



Lecture 7 RF linacs

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S-band (~3 GHz) RF linac





Translate circuit model to a cavity model: Directly driven, re-entrant RF cavity

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Properties of the RF pillbox cavity





- We want lowest mode: with only $\mathbf{E}_{z} \& \mathbf{B}_{\theta}$
- Maxwell's equations are:

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_{\theta}) = \frac{1}{c^2}\frac{\partial}{\partial t}E_z \quad \text{and} \quad \frac{\partial}{\partial r}E_z = \frac{\partial}{\partial t}B_{\theta}$$

Take derivatives

==>

$$\frac{\partial}{\partial t} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(rB_{\theta} \right) \right] = \frac{\partial}{\partial t} \left[\frac{\partial B_{\theta}}{\partial r} + \frac{B_{\theta}}{r} \right] = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}$$

$$\frac{\partial}{\partial r}\frac{\partial E_z}{\partial r} = \frac{\partial}{\partial r}\frac{\partial B_{\theta}}{\partial t}$$

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}$$

For a mode with frequency ω



✤ Therefore,

**

$$E_z'' + \frac{E_z'}{r} + \left(\frac{\omega}{c}\right)^2 E_z = 0$$

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 \succ (Bessel's equation, 0 order)

✤ Hence,

$$E_z(r) = E_o J_o\left(\frac{\omega}{c}r\right)$$

• For conducting walls, $E_z(R) = 0$, therefore

$$\frac{2\pi f}{c}b = 2.405$$



E-fields & equivalent circuits for T₀₂₀ modes



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E-fields & equivalent circuits for T_{ono} modes





T_{0n0} has n coupled, resonant circuits; each L & C reduced by 1/n

Simple consequences of pillbox model





- ✤ Increasing R lowers frequency
 => Stored Energy, $\mathscr{C} \sim \omega^{-2}$
- Beam loading lowers E_z for the next bunch
- Lowering ω lowers the fractional beam loading
- Raising ω lowers $Q \sim \omega^{-1/2}$
- * If time between beam pulses, $T_s \sim Q/\omega$ almost all \mathcal{E} is lost in the walls

Keeping energy out of higher order modes

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Choose cavity dimensions to stay far from crossovers

The beam tube makes the field modes (& cell design) more complicated



- Peak E no longer on axis
 - $E_{pk} \sim 2 3 \times E_{acc}$ $FOM = E_{pk}/E_{acc}$
- ω_0 more sensitive to cavity dimensions
 - Mechanical tuning & detuning

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- Beam tubes add length & €'s w/o acceleration
- Beam induced voltages $\sim a^{-3}$
 - Instabilities

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Cavity figures of merit

Figure of Merit: Accelerating voltage

The voltage varies during time that bunch takes to cross gap

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 \succ reduction of the peak voltage by Γ (transt time factor)



Compute the voltage gain correctly



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The voltage gain seen by the beam can computed in the co-moving frame, or we can use the transit-time factor, Γ & compute V at fixed time

$$V_o^2 = \Gamma \int_{z_1}^{z_2} E(z) dz$$

Figure of merit from circuits - Q



 $Q = \frac{\omega_o \circ Energy \ stored}{Time \ average \ power \ loss} = \frac{2\pi \circ Energy \ stored}{Energy \ lost \ per \ cycle}$

$$\mathscr{O} = \frac{\mu_o}{2} \int_{v} |H|^2 dv = \frac{1}{2} L I_o I_o^*$$
$$\langle \mathscr{O} \rangle = \frac{R_{surf}}{2} \int |H|^2 ds = \frac{1}{2} I_o I_o^* R_{surf}$$

$$R_{surf} = \frac{1}{Conductivity \circ Skin \ depth} \sim \omega^{1/2}$$

$$\therefore Q = \frac{\sqrt{L/C}}{R_{surf}} = \left(\frac{\Delta\omega}{\omega_o}\right)^{-1}$$

Measuring the energy stored in the cavity allows us to measure Q



✤ We have computed the field in the fundamental mode

$$U = \int_{0}^{d} dz \int_{0}^{b} dr 2\pi r \left(\frac{\varepsilon E_{o}^{2}}{2}\right) J_{1}^{2}(2.405r/b)$$
$$= b^{2} d \left(\varepsilon E_{o}^{2}/2\right) J_{1}^{2}(2.405)$$

- To measure Q we excite the cavity and measure the E field as a function of time
- Energy lost per half cycle = $U\pi Q$
- Note: energy can be stored in the higher order modes that deflect the beam

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Figure of merit for accelerating cavity: Power to produce the accelerating field



Resistive input (shunt) impedance at ω_o relates power dissipated in walls to accelerating voltage

$$R_{in} = \frac{\langle V^2(t) \rangle}{\mathscr{P}} = \frac{V_o^2}{2\mathscr{P}} = Q_v \sqrt{L/C}$$

Linac literature commonly defines "shunt impedance" without the "2"

$$\mathcal{R}_{in} = \frac{V_o^2}{\mathcal{P}} \sim \frac{1}{R_{surf}}$$

Typical values 25 - 50 $M\Omega$

Computing shunt impedance



 $\mathcal{R}_{in} = \frac{V_o^2}{\mathcal{P}}$ $\langle \mathcal{P} \rangle = \frac{R_{surf}}{2} \int_{s} |H|^2 ds$

$$R_{surf} = \frac{\mu\omega}{2\sigma_{dc}} = \pi Z_o \frac{\delta_{skin}}{\lambda_{rf}} \text{ where } Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 377\Omega$$

The on-axis field E and surface H are generally computed with a computer code such as SUPERFISH for a complicated cavity shape



Power the cavities so that $E_z(z,t) = E_z(z)e^{i\omega t}$





Return to the picture of the re-entrant cavity



- * Nose concentrate E_z near beam for fixed stored energy
- * Optimize nose cone to maximize V²; I.e., maximize R_{sh}/Q
- Make H-field region nearly spherical; raises Q & minimizes
 P for given stored energy



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Thus, linacs can be considered to be an array of distorted pillbox cavities...



In warm linacs "nose cones" optimize the voltage per cell with respect to resistive dissipation

Q =

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Usually cells are feed in groups not individually.... and

Linacs cells are linked to minimize cost





==> coupled oscillators ==>multiple modes





9-cavity TESLA cell





Example of 3 coupled cavities





 $x_0 \left(1 - \frac{\omega_0^2}{\Omega^2}\right) + x_1 k = 0 \qquad \text{oscillator } n = 0$ $x_1 \left(1 - \frac{\omega_0^2}{\Omega^2}\right) + (x_0 + x_2) \frac{k}{2} = 0 \qquad \text{oscillator } n = 1$

$$x_2\left(1-\frac{\omega_0^2}{\Omega^2}\right)+x_1k=0$$
 oscillator $n=2$

 $x_j = i_j \sqrt{2L_o}$ and Ω = normal mode frequency

Write the coupled circuit equations in matrix form



$$\mathbf{L}\mathbf{x}_{q} = \frac{1}{\boldsymbol{\Omega}_{q}^{2}}\mathbf{x}_{q} \quad \text{where} \quad \mathbf{L} = \begin{pmatrix} 1/\omega_{o}^{2} & k/\omega_{o}^{2} & 0\\ k/2\omega_{o}^{2} & 1/\omega_{o}^{2} & k/2\omega_{o}^{2}\\ 0 & k/\omega_{o}^{2} & 1/\omega_{o}^{2} \end{pmatrix} \quad \text{and} \quad \mathbf{x}_{q} = \begin{pmatrix} x_{1}\\ x_{2}\\ x_{3} \end{pmatrix}$$

Compute eigenvalues & eigenvectors to find the three normal modes

Mode q = 0: zero mode
$$\Omega_0 = \frac{\omega_o}{\sqrt{1+k}}$$
 $\mathbf{x}_0 = \begin{pmatrix} 1\\1\\1 \end{pmatrix}$
Mode q = 1: $\pi/2$ mode $\Omega_1 = \omega_o$ $\mathbf{x}_1 = \begin{pmatrix} 1\\0\\-1 \end{pmatrix}$
Mode q = 2: π mode $\Omega_2 = \frac{\omega_o}{\sqrt{1-k}}$ $\mathbf{x}_2 = \begin{pmatrix} 1\\-1\\1 \end{pmatrix}$

For a structure with N coupled cavities



- ➢ N normal modes, N frequencies
- From the equivalent circuit with magnetic coupling



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$$\omega_m = \frac{\omega_o}{\left(1 - B\cos\frac{m\pi}{N}\right)^{1/2}} \approx \omega_o \left(1 + B\cos\frac{m\pi}{N}\right)$$

where B= bandwidth (frequency difference between lowest & high frequency mode)

• Typically accelerators run in the π -mode



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The tuners change the frequencies by perturbing wall currents ==> changes the inductance ==> changes the energy stored in the magnetic field

$$\frac{\Delta\omega_o}{\omega_o} = \frac{\Delta U}{U}$$

Dispersion diagram for 5-cell structure

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Plii **Power exchange with resonant cavities**



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Beam power out

/ rf



- Define "wall quality factor", Q_w, & "external" quality factor, Q_e
- Power into the walls is $P_w = \omega U / Q_w$.
- If P_{in} is turned off, then the power flowing out $P_e = \omega U/Q_e$
- ♦ Net rate of energy loss = $\omega U/Q_w + \omega U/Q_e = \omega U/Q_{loaded}$

Till time & coupling

Loaded fill time

$$\Gamma_{\rm fill} = 2Q_{\rm L}/\omega$$

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• Critically coupled cavity: $P_{in} = P_w = 1/Q_e = 1/Q_w$

* In general, the coupling parameter $\beta = Q_w / Q_e$



Effects of the rf source & beam at resonance

Voltage produced by the generator is

$$V_{gr} = \frac{2\sqrt{\beta}}{1+\beta} \cdot \sqrt{R_{shunt}} P_{gen}$$

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The voltage produced by the beam is

$$V_{b,r} = \frac{i_{beam}}{Z_{tr}(1+\beta)} \approx \frac{I_{dc}R_{shunt}}{(1+\beta)}$$

Effects of the rf source & beam at resonance

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$$V_{accel} = \sqrt{R_{shunt}} P_{gen} \left[\frac{2\sqrt{\beta}}{1+\beta} \left(1 - \frac{K}{\sqrt{\beta}} \right) \right] = \sqrt{R_{shunt}} P_{wall}$$

where
$$K = \frac{I_{dc}}{2} \sqrt{\frac{R_{shunt}}{P_{gen}}}$$
 is the "loading factor"

 $\Rightarrow = V_{acc} \text{ decreases linearly with increasing beam current}$

Power flow in standing wave linac



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Efficiency of the standing wave linac



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What makes SC RF attractive?

Comparison of SC and NC RF



Superconducting RF

- High gradient
 => 1 GHz, meticulous care
- ♦ Mid-frequencies
 ==> Large stored energy, €
- ★ Large \mathcal{C}_s ==> very small $\Delta E/E$
- Large Q==> high efficiency

Normal Conductivity RF

- High gradient
 => high frequency (5 17 GHz)
- High frequency
 => low stored energy
- Low \mathscr{C}_{s} ==> ~10x larger $\Delta E/E$
- Low Q
 ==> reduced efficiency

Recall the circuit analog





As
$$R_{surf} = > 0$$
, the Q = $> \infty$.

In practice,

$$Q_{\rm nc} \sim 10^4$$
 $Q_{\rm sc} \sim 10^{11}$

Figure of merit for accelerating cavity: Power to produce the accelerating field



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$$R_{in} = \frac{\langle V^2(t) \rangle}{\mathscr{P}} = \frac{V_o^2}{2\mathscr{P}} = Q_v \sqrt{L/C}$$

Linac literature more commonly defines "shunt impedance" without the "2"

$$\mathcal{R}_{in} = \frac{V_o^2}{\mathcal{P}} \sim \frac{1}{R_{surf}}$$

For SC-rf *P* is reduced by orders of magnitude **BUT, it is deposited @ 2K**





Traveling wave linacs

Electromagnetic waves



From Maxwell equations, we can derive

$$\nabla^{2} E_{i} = \frac{\partial^{2} E_{i}}{\partial x^{2}} + \frac{\partial^{2} E_{i}}{\partial y^{2}} + \frac{\partial^{2} E_{i}}{\partial z^{2}} = \frac{1}{c^{2}} \frac{\partial^{2} E_{i}}{\partial t^{2}} \quad i = x, y, z$$

for electromagnetic waves in free space (no charge or current distributions present).

The plane wave is a particular solution of the EM wave equation

$$\overline{E} = \overline{E}_o e^{i(\omega t - ks)} = \overline{E}_o \left[\cos(\omega t - ks) + i \sin(\omega t - ks) \right]$$
Phase of the wave $=\phi$

when

$$\omega = c k$$

Dispersion (Brillouin) diagram: monochromatic plane wave



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The phase of this plane wave is constant for

$$\frac{d\phi}{dt} = \omega - k\frac{ds}{dt} \equiv \omega - kv_{ph} = 0$$

or

$$v_{ph} = \frac{\omega}{k} = c$$

Plane wave representation of EM waves

In more generality, we can represent an arbitrary wave as a sum of plane waves:

$$\overline{E} = \sum_{n=-\infty}^{\infty} \overline{E}_{no} e^{i(n\omega_0 t - ks)} \qquad \overline{E} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt f(\omega) e^{i(\omega t - ks)}$$

Periodic Case

Non-periodic Case

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Exercise: Can the plane wave accelerate the particle in the x-direction?



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Can accelerating structures be smooth waveguides?

✤ Assume the answer is "yes"

- Then $\mathbf{E} = \mathbf{E}(r, \theta) e^{i(\omega t kz)}$ with $\omega/k = v_{ph} < c$
- ♦ Transform to the frame co-moving at $v_{ph} < c$
- Then,
 - The structure is unchanged (by hypothesis)
 - > E is static (v_{ph} is zero in this frame)
 - ==> By Maxwell's equations, H =0
 - $\Longrightarrow \nabla \circ \mathbf{E} = 0$ and $\mathbf{E} = -\nabla \phi$

> But ϕ is constant at the walls (metallic boundary conditions) ==> $\mathbf{E} = 0$

The assumption is false, smooth structures have $v_{ph} > c$

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We need a longitudinal E-field to accelerate particles in vacuum



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- What about traveling waves?
 - \succ Waves guided by perfectly conducting walls can have E_{long}



- But first, think back to phase stability
 - To get continual acceleration the wave & the particle must stay in phase
 - ➤ Therefore, we can accelerate a charge with a wave with a synchronous phase velocity, v_{ph} ≈ v_{particle} < c</p>

University of Ljubljand **Propagating modes & equivalent circuits** FACULTY OF λ A Ro E All frequencies can propagate L2 TRi · (a) C_1 L_1 L2 L-C2 16, C_1 LI Propagation is cut-off C2 at low frequencies TM_{10} (c



d) ==> lower v_{ph}

Similar for TM01 mode in the waveguide



Magnetic flux lines appear as continuous loops Electric flux lines appear with beginning and end points

Figure source: <u>www.opamp-electronics.com/tutorials/waveguide</u> Lessons In Electric Circuits copyright (C) 2000-2002 Tony R. Kuphaldt



Weakly coupled pillboxes

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TM_{0n} modes

Dispersion relation for SLAC structure



Small changes in *a* lead to large reduction in v_g University of Ljubljana

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Notation

 $\beta_g = v_g/c = Relative group velocity$

 $E_a = Accelerating field (MV/m)$

 $E_s = Peak surface field (MV/m)$

 P_d = Power dissipated per length (MW/m)

 $P_t = Power transmitted (MW/m)$

w = Stored energy per length (J/m)

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Structure parameters for TW linacs

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$$r_{shunt} = \frac{E_a^2}{|dP_t/dz|} \quad (M\Omega/m)$$

$$Q = \frac{w\omega}{|dP_t/dz|}$$

$$\frac{r_{shunt}}{Q} = \frac{E_a^2}{w\omega}$$

$$s = \frac{E_a^w}{w}$$
 = Elastance (MQ/m/µs)
 W_{acc} = emergy/length for acceleration





In a structure with a constant geometry,

the inductance & capacitance per unit length are constant

==> constant impedance structure



Why do we need beams?



Collide beams



FOMs: Collision rate, energy stability, Accelerating field

Examples: LHC, ILC, RHIC





In LHC storage rings...

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- Limited space & Large rf trapping of particles
 V/cavity must be high
- ✤ Bunch length must be large (≤ 1 event/cm in luminous region)
 ▶ RF frequency must be low
- Energy lost in walls must be small
 - $> R_{surf}$ must be small

SC cavities were the only practical choice

For ILC, SC rf provides high power, low ε beams at high efficiency



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- To deliver required luminosity (500 fb⁻¹ in 4 years) ==>
 - powerful polarized electron & positron beams (11 MW /beam)
 - tiny beams at collision point ==> minimizing beam-structure interaction
- To limit power consumption ==> high "wall plug" to beam power efficiency
 - Even with SC rf, the site power is still 230 MW !

Why do we need beams?

Intense secondary beams





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1 MW target at SNS

FOM: Secondaries/primary Examples: spallation neutrons, neutrino beams

The Spallation Neutron Source



- I MW @ 1GeV (compare with ILC 11 MW at 500 GeV (upgradeable to 4 MW)
 - ==> miniscule beam loss into accelerator
 - ==> large aperture in cavities ==> large cavities
 - ==> low frequency
 - ==> high energy stability
 - ==>large stored energy
 - = high efficiency at E_z

==> SC RF



Matter to energy: Synchrotron radiation science



Synchrotron light source (pulsed incoherent X-ray emission)

FOM: Brilliance v. λ B = ph/s/mm²/mrad²/0.1%BW

Pulse duration

Science with X-rays Imaging Spectroscopy



Matter to energy: Energy Recovery Linacs Hard X-rays ==> ~5 GeV





Even higher brightness requires coherent emission ==> FEL

Free electron laser



FOM: Brightness v. λ Time structure



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Full range of FEL-based science requires...



- ✤ Pulses rates 10 Hz to 10 MHz (NC limited to ~ 100 Hz)
 - High efficiency
- Pulse duration 10 fs 1 ps
- ✤ High gain
 - Excellent beam emittance
 - ==> Minimize wakefield effect
 - ==> large aperture
 - ==> low frequency
 - Stable beam energy & intensity
 - ==> large stored energy in cavities
 - ==> high Q

 \implies SC RF