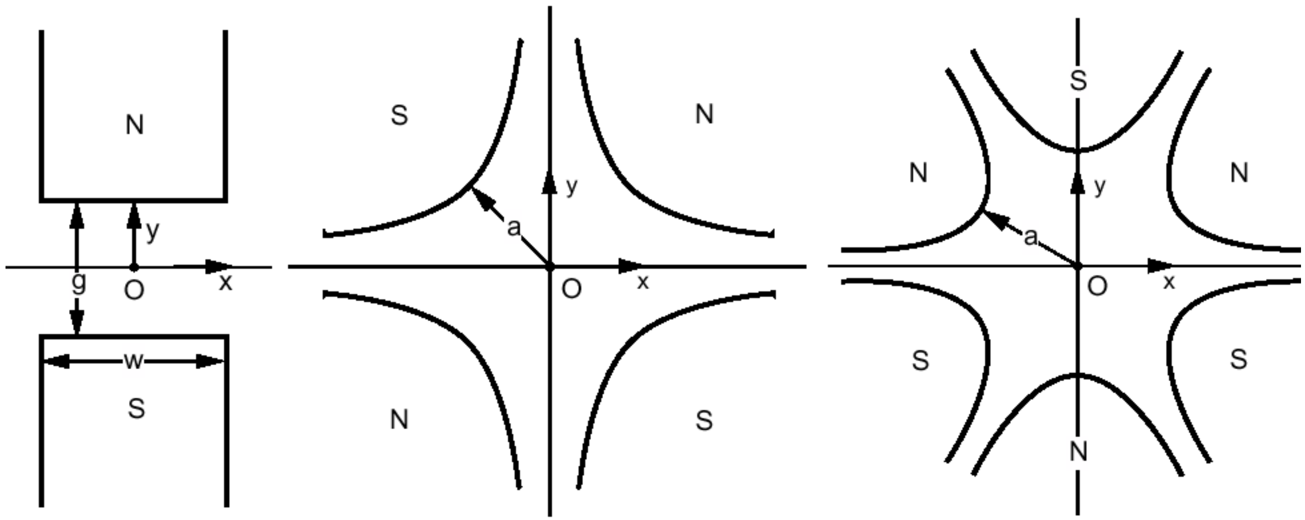
# Final Focus System
# Concepts in Linear Colliders

Mauro Pivi

SLAC

**US Particle Accelerator School**
January 17-21, 2011 in Hampton, Virginia

# What we use to handle the beam



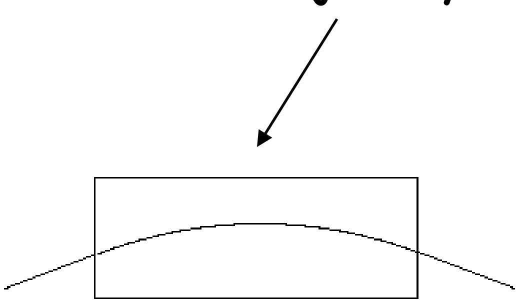

DIPOLE | QUADRUPOLE | SEXTUPOLE

Etc…

**Just bend the trajectory**

**Focus in one plane, defocus in another:**
$x' = x' + G\,x$
$y' = y' - G\,y$

**Second order effect:**
$x' = x' + S\,(x^2 - y^2)$
$y' = y' - S\,2xy$



**Here x is transverse coordinate, x' is angle**

# Introduction

At the risk of oversimplification the basic of the multipole elements can be identified as:

- The purpose of the dipole is to bend the central trajectory of the system (deh..!) and to generate the first-order momentum dispersion

- Quadrupole elements provide the first-order imaging

- Dipole and quadrupoles will also introduce higher-order aberrations. If these aberrations are second order, they may be eliminated or at least modified by the introduction of sextupole elements at appropriate locations.

# Introduction

In general,

- Dipoles introduce both second-order geometric and chromatic aberrations

- Quadrupoles do not generate second-order geometric aberrations but they have strong chromatic (energy dependent) aberrations.

- In regions of zero momentum dispersion, a sextupole will couple with and modify only geometric aberrations. However, in a region where dispersion is present, sextupoles will also couple with and modify chromatic aberrations.

# higher-order optics notation

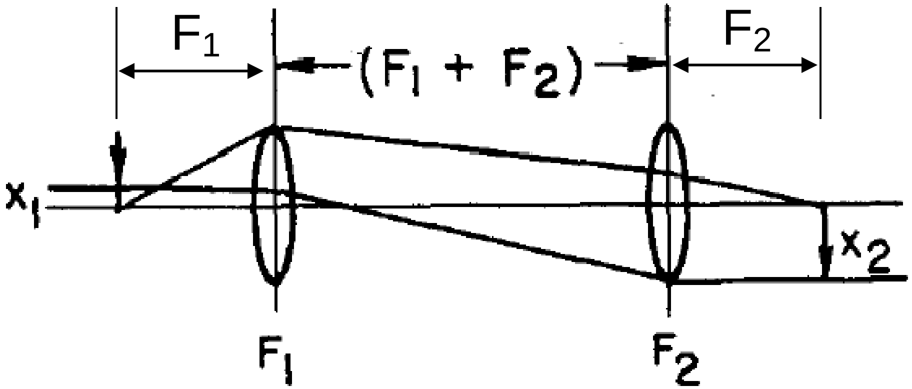
The first, second, third order optics is represented by R, T and U matrix elements

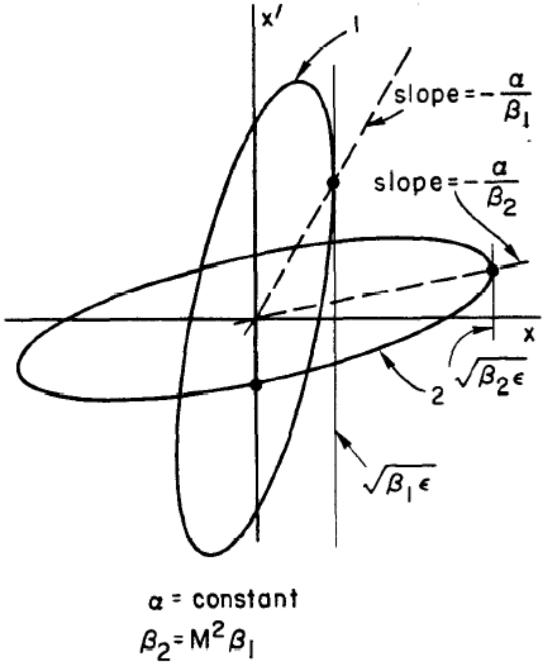
$$x_i = \sum_{j=1}^{6} R_{ij} x_j + \sum_{j,k=1}^{6} T_{ijk} x_j x_k + \sum_{j,k,l=1}^{6} U_{ijkl} x_j x_k x_l + \ldots \qquad \vec{x} = \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix}$$

All terms for which no subsctript is equal to 6 are refered to as geometric terms or geometric aberrations, since they depend only on the central momentum p0.

Any term $R_{ij}$, $T_{ijk}$ or $U_{ijkl}$ where one subscript is equal to 6 will be referred to as chromatic term or chromatic aberration, since the effect depends on the momentum deviation dp/p of the particle

# Telescopic system




The transfer matrix of the optical telescopic system shown for one plane is given by

$$R = \begin{pmatrix} -F_2/F_1 & 0 \\ 0 & -F_1/F_2 \end{pmatrix} = \begin{pmatrix} -M & 0 \\ 0 & -1/M \end{pmatrix}$$

Check point

Where M is the optical magnification defined *$M=F_2/F_1$, with $F_1$ and $F_2$ focusing lengths*

# Telescopic system

Recalling the optics function transformation between two location of the lattice

$$\begin{pmatrix}\beta_2\\ \alpha_2\\ \gamma_2\end{pmatrix} = \begin{pmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -R_{11}R_{21} & 1+2R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{pmatrix}\begin{pmatrix}\beta_1\\ \alpha_1\\ \gamma_1\end{pmatrix}$$

From telescopic system matrix

$$\begin{pmatrix}\beta_2\\ \alpha_2\\ \gamma_2\end{pmatrix} = \begin{pmatrix} M^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/M^2 \end{pmatrix}\begin{pmatrix}\beta_1\\ \alpha_1\\ \gamma_1\end{pmatrix}$$

Then the magnification in terms of optical functions is

$$\boxed{M = \sqrt{\frac{\beta_2}{\beta_1}}}$$

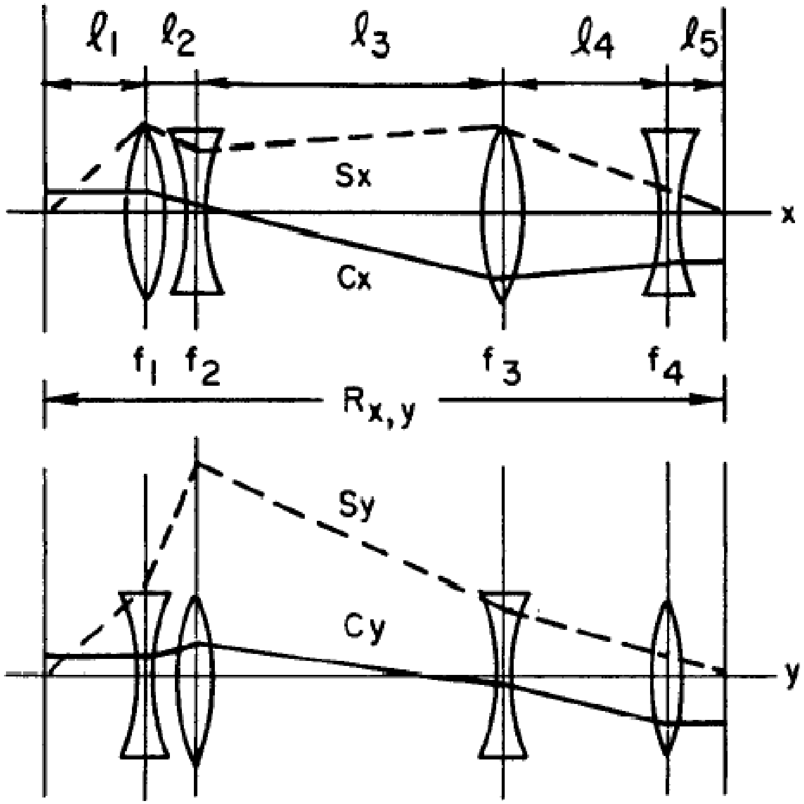
and $\alpha_2 = \alpha_1 = const$

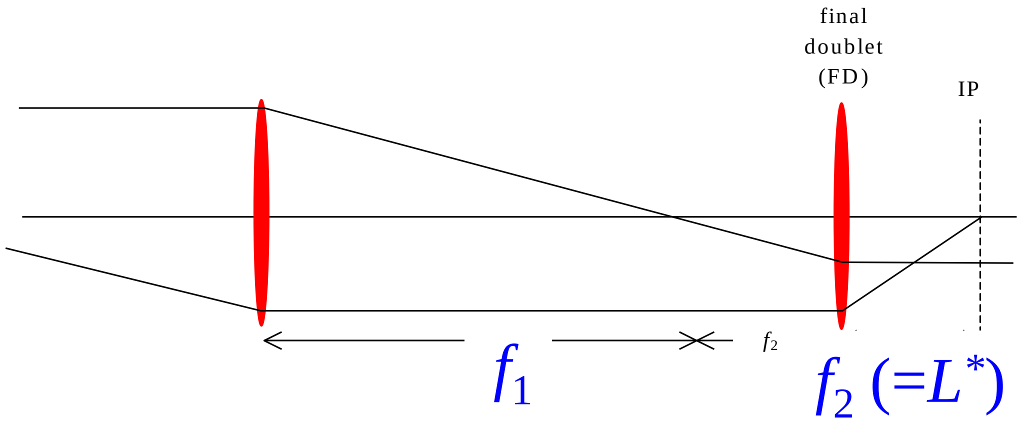
# Telescopic system

In practice, to achieve a telescopic system in both planes we need at least two quadrupoles to simulate each lens of the telescope, and the magnification may be different in each plane.

$$R = \begin{pmatrix} -M_x & 0 & 0 & 0 \\ 0 & -1/M_x & 0 & 0 \\ 0 & 0 & -M_y & 0 \\ 0 & 0 & 0 & -1/M_y \end{pmatrix}$$

$$R = \begin{pmatrix} -\sqrt{\frac{\beta_{2x}}{\beta_{1x}}} & 0 & 0 & 0 \\ 0 & -\sqrt{\frac{\beta_{1x}}{\beta_{2x}}} & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{\beta_{2y}}{\beta_{1y}}} & 0 \\ 0 & 0 & 0 & -\sqrt{\frac{\beta_{1y}}{\beta_{2y}}} \end{pmatrix}$$





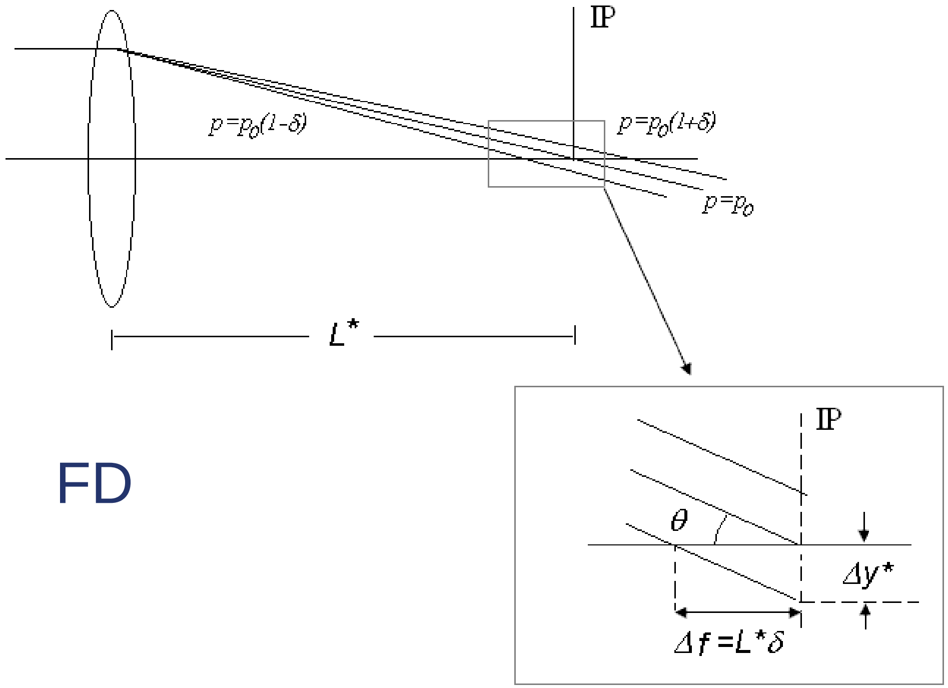

Use telescope optics to demagnify beam by factor $M = f_1/f_2$

*typically* $f_2 = L^*$

The final doublet FD requires magnets with very high quadrupole gradient in the range of ~250 Tesla/m → superconducting or permanent magnet technology.

# Final focus chromaticity

Strong FD lens has high degree of chromatic aberrations




FD

$$\Delta f \approx L^* \delta$$

$$\Delta y^* = \Delta f \,\theta = L^* \delta \theta$$

$$\frac{\Delta y^*_{rms}}{\sigma^*_y} = L^* \delta_{rms} \frac{\theta_{rms}}{\sigma^*_y}$$ using

$$\theta_{rms} = \sqrt{\varepsilon / \beta}$$ and $$\sigma = \sqrt{\varepsilon \beta}$$

$$\boxed{\frac{\Delta y^*_{rms}}{\sigma^*_y} = L^* \frac{\delta_{rms}}{\beta^*_y}}$$

Typically *L*~4m, δ~0.01, β~0.1mm* → $\Delta y^*_{rms} / \sigma^*_y \approx 400$

If uncorrected chromatic aberration of FD would completely dominate the IP spot size! Need compensation scheme.

# Chromaticity correction

Minimization of chromatic distortions: factors that influence the solutions to this problem:

1. a reduction in the momentum spread (not always feasible) would reduce the magnitude of the problem

2. The chromatic distortion of a FFS lattice is a function of the distance *L*.* The closer and stronger the lens the smaller is the distortion.

3. Sextupoles in combination with dipoles (provide dispersion) can be used to cancel chromaticity. Sextupoles introduced as pairs, separated by a –I transform do not generate second order geometric aberrations. However the dipoles introduce emittance growth and energy spread due to synchrotron radiation. Serious constraint.

FF design → Balance between these competing effects

# Chromatic corrections

• The magnetic induction of a quadrupole is a linear function of the variable x, y. A particle with momentum p will be affected differently than a particle with momentum p0. The corresponding strenghts of the quadrupole

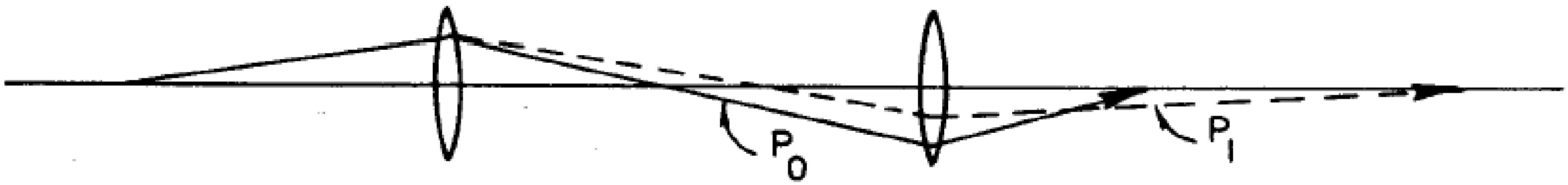
$$\frac{K_1(p)}{K_1(p_0)} = \frac{p_0}{p}$$

the focal strenght of the quadrupole decreases as the momentum increases.

Chromatic properties of a sextupole may be interpreted similarly.

Chromatic effects occur because particles with different momenta respond differently to a given magnetic field.

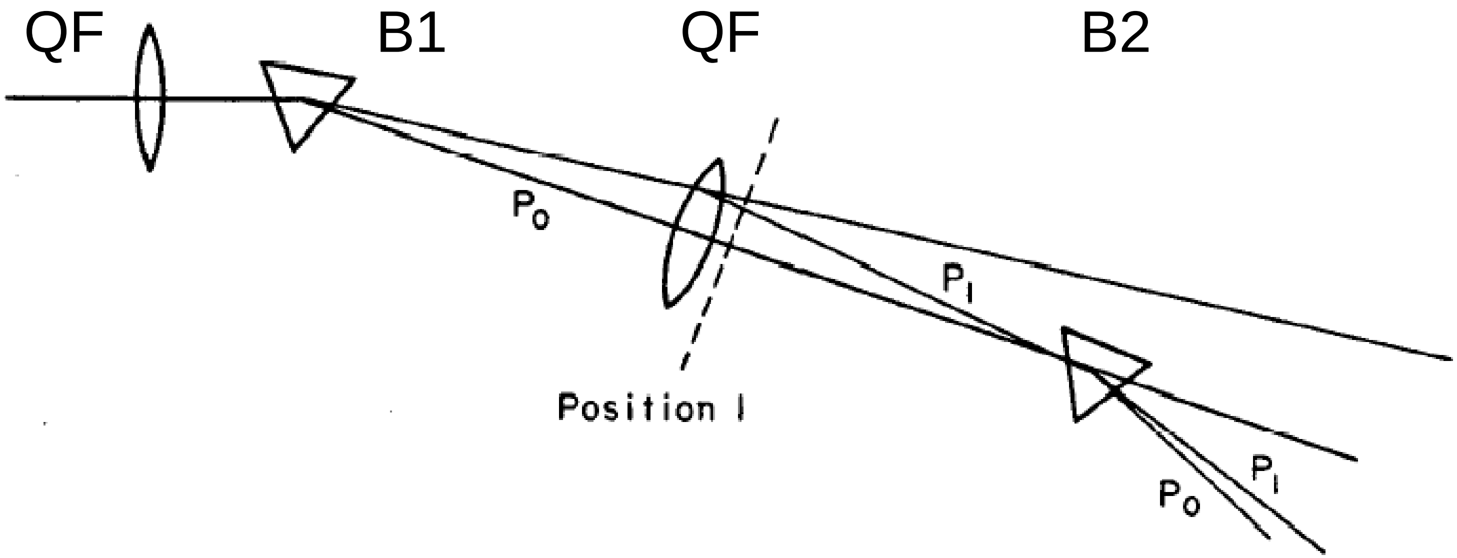
# Chromatic corrections concepts




particle with the same input coordinate but a different momentum p1 see the quadrupoles with strenghts than p0.

• to compensate for this chromatic difference a lattice can be designed where particles of greater momentum encounter an extra quadrupolar field to compensate for the increased momentum. This is achieved by the introduction sextupoles and dipoles into the lattice structures.

# Chromatic corrections concepts




This lattice has the potential of chromatic corrections. While a particle p0 follows the central trajectory, the particle p1 with δ🕒0 will follow the trajectory defined by the function $\delta d_x(s)$. The function is nonzero after the first dipole. At position 1, p1 encountered slightly different quadrupolar strengths than p0. Let's arrange a sextupole at position 1, which is not affecting p0. Particle p1 will experience a gradient proportional to its displacement, therefore proportional to δ.

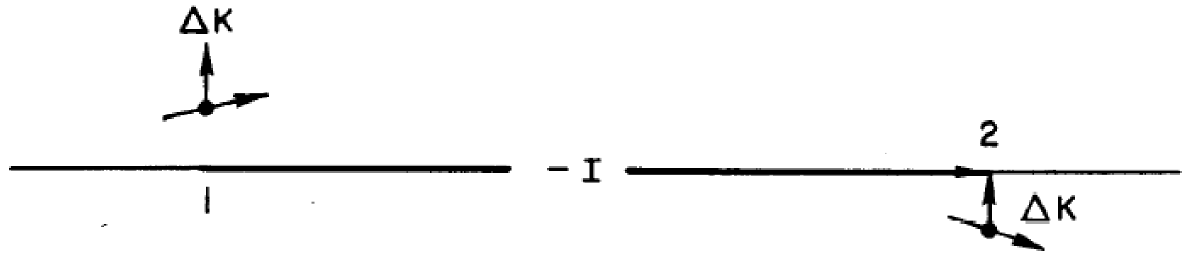
# Chromatic corrections concepts

If proper sextupole strength is chosen, the extra gradient exactly compensates the difference in gradient experienced by particles with different momenta in the preceding quadrupole.

- However, in this process the sextupoles will in general introduce <u>geometric</u> distortions.

A procedure to eliminate chromatic aberrations without introducing second-order geometric aberrations is the following.

# Module for Sextupolar Chromatic corrections general concepts




Consider two FODO cells tuned with a phase advance $\mu_{x,y}$= 90 deg for each cell. Such beam line may be referred to as a –I telescopic transformer, since the transfer matrix in both x and y planes is

$$R_{x,y} = -I = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Check point

A particle at entrance position 1 will emerge at exit position 2 with coordinates

$$x_2 = -x_1 \quad \text{and} \quad x_2' = -x_1'$$

If we place at position 1 a thin magnetic element that produces an angle kick ΔK, the particle will exit at position 2 with

$$x_2 = -x_1 \quad \text{and} \quad x_2' = -x_1' - \Delta K$$

# Module for Sextupolar Chromatic corrections

If we arrange a second magnetic element at position 2 producing another equal angle kick ΔK, the exit coordinates are the same as they were without kick ..

Thus, when for mono-energetic particles with momentum p0 are submitted with equal angle kicks at entrance of a –I transformer, there is no visible effect outside the –I transformer.

Let us arrange two identical elements at entrance and exit of –I transformer:

1) <u>Dipoles:</u> are even-order elements, angle kick is an even function of lateral displacement (assume constant function). Two identical dipole magnet will give no net angular deflection to a particle p0 outside the –I transformer.

# Module for Sextupolar Chromatic corrections

2) Quadrupoles: odd-order elements. The angular kick is an odd function of the lateral position x (angle kick proportional to x). Thus, two identical quadrupoles of opposite polarity will have no net effect for a particle p0 outside the –I transformer. [try it]

Check point

3) Sextupoles: sextupoles are even-order elements, the angular kick is proportional to $x^2$. Thus, pairs of equal strength sextupoles will have no effect outside the –I transformer.

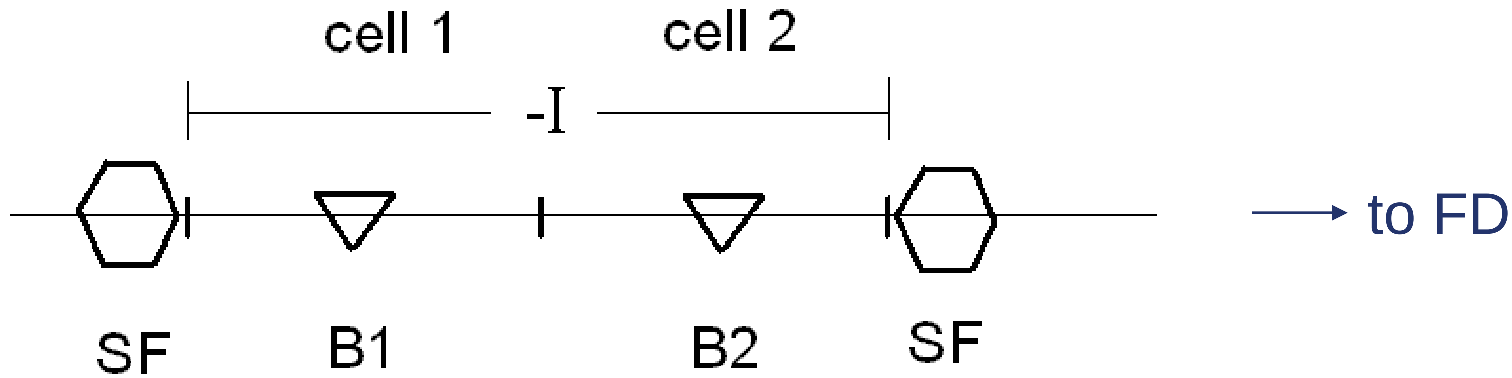
Thus, for the cancellation to be effective, pairs of elements at entrance and exit of –I transformer will have:

odd-order elements → opposite polarity

even-order elements → same polarity

Why do we need such a system?

# Sextupolar Chromatic corrections classical scheme



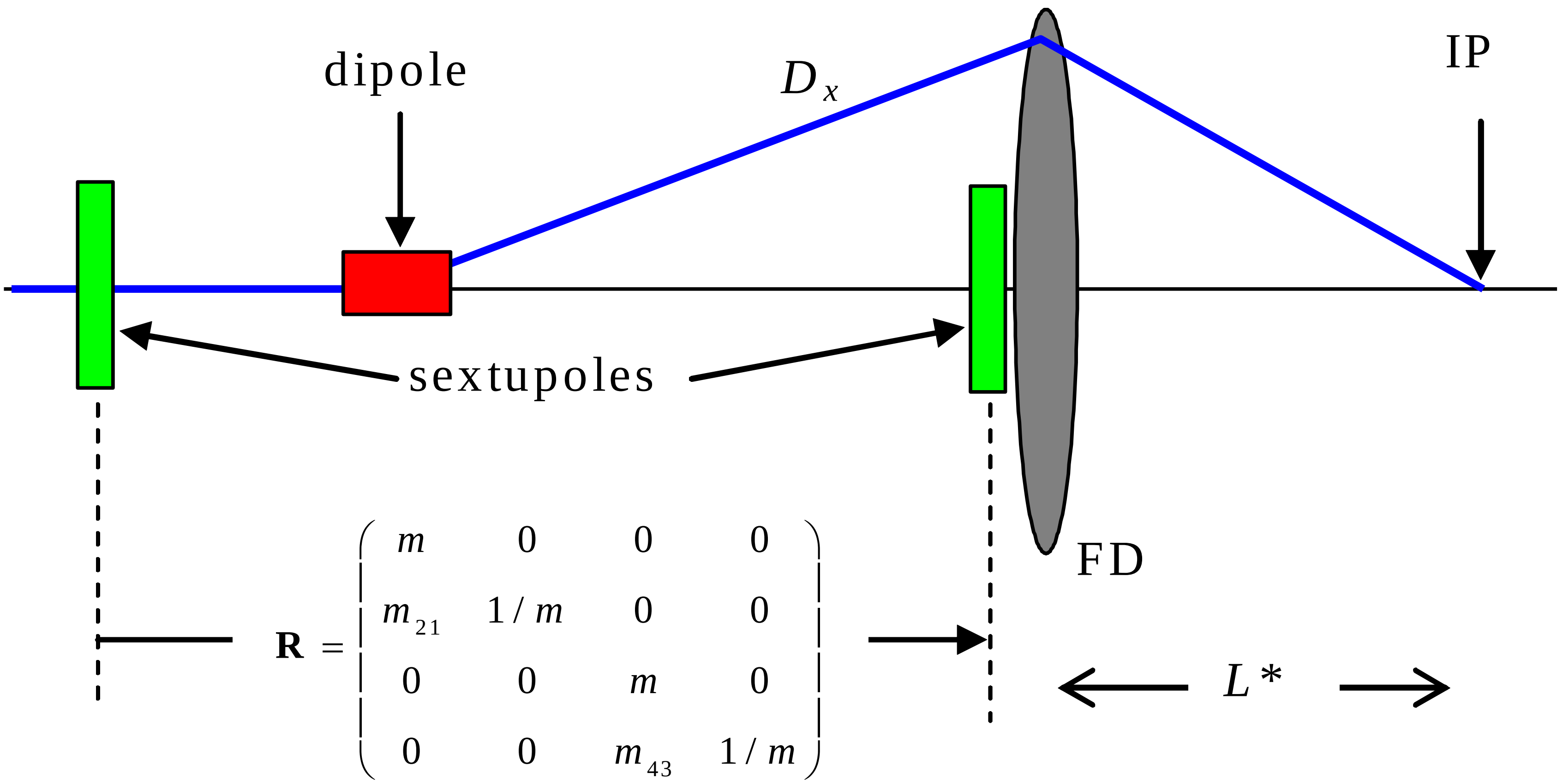

Principle of chromatic corrections:

1. Sextupoles used to correct FD chromaticity, but introduce geometric aberrations

2. Place then two sextupoles of equal strength at entrance/exit of –I transformer, and dipoles inserted in each cell of the transformer. From previous considerations, sextupoles do not introduce geometric aberrations. Dipoles generate dispersion and ensure coupling between sextupoles strength and chromatic behavior of particles.

At least one chromatic correction per plane, sometimes two or more..

Nevertheless, M🕒-I for off energy particles.




P.Raimondi, A.Seryi, originally NLC FF and now adopted by all LC designs.

# Chromaticity correction

Solution to compensate FD chromaticity is to use strong sextupole magnets in a dispersive region of lattice. Horizontal dispersion is generated at the FD location by weak dipoles judiciously placed to cause dispersion to be zero at the IP. A sextupole is placed near the FD. The non-linear kicks from the thin lens sextupole of integrated strength $K_S/2$:

$$\Delta x' = -\frac{1}{2} K_S (x^2 + 2x\eta\delta + \eta^2\delta^2 - y^2)$$

$$\Delta y' = K_S xy + K_S \eta\delta y$$

For a thin lens FD:

$$\Delta x' = \frac{x\delta}{L^*} + \frac{\eta\delta^2}{L^*}$$

$$\Delta y' = -\frac{y\delta}{L^*}$$

Terms in $\delta$ are the first order chromatic kicks and $\delta^2$ is the second order dispersion term

Choosing $\boxed{\frac{1}{L^*} - \eta K_S = 0}$ the first order chromaticity kick vanish

# Chromaticity correction

Note $\eta K_s = \dfrac{1}{L^*}$

Important: to compensate chromaticity, one can choose to run with higher dispersion (higher dipole strength) or higher sextupole strength. Compromise between synchrotron radiation generated in bends and geometric aberrations generated by sextupoles.

Unfortunately, still residual non-linear terms are left which cause aberrations if uncorrected

$$\Delta x' = -\frac{1}{2\eta L^*}(x^2 - y^2) + \frac{\eta}{2L^*}\delta^2$$

$$\Delta y' = \frac{1}{\eta L^*} x y$$

second order dispersion term

Pure geometric terms ($\delta$ independent)

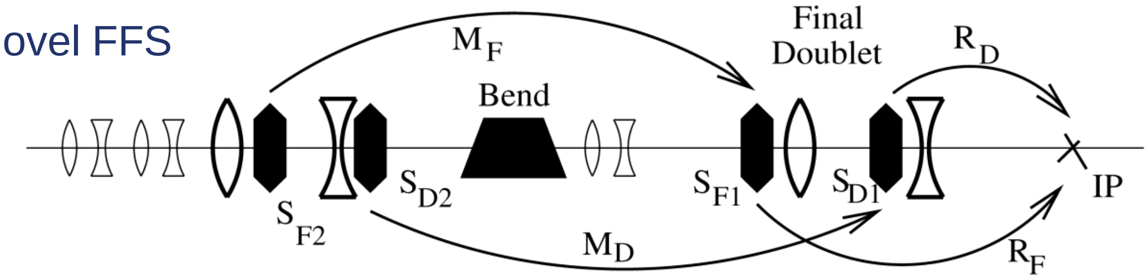
# Chromaticity correction

Thus, the residual non-linear second order dispersion term can be cancelled either by producing X chromaticity in the upstream $\beta$ matching section, so the sextupoles run stronger and cancel the second order dispersion as well (see A. Seryi lecture), or allowing a small dispersion at a sextupole or both.

# Chromaticity correction

The pure geometric (δ independent) term is cancelled by placing sextupole/s upstream –I transformer at the same phase as the FD.

Novel FFS




with, for example:

$$M_F = \begin{pmatrix} F & 0 & 0 & 0 \\ F_{21} & 1/F & 0 & 0 \\ 0 & 0 & F & 0 \\ 0 & 0 & F_{43} & 1/F \end{pmatrix}$$

Improvement: $M_F$ and $M_D$ transformer shown between sextupoles allows better correction of higher order aberrations (with respect to a -I transformer). Also typically a –I transformer suffers of the fact that M≠-I for off energy particles.

# Elements of LC Final Focus System Summary

In Linear Colliders, nanometer size beams are obtained by:

- Final Quadrupole Doublet telescopic system
  - **FD Collateral effects: generate strong chromatic aberrations**

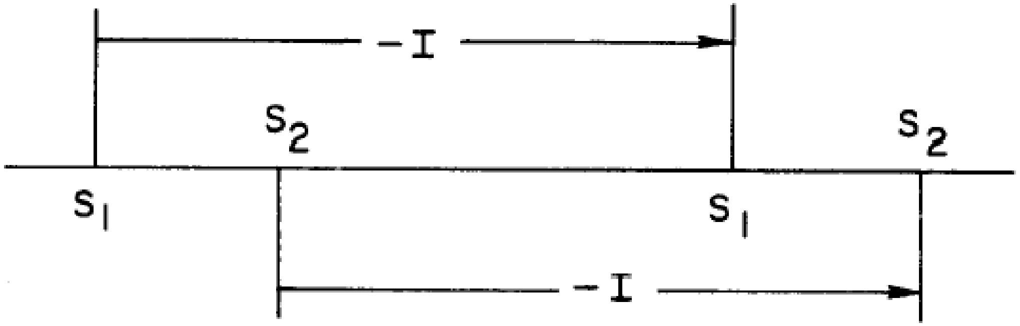
- Sextupoles to correct FD chromatic aberrations
  - **SEXT collateral effects: generate geometric aberrations**

- Sextupoles located at beginning of -I transformer (or equivalent transform) then correct geometric aberrations

- Dipoles to supply dispersion for Sextupoles correction
  - **BEND collateral effects: generate synchrotron radiation**

# Interlaced pairs

Ideally, from second-order geometric aberrations point of view, is to assemble –I transformers that do not interfere between x, y planes (separated in space). This often requires prohibitively long and expensive sections.




Consider interlaced sextupole pairs. A particle arrive at first sextupole S1 with displacement x1. As it gets to the first sextupole of pair S2, its motion is perturbed and particle reaches second sextupole S1 with a displacement not equal to –x1. Not exact cancellation from the second sextupole S1. However, since the disturbance by sextupole S2 is of order two, the uncorrected geometric aberrations of the pair S1 are then of order three and four → fine.