The Fringe Pattern of a Synchrotron-Light Interferometer

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Derivation of the Interference Pattern

- Assumptions:
  - One dimensional (y)
  - Two narrow, parallel slits of width $a$
  - Small slit separation (center to center) $d > a$
  - Slits are far from the source: $z_0s \gg d$
  - Small source size $\sigma_0$, but comparable to slit width
  - Nearly monochromatic light, due to a bandpass filter
Consider a point source at \( Y \) on the \( (X,Y) \) plane.

The light is monochromatic at wavelength \( \lambda = 2\pi/k \).

We calculate the intensity at a point \( y \) on the image plane \( (x,y) \).

The two slits are on the \( (u,v) \) plane.

The electric field at \( y \) is found using a Fraunhofer diffraction integral over the slit plane.

We will need the difference in the length of the optical path for all the rays leaving \( Y \) and arriving at \( y \).
Layout of the Two-Slit Interferometer
Reference Path

- Compare all paths to a reference path that:
  - Leaves the source plane at \(-y/m\)
    - Here \(m\) is the magnification of the lens: \(m = z_{li}/z_{0l}\)
    - Is imaged geometrically to \(y\) on the image plane.
  - All optical paths from \(-y/m\) to \(y\) have equal lengths.
    - A fundamental property of geometric imaging.
    - Applies to the imaged ray passing through the slit at \(v\).
- Consider a ray leaving \(Y \neq -y/m\) that diffracts in the slit at \(v\) with an angle that brings it to \(y\).
  - Its path matches the imaging path from \(-y/m\) to \(v\) to \(Y\) after the slit…but not before.
    - This lets us compute the path difference \(\Delta s\) from the reference.
Difference in the Optical Path Length

\[ \Delta s = \sqrt{(v - Y)^2 + z_{0s}^2} - \sqrt{(v + \frac{z_{0l}}{z_{li}} y)^2 + z_{0s}^2} \]

\[ \approx \frac{1}{2z_{0s}} \left[ -2v \left( Y + \frac{z_{0l}}{z_{li}} y \right) + Y^2 - \left( \frac{z_{0l}}{z_{li}} y \right)^2 \right] \]

\[ = \frac{1}{2z_{0s}} \left[ -2v \left( Y + \frac{f y}{z_{fi}} \right) + Y^2 - \left( \frac{f y}{z_{fi}} \right)^2 \right] \]

\[ = -(gv + h) \]

where \[ \frac{1}{f} = \frac{1}{z_{0l}} + \frac{1}{z_{li}} = \frac{1}{z_{0l}} + \frac{1}{f + z_{fi}} \]
Fraunhofer Diffraction Integral

\[
E(y, Y, k) = \int_{\text{slits}} e(v) \frac{1}{s_{Y, v}} \exp \left[ i k s_{Y, v} + i k s_{v, v} \right] dv \\
= \int_{\text{slits}} \sqrt{A(v)} \exp \left[ i k \Delta s + i \phi(v) \right] dv \\
= ae^{-ikh} \text{sinc} \left( \frac{kga}{2} \right) \left( \sqrt{A_1} e^{-ikgd/2} + \sqrt{A_2} e^{ikgd/2 + i\phi_0} \right)
\]

\[
I(y, Y, k) = a^2 \text{sinc}^2 \left( \frac{kga}{2} \right) \left[ A_1 + A_2 + 2\sqrt{A_1 A_2} \cos \left( kd + \phi_0 \right) \right]
\]

- Envelope from single-slit diffraction
- Individual slits
- Interference between slits
- Phase offset between slits
Fringe pattern of a monochromatic point source at 450 nm, using parameters for the PEP-II HER interferometer at SLAC. $A_1 = A_2 = 1$, $d = 5$ mm, $a = 0.5$ mm.
Diffraction from an Extended Source #1

- Replace point source at $Y$ with a Gaussian:
  \[
  \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left( -\frac{1}{2} \left( \frac{Y-Y_0}{\sigma_0} \right)^2 \right)
  \]
- Source consists of independent electrons: Incoherent.
  - Integrate the intensity, not the electric field.
- Compare arguments: $\text{sinc}^2\left(\frac{kga}{2}\right)$ versus $\cos(kgd + \phi_0)$
- Recall that:
  \[
  g = \frac{1}{z_{0s}} \left( Y + \frac{fy}{z_{fi}} \right) \sim \frac{1}{z_{0s}} \left( \sigma_0 + \frac{fy}{z_{fi}} \right)
  \]
- For $d=5$ mm, $a=0.5$ mm, $\lambda=450$ nm, $\sigma_0=0.3$ mm, $z_{0s}=10$ m:
  - $k\sigma_0a/(2z_{0s}) = 0.105$ Small compared to zero of sinc at $\pi$
  - $k\sigma_0d/z_{0s} = 2.09$ Significant change in cosine phase
We remove the sinc from the integral over the source, getting:

\[
I(y, k) = a^2 \text{sinc}^2 \left( \frac{ka}{2z_{0s}} \left( Y_0 + \frac{fy}{z_{fi}} \right) \right)
\]

\[
\cdot \left\{ A_1 + A_2 + \frac{2\sqrt{A_1 A_2}}{\sqrt{2\pi} \sigma_0} \int \exp \left[ -\frac{1}{2} \left( \frac{Y - Y_0}{\sigma_0} \right)^2 \right] \cos \left[ \frac{kd}{z_{0s}} \left( Y + \frac{fy}{z_{fi}} \right) + \phi_0 \right] dY \right\}
\]

\[
= a^2 \text{sinc}^2 \left( \frac{kg_0 a}{2} \right) \left\{ A_1 + A_2 + 2\sqrt{A_1 A_2} \exp \left[ -\frac{1}{2} \left( \frac{k\sigma_0 d}{z_{0s}} \right)^2 \right] \cos \left( kg_0 d + \phi_0 \right) \right\}
\]

where

\[
g_0 = \frac{1}{z_{0s}} \left( Y_0 + \frac{fy}{z_{fi}} \right) = \frac{fy}{z_{0s} z_{fi}} - \theta_0 = \frac{y}{f + z_{fi} (1 - z_{sl}/f)} - \theta_0
\]
Fringe pattern of an extended source, with $\sigma_0=0.2$ mm
Pass synchrotron light through a narrow Gaussian filter:
\[
\frac{1}{\sqrt{2\pi\sigma_k}}\exp\left[-\frac{1}{2}\left(\frac{k-k_0}{\sigma_k}\right)^2\right]
\]

Compare arguments: \(\text{sinc}^2\left(\frac{k_0a}{2}\right)\) versus \(\cos\left(k_0d + \phi_0\right)\)

Recall that:
\[
g_0 = \frac{y}{f + z_{fi}(1-z_{sl}/f)} - \theta_0
\]

Essentially \(g_0\) is a scaled vertical coordinate with an offset \(\theta_0\).

Compare effect of \(\sigma_k\) to an argument of \(\pi\), where \(\text{sinc} = 0\).

For \(\Delta\lambda = 30\) nm FWHM:
\[
\pi\sigma_k/k_0 = 0.09 \quad \text{Small}
\]

Again we can remove the sinc from the integration over the bandpass filter.
Quasi-Monochromatic Light #2

\[ I(y) = a^2 \text{sinc}^2 \left( \frac{k_0 g_0 a}{2} \right) \]

\[ \cdot \left\{ A_1 + A_2 + \frac{2 \sqrt{A_1 A_2}}{\sqrt{2\pi \sigma_k}} \int \exp \left[ -\frac{1}{2} \left( \frac{k - k_0}{\sigma_k} \right)^2 \right] \exp \left[ -\frac{1}{2} \left( \frac{k \sigma_0 d}{z_{0s}} \right)^2 \right] \cos (k g_0 d + \phi_0) \, dk \right\} \]

\[ = a^2 \text{sinc}^2 \left( \frac{k_0 g_0 a}{2} \right) \left\{ A_1 + A_2 + \frac{2 \sqrt{A_1 A_2}}{\sqrt{1 + \left( \frac{\sigma_k \sigma_0 d}{z_{0s}} \right)^2}} \exp \left[ -\frac{1}{2} \left( \frac{k \sigma_0 d}{z_{0s}} \right)^2 \right] \cos \left( \frac{k \sigma_0 d}{z_{0s}} \right) + \phi_0 \right\} \]

Beam size  Bandwidth
Fringes with a Bandpass Filter

Fringe pattern using a Gaussian bandpass filter with a full width at half maximum (FWHM) of $\Delta \lambda = 30$ nm.
Compare to van Cittert and Zernicke

- We previously looked at the derivation of van Cittert and Zernicke (VCZ). Is this approach the same?

- VCZ:
  - The “degree of coherence” $\mu(A_1,A_2)$ between two apertures due to the finite source.
    - VCZ says that, for a narrow-band source, this expression resembles the diffraction pattern on the slit plane due to the source.
  - The intensity pattern on the camera is then determined from the single-aperture patterns and $\mu$.

- My approach changes the order, but reaches the same result:
  - First do the full calculation through two slits to the camera for a monochromatic point source.
  - Next include the finite size.
  - Finally add the narrow bandwidth.
Interferometry on SPEAR-3
First Mirror (M1) in the HER of PEP-II
Distortion of PEP’s First Mirror

- The first mirror in PEP-II takes a huge heat load, even with grazing incidence to spread out the heat.
  - Extensive (and stiff) stainless-steel water-cooling tubes on the rear add mechanical stress.

- A slot along the midplane of the mirror is meant to allow the narrow and hot x-ray fan to bypass the mirror and hit a separate dump.
  - Some folding of the mirror about the slot.

- Thermal and mechanical stresses reduce image quality and would decrease fringe contrast in an interferometer.
Compensation with a Cylindrical Lens

- Interferometer slits pass light from two thin horizontal stripes along M1.
  - Little of M1’s surface contributes…and we can reduce this more.
- Beam is imaged through the slits onto the camera.
  - Fringes of a vertical (y) interferometer measurement form a series of parallel horizontal lines on the camera.
  - Beam size is calculated from the intensity variation along y.
- The direction along the stripes (x) is less interesting.
  - Change the focal length horizontally to image M1, not the beam.
  - Insert a cylindrical lens to shorten the focal length in x only.
  - Position along each stripe corresponds to an x coordinate of M1.
  - Computer selects x value with best fringe visibility on the camera.
- The interferometer uses only two small rectangles, selected for fringe quality, on M1’s surface.
Adding the Cylindrical Lens

Beam: Source Plane for y Image

First Mirror: Source Plane for x Image

Slit Plane

Spherical Lens

Cylindrical Lens (horizontally focusing)

Image Plane for x and y
LER Interferometer on Tunnel Wall

Cylindrical lens

Slit assembly
Double-Slit Assembly

- Rotation motor
- Spherical lens
- Plate for outer jaws
- Jaw motors
- Plate for inner jaws
Sketch of Slit Assembly

- Inner jaws mounted from lower plate.
- Outer jaws mounted from upper plate.
- Each jaw is on a translation stage with no motor, moving transversely.
- A third, motorized stage on each plate moves longitudinally, spreading the jaws with a wedge.
Cylindrical Lens to Camera

- Cylindrical lens
- Camera
- Polarizer
Polarizer, Filter, and Camera