What do we mean by radiation?

- Energy is transmitted by the electromagnetic field to infinity
  - Applies in all inertial frames
  - Carried by an electromagnetic wave

- Source of the energy
  - Motion of charges
Schematic of electric field

Static charge
Particle moving in a straight line with constant velocity
Consider the fields from an electron with abrupt accelerations

- At $r = ct$, $\exists$ a transition region from one field to the other. At large $r$, the field in this layer becomes the radiation field.
Particle moving in a circle at constant speed

\[ dQ = q \, dl \]

Field energy flows to infinity
Remember that fields add, we can compute radiation from a charge twice as long.

\[ dQ = 2q \, dl \]

The wavelength of the radiation doubles.
All these radiate

Not quantitatively correct because $E$ is a vector; But we can see that the peak field hits the observer twice as often
Current loop: No radiation

Field is static

B field
QED approach: Why do particles radiate when accelerated?

- Charged particles in free space are “surrounded” by virtual photons
  - Appear & disappear & travel with the particles.

- Acceleration separates the charge from the photons & “kicks” photons onto the “mass shell”

- Lighter particles have less inertia & radiate photons more efficiently

- In the field of the dipoles in a synchrotron, charged particles move on a curved trajectory.
  - Transverse acceleration generates the *synchrotron radiation*

\[ \text{Electrons radiate } \sim \alpha \gamma \text{ photons per radian of turning} \]
Radiation field quickly separates itself from the Coulomb field

\[ P_\perp = \frac{q^2}{6\pi\varepsilon_0 m_0^2 c^3} \left( \frac{d\mathbf{p}_\perp}{dt} \right)^2 \]

Radiation field cannot separate itself from the Coulomb field

\[ P_\parallel = \frac{q^2}{6\pi\varepsilon_0 m_0^2 c^3} \left( \frac{d\mathbf{p}_\parallel}{dt} \right)^2 \]

negligible!

\[ P_\perp = \frac{c}{6\pi\varepsilon_0} q^2 \left( \frac{\beta\gamma}{\rho^2} \right)^4 \quad \rho = \text{curvature radius} \]

Radiated power for transverse acceleration increases dramatically with energy

Limits the maximum energy obtainable with a storage ring
Energy lost per turn by electrons

\[ \frac{dU}{dt} = -P_{SR} = - \frac{2 c r_e}{3 \left( m_0 c^2 \right)^3} \frac{E^4}{\rho^2} \quad \Rightarrow \quad U_0 = \int_{finite \ \rho} P_{SR} \, dt \] energy lost per turn

For relativistic electrons:

\[ s = \beta ct \equiv ct \quad \Rightarrow \quad dt = \frac{ds}{c} \quad U_0 = \frac{1}{c} \int_{finite \ \rho} P_{SR} \, ds = \frac{2 r_e E_0^4}{3 \left( m_0 c^2 \right)^3} \int_{finite \ \rho} \frac{ds}{\rho^2} \]

For dipole magnets with constant radius \( r \) (iso-magnetic case):

\[ U_0 = \frac{4 \pi r_e}{3 \left( m_0 c^2 \right)^3} \frac{E_0^4}{\rho} = \frac{e^2}{3 \varepsilon_0} \gamma^4 \]

The average radiated power is given by:

\[ \langle P_{SR} \rangle = \frac{U_0}{T_0} = \frac{4 \pi c r_e}{3 \left( m_0 c^2 \right)^3} \frac{E_0^4}{\rho L} \quad \text{where } L \equiv \text{ring circumference} \]
Energy loss to synchrotron radiation (practical units)

Energy Loss per turn (per particle)

\[ U_{o,\text{electron}} (\text{keV}) = \frac{e^2 \gamma^4}{3 \varepsilon_0 \rho} = 88.46 \frac{E(\text{GeV})^4}{\rho(m)} \]

\[ U_{o,\text{proton}} (\text{keV}) = \frac{e^2 \gamma^4}{3 \varepsilon_0 \rho} = 6.03 \frac{E(\text{TeV})^4}{\rho(m)} \]

Power radiated by a beam of average current \( I_b \): to be restored by RF system

\[ P_{\text{electron}} (\text{kW}) = \frac{e \gamma^4}{3 \varepsilon_0 \rho} I_b = 88.46 \frac{E(\text{GeV})^4 I(A)}{\rho(m)} \]

\[ P_{\text{proton}} (\text{kW}) = \frac{e \gamma^4}{3 \varepsilon_0 \rho} I_b = 6.03 \frac{E(\text{TeV})^4 I(A)}{\rho(m)} \]

Power radiated by a beam of average current \( I_b \) in a dipole of length \( L \) (energy loss per second)

\[ P_e (\text{kW}) = \frac{e \gamma^4}{6\pi \varepsilon_0 \rho^2} L I_b = 14.08 \frac{L(m) I(A) E(\text{GeV})^4}{\rho(m)^2} \]
Radiation is emitted in a cone of angle $1/\gamma$

Therefore the radiation that sweeps the observer is emitted by the particle during the retarded time period

$$\Delta t_{ret} \approx \frac{\rho}{\gamma c}$$

Assume that $\gamma$ and $\rho$ do not change appreciably during $\Delta t$.

At the observer

$$\Delta t_{obs} = \Delta t_{ret} \frac{dt_{obs}}{dt_{ret}} = \frac{1}{\gamma^2} \Delta t_{ret}$$

Therefore the observer sees $\Delta \omega \sim 1/\Delta t_{obs}$

$$\Delta \omega \sim \frac{c}{\gamma^3} \frac{\gamma^3}{\rho}$$
Critical frequency and critical angle

\[ \frac{d^3I}{d\Omega d\omega} = \frac{e^2}{16\pi^3\varepsilon_0 c} \left( \frac{2\omega \rho}{3c\gamma^2} \right)^2 \left( 1 + \gamma^2 \theta^2 \right)^2 \left[ K_{2/3}(\xi) + \frac{\gamma^2 \theta^2}{1 + \gamma^2 \theta^2} K_{1/3}(\xi) \right] \]

Properties of the modified Bessel function \( \Rightarrow \) radiation intensity is negligible for \( x >> 1 \)

\[ \xi = \frac{\omega \rho}{3c\gamma^3} \left( 1 + \gamma^2 \theta^2 \right)^{3/2} >> 1 \]

Critical frequency \( \omega_c = \frac{3c}{2\rho} \gamma^3 \)

\( \approx \omega_{rev} \gamma^3 \)

Critical angle \( \theta_c = \frac{1}{\gamma} \left( \frac{\omega_c}{\omega} \right)^{1/3} \)

For frequencies much larger than the critical frequency and angles much larger than the critical angle the synchrotron radiation emission is negligible.
Integrate over all angles $\Rightarrow$
Frequency distribution of radiation

The integrated spectral density up to the critical frequency contains half of the total energy radiated, the peak occurs approximately at $0.3\omega_c$

where the critical photon energy is

$$\varepsilon_c = \hbar \omega_c = \frac{3}{2} \frac{\hbar c}{\rho} \gamma^3$$

For *electrons*, the critical energy in practical units is

$$\varepsilon_c [\text{keV}] = 2.218 \frac{E[GeV]^3}{\rho[m]} = 0.665 \cdot E[GeV]^2 \cdot B[T]$$
Number of photons emitted

- Since the energy lost per turn is
  \[ U_0 \sim \frac{e^2 \gamma^4}{\rho} \]
- And average energy per photon is the
  \[ \langle \varepsilon_\gamma \rangle \approx \frac{1}{3} \varepsilon_c = \frac{\hbar \omega_c}{3} = \frac{1}{2} \frac{\hbar c}{\rho} \gamma^3 \]
- The average number of photons emitted per revolution is
  \[ \langle n_\gamma \rangle \approx 2\pi \alpha_{\text{fine}} \gamma \]
### Comparison of S.R. Characteristics

<table>
<thead>
<tr>
<th></th>
<th>LEP200</th>
<th>LHC</th>
<th>SSC</th>
<th>HERA</th>
<th>VLHC</th>
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<tbody>
<tr>
<td>Beam particle</td>
<td>e⁺ e⁻</td>
<td>p</td>
<td>p</td>
<td>p</td>
<td>p</td>
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<tr>
<td>Circumference</td>
<td>km</td>
<td>26.7</td>
<td>26.7</td>
<td>82.9</td>
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<td>Beam energy</td>
<td>TeV</td>
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<td>Beam current</td>
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<td>Critical energy of SR</td>
<td>eV</td>
<td>7 x 10⁵</td>
<td>44</td>
<td>284</td>
<td>0.34</td>
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<td>SR power (total)</td>
<td>kW</td>
<td>1.7 x 10⁴</td>
<td>7.5</td>
<td>8.8</td>
<td>3 x 10⁴</td>
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<tr>
<td>Linear power density</td>
<td>W/m</td>
<td>882</td>
<td>0.22</td>
<td>0.14</td>
<td>8 x 10⁻⁵</td>
</tr>
<tr>
<td>Desorbing photons</td>
<td>s⁻¹ m⁻¹</td>
<td>2.4 x 10¹⁶</td>
<td>1 x 10¹⁷</td>
<td>6.6 x 10¹⁵</td>
<td>none</td>
</tr>
</tbody>
</table>

From: O. Grobner CERN-LHC/VAC VLHC Workshop Sept. 2008
Synchrotron radiation plays a major role in electron storage ring dynamics

- Charged particles radiate when accelerated
- Transverse acceleration induces significant radiation (synchrotron radiation) while longitudinal acceleration generates negligible radiation ($1/\gamma^2$).

\[
\frac{dU}{dt} = -P_{SR} = -\frac{2c r_e}{3(m_0c^2)^3} \frac{E^4}{\rho^2}
\]

$r_e$ = classical electron radius
$\rho$ = trajectory curvature

\[
U_0 = \int_{\text{finite } \rho} P_{SR} \, dt \quad \text{energy lost per turn}
\]

\[
\alpha_D = -\left. \frac{1}{2T_0} \frac{dU}{dE} \right|_{E_0} = \frac{1}{2T_0} \frac{d}{dE}\left[ \oint P_{SR}(E_0) \, dt \right]
\]

$\alpha_{DX}, \alpha_{DY}$ = damping in all planes

\[
\frac{\sigma_p}{p_0} = \frac{\epsilon_X, \epsilon_Y}{\epsilon X, \epsilon Y}
\]

$\sigma_p$ = equilibrium momentum spread
$p_0$ = equilibrium momentum
$\epsilon_X, \epsilon_Y$ = equilibrium emittances
Particles change energy according to the phase of the field in the RF cavity

\[ \Delta E = eV(t) = eV_o \sin(\omega_{RF}t) \]

For the synchronous particle

\[ \Delta E = U_0 = eV_0 \sin(\varphi_s) \]
Energy loss + dispersion ==> Longitudinal (synchrotron) oscillations

Longitudinal dynamics are described by

1) \( \varepsilon \), energy deviation, w.r.t the synchronous particle
2) \( \tau \), time delay w.r.t. the synchronous particle

\[
\varepsilon' = \frac{qV_0}{L} \left[ \sin(\phi_s + \omega \tau) - \sin \phi_s \right] \quad \text{and} \quad \tau' = -\frac{\alpha_c}{E_s} \varepsilon
\]

Linearized equations describe elliptical phase space trajectories

\[
\varepsilon' = \frac{e}{T_0} \frac{dV}{dt} \tau \quad \quad \tau' = -\frac{\alpha_c}{E_s} \varepsilon
\]

\[
\omega_s^2 = \frac{\alpha_c eV}{T_0 E_0} \quad \text{angular synchrotron frequency}
\]
Radiation damping of energy fluctuations

The derivative \( \frac{dU_0}{dE} > 0 \) is responsible for the damping of the longitudinal oscillations.

Combine the two equations for \((\varepsilon, \tau)\) in a single 2nd order differential equation:

\[
\frac{d^2 \varepsilon}{dt^2} + \frac{2}{\tau_s} \frac{d\varepsilon}{dt} + \omega_s^2 \varepsilon = 0 \quad \rightarrow \quad \varepsilon = Ae^{-t/\tau_s} \sin \left( \sqrt{\omega_s^2 - \frac{4}{\tau_s^2}} t + \varphi \right)
\]

- \( \omega_s^2 = \frac{\alpha e V}{T_0 E_0} \) \( \text{angular synchrotron frequency} \)
- \( \frac{1}{\tau_s} = \frac{1}{2T_0} \frac{dU_0}{dE} \) \( \text{longitudinal damping time} \)
Damping times

- The energy damping time ~ the time for beam to radiate its original energy

- Typically

\[ T_i = \frac{4\pi R\rho}{C_\gamma J_i E_o^3} \]

- Where \( J_e \approx 2, \ J_x \approx 1, \ J_y \approx 1 \) and \( C_\gamma = 8.9 \times 10^{-5} \text{ meter} - \text{GeV}^{-3} \)

- Note \( \Sigma J_i = 4 \) (partition theorem)
Quantum Nature of Synchrotron Radiation

- Synchrotron radiation induces damping in all planes.
  - Collapse of beam to a single point is prevented by the quantum nature of synchrotron radiation.

- Photons are randomly emitted in quanta of discrete energy.
  - Every time a photon is emitted the parent electron “jumps” in energy and angle.

- Radiation perturbs excites oscillations in all the planes.
  - Oscillations grow until reaching equilibrium balanced by radiation damping.
Energy fluctuations

- Expected $\Delta E_{\text{quantum}}$ comes from the deviation of $\langle N_\gamma \rangle$ emitted in one damping time, $\tau_E$

- $\langle N_\gamma \rangle = n_\gamma \tau_E$

  $\implies \Delta \langle N_\gamma \rangle = (n_\gamma \tau_E)^{1/2}$

- The mean energy of each quantum $\sim \varepsilon_{\text{crit}}$

- $\implies \sigma_\varepsilon = \varepsilon_{\text{crit}}(n_\gamma \tau_E)^{1/2}$

- Note that $n_\gamma = P_\gamma / \varepsilon_{\text{crit}}$ and $\tau_E = E_0 / P_\gamma$
Therefore, …

- The quantum nature of synchrotron radiation emission generates energy fluctuations

\[
\frac{\Delta E}{E} \approx \left( \frac{E_{\text{crit}} E_o}{E_o} \right)^{1/2} \approx \frac{C_q \gamma_o^2}{J \epsilon \rho_{\text{curv}} E_o} \approx \frac{\gamma}{\rho}
\]

where \(C_q\) is the Compton wavelength of the electron

\[C_q = 3.8 \times 10^{-13} \text{ m}\]

- Bunch length is set by the momentum compaction & \(V_{\text{rf}}\)

\[\sigma_z^2 = 2\pi \left( \frac{\Delta E}{E} \right) \frac{\alpha_c R E_o}{e \dot{V}}\]

- Using a harmonic rf-cavity can produce shorter bunches
Transverse cooling:

- Passage through dipoles: $P_{\perp} \text{ less } P_{\parallel} \text{ less}$
- Acceleration in RF cavity: $P_{\perp} \text{ remains less } P_{\parallel} \text{ restored}$

Limited by quantum excitation
At equilibrium the momentum spread is given by:

\[
\left( \frac{\sigma_p}{p_0} \right)^2 = \frac{C_q \gamma_0^2}{J_s} \int 1/\rho^3 \, ds \quad \text{where} \quad C_q = 3.84 \times 10^{-13} \ m
\]

For the horizontal emittance at equilibrium:

\[
\varepsilon = C_q \frac{\gamma_0^2}{J_x} \int H/\rho^3 \, ds \quad \text{where:} \quad H(s) = \beta_T D'^2 + \gamma_T D^2 + 2\alpha_T DD'
\]

In the vertical plane, when no vertical bend is present, the synchrotron radiation contribution to the equilibrium vertical emittance is very small.

Vertical emittance is defined by machine imperfections & nonlinearities that couple the horizontal & vertical planes:

\[
\varepsilon_Y = \frac{\kappa}{\kappa + 1} \varepsilon \quad \text{and} \quad \varepsilon_X = \frac{1}{\kappa + 1} \varepsilon \quad \text{with} \quad \kappa \equiv \text{coupling factor}
\]
Equilibrium emittance & $\Delta E$

- Set

\[ \text{Growth rate due to fluctuations (linear)} = \text{exponential damping rate due to radiation} \]

\[ \Rightarrow \text{equilibrium value of emittance or } \Delta E \sim \gamma^2 \theta^3 \]

\[ \varepsilon_{\text{natural}} = \varepsilon_1 e^{-2t/\tau_d} + \varepsilon_{eq} \left(1 - e^{-2t/\tau_d}\right) \]
Quantum lifetime

- At a fixed observation point, transverse particle motion looks sinusoidal
  \[ x_T = a \sqrt{\beta_n} \sin(\omega \beta_n t + \varphi) \quad T = x, y \]

- Tunes are chosen in order to avoid resonances.
  - At a fixed azimuth, turn-after-turn a particle sweeps all possible positions within the envelope

- Photon emission randomly changes the “invariant” \( a \)
  - Consequently changes the trajectory envelope as well.

- Cumulative photon emission can bring the envelope beyond acceptance at some azimuth
  - The particle is lost

This mechanism is called the transverse quantum lifetime
Several time scales govern particle dynamics in storage rings

- **Damping:** several ms for electrons, ~ infinity for heavier particles
- **Synchrotron oscillations:** ~ tens of ms
- **Revolution period:** ~ hundreds of ns to ms
- **Betatron oscillations:** ~ tens of ns
Interaction of Photons with Matter

Radiography

Diffraction

Photoelectric Effect

Compton Effect

Compton Scattering
Brightness of a Light Source

- Brightness is a principal characteristic of a particle source
  - Density of particle in the 6-D phase space
- Same definition applies to photon beams
  - Photons are bosons & the Pauli exclusion principle does not apply
  - Quantum mechanics does not limit achievable photon brightness

Brightness = \# of photons in given $\Delta \lambda / \lambda$ sec, mrad $\theta$, mrad $\varphi$, mm$^2$

Flux = \# of photons in given $\Delta \lambda / \lambda$ sec

$$\text{Flux} = \frac{d\dot{N}}{d\lambda} = \int \text{Brightness} \, dS \, d\Omega$$
Spectral brightness is that portion of the brightness lying within a relative spectral bandwidth $\Delta \omega / \omega$:

\[ B \]

\[ \frac{\Delta \omega}{\omega} \]

\[ h \omega \]

\[ \text{Photon energy} \]

\[ 10 \text{ eV} \quad 100 \text{ eV} \quad 1 \text{ keV} \quad 10 \text{ keV} \quad 100 \text{ keV} \]

\[ 12 \text{ nm} \quad 1.2 \text{ nm} \quad 0.12 \text{ nm} \]

- 6-8 GeV Undulators
- 1-2 GeV Undulators
- Wigglers
- Bending magnets
How bright is a synchrotron light source?

Remark: the sources are compared at different wavelengths!
Angular distribution of SR

When the electron velocity approaches the velocity of light, the emission pattern is folded sharply forward.
Energy dependence of SR spectrum

SPECTRAL DISTRIBUTION OF SYNCHROTRON RADIATION FROM SPEAR ($\rho = 12.7$ m)

- $\epsilon_c = 0.58$ KeV
- $E_\theta = 1.5$ GeV
Spectrum available using SR

- See smaller features
- Write smaller patterns
- Elemental and chemical sensitivity
Two ways to produce radiation from highly relativistic electrons

Synchrotron radiation

- $10^{10}$ brighter than the most powerful (compact) laboratory source
- An x-ray “light bulb” in that it radiates all “colors” (wavelengths, photons energies)

Undulator radiation

- Lasers exist for the IR, visible, UV, VUV, and EUV
- Undulator radiation is quasi-monochromatic and highly directional, approximating many of the desired properties of an x-ray laser
Relativistic electrons radiate in a narrow cone

Dipole radiation

Frame of reference moving with electrons

Laboratory frame of reference

\[ k' = 2\pi/\lambda' \]

\[ k_z = 2\gamma k_z' \text{(Relativistic Doppler shift)} \]

\[ \theta = \frac{k_x}{k_z} = \frac{k_x'}{2\gamma k_z'} = \frac{\tan \theta'}{2\gamma} = \frac{1}{2\gamma} \]
Third generation light sources have long straight sections & bright e-beams

- Many straight sections for undulators and wigglers
- Brighter radiation for spatially resolved studies (smaller beam more suitable for microscopies)
- Interesting coherence properties at very short wavelengths
Light sources provide three types of SR:

- **Bending magnet radiation**
- **Wiggler radiation**
- **Undulator radiation**
Bend magnet radiation

- **Advantages:**
  - Broad spectral range
  - Least expensive
  - Most accessible
    - Many beamlines

- **Disadvantages:**
  - Limit coverage of hard X-rays
  - Not as bright at undulator radiation

\[ E_c(\text{keV}) = 0.6650E_e^2(\text{GeV})B(\text{T}) \]
For brighter X-rays add the radiation from many small bends

Magnetic undulator (N periods)

Relativistic electron beam, \( E_e = \gamma mc^2 \)

\[ \lambda \approx \frac{\lambda_u}{2\gamma^2} \]

\[ \theta_{cen} \approx \frac{1}{\gamma \sqrt{N}} \]

\[ \left[ \frac{\Delta \lambda}{\lambda} \right]_{cen} = \frac{1}{N} \]

Brightness = \( \frac{\text{photon flux}}{(\Delta A) (\Delta \Omega)} \)

Spectral Brightness = \( \frac{\text{photon flux}}{(\Delta A) (\Delta \Omega) (\Delta \lambda/\lambda)} \)
Undulator radiation: What is $\lambda_{\text{rad}}$?

An electron in the lab oscillating at frequency $f$, emits dipole radiation of frequency $f$.

What about the relativistic electron?
Power in the central cone of undulator radiation

\[ \lambda_x = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right) \]

\[ \bar{P}_{\text{cen}} = \frac{\pi e\gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + \frac{K^2}{2})^2} f(K) \]

\[ \theta_{\text{cen}} = \frac{1}{\gamma^* \sqrt{N}} \]

\[ \left( \frac{\Delta \lambda}{\lambda} \right)_{\text{cen}} = \frac{1}{N} \]

\[ K = \frac{eB_0 \lambda_u}{2\pi m_0 c} \]

\[ \gamma^* = \gamma / \sqrt{1 + \frac{K^2}{2}} \]
Spatial coherence of undulator radiation

\[ \lambda = 13.4 \text{ nm} \]

\[ \lambda = 2.5 \text{ nm} \]

\[ d \cdot \theta = \frac{\lambda}{2\pi} \]

Courtesy of Patrick Naulleau, LBNL / Kris Rosfjord, UCB and LBNL
Characteristics of wiggler radiation

- For $K \gg 1$, radiation appears in high harmonics, & at large horizontal angles $\theta = \pm K/\gamma$
  - One tends to use larger collection angles, which tends to spectrally merge nearby harmonics.
  - Continuum at high photon energies, similar bend magnet radiation,
    - Increased by $2N$ (the number of magnet pole pieces).

\[ \lambda_u = 5 \text{ cm}, N = 89 \]

\[ \text{Undulator radiation (} K \leq 1 \) } 
- Narrow spectral lines
- High spectral brightness
- Partial coherence

\[ \lambda = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right) \]

\[ K = \frac{eB_0\lambda_u}{2\pi m c} \]

\[ \text{Wiggler radiation (} K \gg 1 \) } 
- Higher photon energies
- Spectral continuum
- Higher photon flux ($2N$)

\[ \hbar \omega_c = \frac{3}{2} \frac{\hbar^2 e B_0}{m} \]

\[ n_c = \frac{3K}{4} \left( 1 + \frac{K^2}{2} \right) \]

(Courtesy of K.-J. Kim)
X-ray beamlines transport the photons to the sample

- Photon flux versus photon energy ($h\omega$)
- Grating or crystal
- Monochromator
- Exit slit
- Curved focusing mirror (glancing incidence reflection)

Observe at sample:
- Absorption spectra
- Photoelectron spectra
- Diffraction
- Focusing lens (pair of curved mirrors, zone plate lens, etc.)
To get brighter beams we need another great invention

- The Free Electron Laser (John Madey, Stanford, 1976)

- Physics basis: *Bunched electrons radiate coherently*

- Madey’s discovery: the bunching can be self-induced!
Coherent emission $\Rightarrow$ Free Electron Laser

- 100 MeV linac
- Injector
- Buncher
- High energy linac
- Long wiggler
- To experiment
- Dump

See movie
Laser interaction in the wiggler manipulates electron beam in longitudinal phase

Electron trajectory through wiggler with two periods

In resonance the electrons always “run uphill” against the E field

Energy lost from the electrons augments the electromagnetic field
Electrons see a potential

\[ V(x) \sim |A| \left( 1 - \cos(x + \varphi) \right) \]

where

\[ A \propto B_w \lambda_w E_{\text{laser}} \]

and \( \varphi \) is the phase between the electrons and the laser field.

Imagine an electron part way up the potential well but falling toward the potential minimum at \( \theta = 0 \)

- Energy radiated by the electron increases the laser field & consequently lowers the minimum further.
- Electrons moving up the potential well decrease the laser field.
The equations of motion

- The electrons move according to the pendulum equation
  \[ \frac{d^2 x}{dt^2} = |A| \sin(x + \varphi) \]

- The field varies as
  \[ \frac{dA}{dt} = -J \langle e^{-ix} \rangle \]

where \( x = (k_w - k) z - \omega t \)

The simulation will show us the bunching and signal growth

DownLoad: FEODe.zip
Resonance condition:
- Slip one optical period per wiggler period
- FEL bunches beam on an optical wavelength at \textit{ALL} harmonics
  - Bonifacio et al. NIM A293, Aug. 1990

Gain-bandwidth & efficiency $\sim \rho$
- Gain induces $\Delta E \sim \rho$

\[
\rho = \frac{1}{\gamma} \left( \frac{a_w \omega_p}{4 c k_w} \right)^{2/3} \propto \frac{I^{1/3} B^{2/3} \lambda_{\nu_w}^{4/3}}{\gamma}
\]

1) Emittance constraint
   - Match beam phase area to diffraction limited optical beam

2) Energy spread condition
   - Keep electrons from debunching

3) Gain must be faster than diffraction
FOM 1 from condensed matter studies:
Light source brilliance v. photon energy

Duty factor correction for pulsed linacs
Near term: x-rays from betatron motion and Thomson scattering

Betatron oscillations:

\[ \lambda_x = \frac{\lambda_u}{2\gamma^2} \]

Radiation pulse duration = bunch duration

Strength parameter

Betatron: \( a_\beta = \pi (2\gamma)^{1/2} r_\beta / \lambda_p \)

Thomson scattering: \( a_0 = e / mc^2 A \)

Potential Thompson source from all optical accelerator

Schoenlein et al., Science 1996
Leemans et al., PRL 1996

Gas jet

fs - laser pulse

femtosecond electron beam
50 MeV (γ=98)
4.5 nC/pulse
15 fs

femtosecond laser pulse undulator
800 nm, 50 fs, 100 mJ

femtosecond x-ray pulses
< 20 fs
10 - 30 keV
> 10^5 photons /pulse
Δθ ~ 10 mrad

Experimental sample