Imaging a Beam with Synchrotron Radiation

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Beam Diagnostics Using Synchrotron Radiation:
Theory and Practice
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Diffraction

- Diffraction limits the resolution at long wavelengths.
  - An important consideration when the beam is small (usually in $y$).
    - Image a point near a defocusing quad, where the beam is largest vertically.
  - A cold finger or slot also adds diffraction.

- Small beams drive the design toward shorter wavelengths.
  - Blue rather than red, but often ultraviolet or x rays.
  - More about using these wavelengths later.
Diffraction by an Aperture

![Diagram of diffraction by an aperture]

- Source Plane
- Image Plane
- Aperture
- Point Source
- Diffraction Ray
Diffraction by an Aperture

- All points in an aperture are considered point sources, reradiating light incident from a point source at \((X,Y)\)
  - Wavelength is \(\lambda = 2\pi /k\).
- The field at \((x,y)\) is given by a Fresnel-Kirchhoff integral over the (small) aperture:
  \[
  E(x, y) = -\frac{Ai}{2\lambda} \iint_{\text{aperture}} \frac{e^{ik(r+s)}}{rs} (\cos \alpha + \cos \beta) dS
  \]
  \[
  \approx -\frac{Ai}{2\lambda r_0 s_0} (\cos \alpha + \cos \beta) \iint_{\text{aperture}} e^{ik(r+s)} dS
  \]
  - Everything is essentially constant except the phase from each point in the aperture.
Expanding the Phase

\[ r = \sqrt{(X-u)^2 + (Y-v)^2 + r_0^2} \approx r_0 + \frac{(X-u)^2 + (Y-v)^2}{2r_0} \]

\[ s = \sqrt{(x-u)^2 + (y-v)^2 + s_0^2} \approx s_0 + \frac{(x-u)^2 + (y-v)^2}{2s_0} \]

\[ e^{ik(r+s)} \approx \exp \left[ ik \left( r_0 + s_0 + \frac{X^2 + Y^2}{2r_0} + \frac{x^2 + y^2}{2s_0} \right) \right] \exp \left[ ik \left( \frac{u^2 + v^2}{2r_0} + \frac{u^2 + v^2}{2s_0} \right) \right] \exp \left[ -ik \left( \frac{Xu + Yv}{r_0} + \frac{xu + yv}{s_0} \right) \right] \]

- First factor: Independent of the aperture coordinates \( u, v \).
  - Contributes only an overall phase to the \( uv \) integral over the aperture.
- Second: Quadratic in \( u \) and \( v \). Negligible since the aperture is small.
- Third: Products of \( u, v \) with cosines \( (X/r_0, \text{etc.}) \) of the ray angles from the source or measurement points to the horizontal and vertical axes.
- The only factor that matters in the integral over the aperture is:

\[ e^{ik(r+s)} \approx \exp \left[ -ik \left( pu + qv \right) \right] \]

where \( p = X/r_0 + x/s_0 \) and \( q = Y/r_0 + y/s_0 \)
Spatial Fourier Transform

- The diffraction pattern on the $xy$ plane becomes a **Fourier transform** in the spatial coordinates $uv$ of the aperture:

$$E(x, y) = -\frac{Ai}{2\lambda r_0 s_0} (\cos \alpha + \cos \beta) \int_{\text{aperture}} e^{-ik(pu+qv)} dudv$$

- One example of this principle is a **spatial filter**:
  - Laser light is sometimes focused through a small hole to remove noisy, non-Gaussian parts of the beam’s transverse profile.
  - Since the noise is found at high spatial frequencies, which appear at larger values of $u$ and $v$, it can be clipped by a properly sized hole, which acts as a spatial filter.
Diffraction by a Lens
Diffraction by a Lens: Path Length

- The length $S$ of each optical path from source $(X,Y)$ to image $(x_i,y_i)$ is equal.

$$S = \int_{(X,Y)}^{(x_i,y_i)} n(s) ds$$

- The integral along each path element $ds$ is scaled by the index of refraction $n$.
- This is a fundamental property of geometric imaging.

- The phase difference in the $uv$ integral arises from the different paths from $(X,Y)$ to $(x,y)$, compared to the equal paths from $(X,Y)$ to $(x_i,y_i)$.

- It is helpful to subtract this reference path, so that the phase difference becomes the difference between $(u,v)$ to $(x,y)$ and $(u,v)$ to $(x_i,y_i)$.

$$\sqrt{(x-u)^2 + (y-v)^2 + s_0^2} - \sqrt{(x_i-u)^2 + (y_i-v)^2 + s_0^2}$$

$$\approx -\frac{(x-x_i)u + (y-y_i)v}{s_0} = -\frac{\rho w}{s_0} \cos(\phi - \psi)$$

- Here we used polar coordinates: $(u,v) \rightarrow (w,\psi)$ and $(x-x_i, y-y_i) \rightarrow (\rho,\phi)$
Diffraction by a Lens: Result

- The diffraction integral (neglecting constants) becomes:

\[
E(x, y) = \int_0^{2\pi} \int_0^{D/2} \exp \left[ -ik \frac{\rho w}{s_0} \cos(\phi - \psi) \right] wdw d\psi
\]

\[
= 2\pi \int_0^{D/2} J_0 \left( \frac{k \rho w}{s_0} \right) wdw = \left( \frac{\pi D^2}{4} \right) \frac{2J_1 \left( \frac{k \rho D}{2s_0} \right)}{k \rho D \left( \frac{2s_0}{k \rho D} \right)}
\]

where we have used two Bessel-function identities.

- This is called the Airy diffraction pattern.
Diffraction by a Lens: Airy Pattern

- Concentric circles, with the first minimum at radius $r_A$:
  \[ r_A = 1.22 \frac{s_0}{D} \lambda = 0.61 \frac{\lambda}{\theta} \approx 1.22 \frac{f}{D} \lambda = 1.22F\lambda \]
- $\theta$ is the half angle of the light cone exiting the lens.
- $F$ is called the “F-number” of the lens.
- We plot this pattern for $\lambda = 450$ nm, $D = 50$ mm, and $s_0 = 1$ m
  - Top right: The central circle is saturated by a factor of 30 to highlight the faint rings.
  - Bottom right: The blue curve is multiplied by 10 to highlight the rings.
- $r_A$ is the resolution of the imaging system.
  - Compare it to the size of the geometric image to see if diffraction is a problem.
For the half angle $\theta$, substitute the Gaussian approximation for dipole radiation given earlier:

$$r_d \approx 0.61 \frac{\lambda}{\theta} = \frac{0.61\lambda}{0.60\gamma^{0.062} (\lambda/\rho)^{0.354}} \approx \rho^{\frac{1}{3}} \lambda^{\frac{2}{3}}$$

- Short wavelengths: In the visible, choose blue at 400 nm (or use UV or x rays).
- Large opening angles: In the LHC at high energy, edge radiation is too narrow.
- A difficult case: The HER of PEP-II has $\rho = 165$ m. At 400 nm, $r_d = 0.25$ mm.

More thoroughly, use the SR power spectral density from a point source in a Fraunhofer diffraction integral over the area of the lens illuminated through the beamline aperture, to find the field at $(x', y')$ on the image:

$$E(x', y') = A \int_{-x_a}^{x_a} dx \int_{-y_a}^{y_a} dy \frac{\gamma P_s}{\alpha_c} F_s(\omega, \psi) e^{-ik(ux + vy)}$$

- The first minimum of the intensity then gives the resolution.
- Optics software like Zemax does (monochromatic) diffraction calculations.
A dipole emits light along a gradual arc, not from a single plane.

- What is the source distance?
- Can it all be in focus?
- How do you avoid blurring the measurement?
Depth of Field: A Quick Derivation

- Diameters of A and C images as they cross the $xy$ plane, based on typical rays at angles $\pm \theta/2$:
  \[
  d = 2 \left| \frac{D/4}{2f + \Delta z} (\pm \Delta z) \right| \approx \frac{D\Delta z}{4f} = \theta \Delta z
  \]

- The vertical angle $\theta$ lighting the lens is roughly $2\sigma_\lambda$.

- If we capture a similar portion of a horizontal arc:
  \[
  \Delta z = \rho \sigma_\lambda
  \]
  \[
  d = \theta \Delta z = 2 \rho \sigma_\lambda^2 \approx 0.7 \rho^{\frac{1}{3}} \lambda^{\frac{2}{3}}
  \]

- This expression is similar to the diffraction resolution.

- As before, short wavelengths are preferable.

- But this time, small opening angles are better.
  - If the source is dipole radiation, the angle and the wavelength are not independent.

- But how much of the orbit do we actually capture?
Consider the beam’s orbit both in the horizontal plane \((xz)\) and in horizontal \(phase space\) \((xx')\).

- \(x'\) is the beam’s angle to the direction of motion \(z\).
- Which rays, at which angles, are reflected by \(M1\)?
A point on the orbit near the $xz$ origin is given by:

$$(x, z) = (\rho - \rho \cos \theta, \rho \sin \theta) \approx (\frac{1}{2} \rho \theta^2, \rho \theta) = (\frac{1}{2} \rho x'^2, \rho x')$$

For a point on the orbit, the angle $x'$ to the $z$ axis is equal to $\theta$.

The rays striking the $+x$ and $-x$ ends of M1 are given by:

$$x + x' \left( z_m \pm \frac{L_m}{2} \cos \alpha_m - z \right) = \pm \frac{L_m}{2} \sin \alpha_m$$

$$x + x' z_m \approx \pm \frac{L_m}{2} \sin \alpha_m$$

We plot these curves in phase space, along with the beam’s 1-sigma phase-space ellipse at three points along its orbit.
LHC: $x$ Phase Space

Orbit: $x = \frac{1}{2} \rho x'^2$
and $\pm 1 \sigma$ to each side

Phase-space ellipses
of protons at $s = 30, 155, \text{ and } 280 \text{ cm}$

Mirror edge

Focal-plane slit
set for rays from
$s = 10 \text{ to } 300 \text{ cm}$

$x' \text{[mrad]}$

$x \text{[\mu m]}$
LHC: $x$ Phase Space

- The two mirror edges appear as slanted lines.
- Because the radius of curvature is so long, the mirror receives light from the first 3 m of the dipole.
- The path can be shortened by adding a slit *one focal length* from the first focusing optic (mirror or lens).
  - The position of a ray on this plane corresponds only to its angle $x'$ at the source.
  - We can select light from an adjustable horizontal band across the plot.
- The $x$ positions of the proton ellipses shift along this 3-m path.
  - Project light from each ellipse onto the $x$ axis
  - The combined light is smeared out along $x$ and so blurs the resolution.
- But each proton emits light with an opening angle.
  - We need the photon ellipse, not the proton ellipse.
  - A convolution of the photon ellipse with the opening angle.
LHC: $x$ Phase Space with Light Ellipses

Mirror edge

Focal-plane slit set for rays from $s = 10$ to $300$ cm

Orbit: $x = \frac{1}{2} \rho x'^2$ and $\pm 1 \sigma$ to each side

Phase-space ellipses of light emitted at $s = 30, 155,$ and $280$ cm
The ellipses are much bigger, going well outside the slit.
They get increasing tilted and elongated with distance from the entrance to the dipole.
  * But this plot assumes that the optics are focused at the dipole entrance.
  * Move the focus to the midpoint of the 3-m path.
LHC: $x$ Phase Space, Focus at Midpoint

Orbit: $x = \frac{1}{2} \rho x'^2$
and $\pm 1 \sigma$ to each side

Phase-space ellipses of light emitted at $s = 30, 155, \text{ and } 280 \text{ cm}$

Focal-plane slit set for rays from $s = 10$ to $300 \text{ cm}$

Mirror edge
LHC: $x$ Phase Space, Projection onto $x$

- Slit excludes the right (+$x$) side of the elongated ellipses, but includes the left side.
  - Tail on the left side due to depth of field.
  - Right side of total distribution is a good measure of the true beam size.
- This answer pertains only to the LHC. Each machine requires a careful study of depth of field.
LHC: $y$ Phase Space, Focus at Midpoint

- Mirror edge
- Phase-space ellipse of protons at $s = 30$ cm
- Phase-space ellipses of light emitted at $s = 30, 155, \text{ and } 280$ cm
LHC: $y$ Phase Space, Projection onto $y$

- The elongated ellipses create a tail on both sides.
  - The vertical measurement is more affected by depth of field than the horizontal.
    - Broadens the result by as much as 20% at 7 Tev.
  - Adding a vertical slit does little to reduce the effect.
Photon Emittance (Brightness)

- Accelerator people know that Liouville’s theorem conserves the emittance of a beam in a transport line.
  - The phase-space ellipse changes shape, but not area.
    - At each waist, the size-angle product $\sigma_x \sigma_y$ is constant.
  - (But for electrons in a ring, dissipation by synchrotron radiation allows damping that “cheats” Liouville.)
- Light in an optical transport line has an emittance too.
  - At each image, the product of size and opening angle (light-cone angle) is constant.
    - Magnification makes the image bigger, but the angle smaller.
  - The area of the light’s phase-space ellipse—the brightness of the source—is conserved.
Conservation of Brightness

Lens Equations

\[ \frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_1} \]

\[ m = \frac{s_1}{s_0} \]

\[ y_1 = y_0 m \]

\[ \theta_1 = \frac{\theta_0}{m} \]
The minimum emittance for a light beam is that of the lowest-order Gaussian mode (TEM\textsubscript{00}) of a laser.

- $\omega$ is the beam radius.
  - In the usual definition (where $\omega$ is not the one-sigma value):
    - The electric field follows $E(r) = E_0 \exp(-r^2/\omega^2)$
    - The intensity (power) is the square: $I(r) = I_0 \exp(-2r^2/\omega^2)$

- $\omega_0$ is the radius at the waist (the focus).
  - This size is nonzero due to diffraction.

- $z_R = \pi \omega_0^2/\lambda$ is called the Rayleigh length.
  - Characteristic distance for beam expansion due to diffraction.

- The expansion is given by $\omega^2(z) = \omega_0^2(1+z^2/z_R^2)$

- The angle (for $z \gg z_R$) is $\theta = \omega/\lambda = \omega_0/z_R = \lambda/\pi \omega_0$

- The product of waist size and angle is then $\omega_0 \theta = \lambda/\pi$

- One-sigma values for the size and angle of $I$ give an emittance of $\lambda/4\pi$
Measuring Small Beams

- Third-generation light sources (SLS, SOLEIL, Diamond, SSRF, ALBA, PETRA-III, NSLS2…), future HEP accelerators (ILC damping rings, Super-B, ERLs) and prototypes (ATF at KEK) have very low emittances:
  - Typical emittances: $\varepsilon_x \approx 1$ nm  $\varepsilon_y \approx 10$ pm
  - Typical beam sizes: $\sigma_x < 100$ µm  $\sigma_y < 10$ µm
- An SLM images a beam from many meters away. (It’s really a telescope.)
  - F-number must be large: $F = f / D \approx (10 \text{ m})/(50 \text{ mm}) = 200$
  - Resolution with blue light: $r_A = 1.22 F\lambda \sim 100$ µm
- Techniques for measuring small beams:
  - In this lecture:
    - Imaging with ultraviolet synchrotron radiation
    - Imaging with x rays
    - Other methods that do not use synchrotron radiation (briefly)
  - Wednesday:
    - Synchrotron-light interferometry (~10 µm resolution)
    - Vertical beam size using the null in vertically polarized light
      - Not in Wednesday’s lecture, but similar in concept to an interferometer
Without Synchrotron Light: Wire Scanner

- Methods that don’t use synchrotron light are also useful. They’re outside the scope of this class, but…
- Wire scanner:
  - While a thin stretched wire is scanned across the (wider) beam, measure scattered radiation or lost electrons vs. wire position.
  - Gives a projection of the beam in the scan direction.
  - Three wires at 0, 45, 90 degrees give major and minor axes and tilt of beam ellipse.
  - Wire size ($\geq 4 \mu m$), limited by wire erosion, sets resolution.
  - Multiple measurements: beam jitter
  - Can be destructive to stored beams
Without Synchrotron Light: Laser Wire

- Laser crosses electrons at a waist smaller than the $e$-beam.
- Focus with a small F-number to get resolution $\approx \lambda \geq 300$ nm.
- Like wire scanner, look for scattered radiation.
- Compared to wire scanner, better resolution and non-destructive. Still needs many measurements.
Laser Interferometer

- Split a laser beam. Intersect both parts at an angle as they cross the electron beam.
- Interference fringes with maxima and minima across the electrons.
- Move the beam relative to the fringe pattern.
- When the beam is small compared to the fringe spacing, the scatter is heavily modulated by the shift in the fringes.
- Can measure down to tens of nm
Imaging with UV Synchrotron Light

- Can’t go far into the UV without problems.
  - Window and lens materials become opaque:
    - Glasses (like BK7 at right) are useful above ~330 nm.
    - Fused silica works above ~170 nm.
    - Special materials like MgF₂ work above ~120 nm.
  - Absorption in air below ~100 nm
    - Must use reflective optics in vacuum.
Imaging with Synchrotron X Rays

- The good news: Most of the beam’s emission is in the x-ray region.
- The bad news: How do you form an image?
- We’ll discuss some techniques:
  - Pinhole cameras
  - Zone plates
  - Grazing-incidence optics
  - X-ray lenses
- Labs later today on imaging with pinholes and zone plates (along with ordinary lenses)
Imaging X Rays with a Pinhole Camera

- Resolution $\sigma$ on image plane with a pinhole of radius $r$:
  $\sigma = \sqrt{\sigma_g^2 + \sigma_d^2}$
  - Distances: $a$ from source to pinhole, $b$ from pinhole to image:
  - Geometric optics: Pinhole should be $\ll$ beam size
  $\sigma_g = \frac{r}{\sqrt{3}} \frac{a+b}{a}$
  - Diffraction blurs image if the pinhole is too small:
  $\sigma_d = \frac{5}{8\pi} \frac{\lambda b}{r}$
  - Pinhole size for best resolution:
    - Geometric mean of $\lambda$ (~0.2 nm) and $a$ or $b$ (~10 m): $r \approx 20 \mu m$
  - Optimum resolution on the source plane:
    - Want small $\lambda$, small $a$, and large magnification $b/a$
  $\sigma_{opt}^2 = \frac{5\lambda}{4\pi\sqrt{3}} \left(1 + \frac{a}{b}\right) a$
- On image plane, a scintillator converts x rays to visible light.
- Make “pinhole” with a sheet of heavy metal thick enough to stop x rays.
- X-rays surrounding the hole must be blocked upstream, so that pinhole get too hot and deform.
- Most of the x rays are not used for the image
Their 10-\(\mu\)m pinhole gives a resolution of 13 \(\mu\)m.
X-Ray Pinhole Camera in the PEP-2 LER

- X-ray source in dipole
- Plan View
- Conical and retractable photon stops
- Gate valve
- Detector table
- Elevation View
- Pinhole
- X-ray beamline
- Modified photon stop
- Graphite filter
- Photon BPM
Design of the Pinhole Assembly

- Gold disk for heat transfer
- Pt:Ir (90:10) disk with 4 pinholes.
  - Diameters of 30, 50, 70, and 100 μm.
- Front: Glidcop with 4 larger holes
Photos of the Pinhole Assembly
Imaging X Rays with a Zone Plate

- A diffractive lens, made by microlithography
- Rings of a high-Z metal (gold) deposited on a thin low-Z membrane (SiN)
  - Ring widths as narrow as 50 nm are possible
- Power must be kept low, and bandwidth must be narrow ($\approx 1\%$)
  - Precede with a pair of multilayer x-ray mirrors, which reflect a narrow band and absorb the out-of-band power.
How Does a Zone Plate Work?

- Consider a transmissive diffraction grating.
  - Parallel opaque lines on a clear plate, with period $a$
  - Parallel rays of wavelength $\lambda$ passing through adjacent lines and exiting at an angle $\theta$ have a difference in optical path of $a \sin \theta$.
  - They are in phase if this difference is $n\lambda$, giving the $n^{th}$-order diffraction maximum: $\sin \theta_n = n\lambda/a$

- Now wrap these grating lines into a circle.
  - $1^{st}$ order bends toward center: focusing
  - $-1^{st}$ order bends away from center: defocusing
  - $0^{th}$ order continues straight ahead
  - Make central circle opaque to block $0^{th}$-order light around the focus (a “central stop”).

- But the $1^{st}$-order rays are parallel and so don’t focus
  - Vary the zone spacing as a function of ring radius $r$ so that all the exiting rays meet at a focal point a distance $f$ from the zone plate.
How the Zone Widths Vary

- To focus at $f$, the ray at radius $r_n$ must exit at an angle $\theta_n$ with: $r_n = f \tan \theta_n$
- First-order diffraction gives $\lambda = a_n \sin \theta_n$
- The grating period $a$ now varies too: $a_n = r_{n+1} - r_{n-1}$
- There are many, closely spaced zones, and so we treat $n$ as a continuous variable: $a(n) = \Delta n \frac{dr(n)}{dn} = 2 \frac{dr(n)}{dn}$
- We use the expression for $\tan \theta(n)$ to substitute for $\sin \theta(n)$:

$$\sin^2 \theta = \frac{1}{\cot^2 \theta + 1} = \frac{1}{1 + \frac{f^2}{r^2}} = \frac{\lambda^2}{a^2} = \lambda^2 \left(2 \frac{dr}{dn}\right)^2$$

$$\frac{d}{dn} \left(\frac{r^2}{f^2}\right) = \frac{\lambda}{f} \sqrt{1 + \frac{r^2}{f^2}}$$

$$\int_0^n \frac{\lambda}{f} \, dn' = \frac{\lambda n}{f} = \int_0^{r^2/f^2} \frac{dx}{\sqrt{1 + x}} = 2 \sqrt{1 + \frac{r^2}{f^2} - 2}$$

$$\frac{r^2}{f^2} = \left(\frac{\lambda n}{2f} + 1\right)^2 - 1$$

$$r^2 = n\lambda f + \frac{n^2 \lambda^2}{4}$$
Zone-Plate Formulas

- \( \lambda \) = wavelength (monochromatic light)
- \( \Delta \lambda \) = bandwidth
- \( f \) = focal length of lens at \( \lambda \)
- \( N \) = number of zones
  - Counting both clear and opaque zones
- \( r_n \) = radius of \( n \)th zone boundary
- \( \Delta r = r_N - r_{N-1} \) = thickness of outer zone
- \( D = 2r_N \) = outer diameter
- \( F \) = F-number
- \( r_A \) = (Airy) resolution

\[
\begin{align*}
  r_n &= \sqrt{nf \lambda + n^2 \lambda^2 / 4} \\& \approx \sqrt{nf \lambda} \\
  f &= 4N(\Delta r)^2 / \lambda \\
  D &= 2r_N = 4N\Delta r \\
  F &= f / D = \Delta r / \lambda \\
  r_A &= 1.22F \lambda = 1.22\Delta r \\
  \Delta \lambda &< \lambda / N \text{ to avoid chromatic blurring}
\end{align*}
\]
A zone plate is designed to focus at a single wavelength.

- This is called “strong chromatic aberration”.
- Insert a monochromator, to limit bandwidth and to absorb power at other wavelengths.
- With two crystals, the entering and exiting rays are parallel.
Zone Plate at SPring-8

- Monochromator transmits 8.2-keV photons ($\lambda = 0.151 \text{ nm}$)
- Total magnification = 13.7 (0.2737 by FZP, 50 by XZT)
- 4-µm resolution with the help of the x-ray zooming tube
- Observed a transient in the beam size during top-off operation
SPring-8 Diagnostic Beamline
Zone-Plate Imaging at the ATF at KEK

Injection transient

< 1ms exposure time
Specifications of the ATF Zone Plates

### TABLE II. Specifications of the two FZPs.

<table>
<thead>
<tr>
<th></th>
<th>CZP</th>
<th>MZP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of zone</td>
<td>6444</td>
<td>146</td>
</tr>
<tr>
<td>Radius</td>
<td>1.5 mm</td>
<td>37.3 μm</td>
</tr>
<tr>
<td>Outermost zone width $\Delta r_N$</td>
<td>116 nm</td>
<td>128 nm</td>
</tr>
<tr>
<td>Focal length at 3.24 keV</td>
<td>0.91 m</td>
<td>24.9 mm</td>
</tr>
<tr>
<td>Magnification</td>
<td>$M_{\text{CZP}} = 1/10$</td>
<td>$M_{\text{MZP}} = 200$</td>
</tr>
</tbody>
</table>

- Total magnification = 20
- Detecting 3.24-keV photons ($\lambda = 0.383$ nm)
  - Where’s the monochromator? A pinhole at the intermediate waist can be used to reject defocused light at other wavelengths.
Grazing-Incidence X-Ray Mirrors

- A plasma with electron density $n_e$ has characteristic oscillations of charge and electric field at the “plasma frequency” $\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$
- The index of refraction of the plasma is $n = 1 - \frac{\omega_p^2}{\omega^2} < 1$
- The free electrons in a metal act like a plasma.
  - Visible frequencies are cut off: $\omega \ll \omega_p$ and so $n < 0$
  - X rays are transmitted: $\omega \gg \omega_p$ and so $n$ is slightly below 1
- Snell’s law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$, or $n_1 \cos \alpha_1 = n_2 \cos \alpha_2$
  - Here $\theta_1 = \pi/2 - \alpha_1$ and $\theta_2 = \pi/2 - \alpha_2$ are the ray angles to the normal.
- Total internal reflection occurs in medium 1 when $n_1 \sin \theta_1 > n_2$
  - For x-rays in vacuum striking metal at an angle $\alpha$ to grazing:
    $$\cos \alpha > 1 - \frac{\omega_p^2}{\omega^2}, \quad \text{or} \quad \alpha < \sqrt{2} \frac{\omega_p}{\omega} \ll 1$$
- Mirrors at small angles to grazing can reflect x rays
  - Flat grazing-incidence mirrors
  - Multilayer mirrors that use interference to get a narrow bandwidth
  - Telescopes and imaging systems using off-axis conic surfaces
X-Ray Reflectivity of Materials

Angle to Grazing (deg) vs. Photon Energy (eV)

- C
- Ni

Graphs show reflectivity data for carbon (C) and nickel (Ni) at various photon energies and grazing angles.
Multilayer X-Ray Mirrors

- Multilayer mirror designed for a narrow passband at 8 keV
  - Unpolarized light incident at 1.51 degrees
  - 200 periods with alternating layers of low- and high-Z materials: B\(_4\)C and Mo
  - 3-nm spacing: 2.1 nm of B\(_4\)C and 0.9 nm of Mo, with an interdiffusion thickness of 0.5 nm
Refractive X-Ray Lenses

- The refractive index for x rays is slightly below 1 for any material.
- When a ray in vacuum strikes a material at a non-grazing angle, the transmitted ray enters with a deflection following Snell’s law.
- A spherical surface can make an x-ray lens!
  - Collimated x rays: Use a parabolic surface
- $n < 1$: A focusing lens must be **concave**.
- $1 - n << 1$: Small deflection, long focal length
  - Stack many lenses for a shorter overall focal length.
  - To avoid absorption: Low-Z material (Be)
- $1 - n = \omega_p^2/\omega^2$: Strong chromatic aberration
  - Only monochromatic x-rays can focus.
Beamline Design: Machine Constraints

- Distance to the first mirror (M1)
  - Ports and M1 itself introduce wakefields and impedance.
    - Is M1 flush with the vacuum-chamber wall?
  - The heat load on M1 is reduced by distance.
    - Is the mirror far down a synchrotron-light beamline?

- Distance to the imaging optics
  - In a hutch: Adds distance to get outside the shielding
  - In the tunnel: Inaccessible, but often necessary for large colliders.

- Size and location of the optical table
  - What measurements are needed?
  - Which instruments are available (affordable)?
Choose a source point with a large $y$ size, to lessen effect of diffraction (for visible light).

Magnification: Transform expected beam size to a reasonable size on the camera.
- $6\sigma < \text{camera size} < 12\sigma$: Uses many pixels; keeps the image and the tails on the camera; allows for orbit changes.
- Needs at least two imaging stages: Since the optics are generally far from the source, the first focusing element strongly demagnifies.

Optics: Use standard components whenever possible.
- For example, adjust the design to use off-the-shelf focal lengths from the catalog of a high-quality vendor.
- Use a color filter to avoid dispersion in lenses (or use reflective optics).
- Correct lens focal lengths (specified at one wavelength) for your color.
Basic Design Spreadsheet

- You can iterate a lot of the basic design in a simple spreadsheet.
  - Enter the fixed distances.
  - Specify the desired magnifications.
  - Solve the lens equations, one stage at a time, to find lenses giving the ideal magnifications.
  - Change the lenses to catalog focal lengths.
  - Correct their focal lengths (using the formula for each material as found in many catalogs).
  - Iterate the magnifications and distances.

- Then optimize your design with optics software
  - I used Zemax for the CERN design.