Fluctuation-Based Bunch Length Experiments

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- Motivation
- Time Domain Measurements
- Frequency-based Measurements
- Interferometer-based Measurements
- Introduction to USPAS Simulator
- Alan derived theoretical basis for using statistical fluctuations to measure pulse length

- Each electron is an independent ‘radiator’ with a random, granular distribution along the bunch (shot noise)

- Sometimes the phase of wave packets overlap, sometimes they don’t

- The *mean and variance* (moments) in the signal yields pulse length

- Measurements can be made in the time domain or frequency domain

- We will review some experiments and introduce the USPAS simulator
Sum electric field emission from individual electrons

\[ E(t) = \sum_{k=1}^{N} e(t - t_k) \]

where emission times \( t_k \) are random, Gaussian-distributed numbers

\[ f(t) = \frac{1}{\sqrt{2\pi\sigma_t}} e^{-t^2/2\sigma_t^2} \]

Each wavepacket \( e(t) \) is centered at random time \( t_k \)

Wavepackets superimpose to produce more or less field at time \( t \)

The electromagnetic field intensity is \( E^*E \)

Total pulse energy \( \int E^*Edt \) is therefore random in time.
an incoherent electric field is often not what we were lead to believe -
Fluctuations in Electric Field and Intensity

Each light pulse from the synchrotron has statistical structure

Field fluctuations

Intensity fluctuations $E^*E$

wave packets emitted from individual electrons statistically add or cancel

the correlation length corresponds to the wavepacket coherence length

$$T_{\text{pulse}}/\tau_{\text{coh}} = M \text{ (mode number)}$$
To increase correlation length band-limit the radiation

This increases the coherence length of the individual wave packets

\[ f = \frac{c}{\lambda} \]

\[ \Delta f = -\frac{\Delta \lambda c}{\lambda^2} \]

\[ \Delta t = \frac{1}{\Delta f} = \frac{\lambda^2}{c \Delta \lambda} \]

For 633nm light and a 1nm band pass filter

\[ \Delta t = \frac{\lambda^2}{c \Delta \lambda} = \frac{(633 \times 10^{-9})^2}{(3 \times 10^8) (1 \times 10^{-9})} = 1.3 \text{ ps} \]

For a 15ps bunch, the ‘mode number’ \( M \approx 15 \).
Energy Fluctuation Statistics

**Narrow band filter**

\[ E_1 = \int P_1(t)dt \]

\[ E_2 = \int P_2(t)dt \]

\[ E_3 = \int P_3(t)dt \]

\[ \ldots \]

\[ E_N = \int P_N(t)dt \]

**Wide band filter**

\[ E_1 = \int P_1(t)dt \]

\[ E_2 = \int P_2(t)dt \]

\[ E_3 = \int P_3(t)dt \]

\[ \ldots \]

\[ E_N = \int P_N(t)dt \]

\[ f(E) \]

**signal/noise = 2.4**

\[ \sigma = \text{Sigma (noise)} \]

\[ W = \text{Mean (signal)} \]
Intensity Fluctuation Derivation

Goodman, *Statistical Optics* Chapter 6

Average Value

\[ \overline{W} = \int_{-T}^{T} \overline{I}(t) \, dt \]

Variance

\[ \sigma_W^2 = E\left[\left(\int_{-T}^{T} I(t) \, dt\right)^2\right] - \overline{W}^2 \]

\[ = \int_{-T}^{T} \int_{-T}^{T} \overline{I}(t)\overline{I}(t') \, dt \, dt' - \overline{W}^2 \]

\[ \sigma_W^2 = \int_{-T}^{T} \int_{-T}^{T} \Gamma_I(t-t') \, dt \, dt' - \overline{W}^2 \]

where \( \Gamma \) is the autocorrelation function of \( I(t) \)

in terms of fields

\[ \Gamma_I(\tau) = E\left\{e(t)e^*(t)e(t+\tau)e^*(t+\tau)\right\} \]

‘fourth order correlation’
Intensity Fluctuations (cont'd)

\[ \sigma_W^2 = \int_{-T}^{T} \int \Gamma_I(t - t') dt dt' - \bar{W}^2 \]

\[ \Gamma_I(\tau) = E\{e(t)e^*(t)e(t + \tau)e^*(t + \tau)\} \quad \text{‘fourth order correlation’} \]

But from interferometry \[ \Gamma_I(\tau) = I^2 \cdot (1 + |\gamma(\tau)|^2) \]

Then \[ \sigma_W^2 = \bar{W}^2 \frac{1}{T} \int |\gamma(\tau)|^2 d\tau \]

\[ \frac{\bar{W}^2}{\sigma_W^2} = \left( \frac{1}{T} \int |\gamma(\tau)|^2 d\tau \right)^{-1} = M \quad \text{(same as before)} \]

\[ M = \frac{1}{\frac{1}{T} \int |\gamma(\tau)|^2 d\tau} = \frac{\tau_{\text{pulse}}}{\tau_{\text{coh}}} \quad \text{is the number of modes-per-pulse!} \]

\[ \longrightarrow \text{measurement of } W, \sigma_W \text{ with known } \tau_c \text{ yields } \tau_{\text{pulse}} \]
The ratio of Pulse Time to Coherence Time

\[ M = \frac{T_{\text{pulse}}}{T_{\text{coh}}} \]

Goodman, *Statistical Optics* Chapter 6

**Figure 6-1.** Plots of \( M \) versus \( T/\tau_c \), exact solutions for Gaussian, Lorentzian, and rectangular spectral profiles.
Relation between physics and measurement

\[
\frac{\sigma_W}{\langle W \rangle} = \frac{\text{variance}}{\text{mean}} = \left(\frac{\tau_{\text{coherence}}}{\tau_{\text{pulse}}}\right)^{1/2}
\]

number of modes: \( M \)
Modes-per-pulse: Experimental Evidence, U. Tokyo

\[
\frac{\sigma_W}{<W>} = \frac{\text{variance}}{\text{mean}} = \left(\frac{\tau_{\text{coherence}}}{\tau_{\text{pulse}}}\right)^{1/2}
\]
In the simplest form...

\[ \delta^2 = \frac{\sigma_w^2}{W^2} = \int_{-T}^{T} \int I(t)I'(t')dtdt' \]

fluctuations proportional to intensity correlation

For Gaussian statistics and band pass filter

\[ \delta^2 = \frac{1}{\sqrt{1 + 4\sigma_t^2 \sigma_\omega^2}} \]

Expanding

\[ \delta^2 \approx \frac{1}{2\sigma_t \sigma_\omega} \]

For

\[ \sigma_{coh} \approx \frac{1}{\sigma_\omega} \]

We get

\[ \delta^2 \approx \frac{\sigma_{coh}}{\sigma_t} = \frac{1}{M} \]

Make the coherence length long to reduce the number of modes M
Time-Domain Measurements at Berkeley

Intensity fluctuations, F. Sannibale, et al

\[ \int_{-T}^{T} I(t)I(t')dt dt' \]

LeCroy 3GHz BW, 20Gsample/s calculate average value of AB, CD 5000 samples @ 1.5MHz

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\[ \delta^2 = \frac{\sigma_w^2}{W^2} = \int_{-T}^{T} I(t)I(t')dt dt' \]
Calibration against Streak Camera

\[ \delta^2 = \sqrt{1 + \frac{\sigma_r}{\sigma_{r,c}}} \sqrt{1 + \frac{\sigma_x}{\sigma_{x,c}}} \sqrt{1 + \frac{\sigma_y}{\sigma_{y,c}}} \]

\( \sigma_{x/y,c} \) are transverse coherence sizes - related to transverse EM modes at 633nm - radiation process, including diffraction - ratios about 2 and 0.1

- also shot noise, photodiode noise

Figure 3: Examples of fluctuation and streak-camera bunch length measurements at the ALS for different beam parameters.
Total electric field has a spectral content

\[ f(t) = e(t) \sum \delta(t - t_i) \quad \tilde{E}(\omega) = \tilde{e}(\omega) \sum_{k=1}^{N} e^{i\omega_k} \]

Phasors add up to ‘spike’ at frequencies \( \omega \)

Shot-noise in wavepacket emission causes the spikes

In the frequency domain still have shot-to-shot fluctuations

Width of each spike is inversely proportional to the bunch length
Frequency Domain: An Empirical Argument

\[ f(t) = \sum \delta(t - t_i) \]

Fourier Transform

\[ \tilde{f}(\omega) = \sum \delta(\omega - \omega_i) \]

Convolution

\[ f(t) * g(t) \]

Product

\[ \tilde{f}(\omega) \cdot \tilde{g}(\omega) \]

Product, convolution

\[ (f(t) * g(t)) \times h(t) \]

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Shot-by-Shot Fluctuation Measurements
Empirical Argument (cont’d)

(f(t)*g(t)) x h(t)

convolution, product

\( \frac{1}{t_c} \cdot \frac{1}{t_p} \)

Now make a leap of faith to random emission...

In both domains we have constructive and destructive interference

Figure 2.8: Electric field of Eq. (2.47) consists of \( M \) regular regions (\( M \) longitudinal modes).

Kim, Huang USPAS
Frequency Statistical Analysis

Experimentally can also analyze fluctuations in the frequency domain.

Integrate the power spectrum of each pulse over frequency to find energy:

\[ \varepsilon = \int P(\omega) d\omega \]

The average energy is:

\[ \langle \varepsilon \rangle = \int (P(\omega)) d\omega \]

And the variance is:

\[ \frac{\langle \Delta \varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\langle \varepsilon \rangle^2} \iint \langle (P - \langle P \rangle) \cdot (P' - \langle P' \rangle) \rangle d\omega d\omega' \]

or

\[ \frac{\langle \Delta \varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\langle \varepsilon \rangle^2} \iint \langle PP' \rangle - \langle P \rangle \langle P' \rangle \rangle d\omega d\omega' \]

Need to compute \( \langle P \rangle \) and 4\(^{th}\) order field correlation \( \langle PP' \rangle \) to evaluate variance.
Frequency Domain Analysis (cont’d)

\[
\frac{\langle \Delta \varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\left(\int T d\omega\right)^2} \iint TT' \left| f(\omega - \omega') \right|^2 d\omega \, d\omega'
\]

filter

bunch transform

For a Gaussian filter and a Gaussian bunch

\[
\frac{\langle \Delta \varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\sqrt{1 + 4\Delta \omega^2 \sigma_b^2}} \quad \frac{\langle \Delta \varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{\tau_c}{\tau_b}
\]

(filter characteristic)

(bunch characteristic)

(same as time-domain analysis)
Use a spectrometer to observe spikes in single-shot spectrum

*Sajaev, Argonne Nat’l Labs*

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**Frequency Domain Experiments at APS**

Use a spectrometer to observe spikes in single-shot spectrum

*Sajaev, Argonne Nat’l Labs*
Bunch length proportional to Fourier transform of spectrum autocorrelation

Large frequency correlation
Short bunch length

Small frequency correlation
Long bunch length
Fluctuations in Interference Visibility Pattern

Landmark paper: Zolotorev and Stupakov (1996)

Measure fluctuations in the coherence function of the incoherent electric field

\[ \Gamma(\tau) = \int E(t)E^*(t - \tau)dt \]

Utilizes a two-beam interferometer to measure \( \Gamma(\tau) \)

In simulation, the electric field is represented by \( E(t) = A(t)e(t) \)

pulse envelop  random process
Each pulse of light is a superposition of randomly-phased ‘wave packets’

Simulator generates wave packets at random times $t_k$

Computes wavepacket superposition and resulting intensity $E^*E$

Records statistics of shot-to-shot photon beam energy $U = \int E^* Edt$
to deduce pulse length

Very much like Sinnabale experiment and USPAS laboratory
but you ‘see’ effects not physically observable
Simulator for Pulse-Energy Fluctuations

Single Pulse Intensity

Pulse-by-Pulse Energy

Pulse Energy Histogram

Load Default Parameters

Bunch Length [ps] 2.00e-011
Center wavelength 6.00e-005
Filter Width 2.50e-016
Coherence Time [ps] 2.04
Waves packets/Pulse 100
Number of Pulses 1000
Number of Points 4000

Mean Energy 6.71e-012
Energy Variance 2.25e-012

Histogram Bins 50

Show Wavepacket
Show Single Pulse
Start Acquisition
Stop Acquisition

single wavepacket profile
Part I: Photon beam properties
Calculate wavelength, energy, photon flux, etc.

Part II: Coherence properties
Coherence length with BP filter, etc

Part III: Time-base calculations for simulator code
Need simulate with 1um radiation

Part IV: The simulator interface

Part V: Wavepackets
Study as a function of wavelength, bandwidth, etc

Part VI: Study pulse-to-pulse statistics as a function of
bunch length, filter width, etc

Independent study
Summary Fluctuation Techniques

- Wavepacket emission is a statistically random process

- In the time domain
  use a filter to make coherence length~bunch length
  look for fluctuations in shot-to-shot intensity

- Fluctuations in interferometer visibility pattern

- In the frequency domain
  use a spectrometer to observe fluctuations in spectra

- Simulator for this afternoon