Fluctuation-Based Bunch Length Experiments

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- Motivation
- Time Domain Measurements
- Frequency-based Measurements
- Interferometer-based Measurements
- Introduction to USPAS Simulator

Motivation

- Alan derived theoretical basis for using statistical fluctuations to measure pulse length

- Each electron is an independent ‘radiator’ with a random, granular distribution along the bunch (shot noise)

- Sometimes the phase of wave packets overlap, sometimes they don’t

- The *mean and variance* (moments) in the signal yields pulse length

- Measurements can be made in the time domain or frequency domain

- We will review some experiments and introduce the USPAS simulator

Time Domain View

Sum electric field emission from individual electrons

$$E(t) = \sum_{k=1}^{N} e(t - t_k)$$

where emission times t_k are random, Gaussian-distributed numbers

$$f(t) = \frac{1}{\sqrt{2\pi\sigma_t}} e^{-t^2/2\sigma_t^2}$$

Each wavepacket $e(t)$ is centered at random time t_k

Wavepackets superimpose to produce more or less field at time t

The electromagnetic field intensity is E^*E

Total pulse energy $\int E^* E dt$ is therefore random in time.

Statistically Random Function in Time

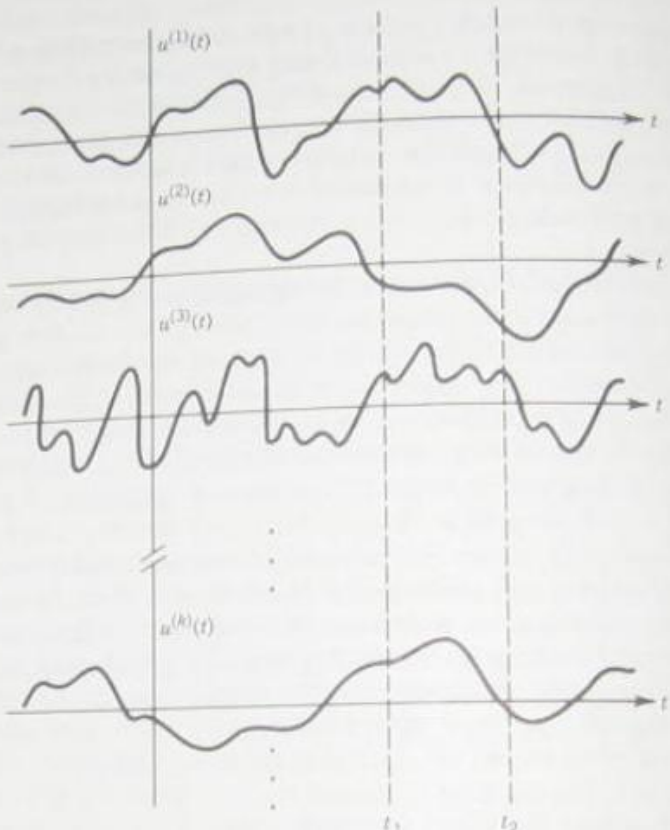


Figure 3-1. An ensemble of sample functions, where t_1 and t_2 are the parameter values for which the joint density function $p_U(u_1, u_2; t_1, t_2)$ is specified.

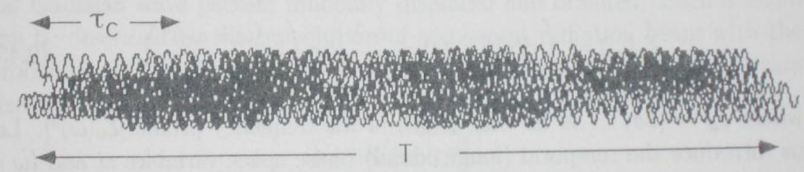


Figure 2.7: An example of chaotic light given by a random superposition of 100 sinusoidal wave packets each six period long. The total duration of the pulse is T.

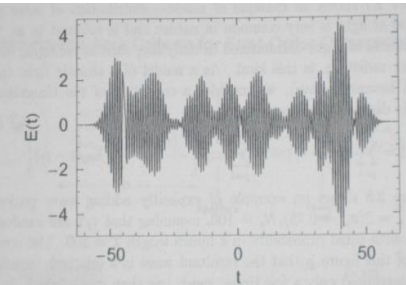


Figure 2.8: Electric field of Eq. (2.47) consists of M regular regions (M longitudinal modes).

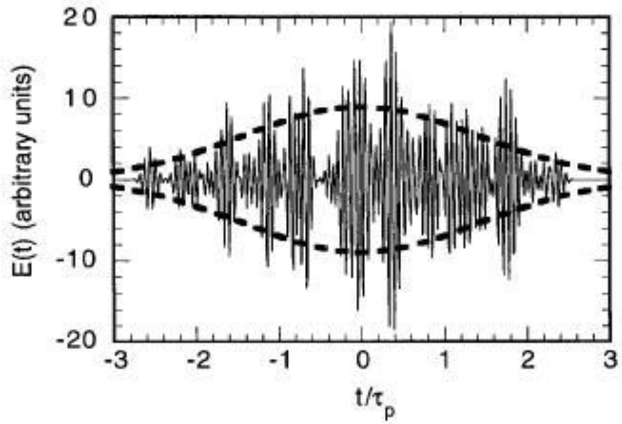
M~10

an incoherent electric field is often not what we were lead to believe -

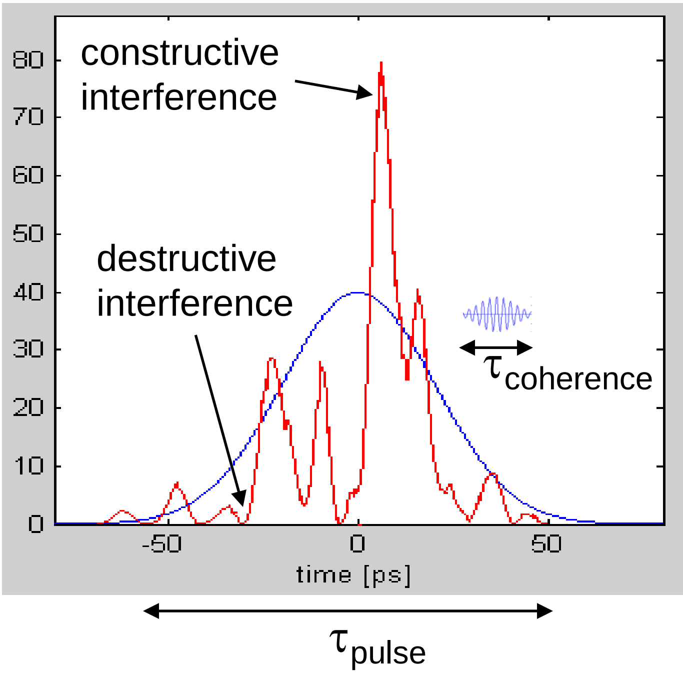
Fluctuations in Electric Field and Intensity

Each light pulse from the synchrotron has statistical structure

Field fluctuations



Intensity fluctuations E^*E



wave packets emittted from individual electrons statistically add or cancel

the *correlation length* corresponds to the wavepacket *coherence length*

T_{pulse}/τ_{coh} = M (mode number)

Coherence Length and Coherence Time

To increase correlation length band-limit the radiation

This increases the coherence length of the individual wave packets

$$f = c/\lambda$$

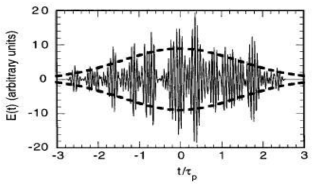
$$\delta f = -\delta\lambda c/\lambda^2$$

$$\delta t = 1/\delta f = \lambda^2/c\delta\lambda$$

For 633nm light and a 1nm band pass filter

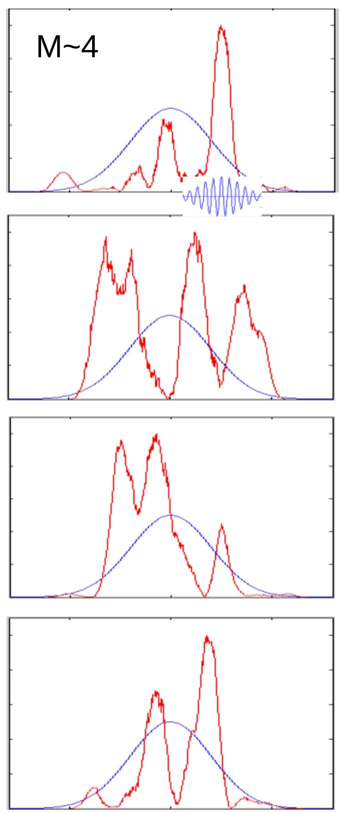
$$\delta t = \lambda^2/c\delta\lambda = \frac{(633*10^{-9})^2}{(3*10^8)(1*10^{-9})} = 1.3ps$$

For a 15ps bunch, the 'mode number' M~ 15.



Energy Fluctuation Statistics

Narrow band filter



$$E_1 = \int P_1(t)dt$$

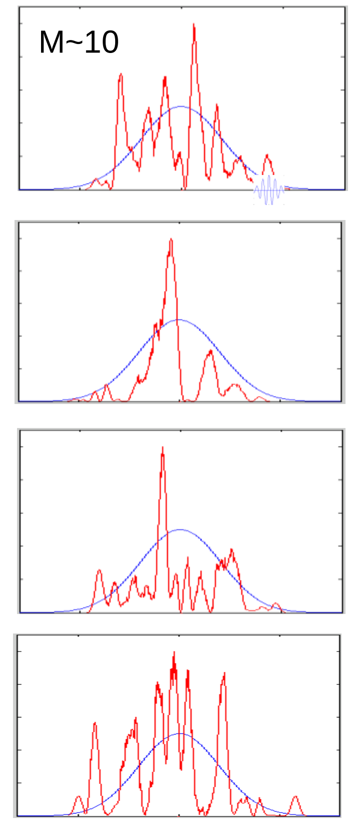
$$E_2 = \int P_2(t)dt$$

$$E_3 = \int P_3(t)dt$$

...

$$E_N = \int P_N(t)dt$$

Wide band filter



$$E_1 = \int P_1(t)dt$$

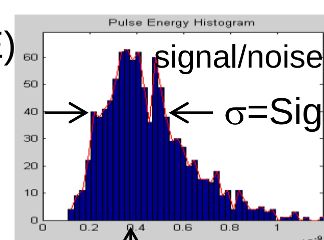
$$E_2 = \int P_2(t)dt$$

$$E_3 = \int P_3(t)dt$$

...

$$E_N = \int P_N(t)dt$$

f(E)



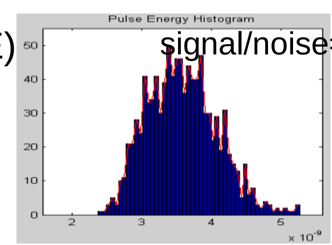
signal/noise=2.4

σ=Sigma (noise)

E

W=Mean (signal)

f(E)



signal/noise=7.1

E

Intensity Fluctuation Derivation

Goodman, *Statistical Optics* Chapter 6

Average Value

$$\overline{W} = \int_{-T}^{T} \overline{I}(t)dt$$

Variance

$$\sigma_W^2 = E\left[\left(\int_{-T}^{T} I(t)dt\right)^2\right] - \overline{W}^2$$

$$= \int_{-T}^{T}\int \overline{I(t)I(t')}dtdt' - \overline{W}^2$$

$$\sigma_W^2 = \int_{-T}^{T}\int \Gamma_I(t-t')dtdt' - \overline{W}^2$$

where Γ is the autocorrelation function of *I(t)*

in terms of fields $\Gamma_I(\tau) = E\{e(t)e^*(t)e(t+\tau)e^*(t+\tau)\}$

'fourth order correlation'

Intensity Fluctuations (cont'd)

$$\sigma_W^2 = \int_{-T}^{T}\int \Gamma_I(t-t')dtdt' - \overline{W}^2$$

$$\Gamma_I(\tau) = E\{e(t)e^*(t)e(t+\tau)e^*(t+\tau)\}$$ 'fourth order correlation'

But from interferometry $\Gamma_I(\tau) = I^2 \cdot \left(1 + |\gamma(\tau)|^2\right)$

Then $$\sigma_W^2 = \overline{W}^2 \frac{1}{T}\int |\gamma(\tau)|^2 d\tau$$

$$\frac{\overline{W}^2}{\sigma_W^2} = \left(\frac{1}{T}\int |\gamma(\tau)|^2 d\tau\right)^{-1} = M$$ (same as before)

$$M = \frac{1}{\frac{1}{T}\int |\gamma(\tau)|^2 d\tau} = \frac{\tau_{pulse}}{\tau_{coh}}$$ is the number of modes-per-pulse!

⟶ measurement of *W*, σ_W with known τ_c yields τ_{pulse}

M: The ratio of Pulse Time to Coherence Time

Goodman, *Statistical Optics* Chapter 6

$$M = \frac{1}{\frac{1}{T}\int |\gamma(\tau)|^2 d\tau} = \frac{\tau_{pulse}}{\tau_{coh}}$$

PROPERTIES OF INTEGRATED INTENSITY 243

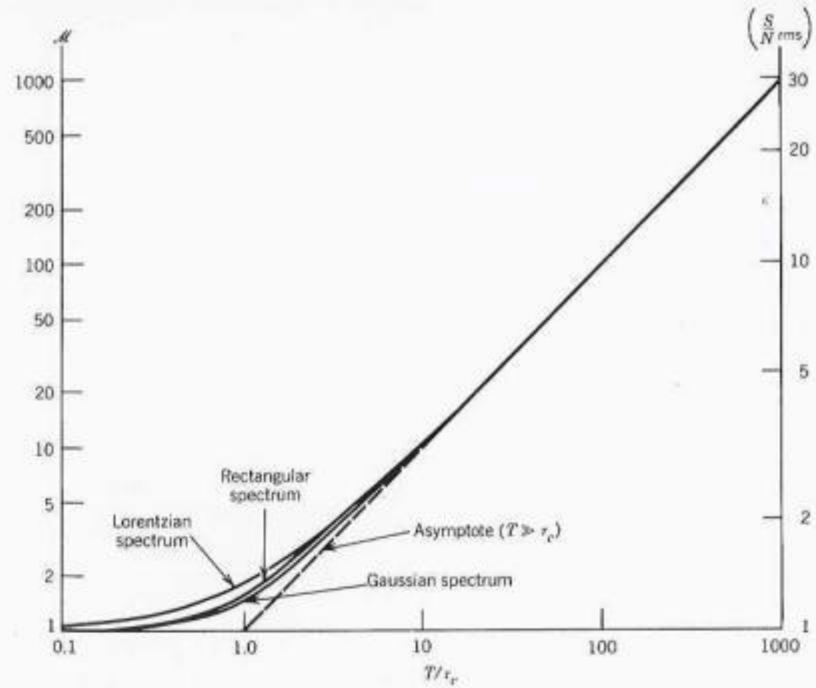


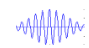
Figure 6-1. Plots of 𝓜 versus T/τ_c, exact solutions for Gaussian, Lorentzian, and rectangular spectral profiles.

Signal/Noise

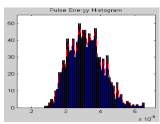
$\tau_{pulse}/\tau_{coherence}$

Relation between physics and measurement

filter



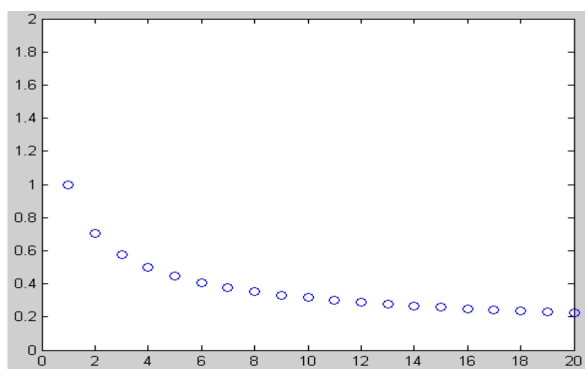
measure



$$\frac{\sigma_W}{< W >} = \frac{variance}{mean} = \left(\frac{\tau_{coherence}}{\tau_{pulse}}\right)^{1/2}$$

measure

calculate

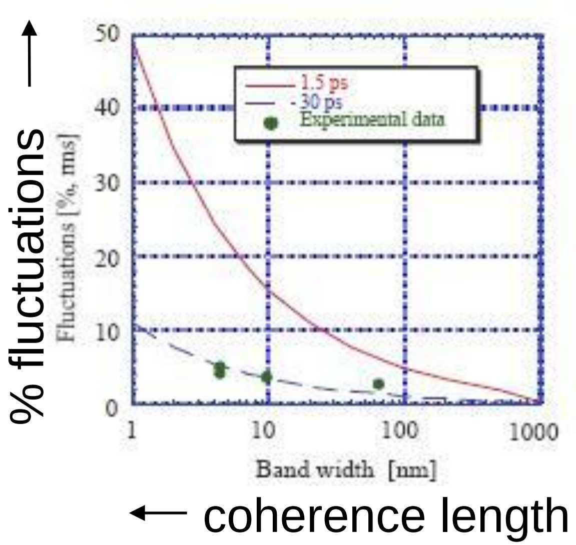


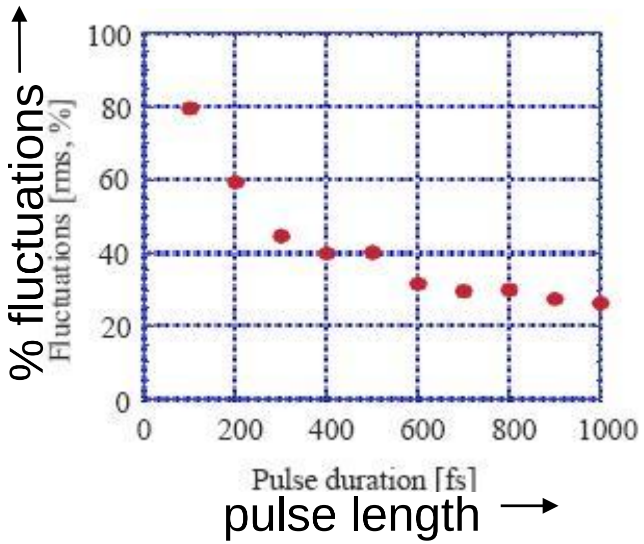
Sigma/mean

number of modes: M

Modes-per-pulse: Experimental Evidence, U. Tokyo

$$\frac{\sigma_W}{< W >} = \frac{variance}{mean} = \left(\frac{\tau_{coherence}}{\tau_{pulse}}\right)^{1/2}$$





Time-Domain Measurements (cont'd)

In the simplest form...

$$\delta^2 = \frac{\sigma_W^2}{\overline{W}^2} = \int_{-T}^{T}\int I(t)I(t')dtdt'$$ fluctuations proportional to intensity correlation

For Gaussian statistics and band pass filter $$\delta^2 = \frac{1}{\sqrt{1+4\sigma_\tau^2\sigma_\omega^2}}$$

Expanding $$\delta^2 \approx \frac{1}{2\sigma_\tau\sigma_\omega}$$

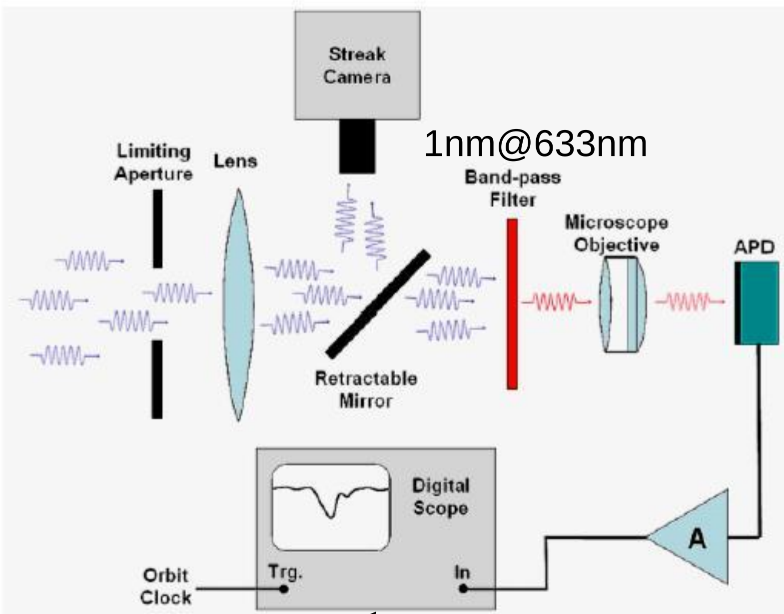
For $$\sigma_{coh} \approx 1/\sigma_\omega$$

We get $$\delta^2 \approx \frac{\sigma_{coh}}{\sigma_\tau} = \frac{1}{M}$$

Make the coherence length long to reduce the number of modes M

Time-Domain Measurements at Berkeley

Intensity fluctuations, F. Sannibale, et al

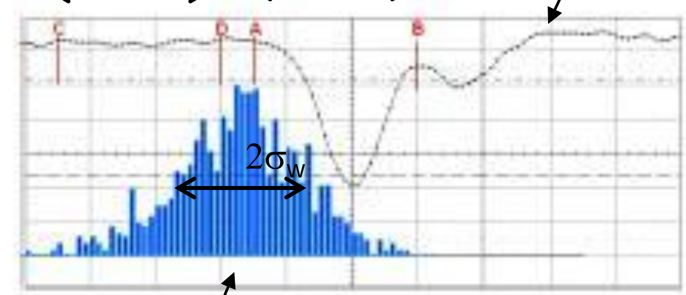


LeCroy 3GHz BW, 20Gsample/s
calculate average value of AB, CD
5000 samples @ 1.5MHz

single pulse

integration interval

background



histogram of pulse energy

$$\delta^2 = \frac{\sigma_W^2}{\overline{W}^2} = \int_{-T}^{T}\int I(t)I(t')dtdt'$$

Calibration against Streak Camera

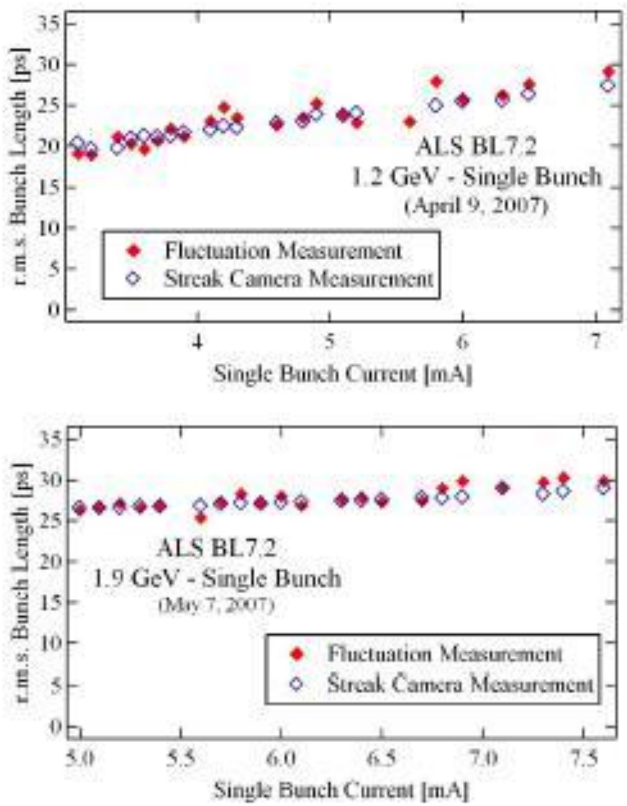


Figure 3: Examples of fluctuation and streak-camera bunch length measurements at the ALS for different beam parameters.

$$\delta^2 = \sqrt{1+\frac{\sigma_\tau}{\sigma_{\tau,c}}}\sqrt{1+\frac{\sigma_x}{\sigma_{x,c}}}\sqrt{1+\frac{\sigma_y}{\sigma_{y,c}}}$$

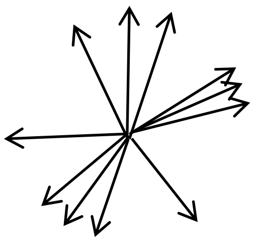
$\sigma_{x/y,c}$ are transverse coherence sizes
-related to transverse EM modes at 633nm
-radiation process, including diffraction
-ratios about 2 and 0.1

- also shot noise, photodiode noise

Frequency Domain View

Total electric field has a spectral content

$$f(t) = e(t)\sum \delta(t - t_i) \qquad \tilde{E}(\omega) = \tilde{e}(\omega)\sum_{k=1}^{N} e^{i\omega t_k}$$



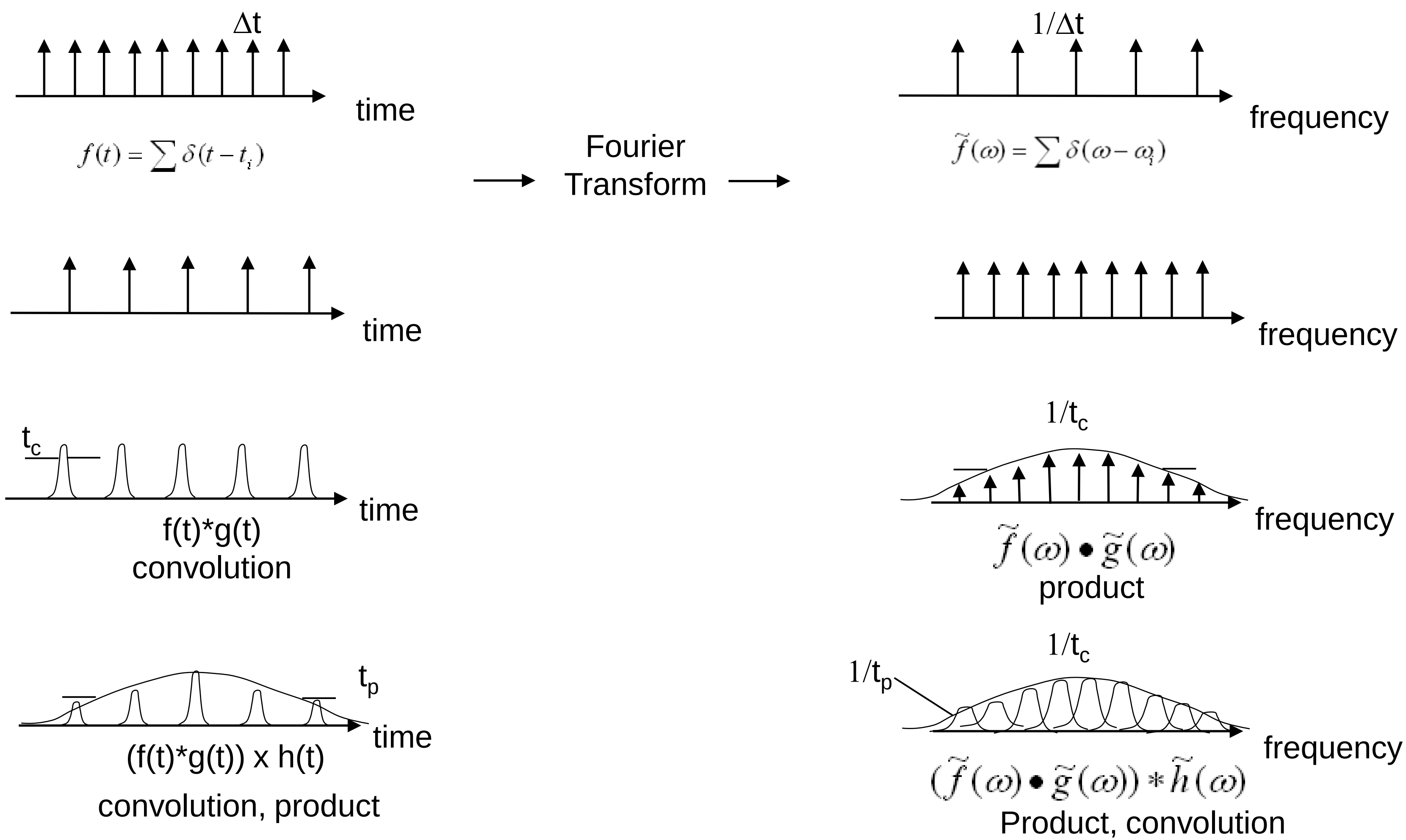
Phasors add up to 'spike' at frequencies ω

Shot-noise in wavepacket emission causes the spikes

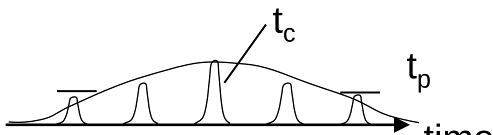
In the frequency domain still have shot-to-shot fluctuations

Width of each spike is inversely proportional to the bunch length

Frequency Domain: An Empirical Argument

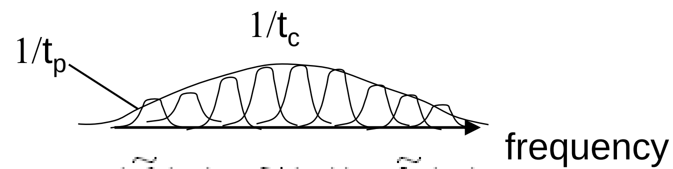


Empirical Argument (cont'd)



(f(t)*g(t)) x h(t)

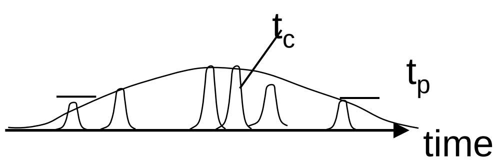
convolution, product

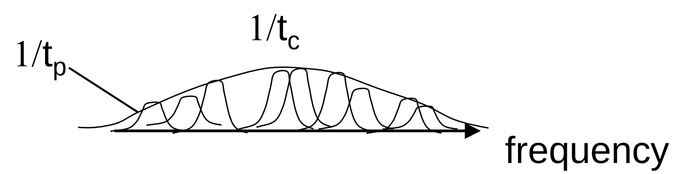


$$(\tilde{f}(\omega) \bullet \tilde{g}(\omega)) * \tilde{h}(\omega)$$

Product, convolution

Now make a leap of faith to random emission…





In both domains we have constructive and destructive interference

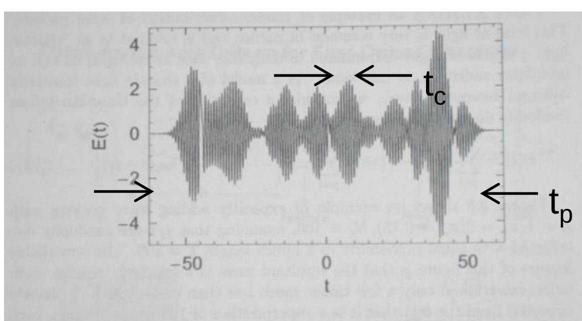
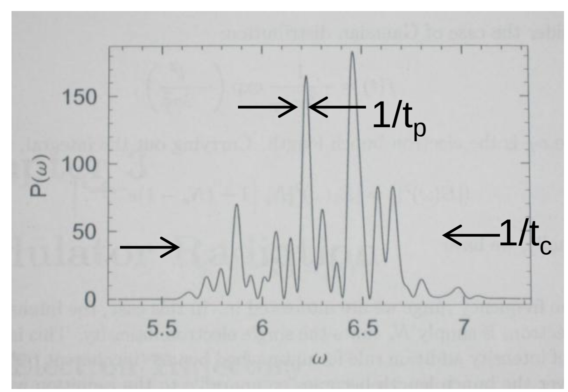


Figure 2.8: Electric field of Eq. (2.47) consists of M regular regions (M longitudinal modes).



Kim, Huang USPAS

Frequency Statistical Analysis

Experimentally can also analyze fluctuations in the frequency domain

Integrate the power spectrum of each pulse over frequency to find energy

$$\varepsilon = \int P(\omega)d\omega$$

The average energy is $\langle\varepsilon\rangle = \int \left(P(\omega)\right) d\omega$

And the variance is

$$\frac{\langle\Delta\varepsilon^2\rangle}{\langle\varepsilon\rangle^2} = \frac{1}{\langle\varepsilon\rangle^2}\iint \left\langle \left[P-\langle P\rangle\right]\cdot\left[P'-\langle P'\rangle\right]\right\rangle d\omega d\omega'$$

or

$$\frac{\langle\Delta\varepsilon^2\rangle}{\langle\varepsilon\rangle^2} = \frac{1}{\langle\varepsilon\rangle^2}\iint \left|\langle PP'\rangle - \langle P\rangle\langle P'\rangle\right| d\omega d\omega'$$

Need to compute <P> and 4th order field correlation <PP’> to evaluate variance

Frequency Domain Analysis (cont'd)

$$\frac{\langle \Delta\varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\left(\int T d\omega\right)^2} \iint TT' \left| f(\omega - \omega') \right|^2 d\omega d\omega'$$

filter

bunch transform

For a Gaussian filter and a Gaussian bunch

$$\frac{\langle \Delta\varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\sqrt{1 + 4\Delta\omega^2 \sigma_b^2}}$$

$$\frac{\langle \Delta\varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{\tau_c}{\tau_b}$$

(same as time-domain analysis)

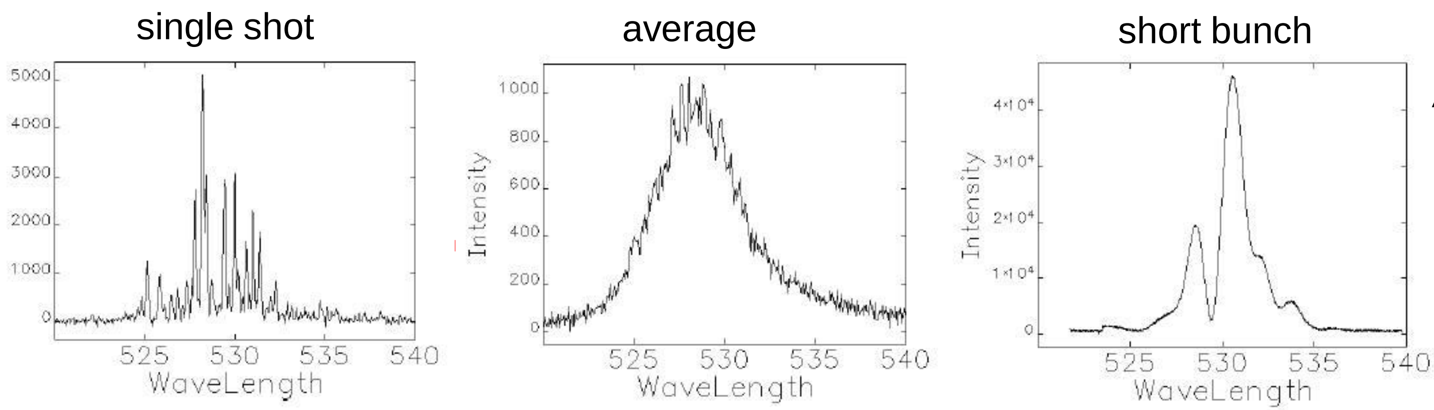
filter characteristic

bunch characteristic

Frequency Domain Experiments at APS

Use a spectrometer to observe spikes in single-shot spectrum

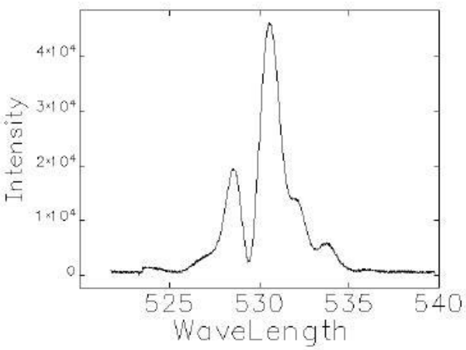
Sajaev, Argonne Nat'l Labs

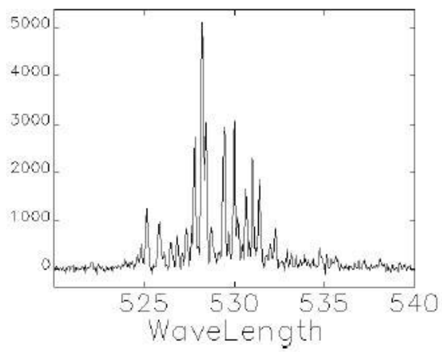


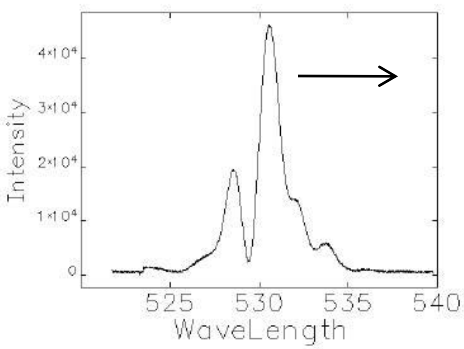
$$\Delta\omega \propto \frac{1}{\sigma_b}$$

Frequency Domain Experiments (cont'd)

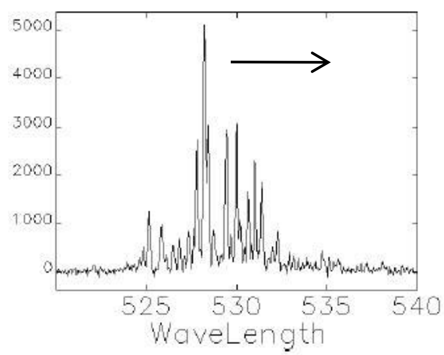
Bunch length proportional to Fourier transform of spectrum autocorrelation



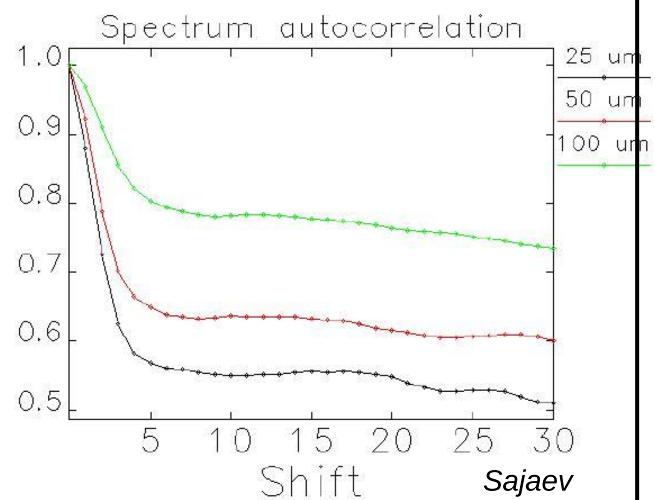




Large frequency correlation
Short bunch length



Small frequency correlation
long bunch length



Sajaev

Fluctuations in Interference Visibility Pattern

Landmark paper : Zolotorev and Stupakov (1996)

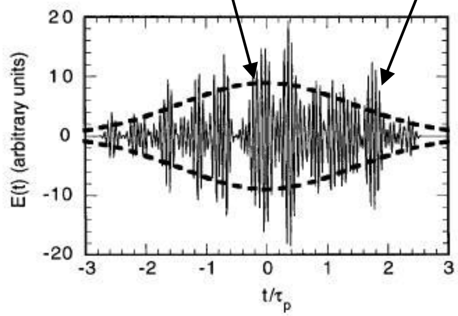
Measure fluctuations in the coherence function of the incoherent electric field

$$\Gamma(\tau) = \int E(t)E^*(t-\tau)dt$$

Utilizes a two-beam interferometer to measure $\Gamma(\tau)$

In simulation, the electric field is represented by $E(t) = A(t)e(t)$

pulse envelop   random process



USPAS Simulator – Pulse Energy Fluctuations

Each pulse of light is a superposition of randomly-phased 'wave packets'

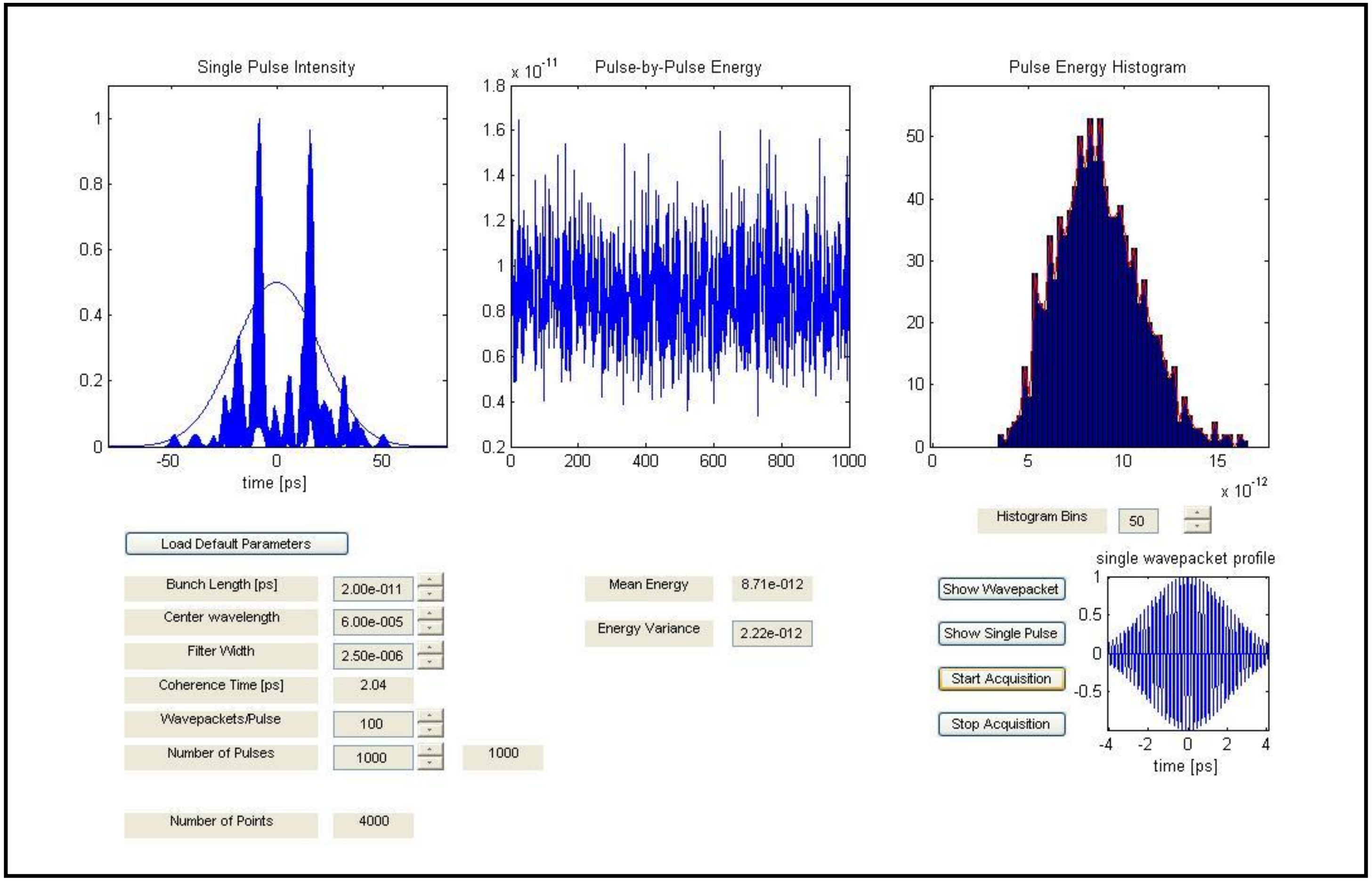
Simulator generates wave packets at random times t_k

Computes wavepacket superposition and resulting intensity E*E

Records statistics of shot-to-shot photon beam energy $U = \int E^* E dt$
to deduce pulse length

Very much like Sinnabale experiment and USPAS laboratory
but you 'see' effects not physically observable

Simulator for Pulse-Energy Fluctuations



USPAS Simulator (cont'd)

Part I: Photon beam properties
Calculate wavelength, energy, photon flux, etc.

Part II: Coherence properties
Coherence length with BP filter, etc

Part III: Time-base calculations for simulator code
Need simulate with 1um radiation

Part IV: The simulator interface

Part V: Wavepackets
Study as a function of wavelength, bandwidth, etc

Part VI: Study pulse-to-pulse statistics as a function of
bunch length, filter width, etc

Independent study

Summary Fluctuation Techniques

- Wavepacket emission is a statistically random process

- In the time domain

  use a filter to make coherence length~bunch length

  look for fluctuations in shot-to-shot intensity

- Fluctuations in interferometer visibility pattern

- In the frequency domain

  use a spectrometer to observe fluctuations in spectra

- Simulator for this afternoon