Chapter 6 Beam Dynamics III Defocusing Effects

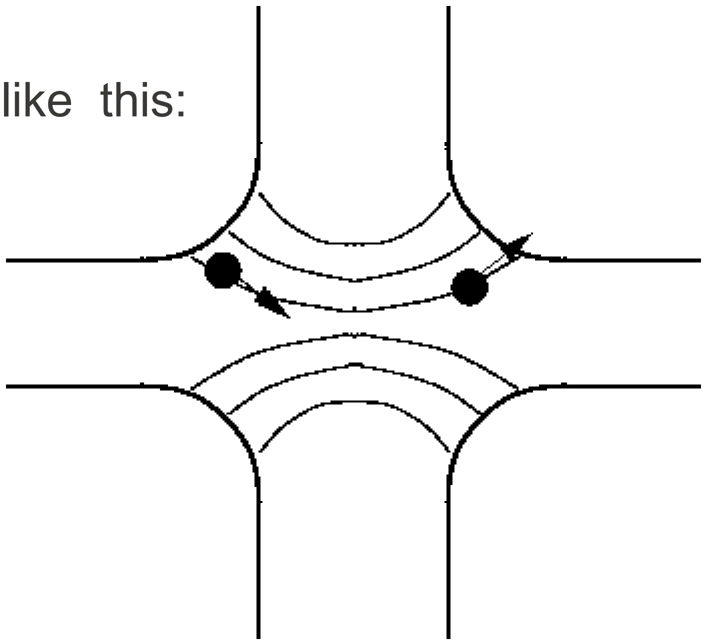
Transverse Defocusing

Space Charge

Trace3D Simulations

Transverse Defocusing in the Acceleration Gap

The electric field pattern in an accelerating gap looks like this:



We know that for longitudinal focusing that the bunch enters the gap when the field is still rising, so the late particles get an extra kick to catch up.

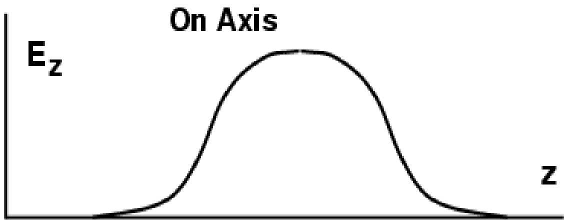
Ions get kicked along the electric field lines, so they get a small kick inward upon entering the gap, but a larger kick outward as they leave because the field is rising to produce longitudinal focusing.

This is a fundamental relationship. Focusing in all three planes (x,y,z) simultaneously is barred by the fact that in the absence of free charges, the field cannot have a local minimum (Earnshaw's theorem): at best, it can be a saddle-shape.

Radial Defocusing

The radial defocusing in the accelerating gap is very significant, particularly at the low-energy entrance region of a linac. It, along with the technologically feasible focusing strength, determines the injection energy of a linac.

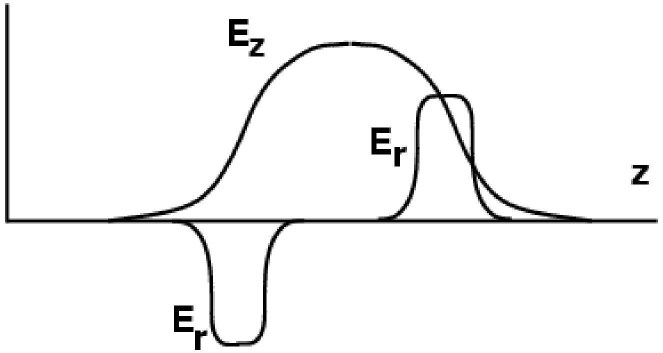
The profile of the accelerating field E_z in the gap is



There is also a radial component to the field, E_r

To find E_r in the gap, expand Gauss' law (with no charges) in cylindrical coordinates.

$$\bar{\nabla}\cdot\bar{E}=0 \quad\rightarrow\quad \frac{1}{r}\frac{d(rE_r)}{dr}+\frac{dE_z}{dz}=0 \quad\rightarrow\quad E_r=-\frac{r}{2}\frac{dE_z}{dz}$$



At the ends of the gap, a radial field exists, that is proportional to *r* and to E_z.

E_r increases in time as the ion crosses the gap, and the polarity is such to defocus the beam.

Radial Momentum Impulse in the Gap

We will calculate the equivalent of a thin defocusing lens for the entire gap. The defocusing strength will depend on the phase of the particle crossing gap with respect to the phase of the RF field.

Let the gap field have the form

$$E_{z\,gap}(z,t) = E_z(z)\cos(\omega t+\phi) = E_z(z)\cos(kz+\phi)$$

$$E_r = -\frac{r}{2}\frac{dE_z}{dz}, \qquad k=\frac{2\pi}{\beta\lambda}$$

The transverse momentum impulse is $\Delta p_r = \int e E_r dt$

Which produces an angular kick r', equivalent to a defocusing lens of focal length f_{gap}

$$\frac{\Delta r'}{r} = \frac{1}{f_{gap}} = -\pi\left(\frac{q e E_0}{mc^2}\right)\frac{T}{\beta^2}\sin\phi$$

This is proportional to $1/\beta^2$, and strong at the beginning of the linac.

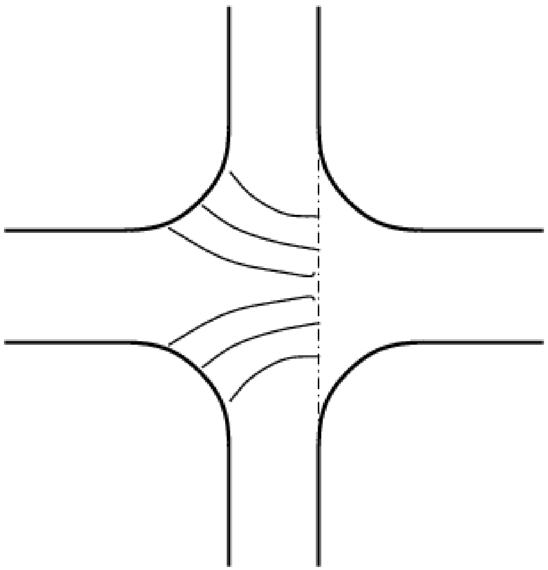
A unitless variable Δ_{gap} is usually defined as $\Delta_{gap} = -\frac{\Delta r'}{r}L_{cell} = -\frac{\Delta r'}{r}\beta\lambda$

$$\Delta_{gap} = \pi\left(\frac{q e E_0}{mc^2}\right)\frac{T\lambda}{\beta}\sin\phi<0 \quad as \quad \phi<0$$

Grid Focusing

Earnshaw's theorem is based upon Laplace's equation with no free charges. Charges may be present in the form of a grid or foil in the gap, altering the field lines.

The field lines do not diverge at the exit of the gap, and the converging lines at the entrance focus the beam.



This scheme was used in early linacs before the invention of strong focusing.

What is wrong with this scheme?

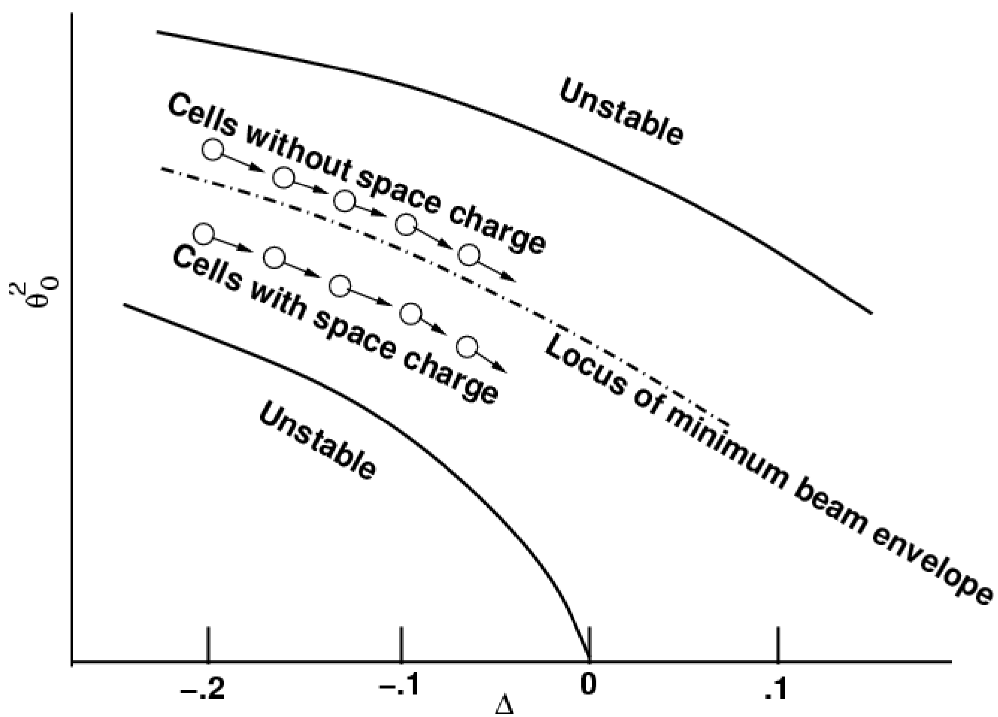
The grid intercepts a fraction of the beam. If 97% of the beam gets past the grids, after 100 drift tubes, only 5% of the beam is left. In addition, the small grid dimensions generate high local fields around them which promote sparking, which may vaporize the grid itself.

Focusing Stability Plot

We have previously seen how an alternating gradient focusing sequence can be represented by an averaged external restoring force.

We have just seen how the gap defocusing field can also be represented by a thin lens.

These two representations are combined in a **stability plot** diagram, whose axes are the smoothed external focusing force, represented by and the defocusing lens strength, represented by Δ_{gap}



The parameter $\theta_0^2 \sim B'_{quadrupole}$ is proportional to the quadrupole strength.

There is a region of stability between the phase advance of 0 and 180 degrees per focusing period.

Gap defocusing requires an external restoring force to maintain stability. Gap defocusing is maximum at the low-energy end of the linac.

Space charge forces also act to defocus the beam.

Using the Focusing Stability Plot

This aid can be useful in choosing the injection parameters for a drift tube linac.

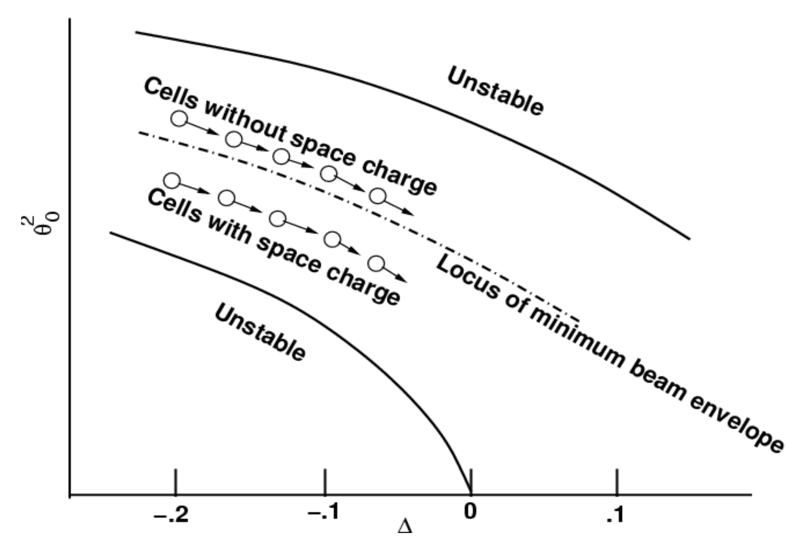
The focusing strength is constrained by the length of the quadrupole that is included in the first drift tube.

The defocusing parameter is determined by the input energy and the average axial acceleration field E_0, as well as the choice of stable phase ϕ_s.

Linac design codes, such as **PARMILA**, allow the user to enter the average focusing strength q_0^2 directly, and compute the quadrupole gradient, and will calculate the gap defocusing force as a function of the cell parameters. The plot permits an informed initial guess of a set of parameters.

The first cell in the linac will be the most critical. The cells will trace out a locus of points in the plot, all of which must be in the stable regime.

Space charge defocusing is not represented explicitly, but has the effect of further reducing the external focusing force.



Alternating Phase Focusing Structure

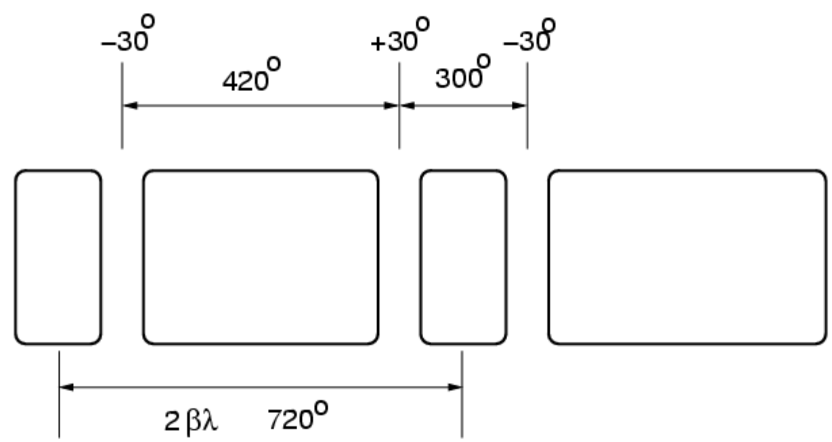
It is possible to eliminate the focusing quadrupoles altogether?

We have already seen that a series of transverse focusing and defocusing lenses can have an overall focusing effect.

The same thing may be applied to longitudinal beam dynamics. We saw that a negative stable phase (ϕ_s < 0) resulting in longitudinal focusing along with transverse defocusing. A positive stable phase (ϕ_s > 0) will cause longitudinal defocusing but transverse focusing.

By alternating the stable phase between negative and positive values, net focusing may be achieved in all phase planes. This technique has been successfully tried in linacs, but the acceptance is small and electric field gradients are high.

One design alternates ϕ_s between +30 and -30 degrees, and a period length of $2\beta\lambda$.



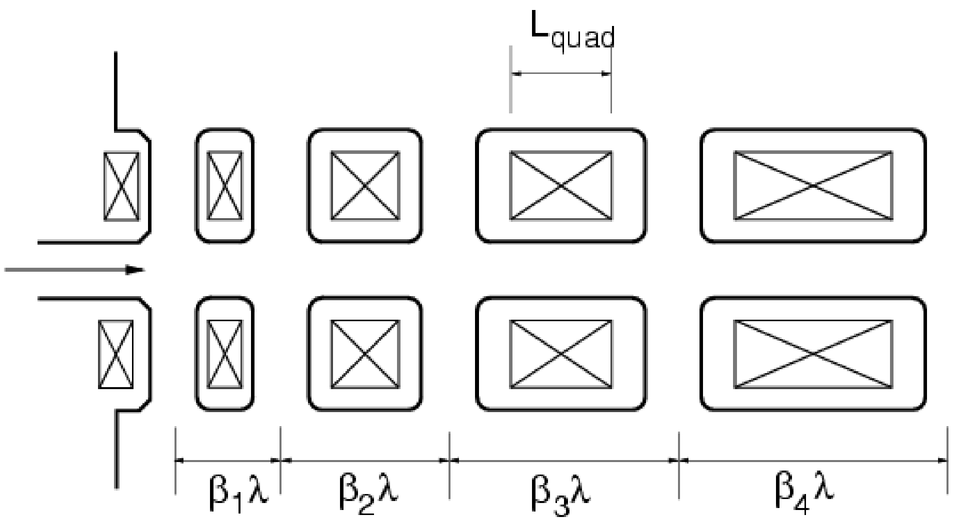
Quadrupoles in Drift Tubes

Drift tube sequence at the low-energy end of an Alvarez linac

The gap-to-gap L_{cell} spacing is $b_i l$.

Typical geometry:

$L_{gap} = ¼\, L_{cell}$
$L_{DT} = ¾\, L_{cell}$
$L_{quad} = ½\, L_{DT}$



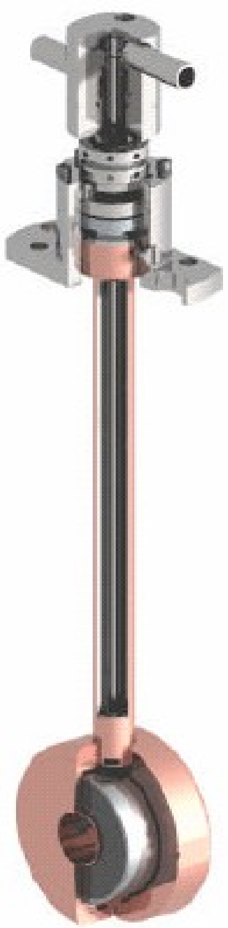
Usually, the integrated quadrupole strength $B'L_{quad}$ is nearly constant throughout the structure. In later drift tubes, the length of the quadrupole is held constant, the gradient B' is constant, which simplifies the construction and the powering of the quadrupoles.

The critical area is at the beginning of the linac, where the quadrupole length is necessarily short, and the gradient B' must be very high. This is usually the defining factor for the selection of the particle energy at the entrance of a quadrupole-focused linac.

Drift Tube Quads Engineering Considerations

The quad gradient $B' = dB/dr = B/a$, where B is the pole-tip field and a is the distance of the (hyperbolic) pole tip from the axis.

Among the constraints and trades involved are:



The length of the quad must fit within the drift tube

The maximum pole tip field is about 2 Tesla

The outer radius of the quad must be smaller than the drift tube outer radius

Enough cooling must be provided through the stem

The current leads must also fit into the stem

The drift tube bore usually has a sleeve, which increases the pole tip radius a

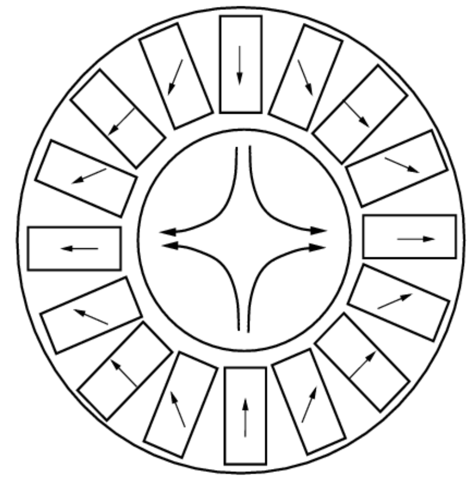
The wire insulation must be radiation resistant

The quad magnetic center must be precisely aligned to an external fiducial

The drift tube assembly (welding) must not destroy the quadrupole

Permanent Magnet Quadrupoles

Recent developments in permanent magnet materials and understanding how they may be configured to give excellent field purity (K. Halbach, LBNL) have led to their introduction to many accelerator-related applications.



Bars of magnetized material with the proper orientation of the field are arranged to produce a quadrupole field within the bore of the magnet.

Advantages:

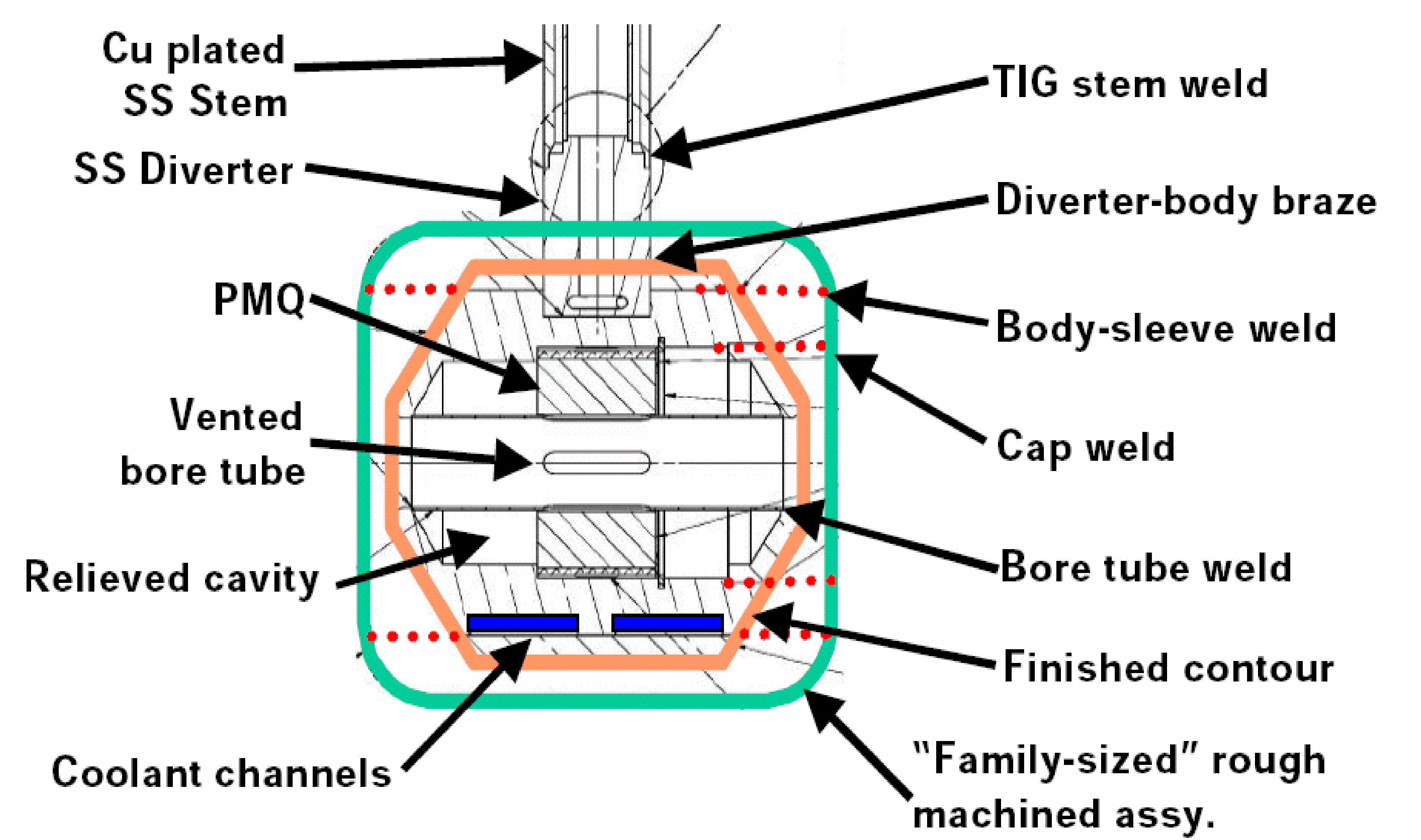
- High field strengths possible
- No electrical excitation needed

Disadvantages:

- Not tuneable
- Difficult to manufacture (e-beam welding)
- Magnetic center may be off-axis
- Radiation hardness unknown

Components of a Drift Tube with Permanent Magnet

This is one of the drift tubes for the SNS drift-tube linac. The outer dimensions were machined from the green to the orange profile after assembly.



Collective Effects (Space Charge)

The design of modern linacs must include the collective effects of the fields of the ions in the bunch on each other (space charge).

We will ignore the interaction of the beam on the accelerating cavity (wakefields, etc), as they are less important in low-energy hadron accelerators than they are in electron linacs, and is not a subject of this course.

We will introduce many approximations as we cannot deal with the fields within the bunch in a microscopic level.

We will introduce the concept of the **macroparticle**, reducing the computational effort by substituting a few particles with large mass and charge representing large ensembles of individual hadrons in the bunch. The charge-to-mass for the macroparticle is the same as for the individual ion being represented in the bunch.

We will also introduce the concept of the **smooth approximation**, where the charges in the bunch produce an average field within the bunch, which adds linearly to external guiding fields.

Smooth Approximation

Here, we will assume that the distribution of particles in the bunch give rise to linear defocusing fields. This implies that there are no non-linear effects and therefore the beam emittance is constant. The space charge force has an equal footing with the linear external restoring forces.

Furthermore, we can collect the external restoring force, usually provided by a FOD lattice, into an averaged external restoring force. The beam envelope flutter is then ignored.

The beam emittance can be interpreted as an outward pressure, along with the space charge forces, that act against the linear external restoring forces.

This approach involves many approximations, but it is useful in investigating the parameter space of beam space charge, emittance, and strength of external restoring force with a semi-analytic expression.

This type of approach is used, with much more sophistication, in programs such as Trace3-D, which use the actual focusing lattice instead of an averaged restoring force.

Macroparticle Approach

A bunch may contain $N=10^{23}$ particles, and calculating all the interactions between the particles would require almost N^2 operations. The quantity q/A occurs in the calculations, q = charge on a particle and A = the mass in AMU.

Macroparticles with much larger mass A and charge q with the same q/A allows one to use a much smaller set of particles in the bunch. The number of macroparticles is chosen so the same current is carried by the beam consistent with the statistical noise acceptable.

An older approach in calculating the interaction of each particle in the bunch with all the others is to directly calculate the force term, which is numerically intensive. Instead, the PIC (Particle in Cell) approach superimposes a 3-D grid over the bunch and deposits charge on the nodes proportional to the particle density in the region of each node. The fields from this charge distribution are easily calculated, and then act on each charge, changing its trajectory. The boundary conditions usually do not include the beam pipe (open boundary).

Recent improvement in codes include intelligent gridding of the problem and application of realistic boundary conditions. Most of the modern codes use this approach.

Envelope Equation with Space Charge (Smooth Approximation)

We will start off with Newton's law $F = m\ddot{x}$ and change to spatial derivative. $\frac{\partial}{\partial t} = \beta c \frac{\partial}{\partial s}$

Separating out three separate force terms, we get the second-order ODE:

$$x'' - k(s) + F_e + F_s = 0$$

x'' is the spatial derivative of the beam envelope
$k(s)$ represents the external restoring force
F_e represents the outward emittance pressure of the beam
F_x represents the outward space charge force

Wangler (p.273-4) gives the derivation of the second moment (width) of the envelope:

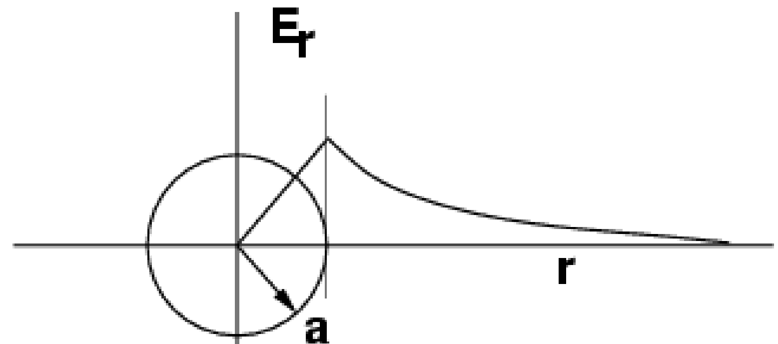
$$a'' + k(s)a - \frac{\epsilon^2}{a^3} + \frac{\overline{xF_s}}{a} = 0$$

where $a = \sigma_x = \sqrt{\overline{x}^2}$

$\sqrt{k(s)} = \frac{\Delta\mu}{L_p}$ is the focusing phase advance per focusing period, which gets rid of the details of the actual focusing lattice.

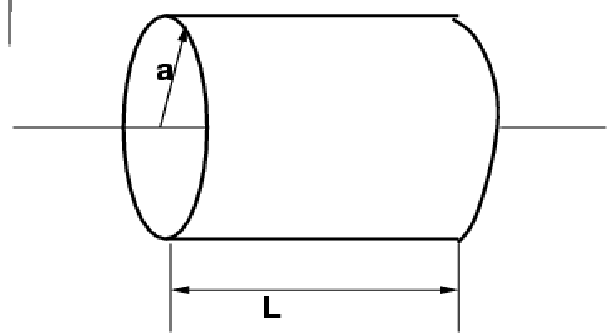
The last term represents the space charge force.

Fields Inside/Outside a DC Bunch



The electric field E_r is linear inside the bunch.

The charge density is calculated for a current *I* moving along *z* for a beam of radius *a*. Over a time *t* the beam moves a distance *L*.

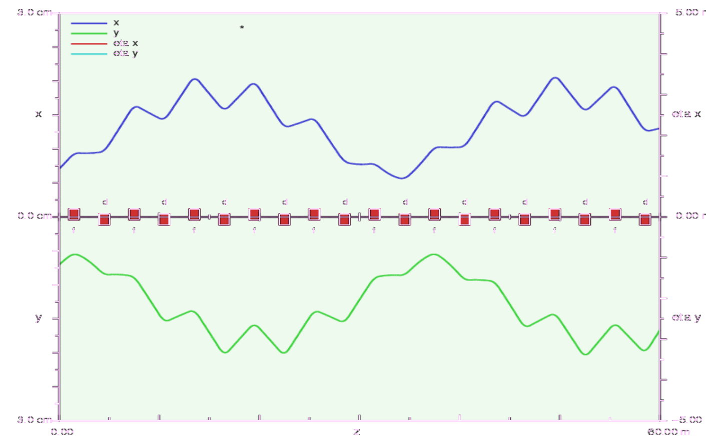


$$\rho = \frac{charge}{volume} = \frac{I\,t}{\pi\,a^2\,\beta\,c\,t} = \frac{I}{\pi\,a^2\,\beta\,c}$$

Find the radial field: $\nabla D = \epsilon_0 \nabla E = \rho = \epsilon_0 \frac{1}{r}\frac{d}{dr} r\,E_r$

Inside the bunch $E_r(r) = \frac{I}{\pi\,a^2\,\beta\,c}\frac{r}{2\epsilon_0} = \frac{\rho\,r}{2\epsilon_0}$ linear with *r*.

Matched Beam in a Periodic Channel



We saw previously that a beam that is mismatched to a periodic channel exhibits envelope breathing.

In the smooth approximation, the space charge and emittance pressures are exactly balanced by the external restoring force. This results in a matched beam condition where there is no envelope breathing. If the envelope is smooth,

$$a''=0 \qquad\qquad k(s)a-\frac{\epsilon^2}{a^3}+\frac{\overline{x F_s}}{a} = 0$$

We can substitute for the focusing and space charge term (see Wangler for details)

$$0 = \left(\frac{\Delta\mu}{L}\right)^2 \bar{a} - \frac{\epsilon^2}{\bar{a}^3} - 60[ohms]\left(\frac{e}{m_p c^2}\right)\frac{I}{\beta^3}\frac{1}{\bar{a}}$$

L = FODO period

Solve for the mean beam size at zero current

$$\bar{a} = \sqrt{\frac{\epsilon}{L\,\Delta\mu}}$$

Current for Matched Channel, DC Beam

$$0 = \left(\frac{\Delta\mu}{L}\right)^2 \bar{a} - \frac{\epsilon^2}{\bar{a}^3} - 60[ohms]\left(\frac{e}{m_p c^2}\right)\frac{I}{\beta^3}\frac{1}{\bar{a}}$$

(What are the units here?)

Hard to solve this one for a, but easy to solve for I, the current that gives a matched beam size *a*

$$I = \frac{1}{60\,ohms}\left(\frac{m_p c^2}{e}\right)\beta^3\left[\left(\frac{\Delta\mu}{L}\right)^2 \bar{a}^2 - \left(\frac{\epsilon}{\bar{a}}\right)^2\right]$$

Sidelight: the top equation is

$$0 = \left(\frac{\Delta\mu}{L}\right)^2 - \frac{\epsilon^2}{\bar{a}^3} - \frac{1}{2\pi\epsilon_0}\left(\frac{e}{m_p c^2}\right)\frac{I}{\beta^3 c}\frac{r}{a^2}$$

But

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega, \quad \frac{1}{\sqrt{\mu_0\epsilon_0}} = c, \quad \rightarrow \quad \frac{1}{\epsilon_0 c} = 377\,\Omega, \quad \frac{1}{2\pi\epsilon_0 c} = 60\,\Omega$$

Bunched Beams

As linacs operate at RF, beams must be bunched to be accelerated. We must then calculate the fields of a 3-dimensional bunch.

The smooth approximation approach is modified to include the effect of a bunched beam.

The bunch frequency, f_b, is the same as or a subharmonic of the linac frequency f_0

$$f_b = \frac{f_0}{N}, \quad N = 1, 2, \ldots$$

The free-space wavelength of the linac operating frequency is $\lambda_0 = \dfrac{c}{f_0}$

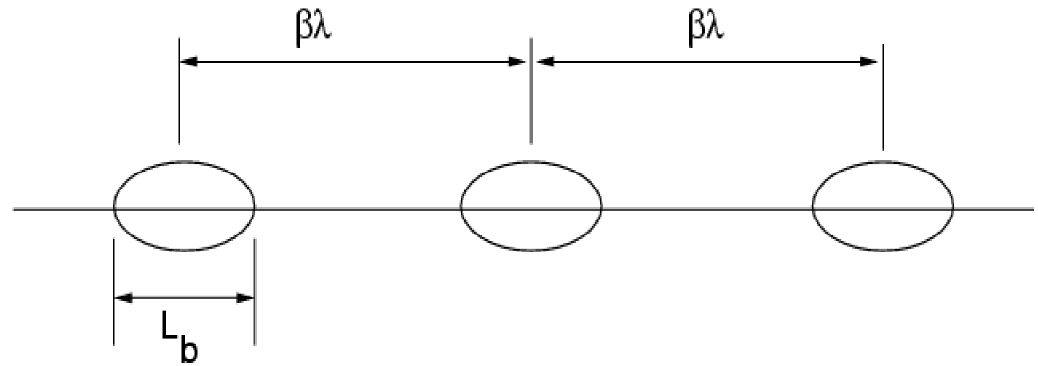
The distance a bunch travels in one linac RF cycle is $\Delta z = \beta \lambda$

For a 1 MeV proton in a 200 MHz linac,

$$\beta = \sqrt{\frac{2T}{m_p c^2}} = 0.0462, \quad \lambda = \frac{c}{200\,MHz} = 1.5\,m, \quad \beta\lambda = 0.0693\,meters$$

For N = 1, the bunch center-to-center spacing is 6.93 cm.

Bunch Length



The distance between bunches is $\beta\lambda$.

The length of an individual bunch is L_b. This definition is a bit fuzzy, as the distribution of charges through the bunch is not constant. The length is often taken at the half-intensity points, or is a rms value.

The current *I* carried by a bunched beam is always the average current, integrated over the bunches. It is sometimes handy to specify the peak current as

$$I_{peak} = I_{average}\left(\frac{bunch\,spacing}{bunch\,length}\right) = I_{average}\frac{\beta\lambda}{L_b}$$

As the phase of the RF wave in a linac is a frequent reference, the bunch length is often expressed in terms of RF phase of *half of the bunch*. This is consistent with expressing the transverse bunch size as a radius, not a diameter.

$$\Delta\phi = \frac{2\pi}{2}\,\frac{L_b}{\beta\lambda}$$

The bunch length is usually expressed in degrees.

Bunched Beam in the Smooth Approximation

We saw that for DC beams, the space charge term in the smooth approximation is

$$60\,\Omega\left(\frac{e}{m_p c^2}\right)\frac{I}{\beta^3 \bar{a}}$$

Let's define the half-length of the bunch $r_z = L_b/2$

The transverse field of the bunch (and the longitudinal field) of the bunch gets modified by a form factor *f,* which we will define empirically with a best-fit recipe.

Recall that for a DC beam $$E_{r,\,DC\,beam} = 60\,\Omega\frac{I\,r}{\beta\,\bar{a}^2}$$

For a bunched beam $$E_{r,\,bunched} = 60\,\Omega\frac{I\,r}{\beta\,\bar{a}^2}\,\frac{3}{4}\left(\frac{\beta\lambda}{r_z}\right)(1-f)$$

$$E_{z,\,bunched} = 60\,\Omega\frac{I\,z}{\beta\,\bar{a}^2}\,\frac{3}{2}\left(\frac{\beta\lambda}{r_z}\right)f$$

For a good enough approximation, $$f = \frac{\bar{a}}{3\,r_z}$$

(See Wangler, p. 276 for further discussion.)

Where to Apply These Estimates in the Accelerator

These equations are recipes to give reasonable estimates of the space charge limit at various places in the accelerator chain.

The beam usually enters the accelerator unbunched, so the peak charge density is equal to the average charge density.

Bunching increases the peak charge density, increasing the effects of beam space charge, but the beam must be bunched to accelerate it. Optimal solutions that maximize the beam acceptance trade bunching and acceleration in a way to minimize loss: this is the primary optimization condition when designing an RFQ accelerator.

What Do These Equations Really Tell Us?

These equations represent the first approach to designing an accelerator to satisfy a set of requirements.

The accelerator physicist starts with a set of user requirements:

- ion species
- energy and energy variability
- intensity or average current
- spot size, energy spread (transverse and longitudinal emittance)
- duty factor
- size of accelerator installation
- cost constraints

Estimates are made of types of accelerator structures, availability of off-the shelf RF source designs (the most expensive component), transition energies between accelerator structures, etc.

These equations help facilitate these decisions. They are at best estimates to be obtained before more quantitative results are obtained by running detailed codes. We have not yet considered the effects of the beam density distribution, generation of halos, beam loss and activation of the accelerator itself. These answers will come after running the more detailed codes.

Beam Transport Codes

The design of beam transport systems is perhaps the most difficult, as there exist more options than designing an accelerator.   A beam transport system becomes a work of art, as it is usually the product of a single designer.

Many transport design codes exist, and we will consider a small subset of them.

Very Partial List of Transport Codes

Code	Method	Space Charge?	Distributions?
Transport	matrix		
Turtle	rays		yes
Trace 3D	envelope	yes	
Parmila	macroparticle	yes	yes
Pro Lab Transport	matrix		
Pro Lab Turtle	rays		yes
Pro Lab Trace 3D	envelope	yes	
Pro Lab Marylie	Lie Algebra		yes
Lattice	matrix		
Marylie	Lie Algebra		yes
COSY	Lie Algebra		yes
MAD	Matrix, Lie		yes

Types of Codes

Most codes are based on the **matrix** element representation of transport elements. This can be extended to higher-order with difficulty. Some codes integrate the particle paths through the fields directly. This may be more accurate, but requires accurate representation of the fields.

Some of the codes include **collective effects** (**space charge**) between the particles. This may be with an **envelope integrator**, such as **Trace-3D**, which assumes a simple distribution, or with a **macroparticle code** such as **Parmila**, which allows any distribution of particles.

Codes that propagate macroparticles or rays, such as Turtle, allow apertures to simulate beam loss and evolution of emittance growth due to nonlinearities or collective effects.

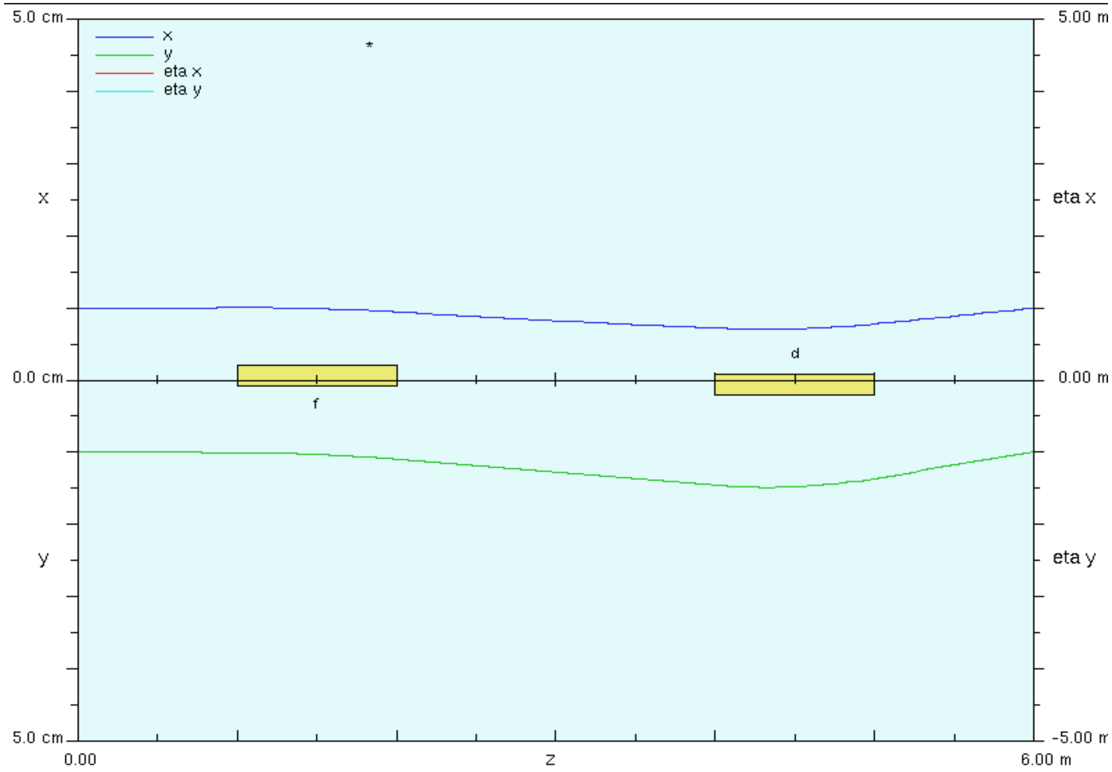
Applications that require very high accuracy, such as millions of turns in a storage ring, use Lie algebraic techniques to insure that the transport elements are simplectic.

Using PARMILA to Simulate Transport Lines

PARMILA has the facilities to model transport lines, usually to carry a beam to or from a linac, but it can model transport lines by themselves. PARMILA includes a space-charge routine, so it is a counterpart to TRACE-3D, which is based on an envelope model with space charge, and PARMILA is based on a macroparticle model with space charge.

PARMILA is one of a few macroparticle codes that includes an array of element types and the ability to include space charge. The transport capability is an add-on to the original code, originally used to calculate transport systems to and from linacs, and it can be used as a stand-alone.

A Simple Quadrupole Transport Line



The beam enters from the left. Two quadrupoles, one focusing the other defocusing in the x-plane are 1 meter long. The drifts are 1, 2 and 1 meter long.

The horizontal and vertical beam envelopes are shown above and below the axis.

The Twiss parameters are printed out after each element.

elemt	lth	sum_l	betax	alphax	etax	eta'x	psix	betay	alphay	etay	eta'y	psiy
-	(m)	(m)	(m)	-	(m)	(rad)	(deg)	(m)	-	(m)	(rad)	(deg)
		0.00	10.00	0.00	0.00	0.00	0.0	10.00	0.00	0.00	0.00	0.0
o	1.00	1.00	10.10	-0.10	0.00	0.00	5.7	10.10	-0.10	0.00	0.00	5.7
f	1.00	2.00	8.93	1.21	0.00	0.00	11.6	12.02	-1.91	0.00	0.00	11.0
o	1.00	3.00	6.80	0.93	0.00	0.00	19.0	16.23	-2.30	0.00	0.00	15.2
o	1.00	4.00	5.21	0.66	0.00	0.00	28.7	21.22	-2.69	0.00	0.00	18.2
d	1.00	5.00	6.09	-1.65	0.00	0.00	39.5	18.58	4.99	0.00	0.00	21.0
o	1.00	6.00	10.00	-2.26	0.00	0.00	46.9	10.00	3.59	0.00	0.00	25.2

Minimum PARMILA Input for a Simple Quadrupole Doublet

This is for a OFOODO sequence, each element 100 cm long, with the quad gradients of 15 and -38 gauss/cm.

The input beam emittance is 1 p cm-mr, and the input Twiss parameter is b = 10 m, a = 0 for a 46.8 MeV proton beam whose rigidity is 1 T-m.

The first drift has zero length, to check on and print out the initial beam statistics.

Note that a frequency must be specified in the **linac** and **structure** lines, even if the beam is unbunched.

Even if space charge is neglected, the **scheff** routine must be included, with zero current.

```
; Transport example
Run 1 1
Title
  Transport example
;linac W0,     Fbunch, Ibeam, Mc^2,     Nq
linac  46.8 100 0 938 1

;--------Beam Particle Distribution Spec---------

Input -8 -1000  0 1000 .001 0 1000 .001 0 180 0.001

;-------------------------------------------------
;Structure id,Nlast,F0,Fsf,deltaphi
Structure   1 0 100 100 0
Transport 0
Bore 10
drift   0 0 1 1
drift 100 0 1 1
quad  100 0 1 15 0
drift 100 0 1 1
drift 100 0 1 1
quad  100 0 1 -38 0
drift 100 0 1 1
scheff .00 .05 20 40 0 0 3
begin
end
```

TRACE 3-D

TRACE 3-D, written by Ken Crandall, calculates the envelope of a bunched beam including space charge. It is one of the earliest programs that uses a graphical interface and is still one of the only programs that incorporates linear space charge in an envelope code along with an optimizer.

The Pro Lab version of TRACE 3-D includes a more friendly interface that eases the learning and use of TRACE 3-D.

Despite its age, TRACE 3-D continues to be useful in cases where space charge is significant and matching (optimization) is required.

Input example for the quad doublet problem:

```
&DATA
 ER = 938.236, Q=1.0, W= 46.731, XI= 0.0
 EMITI= 10.0000, 10.0000, 1756.2623
 BEAMI= 0.0000, 10.0000, 0.0000, 10.0000, 0.00, 1.00
 FREQ= 100.00, ICHROM=0
 XM= 50.00, XPM= 50.00, YM= 50.0000, DPM= 50.00, DWM= 50.00, DPP= 50.00
 N1= 1, N2= 6, SMAX= 2.00, PQSMAX= 1.00
 CMT(  1)='o         ' NT(  1)=  1, A(1,  1)=1000.0,
 CMT(  2)='f         ' NT(  2)=  3, A(1,  2)= 0.1500, 1000.0
 CMT(  3)='o         ' NT(  3)=  1, A(1,  3)=1000.0
 CMT(  4)='o         ' NT(  4)=  1, A(1,  4)=1000.0
 CMT(  5)='d         ' NT(  5)=  3, A(1,  5)=-0.3802, 1000.0
 CMT(  6)='o         ' NT(  6)=  1, A(1,  6)=1000.0
 COMENT='Quadrupole Doublet'
&END
```

TRACE 3-D Plot for Quadrupole Doublet Problem

The appearance of this version differs from the Windows version.

The initial and final beam phase space ellipses are shown (always considered to be elliptical), and the beam envelope is plotted.

Caveat: for the space charge algorithm to work correctly, the beam emittance is specified as 5 times the rms emittance.