Fourth International Accelerator School for Linear Colliders

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Course A4: Damping Ring Design and Physics Issues

Lecture 7

Space Charge, Intrabeam Scattering and Touschek Effects

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Space-charge, intrabeam scattering and Touschek effects

In this lecture, we shall discuss:

- Space-charge forces
- Intrabeam scattering
- Touschek scattering

Space-charge forces and intrabeam scattering tend to lead to emittance growth in the beam, depending on the bunch charge.

Touschek scattering is related to intrabeam scattering, but leads to particle loss, and hence a reduction in the beam lifetime. For low-emittance storage rings (like the ILC damping rings), Touschek scattering is the dominant effect limiting the beam lifetime.

Space-charge forces

In the rest frame of a bunch of charged particles, the bunch will expand rapidly (in the absence of external forces) because of the Coulomb repulsion between the particles.

The electric field around a single particle of charge q at rest is a radial field:

$$E_r = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

Applying a Lorentz boost along the z axis, with relativistic factor γ, the field becomes:

$$E_x = \frac{q}{4\pi\varepsilon_0}\frac{\gamma x}{\left(x^2+y^2+\gamma^2 z^2\right)^{3/2}} \qquad E_y = \frac{q}{4\pi\varepsilon_0}\frac{\gamma y}{\left(x^2+y^2+\gamma^2 z^2\right)^{3/2}} \qquad E_z = \frac{q}{4\pi\varepsilon_0}\frac{\gamma z}{\left(x^2+y^2+\gamma^2 z^2\right)^{3/2}}$$

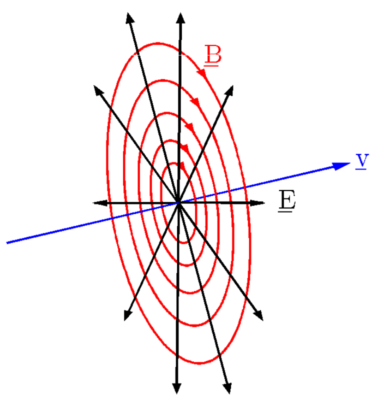
For large γ, the field is strongly suppressed, and falls rapidly away from $z = 0$. In other words, the electric field exists only in a plane perpendicular to the direction of the particle.

Space-charge forces

Associated with the electric field around a moving charged particle is a magnetic field, given by:

$$B_x = -\frac{\beta}{c}E_y \qquad B_y = \frac{\beta}{c}E_x \qquad B_z = 0$$

The magnetic field is similarly “flattened” in the plane perpendicular to the direction of motion of the particle.



Space-charge forces

To calculate the space-charge force on a particle in an ultra-relativistic bunch, we can consider the two-dimensional charge distribution in the x-y plane at the same z location as the particle.

Often, the charge distribution will be Gaussian, or close to Gaussian. In that case, an analytical expression (in terms of the complex error function) for the electric field have been obtained, and is known as the Bassetti-Erskine formula.

M. Bassetti and G.A. Erskine, “Closed expression for the electrical field of a two-dimensional Gaussian charge”, CERN-ISR-TH/80-06 (1980).

The field is highly-nonlinear, and the expressions complicated to work with.

We can make a simple linear approximation to the Bassetti-Erskine formula, which gives:

$$E_x \approx \frac{e\lambda}{2\pi\varepsilon_0} \frac{x}{\sigma_x(\sigma_x + \sigma_y)} \qquad E_y \approx \frac{e\lambda}{2\pi\varepsilon_0} \frac{y}{\sigma_y(\sigma_x + \sigma_y)}$$

where $e\lambda$ is the longitudinal charge density, and σ_x and σ_y are the rms horizontal width and vertical height of the beam, respectively. Note that there are magnetic fields associated with the electric fields.

Effects of space-charge forces

We can write the equation of motion of a particle in a relativistic bunch:

$$\gamma m \ddot{y} = e(E_y + \beta c B_x)$$

where the dots denote the derivative with respect to time. Using:

$$\ddot{y} = \beta^2 c^2 y''$$

(where the primes denote the derivative with respect to path length), and:

$$B_x = -\frac{\beta}{c} E_y$$

we obtain:

$$y'' = \frac{eE_y}{\beta^2 \gamma^3 mc^2}$$

Note that the forces from the electric and magnetic fields almost cancel. This cancellation leads to a suppression (by a factor $1/\gamma^2$) of the space-charge forces.

Effects of space-charge forces

Using the linear expression for the electric field from the Bassetti-Erskine formula, we obtain:

$$\frac{dp_y}{ds} \approx \frac{e\lambda}{2\pi\varepsilon_0} \frac{e}{\beta^2\gamma^3 mc^2} \frac{y}{\sigma_y(\sigma_x+\sigma_y)} = \frac{2r_e\lambda}{\beta^2\gamma^3} \frac{y}{\sigma_y(\sigma_x+\sigma_y)}$$

The (linearised) space-charge force appears as a linear *defocusing* force. To estimate the impact of this force, we can calculate the tune shift associated with it:

$$\Delta\nu_y = \frac{1}{2\pi}\oint \beta_y k_y \, ds$$

where k_y is the vertical linear *focusing* strength, given, in the case of the space-charge force, by:

$$k_y = -\frac{2r_e\lambda}{\beta^2\gamma^3\sigma_y(\sigma_x+\sigma_y)}$$

Note that the longitudinal charge density λ varies with the longitudinal position of the particle in the bunch. What we can calculate is really a *spread* in tune, sometimes referred to as an *incoherent* tune shift.

Effects of space-charge forces

The tune spread from space-charge forces for particles in a Gaussian bunch of N_0 particles and rms bunch length σ_z is given by:

$$\Delta\nu_y = -\frac{2r_e N_0}{(2\pi)^{3/2}\sigma_z\beta^2\gamma^3}\oint \frac{\beta_y}{\sigma_y(\sigma_x+\sigma_y)} ds$$

where the integral extends around the entire circumference of the ring.

Since every particle in the bunch experiences a different tune shift, it is not possible to compensate the tune spread as one could for a *coherent* tune shift (for example, by adjusting quadrupole strengths).

Note that the tune spread gets larger for:

- larger bunch charges
- shorter bunches
- larger beta functions
- lower beam energy (very strong scaling!)
- larger circumference
- smaller beam sizes

Note that the vertical beam size is usually much smaller than the horizontal.

Effects of space-charge forces in the ILC damping rings

How large are the tune shifts in the ILC damping rings? We can make some estimates for different configurations, using the formula on the previous slide.

Lattice	Beam energy	Circumference	Bunch length	$\Delta\nu_x$	$\Delta\nu_y$
PPA	5 GeV	2.8 km	6 mm	-0.001	-0.021
OCS2	5 GeV	6.1 km	6 mm	-0.002	-0.083
BRU	3.74 GeV	6.3 km	9 mm	-0.009	-0.119
MCH	5 GeV	16 km	9 mm	-0.009	-0.176
TESLA	5 GeV	17 km	6 mm	-0.019	-0.313

For tune shifts of order 0.1, we can expect particles in the bunch to cross resonance lines in tune space. This can lead to growth in the amplitude of betatron oscillations for these particles; one consequence can be an overall increase in the emittance of the bunch.

Effects of space-charge forces in the ILC damping rings

The linear approximation to the Bassetti-Erskine formula can significantly over-estimate the size of the space-charge forces at large amplitudes.

What is the real impact of the space-charge forces likely to be in the ILC damping rings?

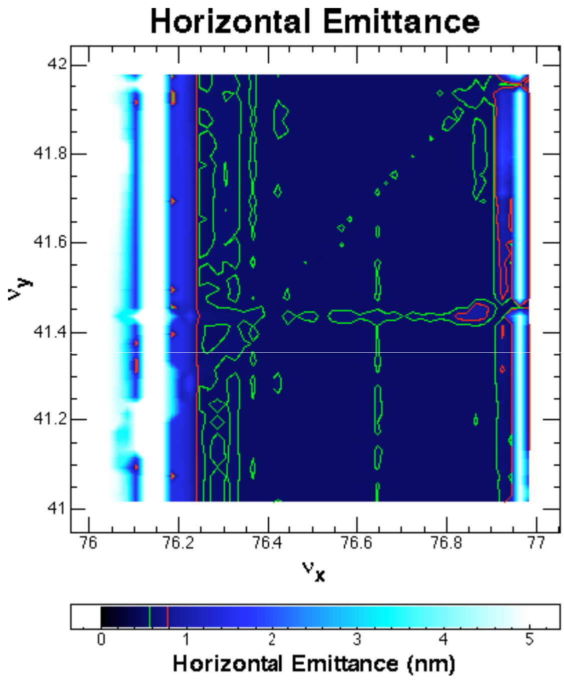
The best way to answer this question is to perform a tracking simulation, in which the space-charge forces are included. Two codes have been applied to the ILC damping rings to study space-charge effects:

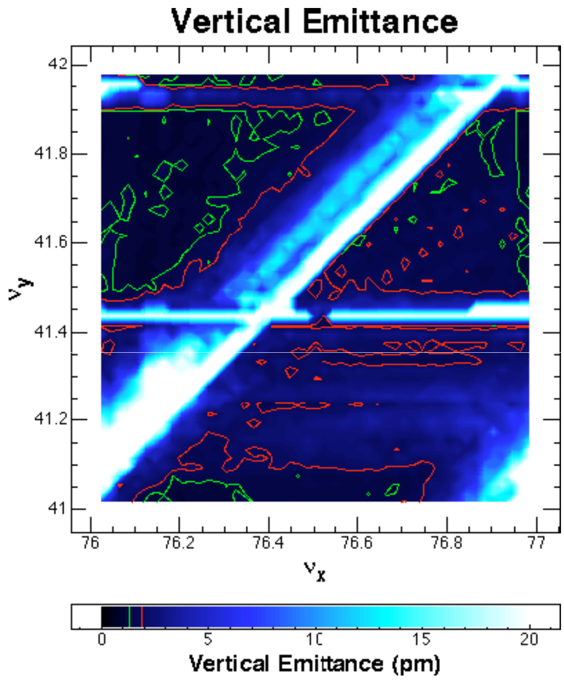
- SAD (studies performed by Katsunobu Oide)
- MaryLie/Impact (studies performed by Marco Venturini)

Both these codes use a “weak-strong” model:

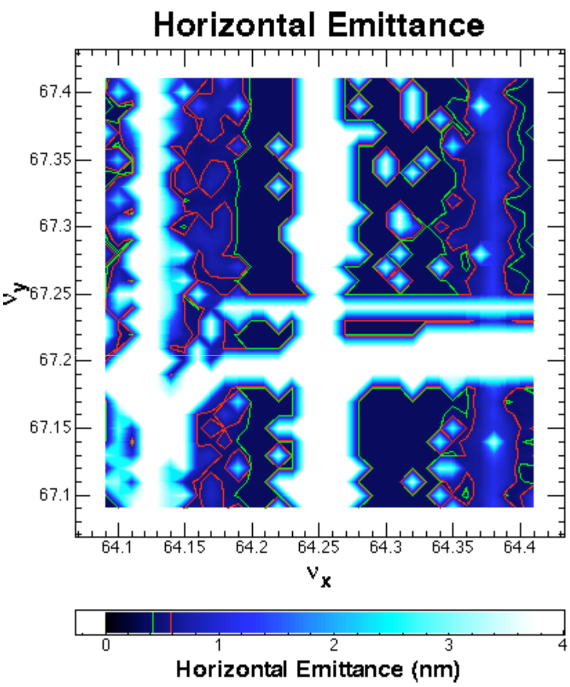
- The (nonlinear) space-charge forces at different locations within a bunch are initially calculated from a (“strong”) charge distribution matched to the lattice in the absence of space-charge.
- A (“weak”) bunch of particles is tracked through the lattice, including the (fixed) nonlinear space-charge forces. The effects of space charge are estimated from the behaviour of the “weak” bunch.

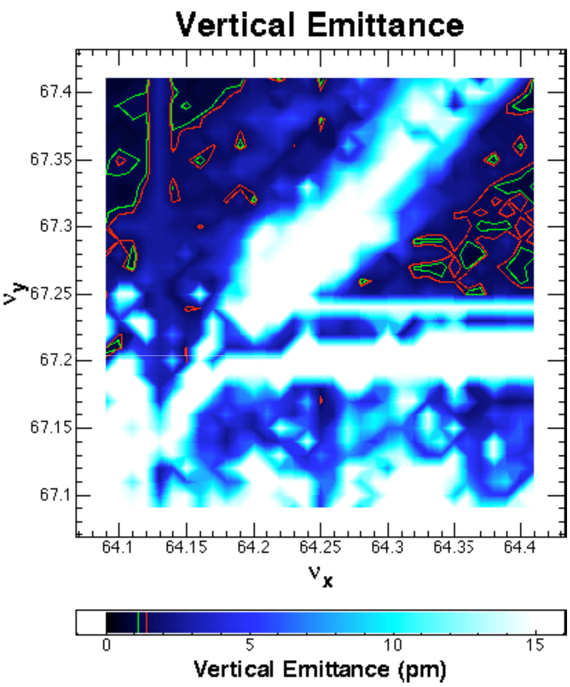
Space-charge emittance growth in the TESLA lattice (from SAD)



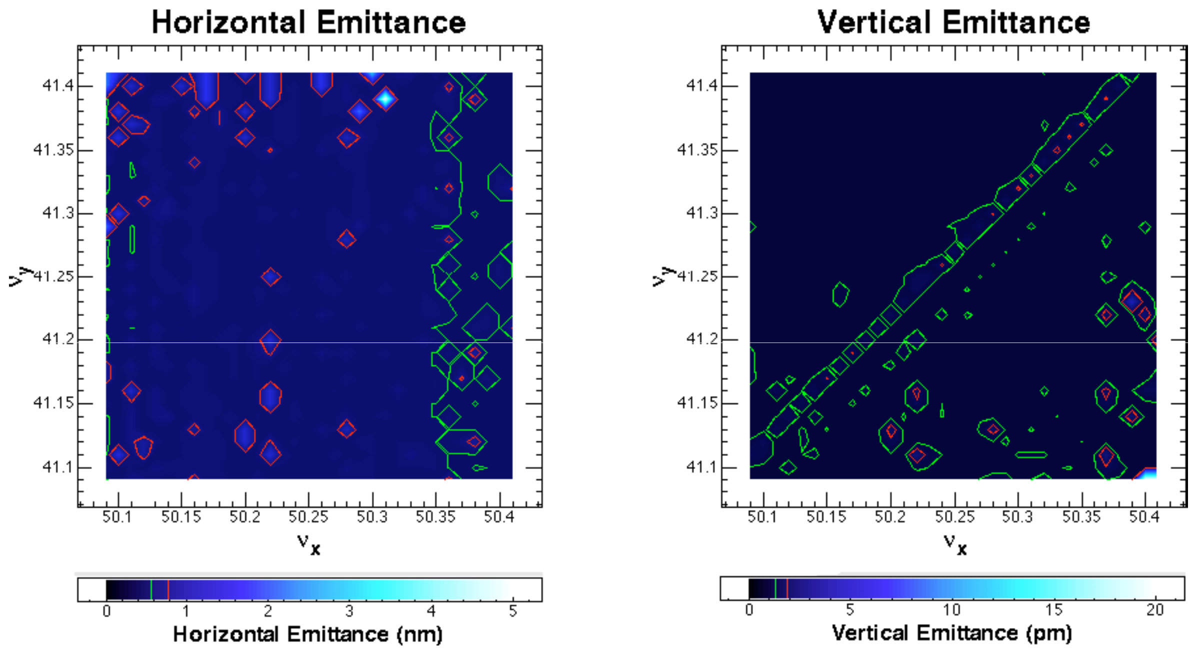


Space-charge emittance growth in the BRU lattice (from SAD)

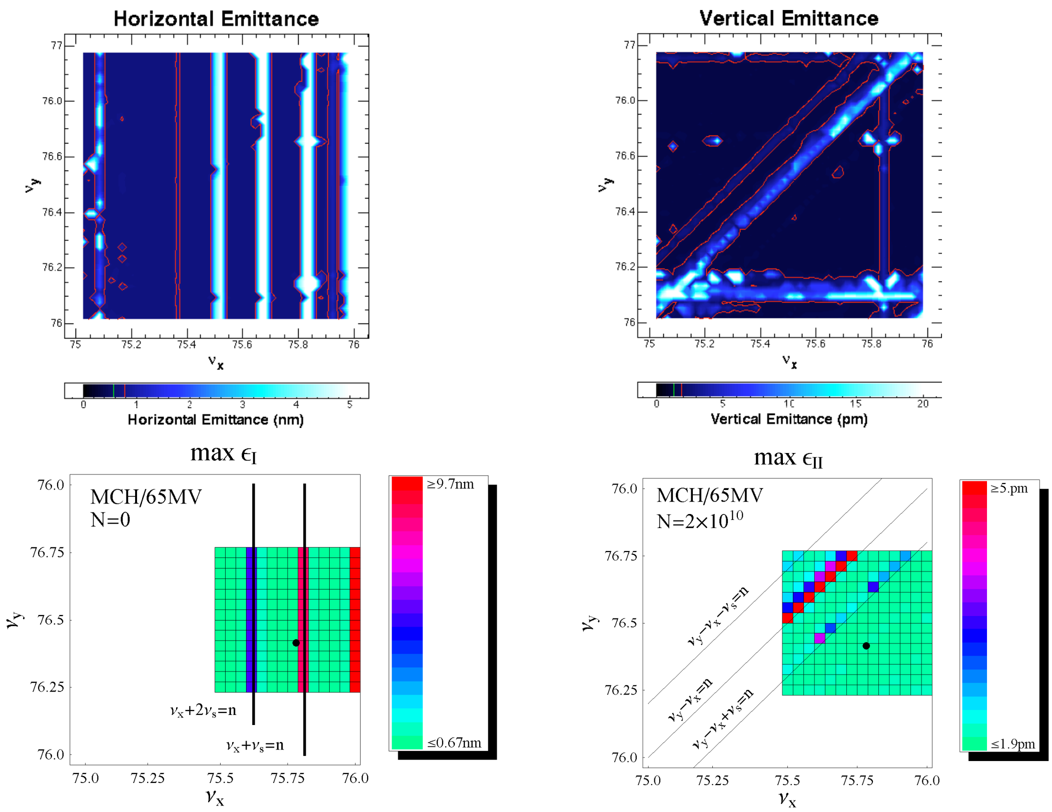




Space-charge emittance growth in the OCS2 lattice (from SAD)



Space-charge simulations in SAD and MaryLie/Impact agree well

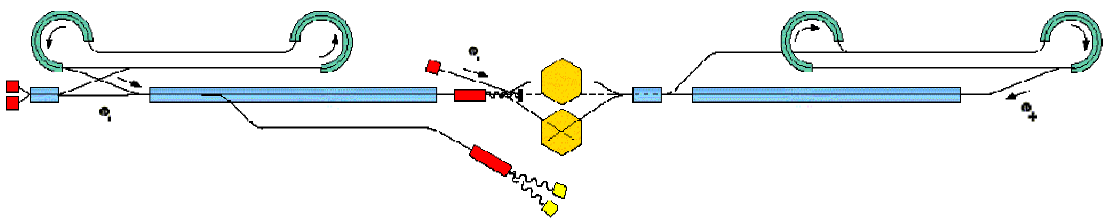


Mitigating space-charge effects with “coupling bumps”

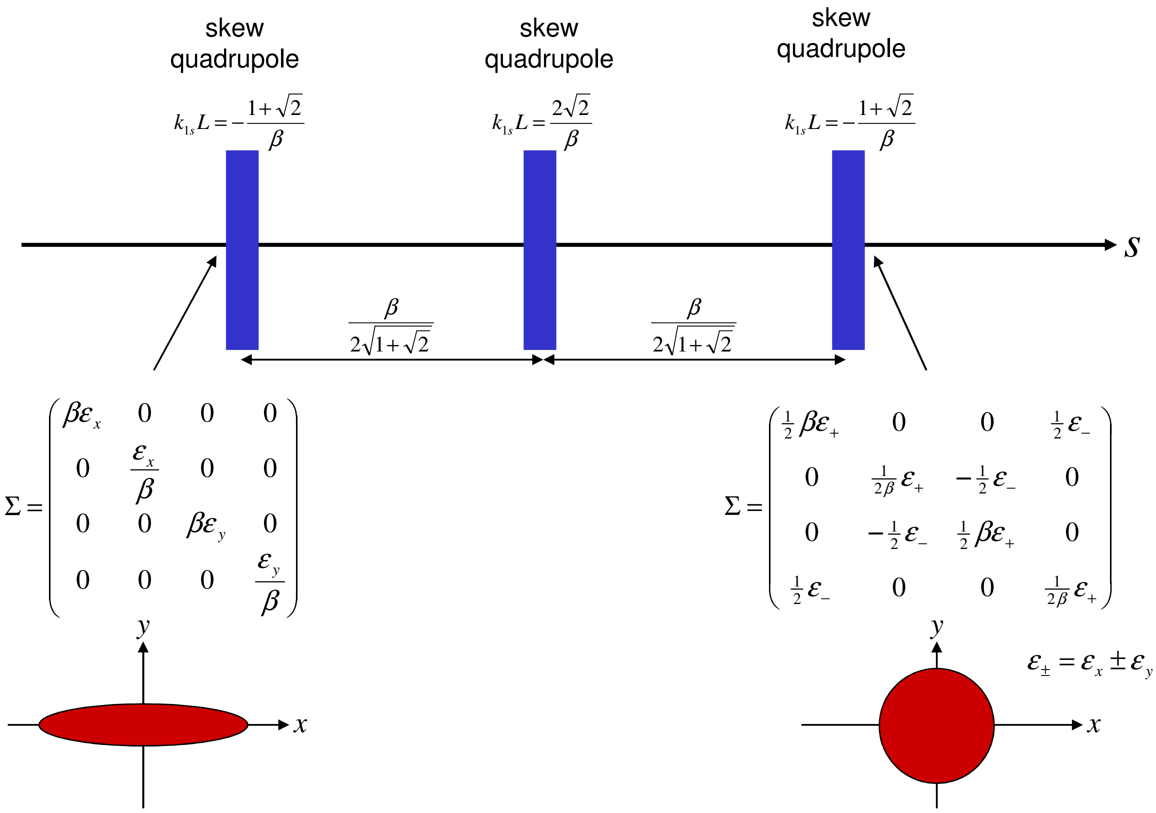
The longer lattices (~ 17 km) tend to suffer more from space-charge effects because the incoherent tune shift scales linearly with the circumference.

A 17 km damping ring would have a “dog-bone” layout, with long straight sections located in the same tunnel as the main linac. These sections would exist purely to increase the circumference: a larger bunch spacing helps with the injection/extraction kickers, and with effects such as electron cloud.

In long straight sections, it is possible to use skew quadrupoles to create local coupling, increasing the vertical beam size over a well-defined section of the damping ring. Since the tune shifts scale with the beam sizes, this mitigates the effects of space charge.

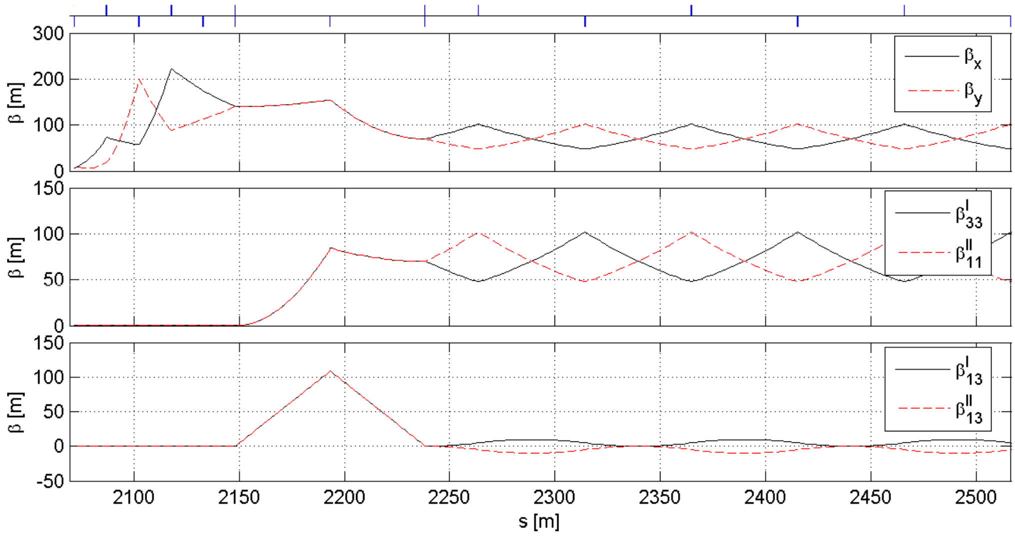


Mitigating space-charge effects with “coupling bumps”



Mitigating space-charge effects with “coupling bumps”

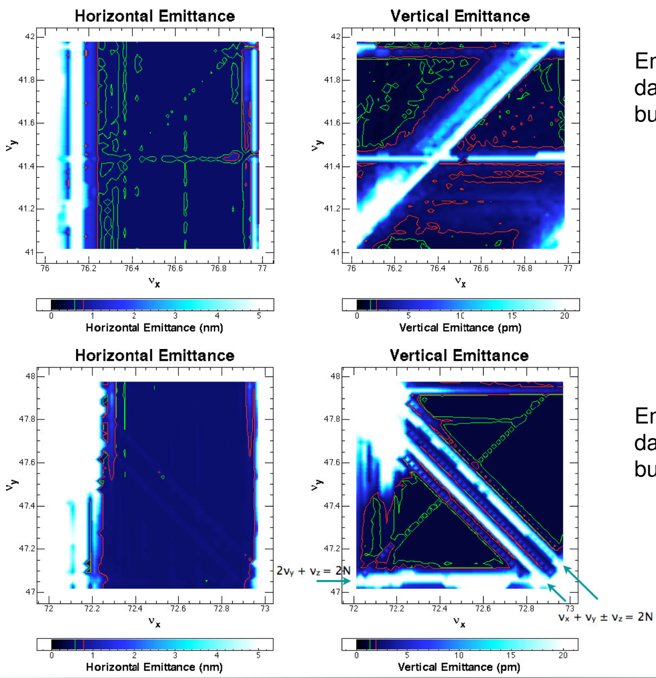
Lattice functions at one end of a “coupling bump”. The bump is closed at the other end by another set of three skew quadrupoles.



$$\langle x^2 \rangle = \beta_x \varepsilon_x + \beta^{II}_{11} \varepsilon_y \qquad \langle xy \rangle = \beta^{I}_{12} \varepsilon_x + \beta^{II}_{12} \varepsilon_y \qquad \langle y^2 \rangle = \beta^{I}_{33} \varepsilon_x + \beta_y \varepsilon_y$$

Mitigating space-charge effects with “coupling bumps”

Tracking simulations show that coupling bumps can mitigate the space-charge effects, but drive coupling resonances that can themselves lead to vertical emittance growth.



Emittance growth in the TESLA damping ring lattice without coupling bumps (from tracking in SAD).

Emittance growth in the TESLA damping ring lattice with coupling bumps (from tracking in SAD).

Space-charge effects: conclusions

The effects of space charge forces can be significant in the ILC damping rings because:

- the vertical beam size (at equilibrium) is very small;
- the rings have relatively large circumference for the beam energy;
- the peak charge density in a bunch is reasonably large.

Calculating the tune shifts from a model based on linear space-charge forces (obtained by expanding the Bassetti-Erskine formula to first-order in the transverse coordinates) tends to give a pessimistic estimate of the impact of space-charge, in terms of the expected emittance growth.

Tracking simulations (using a weak-strong model for the nonlinear space charge forces) indicate a strong dependence on the lattice design, and on the horizontal and vertical tunes of the lattice.

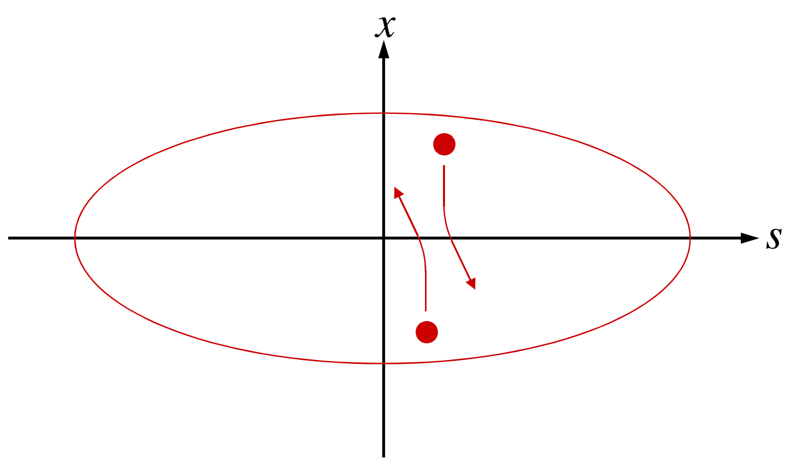
It may be possible to mitigate space-charge forces in a 17 km (dog-bone) damping ring by using coupling bumps to increase the vertical beam size in the long straights. However, the coupling bumps may drive other resonances, which can themselves lead to an increase in the emittance.

It appears to be possible to design a 5 GeV, 6 km damping ring that does not suffer from strong space-charge effects.

Intrabeam scattering

Particles within a bunch can collide with each other as they perform betatron and synchrotron oscillations. The collisions lead to a redistribution of the momenta within the bunch, and hence to a change in the emittances.

If a collision results in the transfer of transverse to longitudinal momentum at a location where the dispersion is non-zero, the result (after many scattering events) can be an increase in both transverse and longitudinal emittance, in a way similar to the increase in emittance by quantum excitation.



Intrabeam scattering

A large change in momentum (“large-angle scattering”) can lead to the energy deviation of particles becoming larger than the energy acceptance of the ring, in which case the particles will be lost. This is the Touschek effect, which limits the lifetime of the beam, and that we will discuss shortly.

Multiple small-angle scattering processes lead to an increase in beam emittance. This effect of intrabeam scattering (IBS) is well known in proton machines, where it can limit the luminosity lifetime of a hadron collider. However, the growth rates are generally slow enough that in electron machines, IBS is completely counteracted by radiation damping.

If the particle density in a bunch is high enough (large bunch population, small emittances) and the energy is not too high, then emittance growth from IBS may be significant in electron machines.

Observations of IBS have been made in the KEK-ATF, and in the LBNL-ALS. Emittance growth from IBS is expected to be significant in the ILC damping rings.

Calculating IBS emittance growth rates

The basic theory of IBS has been developed by Piwinski, and extended by Bjorken and Mtingwa.

A. Piwinski, Proc. 9th Int. Conf. on High Energy Acc., Stanford (1974) 405.

J.D. Bjorken and S.K. Mtingwa, Particle Accelerators 13 (1983) 115.

The detailed analysis of IBS is complicated, and leads to formulae that generally involve complicated integrals that are slow to evaluate numerically.

Various approximations (usually valid for high-energy regimes) have been given by numerous researchers.

Kubo *et al* have obtained formulae (“completely integrated modified Piwinski”, or CIMP formulae) based on the Piwinski formalism that avoid the need for performing complicated integrals.

K. Kubo, S.K. Mtingwa and A. Wolski, “Intrabeam Scattering Formulas for High Energy Beams”, Phys. Rev. ST Accel. Beams **8**, 081001 (2005).

Calculating IBS emittance growth

The horizontal and vertical IBS growth times (respectively, T_x, T_y) are defined so that:

$$\frac{d\varepsilon_x}{dt} = \frac{2}{T_x}\varepsilon_x$$

$$\frac{d\varepsilon_y}{dt} = \frac{2}{T_x}\varepsilon_y$$

The longitudinal (energy spread) growth time T_δ is defined so that:

$$\frac{d\sigma_\delta}{dt} = \frac{1}{T_\delta}\sigma_\delta$$

Calculating IBS emittance growth

The CIMP formulae give for the horizontal and vertical growth rates (averaged around the lattice):

$$\frac{1}{T_x} \approx 2\pi^{3/2}(\log)A\left\langle \frac{\mathcal{H}_x\sigma_H^2}{\varepsilon_x}\left(\frac{1}{a}g\left(\tfrac{b}{a}\right)+\frac{1}{b}g\left(\tfrac{a}{b}\right)\right) - a\ g\left(\tfrac{b}{a}\right)\right\rangle$$

$$\frac{1}{T_y} \approx 2\pi^{3/2}(\log)A\left\langle \frac{\mathcal{H}_y\sigma_H^2}{\varepsilon_y}\left(\frac{1}{a}g\left(\tfrac{b}{a}\right)+\frac{1}{b}g\left(\tfrac{a}{b}\right)\right) - b\ g\left(\tfrac{a}{b}\right)\right\rangle$$

where:

$$\mathcal{H}_x = \gamma_x\eta_x^2 + 2\alpha_x\eta_x\eta_{px} + \beta_x\eta_{px}^2 \qquad \mathcal{H}_y = \gamma_y\eta_y^2 + 2\alpha_y\eta_y\eta_{py} + \beta_y\eta_{py}^2$$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_\delta^2} + \frac{\mathcal{H}_x}{\varepsilon_x} + \frac{\mathcal{H}_y}{\varepsilon_y}$$

$$a = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_x}{\varepsilon_x}} \qquad b = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_y}{\varepsilon_y}}$$

Calculating IBS emittance growth

The function g is given in terms of associated Legendre functions:

$$g(\omega) = \sqrt{\frac{\pi}{\omega}}\left[P^{0}_{-1/2}\left(\frac{\omega^2+1}{2\omega}\right) \pm \frac{3}{2}P^{-1}_{-1/2}\left(\frac{\omega^2+1}{2\omega}\right)\right]$$

where we take the plus sign for $\omega \geq 1$, and the minus sign for $\omega \leq 1$.

The variable A is given in terms of the bunch parameters (in the usual notation) by:

$$A = \frac{r_e^2 c N_0}{64\pi^2\gamma^4\varepsilon_x\varepsilon_y\sigma_z\sigma_\delta}$$

Note the strong (γ^4) energy scaling. The "Coulomb log" factor is given by:

$$(\log) \approx \ln\left(\frac{\gamma^2\sigma_y\varepsilon_x}{r_e\beta_x}\right)$$

Calculating IBS emittance growth

The CIMP formula for the energy spread growth rate (averaged around the lattice) is:

$$\frac{1}{T_\delta} \approx 2\pi^{3/2}(\log)A\left\langle\frac{\sigma_H^2}{\sigma_\delta^2}\left(\frac{1}{a}g\left(\tfrac{b}{a}\right)+\frac{1}{b}g\left(\tfrac{a}{b}\right)\right)\right\rangle$$

Note that the expressions for the transverse growth rates:

$$\frac{1}{T_x} \approx 2\pi^{3/2}(\log)A\left\langle\frac{\mathcal{H}_x\sigma_H^2}{\varepsilon_x}\left(\frac{1}{a}g\left(\tfrac{b}{a}\right)+\frac{1}{b}g\left(\tfrac{a}{b}\right)\right)-a\,g\left(\tfrac{b}{a}\right)\right\rangle$$

$$\frac{1}{T_y} \approx 2\pi^{3/2}(\log)A\left\langle\frac{\mathcal{H}_y\sigma_H^2}{\varepsilon_y}\left(\frac{1}{a}g\left(\tfrac{b}{a}\right)+\frac{1}{b}g\left(\tfrac{a}{b}\right)\right)-b\,g\left(\tfrac{a}{b}\right)\right\rangle$$

include both "direct" effects of the scattering (which lead to damping of the transverse emittances, as momentum is transferred from the transverse to the longitudinal planes) and "indirect" effects (which lead to a growth of the transverse emittances, when the energy deviation is modified at locations of non-zero dispersion).

IBS, radiation damping and equilibrium emittances

Including IBS (with growth time T_x) and radiation (with damping time τ_x), the horizontal emittance evolves according to:

$$\frac{d\varepsilon_x}{dt} = -\frac{2}{\tau_x}\left(\varepsilon_x - \varepsilon_{x0}\right) + \frac{2}{T_x}\varepsilon_x$$

where ε_{x0} is the natural emittance (the equilibrium horizontal emittance with zero bunch charge). Note that the IBS growth time T_x is a function of the emittances (horizontal, vertical and longitudinal).

The equilibrium emittance for a given bunch charge is reached when:

$$\frac{d\varepsilon_x}{dt} = 0$$

which is achieved at an emittance ε'_{x0} given by:

$$\varepsilon'_{x0} = \frac{T_x}{T_x - \tau_x}\varepsilon_{x0}$$

For self-consistency, the IBS growth time should be evaluated at the equilibrium emittances ε'_{x0}, ε'_{y0} and the equilibrium bunch length and energy spread σ'_z, σ'_δ. In practice, we need to iterate the calculation (in all planes simultaneously) to find the equilibrium with IBS.

IBS emittance growth and betatron coupling

In the horizontal plane, there is large dispersion from the dipole magnets. This means that the "indirect" effects of IBS tend to dominate, and there is growth of emittance in both the longitudinal and the horizontal planes.

In a perfectly aligned storage ring, where there is no vertical dispersion and no betatron coupling, the only IBS effect is a "direct" effect which can shrink the vertical emittance (which would already be very small).

However, in general, there is a significant amount of vertical emittance generated by both vertical dispersion and betatron coupling. The effect of IBS on the vertical emittance depends on the relative contributions of the dispersion and the coupling…

IBS emittance growth and betatron coupling

If there is no vertical dispersion and the vertical emittance is generated entirely by betatron coupling, then the "indirect" effects of IBS in the vertical plane are small. However, the ratio of vertical to horizontal emittance remains constant; so any growth in the horizontal emittance appears in the same proportion in the vertical emittance.

If there is no betatron coupling, and the vertical emittance is generated entirely by vertical dispersion, then the IBS emittance growth in the vertical plane is independent of the emittance growth in the horizontal plane.

In general, if the betatron coupling contributes a fraction r of the vertical emittance, then the equilibrium vertical emittance with IBS is given by:

$$\varepsilon'_{y0} = \left[(1-r)\frac{T_y}{T_y - \tau_y} + r\frac{T_x}{T_x - \tau_x} \right] \varepsilon_{y0}$$

For the ILC damping rings, we generally assume that $r \approx 0.5$.

Observations of IBS emittance growth in the KEK-ATF

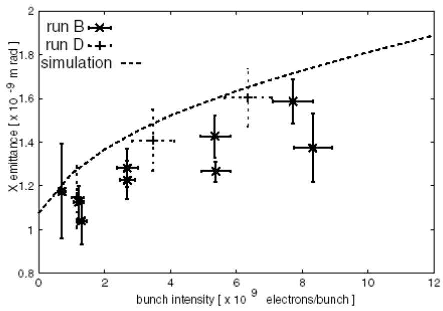


FIG. 3. Current dependence of the horizontal emittance: Data for the smallest emittance cases (runs B and D) are shown. The result of a SAD simulation for 0.4% coupling is superimposed.

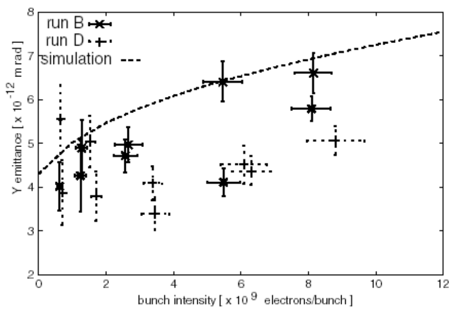
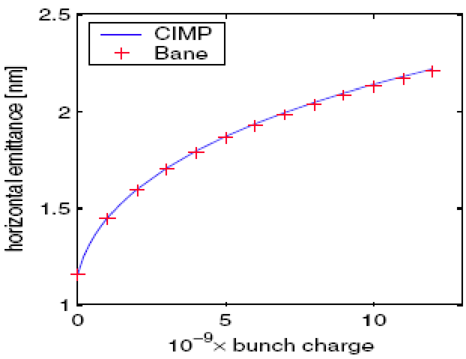
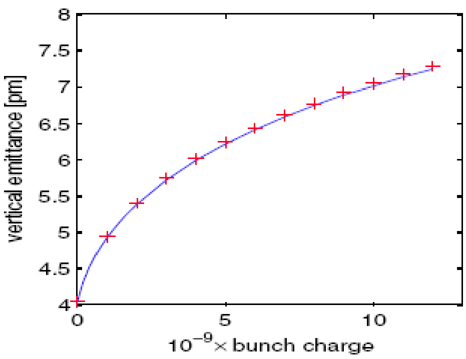


FIG. 2. Current dependence of the vertical emittance: Data for the smallest emittance cases (runs B and D) are shown. The result of a SAD simulation for 0.4% coupling is superimposed.





Observations of IBS emittance growth in the KEK-ATF

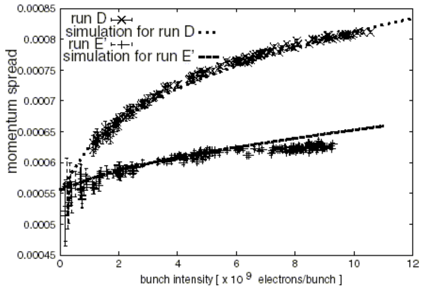


FIG. 4. Current dependence of the momentum spread: Data for the smallest (run D) and the largest emittance case (run E') are shown. (E' means that the data were taken in same condition as E, but on another day.) The results of SAD simulations for 0.4% and 6% coupling are superimposed.

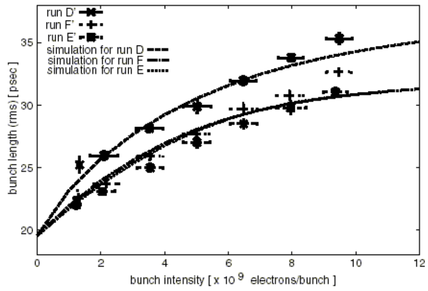
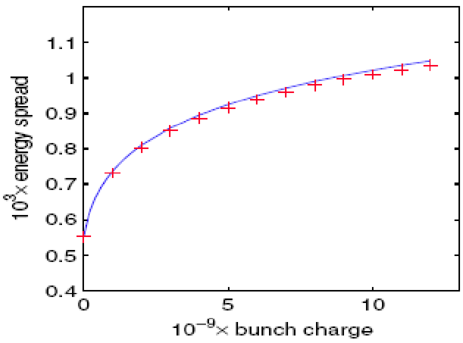
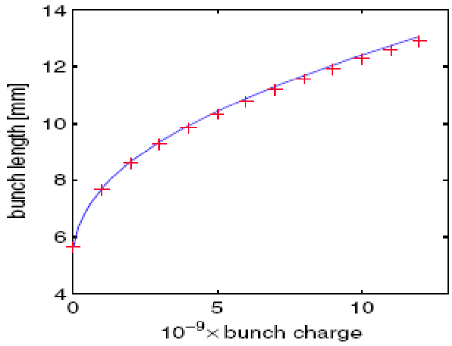


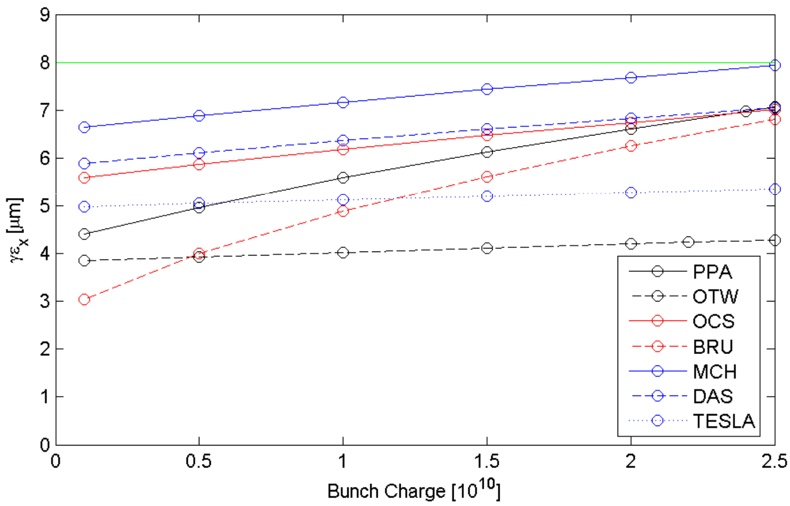
FIG. 5. Current dependence of the bunch length: Data are shown for the runs D', F', and E' (the symbols D', E', and F' indicate that the data were taken for the same condition as D, E, and F, but on another day.) The results of SAD simulations for 0.4%, 6%, and 3% coupling are superimposed.





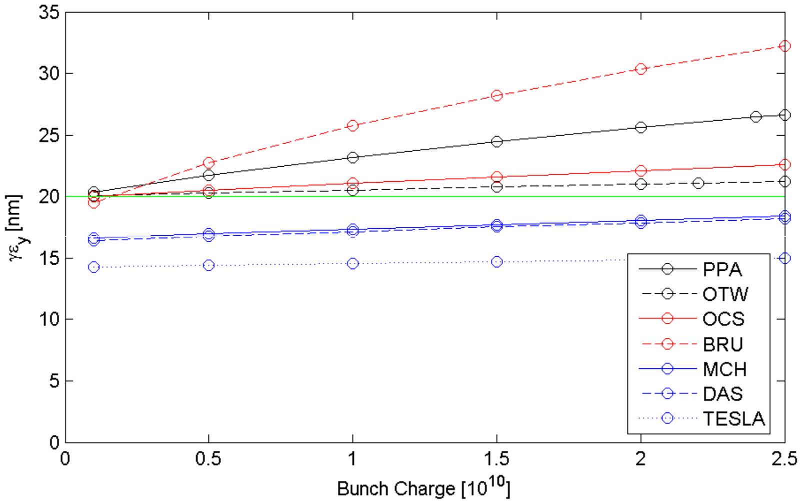
Predictions of IBS emittance growth for the ILC damping rings

Damping ring lattices are usually designed with a natural emittance below the specification for the extracted emittance (8 μm normalised) to allow for emittance growth from IBS (and other possible effects).

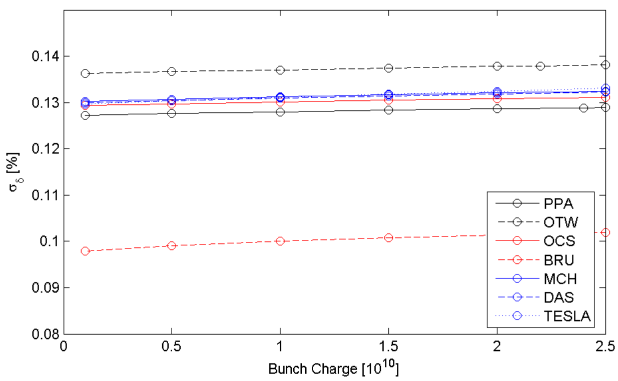


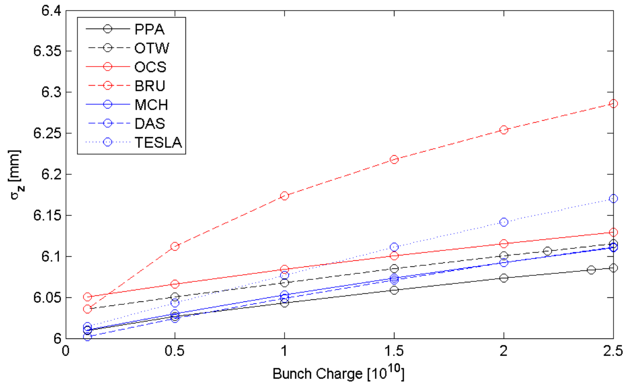
Predictions of IBS emittance growth for the ILC damping rings

Similarly, in the vertical plane, we may need to aim for coupling correction at the level to reduce the zero-charge vertical emittance below the nominal extracted emittance specification of 20 nm.



Predictions of IBS emittance growth for the ILC damping rings

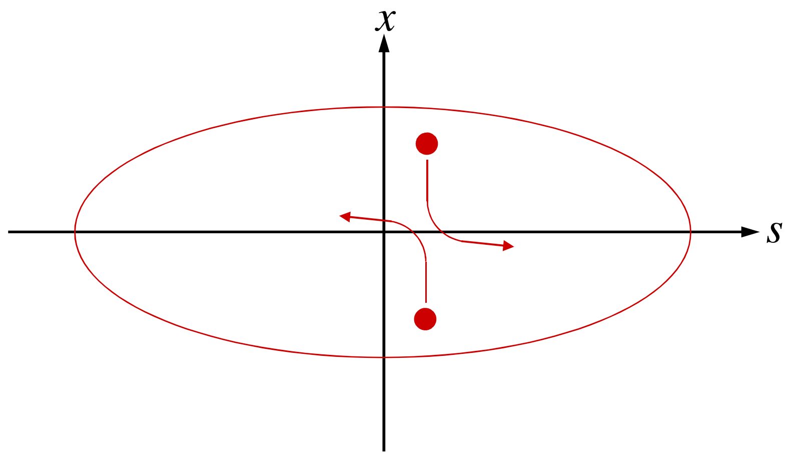




IBS effects in the longitudinal plane are expected to be small.

Touschek scattering

The Touschek effect is related to intrabeam scattering, but refers to scattering events in which there is a large transfer of momentum from the transverse to the longitudinal planes. IBS refers to multiple small-angle scattering; the Touschek effect refers to single large-angle scattering events.



If the change in longitudinal momentum is large enough, the energy deviation of one or both particles can be outside the energy acceptance of the ring, and the particles will be lost from the beam.

Touschek lifetime

Particle loss from the Touschek effect tends to be the dominant limitation on the beam lifetime in low-emittance storage rings, such as those in third-generation synchrotron light sources; and is expected to be the dominant limitation on lifetime in the ILC damping rings.

During regular operations, any given bunch is stored in the damping rings for only 200 ms. Generally, we expect a Touschek lifetime of the order of an hour; so Touschek scattering is not likely to be an operational limitation for the damping rings.

However, during commissioning and tuning, there are likely to be situations where we will want to work with a beam stored for a long period (for example, to avoid issues related to injection and extraction transients). For commissioning and tuning, a reasonable beam lifetime is desirable.

Touschek lifetime

We do not analyse Touschek scattering in detail, but (as for IBS) simply quote the result. The Touschek lifetime is given by:

$$\frac{1}{\tau} = -\frac{1}{N}\frac{dN}{dt} = \frac{r_e^2 cN}{8\pi\sigma_x\sigma_y\sigma_z}\frac{1}{\gamma^2\delta_{\max}^3} D\left(\left[\frac{\delta_{\max}\beta_x}{\gamma\sigma_x}\right]^2\right)$$

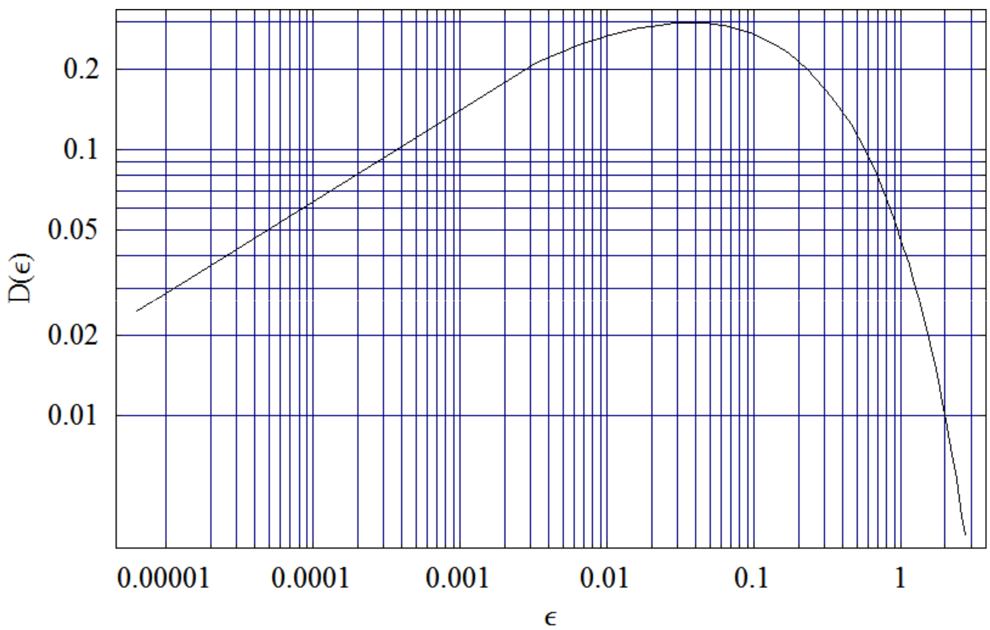
where N is the number of particles in a bunch, σ_x, σ_y, σ_z are the rms horizontal and vertical beam sizes and bunch length, and $\delta_{\max}$ is the energy acceptance of the ring.

Note that the energy acceptance may be limited by the RF acceptance (which depends on the RF voltage, and is typically 2% or more) or by the nonlinear dynamics (which may give a limitation as low as 1%).

The function $D(\varepsilon)$ is given by:

$$D(\varepsilon) = \sqrt{\varepsilon}\left[-\tfrac{3}{2}e^{-\varepsilon} + \frac{\varepsilon}{2}\int_\varepsilon^\infty \frac{\ln u}{u}e^{-u}du + \tfrac{1}{2}(3\varepsilon - \varepsilon\ln\varepsilon + 2)\int_\varepsilon^\infty \frac{e^{-u}}{u}du\right]$$

Touschek lifetime



Touschek lifetime in the ILC damping rings

The energy acceptance is generally a function of position in the lattice. However, we can make a rough estimate of the expected lifetime by assuming a fixed energy acceptance of 1%.

Lattice	Beam energy	Particles per bunch	Bunch length	Touschek lifetime
OCS	5 GeV	2×10^{10}	6 mm	33 min
BRU	3.74 GeV	2×10^{10}	9 mm	18 min
MCH	5 GeV	2×10^{10}	9 mm	68 min
TESLA	5 GeV	2×10^{10}	6 mm	50 min

Note that, in the parameter regime $(\varepsilon << 1)$ relevant for the damping rings:

$$D(\varepsilon) \propto \sqrt[3]{\varepsilon}$$

in which case:

$$\tau \propto \delta_{\max}^2$$

Intrabeam scattering and Touschek effect: conclusions

Intrabeam scattering will likely lead to some increase in the beam emittances in the ILC damping rings. We can design the ring with a natural emittance slightly lower than the specified extracted horizontal emittance to allow for IBS emittance increase.

In the vertical plane, we will likely need to do even better than the (already demanding) specification on the vertical emittance, to allow for IBS emittance growth.

IBS effects on the bunch length and energy spread are likely to be small in the ILC damping rings.

A reasonable Touschek lifetime is desirable for commissioning and tuning. With a 5 GeV beam energy and energy acceptance of 1%, we can achieve a beam lifetime between ½ hour and 1 hour.