$J_c = I_c/A_{sc}$ ;  $A_{sc}$  may include material that is not superconducting, but not low resistivity



# Oxide superconductors (BSCCO & YBCO)

- Ag/BSCCO PIT available from industry but is difficult to process & handle (high cost)
- poor mechanical properties, anisotropic superconducting properties
- Requires "wind and react" approach
- Useful for  $T > 4$  K operation at low to moderate field
- Useful for  $T \approx 4$  K at high fields ( $B > 20$  T)



Conductor cross section:  
~ 4 mm x 0.5 mm

# Examples of High Temperature Superconductors (HTS)

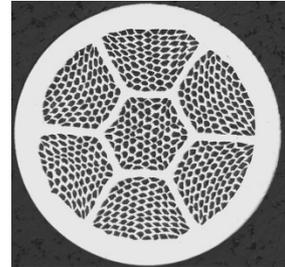
$\text{Bi}_2\text{Sr}_2\text{Ca}_1\text{Cu}_2\text{O}_x / \text{AgMg}$  tape

$\text{Bi}_2\text{Sr}_2\text{Ca}_1\text{Cu}_2\text{O}_x / \text{AgMg}$  round wire

(B) Cross-section, 5 by 0.2 mm



(D)



Cross-sections are roughly to scale

$\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_x / \text{Ag}$

(A)



(C)



(F)



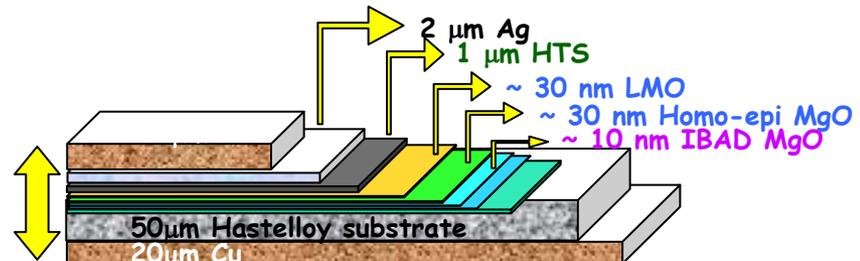
*BSCCO: Structure of ceramic filaments in silver-alloy*

Can be partially or fully substituted with Rare Earth metals: Y, Gd, Sm, Eu, Dy, Nd

$\text{Y}_1\text{B}_2\text{C}_3\text{O}_x$ : "Coated Conductor"

USPAS Cryogenics Short Course

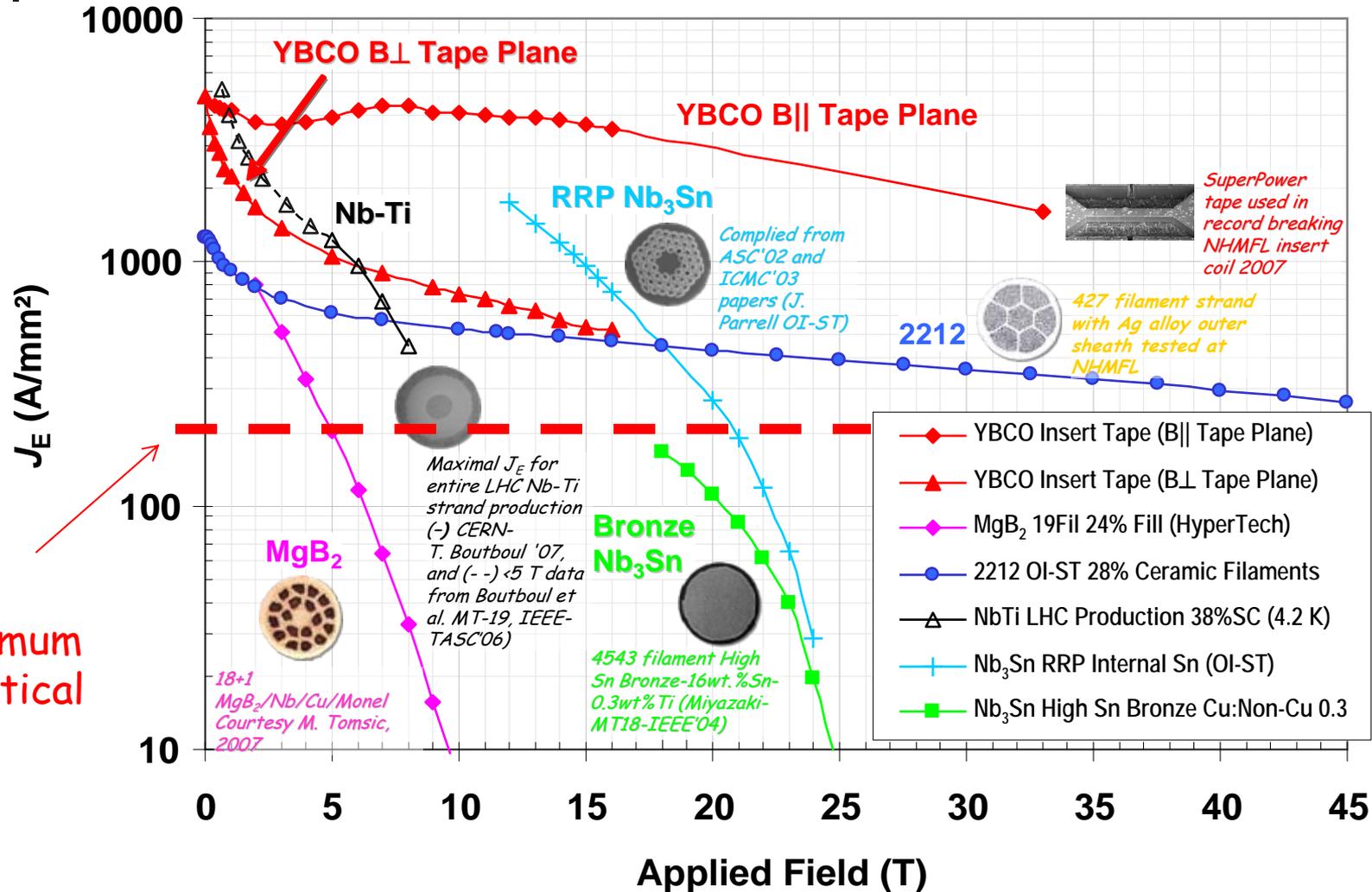
< 0.1 mm



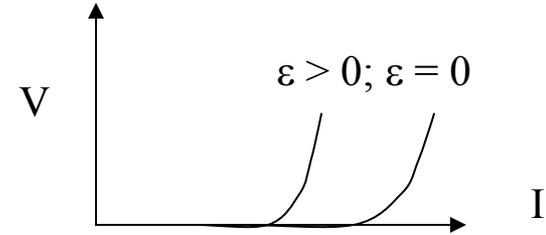
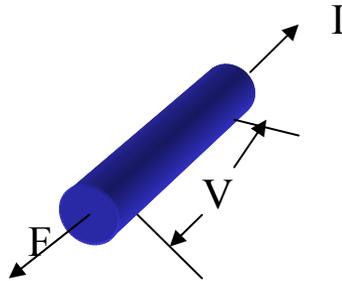
Boston, MA 6/14 to 6/18/2010

# $J_c$ of High Temperature Superconductors

4.2 K

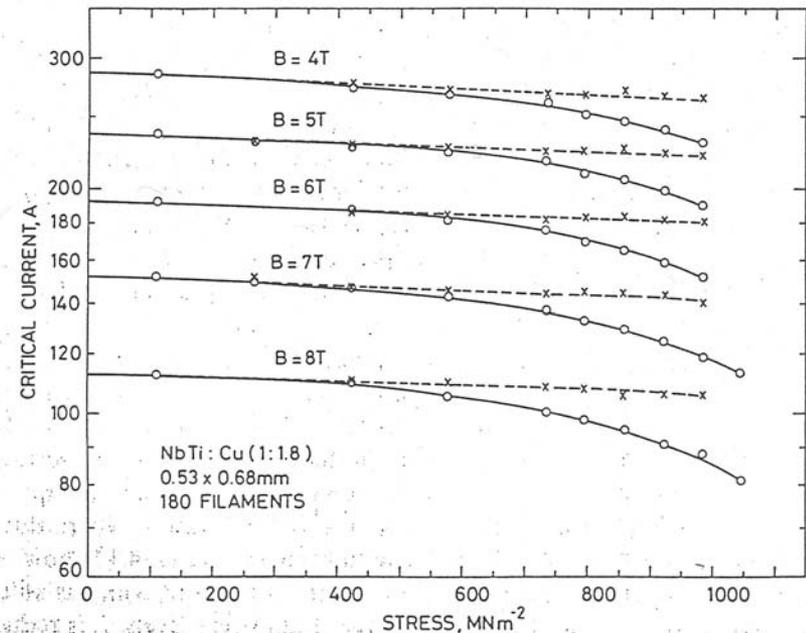


# Stress effects in SC?



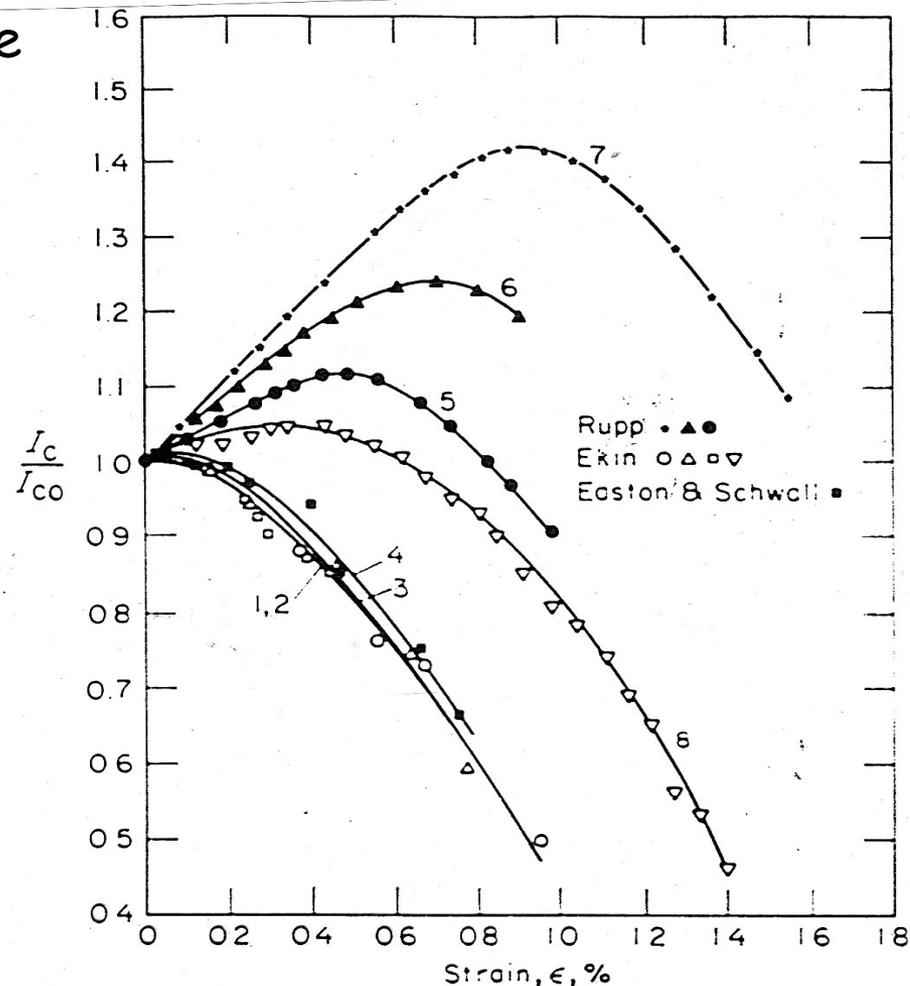
- In NbTi, the stress effects are largely reversible up to  $\approx 2\%$  until filament breakage

$E_y$  (NbTi)  $\approx 82$  GPa  
 $\sigma_U$  ( $\epsilon = 2\%$ )  $\approx 2200$  MPa  
 $E_y$  (NbTi)  $\approx 82$  GPa  
 $\sigma_U$  ( $\epsilon = 2\%$ )  $\approx 130$  MPa  
 Composite is average between  
 Copper and NbTi

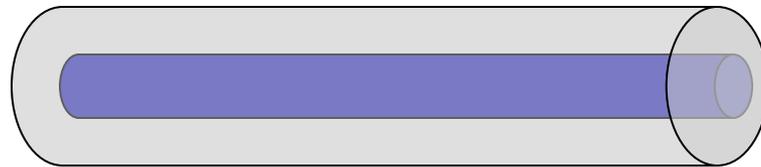


# Stress Effects in Nb<sub>3</sub>Sn

- For Nb<sub>3</sub>Sn,  $I_c$  can increase
- with strain- reversible effect
- $I_c$  increase depends on ratio
- Nb<sub>3</sub>Sn to Bronze (Cu)
- Bronze content in 1-4 is greater than in 5-7
- irreversible degradation > 0.5% past peak



# Precompression of Nb<sub>3</sub>Sn



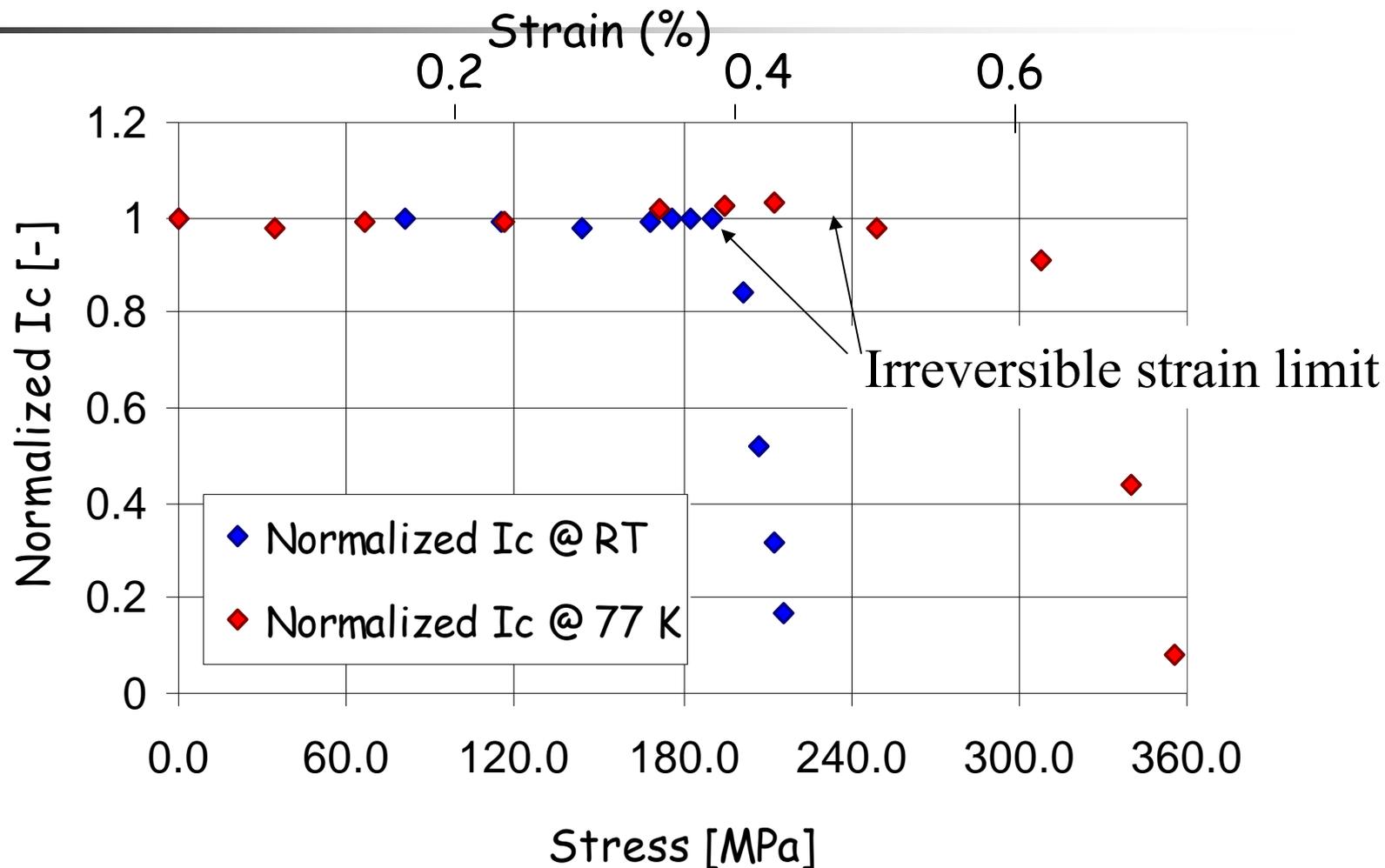
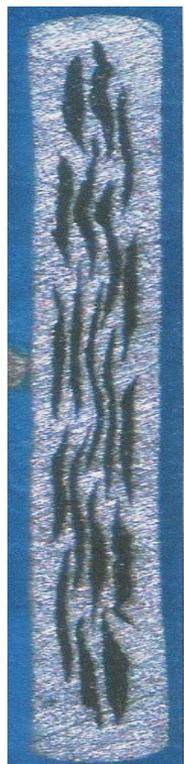
Bronze

Nb<sub>3</sub>Sn

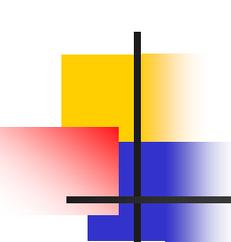
- Composite of bronze and Nb<sub>3</sub>Sn is processed at high temperature  $\approx 700\text{ C}$  (1000 K) then cooled to near 0 K for operation
- Integrated thermal contraction:  
 $\Delta L/L$  (1000 K to 0 K)  $\approx 0.8\%$  for Nb<sub>3</sub>Sn & 1.8% for Br

$$\sigma_{Nb_3Sn} = \underbrace{\left( \frac{\Delta L}{L} \Big|_{Nb_3Sn} - \frac{\Delta L}{L} \Big|_{Br} \right)}_{\approx 1\%} \left( \frac{E_{Nb_3Sn} E_{Br}}{EA_{Nb_3Sn} + EA_{Br}} \right) A_{Br}$$

# HTS 3-Ply Conductor, Tension RT and 77 K



Critical stresses/strains ~ 200 MPa/0.4% (RT), 300 MPa/0.6% (77 K)



# Superconducting Materials Summary

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- NbTi/Cu conductor technology is well developed, with good mechanical and electromagnetic properties up to moderate magnetic fields  $\approx 12$  T
- Nb<sub>3</sub>Sn/Cu conductor is also readily available although is less reliable, costs more and is more challenging to incorporate in magnets. Significant strain degradation.
- BSCCO (2212 & 2223)/Ag and YBCO are the most advanced HTS conductors and are available in long lengths. Strain degradation is an issue.
- Other superconductors have been and are being applied in special cases, but reliable conductor technology is not available.

# Low Temperature Materials- Summary

- Selection of proper material is important to proper design of cryogenic systems
  - Extensive data base of materials properties
    - Cryocomp™ (thermal and transport properties) - available for download from Blackboard
    - NIST documentation
    - Others??
  - New materials are being developed and introduced into cryogenic systems
    - Composites (Zylon, etc.)
    - Alloys
  - Other issues
    - Contact resistance
    - Mechanical properties of bonds, welds, etc..
- } Knowledge base is incomplete