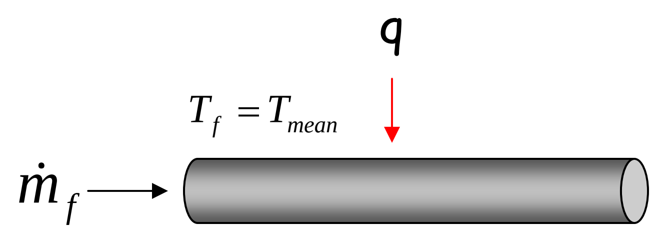
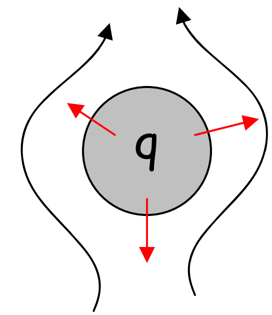
3.2 Cryogenic Convection Heat Transfer

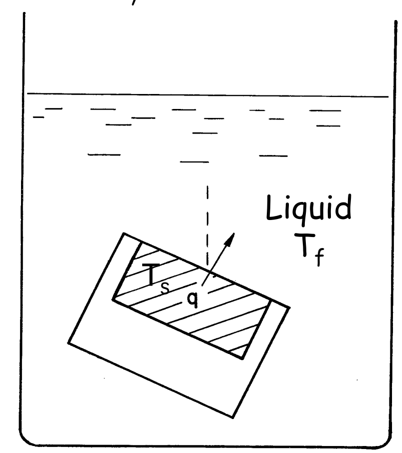
- Involves process of heat transfer between solid material and adjacent cryogenic fluid
- Classic heat transfer problem (Newton's law)

  $$q(kW/m^2) = h\,(T_s - T_f)$$

- Configurations of interest
  - Internal forced flow (single phase, $T_f = T_{mean}$)
  - Free convection (single phase, $T_f = T_\infty$)
  - Internal two phase flow
  - Pool boiling (two phase)
- Understanding is primarily empirical leading to correlations based on dimensionless numbers
- Issue is relevant to the design of:
  - Heat exchangers
  - Cryogenic fluid storage
  - Superconducting magnets
  - Low temperature instrumentation

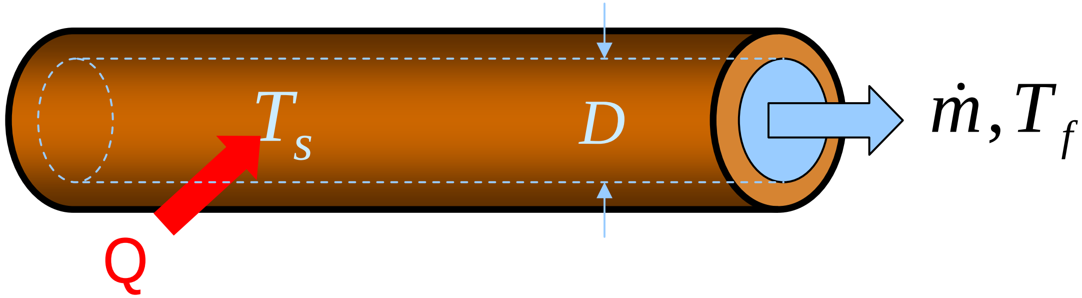






Single phase internal flow heat transfer

Forced Convection



Classical fluid correlations

- The heat transfer coefficient in a classical fluid system is generally correlated in the form where the Nusselt number,
  where, $Nu_D \equiv hD/k_f$ and D is the characteristic length
- For laminar flow, Nu_D = constant ~ 4 (depending on b.c.)
- For turbulent flow (Re_D > 2000)

$$Nu_D = f(\mathrm{Re}_D, \mathrm{Pr}) = C\,\mathrm{Re}_D^n\,\mathrm{Pr}^m \quad \text{and} \quad \mathrm{Pr} \equiv \frac{\mu_f C_p}{k_f} \quad \text{(Prandtl number)}$$

- Dittus-Boelter Correlation for classical fluids (+/- 15%)

$$Nu_D = 0.023\,\mathrm{Re}^{4/5}\,\mathrm{Pr}^{2/5}$$

Note that fluid properties should be computed at T_f (the "film temperature"): $T_f \equiv \dfrac{T_s + T_f}{2}$

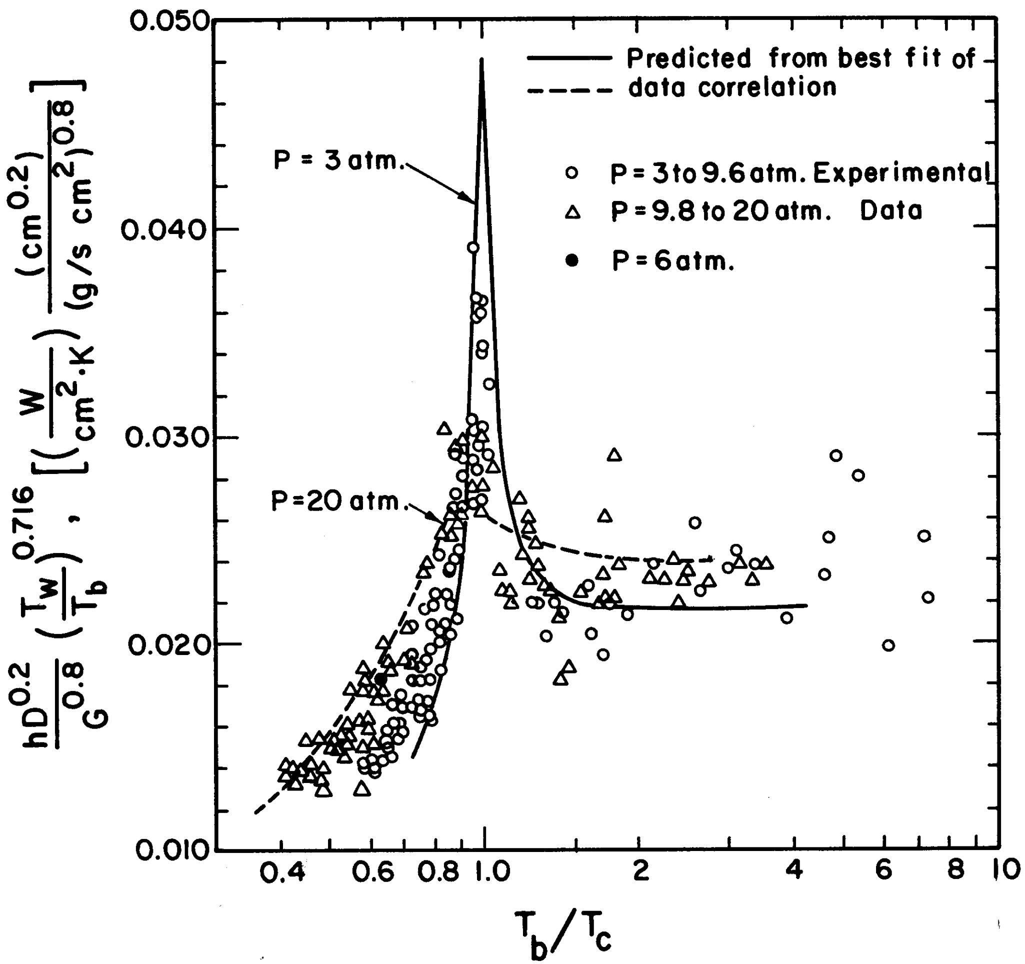
Johannes Correlation (1972)

- Improved correlation specifically for helium (+/- 8.3%)

$$Nu_D = 0.0259\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{2/5}\left(\frac{T_s}{T_f}\right)^{-0.716}$$

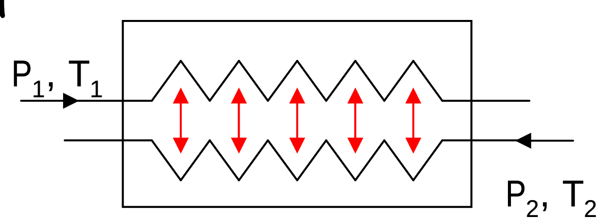
- Last factor takes care of temperature dependent properties
- Note that one often does not know T_f, so iteration may be necessary.

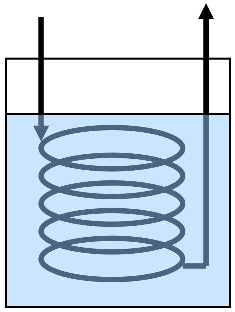
Example

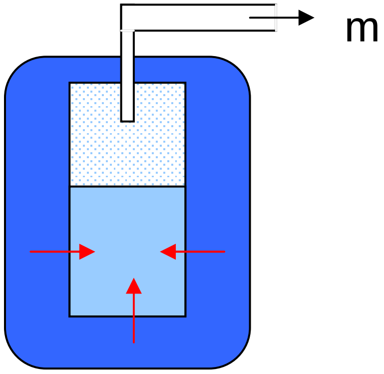


Application: Cryogenic heat exchangers

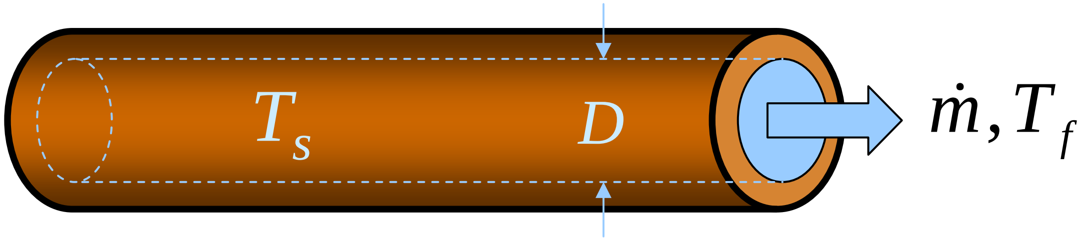
- Common types of heat exchangers used in cryogenic systems
  - Forced flow single phase fluid-fluid
    - E.g. counterflow heat exchanger in refrigerator/liquefier
  - Forced single phase flow - boiling liquid (Tube in shell HX)
    - E.g. LN_2 precooler in a cooling circuit
  - Static boiling liquid-liquid
    - E.g. Liquid subcooler in a magnet system







Simple 1-D heat exchanger

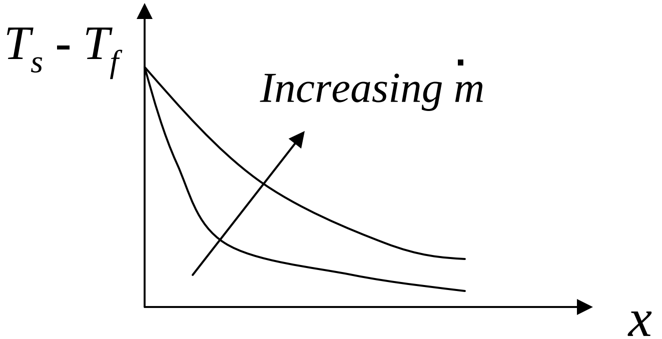


- Differential equation describing the temperature of the fluid in the tube:

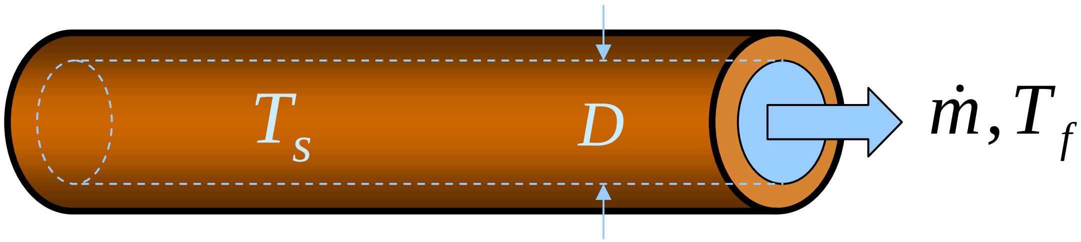
$$\dot{m}C\frac{dT_f}{dx} + hP\left(T_f - T_s\right) = 0$$

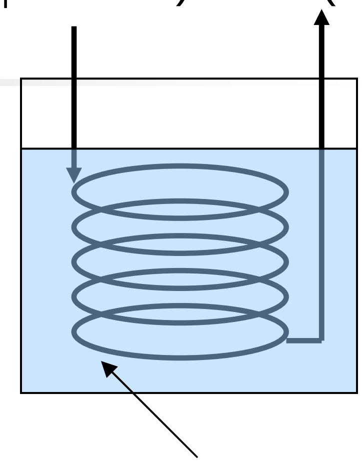
- For constant T_s, the solution of this equation is an exponential

$$T_s - T_f = (T_s - T_f)_0 \exp\left(-\frac{hP}{\dot{m}C}x\right)$$



Liquid nitrogen precooler





- Assumptions & givens
  - T_s is a constant @ 77 K (NBP of LN_2)
  - Helium gas (C_p = 5.2 kJ/kg K; μ = 15 x 10^{-6} Pa s; ρ = 0.3 kg/m^3, k = 0.1 W/m K)
  - Allowed pressure drop, Δp = 10 kPa
  - Helium mass flow rate = 1 g/s

Properties are average values between 300 K and 80 K

- Find the length and diameter of the HX (copper tubing)
- Total heat transfer:

$$Q = \dot{m} C_p \left(T_{in} - T_{out} \right)$$

= 1 g/s x 5.2 J/g K x 220 K
= 1144 W

- Log mean ΔT:

$$\Delta T_{lm} = \frac{\Delta T_f (x=0) - \Delta T_f (x=L)}{\ln\left(\Delta T_f (x=0) \big/ \Delta T_f (x=L) \right)} = \frac{(300-77) - (80-77) \text{ K}}{\ln [(300-77)/(80-77)]}$$

= 51 K

Liquid nitrogen precooler (continued)

$$UA = h\pi DL = Q/\Delta T_{lm}$$ = 1144 W/51 K = 22.4 W/K

- The heat transfer coefficient is a function of Re_D and Pr = 0.67
- Assuming the flow is turbulent and fully developed, use the Dittus Boelter correlation

$$Nu_D = \frac{hD}{k_f} = 0.023\,Re_D^{0.8}\,Pr^{0.3} \quad \text{and} \quad Re_D = \frac{4\dot{m}}{\pi D\mu}$$

Substituting the Re_D and solving for h

$$h = 0.023\frac{k_f}{D}\left(\frac{4\dot{m}}{\pi D\mu}\right)^{0.8} Pr^{0.3} = \frac{0.0247 \times 0.1 \text{ W/m K} \times (10^{-3}\text{ kg/s})^{0.8}}{(15 \times 10^{-6}\text{ Pa s})^{0.8} \times D^{1.8}}$$

$$= 0.07/D(m)^{1.8}$$

$h\pi DL$ = 0.224 x ($L/D^{0.8}$) = 22.4 W/K

or $L/D^{0.8}$ = 100 $m^{0.2}$ ← 1 equation for two unknowns

Liquid nitrogen precooler (continued)

- Pressure drop equation provides the other equation for L & D

$$\Delta p = f\,\frac{L}{2\rho D}\left(\frac{\dot{m}}{A_{flow}}\right)^2$$

$;\ A_{flow} = \pi D^2/4$ *and* $f \sim 0.02$ *(guess)*

$$\mathrm{Re}_D = \frac{4\dot{m}}{\pi D \mu}$$

$$\frac{4 \times 0.001\ \text{kg/s}}{\pi \times 0.0062\ \text{m} \times 15 \times 10^{-6}\ \text{Pa s}}$$

~ 13,700

Substituting for Re_D *and* f

$$\Delta p \approx 0.016\,\frac{\dot{m}^2}{\rho}\,\frac{L}{D^5} = \frac{0.016 \times (10^{-3}\ \text{kg/s})^2}{0.3\ \text{kg/m}^3}\left(\frac{L}{D^5}\right)$$

$$\Delta p = 5.33 \times 10^{-8}\left(\frac{L}{D^5}\right)$$

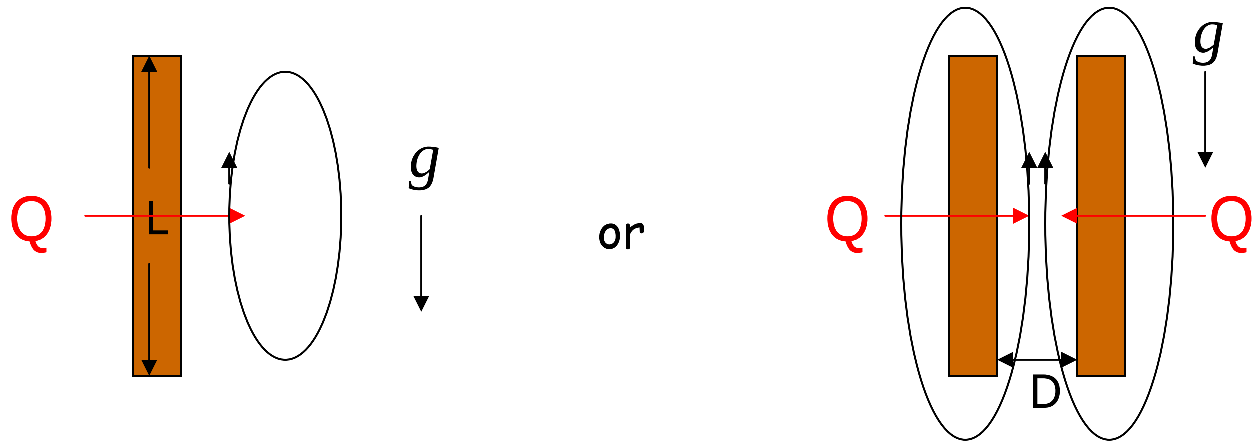
2nd equation for two unknowns

Substitute:

Eq. 1: $L = 100\ D^{0.8} \longrightarrow \Delta p = 5.33 \times 10^{-6}/D^{4.2}$ with $\Delta p = 10{,}000$ Pa

$D = [5.33 \times 10^{-6}/\Delta p]^{1/4.2} =$ 6.2 mm and $L = 100 \times (0.0062\ \text{m})^{0.8} =$ 1.7 m

Single phase free convection heat transfer



- Compressible fluid effect: Heat transfer warms the fluid near the heated surface, reducing density and generating convective flow.
- Free convection heat transfer is correlated in terms of the Rayleigh number,

$$Nu_L = f\left(Gr, \Pr\right) \sim C Ra_L^n \quad \text{where} \quad Ra_L \equiv Gr \Pr = \frac{g\beta \Delta T L^3}{D_{th} \nu}$$

where g is the acceleration of gravity, β is the bulk expansivity, ν is the kinematic viscosity (μ/ρ) and D_{th} is the thermal diffusivity ($k/\rho C$). L (or D) are the scale length of the problem (in the direction of $\vec{g}$)

Free convection correlations

- For very low Rayleigh number, Nu = 1 corresponding to pure conduction heat transfer
- For Ra < 10^9, the boundary layer flow is laminar (conv. fluids)

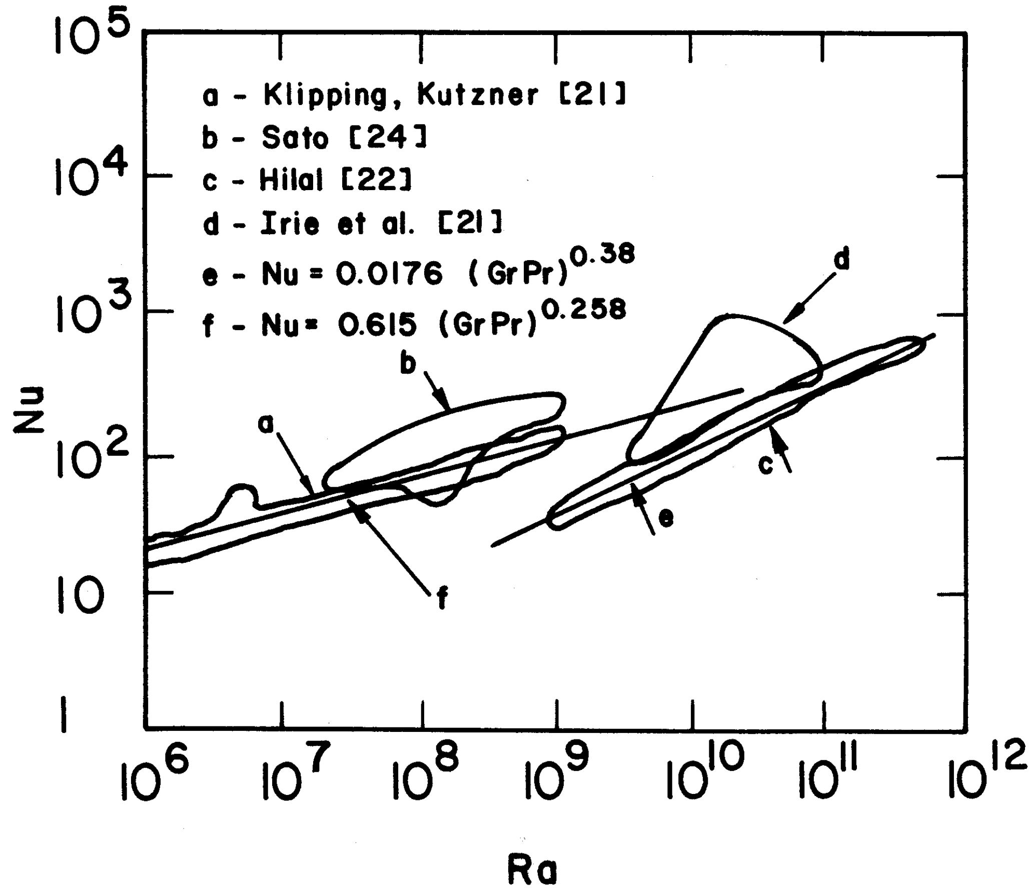
$$Nu_L \approx 0.59 Ra_L^{0.25}$$

- For Ra > 10^9, the boundary layer is turbulent

$$Nu_L \approx 0.1 Ra_L^{0.33}$$

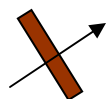
Example

Free convection correlation for low temperature helium

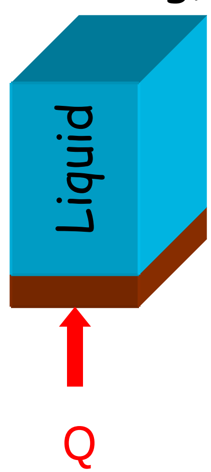


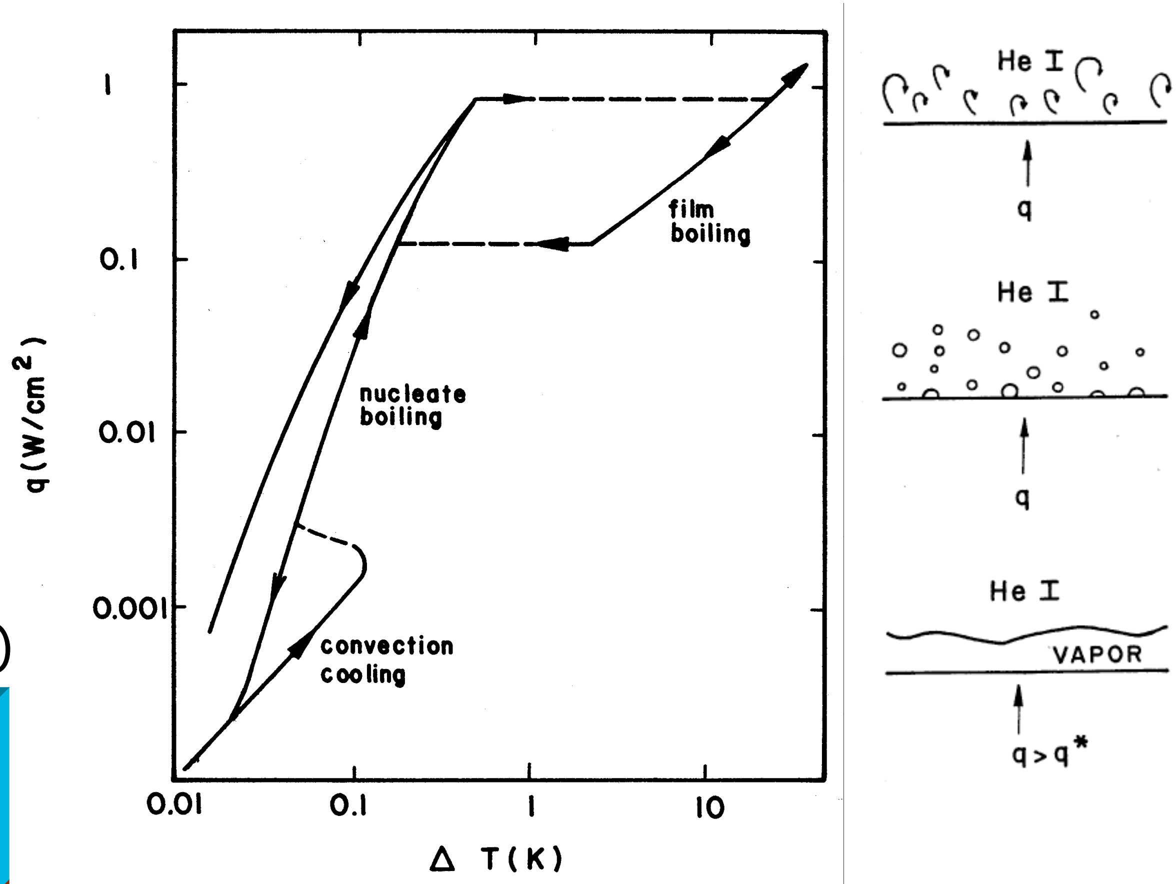
Pool Boiling Heat Transfer (e.g. Helium)

<u>Factors affecting heat transfer curve</u>

- Surface condition (roughness, insulators, oxidation)
- Orientation 
- Channels (circulation)
- Time to develop steady state (transient heating)

Heated Surface





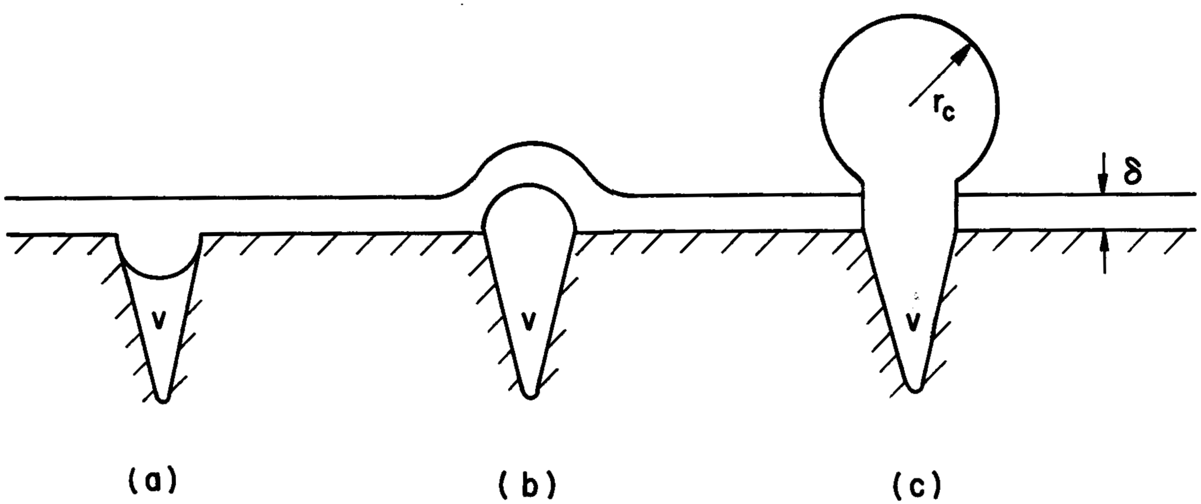
Note: Other cryogenic fluids have basically the same behavior, although the numerical values of q and ΔT are different.

Nucleate Boiling Heat Transfer

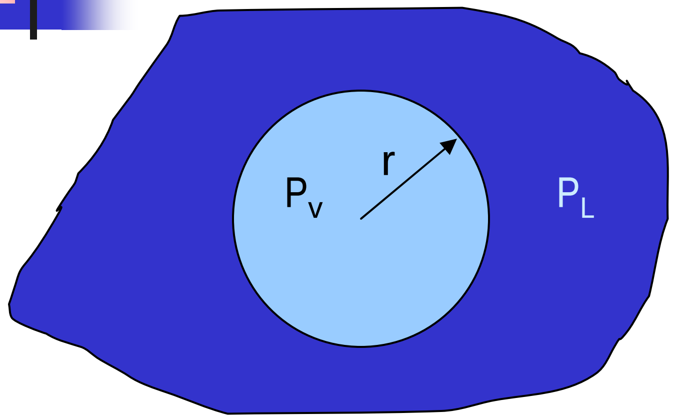
- Nucleate boiling is the principal heat transfer mechanism for static liquids below the peak heat flux ($q^* \sim 10$ kW/m^2 for helium)
- Requirements for nucleate boiling
  - Must have a thermal boundary layer of superheated liquid near the surface

$$\delta_{th} = \frac{k_f \Delta T}{q} \quad \sim 1 \text{ to } 10\ \mu\text{m for helium}$$

  - Must have surface imperfections that act as nucleation sites for formation of vapor bubbles.



Critical Radius of Vapor Bubble



$$P_v = P_L + \frac{2\sigma}{r}$$

σ is the surface tension

- Critical radius: For a given T, p, the bubble radius that determines whether the bubble grows of collapses
  - $r > r_c$ and the bubble will grow
  - $r < r_c$ and the bubble will collapse
- Estimate the critical radius of a bubble using thermodynamics
  - Clausius Clapeyron relation defines the slope of the vapor pressure line in terms of fundamental properties

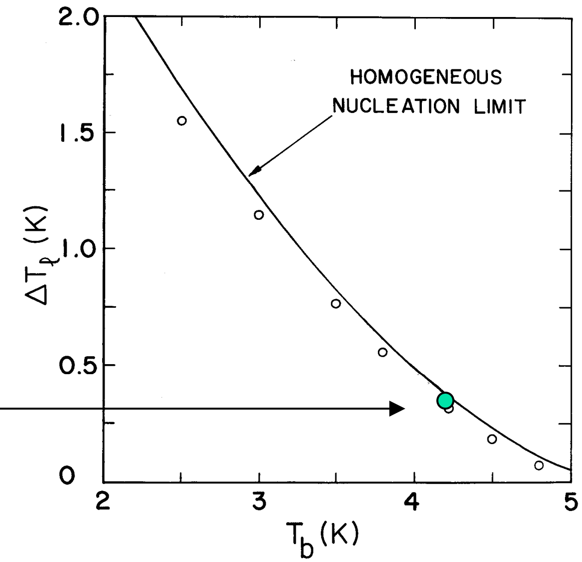
$$\left.\frac{dp}{dT}\right)_{sat} = \frac{\Delta s}{\Delta v} = \frac{h_{fg}}{T(v_v - v_L)} \approx \frac{h_{fg}\, p}{RT^2}$$

If the gas can be approximated as ideal and $v_v >> v_L$

Critical radius calculation

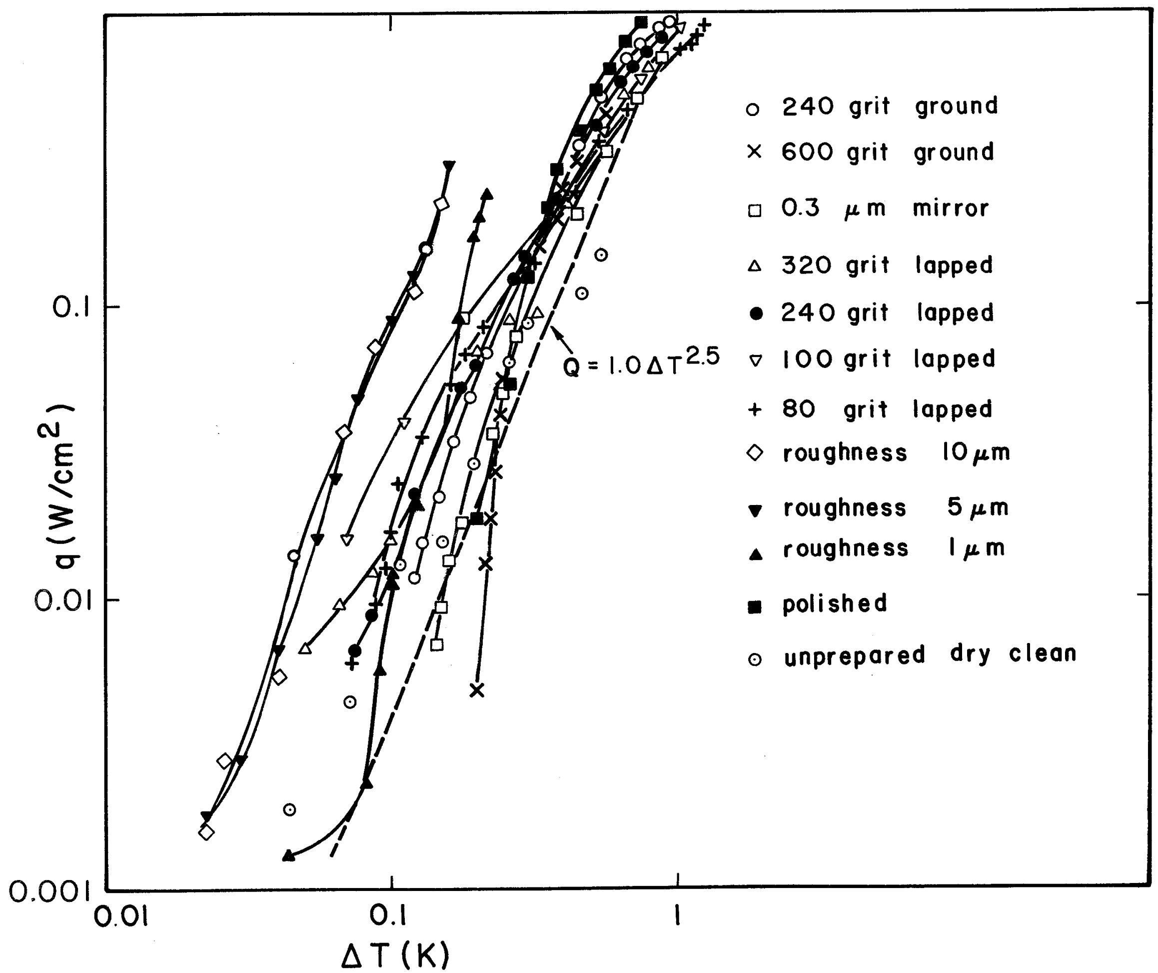
- Integrating the Clausius Clapeyron relation between p_s and $p_s + 2\sigma/r$

$$r_c = \frac{2\sigma}{p_s}\left(e^{h_{fg}\Delta T/RT^2} - 1\right)^{-1} \approx \frac{2\sigma R T_s^2}{h_{fg} p_s \Delta T}$$



- Example: helium at 4.2 K (NBP)
  - Empirical evidence indicates that $\Delta T \sim 0.3$ K
  - This corresponds to $r_c \sim 17$ nm
  - Number of helium molecules in bubble ~ 10,000
  - Bubble has sufficient number of molecules to be treated as a thermodynamic system
- Actual nucleate boiling heat transfer involves heterogeneous nucleation of bubbles on a surface. This is more efficient than homogeneous nucleation and occurs for smaller ΔT.

Nucleate Boiling Heat Transfer He I



Note that h_{nb} is not constant because $Q \sim \Delta T^{2.5}$

Nucleate Boiling Heat Transfer Correlations

- The mechanism for bubble formation and detachment is very complex and difficult to model
- Engineering correlations are used for analysis

- Kutateladse correlation

$$\frac{h}{k_l}\left(\frac{\sigma}{g\rho_l}\right)^{1/2} = 3.25\times 10^{-4}\left[\frac{qC_{pl}\rho_l}{h_{fg}\rho_v k_l}\left(\frac{\sigma}{g\rho_l}\right)^{1/2}\right]^{0.6}\times\left[g\left(\frac{\rho_l}{\mu_l}\right)^2\left(\frac{\sigma}{g\rho_l}\right)^{3/2}\right]^{0.125}\left(\frac{p}{(\sigma g\rho_l)^{1/2}}\right)^{0.7}$$

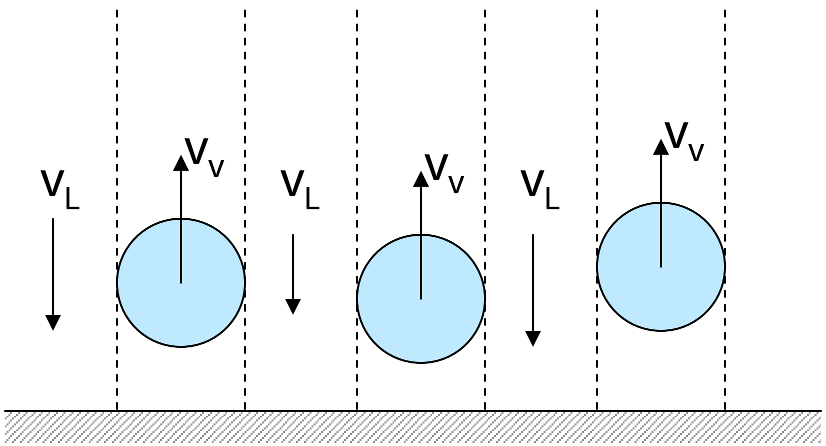
- Rearranging into a somewhat simpler form,

$$q = 1.90\times 10^{-9}\left[g\left(\frac{\rho_l}{\mu_l}\right)^2\chi^3\right]^{0.3125}\left(\frac{p\chi}{\sigma}\right)^{1.75}\left(\frac{\rho_l}{\rho_v}\right)^{1.5}\times\left(\frac{C_p}{h_{fg}}\right)^{1.5}\left(\frac{k_l}{\chi}\right)(T_s - T_b)^{2.5}$$

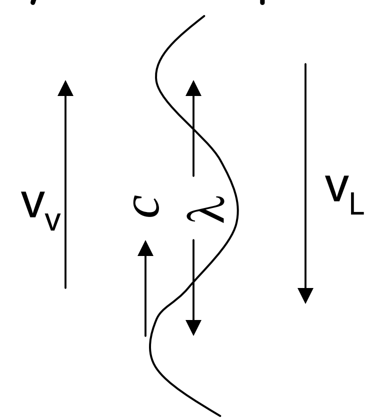
where $\chi = \left(\frac{\sigma}{g\rho_l}\right)^{1/2}$ $\qquad q(W/cm^2) = 5.8\Delta T^{2.5}$ For helium at 4.2 K

Peak Heat Flux (theory)

- Understanding the peak nucleate boiling heat flux is based on empirical arguments due to instability in the vapor/liquid flow



Instability in the vapor-liquid boundary



- Instability due to balance between surface energy and kinetic energy

$$c^2 = \frac{2\pi\sigma}{\lambda(\rho_L + \rho_v)} - \frac{\rho_L \rho_v}{(\rho_L + \rho_v)^2}(v_v - v_L)^2$$

Transition to unstable condition when $c^2 = 0$

Peak Heat Flux Correlations

- Zuber correlation:

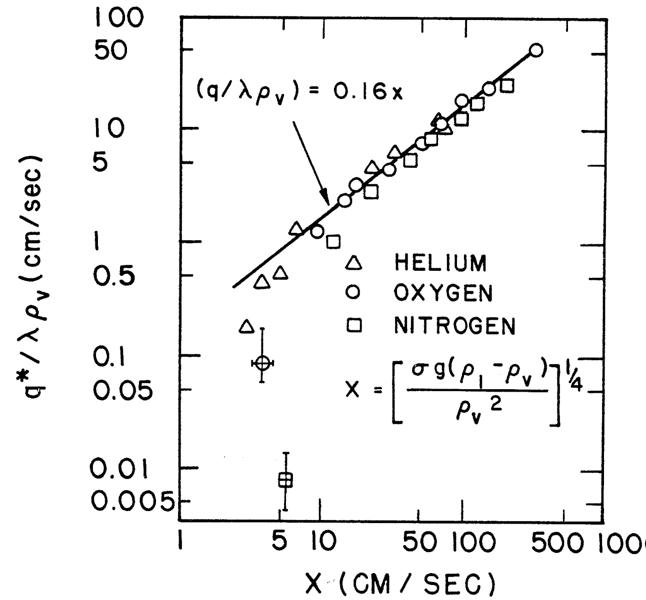
$$q^* \approx K h_{fg} \rho_v \left[\frac{\sigma(\rho_l - \rho_v) g}{\rho_v^2}\right]^{1/4} \left[\frac{\rho_l}{\rho_l + \rho_v}\right]^{1/2}$$

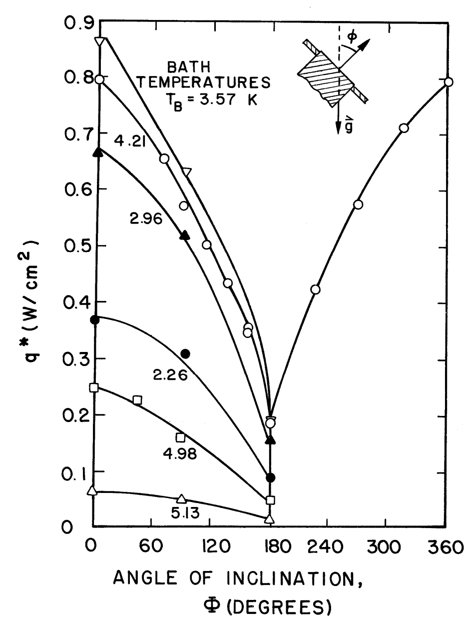
- Empirical based on Zuber Correlation

$$q^* \approx 0.16 h_{fg} \rho_v^{1/2} \left[\sigma g (\rho_l - \rho_v)\right]^{1/4}$$

~ 8.5 kW/m² for He I at 4.2 K

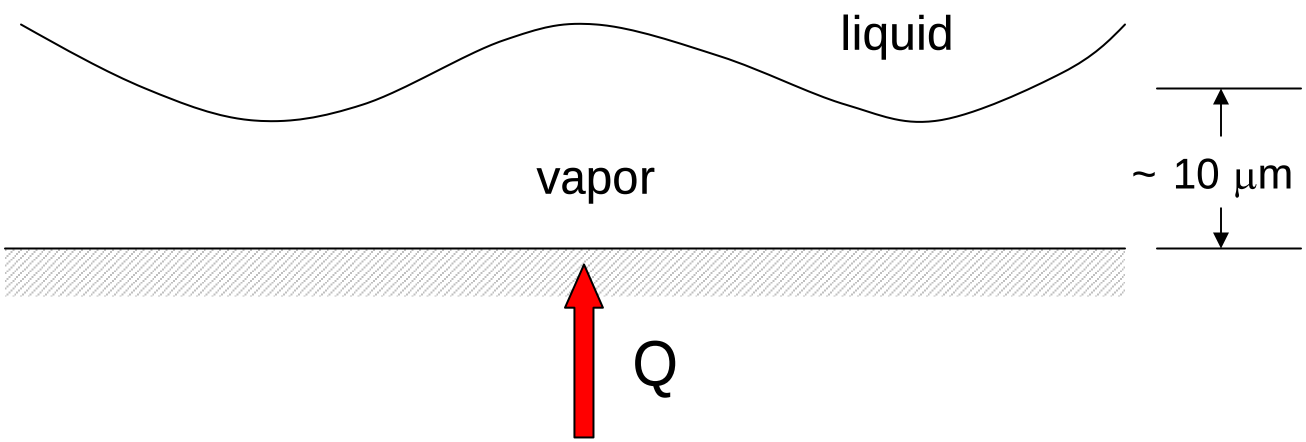
- Limits:
  - $T \longrightarrow T_c$; $q^* \rightarrow 0$ since $h_{fg} \rightarrow 0$ and $\sigma \rightarrow 0$
  - $T \longrightarrow 0$; $q^* \sim \rho_v^{1/2}$ (decreases)
  - q^*_{max} near 3.6 K for LHe





Film Boiling

- Film boiling is the stable condition when the surface is blanketed by a layer of vapor
  - Film boiling heat transfer coefficient is generally much less that that in nucleate boiling
  - Minimum film boiling heat flux, q_{mfb} is related to the stability of the less dense vapor film under the more dense liquid



"Taylor Instability" governs the collapse of the vapor layer

Film Boiling Heat Transfer Correlations

- Factors affecting the process
  - Fluid properties: C_p, h_{fg}, σ, ρ_l, ρ_v
  - Fluid state: saturated or pressurized (subcooled)
  - Heater geometry (flat plate, cylinder, etc.)
- Breen-Westwater correlation

$$h_{fb}\left(\frac{\sigma}{g(\rho_l-\rho_v)}\right)^{1/8}\left(\frac{\mu_v(T_s-T_b)}{k_v^3\rho_v(\rho_l-\rho_v)g\lambda'}\right)^{1/4} = 0.37+0.28\left(\frac{\sigma}{gD^2(\rho_l-\rho_v)}\right)^{1/2}$$

Where, $$\lambda' = \frac{\left[h_{fg}+0.34C_{pv}(T_s-T_b)\right]^2}{h_{fg}}$$

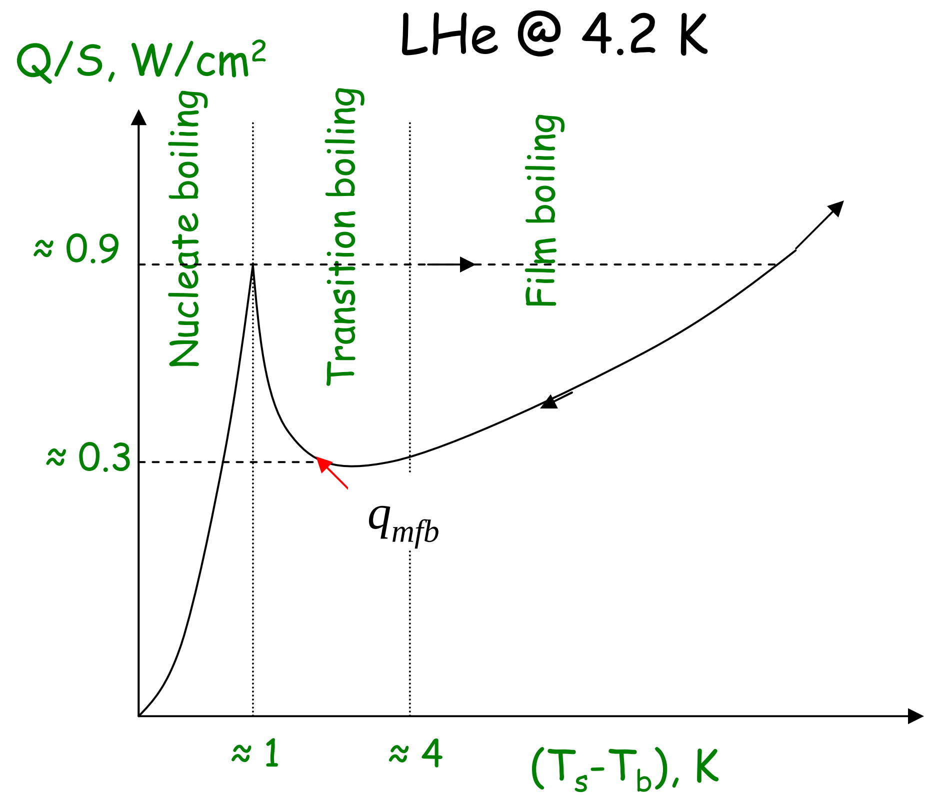
- Simplified form for large diameter

$$h_{fb} = 0.37\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/8}\left(\frac{k_v^3\rho_v(\rho_l-\rho_v)g\lambda'}{\mu_v(T_s-T_b)}\right)^{1/4} \Longrightarrow q \approx (T_s-T_f)^{3/4}$$

Minimum film boiling heat flux

- Minimum film boiling heat flux is less than the peak heat flux
- Recovery to nucleate boiling state is associated with Taylor Instability.

$$q_{mfb} = 0.16 h_{fg} \rho_v \left[\frac{g\sigma(\rho_l - \rho_v)}{(\rho_l + \rho_v)^2} \right]^{1/4}$$

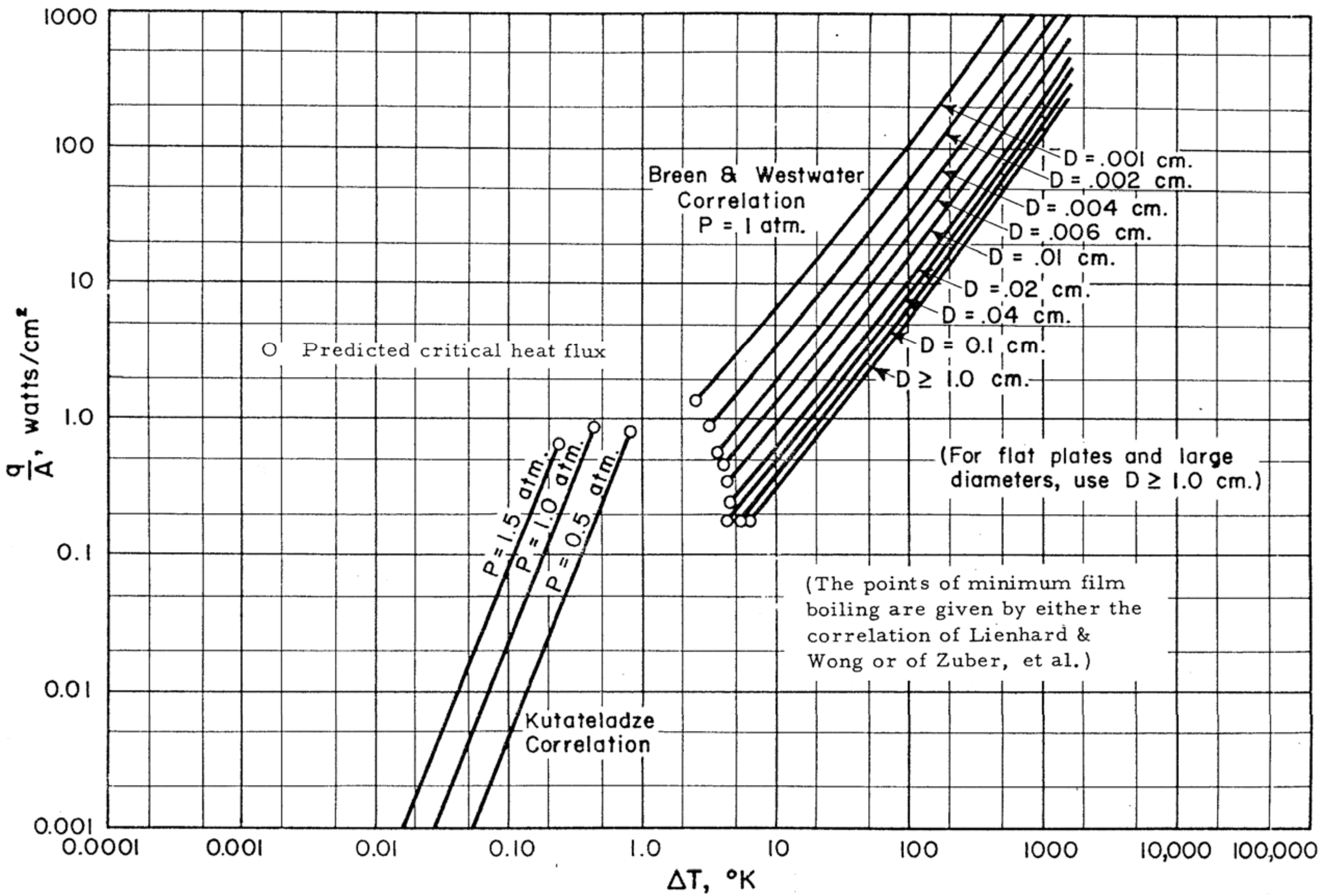


- Dimensionless ratio:

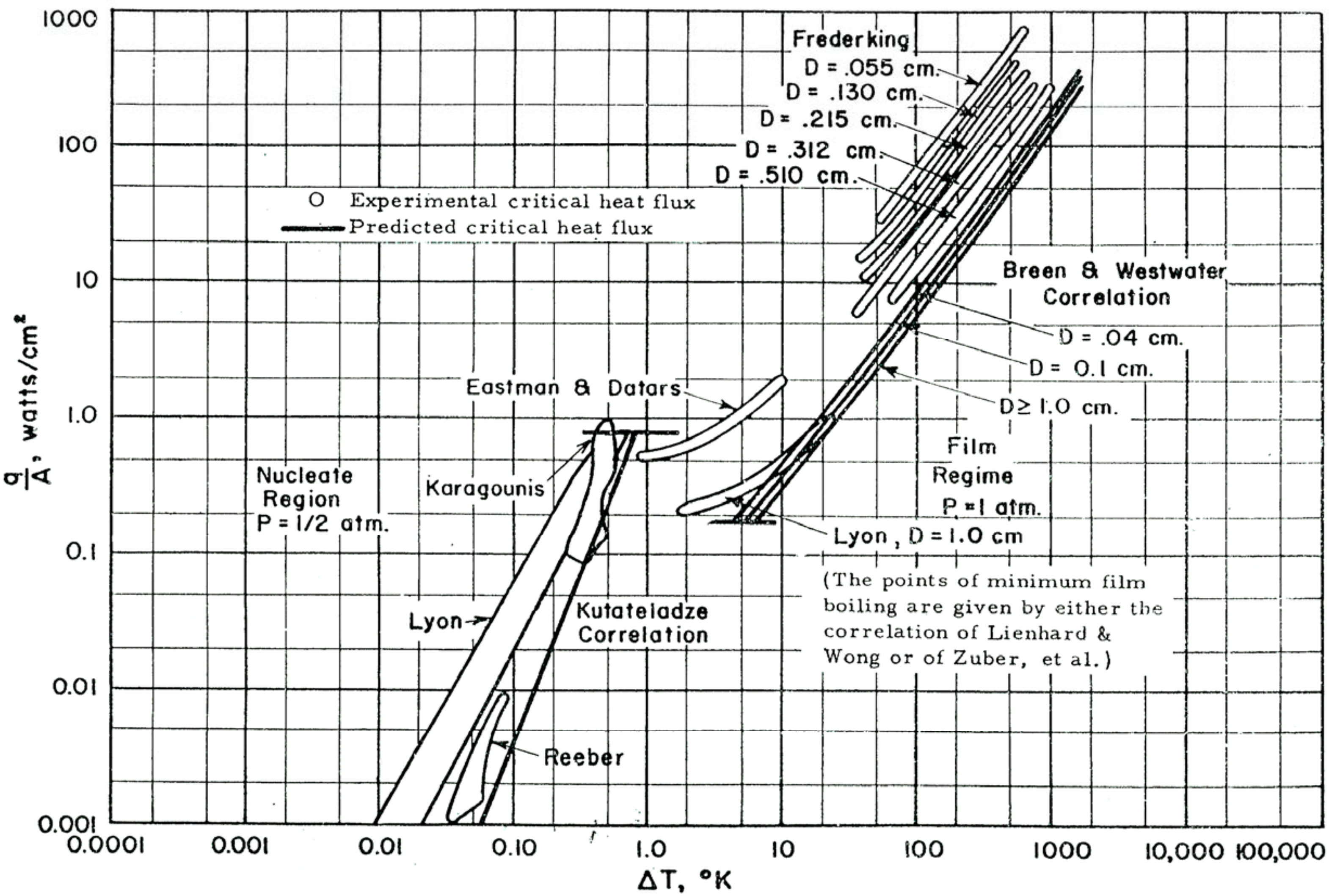
$$\frac{q_{mfb}}{q^*} = \left[\frac{\rho_v}{\rho_l + \rho_v} \right]^{1/2}$$

~ 0.36 for LHe @ 4.2 K
~ 0.1 @ 2.2 K
~ 1 @ T_c = 5.2 K
~ 0.09 for LN_2 @ 80 K where q* ~ 200 kW/m²

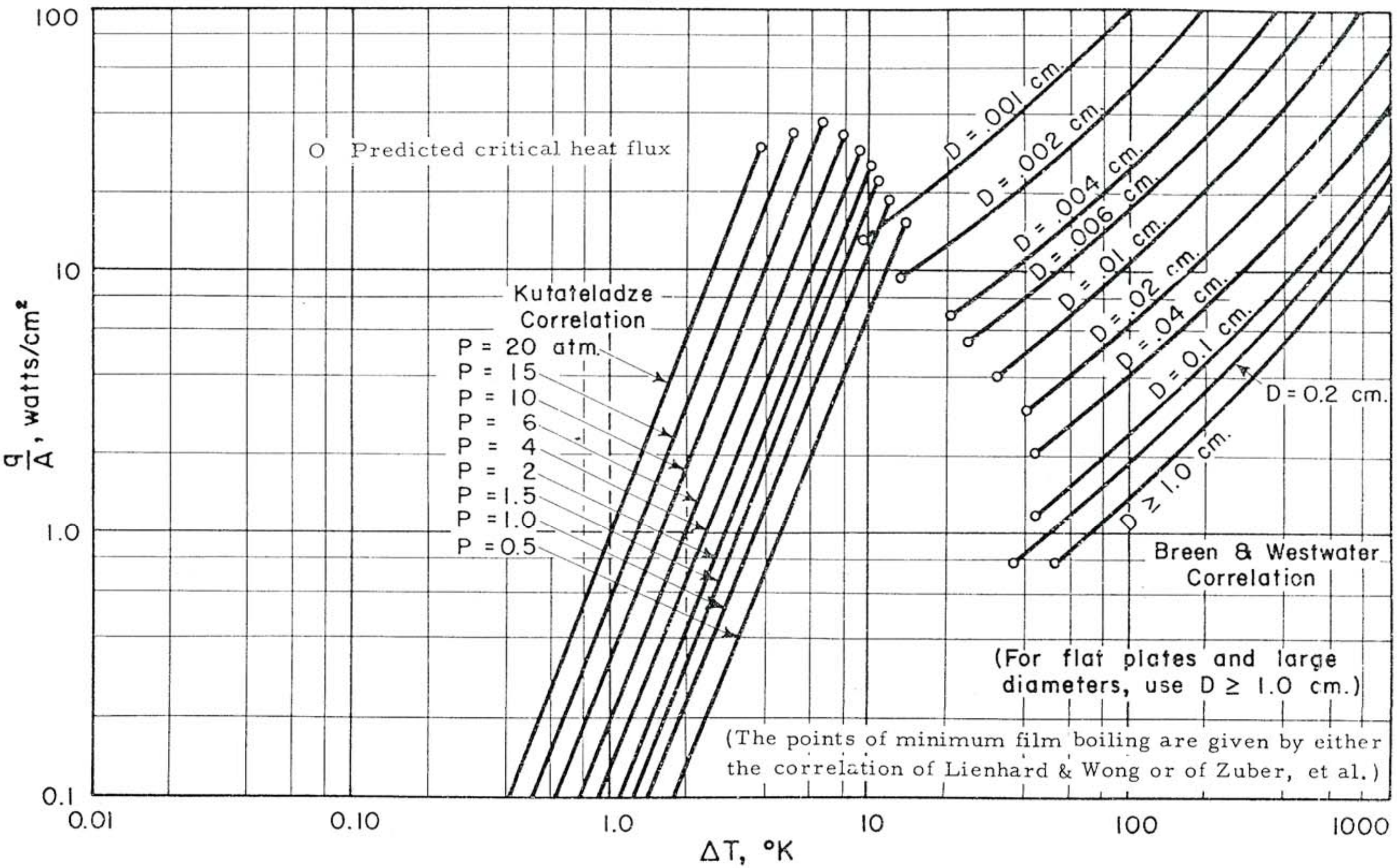
Prediction of Nucleate/Film Boiling for Helium



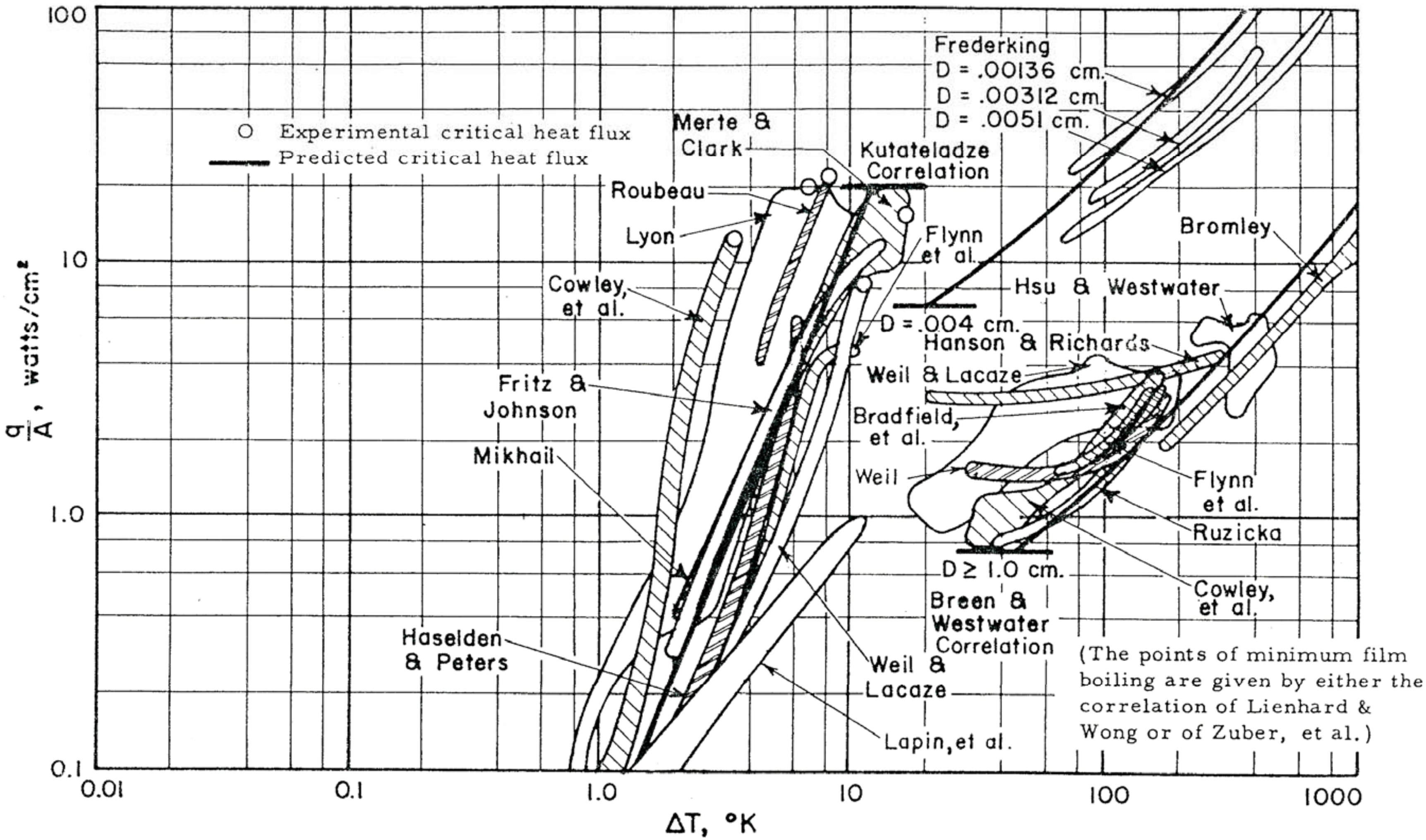
Experimental Heat Transfer (Helium)



Prediction of Nucleate/Film Boiling for Nitrogen



Experimental Heat Transfer (Nitrogen)



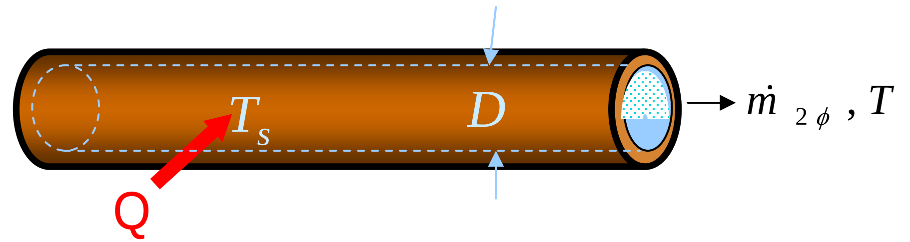
Internal two phase flow

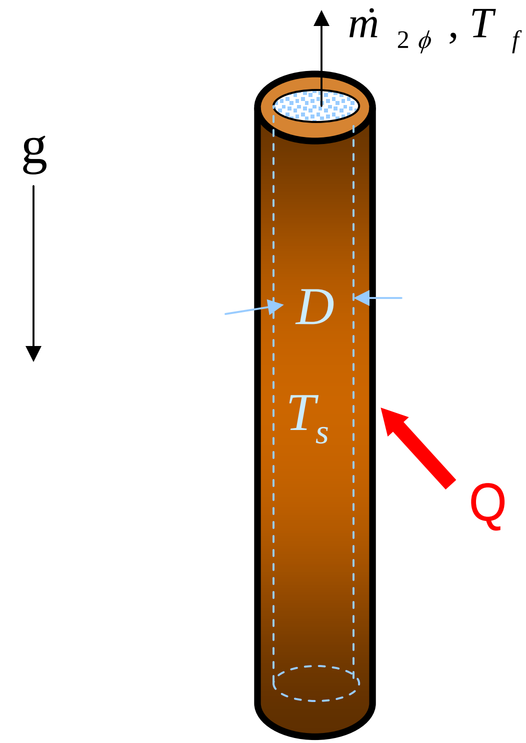
- Heat transfer depends on various factors
  - Mass flow rate
  - Orientation w/r/t gravity
  - Flow regime
  - Quality (χ)
  - Void fraction (α)
- Total heat transfer rate

$$Q_T = Q_{fc} + Q_b$$

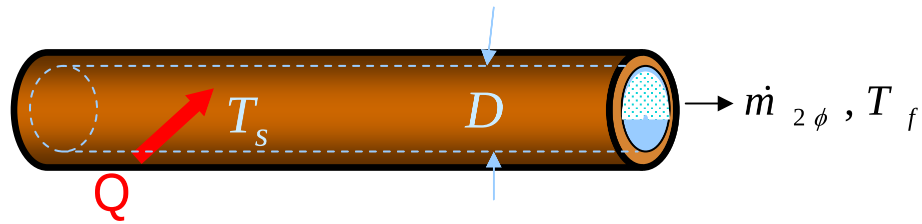
where: Q_{fc} is convective and Q_b is gravity enhanced boiling.

Depending on factors above, either contribution may dominate





Horizontal flow two phase heat transfer



- Consider the case where gravitational effects are negligible
  - Horizontal flow at moderate Re so that inertial forces dominate
- Correlation based on enhanced Nusselt number

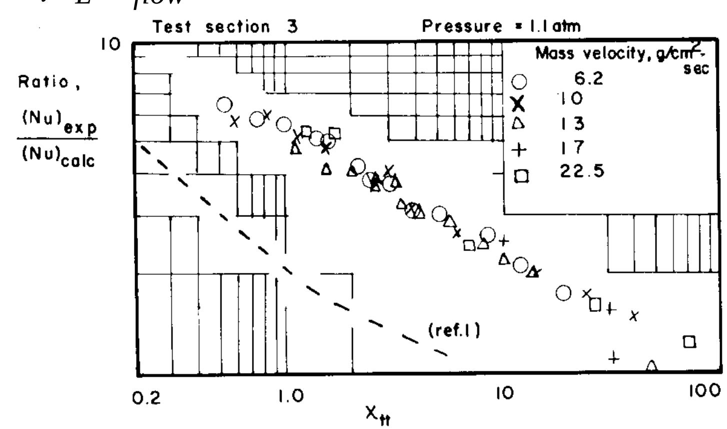
$$\frac{Nu_{2\phi}}{Nu_L} = f(\chi_{tt}) \quad \text{where} \quad \chi_{tt} = \left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\mu_L}{\mu_v}\right)^{0.1}\left(\frac{\rho_v}{\rho_L}\right)^{0.5}$$

$$\text{and} \quad Nu_L = 0.023\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}_L^{0.4} \quad ; \quad \mathrm{Re}_L = \frac{\dot{m}(1-x)}{\mu_L A_{flow}}$$

Typical correlation (de la Harpe):

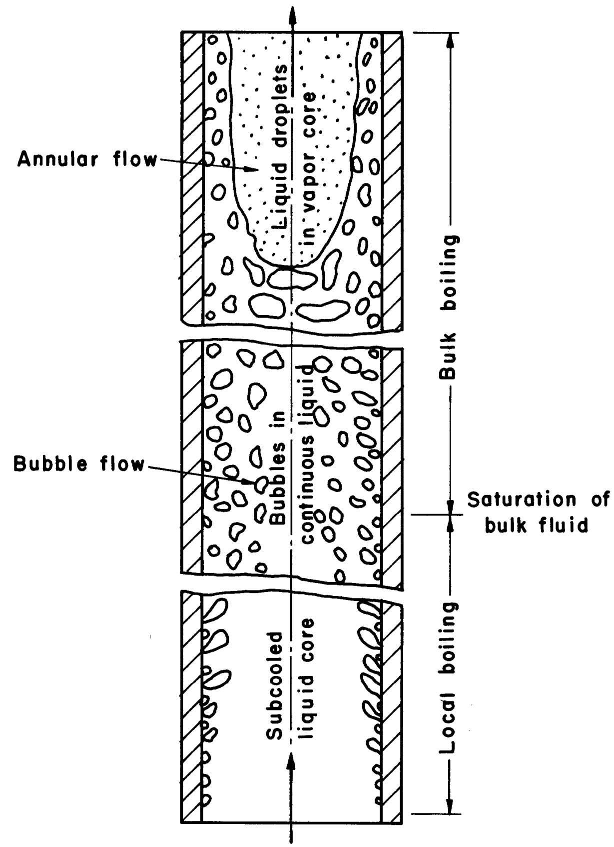
$$\frac{Nu_{2\phi}}{Nu_L} \approx A\chi_{tt}^{-m} \longrightarrow \quad 1 \text{ for } \chi_{tt} \text{ large}$$

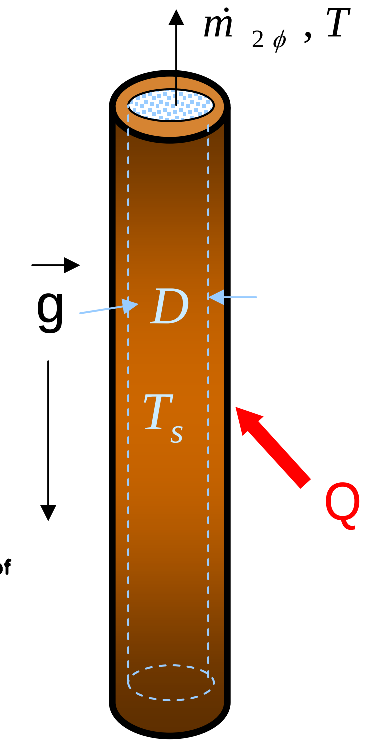
with m ~ 0.385 and A ~ 5.4



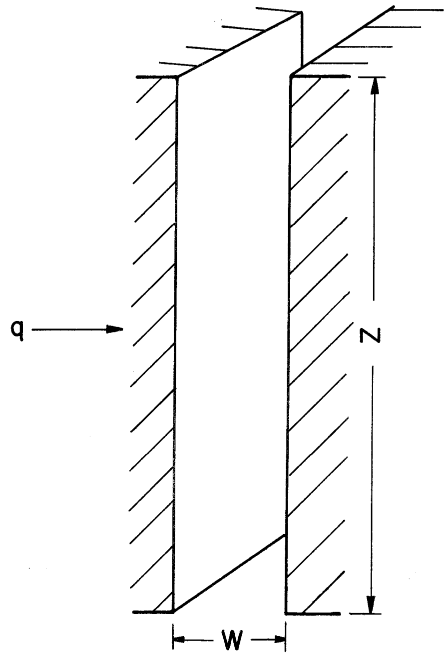
Vertical channel heat transfer

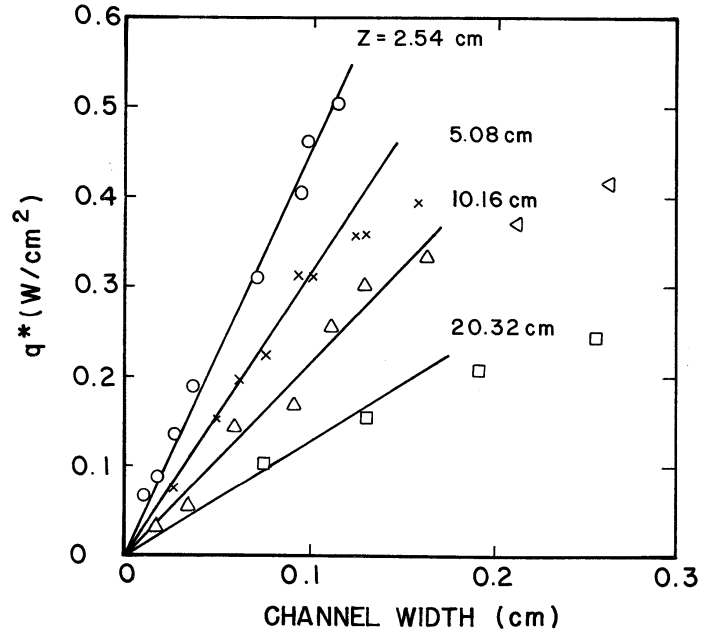
- Main difference between this problem and pool boiling is that the fluid is confined within channel
- At low mass flow rate and self driven flows (natural circulation) the heat transfer is governed by buoyancy effects
- Process is correlated against classical boiling heat transfer models
- In the limit of large D the correlation is similar to pool boiling heat transfer (vertical surface)





Vertical channel maximum heat flux





Critical flow in an evaporator

Empirical observations

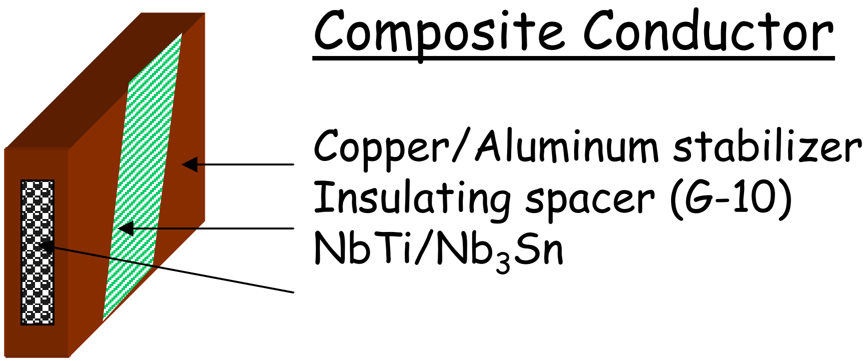
- q* ~ w for small w
- q* ~ $z^{-1/2}$ for w/z < 0.1
- q* ~ constant for w/z >> 1

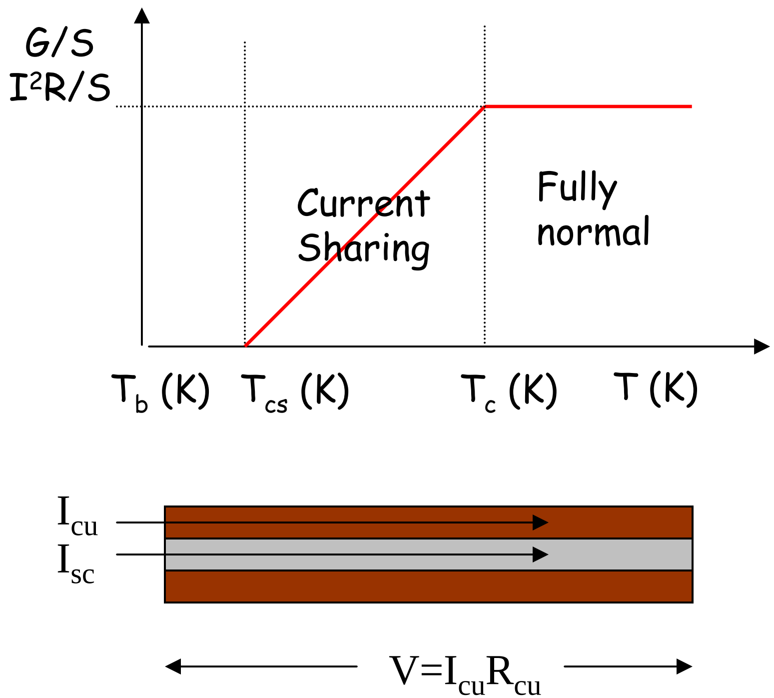
$$q^* = \frac{w}{\sqrt{z}}\frac{h_{fg}\rho_v}{2}\left[\frac{g\chi_c}{\beta-1}\left(1-\frac{\ln[1+\chi_c(\beta-1)]}{\chi_c(\beta-1)}\right)\right]^{1/2} \qquad \beta = \frac{\rho_l}{\rho_v}$$

and $\chi_c = \left.\dot{m}_v \middle/ \dot{m}\right)_{\max}$ "Critical quality" ~ 0.3 for helium

Example: Cryogenic Stability of Composite Superconductors (LTS in LHe @ 4.2 K)

- Used in large magnets where flux jumping and other small disturbances are possible and must be arrested
- General idea: in steady state ensure that cooling rate exceeds heat generation rate (Q > G)
- Achieved by manufacturing conductor with large copper (or aluminum) fraction and cooling surface
- Lower overall current density
- Potentially high AC loss (eddy currents)

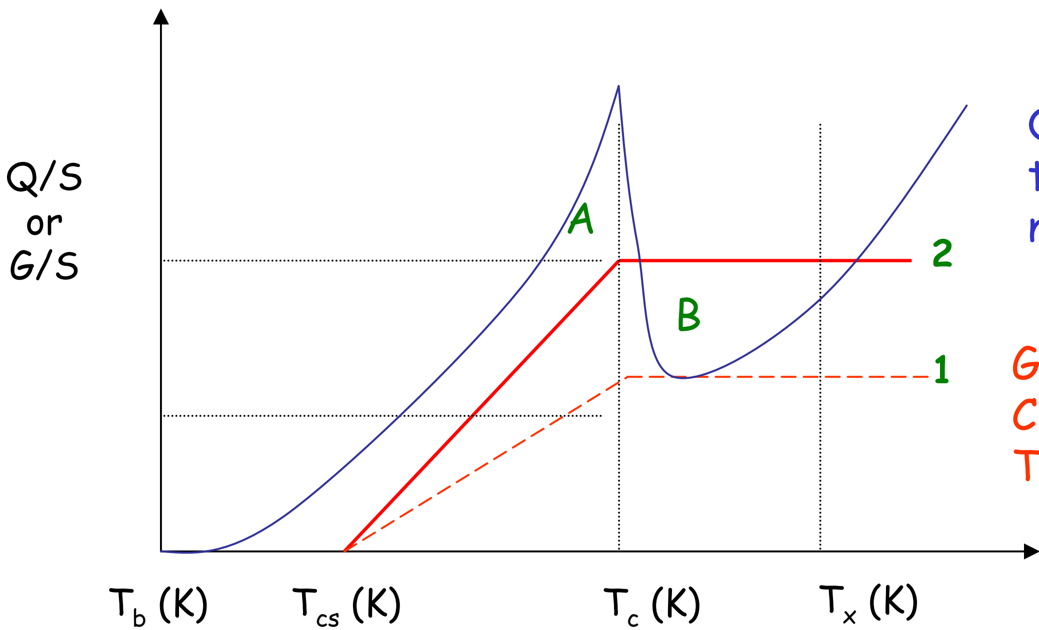




Cryogenic Stability (LHe @ 4.2 K)

Case 1: Unconditional stability, recovery to fully superconducting state occurs uniformly over length of normal zone

Case 2: Cold End recovery (Equal area criterion): Excess cooling capacity (area A) > Excess heat generation (area B)



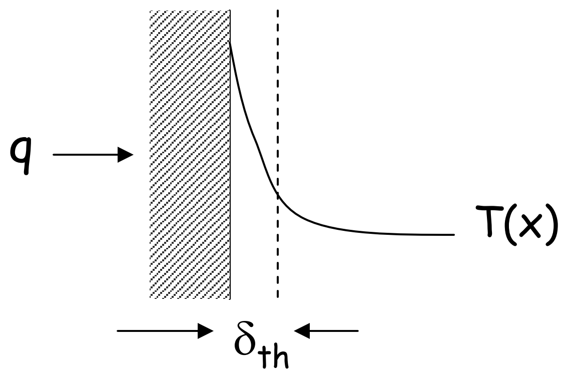
Q/S is the LHe boiling heat transfer curve for bath cooling normalized per surface area

G/S is the two part heat generation Curve for a composite superconductor

T_{cs} is the temperature at which $T_{op} = T_c$

Transient Heat Transfer

- Heat transfer processes that occur on time scale short compared to boundary layer thermal diffusion. Why is this important in cryogenics? (D_{th} (copper) ~ 10^{-4} m²/s @ 300 K; ~ 1 m²/s @ 4 K)
- Normal liquid helium has a low thermal conductivity and large heat capacity



$$D_f = \frac{k_f}{\rho C}$$ ~ 3 x 10^{-8} m²/s (LHe @ 4.2 K)

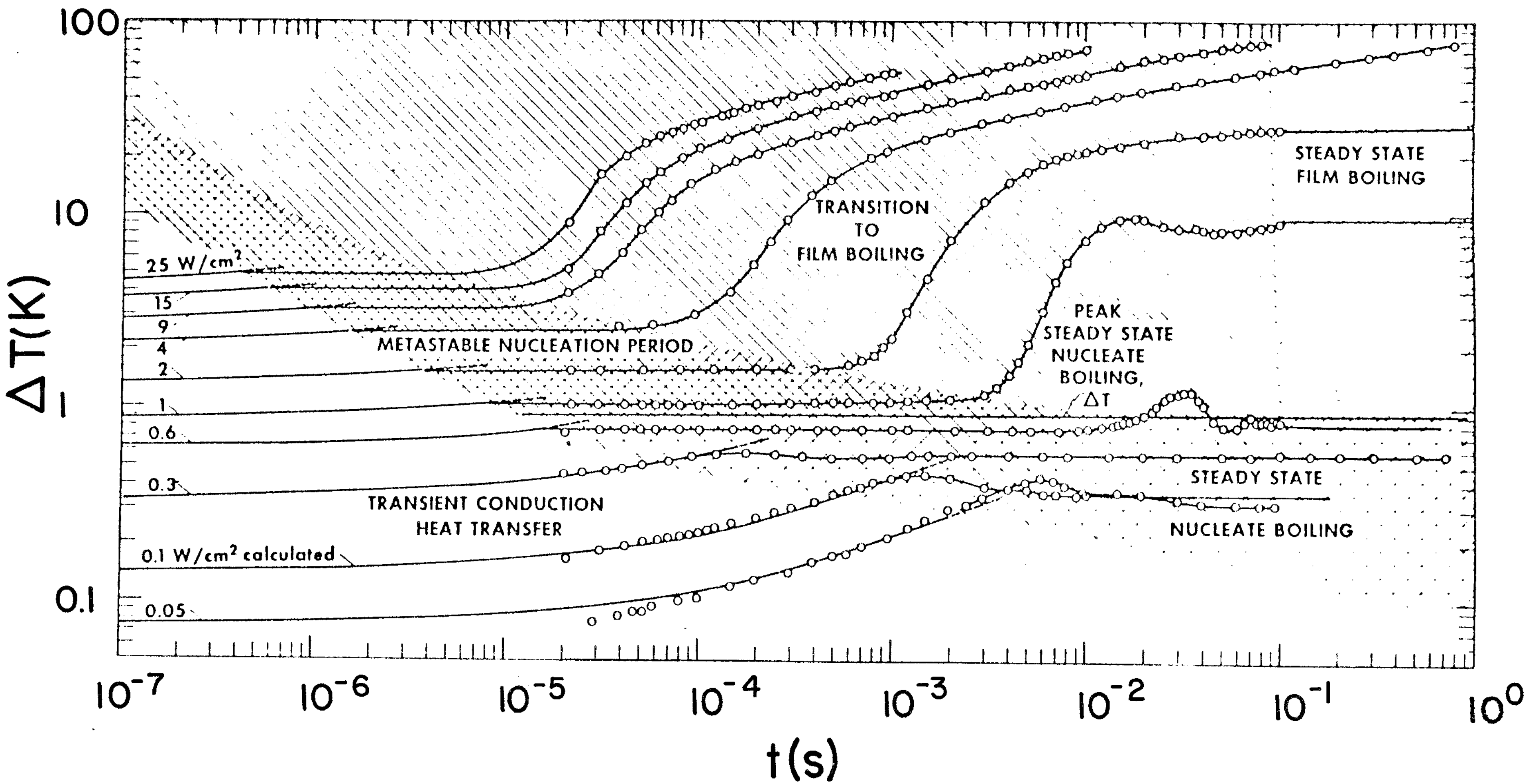
$$\delta_{th} = \frac{\pi}{2}\left(D_f t\right)^{1/2}$$ ~ 3 x 10^{-4} $t^{1/2}$ [m] for LHe @ 4.2 K
~ 1.5 $t^{1/2}$ [m] for copper @ 4.2 K

- Lumped capacitance condition: $Bi = \frac{hL}{k} << 1$ ~ 10L [m]

- Note that this subject is particularly relevant to cooling superconducting magnets, with associated transient thermal processes
- Important parameters to determine
  - $\Delta E = q\,\Delta t^*$ (critical energy)
  - T_s – surface temperature during heat transfer

Transient Heat Transfer to LHe @ 4.2 K

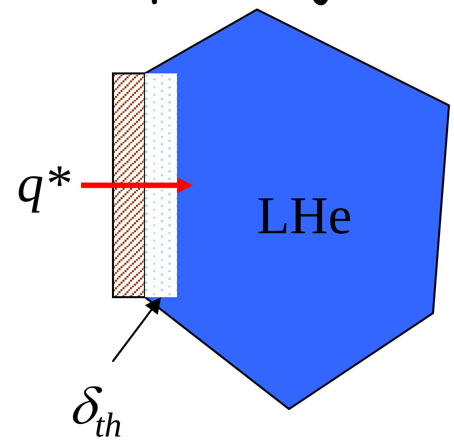
Time evolution of the temperature difference following a step heat input:
Steady state is reached after ~ 0.1 s



Critical Energy for Transition to Film Boiling

Hypothesis: The "critical energy" is determined by the amount of energy that must be applied to vaporize a layer of liquid adjacent to the heated surface.

- Energy required $$\Delta E = \rho_l h_{fg} \delta_{th}$$



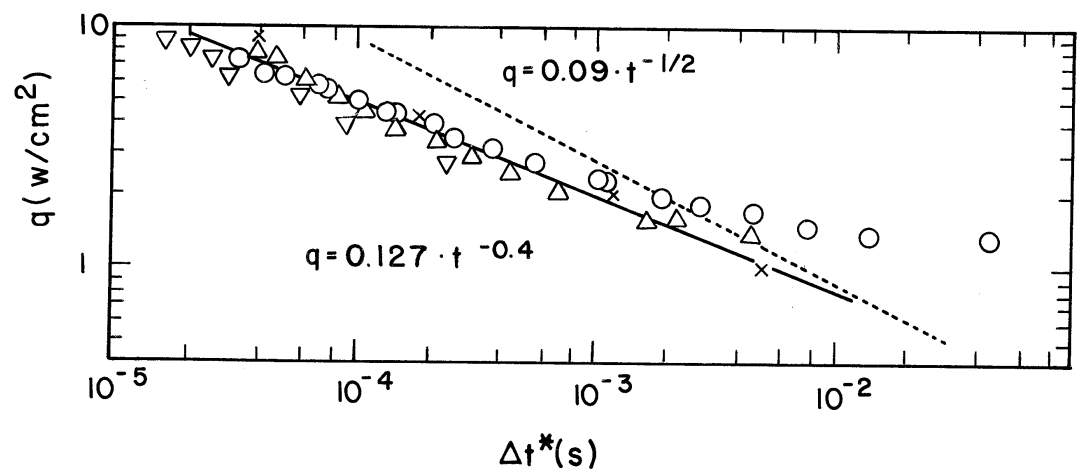
- Layer thickness determined by diffusion

$$\delta_{th} = \frac{\pi}{2}\left(D_f t\right)^{1/2}$$

- Critical flux based on heat diffusion:

$$q^* = \frac{\pi}{2}\rho_l h_{fg}\left(\frac{k_l}{\rho_l C \Delta t^*}\right)^{1/2}$$

$\sim 0.09\ \Delta t^{-1/2}$ [W/cm^2] @ 4.2 K



Surface Temperature (Transient HT)

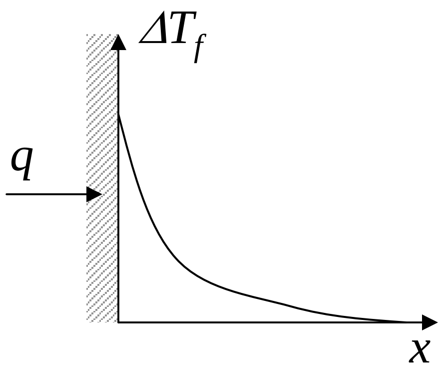
- During transient heat transfer, the surface temperature will be higher than the surrounding fluid due to two contributions:
  - Fluid layer diffusion: transient conduction in the fluid layer will result in a finite temperature difference
  - Kapitza conductance: At low temperatures, there can be a significant temperature difference, ΔT_k, due to thermal impedance mismatch (more on this subject later). This process is dominant at very low temperatures, but is small above ~ 4 K, so is normally only important for helium systems.
- Fluid layer diffusion equation:

$$\frac{\partial^2 \Delta T_f}{\partial x^2} = \frac{1}{D_f} \frac{\partial \Delta T_f}{\partial t}; D_f = \frac{k}{\rho C_p}$$

- Boundary conditions:
  - $\Delta T_f (x,0) = 0$; initial condition
  - ΔT_f (infinity, t) = 0; isothermal bath
  - $q = -k_f \, d\Delta T_f/dt)_{x=0}$; heat flux condition
  - Solid is isothermal (Bi = hL/k << 1)

Solution is a standard second order differential equation with two spatial and one time boundary condition.

Transient diffusion solution



- Integrating the diffusion equation:

$$\Delta T_f = \frac{q}{k_f}\left[2\left(\frac{D_f t}{\pi}\right)^{1/2}\exp\left(-\frac{x^2}{4D_f t}\right) - x\,erfc\left(\frac{x}{2(D_f t)^{1/2}}\right)\right]$$

- Evaluating at x = 0 (surface of heater)

$$\Delta T_f(x=0) = \frac{2q}{\sqrt{\pi}}\left(\frac{t}{\rho C_p k_f}\right)^{1/2}$$

- The transient heat transfer coefficient can then be defined

$$h = \frac{q}{\Delta T_f} = \frac{\sqrt{\pi}}{2}\left(\frac{\rho C_p k_f}{t}\right)^{1/2}$$

$$h \approx \frac{0.1}{\sqrt{t}}$$; kW/m²K for helium near 4 K

At t = 10 μs; h ~ 30 kW/m² K and for q = 10 kW/m²; ΔT_f ~ 0.3 K
Note: this value of h >> h_{nb} (nucleate boiling HT coefficient)

Summary Cryogenic Heat Transfer

- Single phase heat transfer correlations for classical fluids are generally suitable for cryogenic fluids
  - Free convection
  - Forced convection
- Two phase heat transfer also based on classical correlations
  - Nucleate boiling
  - Peak heat flux
  - Film boiling
- Transient heat transfer is governed by diffusive process for ΔT and onset of film boiling