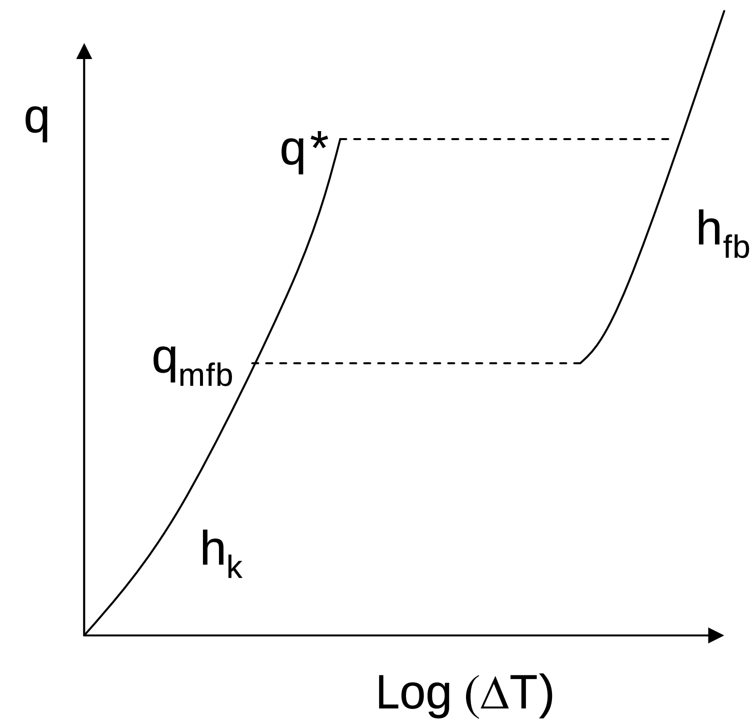
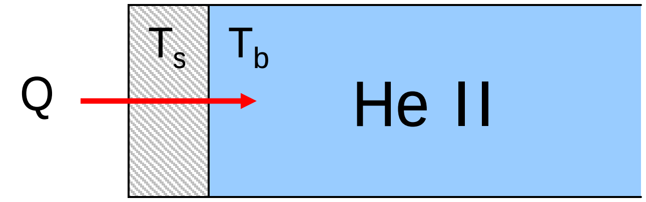
3.3 He II Heat and Mass Transfer

- Heat transfer characteristics of He II are sufficiently unique that conventional heat transfer (convection, conduction) do not apply.
- Important questions
  - What is the limit to heat transfer, q^*, critical energy, ΔE?
  - What is the associated thermal gradient in the He II?
  - How is the surface temperature determined?
- Understanding and modeling must be based on transport properties of He II
- Examples to be considered
  - Thermal stability of He II cooled magnet
  - Design of a He II bath heat exchanger

Surface heat transfer – general form

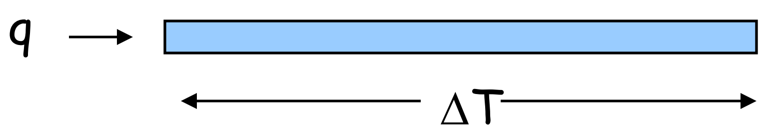
- Surface heat transfer characteristics look similar to ordinary fluids
- Physical interpretation different
  - h_k is the Kapitza conductance regime (non-boiling)
  - q^* is the peak heat flux
  - q_{mfb} minimum film boiling heat flux
  - h_{fb} film boiling heat transfer coefficient
- All above processes are similar to boiling heat transfer in normal liquids, but the physical interpretation is different

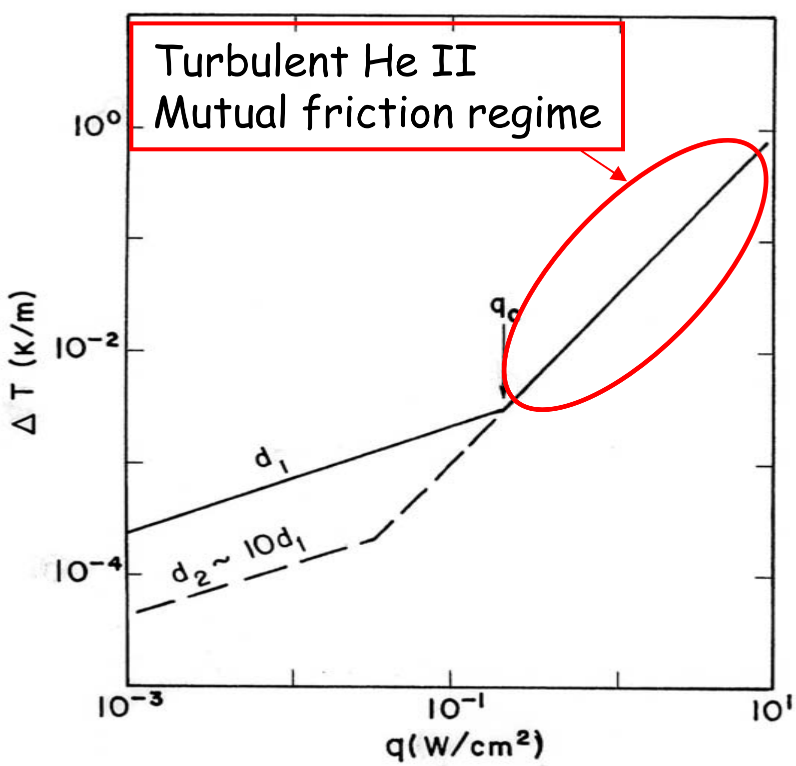




Heat Conductivity of He II

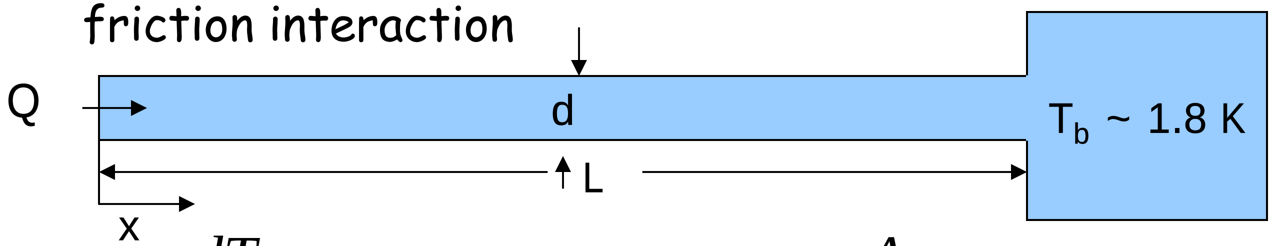
- Anomalous heat transport
  - Effective heat conductivity comparable to that of high purity metals
  - Low flux regime dT/dx ~ q
  - High flux regime dT/dx ~ q^3
  - Transition between two regimes depends on the diameter of channel
- Heat transport in He II can be understood in terms of the motion of two interpenetrating fluids. This "Two Fluid" model effectively describes the transport properties
- High heat flux regime of greatest technical interest





Peak Heat Flux in He II Channel

- Thermal gradient in He II channel allows T(x = 0) to increase above T_b
  - For practical channel dimensions, gradient is controlled by mutual friction interaction



$$\frac{dT}{dx} = -f(T)q^3 \qquad f(T) = \frac{A\rho_n}{\rho_s^3 s^4 T^3}$$

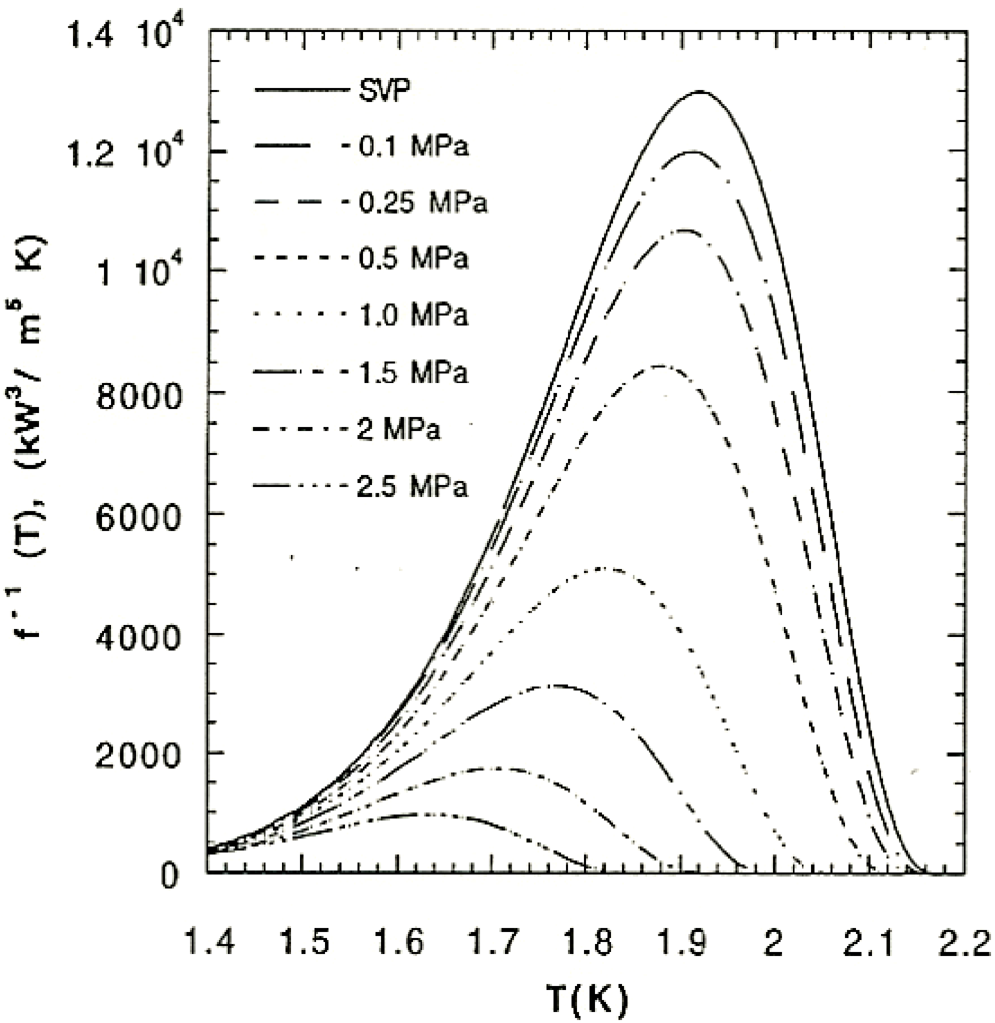
Thermal resistivity function

$$q = -\left[f^{-1}(T)\frac{dT}{dx} \right]^{1/3}$$

Non-linear conduction equation

- Heat flux is limited by maximum allowable temperature at x = 0 (usually T_λ)
  - Steady state gradient
  - Thermal diffusion

He II Heat Conductivity Function, $f^{1}(T,p)$



- Correlation based on He II turbulent flow

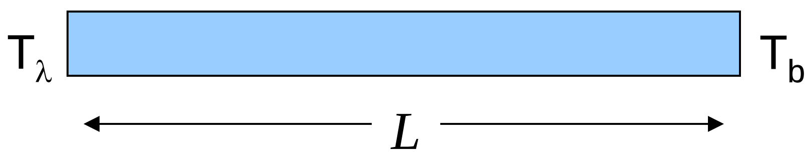
$$f^{-1}(T) \cong \frac{\rho^2 s_\lambda^4 T_\lambda^3}{A_\lambda}\left[t^{5.6}\left(1-t^{5.6}\right)\right]^3; t = T/T_\lambda(p)$$

Where A_λ ~ 145 cm s/g

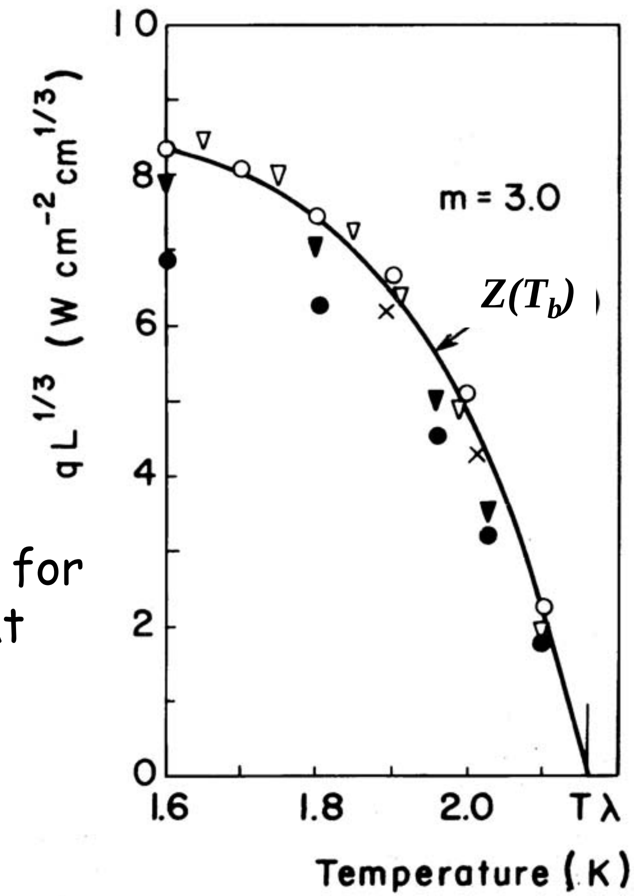
- Above correlation is good to about 10%
- Results indicate that the peak heat flux, Q*, should decrease with increasing p and T.
- Improved correlation by Sato (2003)

Maximum Steady State Heat Flux (q*)

Integrating the heat transport equation over the length of the channel



$$q^* = \left[\frac{1}{L} \int_{T_c}^{T_\lambda} \frac{dT}{f(T)} \right]^{1/3} \equiv \frac{Z(T_b)}{L^{1/3}}$$

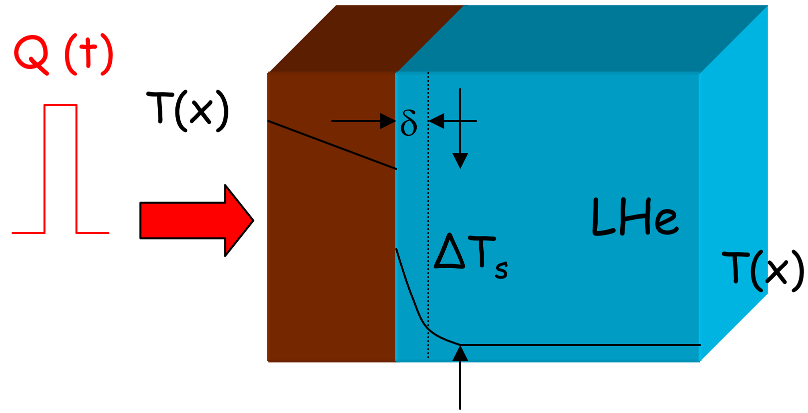


This expression is used to compute the maximum q for a channel heated at one end and containing He II at moderate pressure

Example: L = 10 cm at T_b = 1.8 K → q* = 5.9 W/cm^2

Transient Heat Transfer in He II

- Heat pulse diffuses through conductor and is transferred to He II by conduction
- δ = thermal diffusion length into LHe ≈ cm rather than μm as in He I
- Surface temperature difference $\Delta T_s = \Delta T_K + \Delta T_{He\ II}$ where $\Delta T_K = Q/h_K S$ (Kapitza Conductance)
- Take-off power is equivalent to energy to raise local He II temperature to T_λ = 2.2 K



Kapitza Conductance

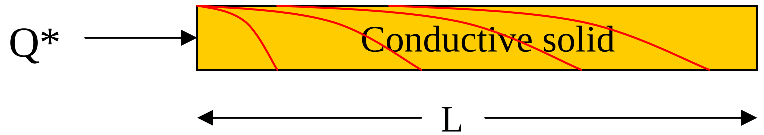
Def: Interfacial temperature difference due to thermal mismatch between two media.

In He II heat transfer, $h_K \approx 5\ \mathrm{kW/m^2K}$

$$Q \cdot \Delta t = \int_{T_b}^{T_\lambda} \rho C \frac{dT_{HeII}}{dx} dx$$

Note: this problem has significant implications to the thermal stability of He II cooled superconducting magnets.

Thermal diffusion - solids



- Heat transport is described by the diffusion equation

$$\frac{\partial T}{\partial t} = D_{th} \frac{\partial^2 T}{\partial x^2} \quad \text{where} \quad D_{th} = \frac{k}{\rho C}; \left[\frac{m^2}{s}\right]$$

- Characteristic time for diffusion over length L:

$$\tau_D \sim \frac{L^2}{D_{th}}$$ Corresponds to a Fourier number = 1 $\left(Fo \equiv \frac{D_{th} t}{L^2}\right)$

- Metals at low temperature
  - Copper: $D_{th} \sim 1\ m^2/s$; for L = 1 m then $\tau_D \sim 1$ s
  - Stainless steel: $D_{th} \sim 3 \times 10^{-3}\ m^2/s$; for L = 1 m then $\tau_D \sim 3000$ s

Thermal "diffusion" - He II

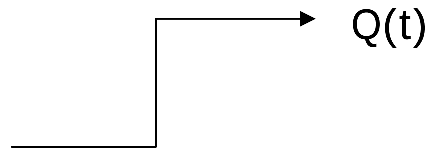
$$\frac{dT}{dx} = f(T)q^3 \Rightarrow k_{eff} = \frac{1}{f(T)q^2}$$

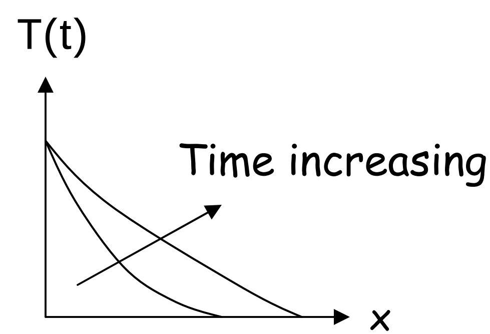
- The effective thermal conductivity of He II is heat flux dependent
  - At T = 1.8 K, $f^{-1}(T)$ ~ 10000 kW^3/m^5 K
  - For heat flux q = 10 kW/m^2, k_{eff} ~ 10^5 W/m K
  - For heat flux q = 100 kW/m^2, k_{eff} ~ 1000 W/m K ~ k_{cu} @ 2 K
- Volume heat capacity of He II is much larger than that of metals at low temperature ($\rho C_{He\ II}$ ~ 1000 kJ/m^{3}K; ρC_{cu} ~ 0.2 kJ/m^{3}K)
- Effective thermal diffusivity for He II
  - $D_{eff} = k_{eff}/\rho C$ ~ 0.1 m^2/s @ 10 kW/m^2
  - Characteristic diffusion time (L = 1 m): $\tau_D \sim L^2/D_{eff}$ ~ 10 s
- **Significance**: thermal diffusion is an important heat transport mechanism in He II (contrary to normal liquids except at very short times)

He II thermal diffusion equation

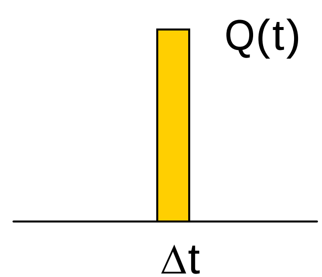
$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(f^{-1} \frac{\partial T}{\partial x} \right)^{\frac{1}{3}} \qquad \text{where} \qquad \tau_d \approx \rho C f^{\frac{1}{3}} L^{\frac{4}{3}} \Delta T^{\frac{2}{3}}$$

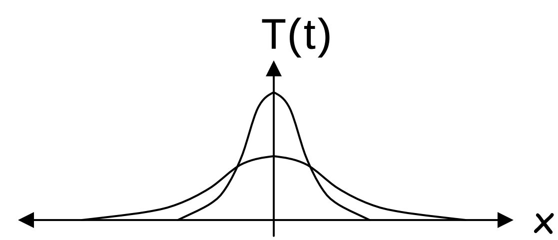
- Non-linear partial differential equation
- Methods of solution
  - Approximate methods (similarity solution)
  - Numerical methods
- Problems of interest
  - Step function heat flux



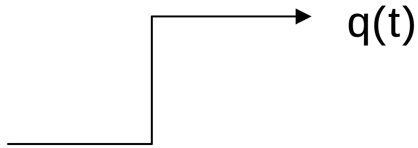


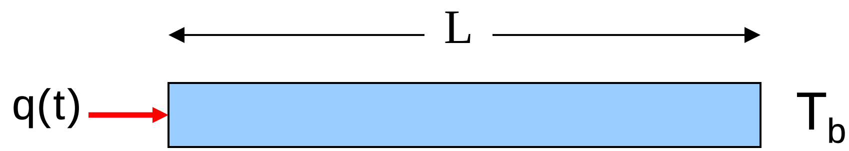
  - Heat pulse





Step Function Heat Flux (Clamped flux)





- Non-linear diffusion equation that can be solved by numerical methods of approximate methods (constant properties)

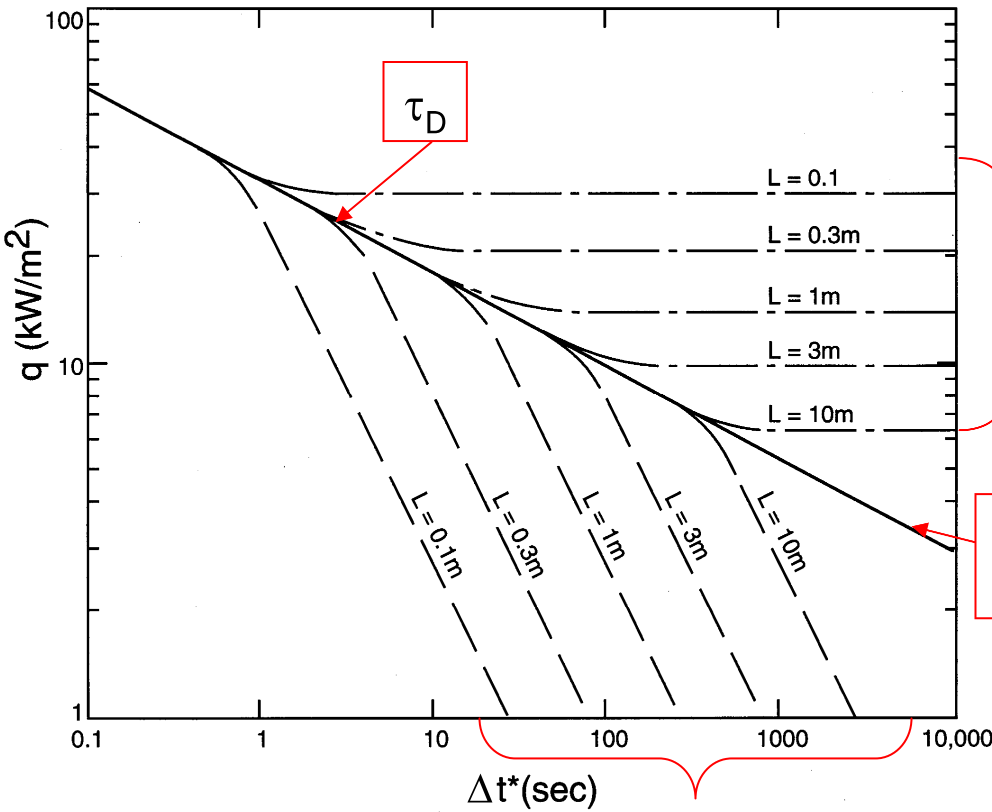
$$\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial x}\left[\frac{\partial \theta}{\partial x}\right]^{1/3}$$

where $\theta \equiv \frac{T - T_b}{T_\lambda - T_b}$ and $\tau \equiv \frac{t}{f^{1/3}\rho C\left(T_\lambda - T_b\right)^{2/3}}$

- Boundary conditions:
  - Heat transfer b.c. at x = 0: $\left.\frac{\partial \theta}{\partial x}\right|_{x=0} = -\frac{q^3 f}{T_\lambda - T_b} \quad \forall t > 0$
  - T = T_b or Θ = 0 @ x = infinity (approximate solution for x << L)
  - For finite length, L, numerical solution is required. Solution should asymptotically approach steady state profile

Boundary conditions @ x = L

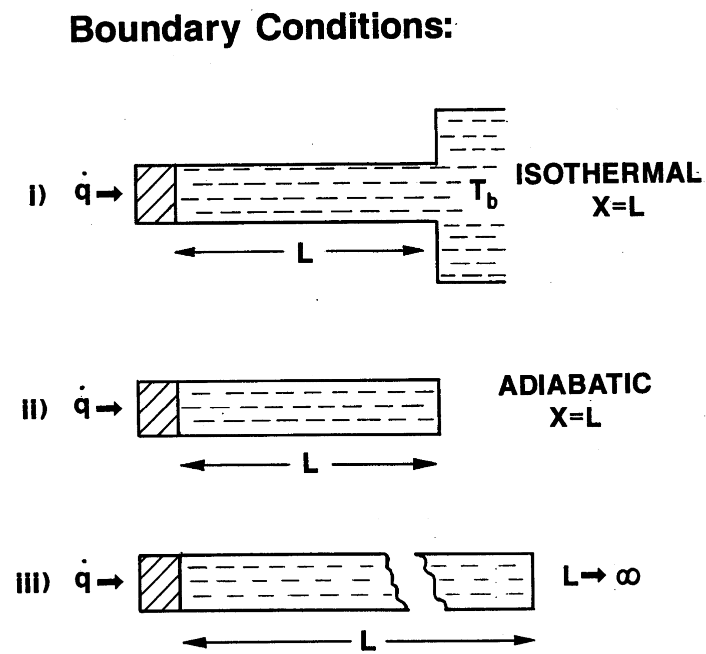
Time to onset of film boiling, Δt^* depends on boundary condition at x = L.



Isothermal B.C. Diffusion

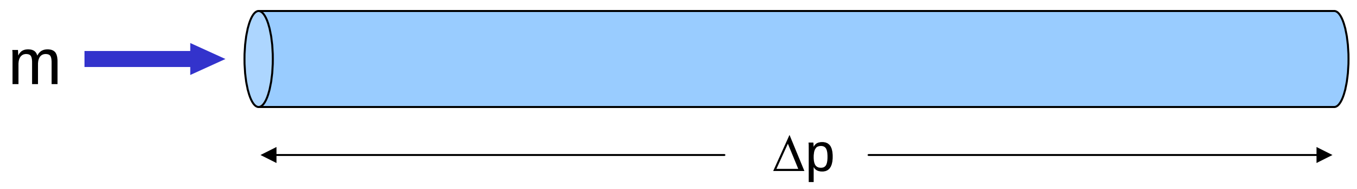
Infinite channel

Adiabatic B.C. Diffusion



Diffusion time (τ_D) is indicated by point where boundary condition at x = L determines Δt^*

Forced flow He II pressure drop



- Recall for ideal superflow the (laminar) pressure drop is associated with the normal fluid viscous drag (fountain effect)
- In turbulent flow (Re_D > 1200) He II behaves more as a classical fluid

$$\mathrm{Re}_D = \frac{\rho v D}{\mu_n}$$

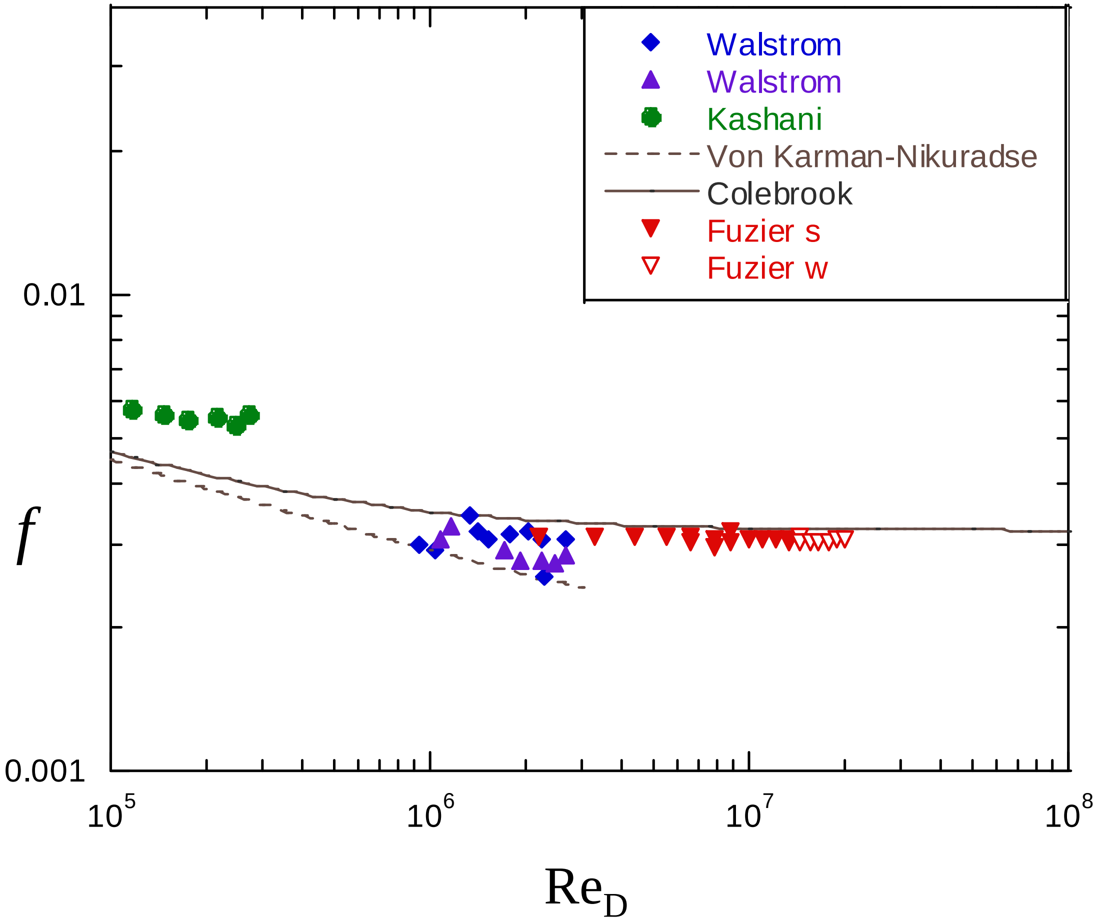
Since μ_n is small, turbulent flow is common in helium

- Associated pressure drop is given by the classical expression

$$\frac{dp}{dx} = -4\frac{f_d}{d}\left(\frac{1}{2}\rho v^2\right)$$

where the friction factor, $f_d(\mathrm{Re}_d)$

Friction factor for He II



Blausius correlation

$$f = \frac{0.079}{Re_D^{1/4}}$$

Von Karman-Nikuradse

$$\frac{1}{\sqrt{f}} = 1.737 * \ln\left(Re * \sqrt{f}\right) - 0.396$$

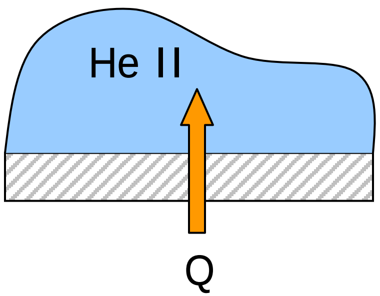
Colebrook for ε = 1.4 x 10^{-4}

$$\frac{1}{\sqrt{f}} = -4\log\left(\frac{\varepsilon}{3.7D} + \frac{1.25}{Re\sqrt{f}}\right)$$

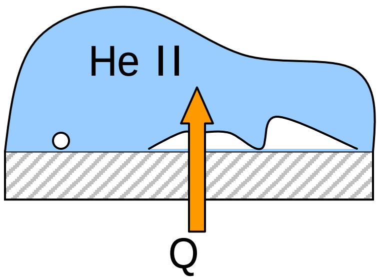
Surface Heat Transfer (He II)

There are three regimes of heat transfer that can occur at a heated surface in He II

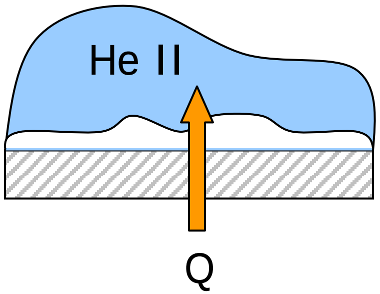
1. Kapitza Conductance (non-boiling)
   - Temperature difference occurs at surface, $\Delta T_s \sim 1$ K
   - Due to a surface thermal impedance



2. Transition to film boiling (unstable)
   - Exchange between boiling and non-boiling condition



3. Film boiling
   - Vapor layer covering surface
   - ΔT can be large ~ 10 to 100 K

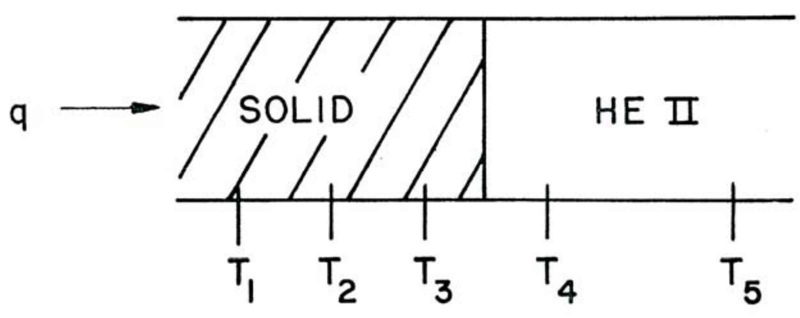


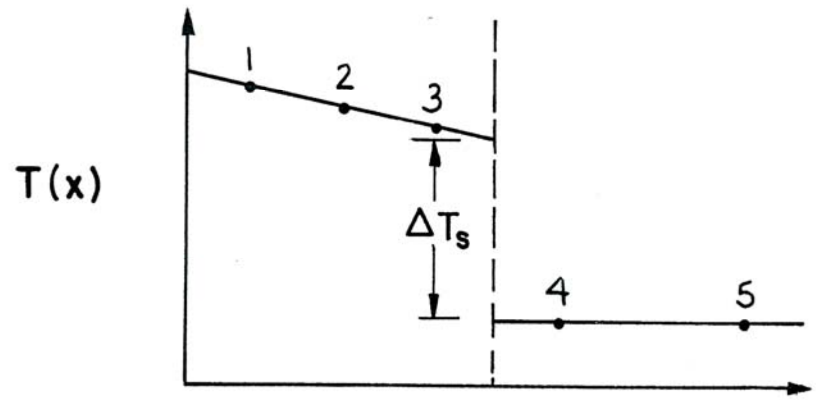
Kapitza Thermal Boundary Conductance

- **discovered by Kapitza (1941) while studying heat flow around a heated Cu block in He II**

- **general term associated with thermal resistance at low temperatures**

How measured: $h_k \equiv \lim_{\Delta T \to 0} \frac{q}{\Delta T_s}$



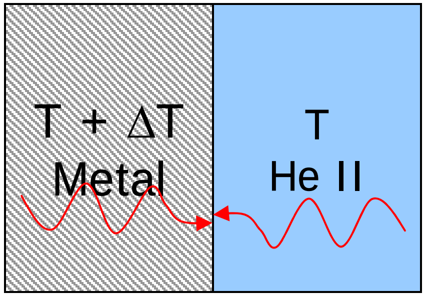


Practical Significance of Kapitza Conductance

- Kapitza conductance causes largest ΔT in a non-boiling heat transfer process in He II
  - $h_k \sim 10$ kW/m^2 K ⟶ $\Delta T_s(q = 10$ kW/m^2 K$) \sim 1$ K
  - $dT/dx_{He\ II} \sim 100$ mK/m $\Delta T_{He\ II} \sim (T_\lambda - T_b) \sim 400$ mK over 4 m
- Kapitza conductance is important in the design of numerous technical devices
  - He II heat exchangers
  - Composite superconductors stability
  - Low temperature refrigerators and instrumentation

Theory of Kapitza Conductance

- Phonon Radiation Limit
  - Heat exchange occurs by phonons (quantized lattice vibrations) impinging on the interface
  - Analogue to radiation heat transfer



$$q = \sigma\left(T + \Delta T\right)^4 - \sigma T^4$$

where
$$\sigma = \frac{\pi^4}{10\hbar}\left(\frac{k_B}{\Theta_D}\right)^2\left(\frac{3N}{4\pi V}\right)^{2/3}$$

- Expand

$$q = \underbrace{4\sigma T^3}_{h_k}\Delta T\left[1 + \frac{3}{2}\frac{\Delta T}{T} + \left(\frac{\Delta T}{T}\right)^2 + \frac{1}{4}\left(\frac{\Delta T}{T}\right)^3\right]$$

- Note: $h_k \sim T^3/\Theta_D^{-2}$

Comparison of Highest Experimental Values for the Kapitza Conductance with the Phonon Radiation Limit[a]

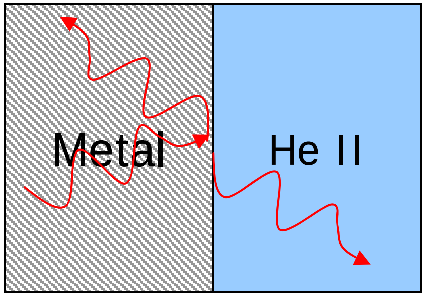
Solid	Θ_D (K)	h_K^P(1.9 K) (kW/m²·K)	h_K(1.9 K) (kW/m²·K)
Hg	72	440	30
Pb	100	190	32
In	111	171	11
Au	162	155	8.8
Ag	226	55	6
Sn	195	54	12.5
Cu	343	30	7.5
Ni	440	19	4.0
W	405	18	2.5
KCl	230	22	6.9
SiO_2 (quartz)	290	19	5.7
Si	636	6.4	4.2
LiF	750	5.1	4.5
Al_2O_3	1000	1.5	1.6

[a] Compiled by Snyder.[31]

Theory of Kapitza Conductance (cont.)

- Phonon radiation limit is an upper limit because it does not take into consideration boundary reflections due to dissimilarity between solid & He II
- Acoustic mismatch theory
  - Based on impedance mismatch between dissimilar materials
  - Similar to optical transmission between media with different refractive indices
- Similar expression to Phonon Radiation Limit

$$q = \sigma\left(T + \Delta T\right)^4 - \sigma T^4$$



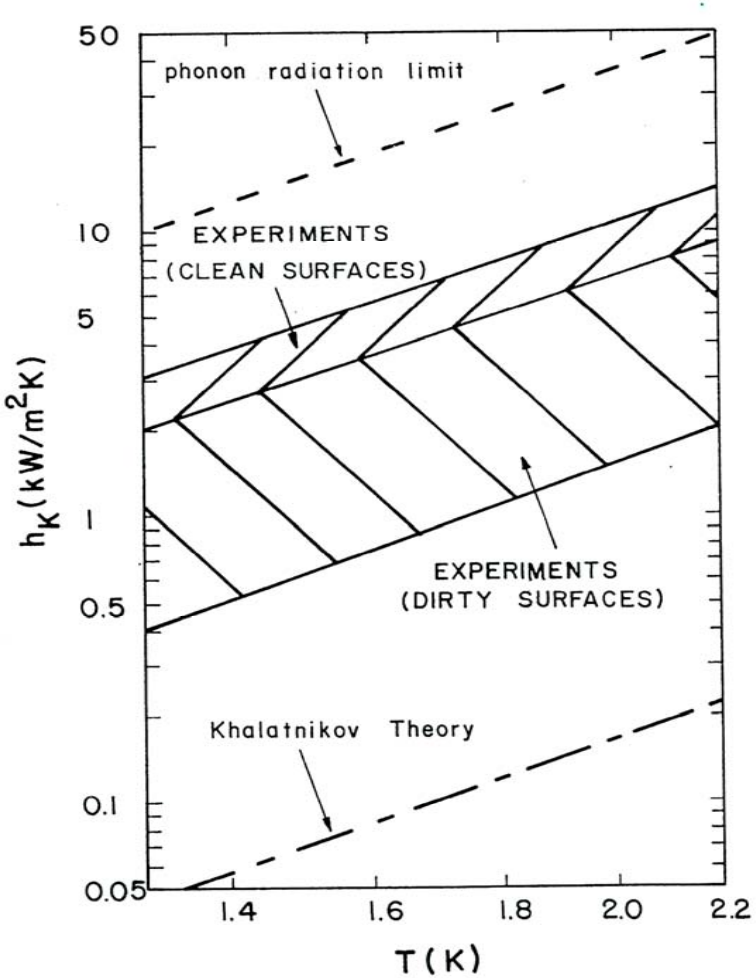
Where in this case,

$$\sigma = \frac{4\pi^5 k_B^4 \rho_L c_L}{15\hbar^3 \rho_s c_s^3} \approx \Theta_D^{-3} \quad \text{and} \quad h_k \sim T^3 / \theta_D^3$$

- This is a lower limit to heat transfer

Experimental values for Kapitza Conductance

Example Copper:



i) Phonon radiation limit

$h_K^{ph} = 4.4\ T^3\ \mathrm{kW/m^2K}$

ii) Acoustic mismatch theory

$h_K^A = 0.021\ T^3\ \mathrm{kW/m^2K}$

iii) Experimental results

a) clean surfaces

$h \sim 0.9\ T^3\ \mathrm{kW/m^2K}$

b) dirty surfaces

$h \sim 0.4\ T^3\ \mathrm{kW/m^2K}$

Large variations $\Rightarrow$

Kapitza conductance is an empirical quality.

Kapitza Conductance for ΔT/T ~ 1

- Expand from theory

$$q = \sigma(T + \Delta T)^4 - \sigma T^4$$

$$q = 4\sigma T^3 \Delta T\left[1 + \frac{3}{2}\frac{\Delta T}{T} + \left(\frac{\Delta T}{T}\right)^2 + \frac{1}{4}\left(\frac{\Delta T}{T}\right)^3\right] \approx h_k \Delta T$$

- Alternate correlation

$$q = a\left(T_s^n - T_b^n\right) \quad \text{for finite } \Delta T$$

where a and n are empirically determined

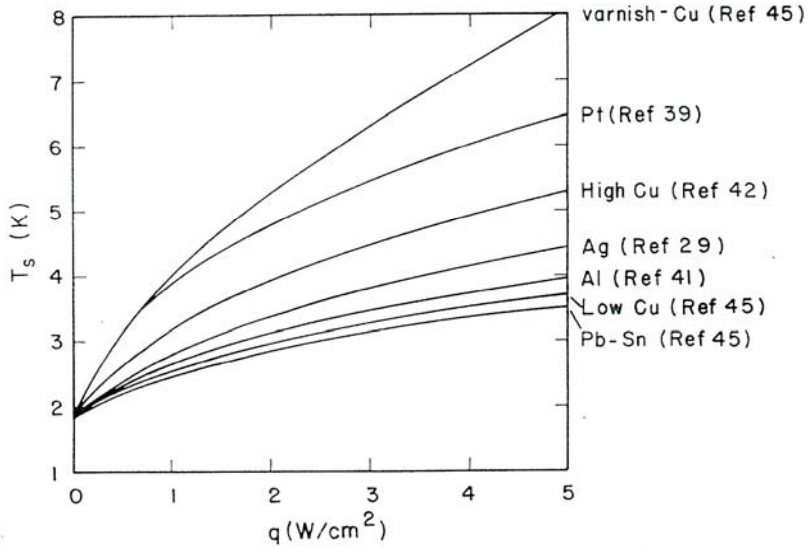
T_s: surface temperature = $T_b + \Delta T$

T_b: fluid temperature near surface

Large Heat Flux Kapitza Conductance

Empirical correlation

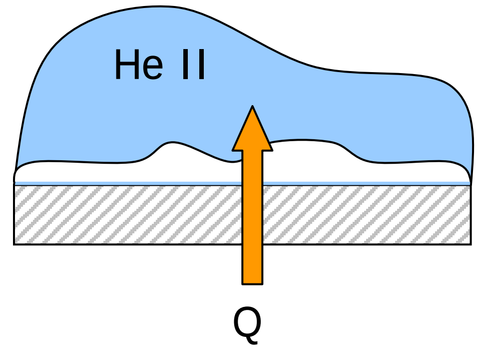
$$q = a\left(T_s^n - T_b^n\right)$$



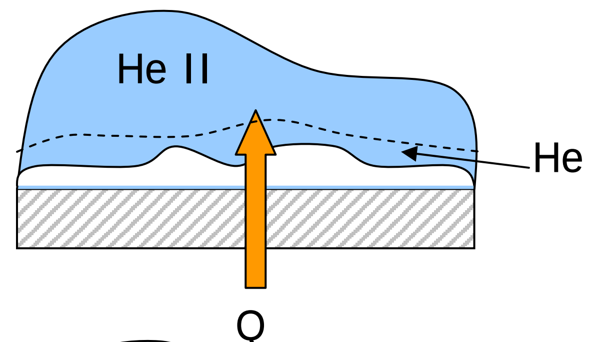
Metal	Surface condition	T_s at 1 W/cm²	a (W/cm² · K)	n
Cu	As received	3.1	0.0486	2.8
	Brushed and baked	2.85	to	
	Annealed	2.95	0.02	3.8
	Polished	2.67	0.0455	3.45
	Oxidized in air for 1 month	2.68	0.046	3.46
	oxidized in air at 200° C for 40 min	2.46	0.052	3.7
	50–50 PbSn solder coated	2.43	0.076	3.4
	Varnish coated	4.0	0.0735	2.05
Pt	Machined	3.9	0.019	3.0
Ag	Polished	2.8	0.06	3.0
Al	Polished	2.66	0.049	3.4

Film Boiling Heat Transfer Modes

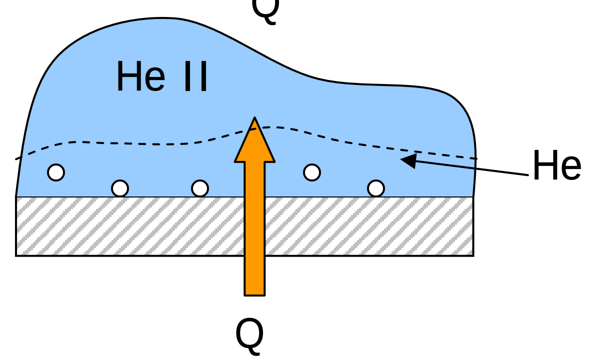
1. Near saturation ($p < p_\lambda$)
   Low density vapor blankets surface significantly reducing heat transfer



2. Pressurized to $p > p_\lambda$
   Triple phase phenomena (He II, He I, vapor)



3. Near T_λ permits nucleate boiling in He I phase w/o exceeding q^*



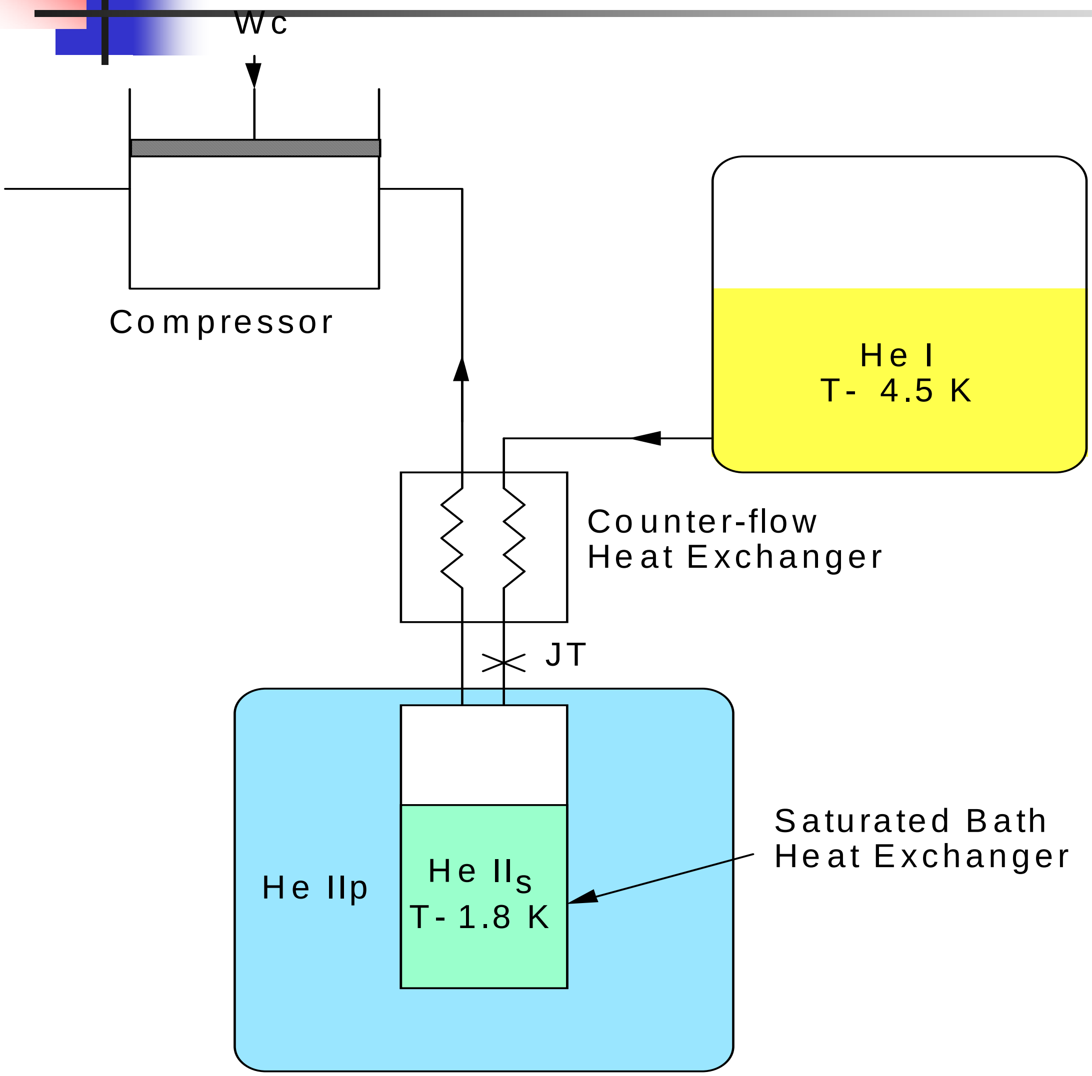
Film Boiling Heat Transfer (He II)

Sample	T_b (K)	T_s (K)	Δp (kPa)[a]	h(kW/m$^2\cdot$K)
Wire ($d = 76\ \mu$m)	1.8	150	0.42	1.1
Wire ($d = 25\ \mu$m)	1.8 K	150	0.56	2.2
Flat rectangular plate	1.8	75	0.14	0.22
(39 mm × 11 mm)	1.8	75	0.28	0.3
	1.8	75	0.84	0.55
Flat surface ($d = 13.7$ mm)	2.01	40	0.13	0.69
	2.01	25	0.237	0.98
Horizontal cylinder	1.88	40	0.10	0.2
($d = 14.6$ mm)	2.14	40	0.10	0.2
Wire ($d = 200\ \mu$m)	2.05	150	0.14	0.66
Cylinder ($d = 1.45$ mm)	1.78	80	0.06	0.22

[a] 1 kPa = 7.5 torr = 70.3 cm He.

- **Typical value h_{fb} ~ 0.5 kw/m^{2}K**
- **h_{fb} (flat plates) < h_{fb} (wires)**
- **h_{fb} increases with pressure**

Subcooled He II Refrigerator



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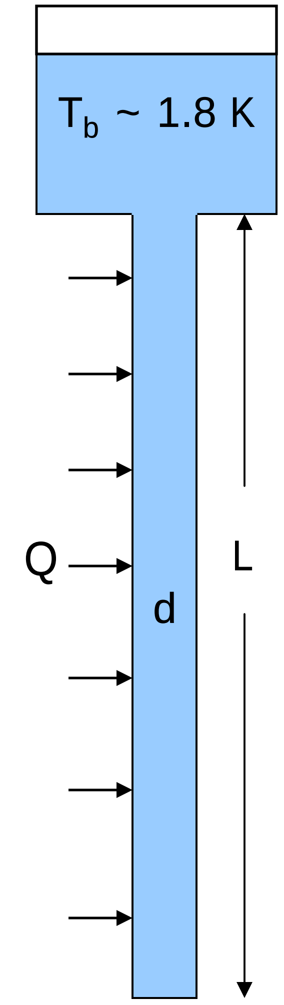
Application: He II Heat Exchanger

- Performance of He II heat exchangers are governed by several processes:
  - Surface heat transfer coefficient due to Kapitza conductance
  - Thermal gradient in He II channel allows T(x = 0) to increase above T_b
  - Solution is similar to a conductive fin problem

$$\frac{d}{dx}\left(f^{-1}\frac{dT}{dx}\right)^{1/3} + \frac{PU}{A}\left(T_b - T\right) = 0$$

- Heat flux is limited by the temperature along the heat exchanger exceeding the local saturation temperature



He II Heat Exchanger Requirements

1. The heat exchanger must have sufficient surface area

$$A_s \geq \frac{Q}{U(T_b - T_0)}$$

U = overall heat transfer coefficient

2. Bulk boiling in the heat exchanger should be avoided.

$T < T_{Sat}$ everywhere within the heat exchanger

3. Temperature gradient along heat exchanger should be minimized to not degrade performance

$T_L < T_b$ otherwise heat transfer at the end is poor (low effectiveness)

Summary of He II heat transfer

- Heat conductivity of He II is very high and thus models to interpret heat transfer are different from classical fluids
- Thermal gradient (dT/dx) in He II governed by two mechanisms
  - Normal fluid viscous drag (μ_n) yielding dT/dx ~ q
  - Turbulent "mutual friction" (dT/dx ~ q^3)
- Peak heat flux is determined by the He II near the heater reaching a maximum with onset of local boiling
- Thermal diffusion-like mechanism controls heat transfer for short times and can result in significantly higher peak heat flux
- Forced flow He II pressure drop is similar to that of classical fluids
- Non-boiling heat transfer controlled by Kapitza conductance process (thermal impedance mismatch)
- Boiling heat transfer forms vapor film over surface.