Chapter 4:

RF/Microwave interaction and beam loading in SRF cavity

4.1 RF field in SRF cavity

4.2 Beam loading

4.3 Dynamic detuning (microphonics, Lorentz force detuning, etc)

4.4 Basics on RF control

-develop equivalent circuit for rf system, cavity and beam

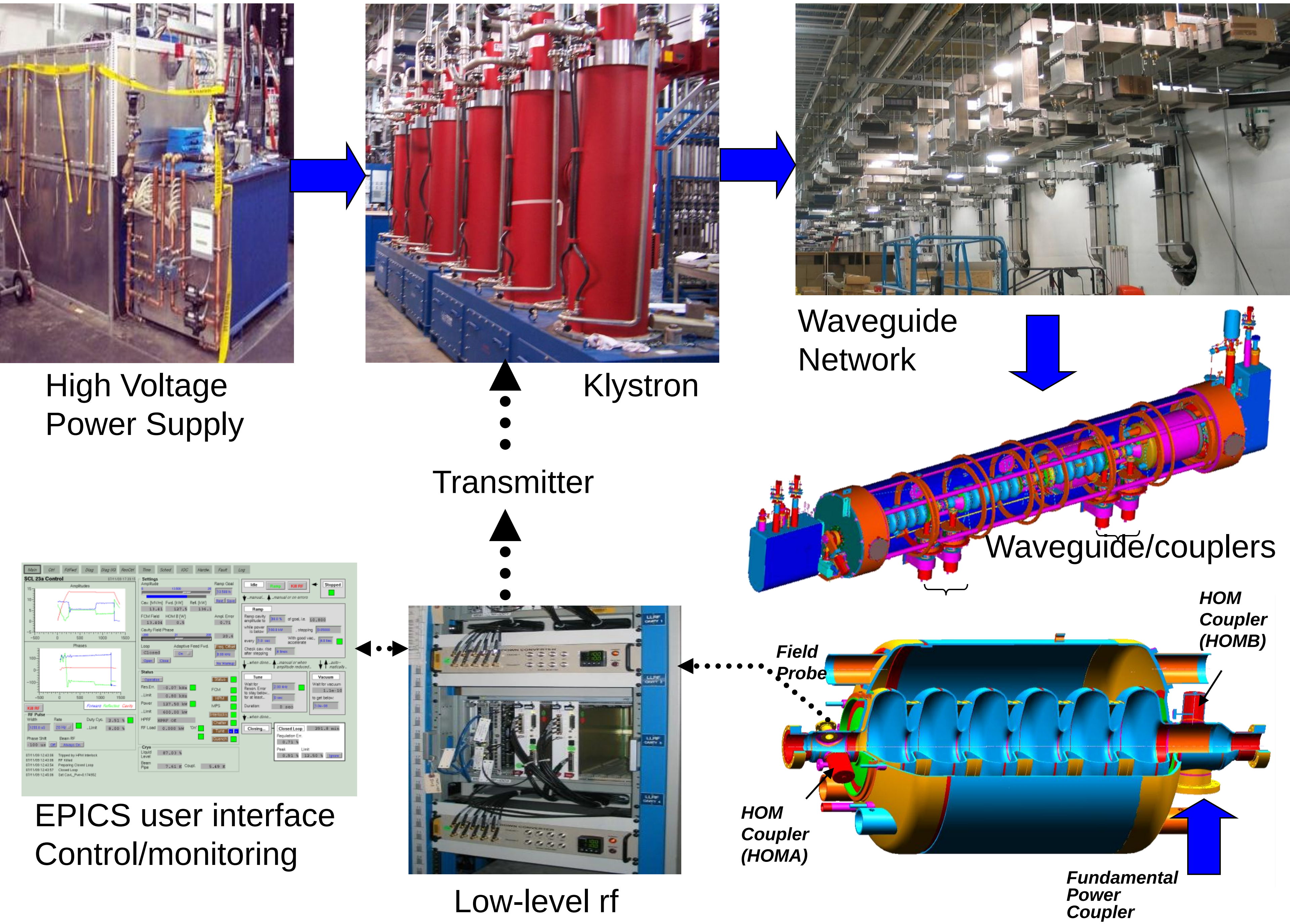
-develop equations for steady state and transient

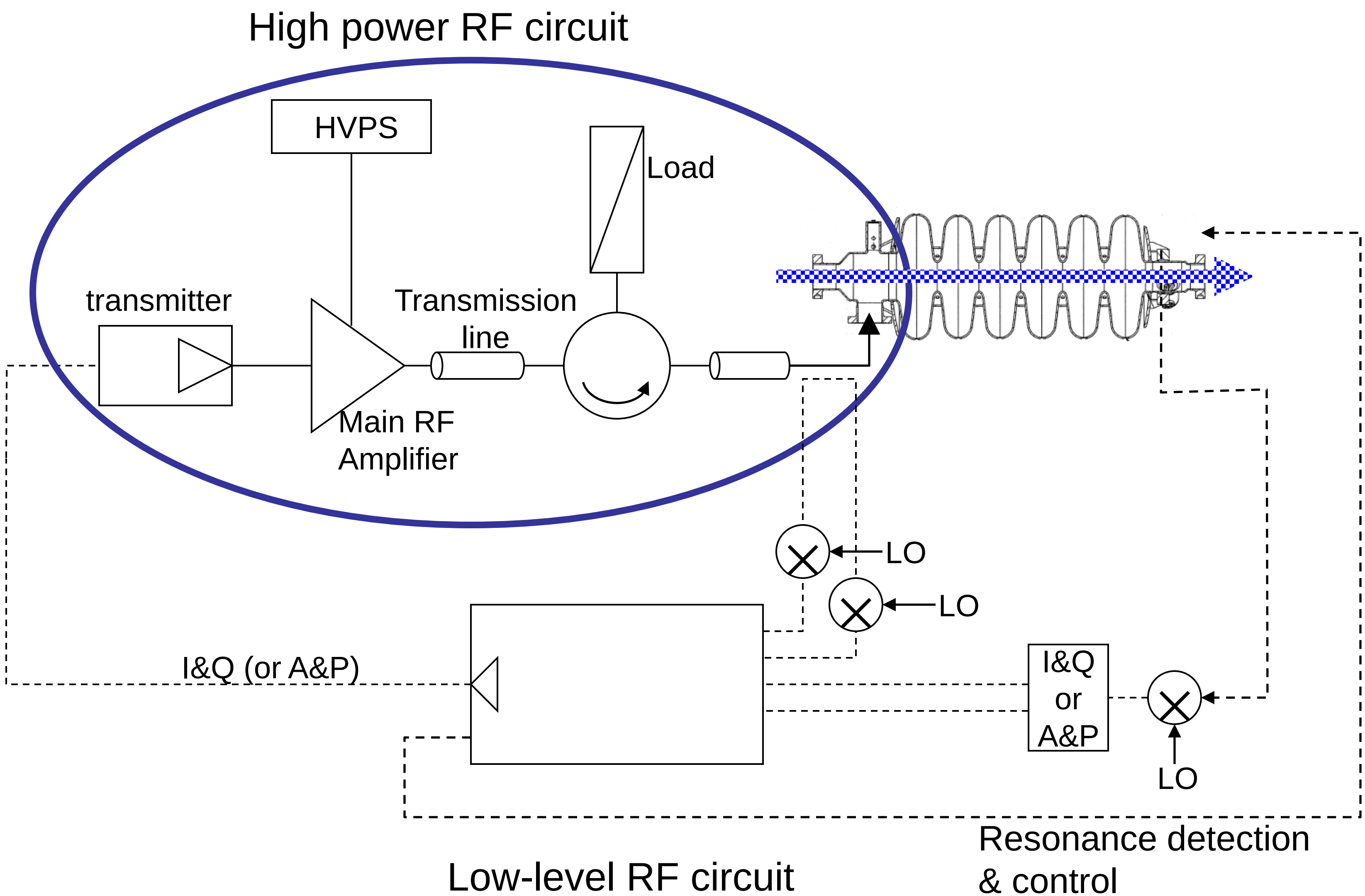
-develop concept for the LLRF control

RF circuit modeling

RF components

- RF source; klystrons are the most popular devices for f>300MHz.
  - tetrode, solid state amplifier for low power and/or low frequency
- RF transmission; Waveguides or coaxial cables
- Circulator; usually used as an isolator with matched load to protect RF source
- Power coupler; feed RF power to a cavity
- Cavity; electro-magnetic energy storage device
- RF control; control cavity field and phase


High Voltage
Power Supply
Klystron
Waveguide
Network
Transmitter
Waveguide/couplers
EPICS user interface
Control/monitoring
Low-level rf
Field
Probe
HOM
Coupler
(HOMB)
HOM
Coupler
(HOMA)
Fundamental
Power
Coupler



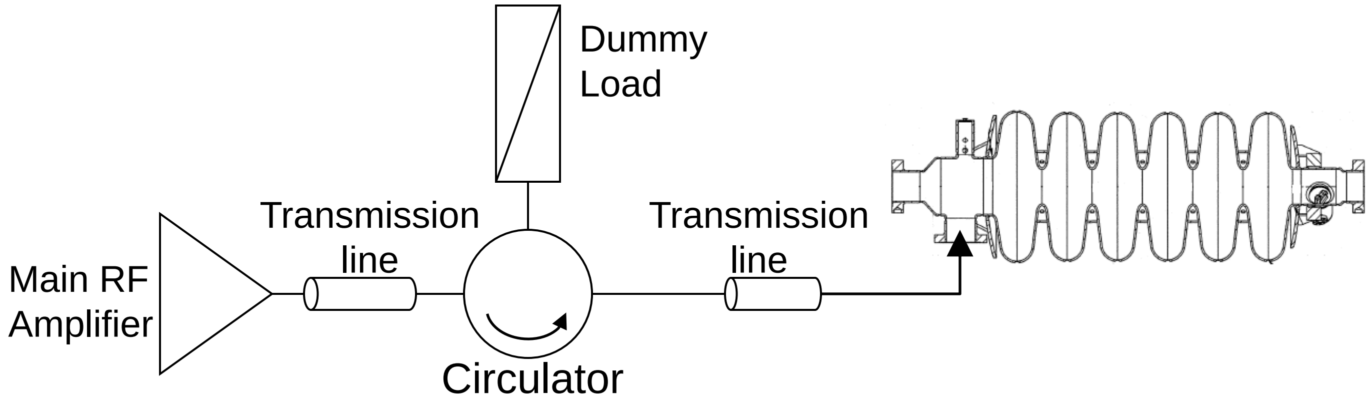
AND…

Timing system & Synchronization

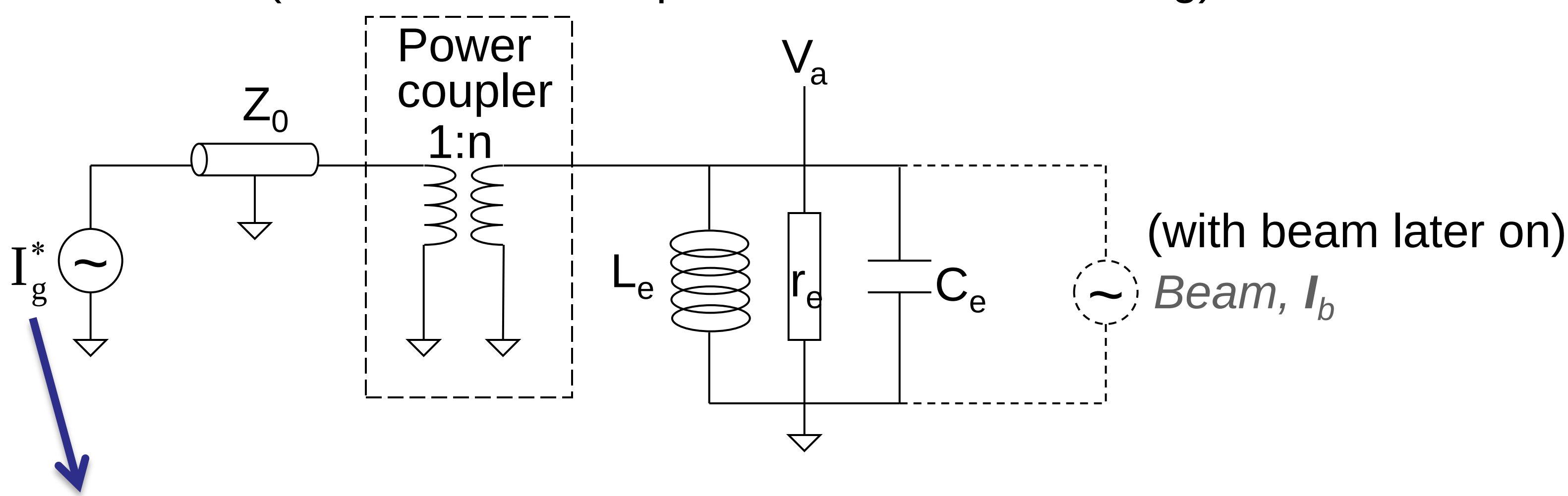
Protection system (machine & personal)

Diagnostics & user interfaces

First, main high power RF circuit and cavity responses without beam.

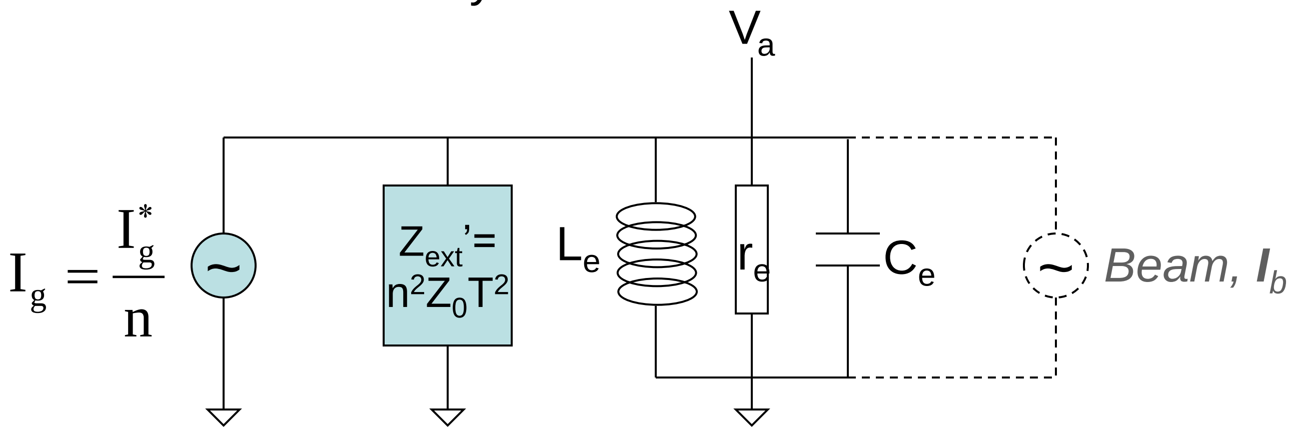


Equivalent circuit (will use effective quantities for the modeling)

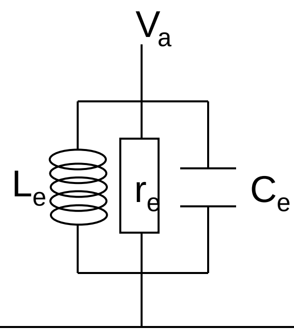


Due to the circulator, this is not an exactly equivalent for generator current.
So introduced I_g^* → twice of equivalent generator current.

Covert the model to the cavity side

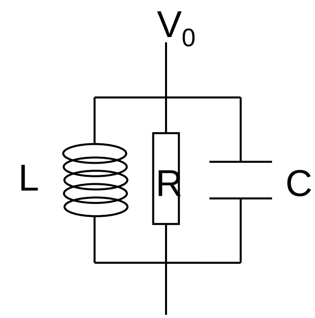


defined in Chap. 2

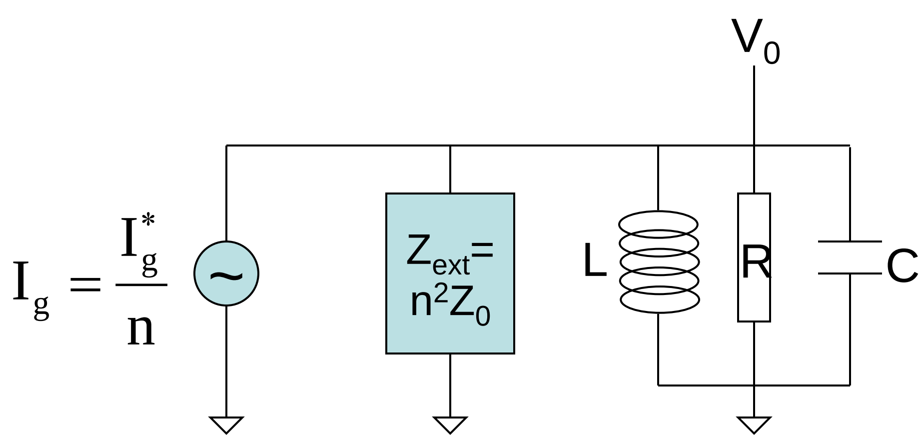


$$P_c = \frac{V_a^2}{2r_e} = \frac{V_a^2}{r_{sh}} = \frac{(V_0T)^2}{r_{sh}} = \frac{V_0^{\ 2}}{R_{sh}} = \frac{V_0^2}{2R} \ [\mathrm{W}]$$

$$r_{sh} = \frac{(E_0TL)^2}{P_c} = \frac{(V_0T)^2}{P_c} = \frac{V_a^2}{P_c} = R_{sh}T^2 \ [\Omega]$$



Remember that equivalent circuit parameters are defined by references (V_0, V_a).



Coupling factor β

Coupling between cavity and the transmission line through a coupler, β

$$\beta = \frac{r_e}{Z'_{ext}} = \frac{RT^2}{Z_{ext}T^2} = \frac{R}{Z_{ext}} = \frac{R}{n^2 Z_0} \rightarrow Z_{ext} = \frac{R}{\beta},\ Z'_{ext} = \frac{r_e}{\beta}$$

$$\frac{1}{r_L} = \frac{1}{r_e} + \frac{\beta}{r_e} \rightarrow r_L = \frac{r_e}{1+\beta} : \text{effective loaded shunt impedance}$$

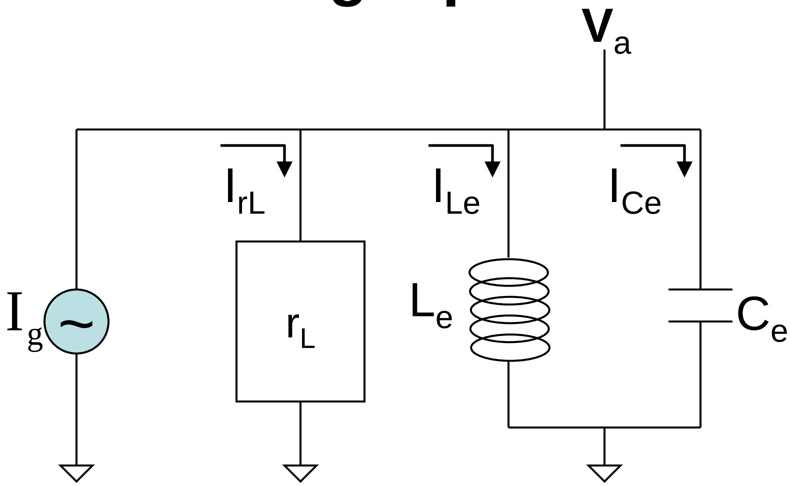
Similarly we can define $\frac{1}{R_L} = \frac{1}{R} + \frac{\beta}{R} \rightarrow R_L = \frac{R}{1+\beta} : \text{loaded shunt impedance}$

$$Q_L = \omega_0 U/(P_{ex} + P_c) \rightarrow \frac{1}{Q_L} = \frac{1}{Q_{ex}} + \frac{1}{Q_0} \rightarrow Q_L = \frac{Q_0}{1+\beta} : \text{Loaded } Q$$

$$Q_{ex} = \omega_0 U/P_{ex}, \quad Q_0 = \omega_0 U/P_c$$

As it should be, coupling factor and Q's are not function of particle velocity.
It is function of coupler geometry at a given mode (field profile).
That means Q_{ex} will be different when there's a field tilt (field flatness).

Governing equation for RF field in a cavity



$$\mathbf{I}_{C_e} = C_e \dot{\mathbf{V}}_a$$

$$\mathbf{I}_{r_L} = \mathbf{V}_a / r_L$$

$$\mathbf{I}_{L_e} = \int \mathbf{V}_a dt / L_e$$

$$\dot{\mathbf{I}}_{C_e} + \dot{\mathbf{I}}_{r_L} + \dot{\mathbf{I}}_{Le} = \dot{\mathbf{I}}_g$$

$$C_e \ddot{\mathbf{V}}_a + \frac{1}{r_L}\dot{\mathbf{V}}_a + \frac{1}{L_e}\mathbf{V}_a = \dot{\mathbf{I}}_g \rightarrow \ddot{\mathbf{V}}_a + \frac{1}{r_L C_e}\dot{\mathbf{V}}_a + \frac{1}{L_e C_e}\mathbf{V}_a = \frac{1}{C_e}\dot{\mathbf{I}}_g$$

We can eliminate non-practical parameters (C_e, L_e, C, L) using the relations:

$$Q_L = \frac{\omega_0 U}{P_{ex} + P_c} = \omega_0 \frac{\frac{1}{2} C_e V_a^2}{\frac{1}{2}\frac{V_a^2}{r_L}} = \omega_0 C_e r_L = \frac{r_L}{\omega_0 L_e} = \frac{R_L}{\omega_0 L} = \omega_0 C R_L$$

$$\boxed{\ddot{\mathbf{V}}_a + \frac{\omega_0}{Q_L}\dot{\mathbf{V}}_a + \omega_0^2 \mathbf{V}_a = \frac{\omega_0 r_L}{Q_L}\dot{\mathbf{I}}_g}$$

If we use the equivalent circuit with V_0, r_L should be replaced with R_L

Steady state solution with RF only

$$\ddot{\mathbf{V}}_a + \frac{\omega_0}{Q_L}\dot{\mathbf{V}}_a + \omega_0^2 \mathbf{V}_a = \frac{\omega_0 r_L}{Q_L}\dot{\mathbf{I}}_g$$

Typical damped driven oscillator equation

Generator current is the only source → generator induced voltage V_g=V_a
Particular solution in steady state of second order differential equation

$$\mathbf{V}_a(t) = V_a e^{i(\omega t+\psi)} \quad \text{at} \quad \mathbf{I}_g(t) = I_g e^{i\omega t}$$

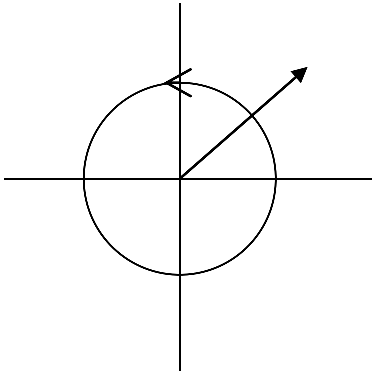
$$\mathbf{V}_a(t) = \frac{r_L}{\sqrt{1+Q_L^2\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)^2}} I_g e^{i(\omega t+\psi)} = \frac{r_L}{\sqrt{1+\tan^2\psi}} I_g e^{i(\omega t+\psi)} (= r_L I_g e^{i\omega t},\ \text{if } \Delta f = 0)$$

$$\tan\psi = Q_L\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right) \approx 2Q_L\frac{\omega_0-\omega}{\omega_0} = 2Q_L\frac{\Delta f}{f_0} = 2Q_L\delta, \text{when } |\delta| << 1$$

ψ : detuning angle

Phasor representation

To have the total voltage we need to add/subtract generator current/voltage and beam current/induced voltage. Linear superposition works from the linearity of Maxwell's equations. But one should take the relative phase into account.



In general, fields can be expressed as

$$\mathbf{A} = Ae^{i(\omega t+\theta)}, \ \ A: \text{amplitude}, \ \omega t + \theta : \text{phase}$$

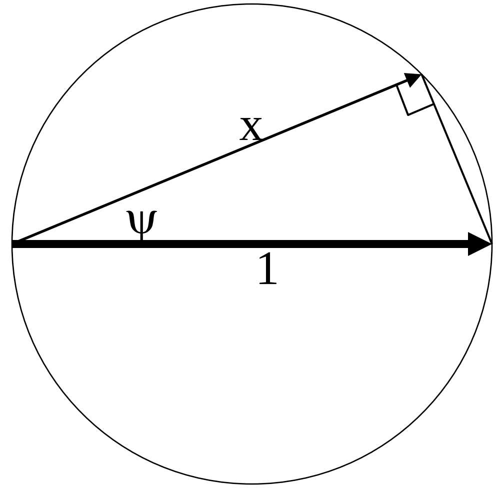
If we choose a frame of reference that is rotating at a frequency ω, the phasor will be stationary in time.

References can be arbitrary but it is convenient to have:

Reference frequency ω: operating frequency (rf source frequency) since all other fields are around operating frequency.

Reference phase: beam arrives at the electrical center of cavity → zero phase (or real axis). How can we represent 'beam' at the reference frequency?

$$x = \frac{1}{\sqrt{1+\tan^2\psi}} = \cos\psi$$



$$\mathbf{V}_a(t) = \frac{r_L}{\sqrt{1+\tan^2\psi}} I_g e^{i(\omega t+\psi)} = r_L I_g \cos\psi \; e^{i\psi} \; e^{i\omega t}$$

common rotating term

Amplitude decrease due to detuning

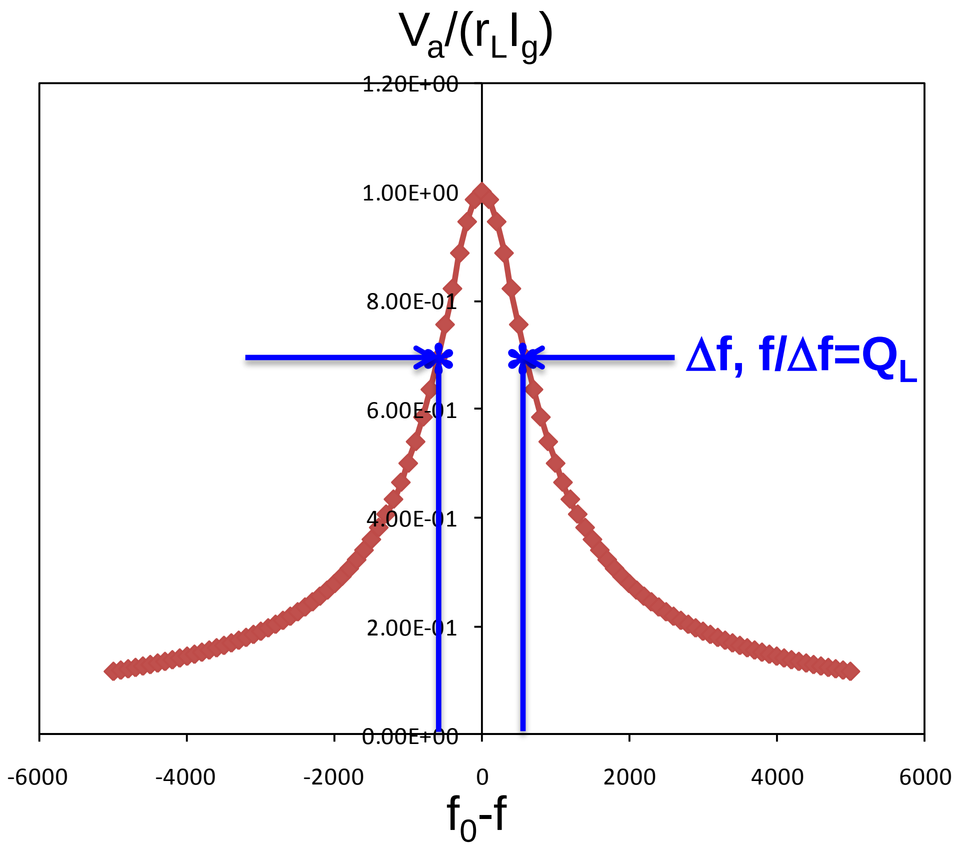
relative phase change due to detuning

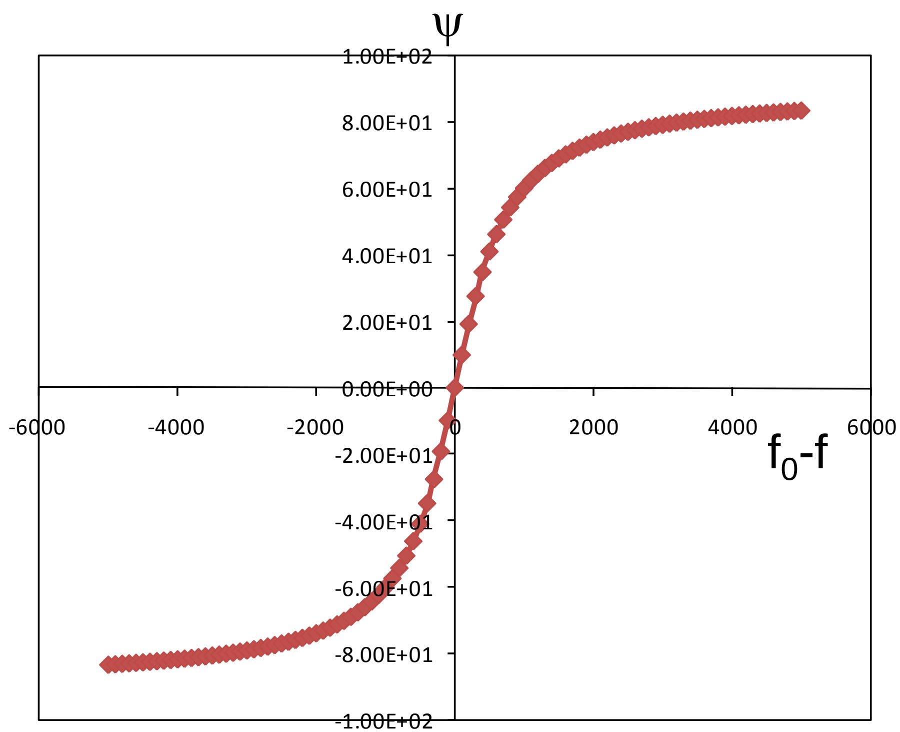
Total impedance of the equivalent model including detuning without beam

$$Z_{tot} = \frac{\mathbf{V}_a(t)}{\mathbf{I}_g(t)} = r_L \cos\psi \cdot e^{i\psi} = \frac{r_e}{1+\beta}\cos\psi \cdot e^{i\psi} = \frac{r_{sh}}{2(1+\beta)}\cos\psi \cdot e^{i\psi}$$

Ex) $Q_L=7\times10^5$, f=805 MHz

plot the normalized V_a and the detuning angle as a function of cavity detuning





Bandwidth at -3bB: $10\log_{10}(P/P_{ref})$ for power, $20\log_{10}(V/V_{ref})$ for voltage

$20\log_{10}(1/\mathrm{sqrt}(2))=-3.01 \Leftrightarrow \psi=\pm\pi/4$

Half width at -3dB: $\omega_{1/2}=\omega_0/(2Q_L)=2\pi\cdot575$ Hz in this example

τ (time constant of loaded cavity)$=1/\omega_{1/2}=2Q_L/\omega_0=277\mu s$

RF power without beam loading

As mentioned,

'due to the circulator, this is not an exactly equivalent for generator current.

So introduced $\mathbf{I_g^*}$ → twice of equivalent generator current.'

To calculate forward power in the transmission line (waveguide or coaxial cable)

$$\text{Forward current}: \mathbf{I}_{for} = \mathbf{I}_g/2$$

$$\text{Voltage}: \mathbf{V}_{for} = (\mathbf{I}_g/2)\cdot(r_e/\beta), \text{ this corresponds to actual forward power.}$$

$$\text{Don't be confused with generator induced voltage in the cavity } \mathbf{V}_g$$

So, the time averaged forward power in the transmission line from the generator is

$$P_g = \frac{1}{2}\left(\frac{I_g}{2}\right)\left(\frac{I_g}{2}\cdot\frac{r_e}{\beta}\right) = \frac{I_g^2 r_e}{8\beta} \qquad \text{with } \mathbf{V}_a = \frac{r_L}{\sqrt{1+\tan^2\psi}} I_g e^{i\psi} = r_L I_g \cos\psi\, e^{i\psi},\ r_L = \frac{r_e}{1+\beta}$$

One can calculate 'Forward power' needed to get V_a is:

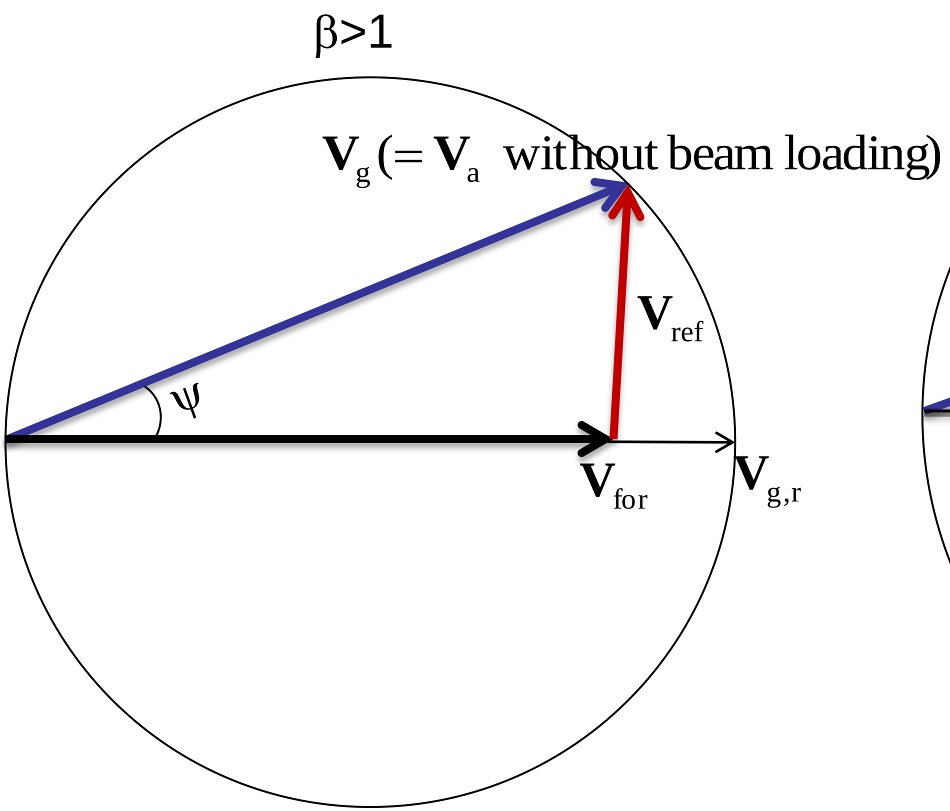
$$P_g = \frac{I_g^2 r_e}{8\beta} = V_a^2 \frac{(1+\beta)^2}{4\beta}\frac{1}{2r_e}\frac{1}{\cos^2\psi} = P_c \frac{(1+\beta)^2}{4\beta}(1+\tan^2\psi) \quad \because 2r_e = r_{sh},\ r_{sh} = \frac{V_a^2}{\omega_0 U} Q_0 = \frac{V_a^2}{P_c}$$

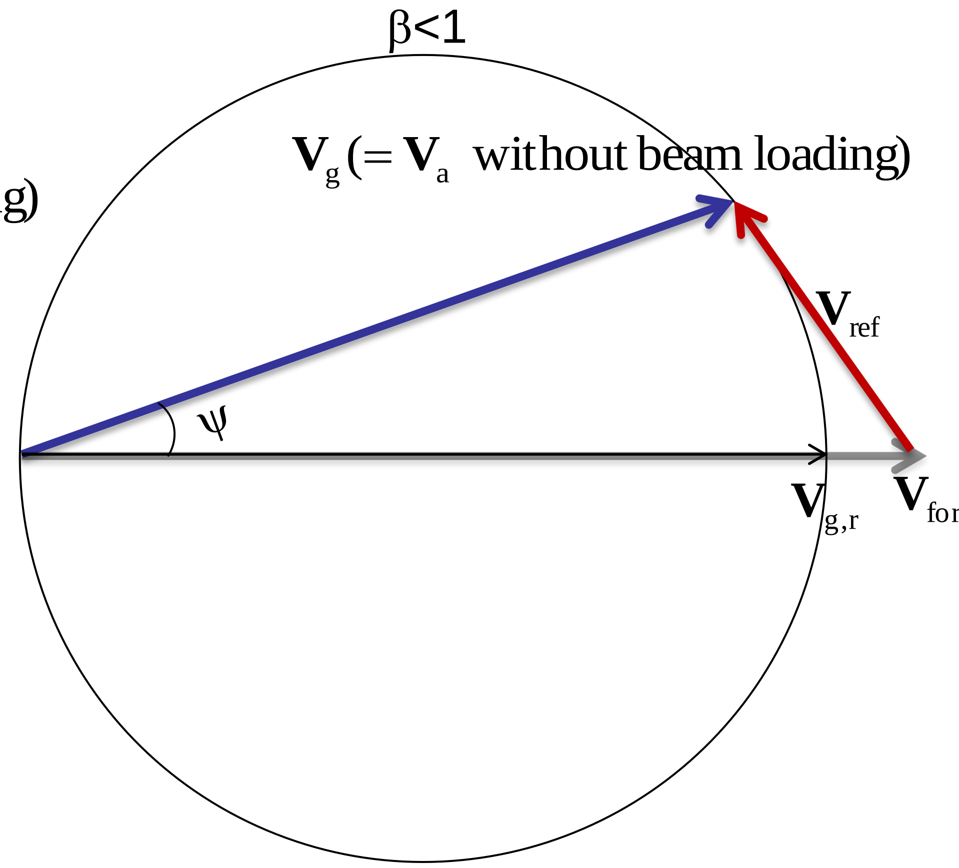
This is a useful equation when P_c & β are well defined, as for normal conducting cavity.

$$\mathbf{V}_{for} = \frac{r_e}{2\beta}\mathbf{I}_g$$

$$\mathbf{V}_{g,r} = r_L \mathbf{I}_g = \frac{r_e}{\beta+1}\mathbf{I}_g \text{ : generator induced voltage on resonance}$$

$$\frac{\mathbf{V}_{for}}{\mathbf{V}_{g,r}} = \frac{\beta+1}{2\beta}, \text{ in phase}$$





In superconducting cavity, more practical parameters are Q_L, r/Q, V_a (or V_0), $f_{1/2}$ since Q_0 is much bigger than Q_{ex}, P_c is not well-defined, etc.

Forward power needed to get V_a is :

$$P_g = \frac{I_g^{\,2} r_e}{8\beta} = V_a^2 \frac{(1+\beta)}{8\beta}\frac{(1+\beta)}{r_e}(1+\tan^2\psi) \approx V_a^2 \frac{1}{8}\frac{1}{r_L}\frac{1}{\cos^2\psi} = V_a^2 \frac{1}{4}\frac{1}{(r/Q)Q_L}\frac{1}{\cos^2\psi}$$

$\because \beta >> 1 \rightarrow \beta + 1 \approx \beta.$ (when one uses this assumption, check the validity of this.)

$$\because \frac{r}{Q} = \frac{V_a^2}{\omega U} = \frac{V_a^2}{P_c}\cdot\frac{P_c}{\omega U} = \frac{r_{sh}}{Q_0} = \frac{2r_e}{Q_0} = \frac{2r_e}{\beta+1}\frac{\beta+1}{Q_0} = 2\frac{r_L}{Q_L} \rightarrow \frac{r_e}{1+\beta} = r_L = \frac{1}{2}\left(\frac{r}{Q}\right)Q_L$$

And as defined earlier, $\tan\psi = 2Q_L \dfrac{\Delta f}{f_0} = \dfrac{\Delta f}{f_{1/2}}$, $\left(f_{1/2} = \dfrac{f_0}{2Q_L}\right)$

Any set of P_x and corresponding Q_x makes the same relation.

$$\left|\mathbf{V}_a\right| = \frac{r_L}{\sqrt{1+\tan^2\psi}} I_g = r_L I_g \cos\psi = 2\sqrt{P_g (r/Q) Q_L}\cos\psi = 2\sqrt{P_x (r/Q) Q_x}\cos\psi$$

$$\boxed{P_g = \frac{1}{4}\frac{V_a^2}{(r/Q)Q_L}\left[1+\left(\frac{\Delta f}{f_{1/2}}\right)^2\right] = \frac{1}{4}\frac{V_o^2 T^2}{(R/Q)T^2 Q_L}\left[1+\left(\frac{\Delta f}{f_{1/2}}\right)^2\right] = \frac{1}{4}\frac{V_o^2}{(R/Q)Q_L}\left[1+\left(\frac{\Delta f}{f_{1/2}}\right)^2\right]}$$

Don't be confused with other passive couplings.

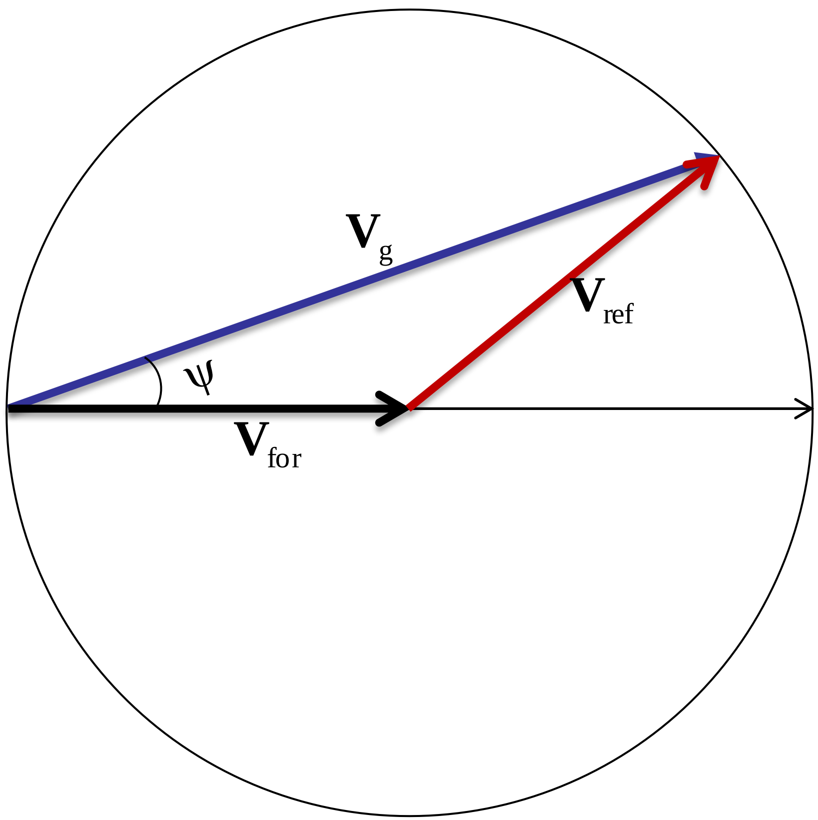
Quiz) when we measure Ea through field probe, how?

$$\mathbf{V}_g (= \mathbf{V}_a \text{ without beam loading}) = r_L I_g \cos\psi \, e^{i\psi}$$

When $\beta >> 1$

$$\mathbf{V}_{for} = \frac{r_e}{2\beta}\mathbf{I}_g \approx \frac{r_e}{2(\beta+1)}\mathbf{I}_g = \frac{r_L}{2}\mathbf{I}_g$$

$$|\mathbf{V}_{for}| \approx |\mathbf{V}_{ref}|$$



$\mathbf{V}_{g,r} = r_L \mathbf{I}_g$ Generator induced cavity field when the system is on resonance

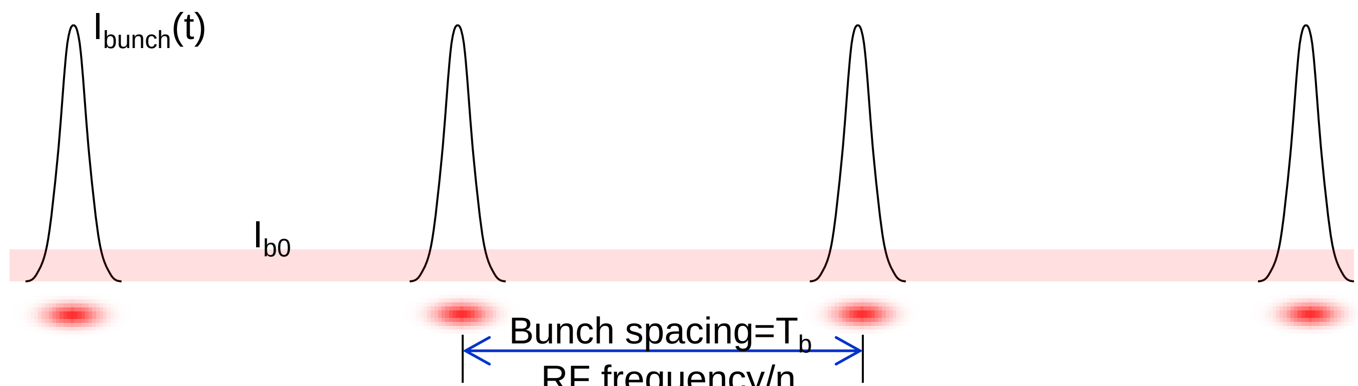
HOMEWORK 4-1

for f=805 MHz, Ea=10MV/m, L=0.68m, r/Q=279Ω, and
$Q_L=7\times10^5$, $Q_L=1\times10^6$, $Q_L=2\times10^6$,

1. Plot required forward power as a function of detuning (-500 Hz~500 Hz) using spreadsheet 4_1.xlsx
2. If $Q_0=1\times10^{10}$, What is cavity wall loss, Pc? What does that mean?

Equivalent beam in a RF circuit model

Micro-pulse



When we say 'Beam current', it is an time averaged DC current.

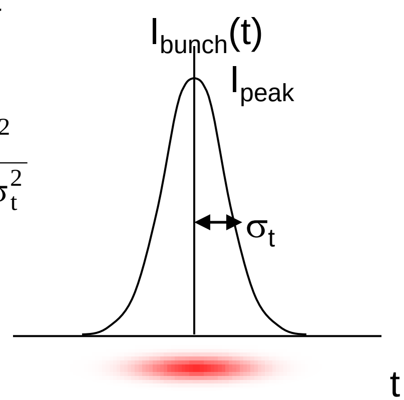
Ex) I_{b0}=40 mA CW beam at bunch spacing 402.5 MHz

T_b=1/402.5 MHz~2.5ns,

Q (charge per bunch)=I_{b0} (C/s) x T_b (s)=0.04 x 2.5e-9 = 100 pC

Temporal distribution of beam can be described by a Gaussian distribution with standard deviation σ_t

$$I_{bunch}(t) = \frac{Q}{\sqrt{2\pi}\sigma_t} e^{-\frac{t^2}{2\sigma_t^2}}$$



If σ_t is 1.0 degree of 402.5 MHz (7 ps),

I_{peak}~5.7 A

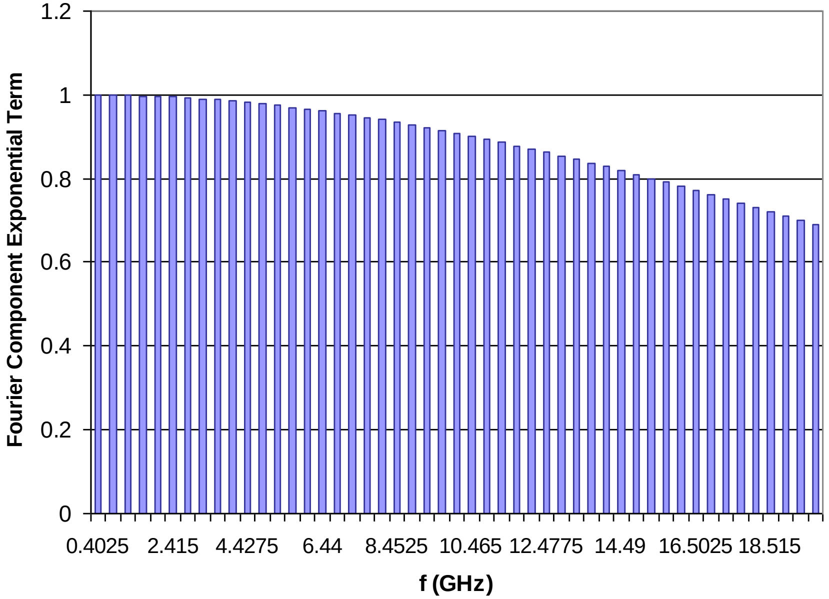
Fourier decomposition from $-T_b/2$ to $T_b/2$

$$I_{bunch}(t) = \frac{Q}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{t^2}{2\sigma_t^2}\right) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_b t),$$

$$a_n = 2I_{peak}\sqrt{2\pi}\frac{\sigma_t}{T_b}\exp\left(-\frac{n^2\omega_b^2\sigma_t^2}{2}\right) = 2I_{b0}\exp\left(-\frac{n^2\omega_b^2\sigma_t^2}{2}\right),\ n = 0, 1, 2, 3, \cdots$$

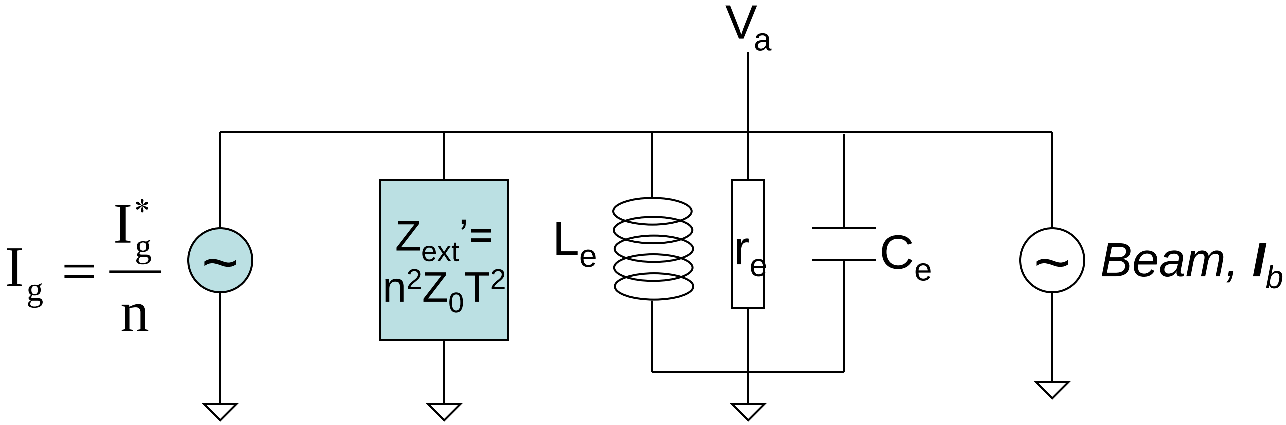
$$\omega_b = \frac{2\pi}{T_b}$$

If $\frac{1}{n\omega_b} >> \sigma_t$, $\exp\left(-\frac{n^2\omega_b^2\sigma_t^2}{2}\right) \approx 1$



Ex) Bunch spacing=402.5 MHz, Operating RF frequency=805 MHz:
So the Fourier component (n=1, 2, 3,…) of bunched beam at operating frequency is simply $2I_{b0}$.

Steady state with beam loading



- We added ‘beam’ as a current source like the RF generator. The beam energy effect is in the effective quantities.
- Beam current at the operating frequency is

$$\left|\mathbf{I}_b\right| = 2I_{b0},\ I_{b0}: \text{DC current of beam}$$

- The cavity voltage is the sum of generator induced voltage and beam induced voltage in a cavity.

$$\ddot{\mathbf{V}}_a + \frac{\omega_0}{Q_L}\dot{\mathbf{V}}_a + \omega_0^2\mathbf{V}_a = \frac{\omega_0 r_L}{Q_L}(\dot{\mathbf{I}}_g + \dot{\mathbf{I}}_b) \Rightarrow \mathbf{V}_a = \mathbf{V}_g + \mathbf{V}_b$$

-As mentioned earlier, set the reference phase for beam phase at the center of electric center → beam induced image current sits on negative real axis.

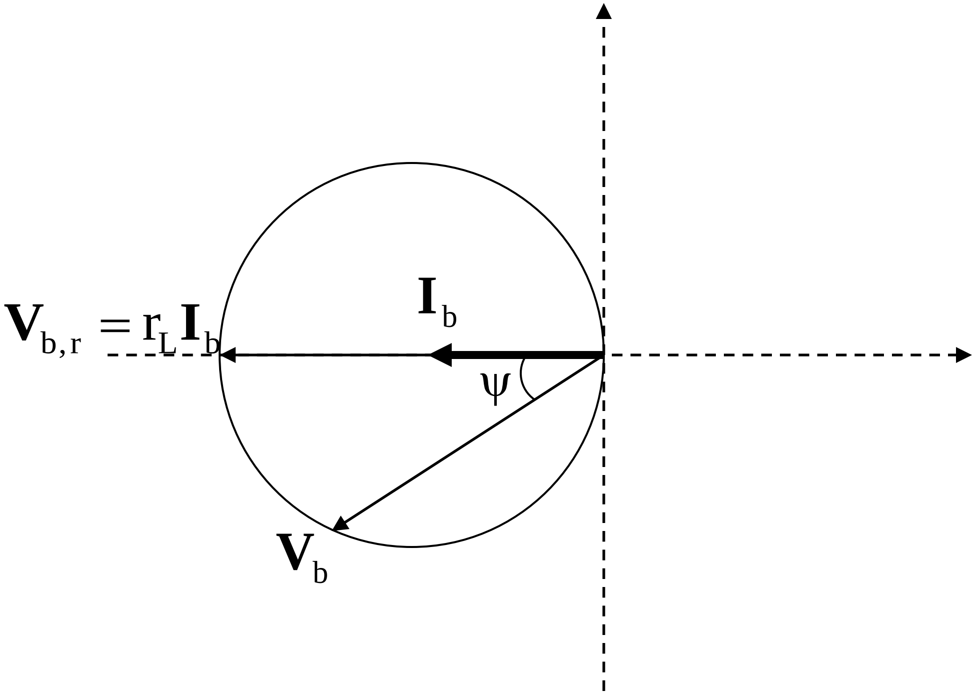
-Beam induced voltage has the same form as generator induced voltage.

$$\text{beam induced current, } \mathbf{I}_b = I_b e^{i\pi}$$

$$\mathbf{V}_b = \frac{r_L}{\sqrt{1+\tan^2\psi}} I_b e^{i(\pi+\psi)} = r_L I_b \cos\psi \, e^{i(\pi+\psi)}$$

$$\tan\psi = Q_L\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right) \approx 2Q_L \frac{\omega_0 - \omega}{\omega_0} = 2Q_L \frac{\Delta f}{f_0}$$

In this example ψ is positive, Which means the resonance frequency is higher than generator frequency.

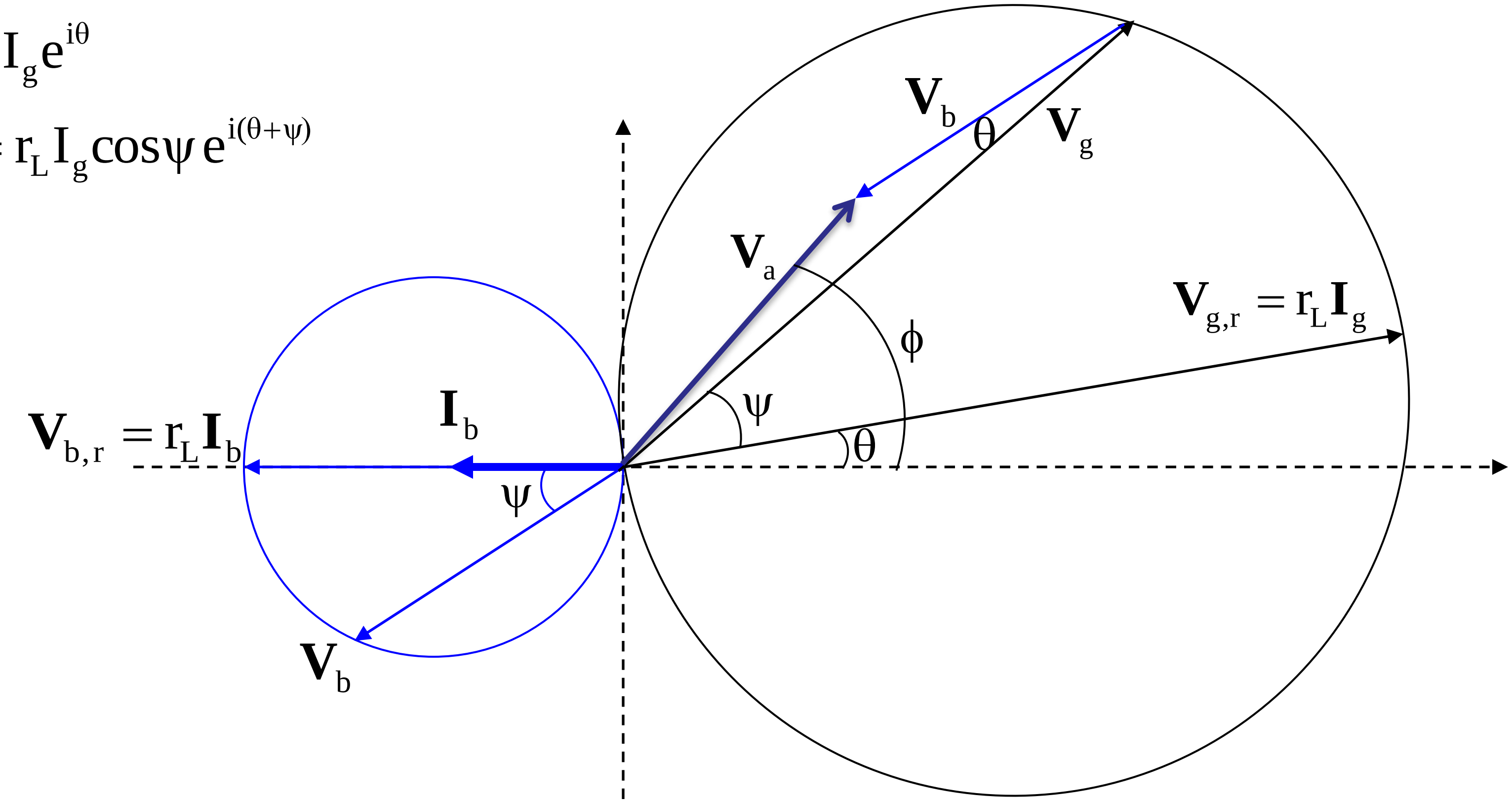


Generator induced voltage is about same as before for the RF only case except relative phase of generator current.

Let's start with arbitrary phase first.

$$\mathbf{I}_g = I_g e^{i\theta}$$

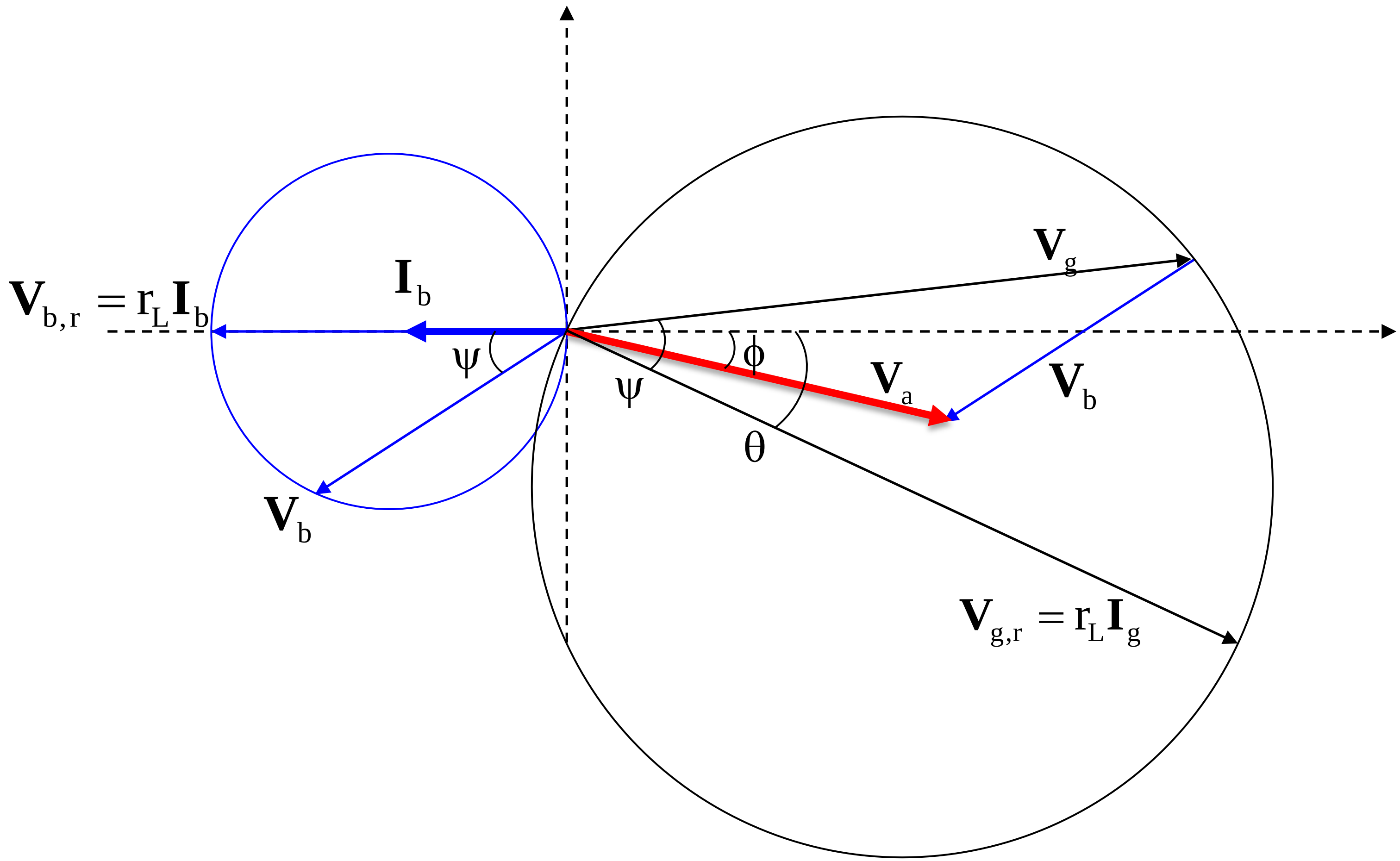
$$\mathbf{V}_g = r_L I_g \cos\psi \, e^{i(\theta+\psi)}$$



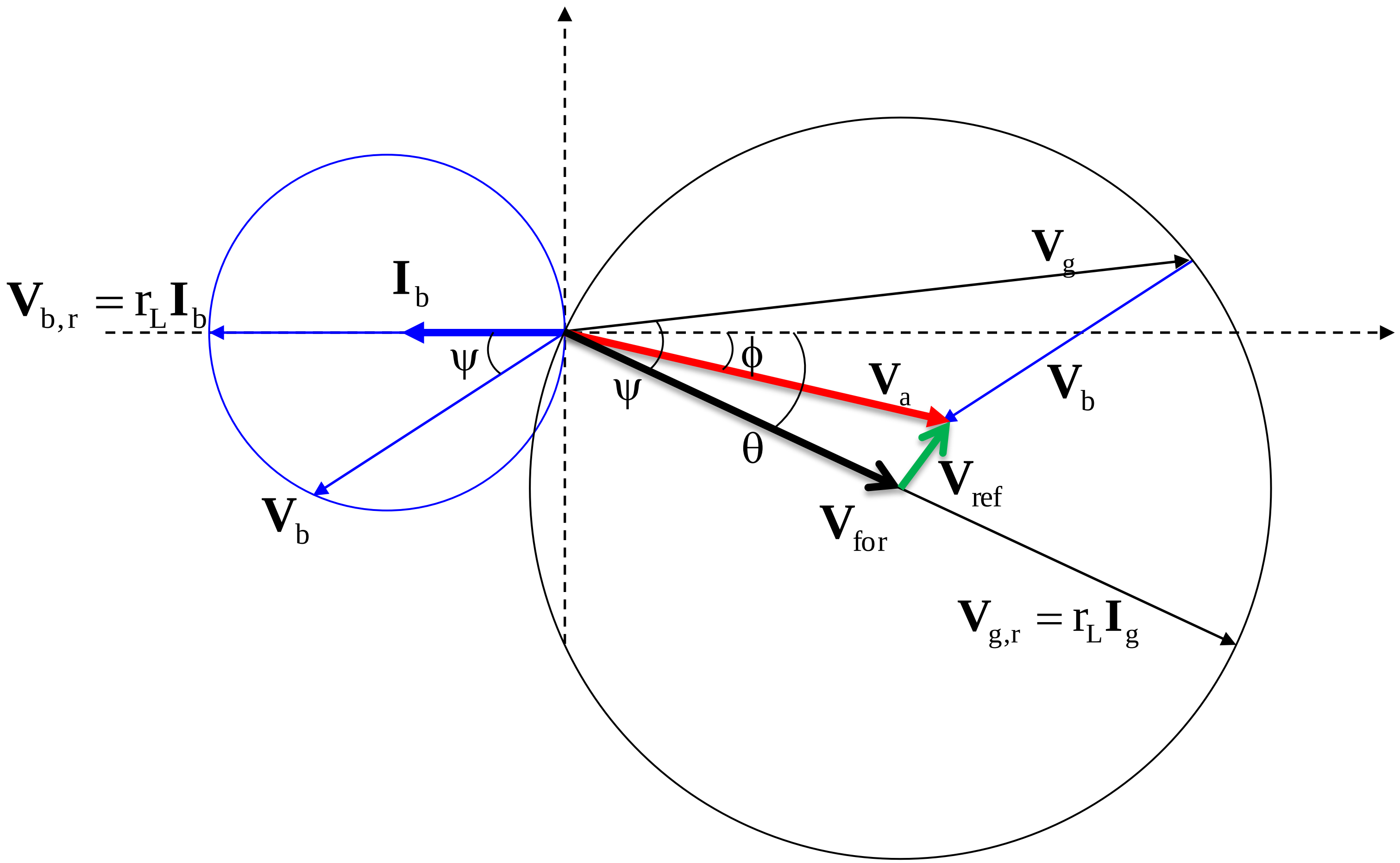
The black circle is rotating around the origin when generator phase (RF phase) changes.

The real component of $\mathbf{V}_a$ is for acceleration: $\mathbf{V}_a \cos\phi$

-To get required accelerating voltage V_a and synchronous phase ϕ for a beam current I_b at a fixed **cavity detuning (ψ)** and **Loaded Q (Q_L)**, the generator current $\mathbf{I}_g$ (RF power and phase) is uniquely determined.



How about forward voltage ($\mathbf{V}_{for}$) and reflected voltage (V_{ref}) of the system with β>>1



$\mathbf{V}_a = \mathbf{V}_{for} + \mathbf{V}_{ref}$: P_{for} and P_{ref} can be directly calculated from $\mathbf{V}_{for}$ and $\mathbf{V}_{ref}$.

$\mathbf{V}_a = \mathbf{V}_g + \mathbf{V}_b$

Generator power

$$P_g = V_a^2 \frac{(1+\beta)}{8\beta} \frac{1}{r_L} \left[\left(1 + \frac{I_g r_L}{V_a} \cos\phi \right)^2 + \left(\tan\psi + \frac{I_g r_L}{V_a} \sin\phi \right)^2 \right]$$

For SRF cavities where $\beta >> 1$, using the relations

$$r_L = \frac{1}{2} \left(\frac{r}{Q} \right) Q_L, \; I_b = 2I_{b0}, \; \frac{1}{Q_b} \equiv \frac{I_{b0} V_a \cos\phi}{\omega U} = \frac{(r/Q) I_{b0}}{V_a} \cos\phi, \; P_b = I_{b0} V_a \cos\phi$$

$$P_g = \frac{V_a^2}{4(r/Q) Q_L} \left[\left(1 + \frac{Q_L}{Q_b} \right)^2 + \left(\frac{\Delta f}{f_{1/2}} + \frac{Q_L}{Q_b} \tan\phi \right)^2 \right]$$

Power balance $P_g = P_c + P_b + P_{ref}$ in steady state

Optimum cavity detuning (ψ_{opt}) and Loaded Q (Q_L),

If $\frac{\Delta f}{f_{1/2}} = -\frac{Q_L}{Q_b} \tan\phi \rightarrow \Delta f_{opt} = -\frac{f_0}{2Q_b} \tan\phi$: optimum detuning

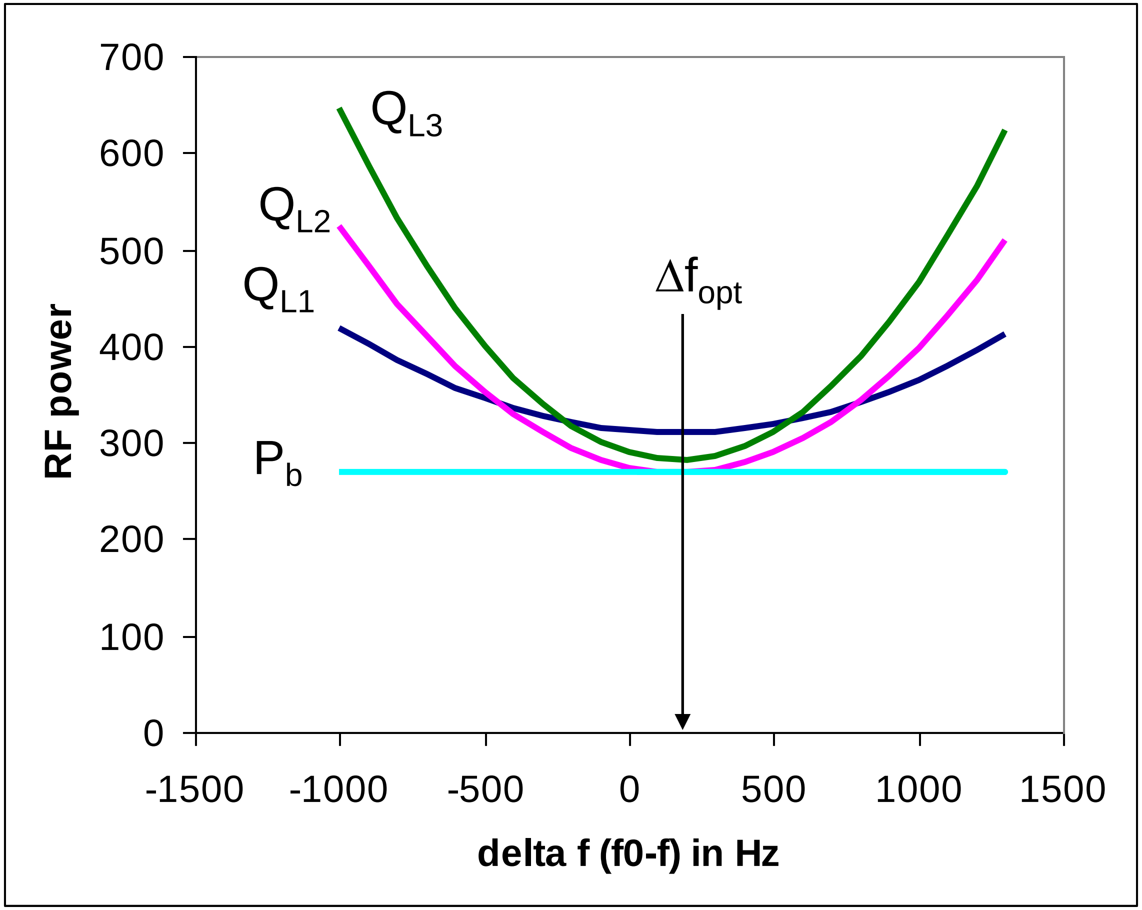
If $Q_L = Q_b$ & at $\Delta f_{opt} \rightarrow P_g = V_a I_{bo} \cos\phi$ No reflected power

HOMEWORK 4-2, Play with the spreadsheet

Ex) Using parameters in the table, (at particle beta=0.61) calculate Va, Qb, and optimum Δf, and

generate table of required RF power as a function of Δf (-1000 Hz ~ 1300 Hz) for Q_{L1}=3e5, Q_{L2}=7e5, Q_{L3}=1e6.

r/Q(at β=0.61)=	279	Ohm
TTF (at β=0.61)=	0.68	
I_{b0}=	0.04	A
Syn Phase=	-15	degree
E_0=	15	MV/m
Length=	0.6816	m
Q_{L1}=	3.00E+05	
Q_{L2}=	7.00E+05	
Q_{L3}=	1.00E+06	
f=	8.05E+08	Hz

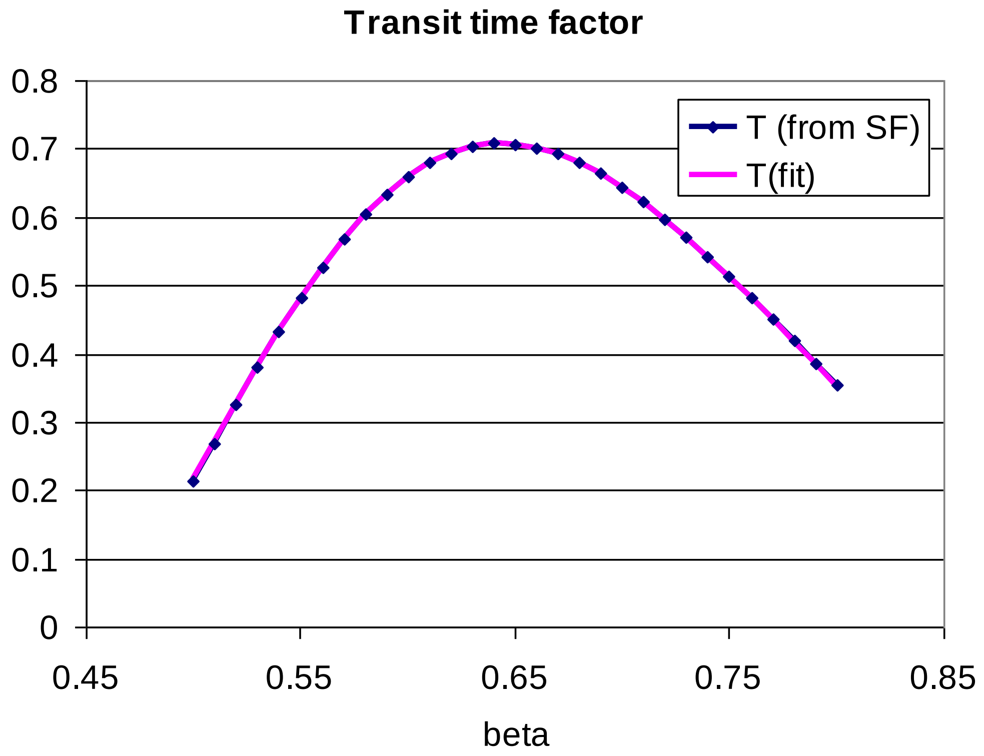


(4_2.xlsx)
change parameters in blue

HOMEWORK 4-3, Play with the spreadsheet

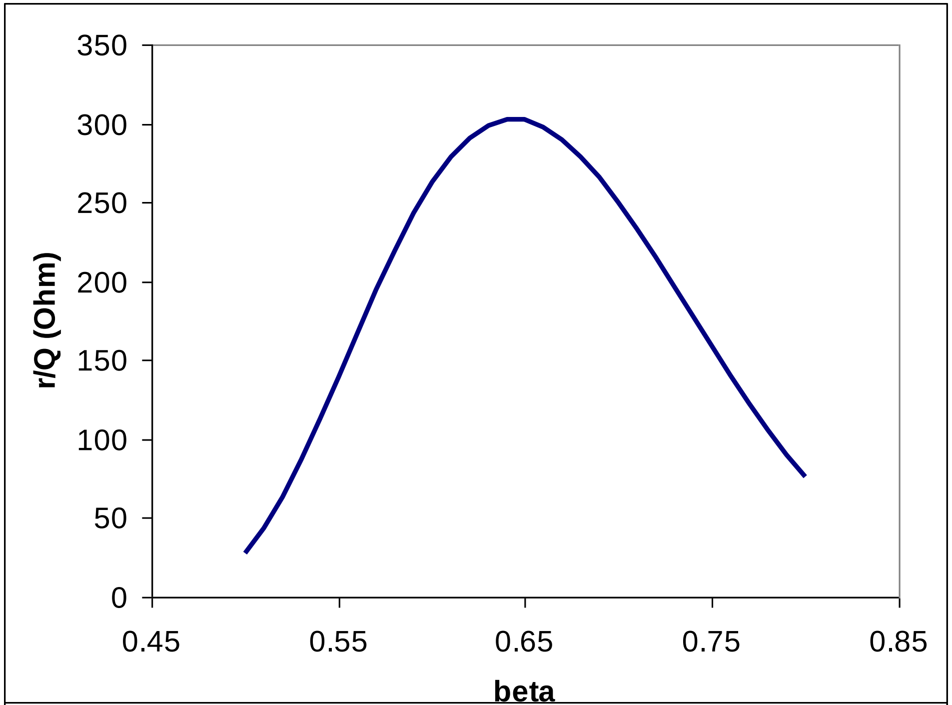
Ex) Using parameters in the table and TTF data, generate (r/Q), Q_b, RF power required, optimum Δf, required RF power at optimum Δf as a function of particle velocity from β=0.5 to β=0.8.

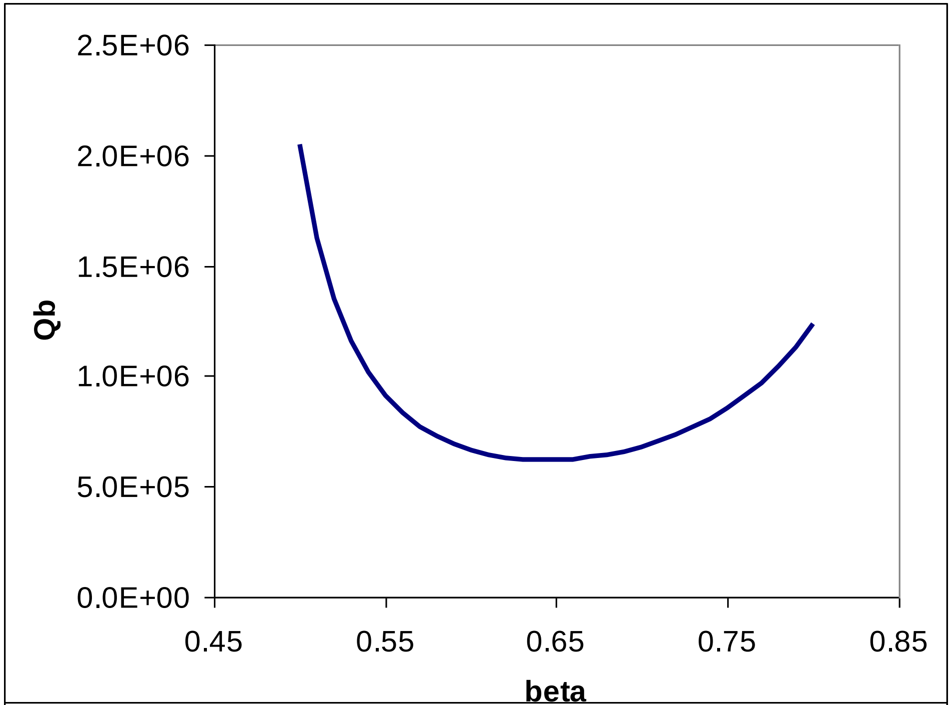
r/Q(at beta=0.61)=	**279**	**Ohm**
Ib0=	**0.04**	**A**
Syn Phase=	**-15**	**degree**
E0=	**15**	**MV/m**
Length=	**0.6816**	**m**
QL=	**7.00E+05**	
delta f=	**-300**	**Hz**
f=	**8.05E+08**	**Hz**

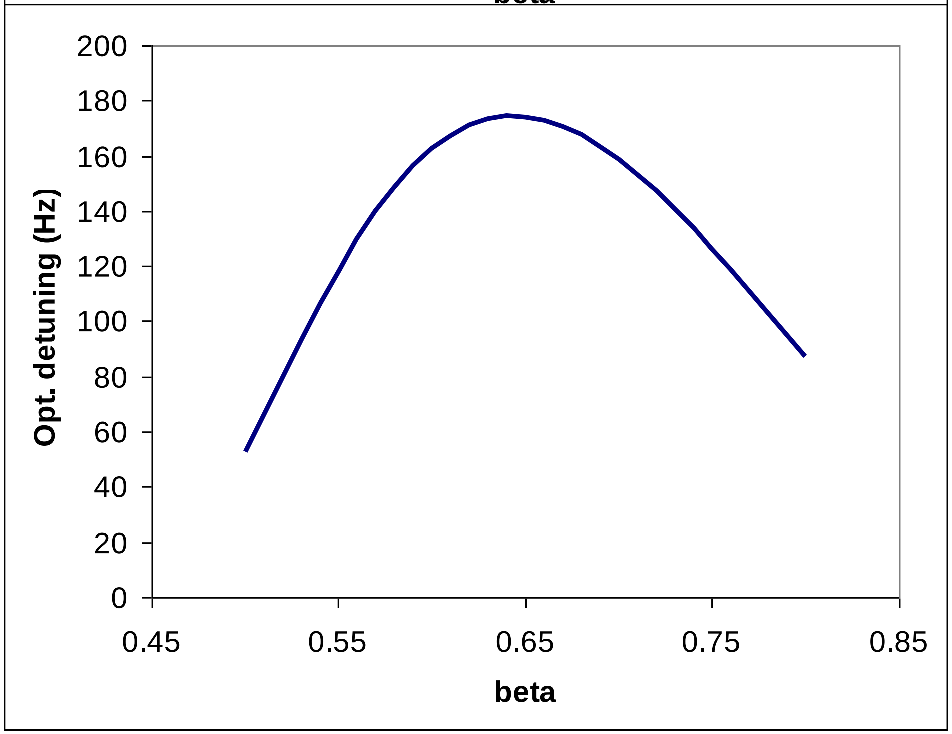


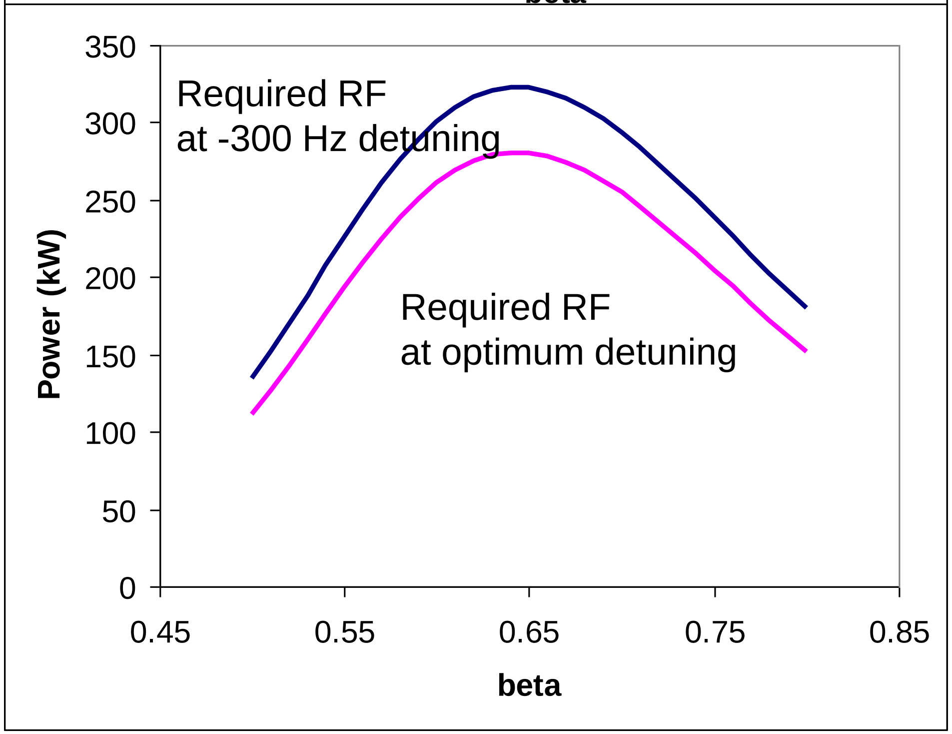
TTF=1540.22897*b^6 - 6951.52591*b^5 + 12957.19469*b^4 - 12724.40926*b^3 + 6910.19655*b^2 - 1956.643*b + 224.88436

(4_3.xlsx); change parameters in blue


350
300
250
200
150
100
50
0
r/Q (Ohm)
0.45
0.55
0.65
0.75
0.85
beta


2.5E+06
2.0E+06
1.5E+06
1.0E+06
5.0E+05
0.0E+00
Qb
0.45
0.55
0.65
0.75
0.85
beta


200
180
160
140
120
100
80
60
40
20
0
Opt. detuning (Hz)
0.45
0.55
0.65
0.75
0.85
beta


350
300
250
200
150
100
50
0
Power (kW)
Required RF
at -300 Hz detuning
Required RF
at optimum detuning
0.45
0.55
0.65
0.75
0.85
beta

Transient behavior without beam

$$\ddot{\mathbf{V}}_a + \frac{\omega_0}{Q_L}\dot{\mathbf{V}}_a + \omega_0^2 \mathbf{V}_a = \frac{\omega_0 r_L}{Q_L}\dot{\mathbf{I}}_g$$

Transient term

Steady state term

General Solution:

initial condition problem

$$V_a(t) = e^{-\frac{t}{\tau}}\left(c_1 e^{i\omega_1 t} + c_2 e^{-i\omega_1 t}\right) + r_L I_g \cos\psi e^{i(\omega t+\psi)}$$

$$\text{where } \tau \text{ (time constant)} = \frac{2Q_L}{\omega_0} = \frac{1}{\omega_{1/2}},\ \omega_1 \text{ (system resonance frequency)} = \omega_0\sqrt{1-\frac{1}{4Q_L^2}} \approx \omega_0$$

On resonance case, $\omega_0 = \omega$. All are in phase. If we set $\mathbf{I}_g = I_g e^{i\omega t}$, all are on real axis.

1) at $t = 0$, $\mathbf{V}_a(0) = 0$ & turn on RF $\mathbf{I}_g = I_g e^{i\omega t}$

$$V_a(t) = r_L I_g (1 - e^{-t/\tau}) = 2\sqrt{P_g (r/Q) Q_L}\,(1 - e^{-t/\tau})$$

$$\frac{1}{2} V_{g,r} = V_{for} = r_L I_g / 2 = \sqrt{P_g (r/Q) Q_L}$$

$$\mathbf{V}_{for} + \mathbf{V}_{ref} = \mathbf{V}_a \rightarrow V_{ref}(t) = \sqrt{P_g (r/Q) Q_L}\,(1 - 2e^{-t/\tau})$$

$$P_{ref}(t) = \frac{V_{ref}^2(t)}{2r_L} = P_g (1 - 2e^{-t/\tau})^2$$

2) at $t = t_1$, $\mathbf{V}_a(t_1) = 2\sqrt{P_g(r/Q)Q_L}\,(1 - e^{-t_1/\tau})e^{i\delta}$ & turn off RF

$\mathbf{V}_a(t) = \mathbf{V}_a(t_1)e^{-(t-t_1)/\tau}, \; t > t_1$

General power balance : $P_g = P_c + P_{ref} + P_b + dU/dt$

In this example, RF is turned off at t_1 and P_c is negligible $(\beta >> 1)$

$\rightarrow P_{ref}(t) = -dU/dt$ in this example

$$\frac{r}{Q} = \frac{V_a^2}{\omega_0 U} \rightarrow U = \frac{V_a^2}{\omega_0 (r/Q)} \rightarrow dU/dt = \frac{2V_a(t)}{\omega_0 (r/Q)} \frac{dV_a(t)}{dt}$$

$P_{emit} (= P_{ref}) = 4P_g(1 - e^{-t_1/\tau})^2 e^{-2(t-t_1)/\tau}$

The direction of this power after RF off is same as that of reflected power. This power is a release of stored energy from cavity. The mechanism is different. This power is called ‘emitted power’.

Some useful information can be taken just from decay curves of the V_a and P_{emit} after RF off.

$$-\int_{t_1}^{\infty} \mathrm{Pemit}(t)dt = U\,(\text{stored energy}),$$

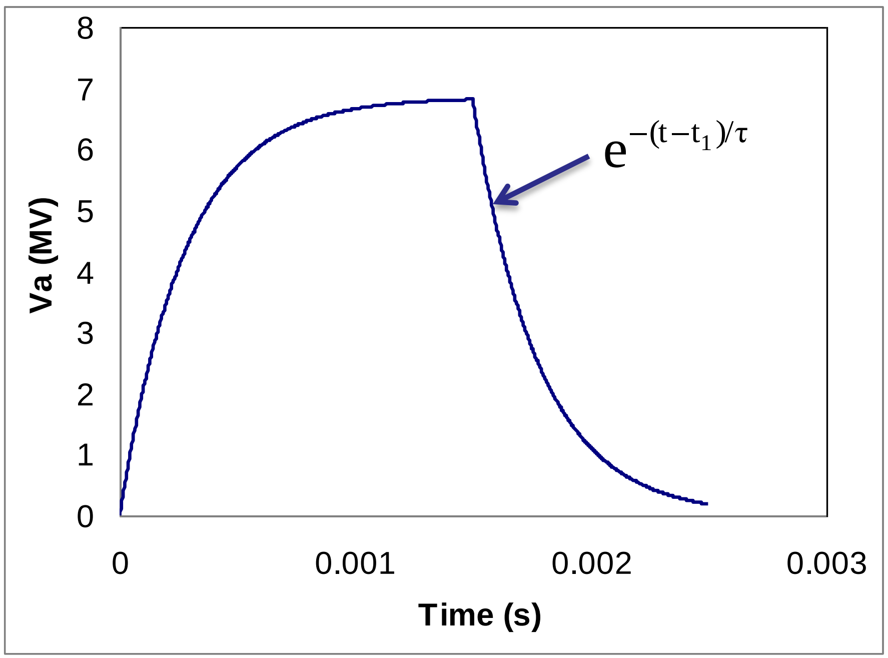
$$\text{when we know U, Va can be calculated using } \frac{r}{Q} = \frac{V_a^2}{\omega_0 U}\ (r/Q : \text{cavity property})$$

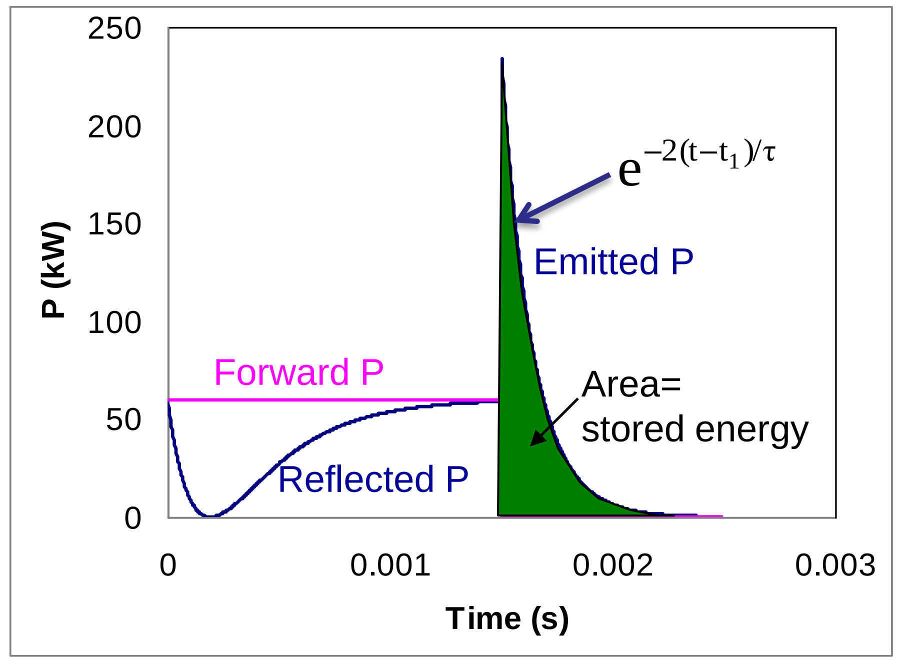
$$-\text{with fit at decay of either Va or Pemit} \rightarrow \tau \rightarrow Q_L$$

$$-\text{detuning amount; we will look at it shortly}$$

Ex) (transient1.xls); change parameters in blue

Pg=	60	kW
r/Q=	279	Ohm
QL=	7.00E+05	
t1=	0.0015	s
f=	8.05E+08	Hz





Transient behavior with detuning & beam loading

-For general expressions including time-varying detuning and beam loading, the system equation will be developed.

-Using finite difference method, cavity behaviors will be explored.

-First, separate out fast rotating terms and build up model in vector space (real & imaginary)

$$\mathbf{V}_a(t) = \hat{V}_a(t) \cdot e^{i\omega t},$$

$$\mathbf{I}(t) = \hat{I}(t) \cdot e^{i\omega t}$$

Complex envelope (slowly varying)

RF term (fast oscillation)

$$\ddot{\mathbf{V}}_a + \frac{\omega_0}{Q_L}\dot{\mathbf{V}}_a + \omega_0^2 \mathbf{V}_a = \frac{\omega_0 r_L}{Q_L}\dot{\mathbf{I}}, \text{ where } \dot{\mathbf{I}} = \dot{\mathbf{I}}_g + \dot{\mathbf{I}}_b$$

$$\ddot{\hat{V}}_a + 2(\omega_{1/2} + i\omega)\dot{\hat{V}}_a + \left[2\omega(\Delta\omega + i\omega_{1/2}) + (\Delta\omega)^2\right]\hat{V}_a = 2r_L\omega_{1/2}(\dot{\hat{I}} + i\omega\hat{I})$$

where $\Delta\omega = \omega_0 - \omega,\ \omega_{1/2} = \omega_0/(2Q_L)$

$$\Rightarrow \frac{1}{2i\omega}\ddot{\hat{V}}_a + \dot{\hat{V}}_a - i(\Delta\omega + i\omega_{1/2})\hat{V}_a = r_L\omega_{1/2}\hat{I}$$

$$\dot{\hat{V}}_a - i(\Delta\omega + i\omega_{1/2})\hat{V}_a = r_L \omega_{1/2} \hat{I}$$

insert $\hat{V}_a = V_{ar} + iV_{ai}, \hat{I} = I_r + iI_i$ in the equation above, and arrange

$$\text{Real term}: \frac{dV_{ar}}{dt} = -\omega_{1/2} V_{ar} - \Delta\omega V_{ai} + r_L \omega_{1/2} I_r$$

$$\text{Imaginary}: \frac{dV_{ai}}{dt} = \Delta\omega V_{ar} - \omega_{1/2} V_{ai} + r_L \omega_{1/2} I_i$$

System Equation for SRF cavity: First order ordinary differential equation.
Initial condition problem

This equation set describes SRF cavity field in complex space.
Using following relations sets of power and voltage can be calculated.

$$\text{Input constant}: \omega_{1/2} = \omega_0/(2Q_L),\ \Delta\omega = \omega_0 - \omega,\ r_L = \frac{Q_L}{2}(r/Q)$$

slowly varying $\Delta\omega$ in time can be an input too.

$$\text{driving source}: \mathbf{I} = \mathbf{I}_g + \mathbf{I}_b$$

$$\mathbf{V}_{for} = \frac{r_L}{2}\mathbf{I}_g,\ P_g = \frac{|\mathbf{V}_{for}|^2}{2r_L},\ |\mathbf{I}_b| = |2I_{b0}|$$

$V_{ar}, V_{ai} \Rightarrow$ cavity voltage and phase

$V_{fr}, V_{fi} \Rightarrow$ forward votage, phase & forward power

$V_{rr}, V_{ri} \Rightarrow$ reflected votage, phase & reflected power

$$\mathbf{V}_a = \mathbf{V}_{for} + \mathbf{V}_{ref}$$

One simple way using finite difference method (FDM)

$$V_{ar,t+\Delta t} = V_{ar,t} + (K_{11,t} + K_{21,t})/2$$

$$V_{ai,t+\Delta t} = V_{ai,t} + (K_{12,t} + K_{22,t})/2$$

$$K_{11,t} = \Delta t(-\omega_{1/2} V_{ar,t} - \Delta\omega V_{ai,t} + r_L \omega_{1/2} I_{r,t})$$

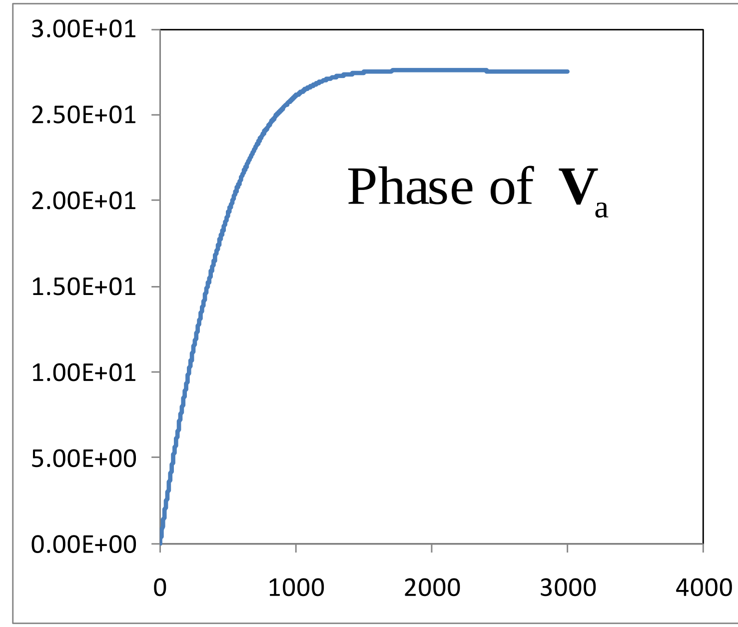
$$K_{12,t} = \Delta t(\ \ \Delta\omega V_{ar,t} - \omega_{1/2} V_{ai,t} + r_L \omega_{1/2} I_{i,t})$$

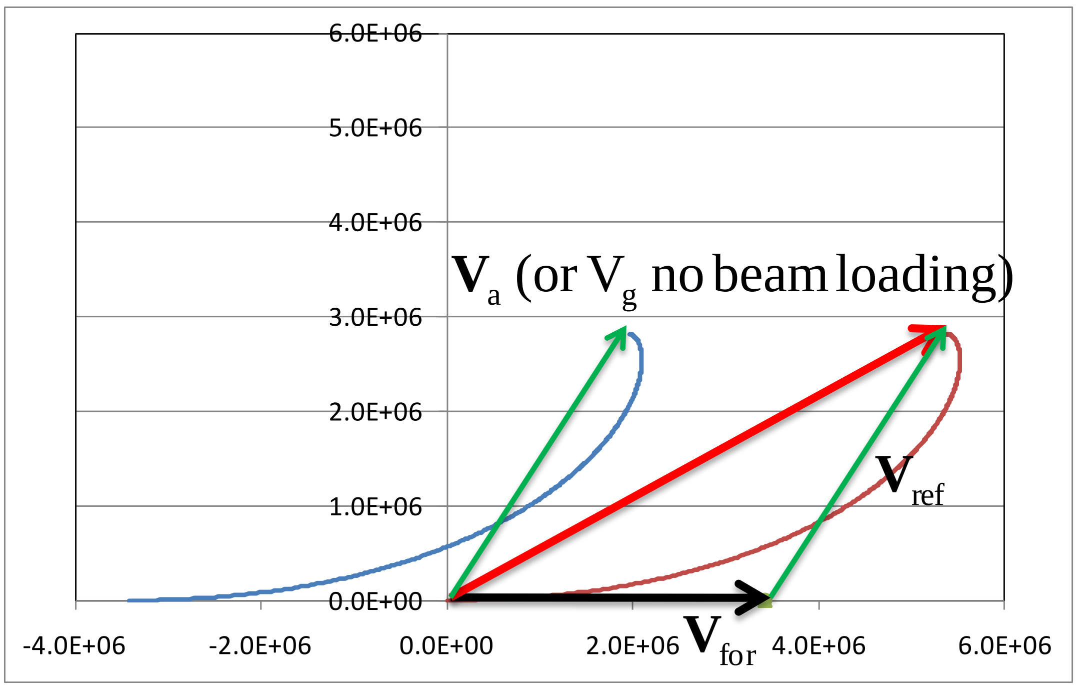
$$K_{21,t} = \Delta t\left\{-\omega_{1/2}(V_{ar,t} + K_{11,t}) - \Delta\omega(V_{ai,t} + K_{12,t}) + r_L \omega_{1/2} I_{r,t}\right\}$$

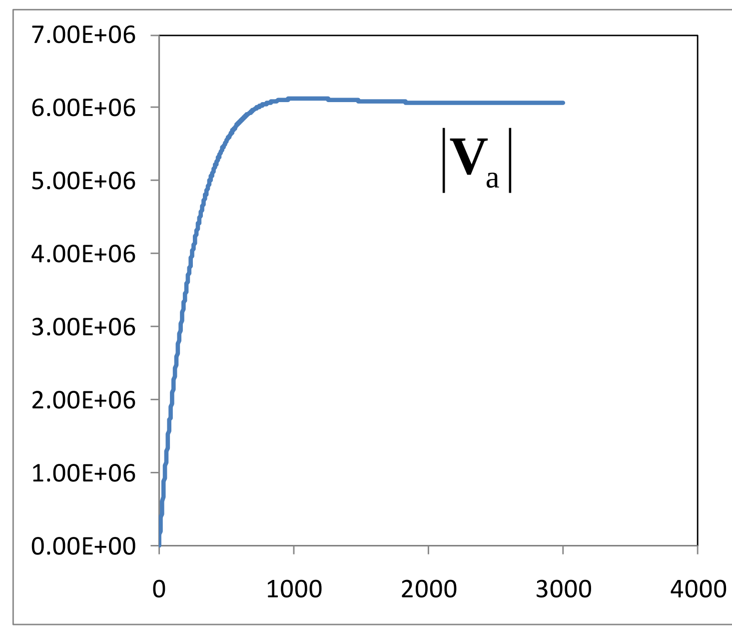
$$K_{22,t} = \Delta t\left\{\ \ \Delta\omega(V_{ar,t} + K_{11,t}) - \omega_{1/2}(V_{ai,t} + K_{12,t}) + r_L \omega_{1/2} I_{i,t}\right\}$$

Ex) RF only

RF power	60000
Phase1	0
F	8.05E+08
w	5.06E+09
QL	7.00E+05
r/Q	279
Cl	0.6816
rL	97650000
w1/2	3.61E+03
detuning end of pulse	300
slope Hz/ms	0
filling time us	3.00E+03
dt	2.00E-06



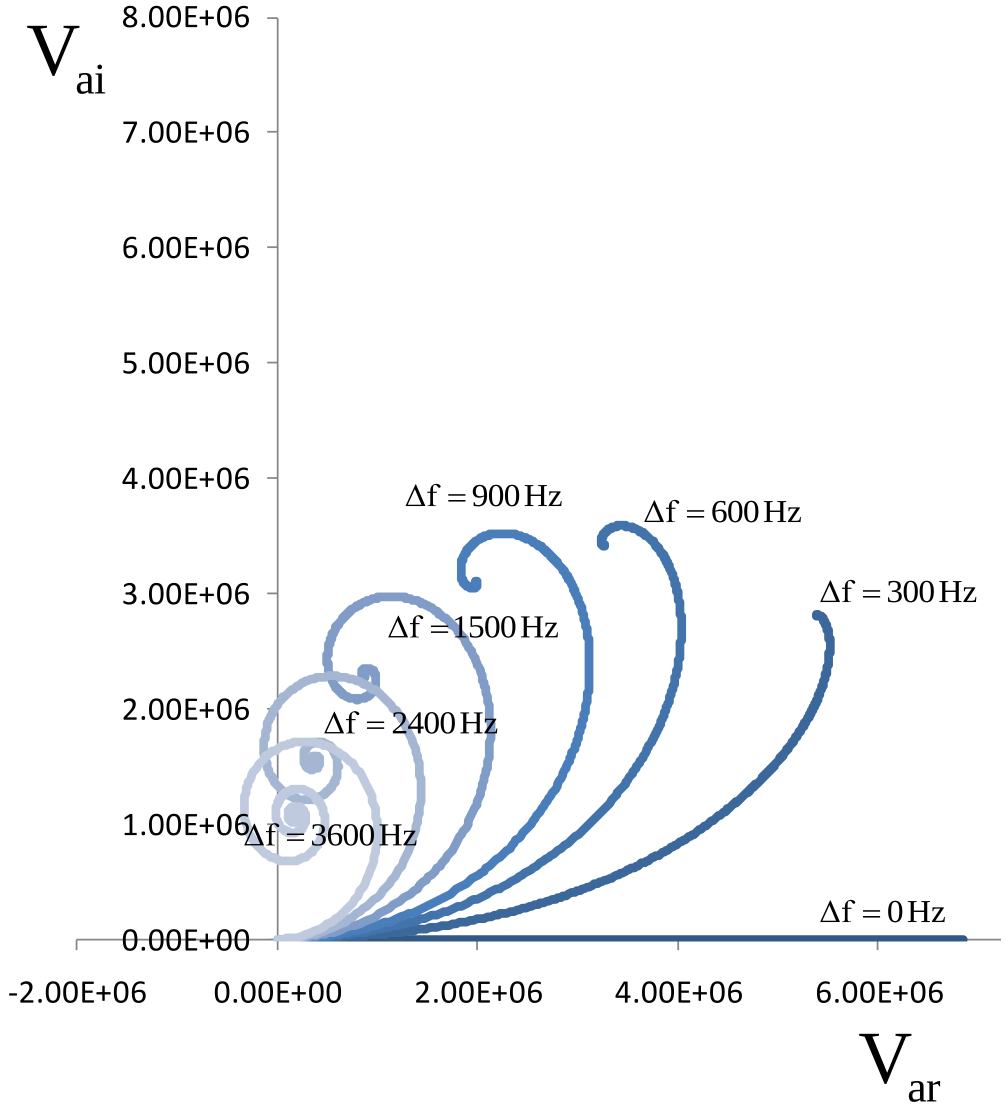


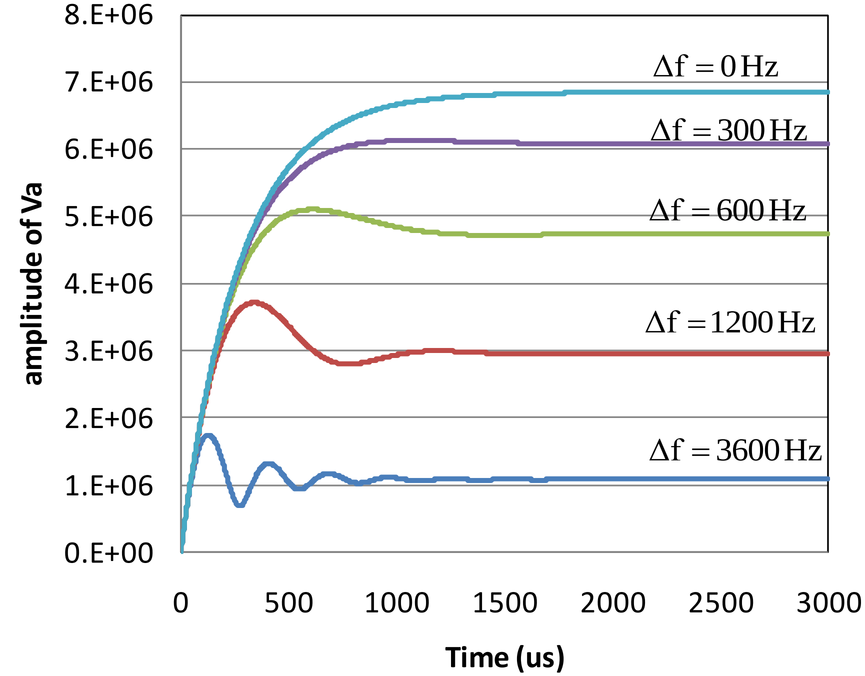


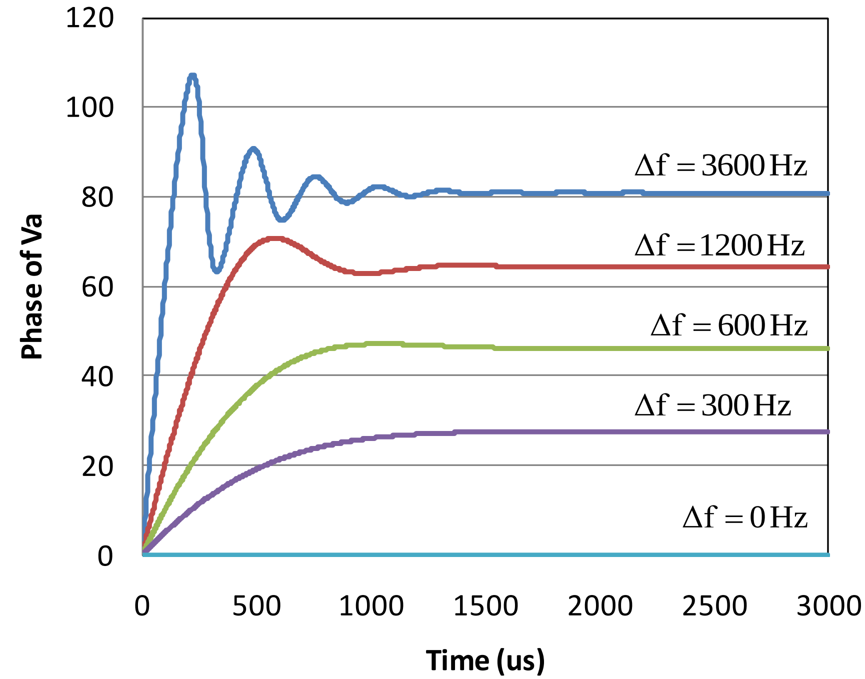
(filling_test_FDM.xls); change parameters in blue

HOMEWORK 4-4)

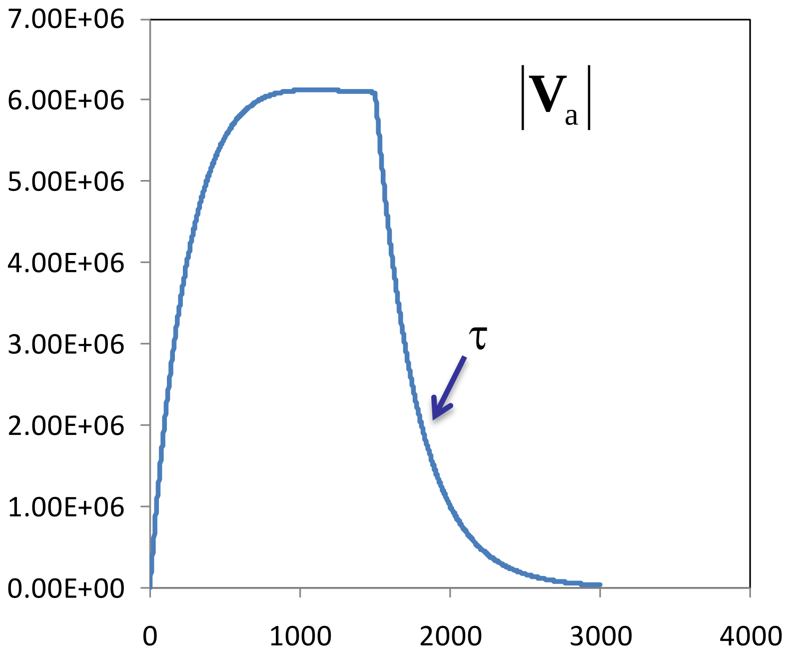
Generate these three plots $\mathbf{V}_a$ at various detuning using filling_test_FDM.xlsx (RF only)



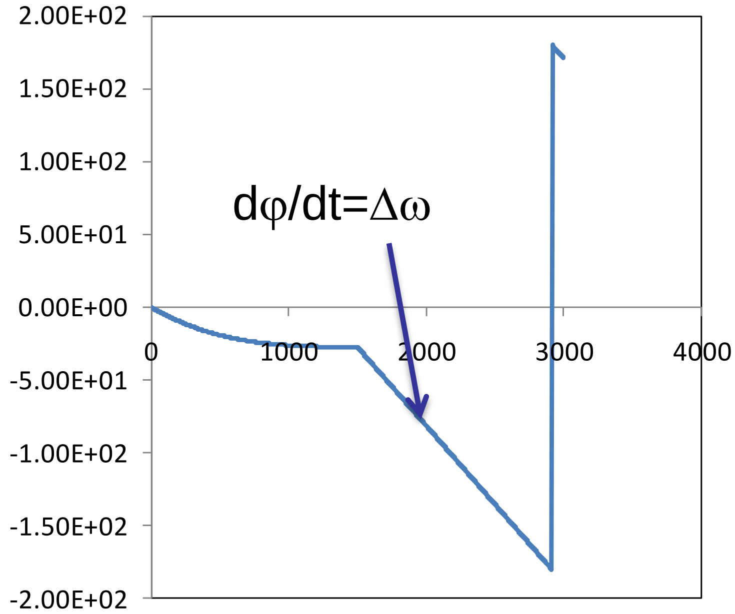


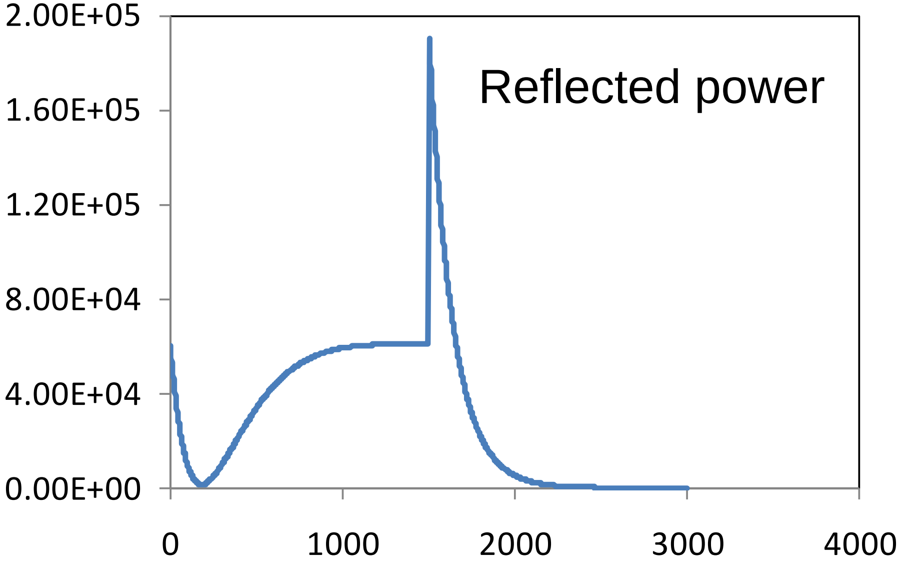


Ex) pulsed operation. RF pulse length 1500 μs, detuning -300 Hz
other parameters are same as in the previous example



Phase of $\mathbf{V}_a$



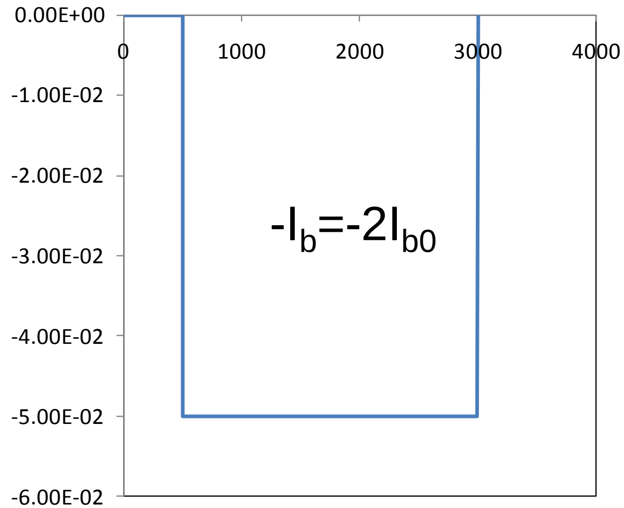


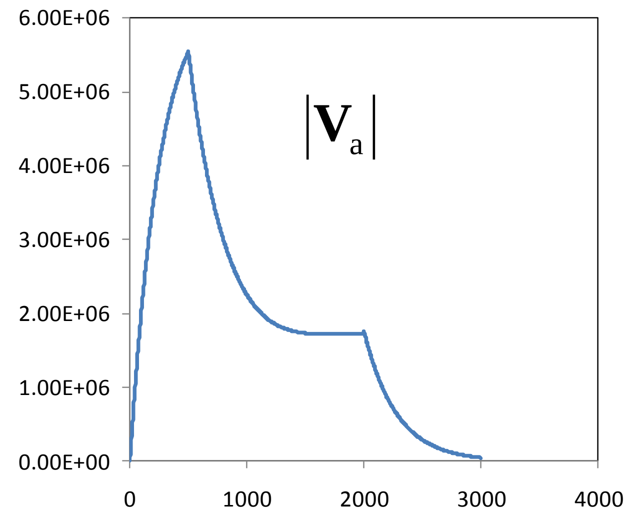
(filling_test_FDM.xls)
change parameters in blue

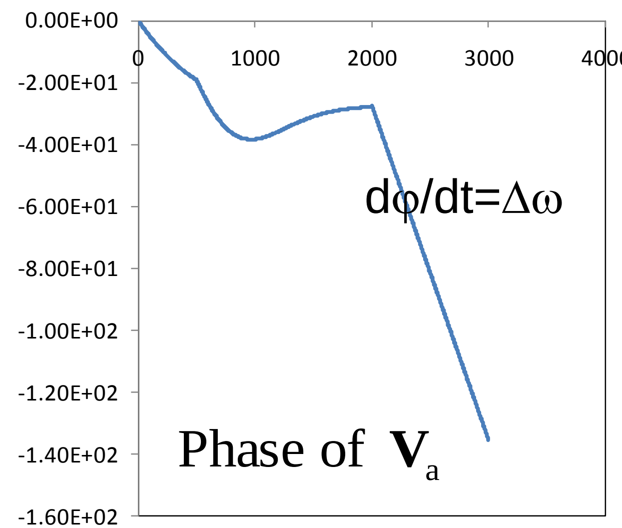
Ex) open loop with beam loading

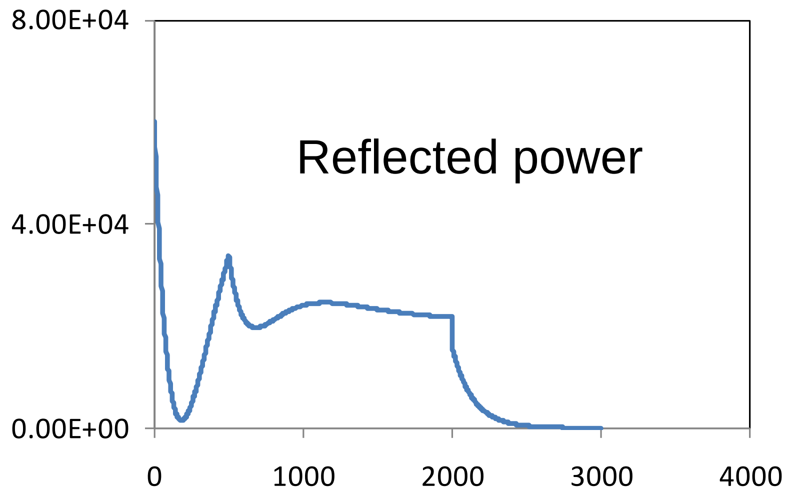
RF power (W)	**60000**
Vfor phase (degree)	**2**
f (Hz)	**8.05E+08**
w	5.06E+09
QL	**7.00E+05**
r/Q (Ohm)	**279**
Cl (m)	**0.6816**
rL	97650000
w1/2	3.61E+03
rL*w1/2	3.53E+11
detuning end of pulse	**-300**
slope Hz/ms	**0**
rf pulse length us	**2.00E+03**
Ib0 (A)	**2.50E-02**
beam enters at t (us)	**500**
bPhase (degree)	**0**
dt	**2.00E-06**

(pulse_beam_test_FDM.xls)
change parameters in blue









RF control

To have a beam with a required quality (emittance, energy spread, etc.) cavity field amplitude and phase should be maintained within a certain (machine specific) ranges.
For example, <1% in amplitude and <1 degree in phase are typical values.

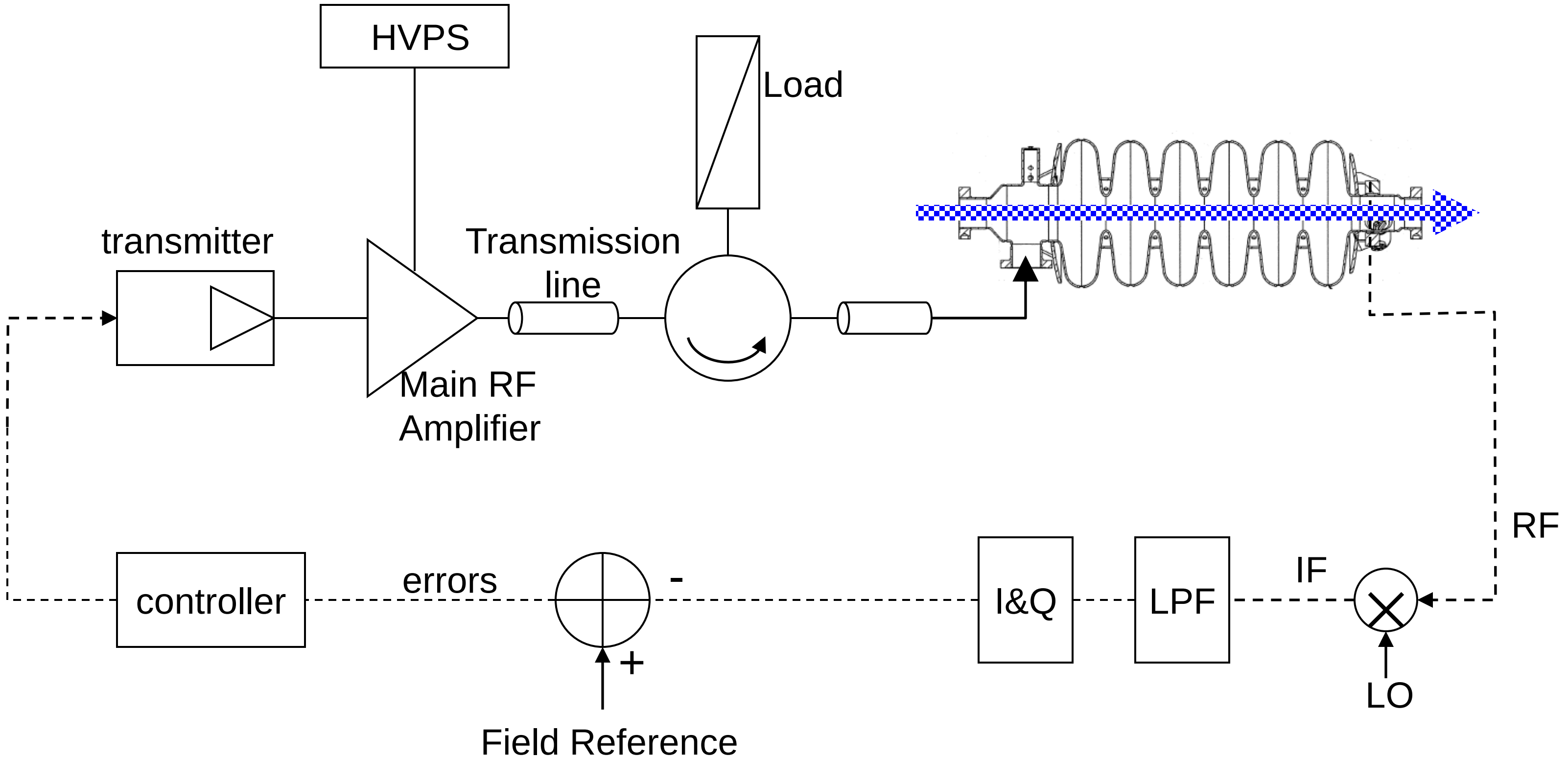
In modern digital LLRF systems, feedback control is an essential part and feed forward control becomes more popular.

LLRF control system should provide required cavity field stability against;

Beam loading (transient including jitter and/or CW)
Beam fluctuations
HPRF droop/ripple (mainly from HVPS)
Dynamic cavity detuning (Lorentz force detuning, microphonics)
Loop delay of RF control system
Reference RF phase/amplitude fluctuations
Electron loading conditions in a cavity and/or at around a power coupler
Any changes that affects characteristics of control system

    matching condition in HPRF transmission line
    thermal drift (electronic board, cables, etc.)

Field detection and control: digital I/Q mostly in modern control system
this uses conceptually same as the phasor relations we learned

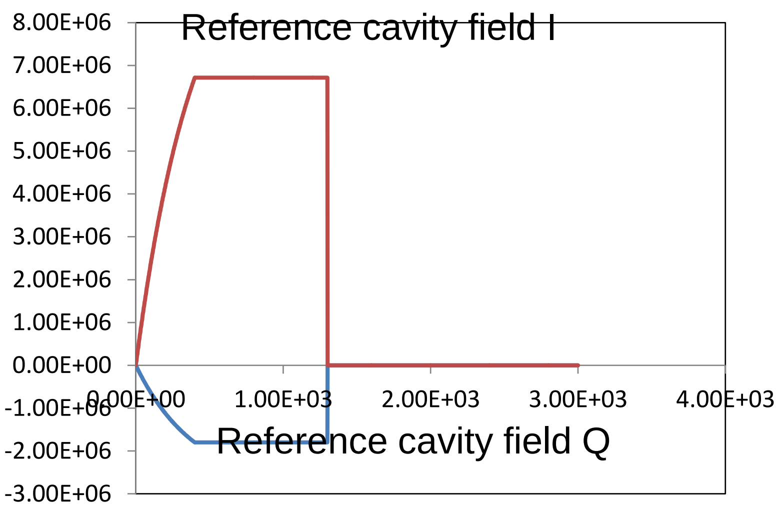


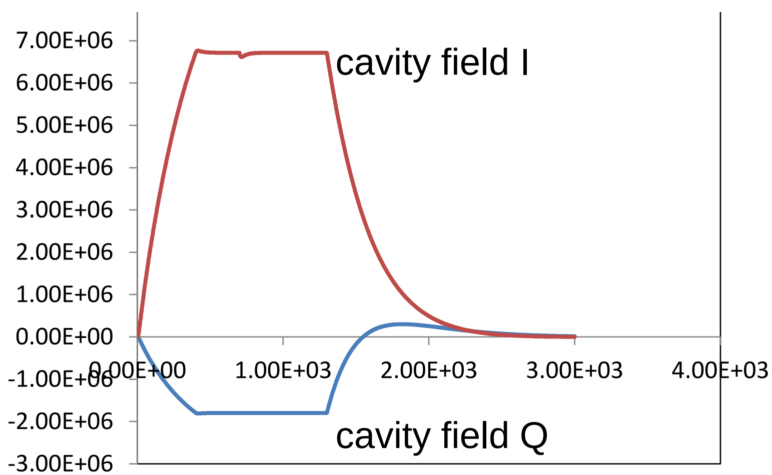
I: in-phase (corresponds real component),
Q: quadrant (corresponds imaginary component)

Ex) Feedback control example:

The real world has many complex practical issues. One can develop a conceptual understandings of the rf control from this example.

(Feed_back_test_FDM.xls) change parameters in blue





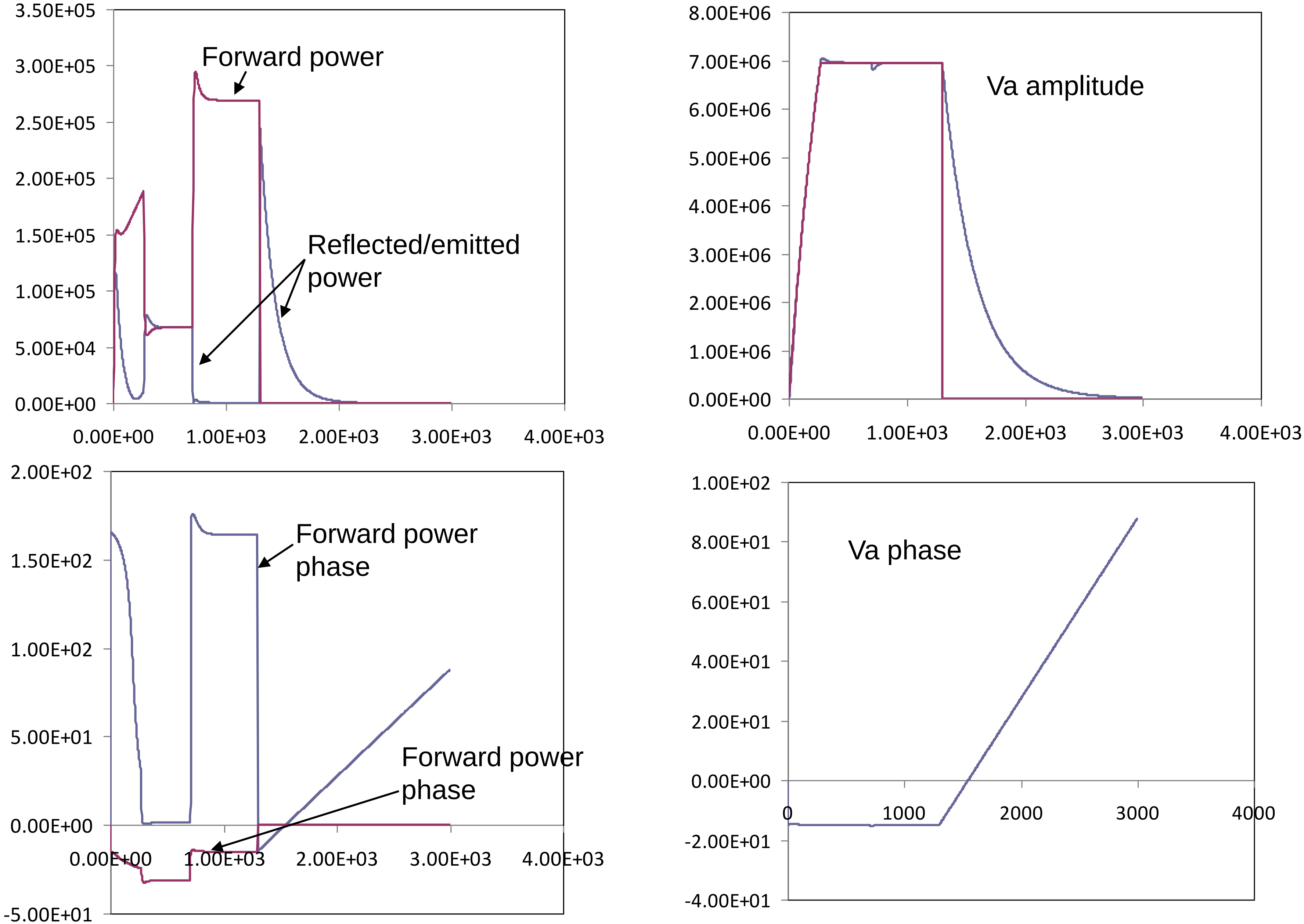
Parameters used
In this example

f (Hz)	**8.05E+08**
w	5.06E+09
QL	**7.00E+05**
r/Q (Ohm)	**279**
Cl (m)	**0.6816**
Vcav MV	**6952320**
filling time (s)	**4.00E-04**
(1-exp(-1))^-1	1.5819767
rL	97650000
w1/2	3.61E+03
rL*w1/2	3.53E+11
detuning end of puls	**168**
slope Hz/ms	**0**
rf pulse length us	**1.30E+03**
Ib0 (A)	**2.60E-02**
beam enters at t (us)	**700**
bPhase (degree)	**-15**
proportional gain K	**20**
integral gain Ki	**1**
dt	**2.00E-06**

For reference During filling → filling time (s)

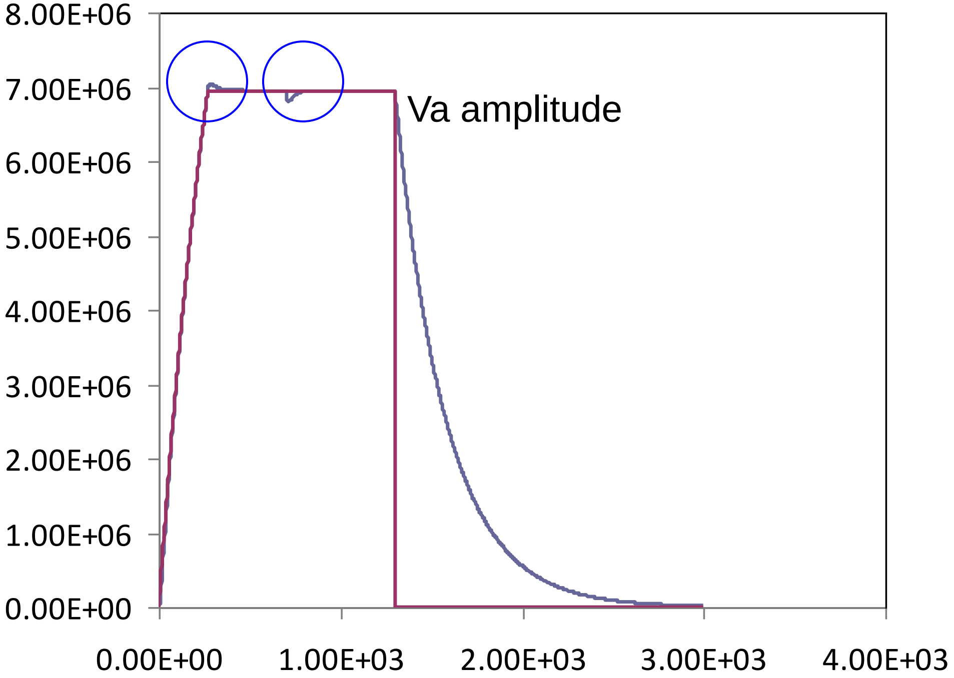
Constant detuning → detuning end of puls

Feedback gains → proportional gain K



RF control for constant field/phase:

One can say that it forces the system into steady state condition quickly and stably



There are still errors with feedback control from many other sources.
the errors are repetitive, one can generate error tables including loop delay information and apply to the next pulses. → feed forward

Cavity detuning due to the Lorentz force (dynamic)

Vibrations, resonances and damping

-Vibration source; RF pulse → repetitive hammering by radiation pressure
with frequencies of repetition rate and harmonics

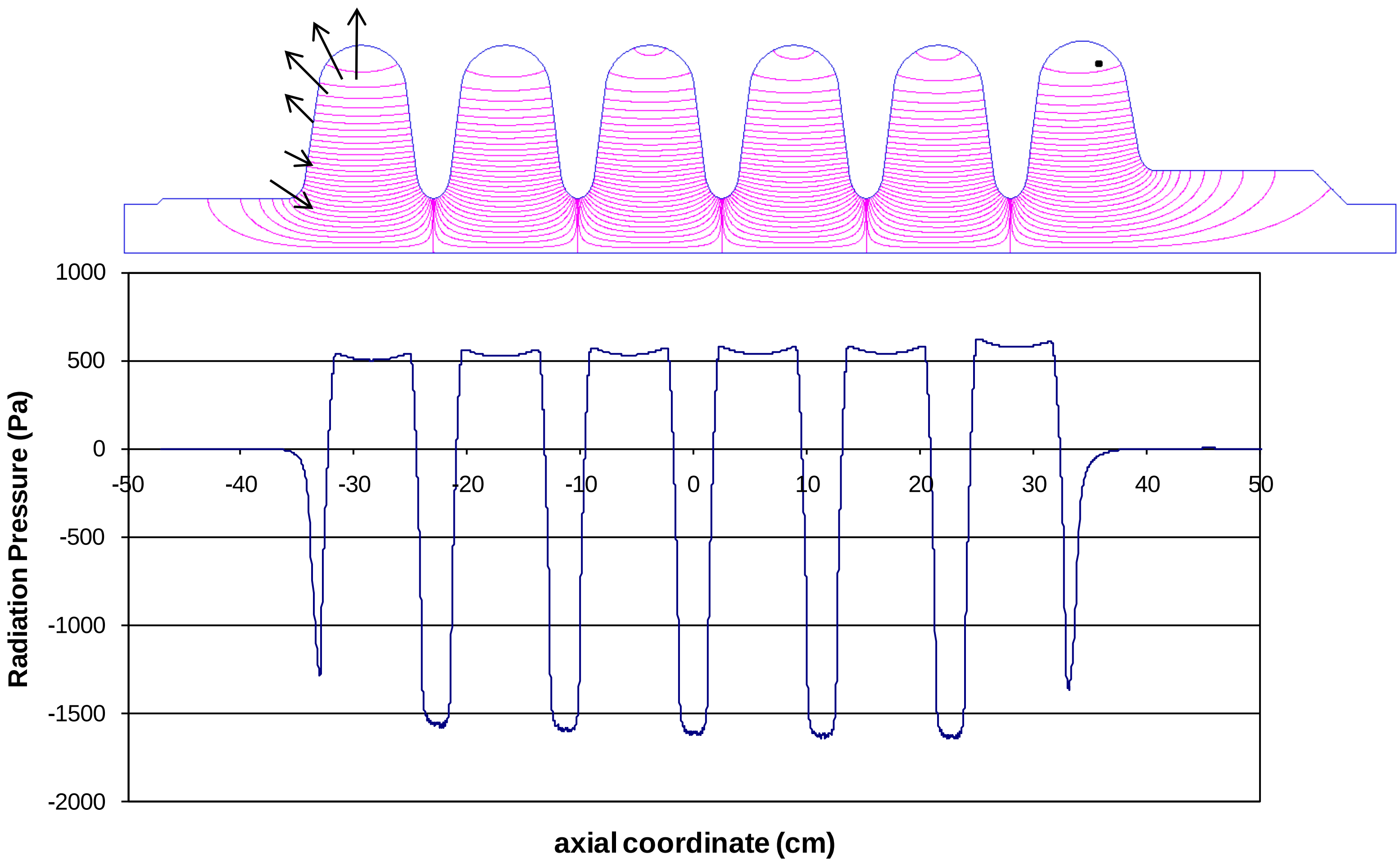
-Mechanical resonance frequencies (w_n)
determined by the equivalent mass of each mechanical mode
& equivalent stiffness of the system

-Resonance; source term hits around the mechanical resonance frequency

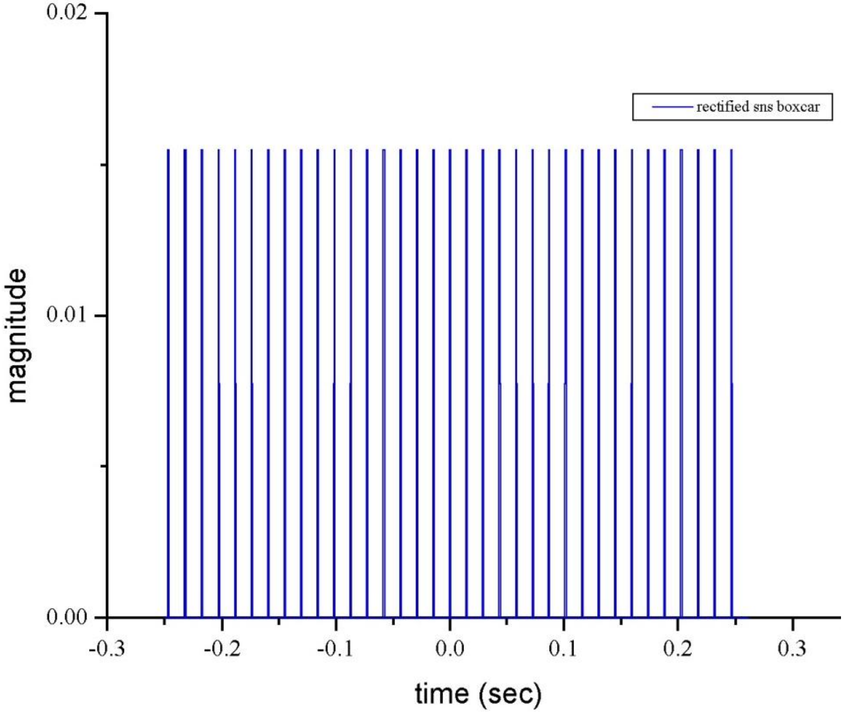
-Damping; determined by the whole system
energy transfer; sound wave radiation, internal/structural damping,
helium, heat dissipation, transfer to other system thru propagation

-Vibration amplitude; determined by the relation between the damping and the resonance.
generally amplitude is smaller at higher frequency and higher damping.
This will add initial frequency off-set.

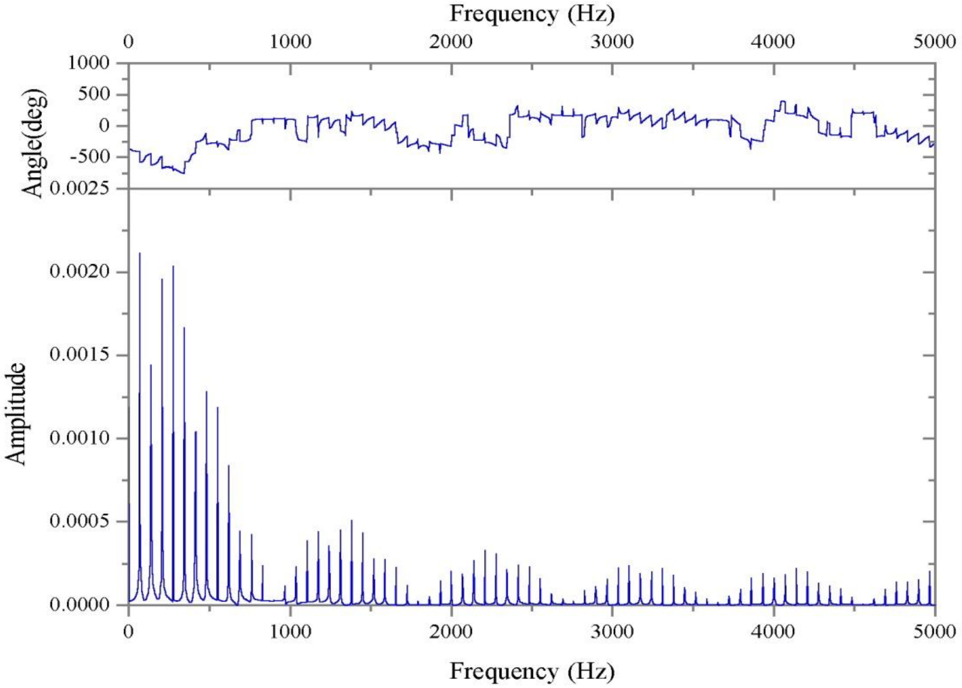
Radiation Pressure



Radiation Pressure in time and frequency domain (vibration source)

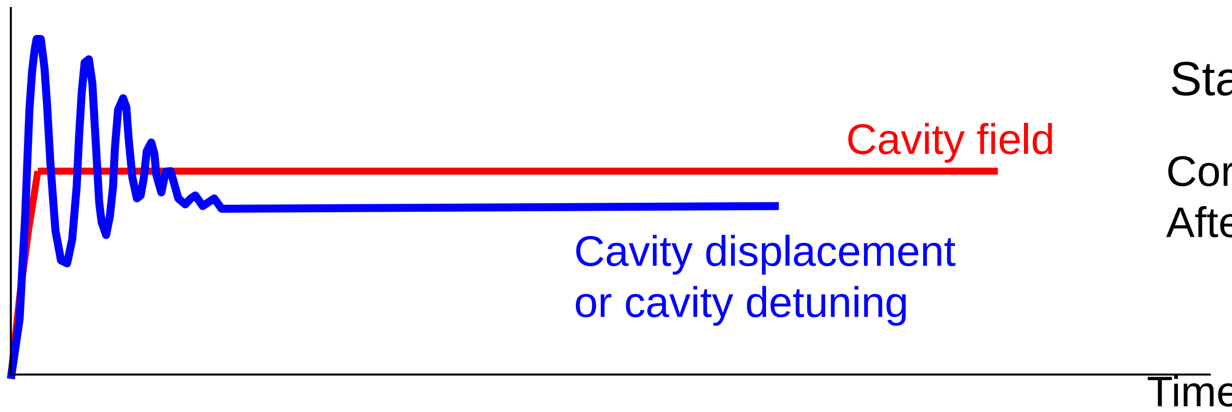


Impulsive Forcing Function
(time domain)



Transformed Forcing Function
(Frequency Domain)

CW

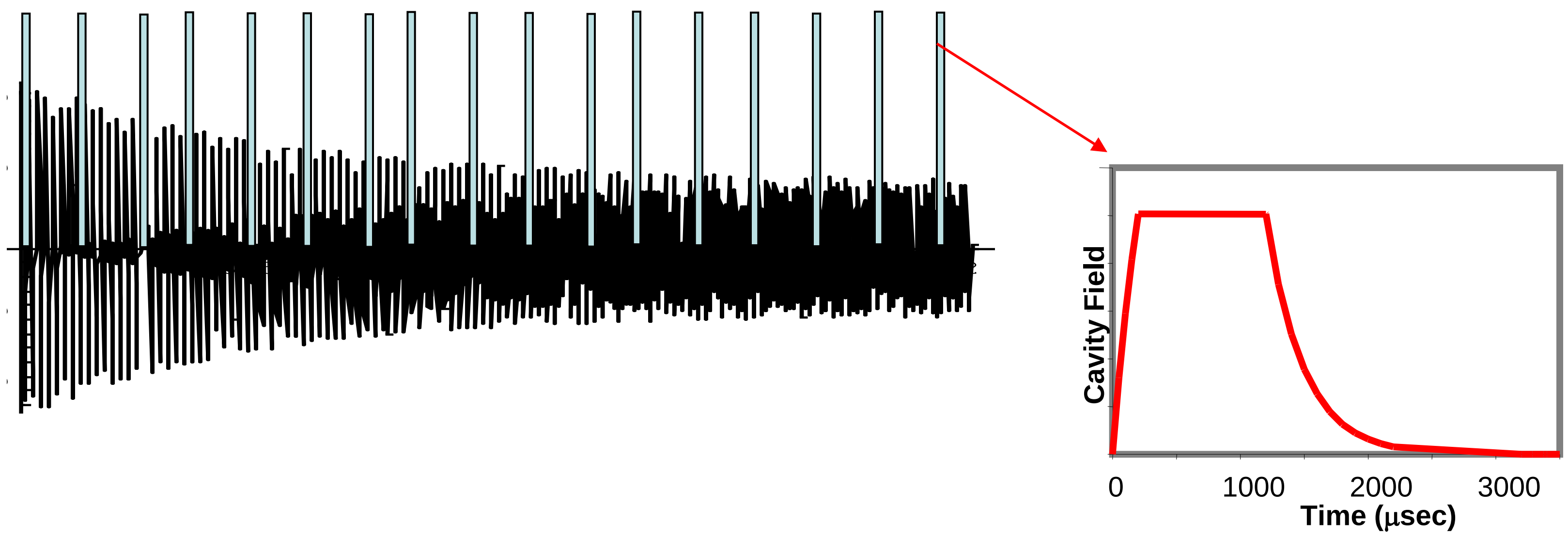


Static deformation

Corresponds static K_L
After transient disappear

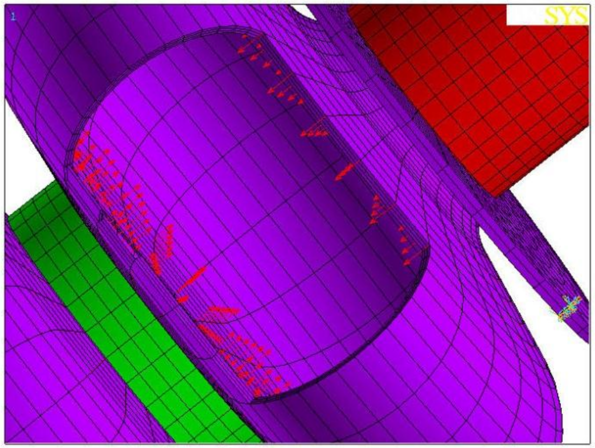
Dynamic vibrations. Reaches steady state after transient period.

Pulsed mode



Mechanical modes of the system

$$\Delta\ddot{\omega}_l + \frac{\omega_l}{Q_l}\Delta\dot{\omega}_l + \omega_l^2\Delta\omega_l = -k_l V_{cav}^2 \omega_l^2$$

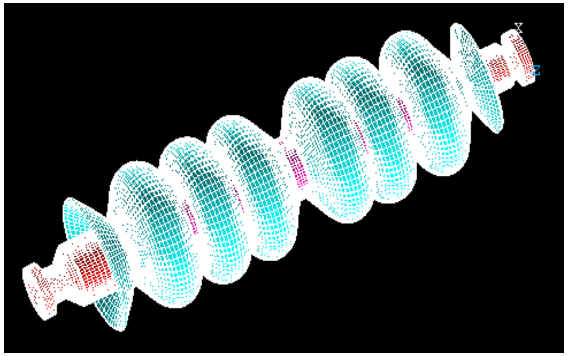


ω_l : resonance frequency of mechanical mode l

Q_l : Quality factor of mechanical mode l

$$\Delta\omega_{total} = \sum_l \Delta\omega_l ,$$

k_l: dynamic detuning coeff. of mode l



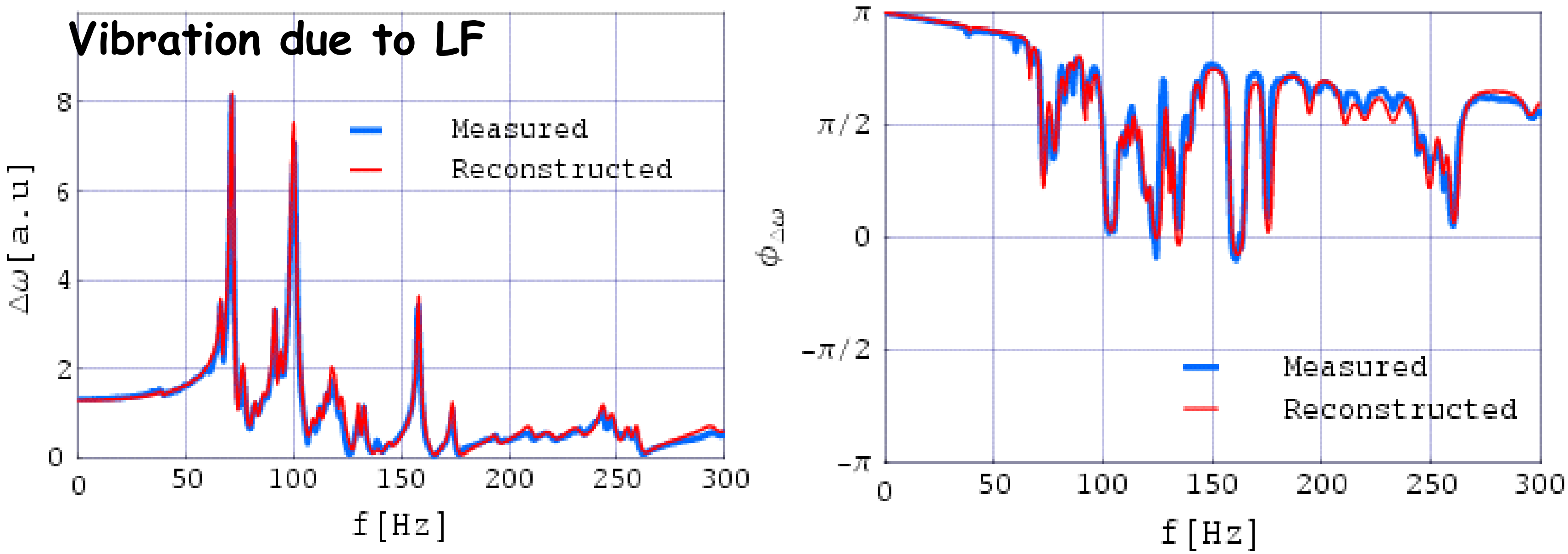
Resonance system

Mode, damping, & modal mass finding strongly depends on boundary conditions and whole mechanical system details.

→ Mode frequencies & Q of the mode can have large spread. → large error

Once mechanical responses (amplitude and phase) of one selected cavity are measured, quite accurate prediction or reconstruction is possible for that cavity.

But due to the sensitivity of mechanical mode characteristics, there is large scattering. Sometimes unpredicted mode could be found.

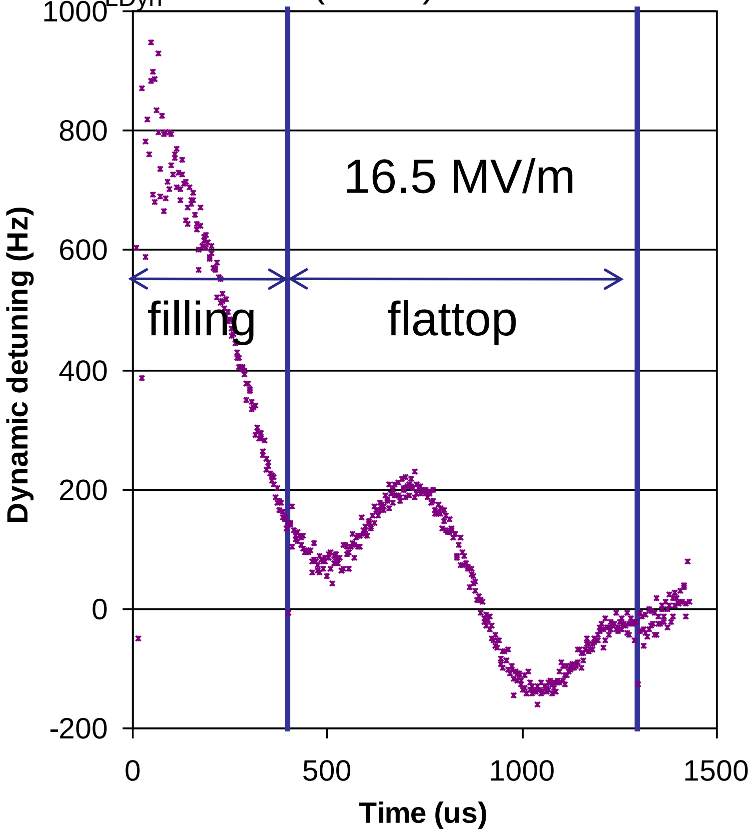


Measured transfer function by Lorentz force

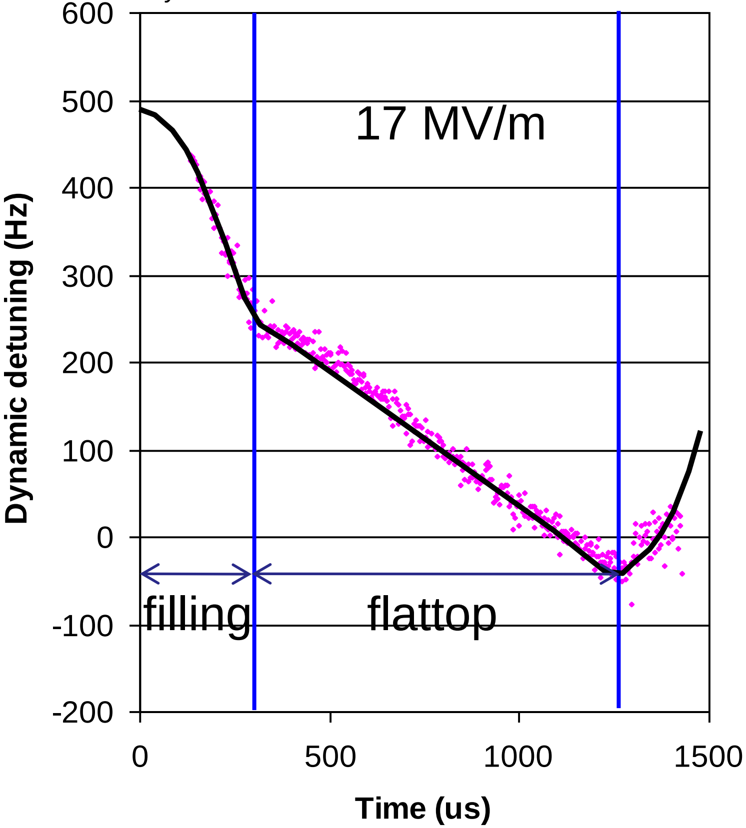
Ex) SNS cavities

Unpredicted 1.6kHz component sits on nominal low frequency response in medium beta cavities.

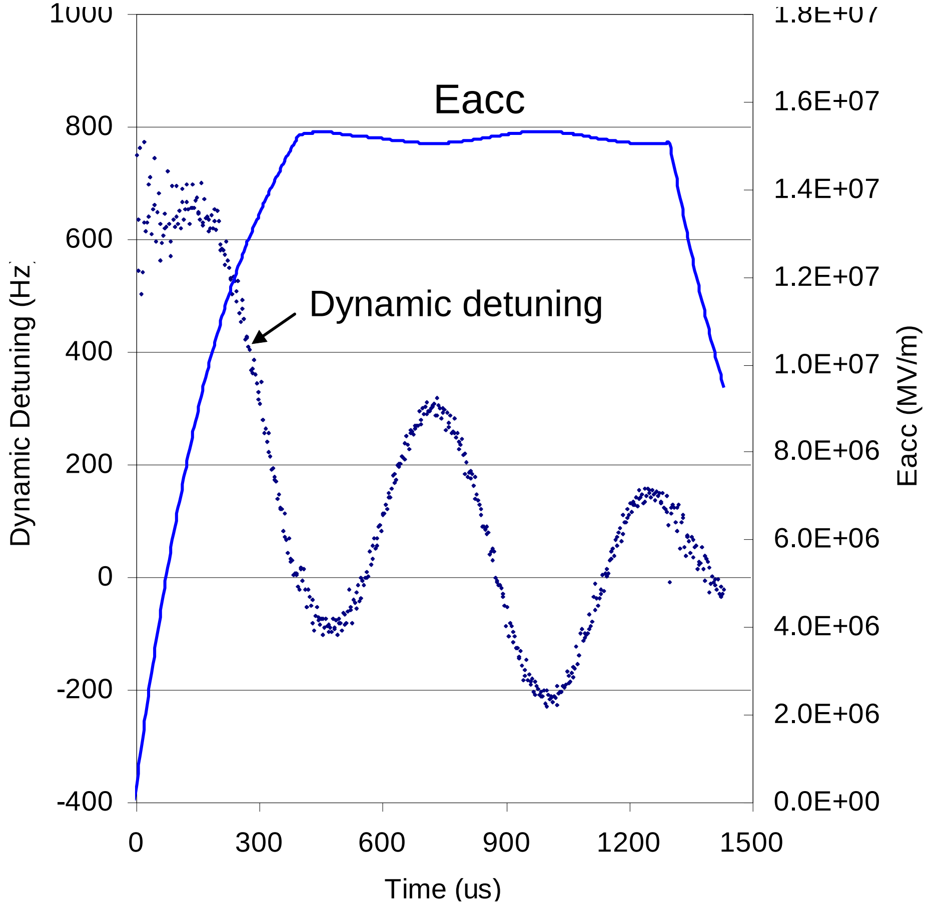
Medium beta cavity (installed cavity)
K_{LDyn}: 3~4 Hz/(MV/m)^2



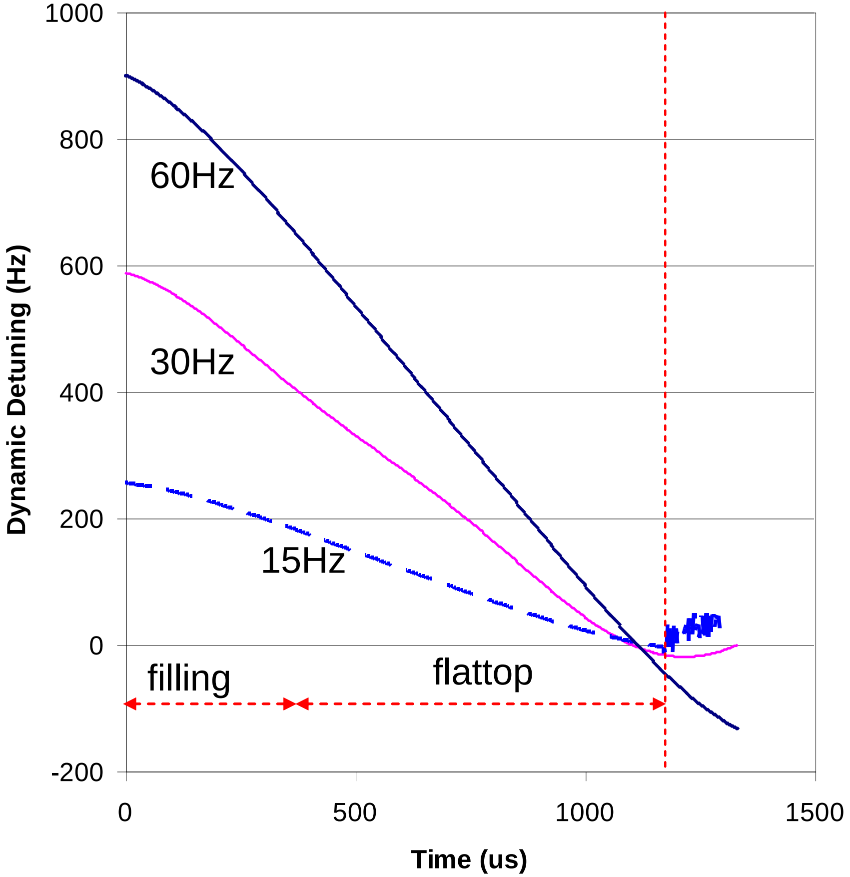
High beta cavity (installed cavity)
K_{LDyn}: 1~2 Hz/(MV/m)^2



some cavities show bigger resonance phenomena
repetition rate dependent



The 1.6 kHz components shows resonances at higher repetition rate in some of medium beta cavities



In this example the accelerating gradient is 12.7 MV/m. (high beta cavity)

Static driving forces are proportional to ‘square of cavity field’ (Lorentz force).

Dynamic responses could be quite different depending modal mass, modal boundary conditions, driving force spectrums.

Low Qex and high beam loading structure, not a big issue (will need some extra RF power).

Very important in pulsed machine especially in pulsed high Qex structure.

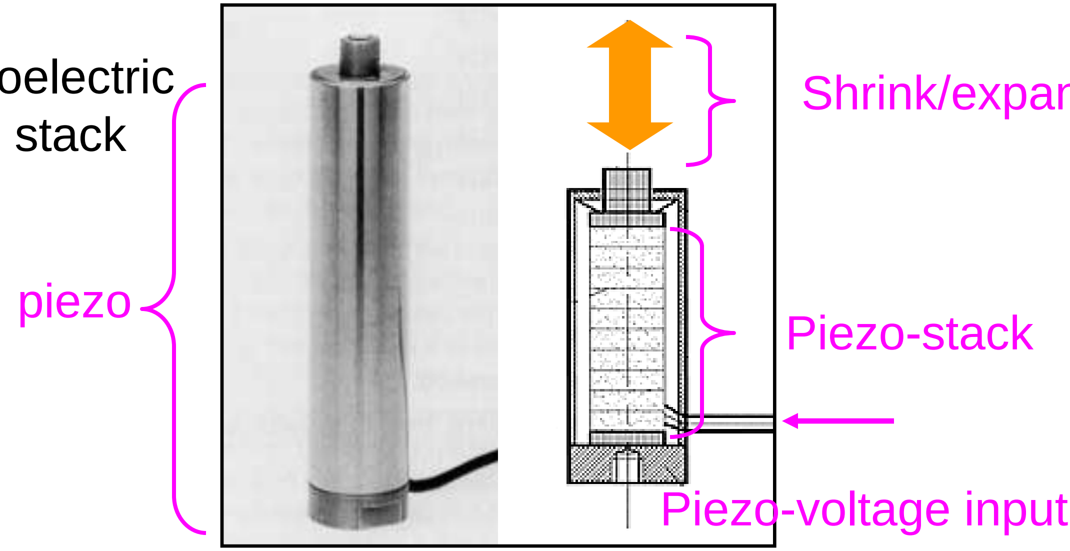
Generate steady state vibration pattern. Repetitive from pulse to pulse.

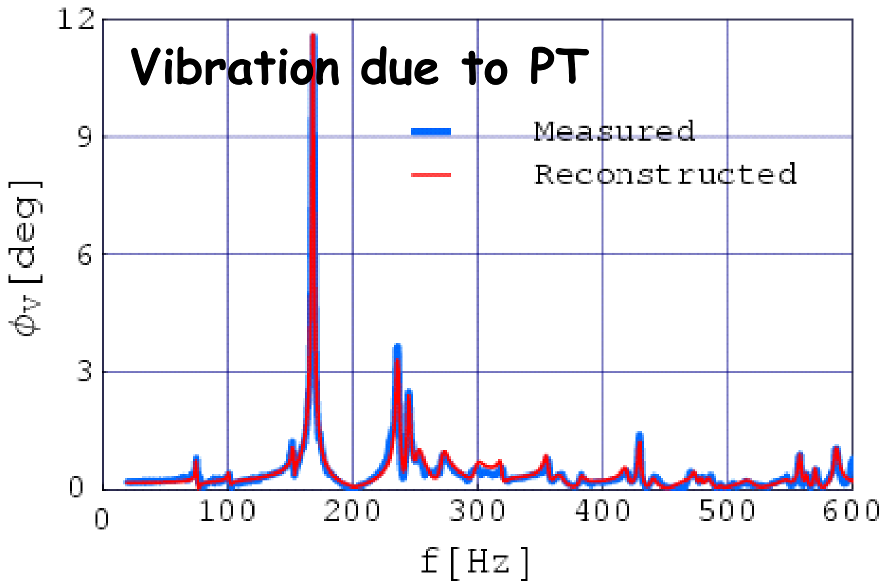
Counter vibration (compensation) for the biggest frequency components can correct quite efficiently. Demonstrations have been done using piezo-electric actuators.

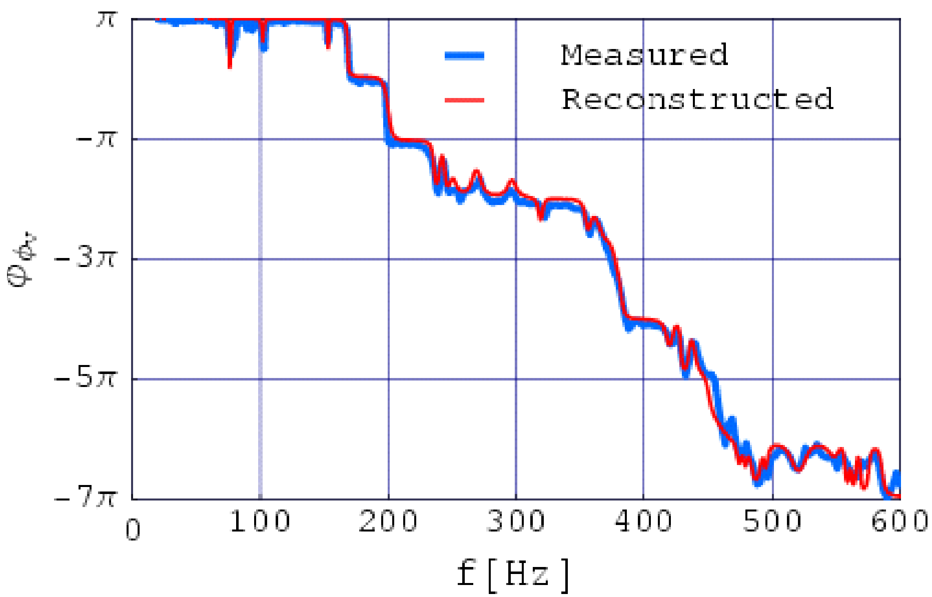
LFD compensation

An input voltage is applied to the piezoelectric actuator device which make the piezo stack shrink/expand

Since the forcing mechanisms are different between LFD and piezo detuning system responses are not same.







Building a virtual cavity for dynamic detuning (complete set of modeling)

General RF eq. of Cavity field

$$\int_0^t \tilde{I}\, e^{j\int_{t'}^{t} \tilde{\omega}\, dt''}\, dt'$$

Dynamic detuning Due to LF

$$\Delta\omega_{L,m} = -\Omega_m^2 k_L$$

Dynamic detuning Due to Piezo tuner

$$\Delta\ddot{\omega}_{P,m} + \frac{\Omega_m}{Q_m}\Delta\dot{\omega}_{P,m} + \Omega_m^2 \Delta\omega_{P,m} = -\Omega_m^2 k_{P,m} V_P$$

$$\Delta\omega_P(t) = \Delta\omega_{osc} \sin \omega_{osc} t$$

$$\sigma = \frac{\Delta\omega_{osc}}{\omega_{osc}} \quad ; \quad \psi_i = \frac{\omega_{1/2}}{\omega_{osc}} \quad ; \quad \theta = \omega_{osc} t$$

$$\tilde{v}(\theta) = e^{-j\sigma\cos\theta} \sum_n \tilde{P}_n(\sigma) \cos\theta_n \cos(n\theta - \theta_n) \ldots /2))$$

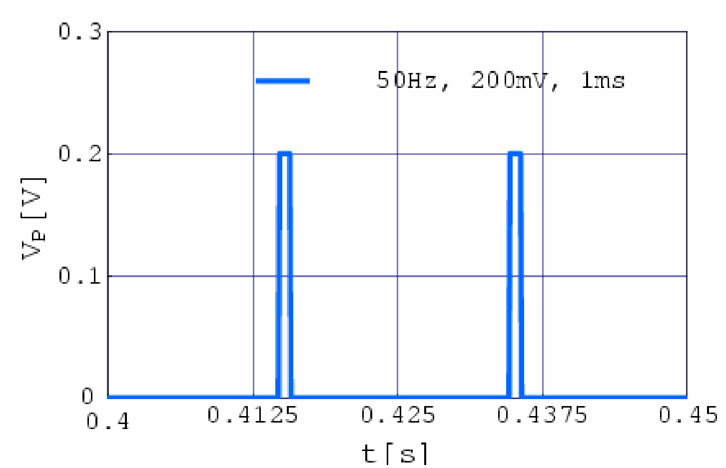
$\tilde{P}_n(\sigma)$ polynomes of σ

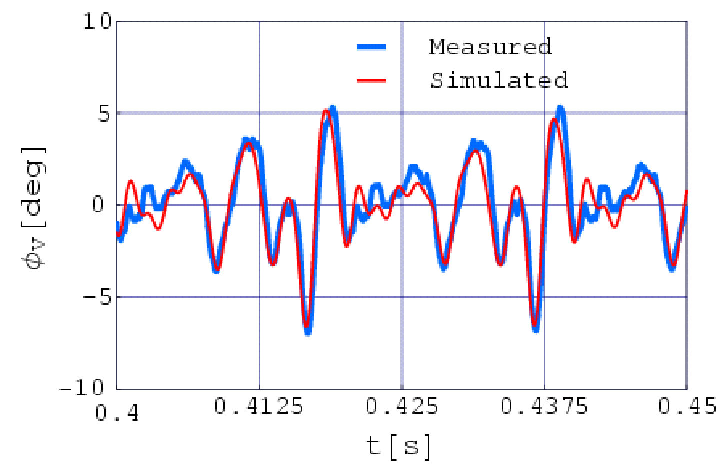
$$\tan\theta_n = \frac{n}{\psi_i}$$

Virtual Cavity (analytic basis; very fast and provide general view

Many useful and practical tools
: optimization of compensation for LFD
microphonics compensation study in high Qex cavity
ponderomotive oscillation study for RF system…

Verification of Virtual Cavity



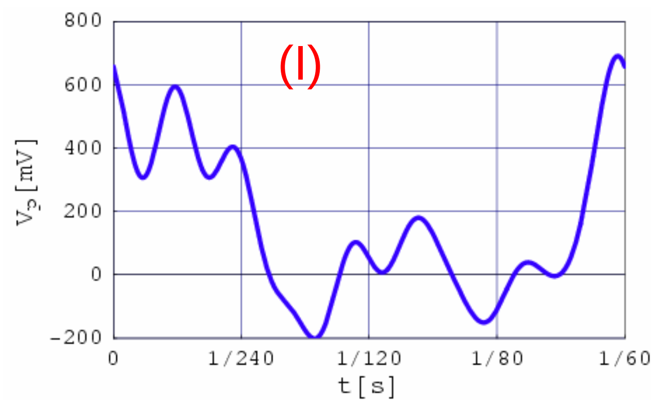


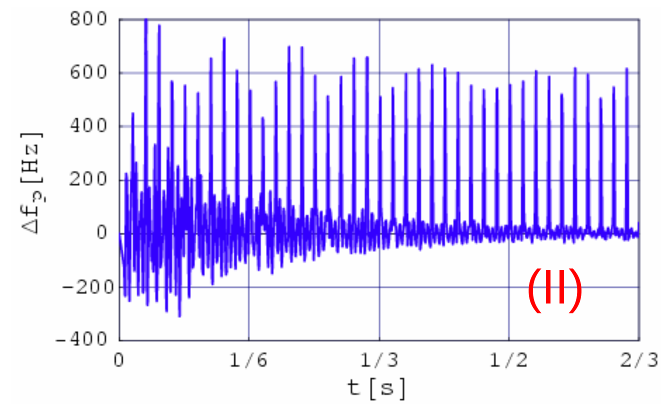
Ex. LFD compensation (optimization study)

LF only contains harmonics of the repetition rate.
Ideally, perfect compensation is possible.
Decompose the LFD into harmonics → find corresponding piezo signal components

(I) Piezo. Tuner input voltage

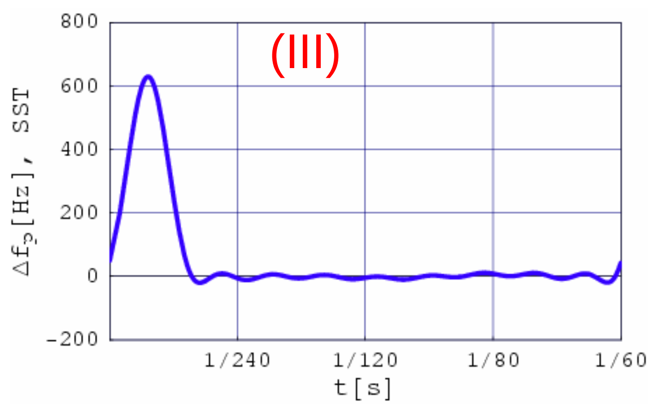
(II) Generated detuning (including transient)

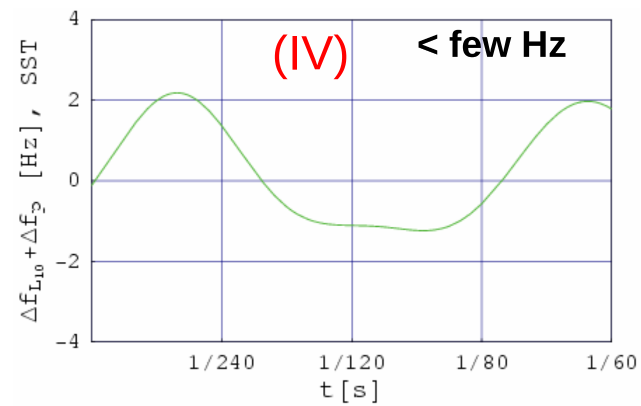




(III) Detuning generated by the Piezo. Tuner in SST

(IV) Its sum with the initially Targeted portion of the Lorentz Detuning





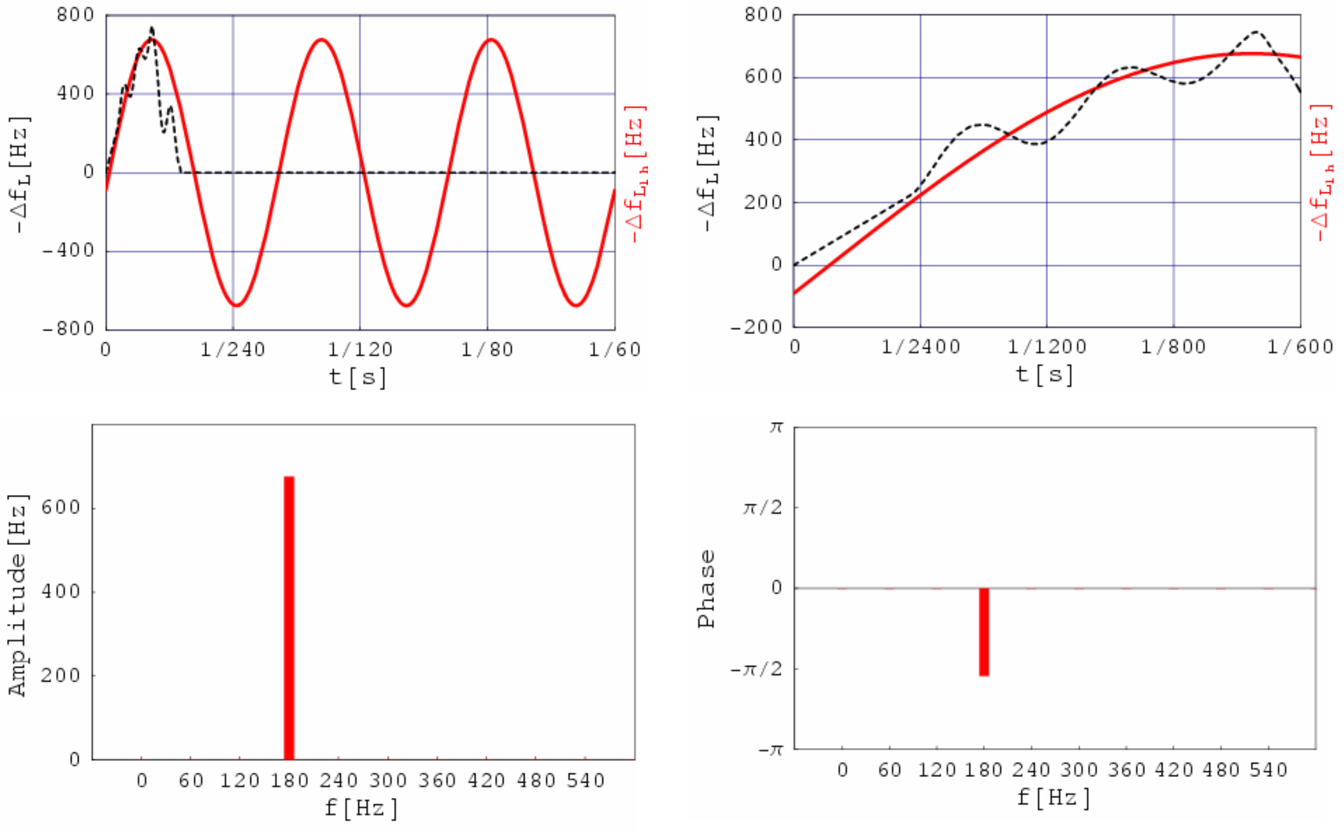
Seems to be complex to apply for the real system

Practically applicable and straightforward compensation scheme (simple harmonic compensation)

-Only the Lorentz Detuning during the RF turn-on transient and the beam pulse is tried to be compensated

-A single harmonic seems sufficient to obtain a satisfying compensation

-The Piezo. Tuner input voltage contains only this harmonic
→ Simple waveform



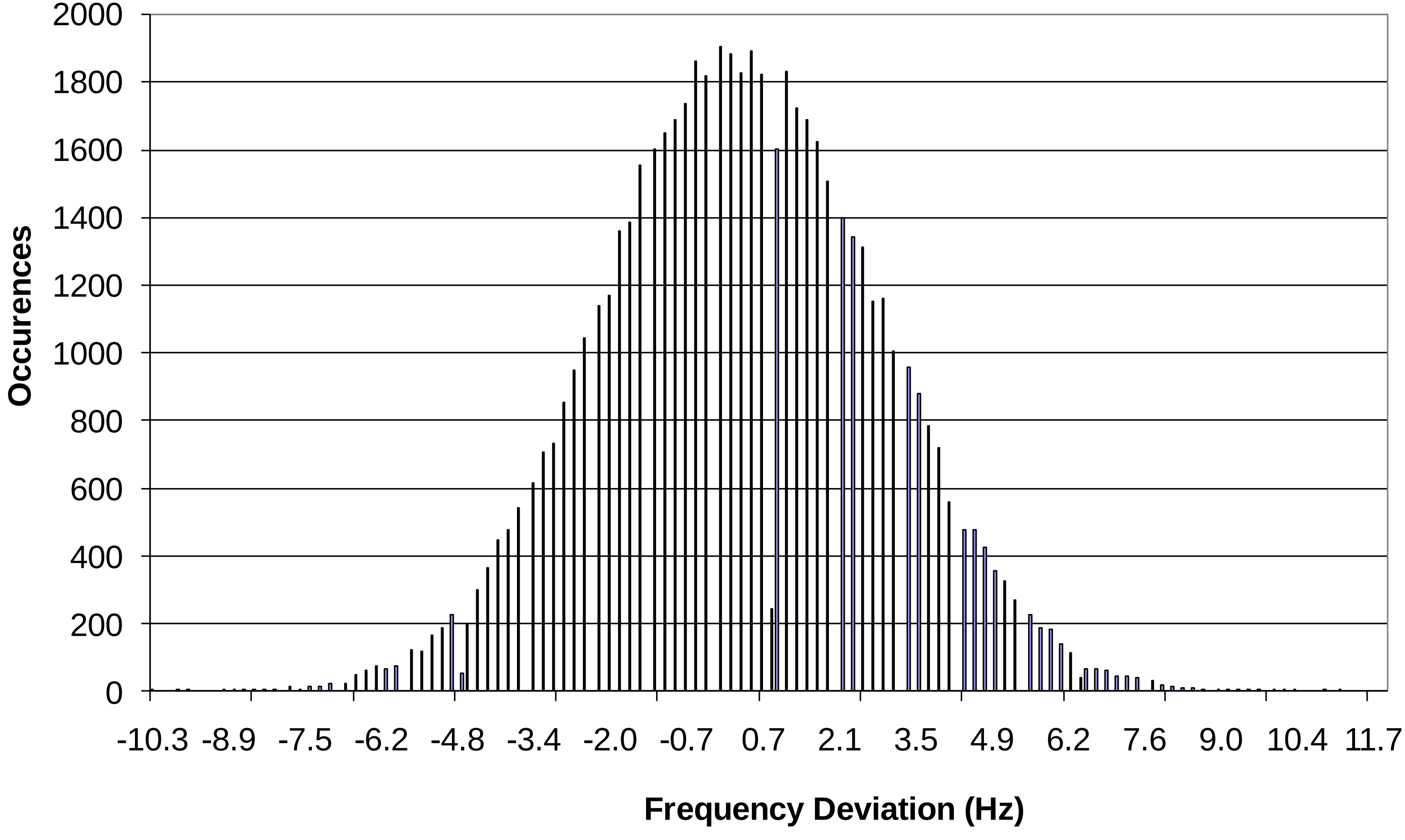
Ex. Analysis with dynamic detuning (feed_back_test_FDM_dyn_detuning.xls)

Microphonics

There are always mechanical vibrations from environments.
These vibrations can shake cavities.
Responses of cavities are function of dynamic/modal characteristics of the system (not only by cavity mechanical properties).
Qex is normally higher in low beam current machines. Cavity bandwidths are getting narrower as Qex's get higher.
If HPRF does not have enough margin, a cavity field may not reach its operating setpoint.

Examples of vibration sources
- helium pressure fluctuations
- pumps (water, mechanical vacuum)
- ground vibration including ocean waves (1/7 Hz)
- traffics
- etc.

Ex) Microphonics measurement at JLab



Beam current in proposed CW superconducting linacs is < several mA.
→ Q_b is mid 10^7 range

If loaded Q Q_L is 5×10^7 (for RF efficiency), $\omega_{1/2} = \omega_0/(2Q_L)$ will be in comparable ranges of microphonics.
f=650 MHz → $f_{1/2}$=6.5 Hz
f=80.5 MHz → $f_{1/2}$=0.8 Hz

If $\omega_{1/2}$ is too small,
a few bandwidth of cavity frequency variation will cause large cavity phase variations, and/or cavity field can not be kept at operating point.

It may need to make stiff cavities to push mechanical frequencies as high as possible so they don't couple to the low frequency mechanical noise that has the largest amplitudes. But required force of mechanical tuner should be in a reasonable range.

Amplifier may need to have a certain amount margin that can cover detuned cavity operation.

Active feedback/feed forward control may need to be used.