

Electron Laser interaction

System consists of

Relativistic electron beam

Magnetic field

Laser beam

Following section describes interaction of the electron beam with either magnetic field, laser beam or both

References:

Classical Electrodynamics, Jackson, Ch. 9, 12, 14,

Free Electron Lasers, C. H. Brau, Ch.1, 2

High energy free electron laser accelerator, E. D. Courant, C. Pellegrini, W. Zakowicz, Phys. Rev A 32(1985) 2813

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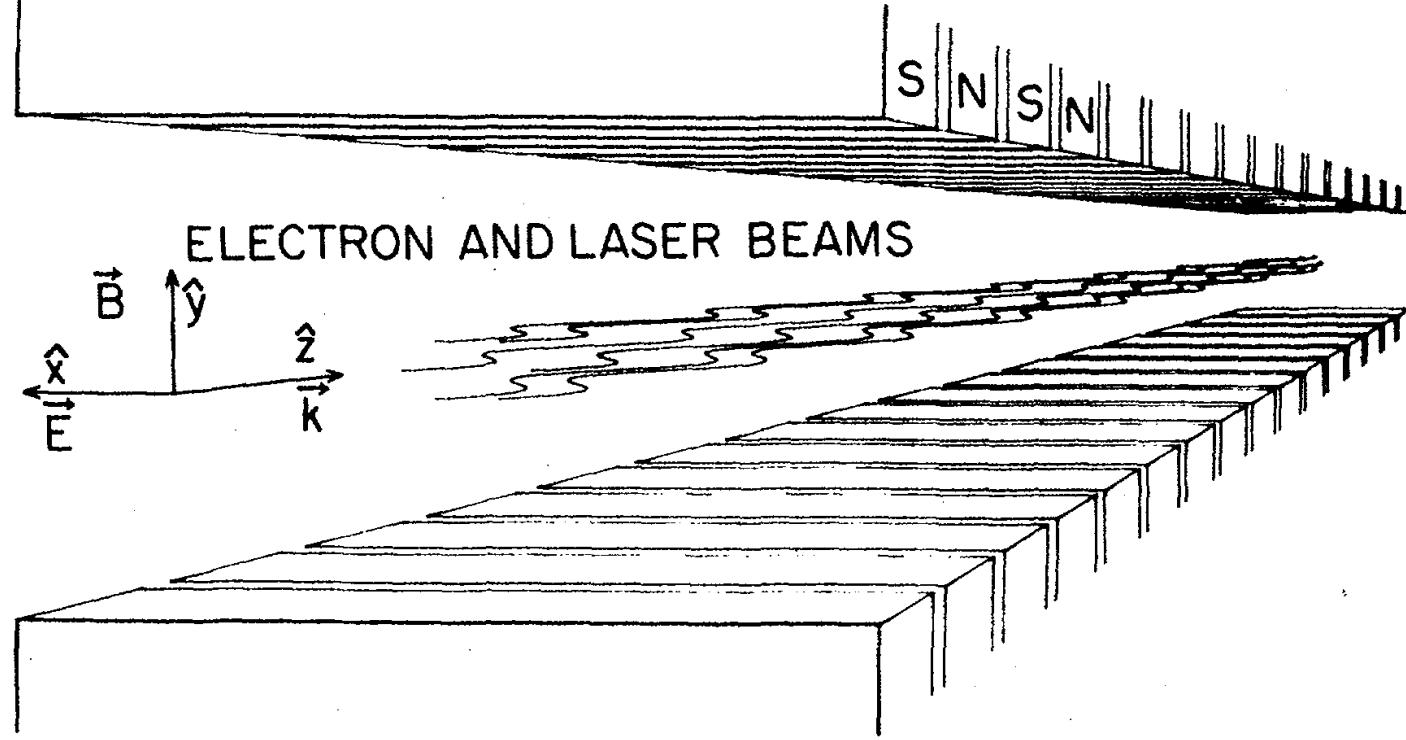


FIG. 1. Schematic view of IFEL accelerator.

Let us consider a system where relativistic electrons are moving along a magnetic wiggler in the field of a laser. Let us further assume that the Poynting vector of the laser, the electron propagation direction, and the wiggler axis are parallel. The Lorentz equation of motion of the electron, including the force of radiation reaction, F_{reac} , can be written as

$$m \frac{d(\vec{\gamma})}{dt} = e \left[\vec{E}_l + \frac{\vec{\nu}}{c} \times (\vec{B}_l + \vec{B}_w) \right] + \vec{F}_{reac}$$

\vec{E}_l Is the electric field of the laser,

\vec{B}_l & \vec{B}_w are the magnetic field associated with the laser and the wiggler respectively,

$$\gamma = (1 - \beta^2)^{-1} \quad \text{and}$$

$$\beta = v / c$$

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For a transverse EM wave such as the laser beam,

$$\vec{B}_l = \vec{k} \times \vec{E}_l = \vec{z} \times \vec{E}_l$$

$$m \frac{d(\vec{\mathcal{W}})}{dt} = e \left[\vec{E}_l (1 - \beta_z) + \hat{z} (\vec{\beta} \bullet \vec{E}_l) + \vec{\beta} \times \vec{B}_w \right] + F_{reac}$$

For

$$\beta_T \ll \beta_z \ll 1$$

the laser field is not extremely strong

the radiation loss (due to) is small compared to the electron energy

Conservation of canonical transverse momentum
dictates

$$p_T = m\gamma v_T + e(A_l + A_w) = \text{const}$$

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The longitudinal component describes the change in the energy of the electron and can be written as

$$mc^2 \frac{d\gamma}{dt} = e \vec{v}_T \bullet \vec{E}_l - \frac{dP_{rad}}{dt}$$

$$\frac{dP}{dt} = \frac{2}{3} \frac{e^2}{c} \gamma^6 \left[\dot{\vec{\beta}}^2 - (\vec{\beta} \times \dot{\vec{\beta}})^2 \right]$$

Energy exchange between the electron and laser beam

Electron to laser : FEL

Laser to electron: IFEL

Assume rate of change of radiative loss is zero: $\frac{dP_{rad}}{dt} = 0$

$$mc^2 \frac{d\gamma}{dt} = e\vec{v}_T \bullet \vec{E}_l$$

$$\frac{d\gamma}{dz} = \frac{1}{c} \frac{d\gamma}{dt}$$

Assume a helical wiggler and a circularly polarized laser or planar wiggler with linearly polarized laser. One can calculate the transverse velocity v_T of the electron in the magnetic field and the scalar product of v_T and E_l

Courtesy:

E. D. Courant, C. Pellegrini and W. Zakowicz, Phys Rev A 32 (1985) 2813

R. B. Palmer J. Appl. Phys. 43, (1972) 3614

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$$\frac{d\gamma}{dz} = E_0 \alpha e^{\left\{2\pi iz\left[\frac{1}{\Lambda} + \frac{1}{\lambda}\left(1 - \frac{1}{\beta \cos \alpha}\right)\right] + i\phi\right\}}$$

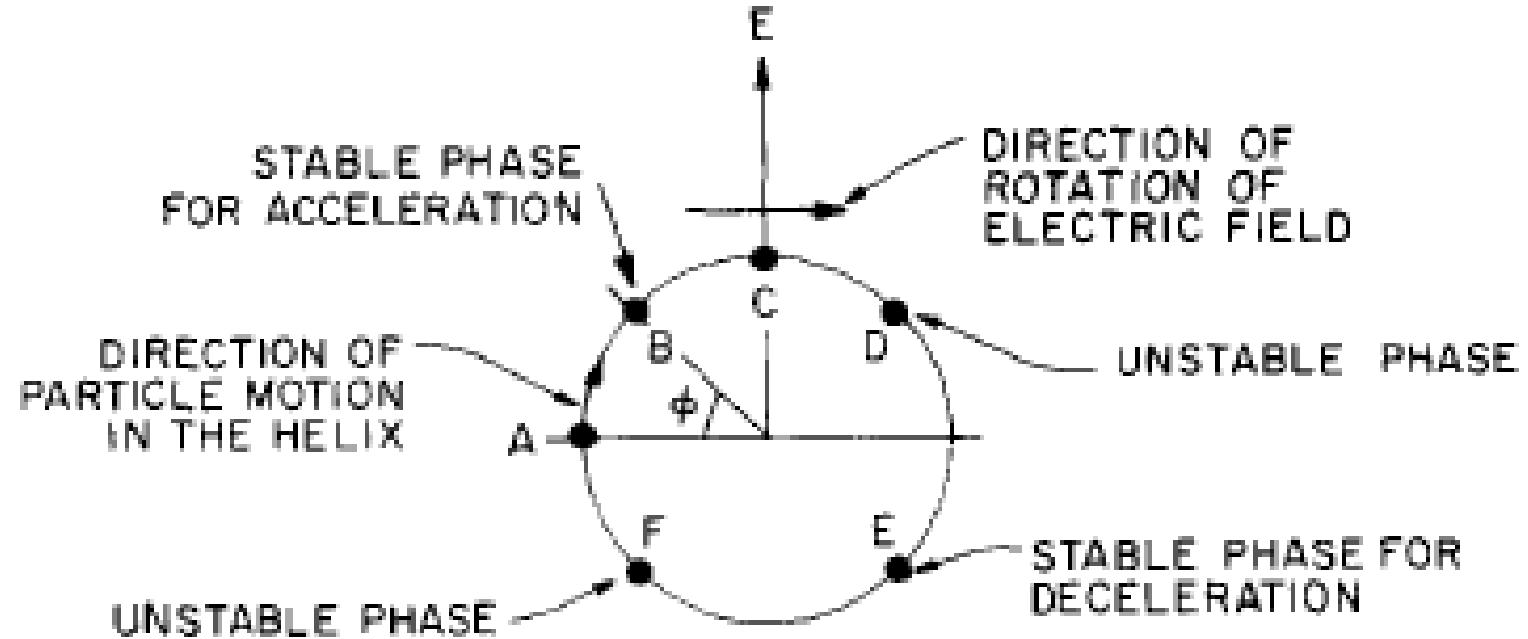
where E_0 is the amplitude of the laser field, α is the angle between the particle and the z axis, Λ is the wiggler period, λ is the wavelength of the laser and Φ is the relative phase of the particle in the helix to the electric field of the laser.

If the wiggler is designed such that

$$\frac{1}{\Lambda} = -\frac{1}{\lambda} \left(1 - \frac{1}{\beta \cos \alpha}\right)$$

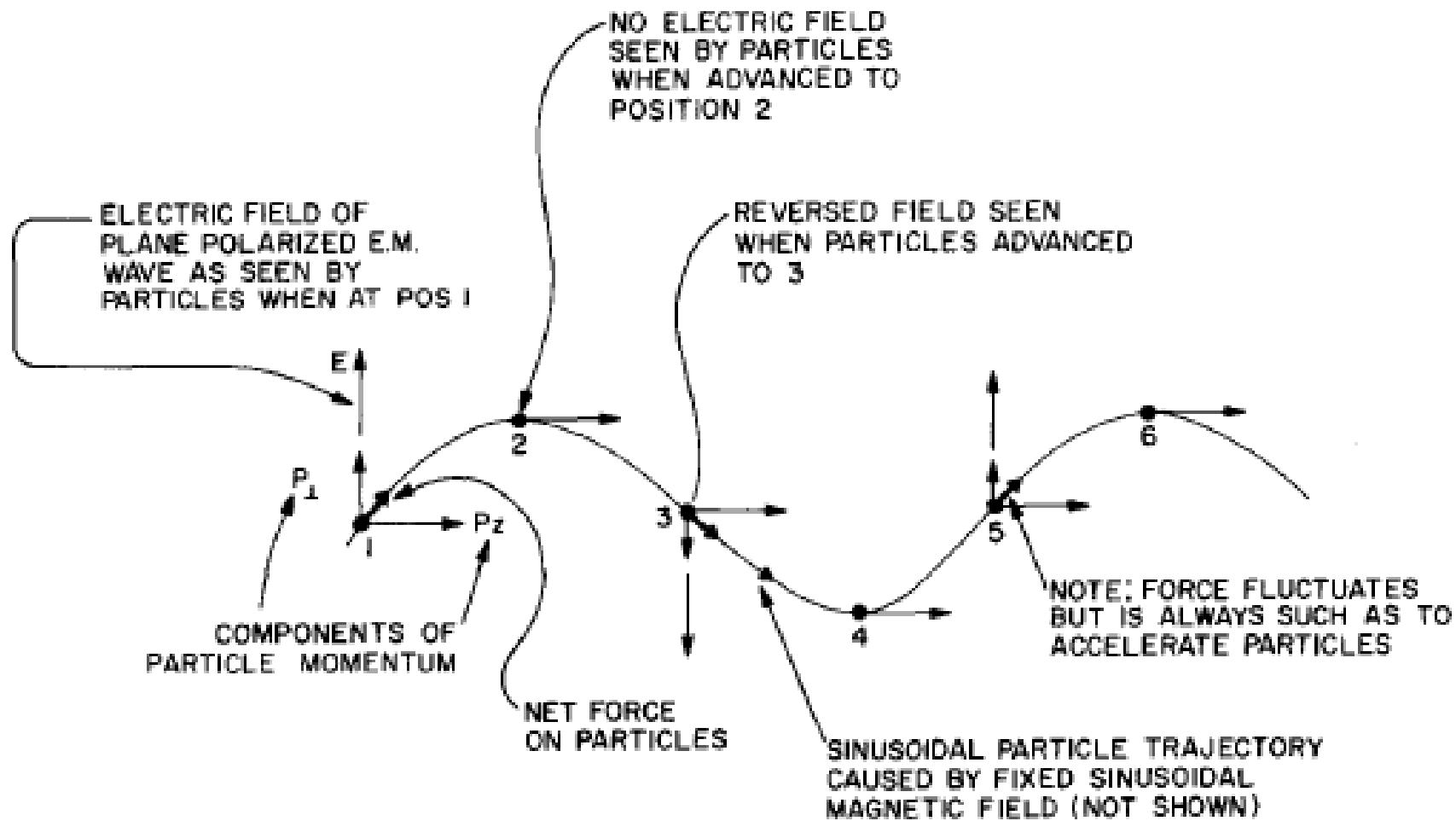
then,

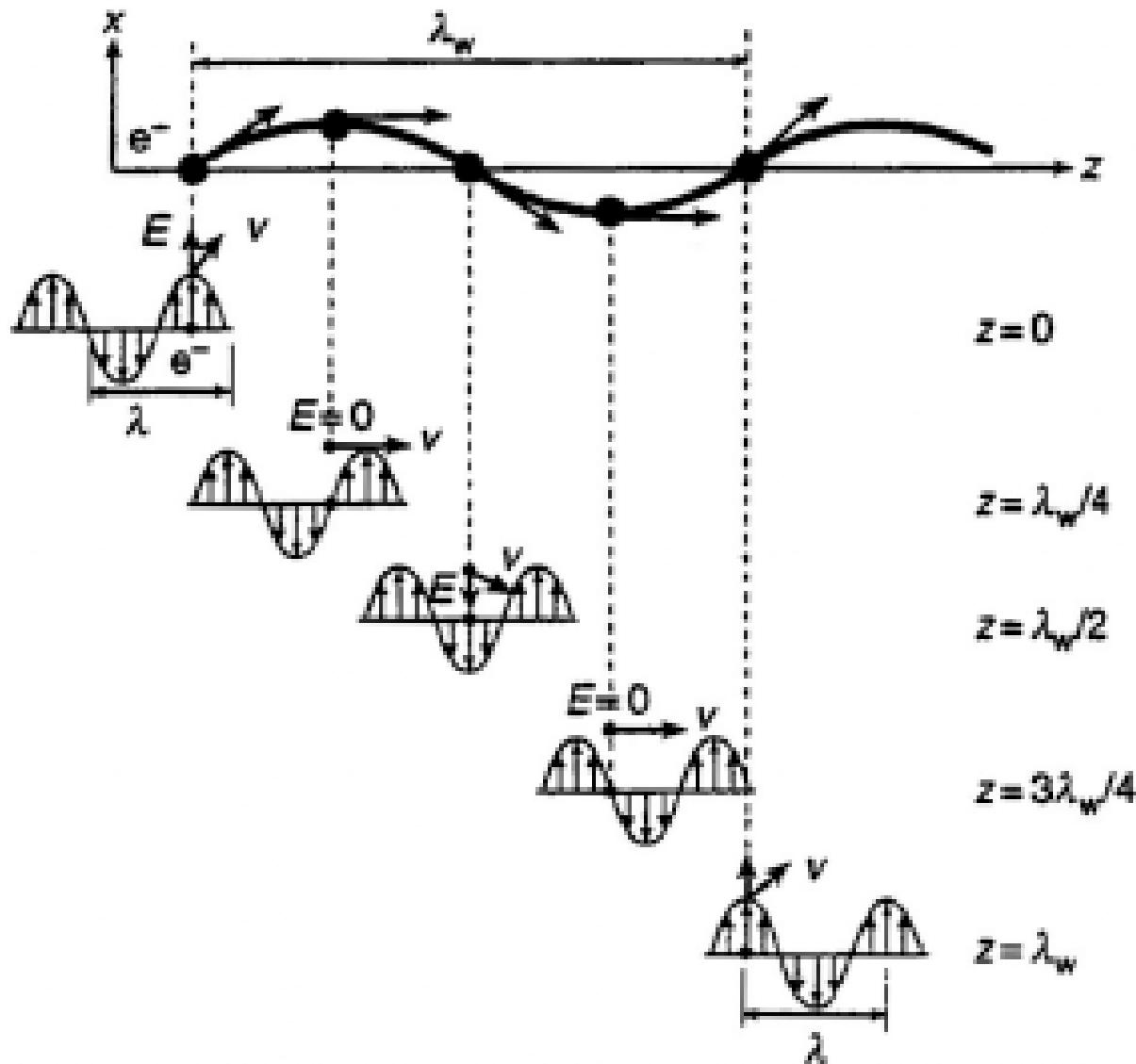
$$\frac{d\gamma}{dz} = \alpha E_0 \cos \phi$$



Particles at

- A: Max acceleration, but unstable
- B: Medium acceleration, stable-slow ones speed up, fast ones slow down-IFEL condition- bunching, coherence
- C: No acceleration
- E: Medium deceleration, FEL condition



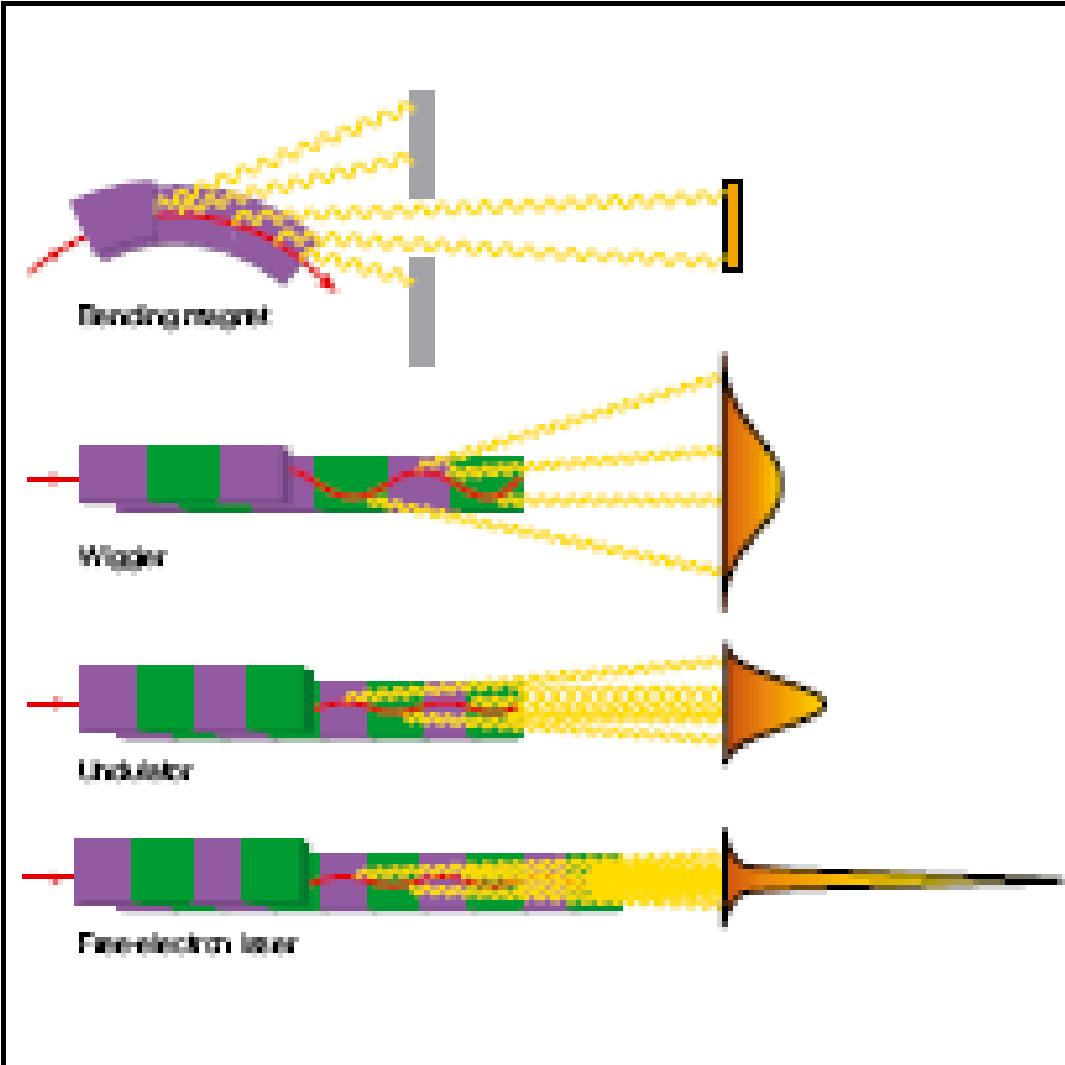


Snap shot of electron trajectory and electric field of laser for FEL
as they progress along one period of the wiggler

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Courtesy: http://reu.physics.ucla.edu/common/papers/2007/affolter_matthew.pdf

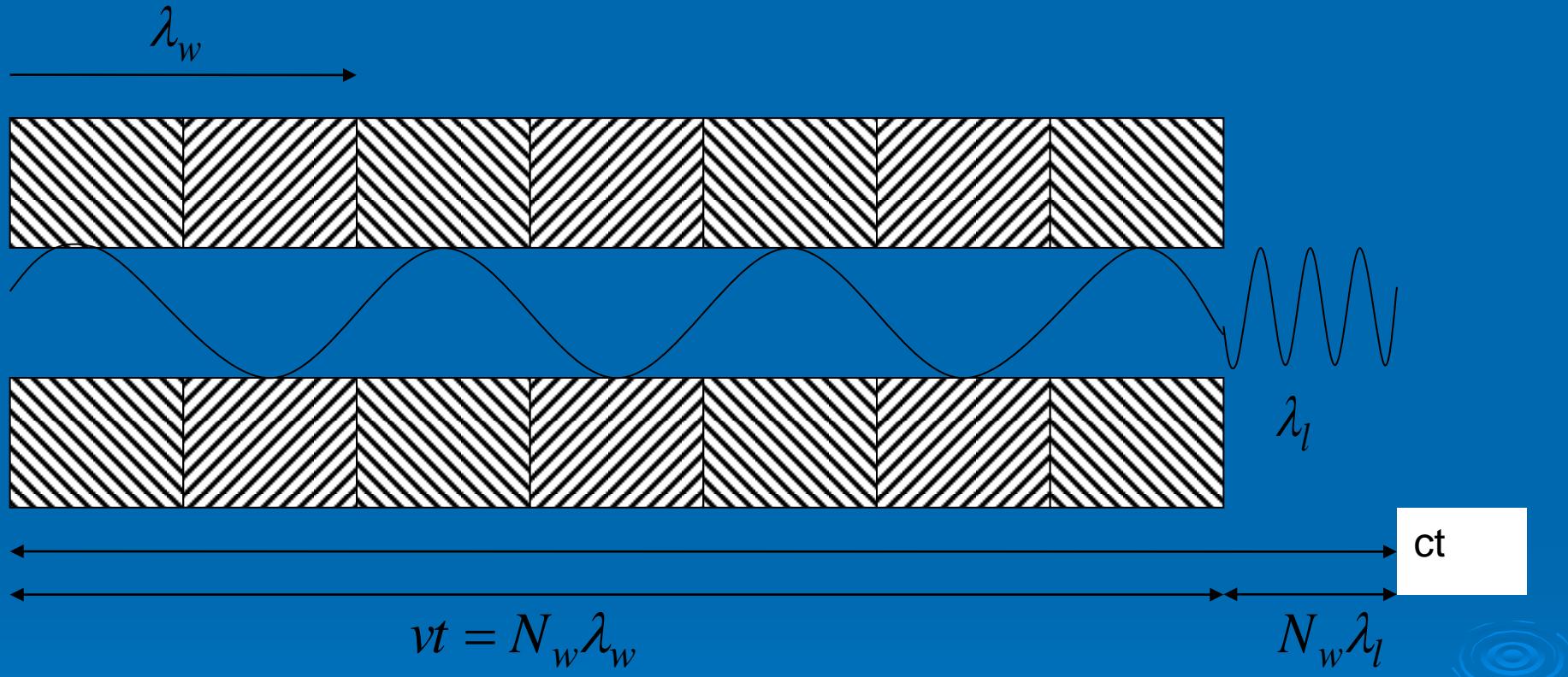


Generation of radiation in various types of

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Spontaneous radiation in a wiggler



EM field can be assumed to be negligible and the radiated intensity per unit solid angle $d\Omega$ per unit frequency $d\omega$ can be expressed as

$$\frac{d^2 I}{d\Omega d\omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{+\infty} \vec{n} \times (\vec{n} \times \vec{\beta}) e^{i\omega(t - \vec{n} \cdot \frac{\vec{r}}{c})} dt \right|^2$$

$$\lambda_l \approx \frac{\lambda_w}{2\gamma^2}$$

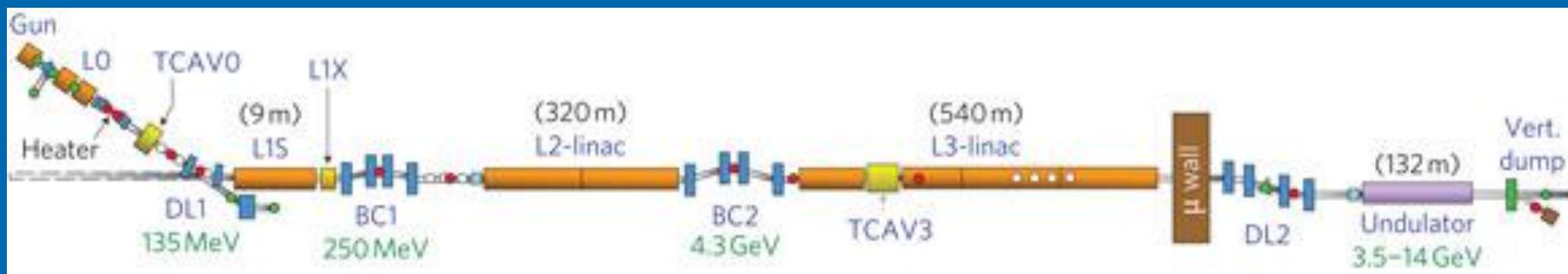
$$\lambda_l = \frac{\lambda_w}{2\gamma^2} \left(1 + \left(\frac{eB_w \lambda_w}{mc^2} \right)^2 \right)$$

Intensity Spectrum is given by

$$I(\omega) \propto \left| \int_0^{N_w \lambda_l / c} e^{(-i(\omega - \omega_l)t)} dt \right|^2$$

Line width is $\propto N_w^{-1}$

Example: LCLS X-Ray FEL



P. Emma, R. Akre et al. Nature Photonics 4, 641 - 647 (2010)

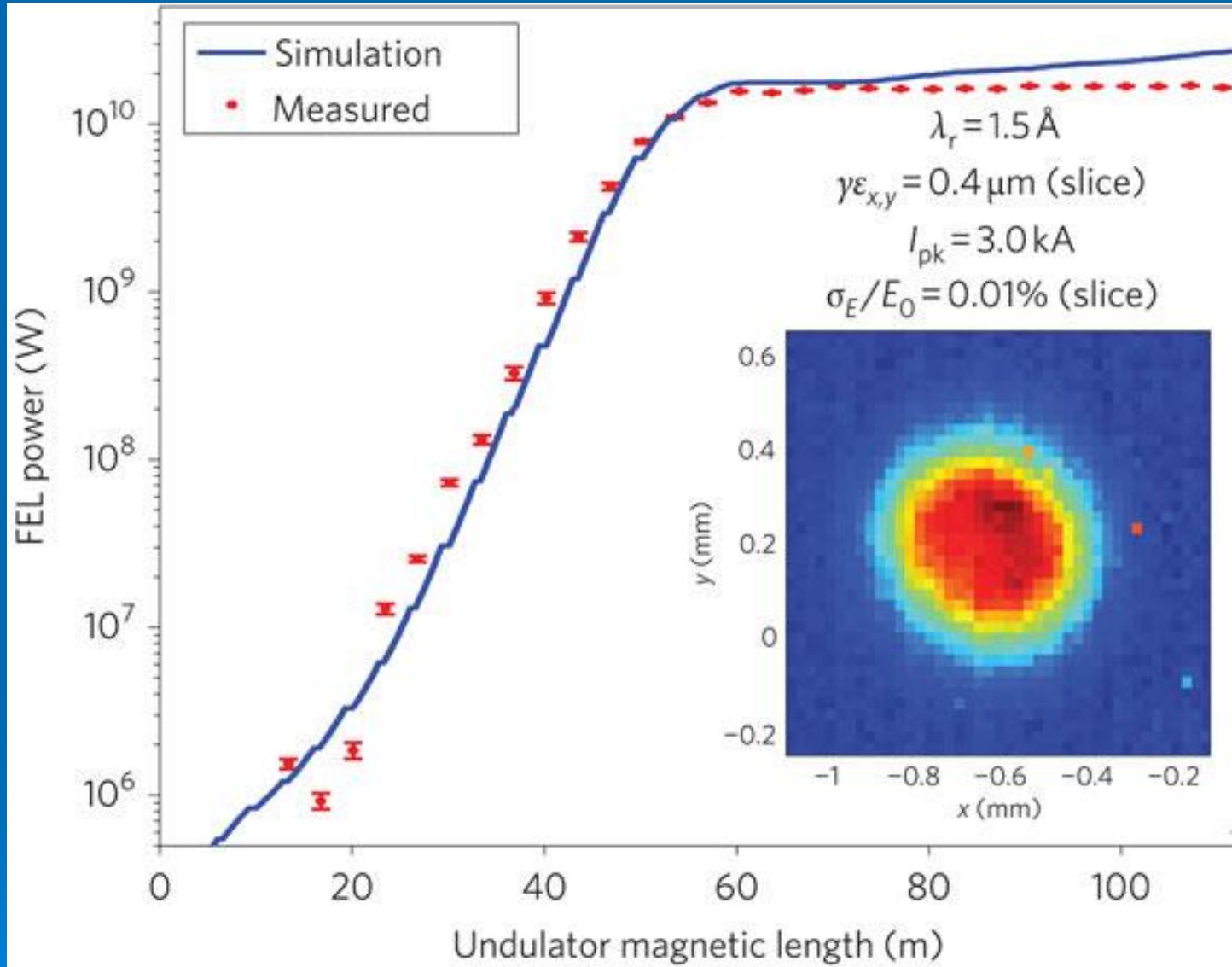
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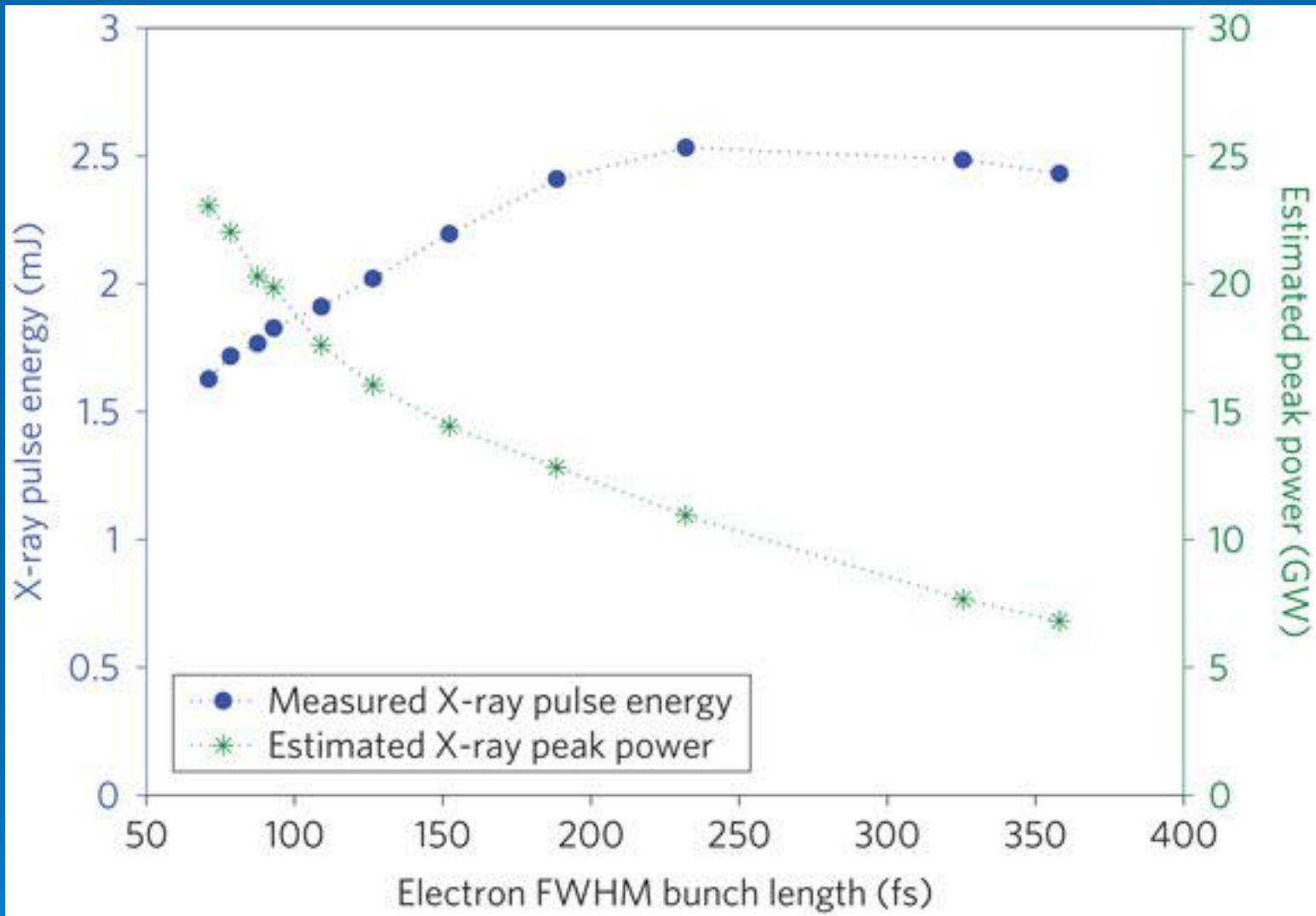
Table 1 | Design and typical measured parameters for both hard (8.3 keV) and soft (0.8–2.0 keV) X-rays. The ‘design’ and ‘hard’ values are shown only at 8.3 keV. Stability levels are measured over a few minutes.

Parameter	Design	Hard	Soft	Unit
Electrons				
Charge per bunch	1	0.25	0.25	nC
Single bunch repetition rate	120	30	30	Hz
Final linac e^2 energy	13.6	13.6	3.5–6.7	GeV
Slice [†] emittance (injected)	1.2	0.4	0.4	mm
Final projected [†] emittance	1.5	0.5–1.2	0.5–1.6	mm
Final peak current	3.4	2.5–3.5	0.5–3.5	kA
Timing stability (r.m.s.)	120	50	50	fs
Peak current stability (r.m.s.)	12	8–12	5–10	%
X-rays				
FEL gain length	4.4	3.5	'''1.5	m
Radiation wavelength	1.5	1.5	6–22	A
Photons per pulse	2.0	1.0–2.3	10–20	10^{12}
Energy in X-ray pulse	1.5	1.5–3.0	1–2.5	mJ
Peak X-ray power	10	15–40	3–35	GW
Pulse length (FWHM)	200	70–100	70–500	fs
Bandwidth (FWHM)	0.1	0.2–0.5	0.2–1.0	%
Peak brightness (estimated)	8	20	0.3	$10^{32} *$
Wavelength stability (r.m.s.)	0.2	0.1	0.2	%
Power stability (r.m.s.)	20	5–12	3–10	%

*Brightness is photons per phase space volume, or photons $s^{-21} \text{ mm}^{22} \text{ mrad}^{22}$ per 0.1% spectral bandwidth.

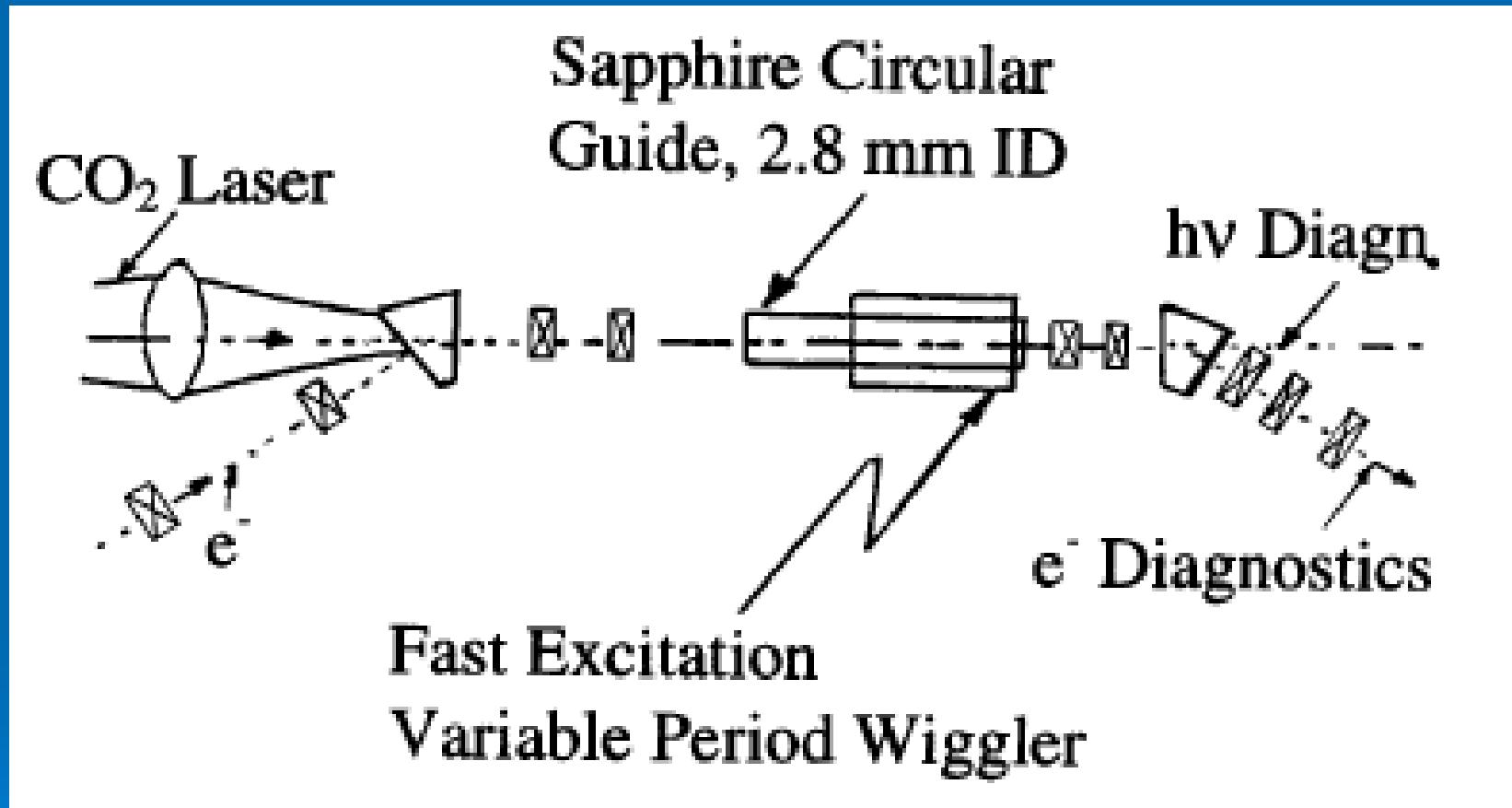
[†]‘Slice’ refers to femtosecond-scale time slices and ‘projected’ to the full time-projected (that is, integrated) emittance of the bunch.





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Example of IFEL



Courtesy: A. van Steenbergen et al. Phys Rev. Lett. 77 (1996), 2690

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TABLE I. Design parameters of the IFEL accelerator.

Electron beam	
Injection energy (MeV)	40.0
Exit energy (MeV)	42.3
Mean accelerator field (MV/m)	4.9
Current, nominal (mA)	5
N (bunch)	10^9
I_{\max} (A)	30
$\Delta E/E(1\sigma)$	$\pm 3 \times 10^{-3}$
rms emittance (m rad)	7×10^{-8}
Beam radius (mm)	0.3
Wiggler	
L_w (m)	0.47
Section length (m)	0.6
Period λ_w (cm)	2.89–3.14
Gap (mm)	4
B_w^{\max} (T)	1.0–1.024
Beam oscillation $a_{1/2}$ (mm)	0.16–0.19
Laser beam	
Power W_l (GW)	1
Wavelength λ (μm)	10.6
Maximum field E_0 (MV/m) ^a	0.78×10^3
Guide loss α (m^{-1})	0.05
Field attenuation (dB/section)	0.26
τ , FWHM (ps)	200–300
Normal field A (m^{-1})	1.53×10^3
Beam waist $r_0(L_w/2)$ (mm)	1.0

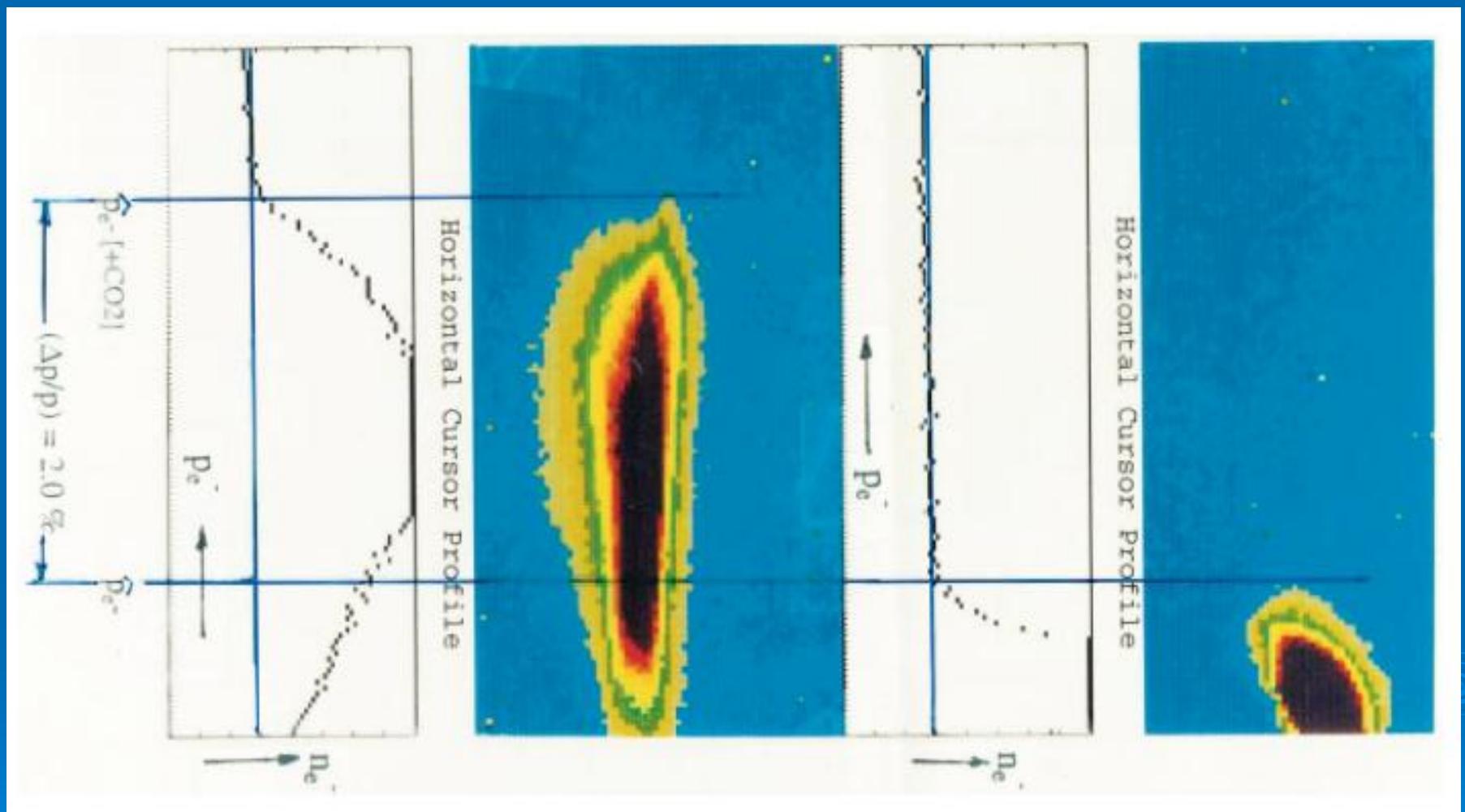
^a $E_0 = (\pi W_l Z_0)^{1/2}/R_0$, $Z_0 = 377 \Omega$, and R_0 [Feng Rao et al., *SPAS 2013*, Durham]

CO₂ laser

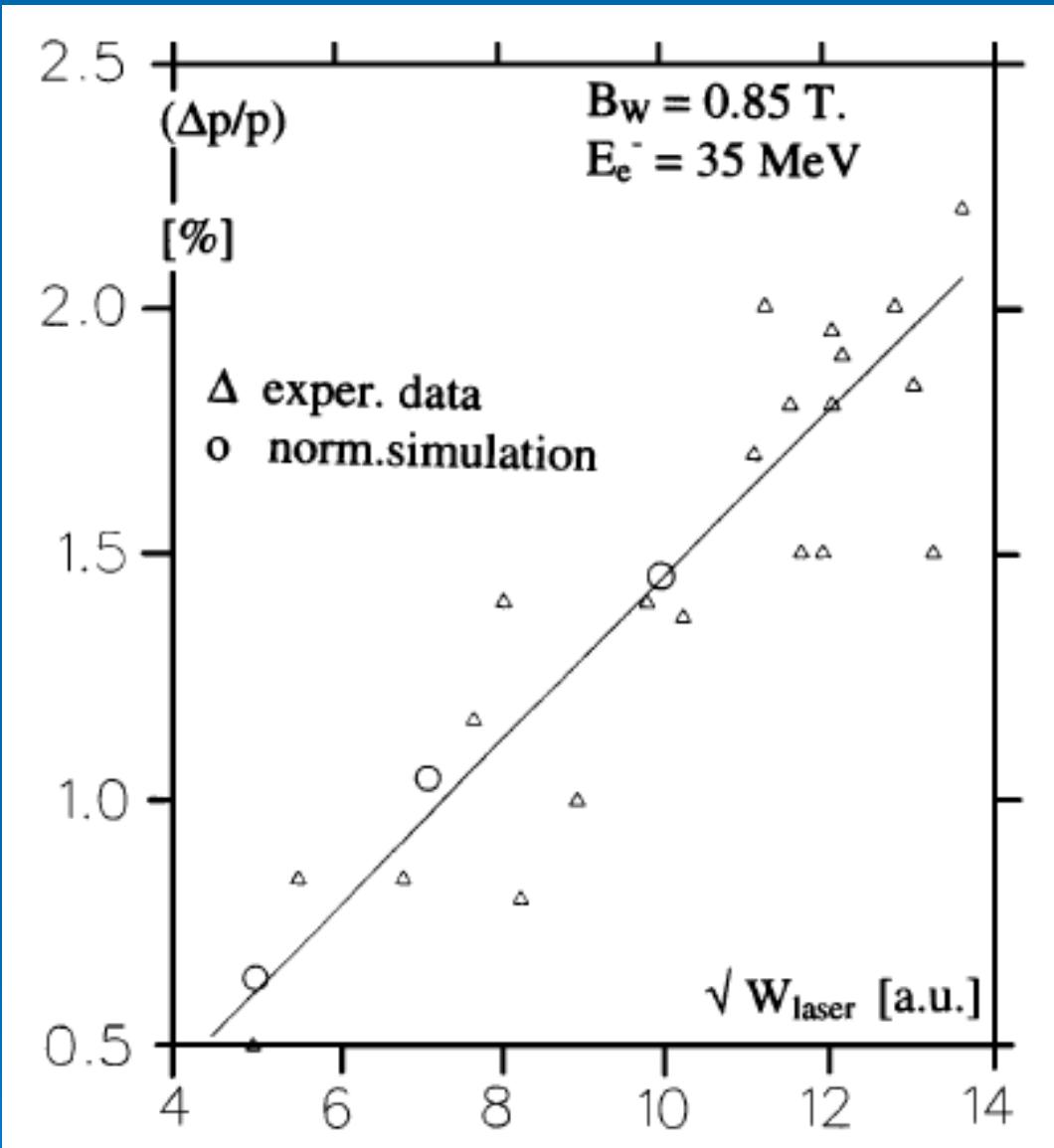
EH₁₁ mode stabilized
with in Rayleigh length
from entrance

2.8 mm ID, 0.6 m long
sapphire circular
waveguide

Evidence of Acceleration

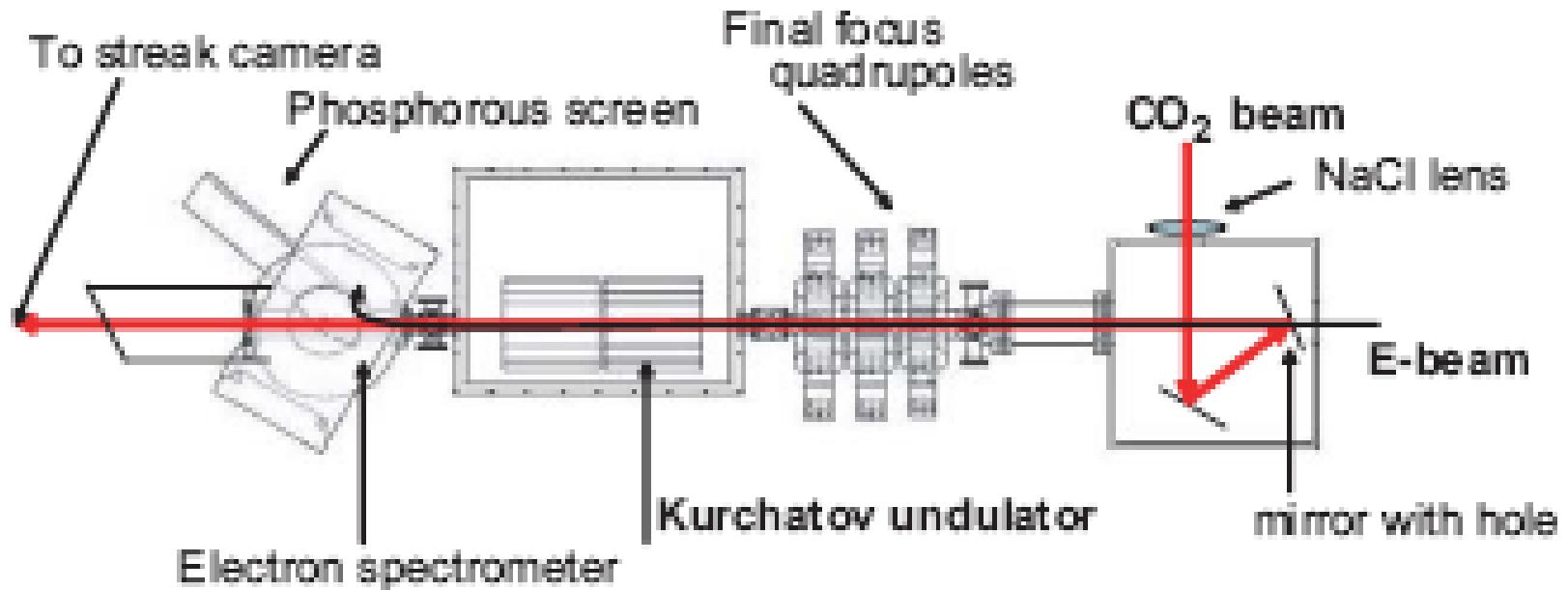


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Acceleration Linear in electric field of laser

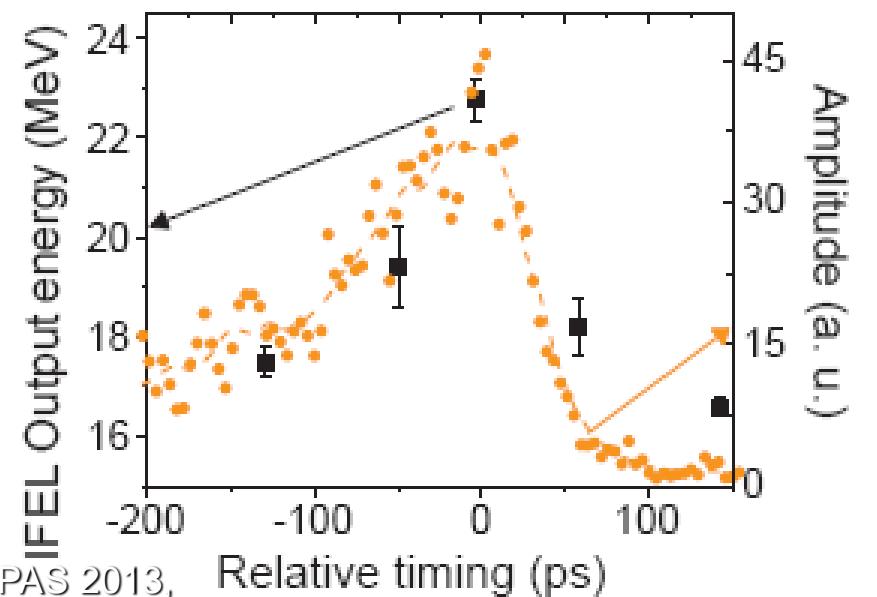
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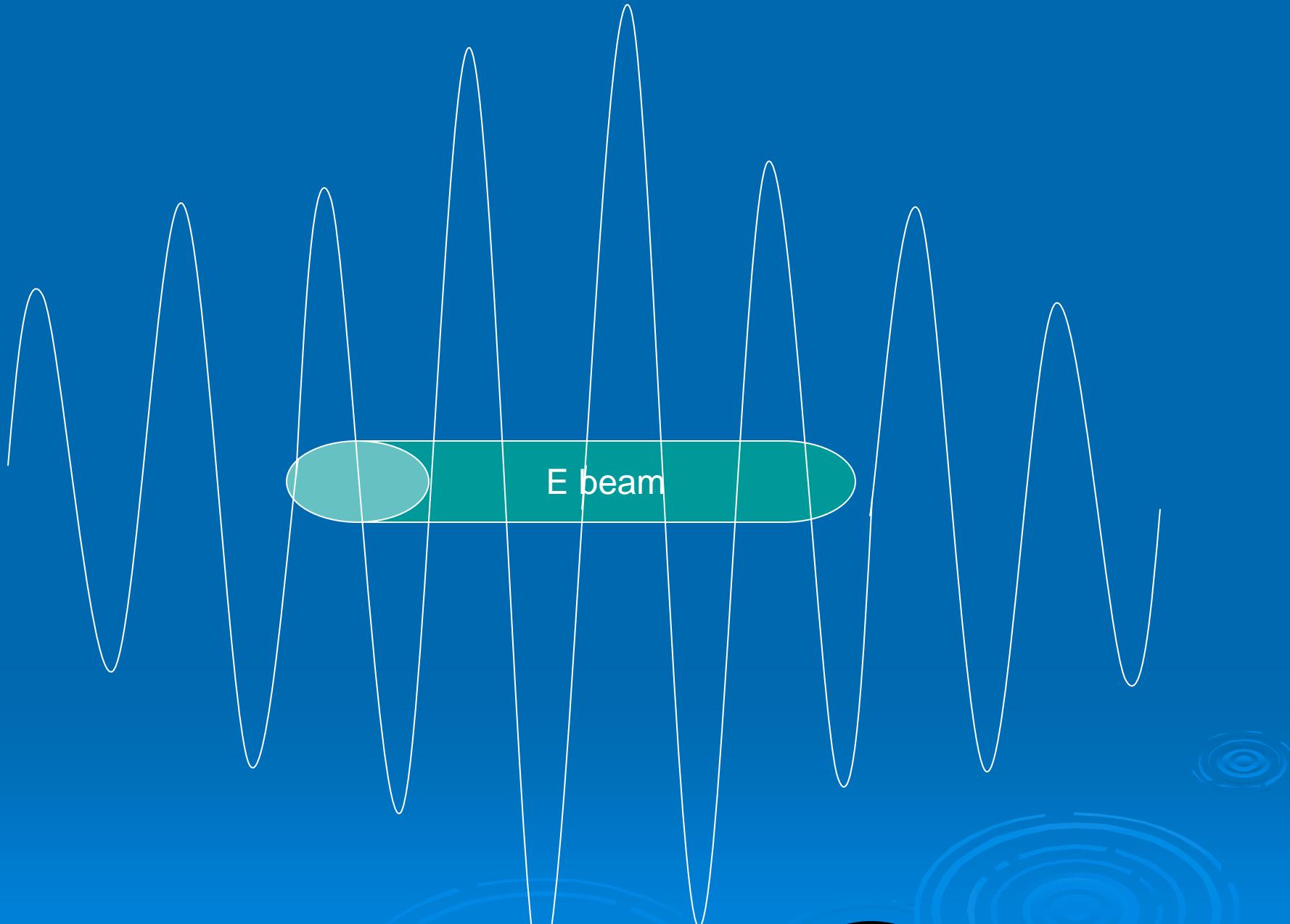


Courtesy: P. Musumeci et al., Proc. Of 2005 PAC P. 500
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Electron beam	
energy	14.5 MeV
charge	0.3 nC
emittance	5 mm-mrad
pulse length (rms)	6 ps
σ_{rms} at focus	120 μ m
CO ₂ laser	
power	400 GW
wavelength	10.6 μ m
pulse length (FWHM)	240 ps
spot size ($1/e^2$)	240 μ m

- >20 MeV (150%) energy gain
- >70Mv/m gradient
- >5% of particles trapped for acceleration



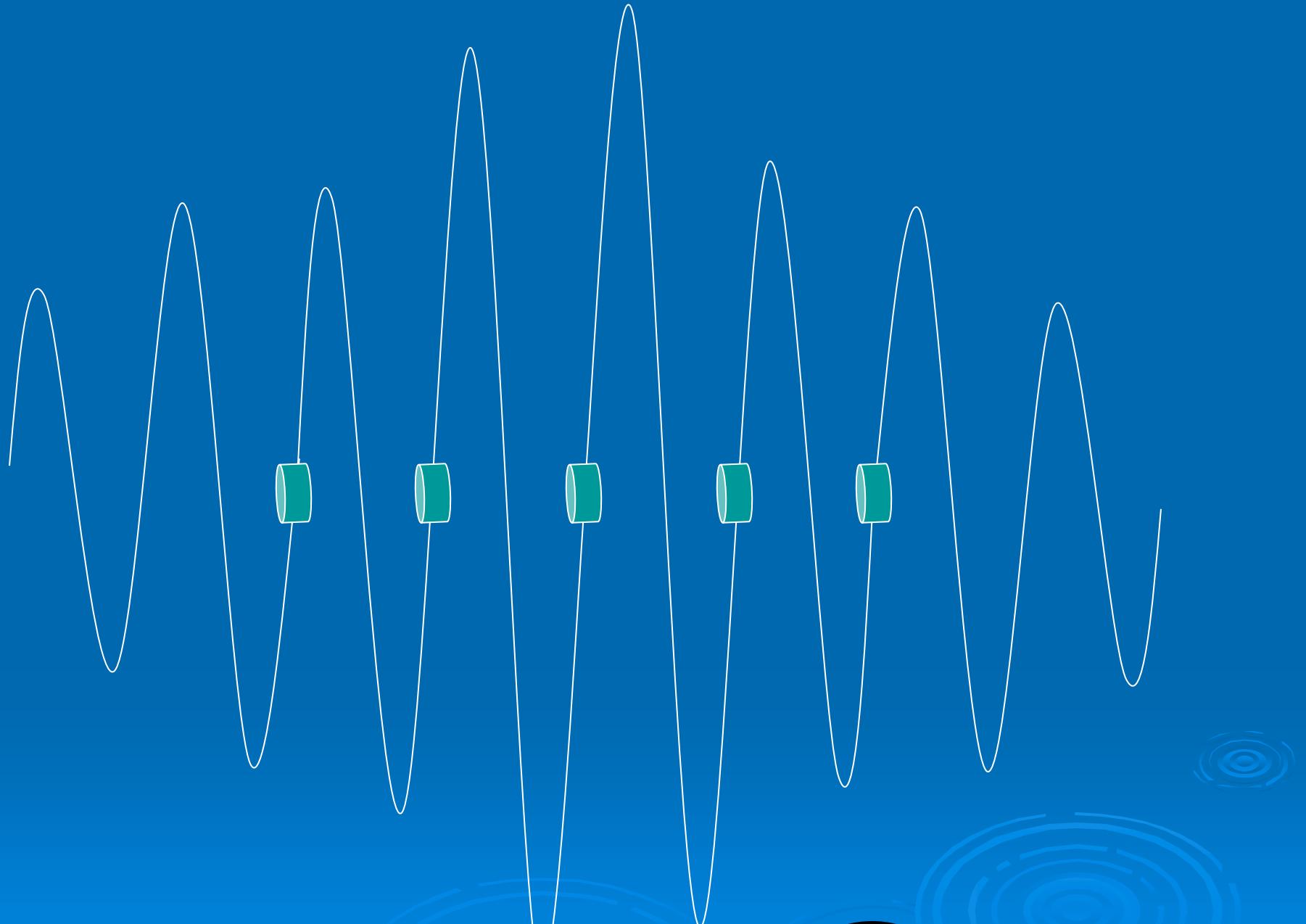


Acceleration and Deceleration → Electron bunching needed

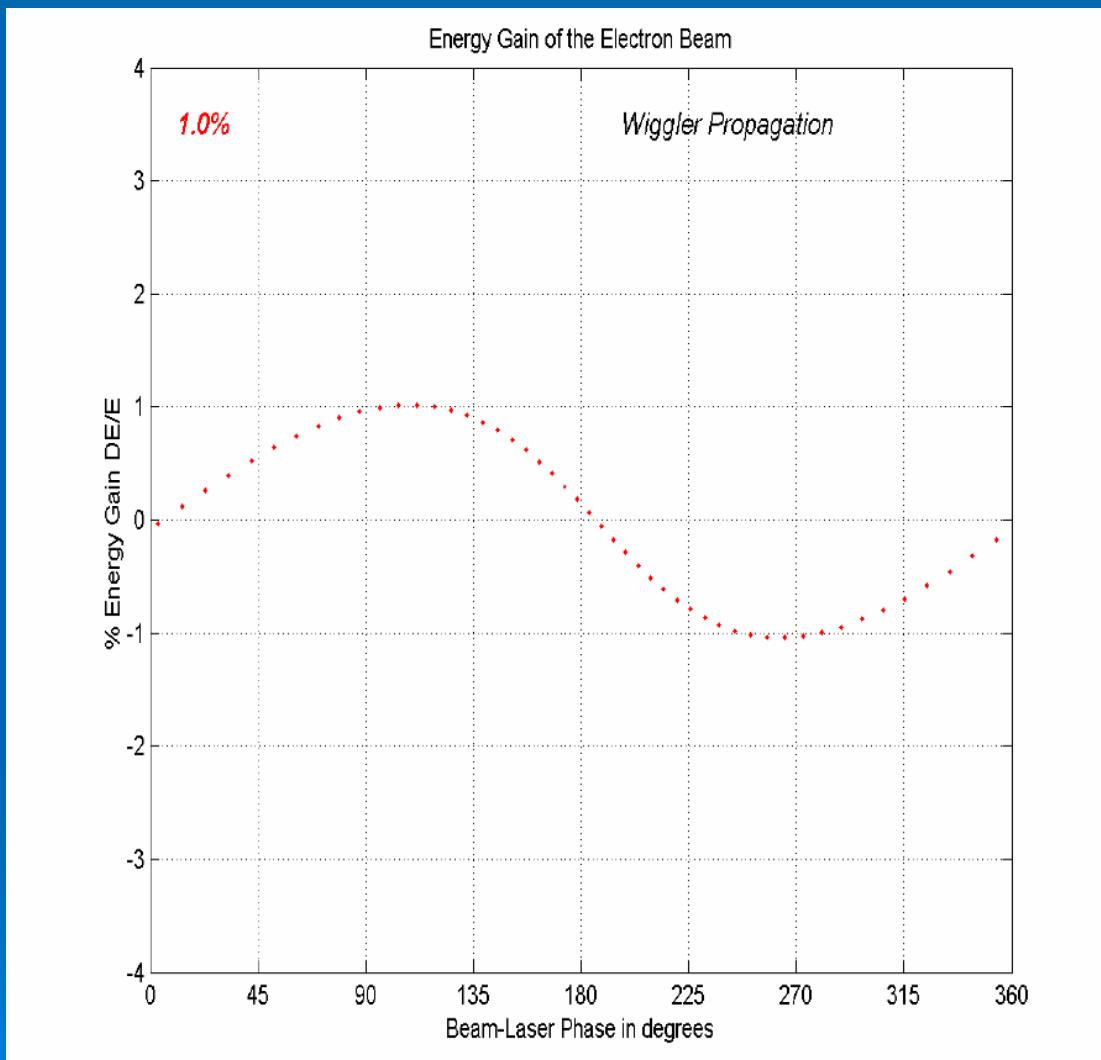
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Microbunching

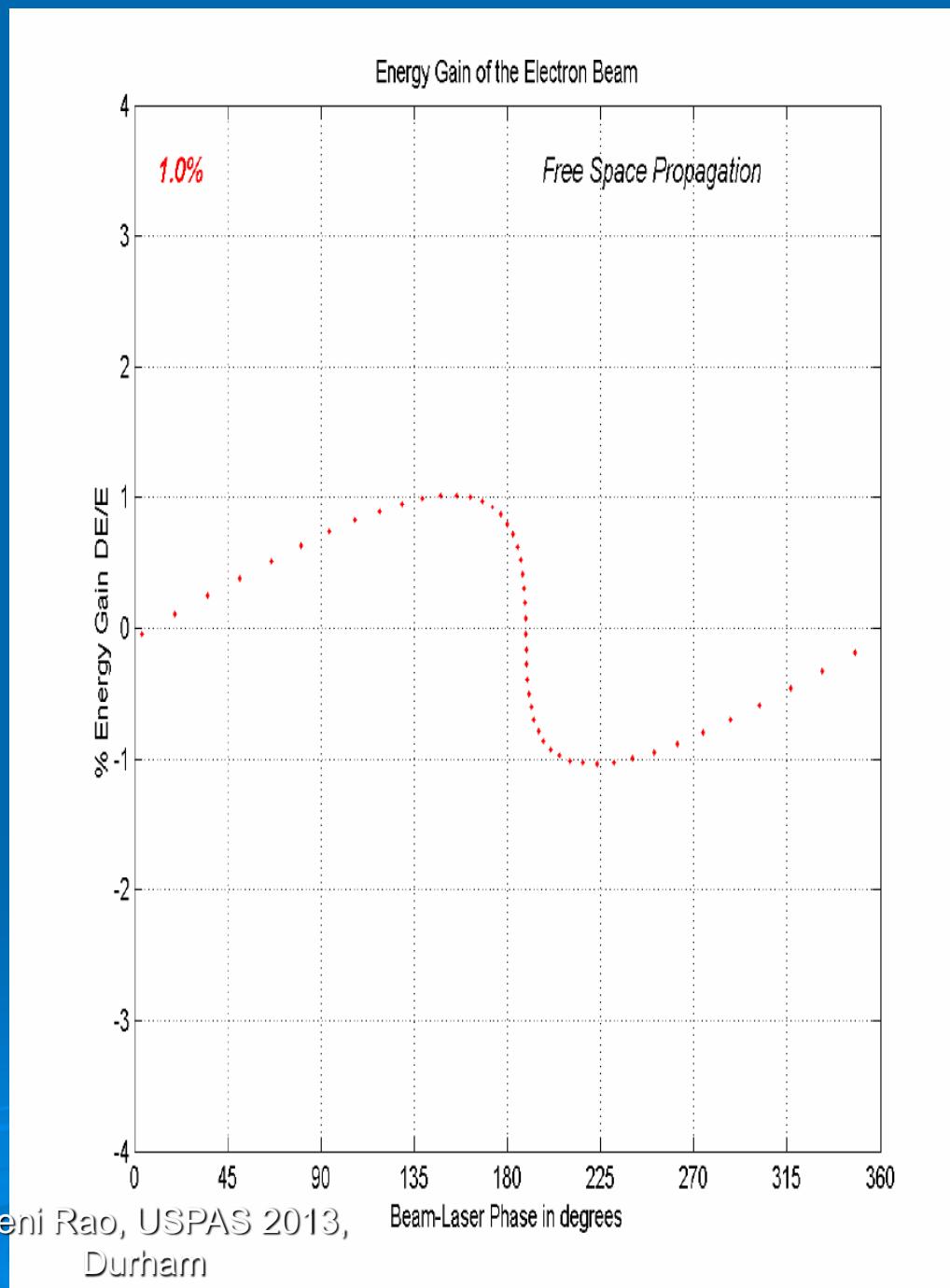
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Energy modulation in wiggler

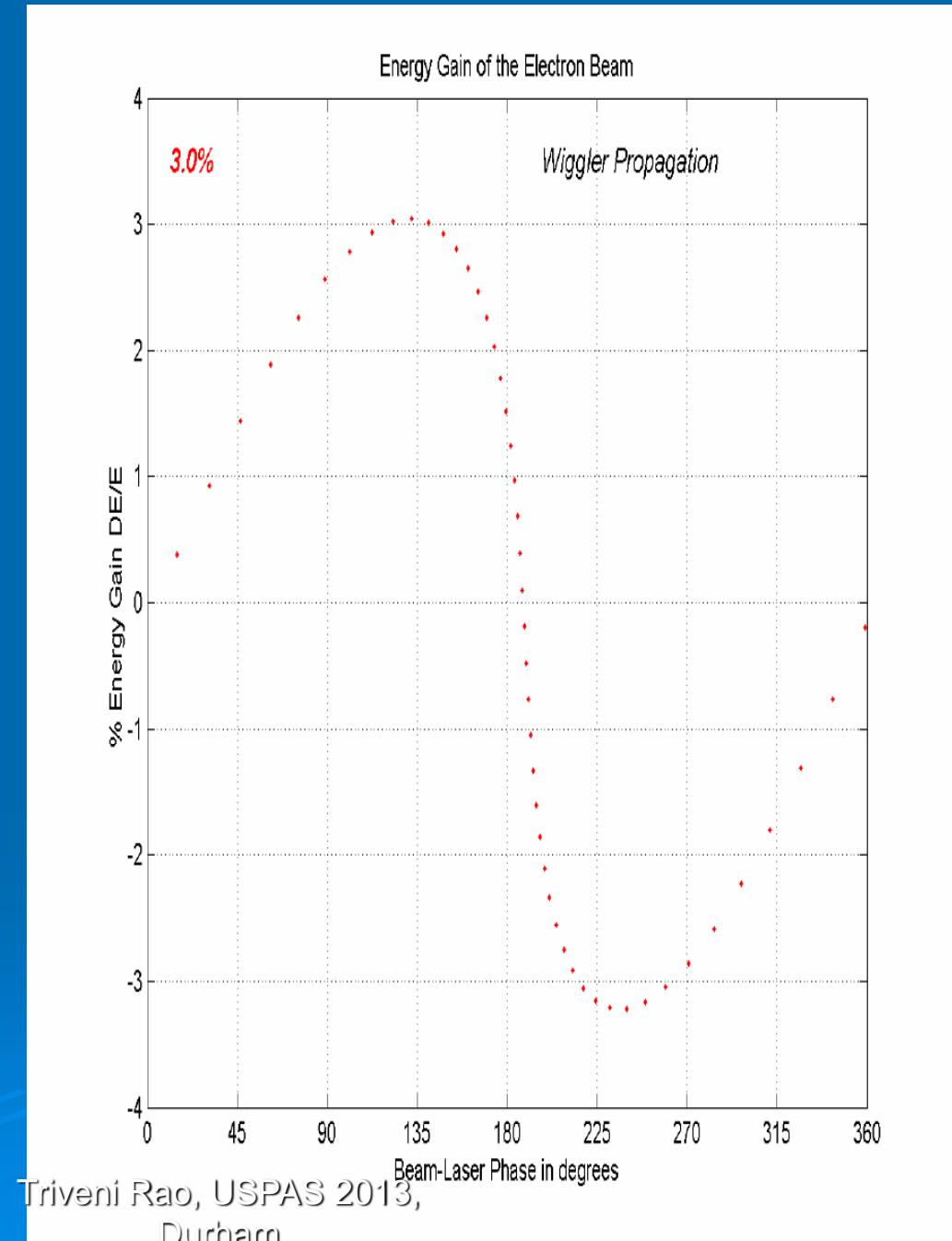


Charge distribution after transit- left hand side faster e catch up with right hand side slower e

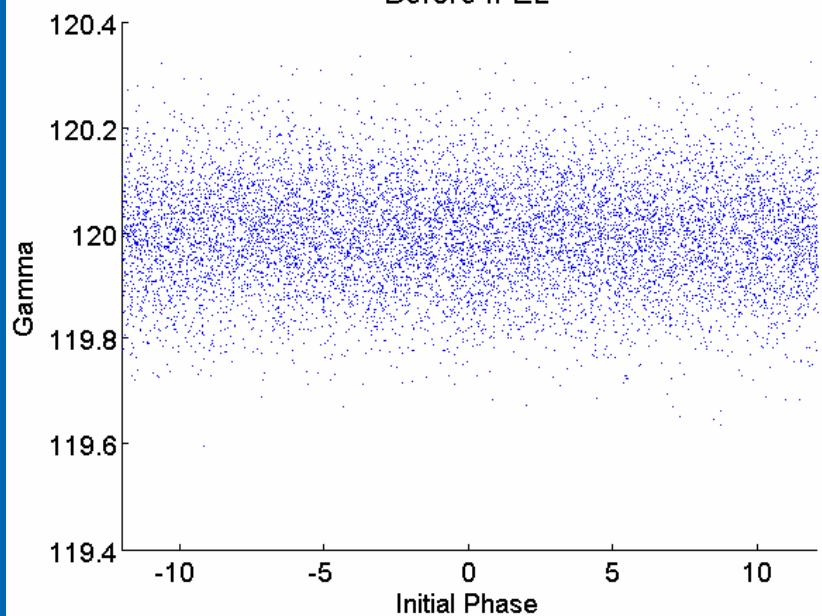


Further transport

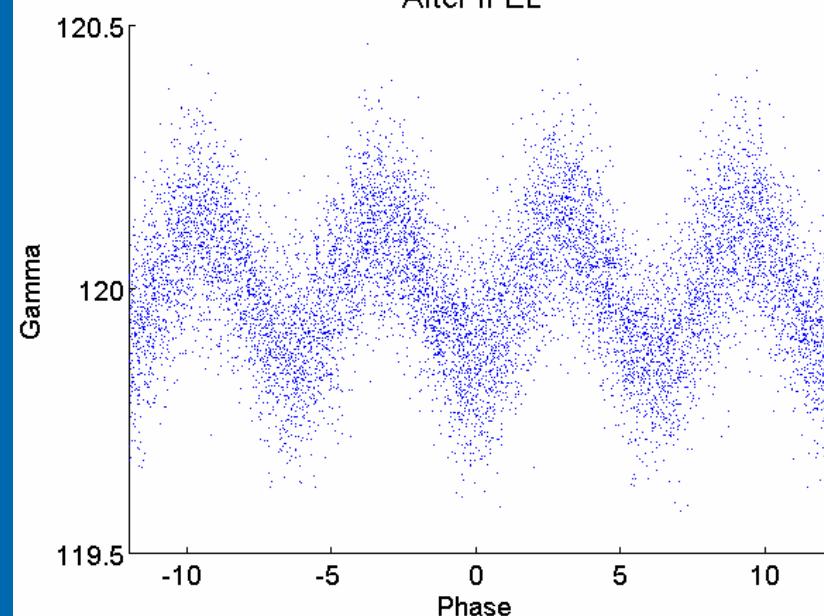
Optimal
positioning of
next stage



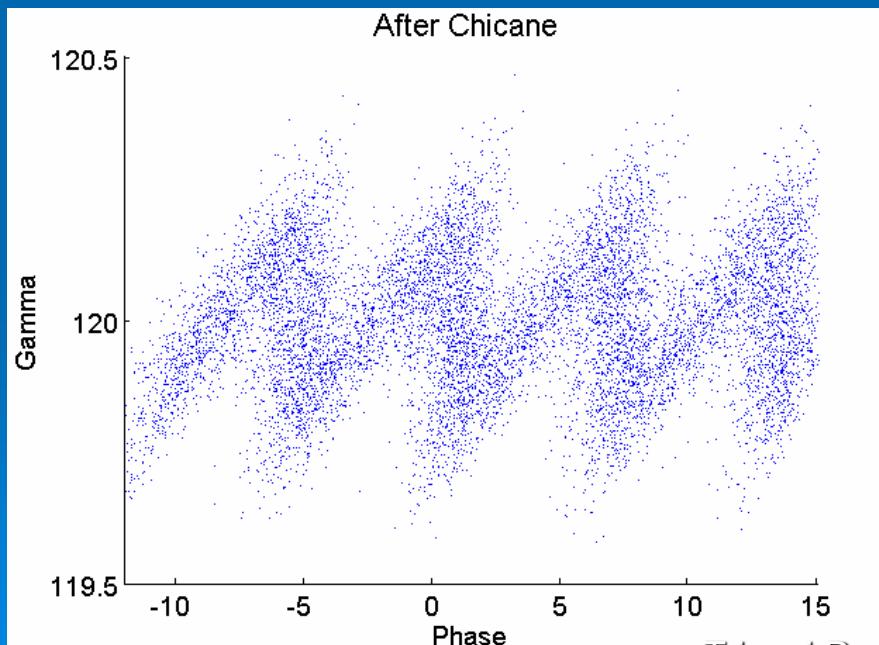
Before IFEL



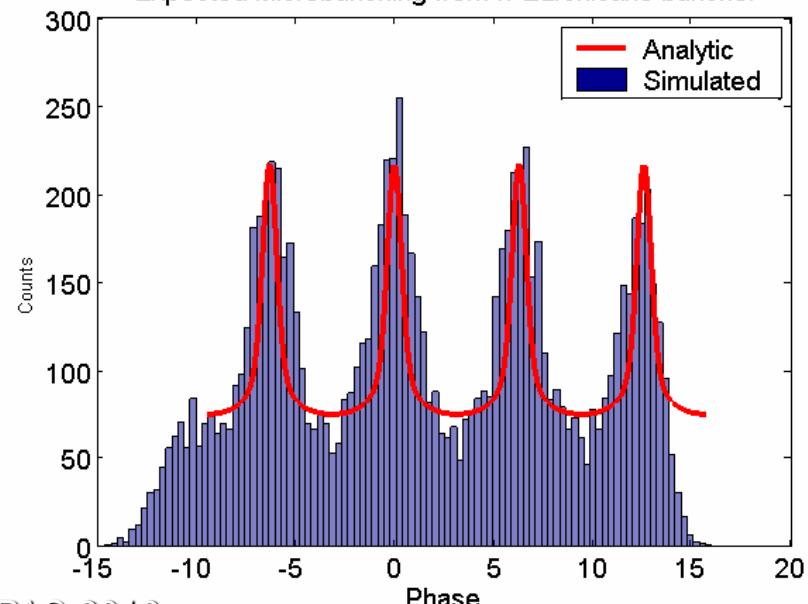
After IFEL



After Chicane

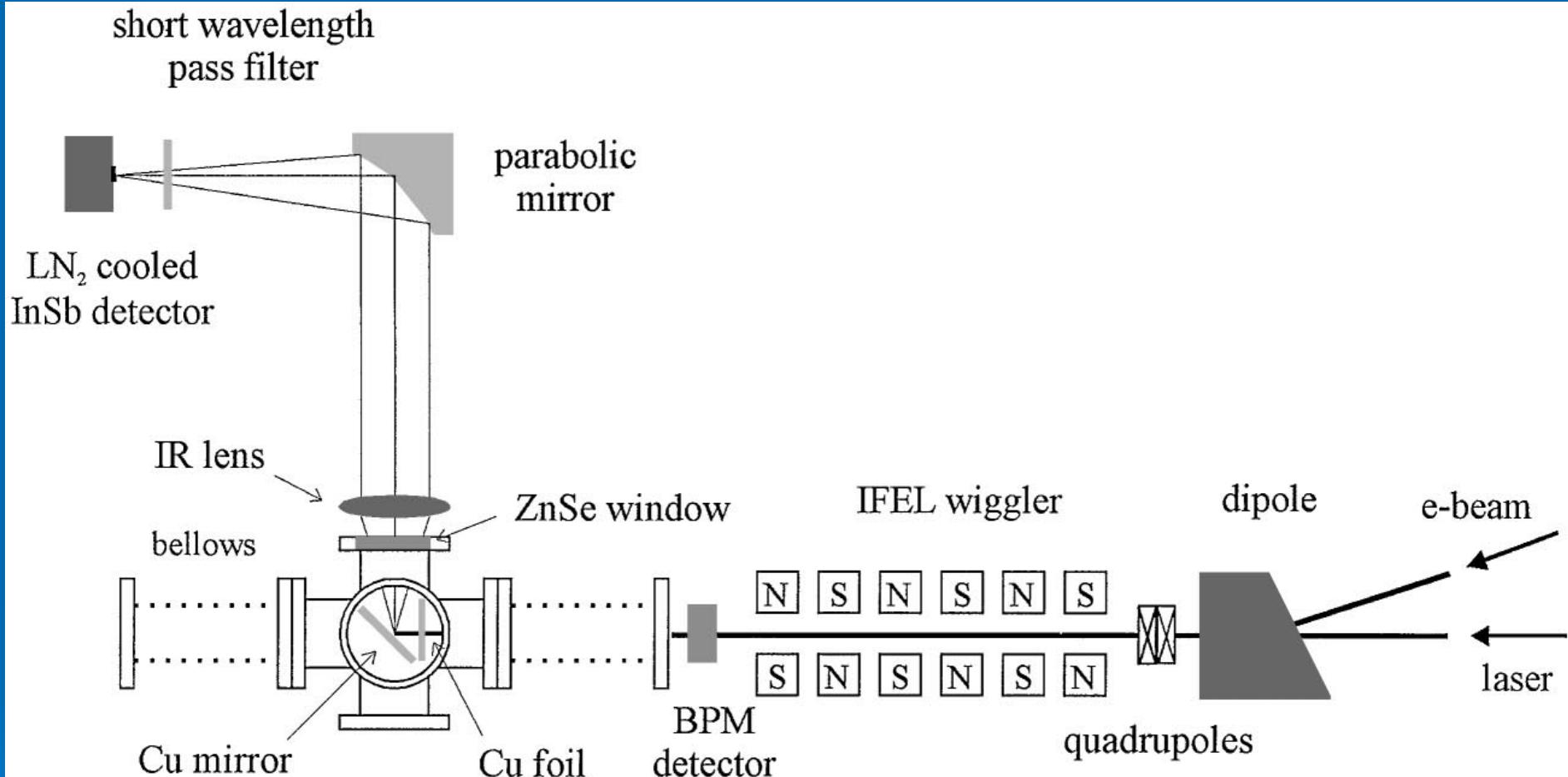


Expected Microbunching from IFEL/chicane buncher

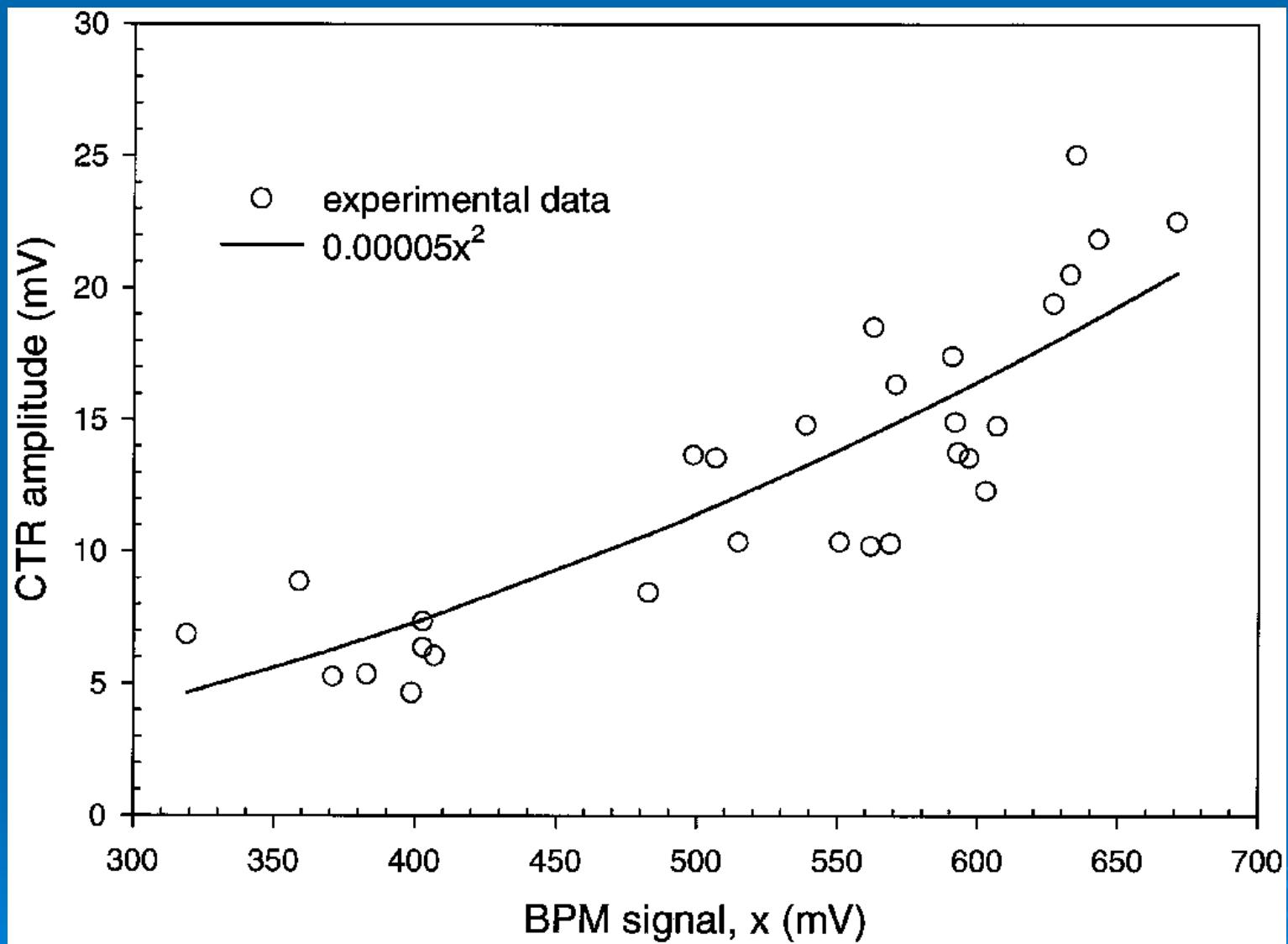


Applications of Micro-bunching

- Seeder for staged acceleration
 - Staged IFEL, ICA, LW
- Seeder for FEL
 - Normal FEL at fundamental wavelength
 - High harmonic generation
- Source of short bunch length electrons
 - Attosecond e beam generation
- Source of coherent radiation
 - CTR, THZ (300 μ m)

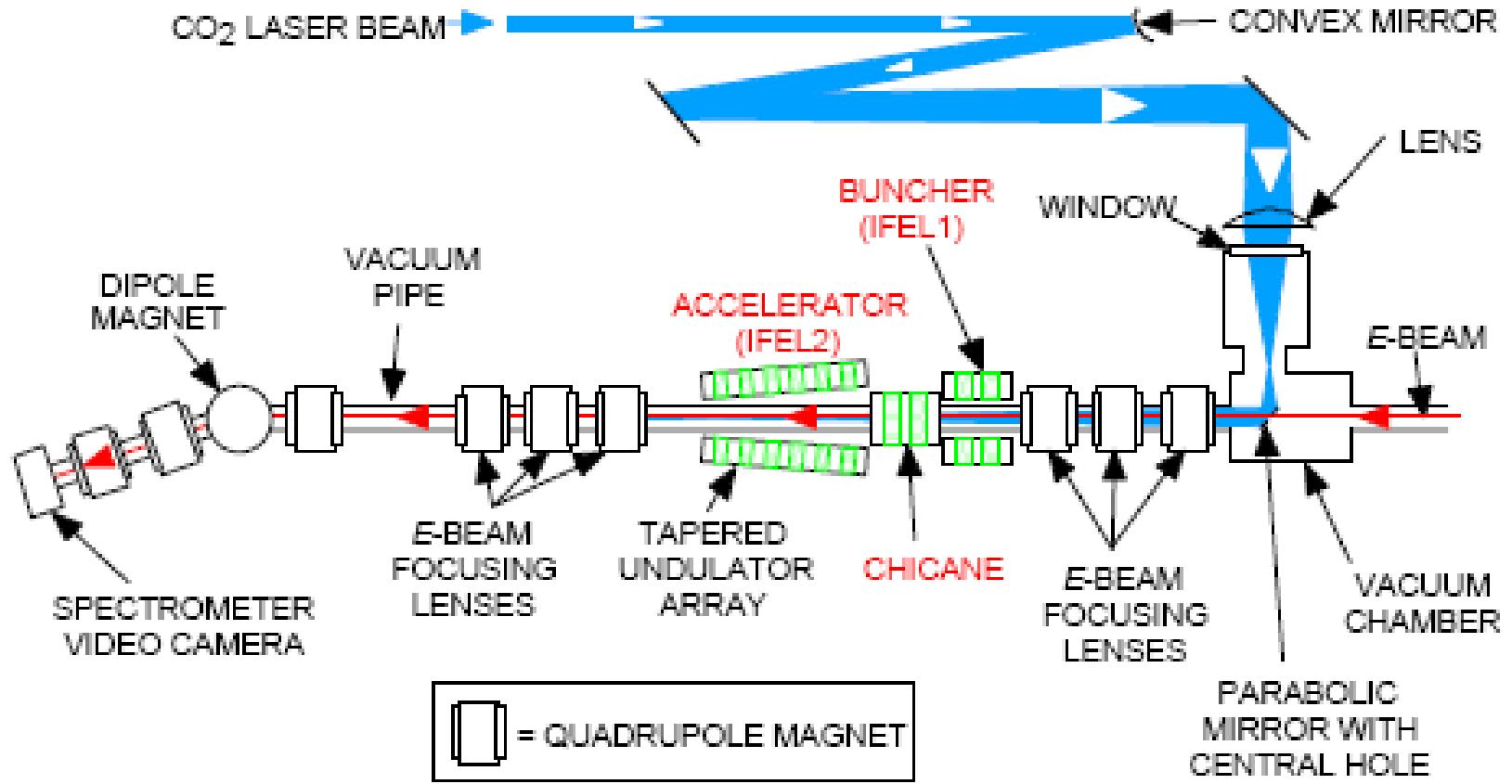


Courtesy: Y. Liu et al Phys Rev. Lett. 80, (1998) 4418
Triveni Rao, USPAS 2013,
Durham



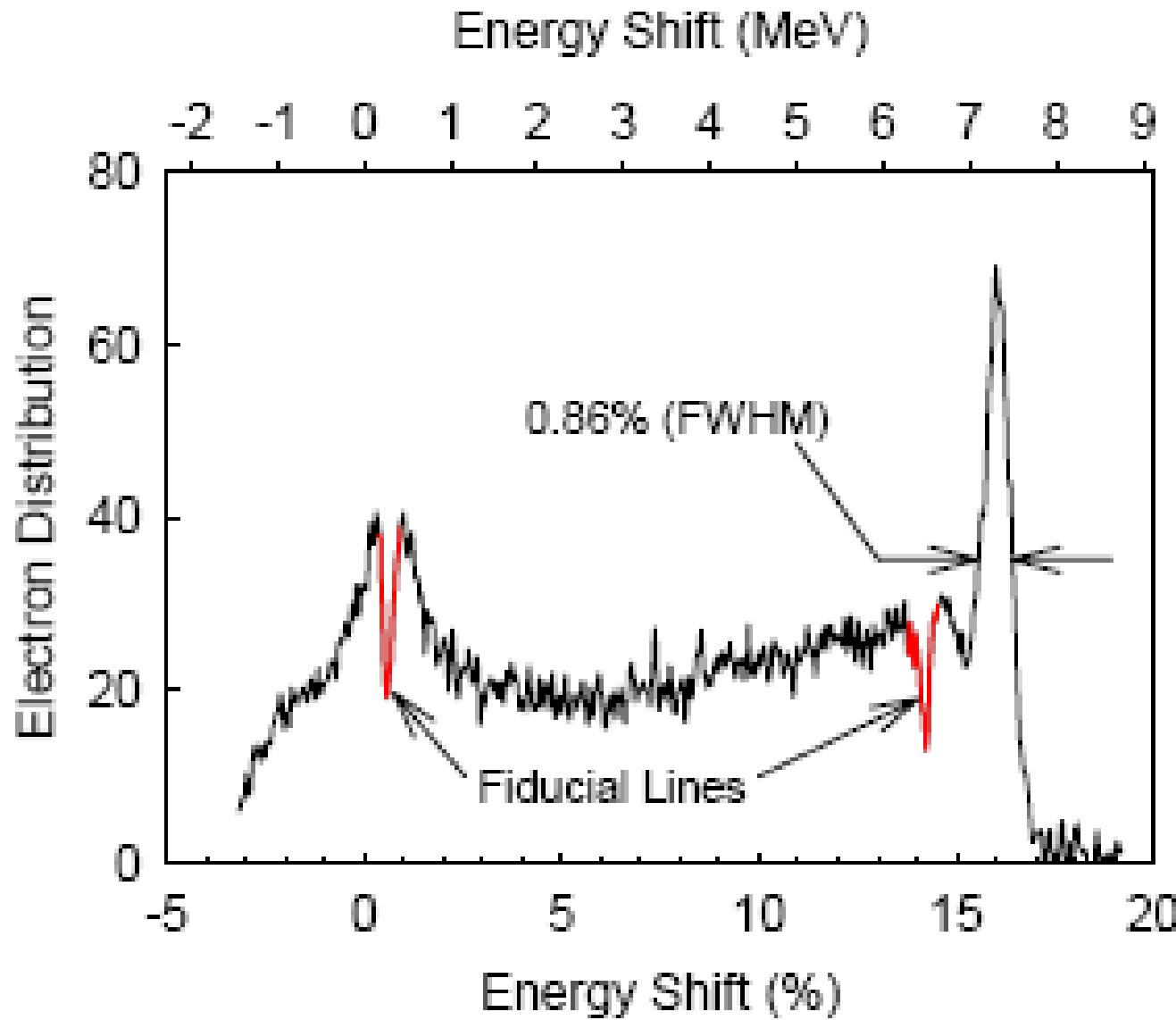
Coherent transition radiation signal
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Seeder for staged acceleration

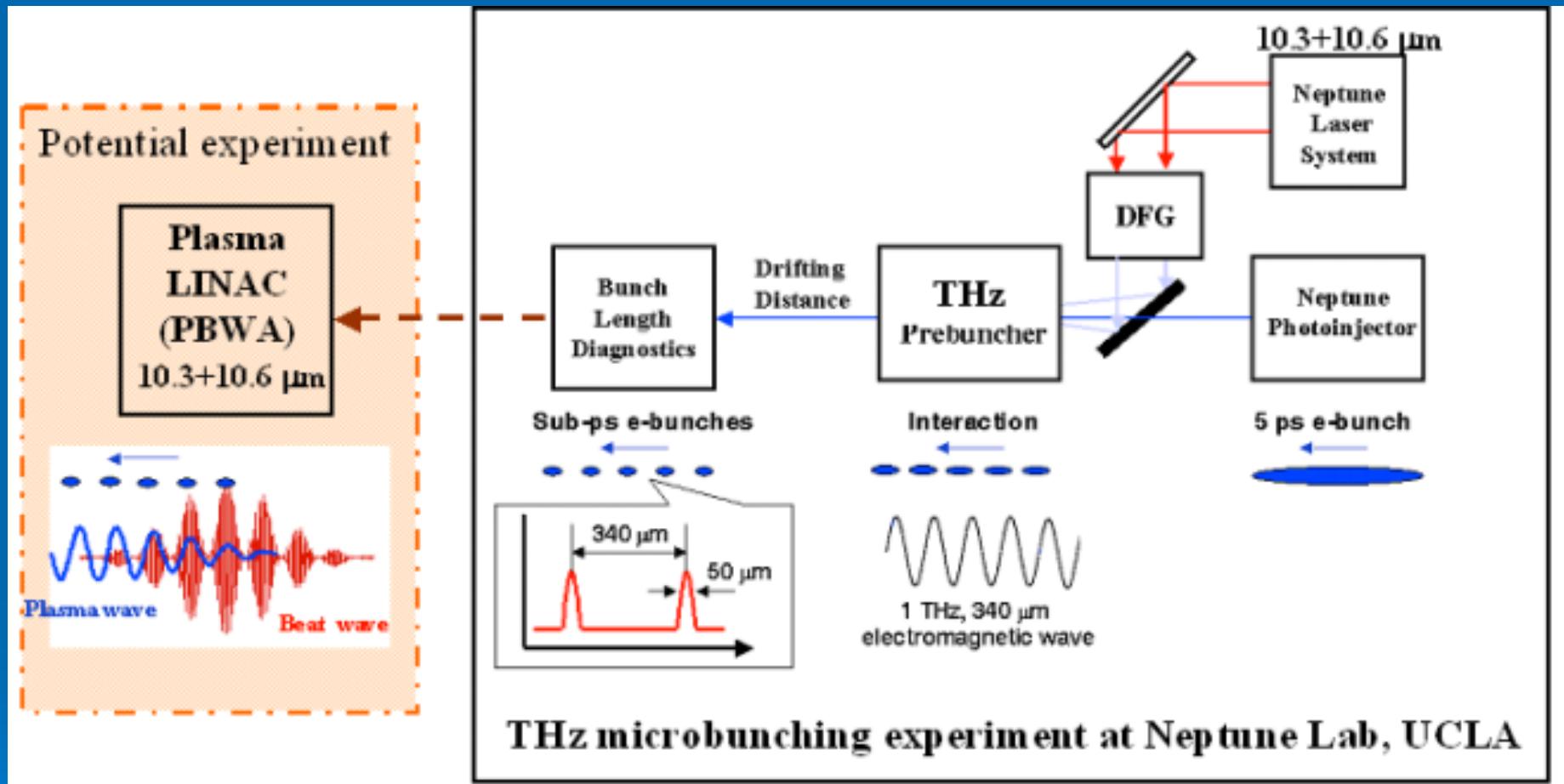


Courtesy: W. Kimura et al. Proc. Of 2003 PAC, 1909
Triveni Rao, USPAS 2013,
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Parameter	Value
E-beam energy	45.6 MeV
E-beam intrinsic energy spread	~0.04%
E-beam normalized emittance	1.5 mm-mrad
E-beam charge	~0.1 nC
E-beam pulse length	~3 ps
Laser wavelength	10.6 μm
Laser pulse length	~180 ps
Laser pulse energy	>5 J



FEL SEEDER

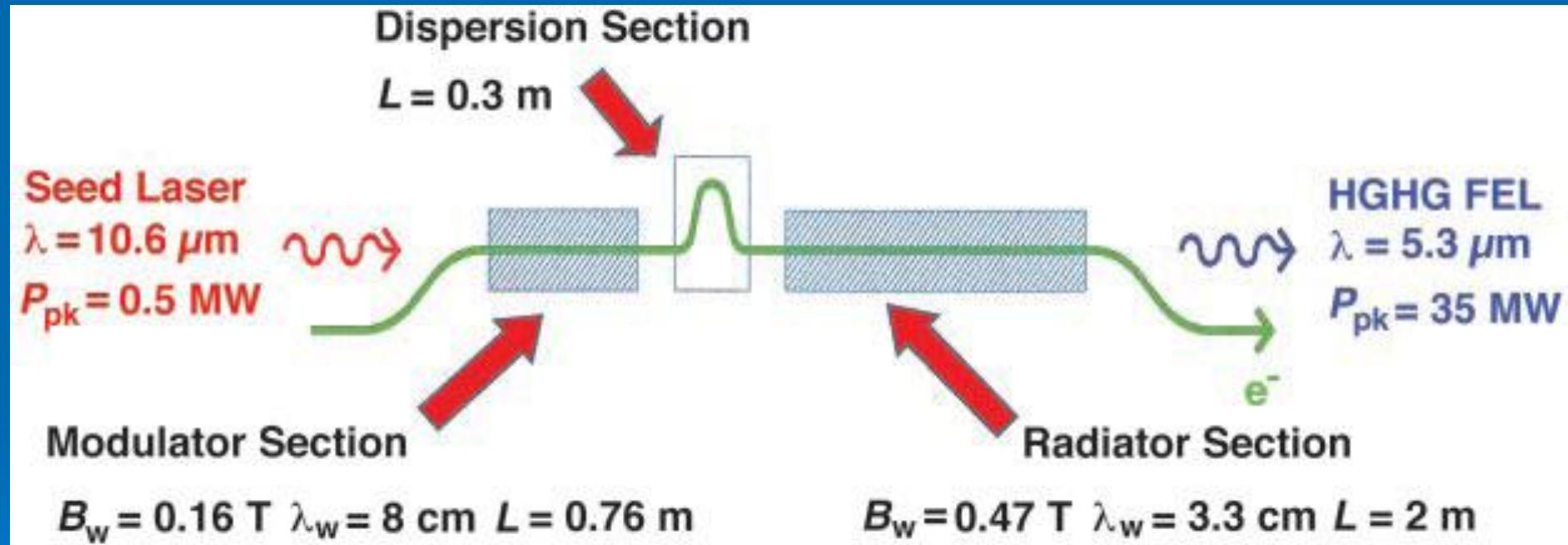


Micro bunches $340 \mu\text{m}$ apart \rightarrow seed for THZ FEL

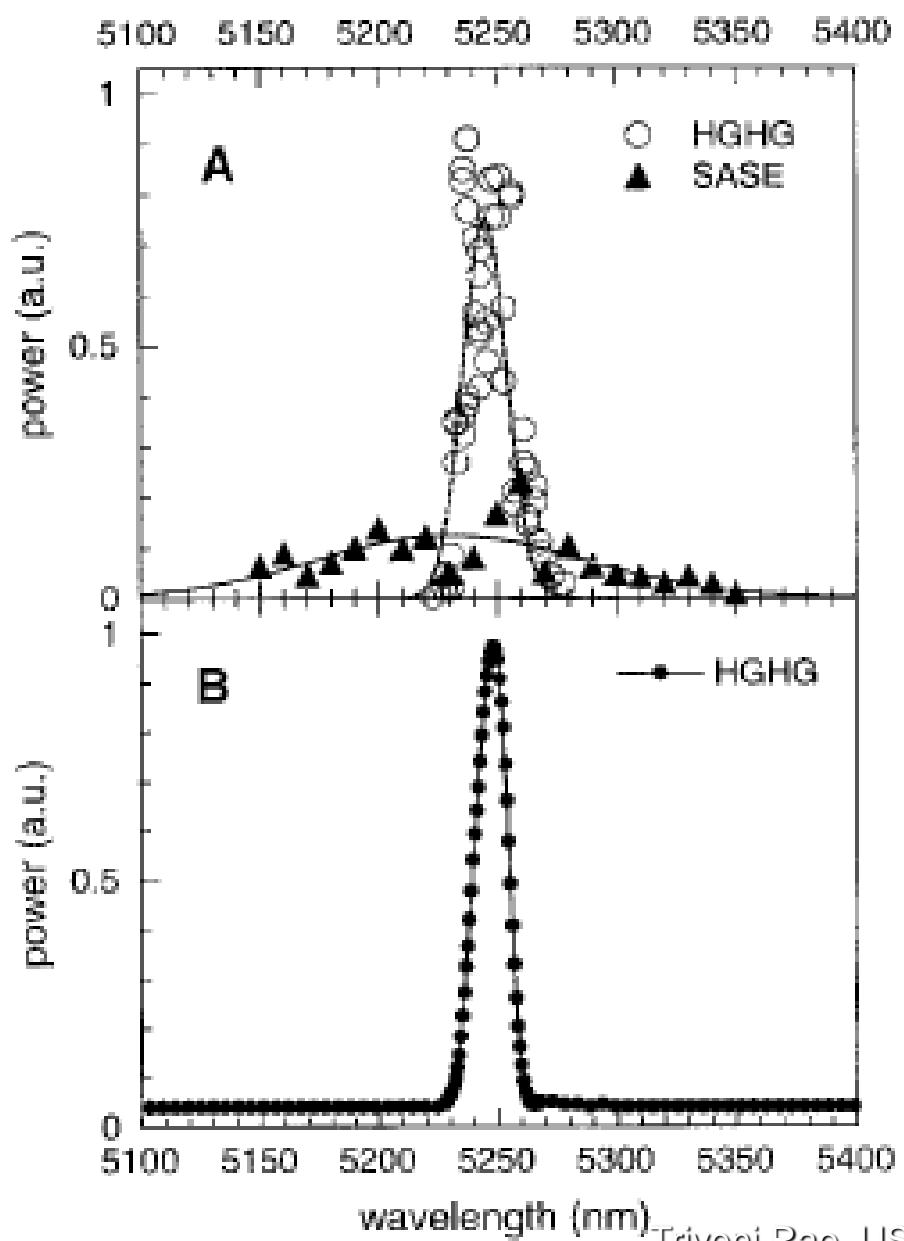
Courtesy: C. Sung et al. Proc. Of 2005 PAC, P. 2812

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High Gain Harmonic Generator



Courtesy: L. H. YU et al. Science 289, (2000) 932
Trivent Rao, USPAS 2013,
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A: Comparison of HGHG and SASE

Spectral width of HGHG 15 nm,<< SASE (90 nm)

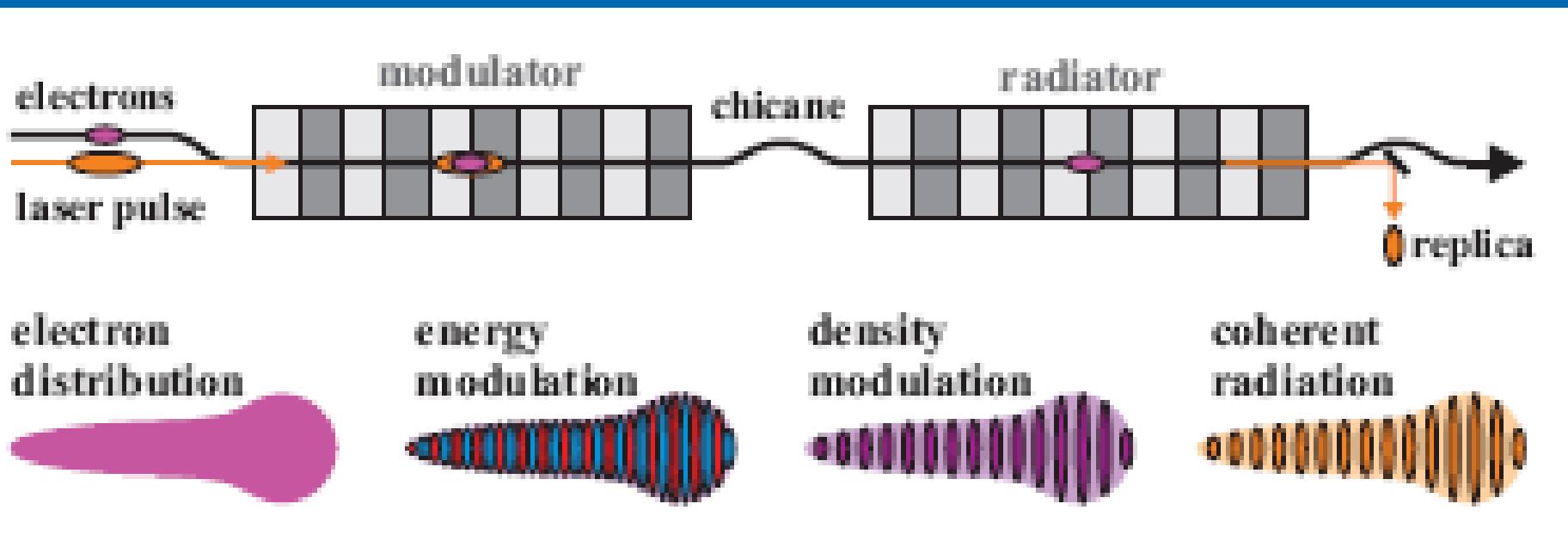
Amplitude of HGHG is $>10^6$ of SASE

B: Spectral distribution of HGHG signal

Technique used to generate VUV, X ray FEL beams: BESSY, SDL @ BNL, LBL, UCB

Source of radiation and use of radiation for electron diagnostics

Slice emittance, longitudinal distribution of short (100 fs) electron bunch can not be measured by standard techniques → generate optical replica of the electron beam

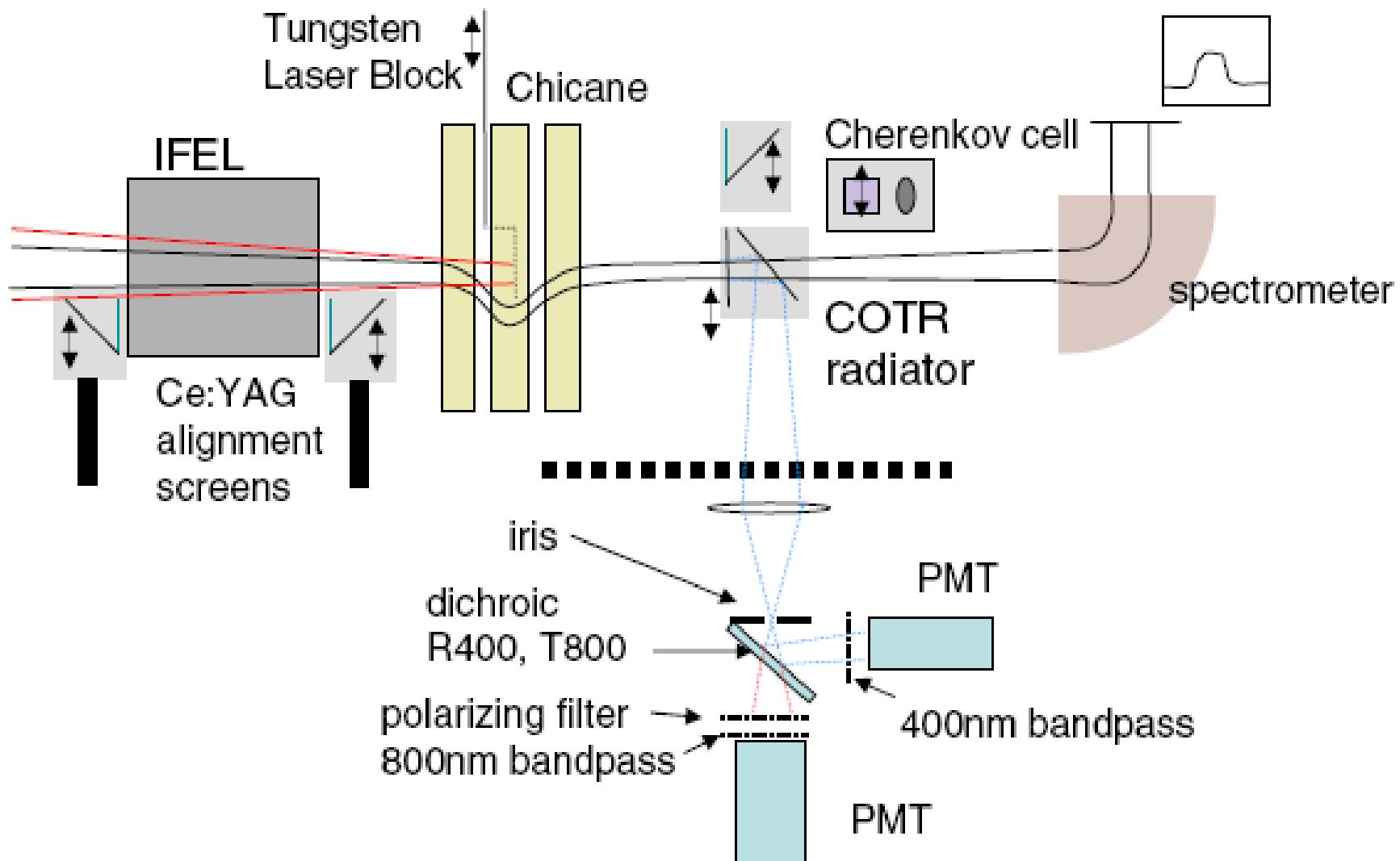


Use standard optical technique to measure beam parameters

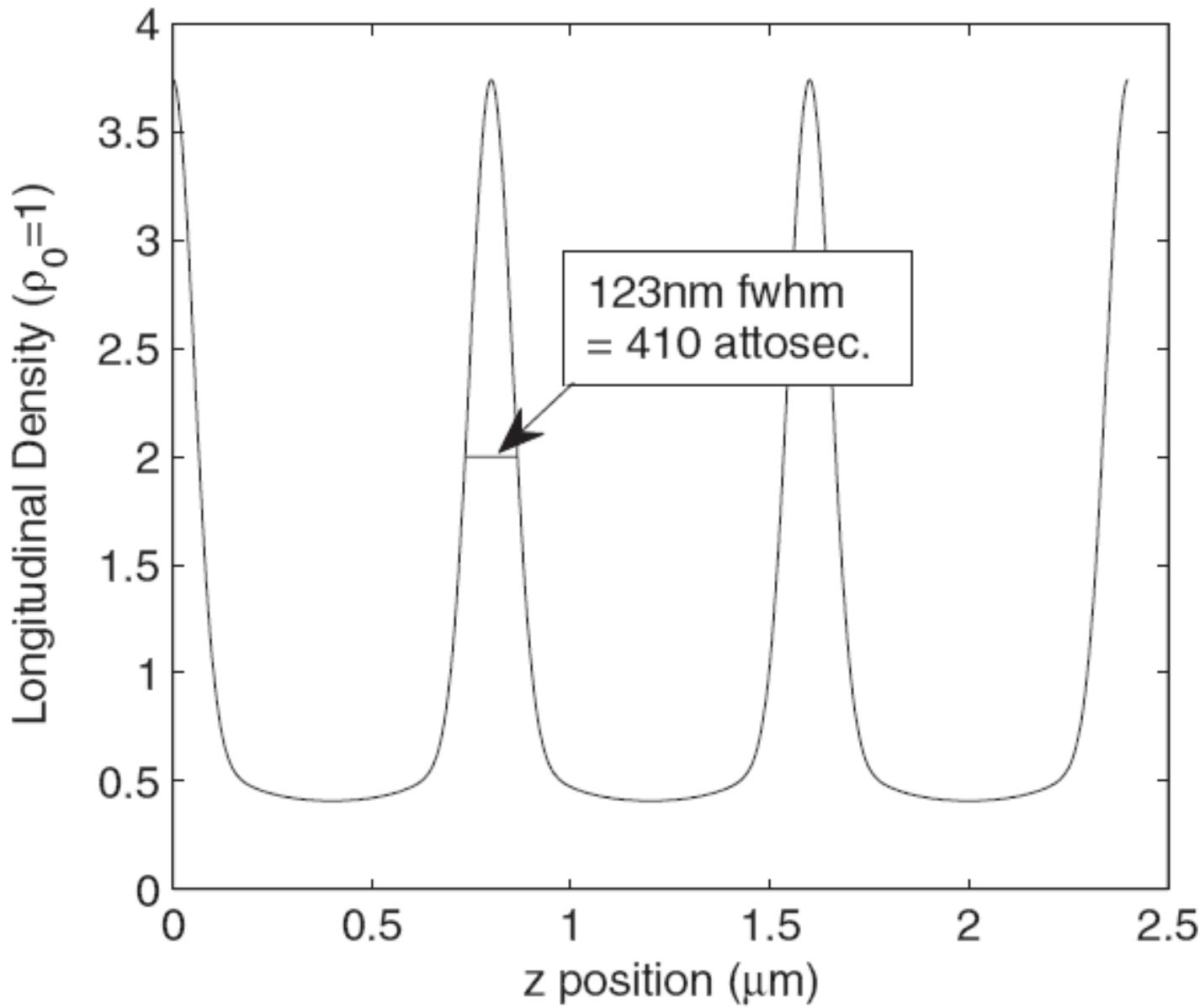
Attosecond electron beam generation

TABLE I. Experimental parameters for attosecond bunch train production. All widths are given as FWHM.

Parameter	Value
Electron energy	60 MeV
Electron energy spread	30 keV (typical)
Electron energy jitter	6 keV
Electron pulse length	0.8 ps ^a (typical)
Electron timing jitter	<0.2 ps ^a
Electron spot size	100 μ m (nominal)
Electron transverse jitter (<i>x</i> and <i>y</i>)	25 μ m
Bunch charge	1 pC (nominal)
Laser wavelength	785 nm
Laser energy	0.65 mJ/pulse
Laser pulse length	0.55 ps
Laser spot size	200 μ m
Undulator period	1.8 cm
Number of periods	3
Undulator strength (a_w)	0.46
Chicane R_{56}	0.04–0.16 mm



Courtesy: C. Sears et al. Phys. Rev. Spec. Topics AB 11, (2008), 061301
 Triveni Rao, USPAS 2013,
 Durham



Opens up possibility of accelerating with high intensity Ti:Sa laser

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Durham

Linear Thomson scattering: Laser counter propagating to e beam

Assume the system to be the electrons moving in the EM field of the laser. The transverse field of the laser is equivalent to the wiggler with wiggler period equal to the periodicity of the laser (is the length of the one period of the wiggler). Since the laser and the electron beam are counter propagating, the wavelength of the scattered radiation in the forward direction (direction of motion of the electron) is now modified to

$$\lambda_l = \frac{\lambda_{laser}}{4\gamma^2}$$

The power radiated by a single electron interacting with the laser beam is

$$P_s = 21.3 \gamma_0^2 \left(\frac{r_e}{r_0} \right)^2 P_0$$

γ_0 is the initial energy of the electron in units of its rest mass

r_e is the classical electron radius = 2.82×10^{-9} μm

r_0 is the spot size of the laser beam

$$P_0(GW) = 21.5 \left(\frac{a_0 r_0}{\lambda_l} \right)^2$$

$a_0 = |e| \frac{A_0}{m_0 c^2}$ is the normalized peak amplitude of the vector potential A_0 and is analogous to the wiggler strength parameter

The total radiated power in practical unit can be given as

$$P_T(W) = 2.11 * 10^{-2} \frac{L_0}{Z_R} \lambda_l(\mu m) I_b(A) E_b^2(MeV) P_0(GW)$$

L_0 is the laser pulse length,

Z_R is the Rayleigh length of the incident laser,

I_b is the electron beam current in Amperes,

E_b is the electron energy in MeV

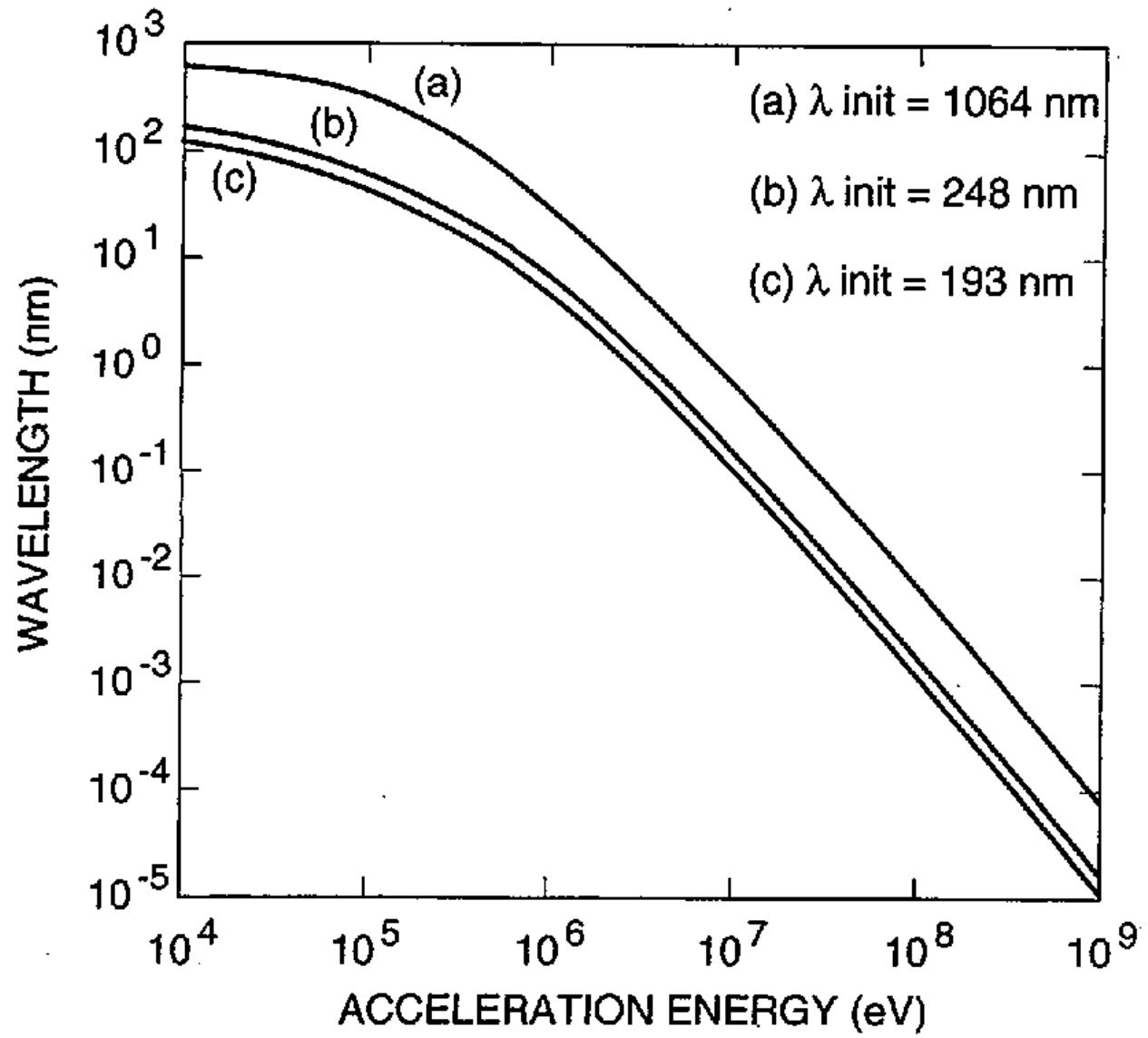
P_0 is the laser power in GW

This radiation is emitted in a cone angle $\theta = \frac{1}{\gamma_0}$

The contribution to the spectral width of the radiation comes from three sources: the finite number of wiggler period $N_w = L_0 / \lambda_l$, emittance of the electron beam \mathcal{E}_b and the energy spread of the electron beam δE .

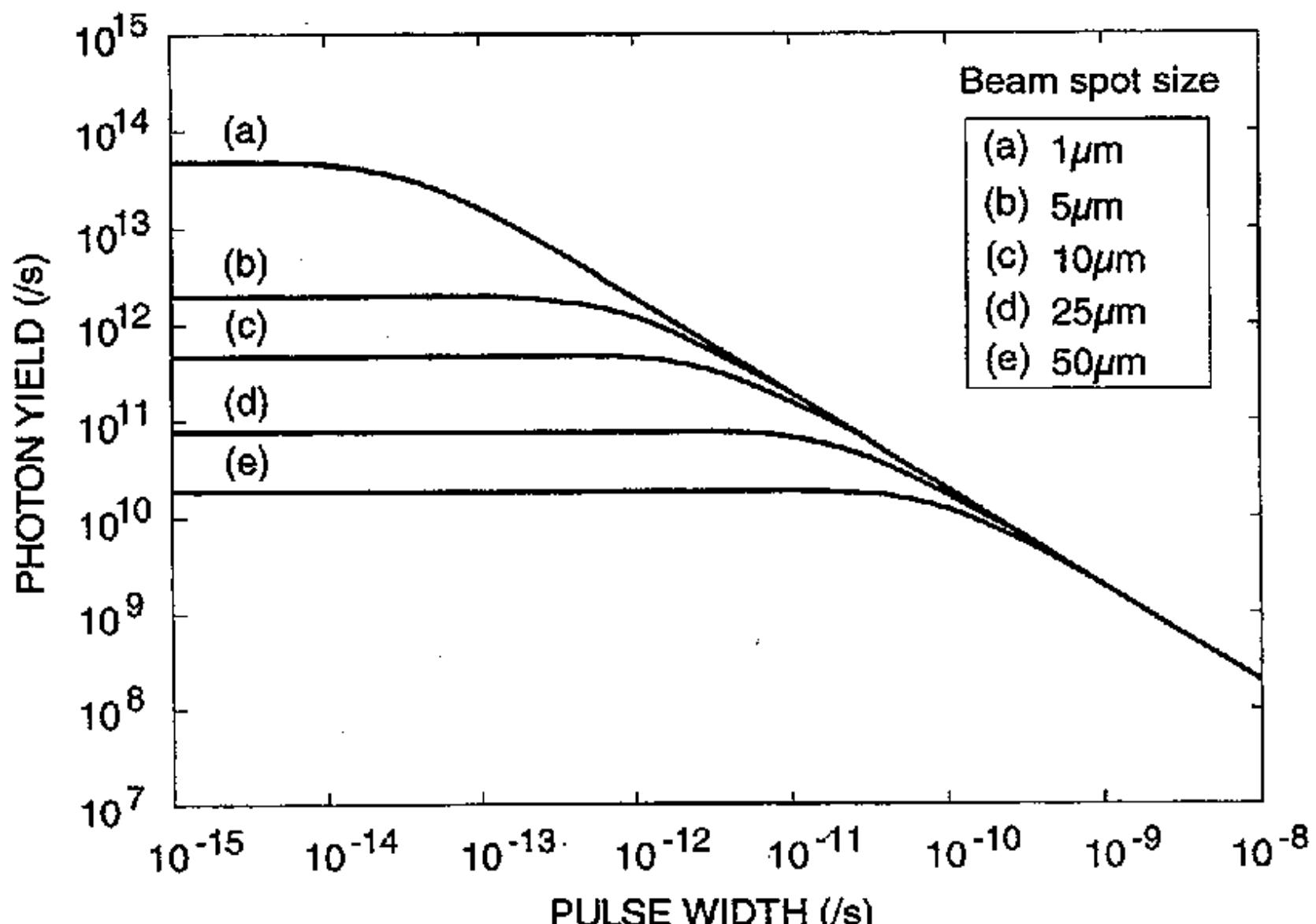
The total width can be written as

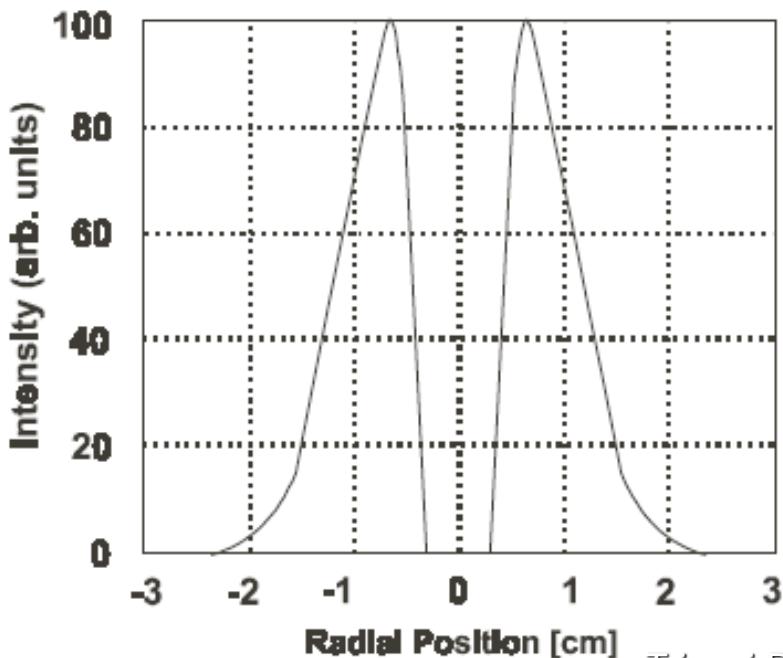
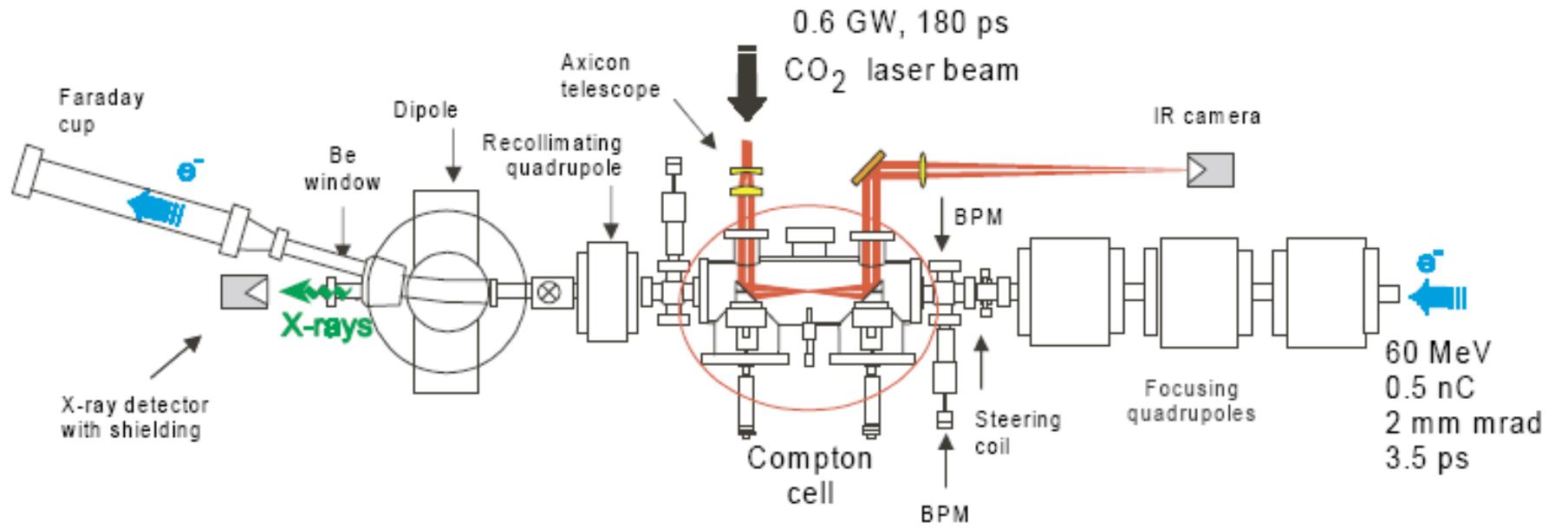
$$\left(\frac{\delta\omega}{\omega} \right)_T = \left(\frac{1}{N_w^2} + \frac{\varepsilon^4}{r_b^4} + 4 \frac{\delta E^2}{E^2} \right)^{1/2}$$



Courtesy:

Jpn. J. Appl. Phys. Vol. 37 (1998) pp. L 184–L 186
Triveni Rao, USPAS 2013,
Part 2, No. 2A, 1 February 1998



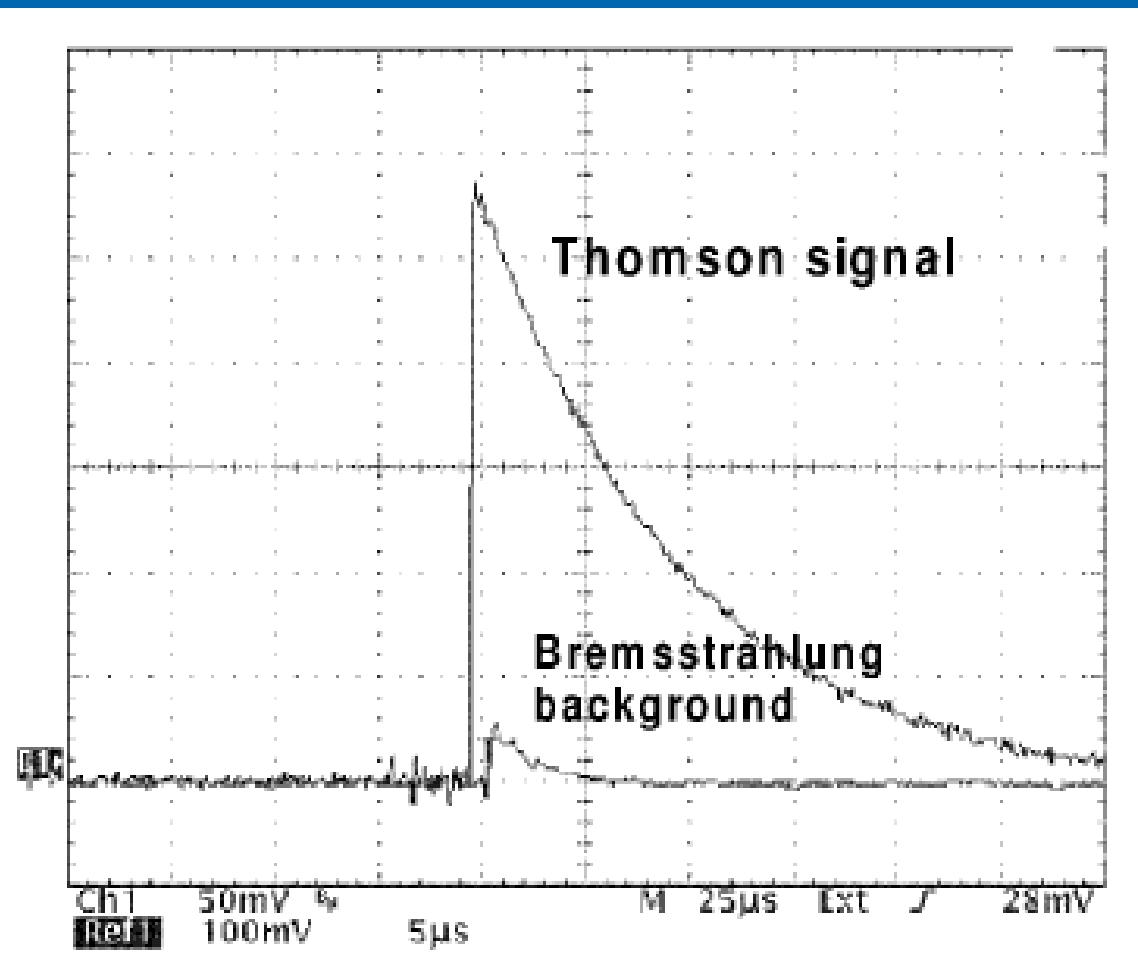


e⁻:

60 MeV, 0.5 nC, 140 A, 3.5 ps,
2 mm mrad, 32 μm spot

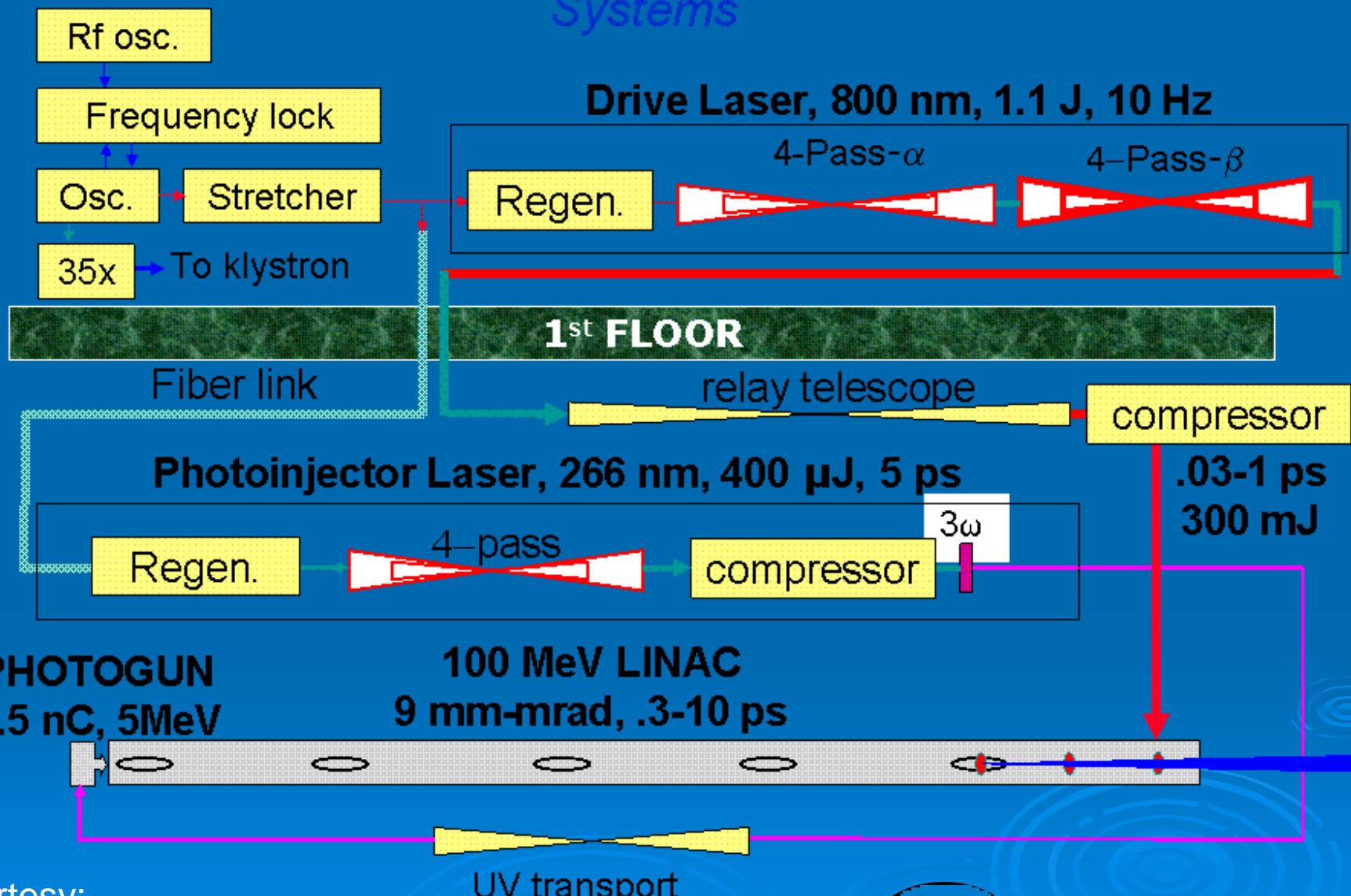
Laser:

10.6 μm, 600 MW, 180 ps, lin.
Pol., 32 μm annular spot



$$\begin{aligned}\lambda &= 1.8 \text{ \AA} \\ t &= 3.5 \text{ ps} \\ n/\text{pulse} &= 2.8 \times 10^7 \\ n_{\text{pk}} &= 8 \times 10^{18}\end{aligned}$$

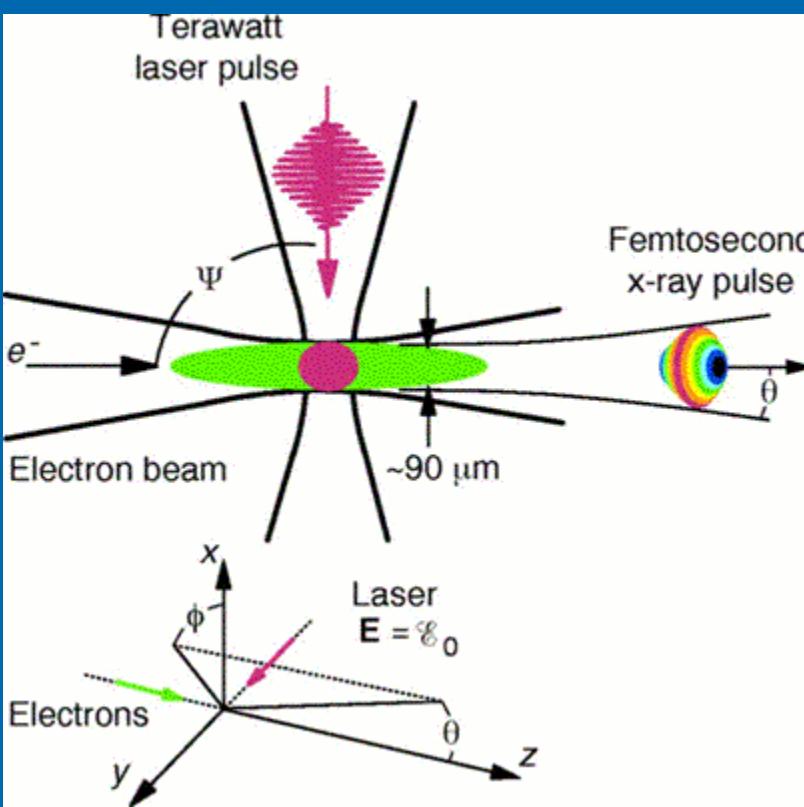
Terawatt Falcon Laser & Linac Photoinjector Laser Systems



Courtesy:

http://pbpl.physics.ucla.edu/Research/Experiments/Beam_Radiation_Interaction/Thoms
son_Scattering/

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Laser Parameters:

170 mJ

50 fs FWHM

36 μm $1/e^2$ diameter

$M^2 = 1.4$

Electron Beam Parameters:

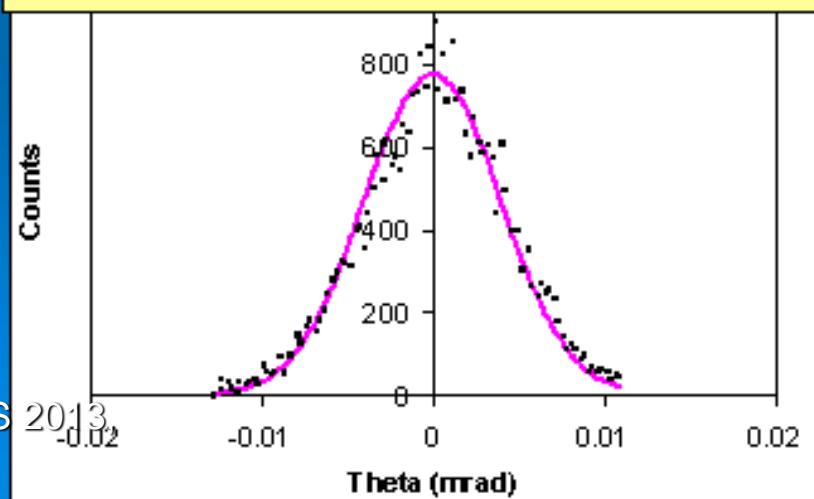
57 MeV

0.275 nC

$\sigma_x = \sigma_y =$ Triveni Rao, USPAS

$\epsilon_n = 10 \text{ mm-mrad}$ Durham

Integrated Dose: 2.3×10^6 photons



Nonlinear Thomson Scattering

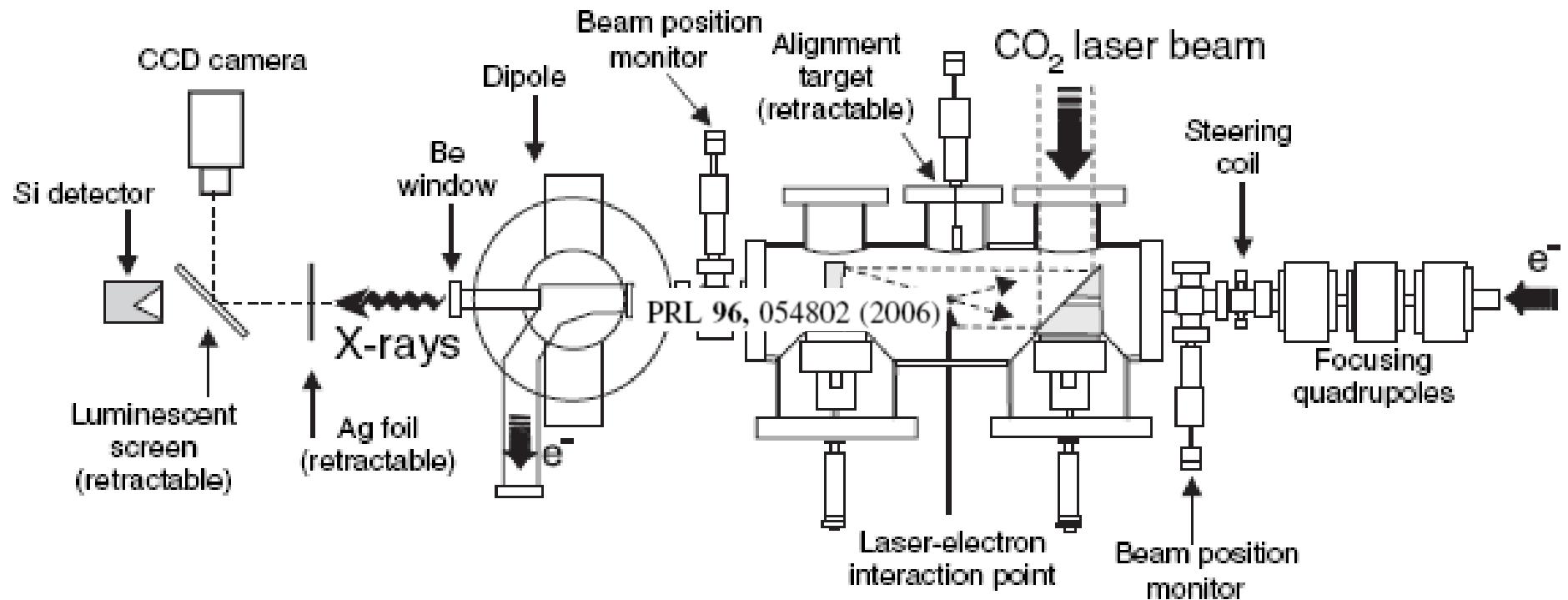
When the strength of the laser field is large, nonlinear effects can be seen.

Laser strength parameter a_0 is defined as

$$\begin{aligned} a_0 &= eA_0/m_e c^2 \\ &= 0.85 \cdot 10^{-9} \cdot \lambda(\mu\text{m}) \cdot \sqrt{I_0(\text{W/cm}^2)} \end{aligned}$$

$a_0 \ll 1$: Linear Thomson scattering: radiation at fundamental frequency

$a_0 \approx 1$, Nonlinear Thomson scattering: radiation at harmonic frequencies



Courtesy: M. Babzein et al. Phys . Rev. Lett. 96, 54802 (2006)

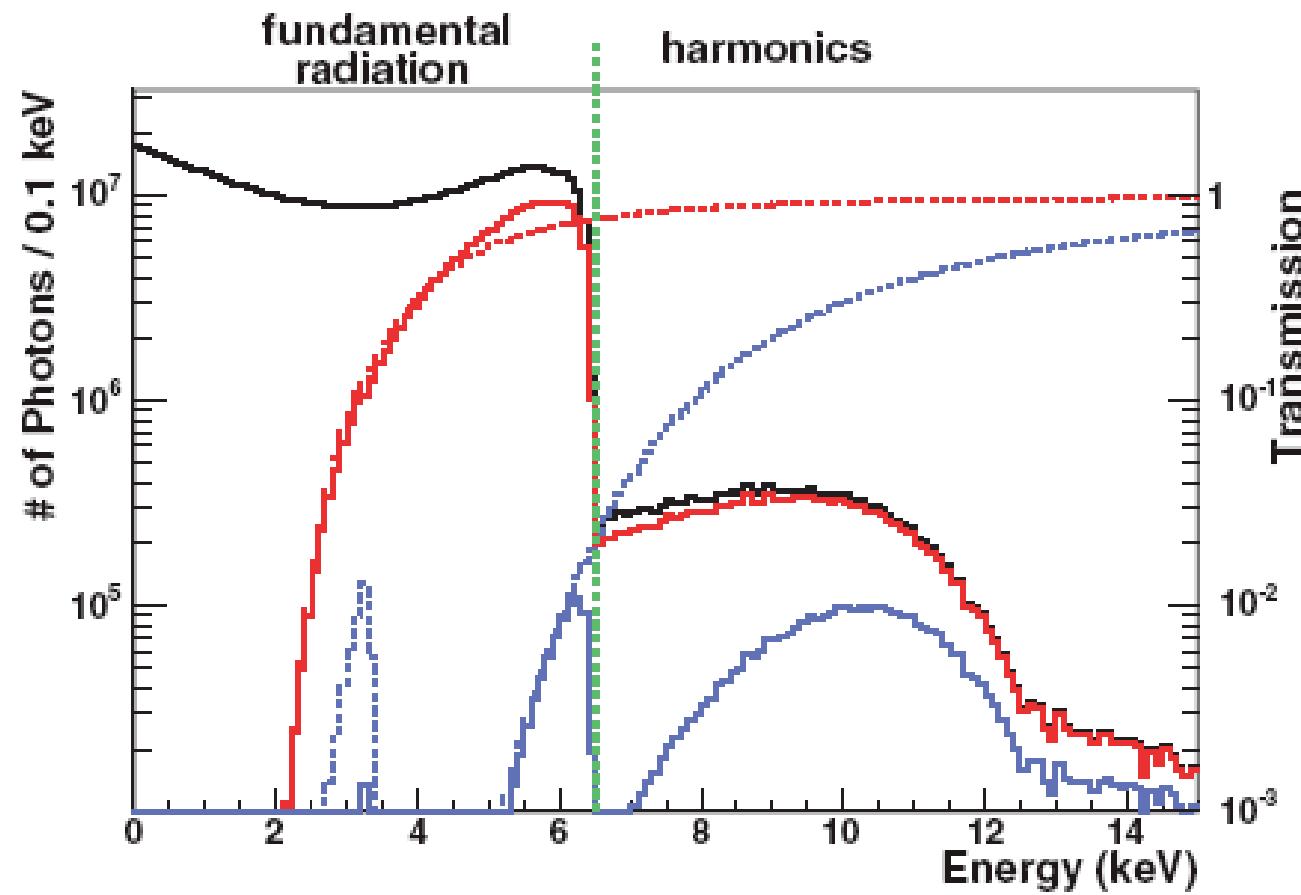
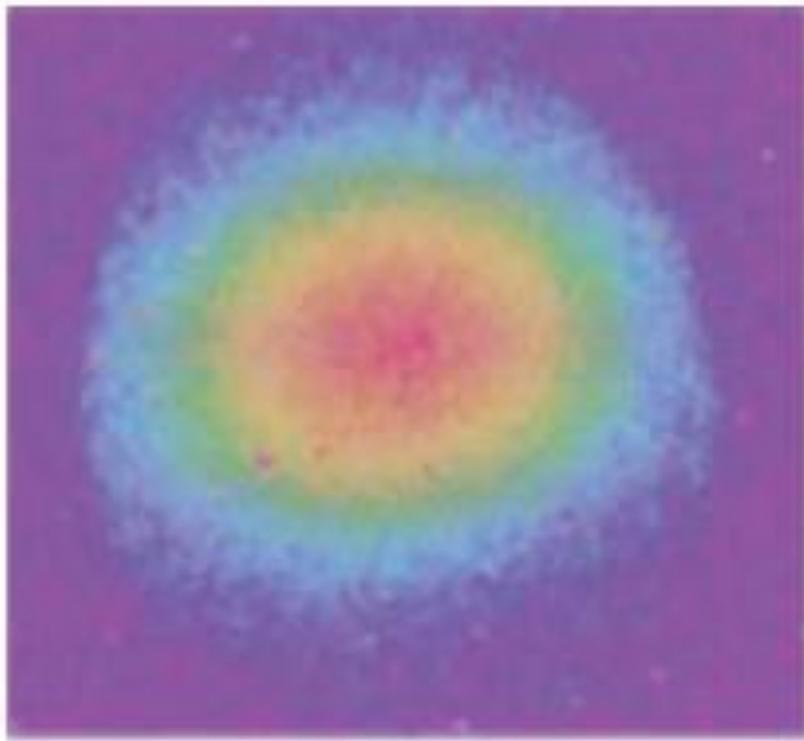
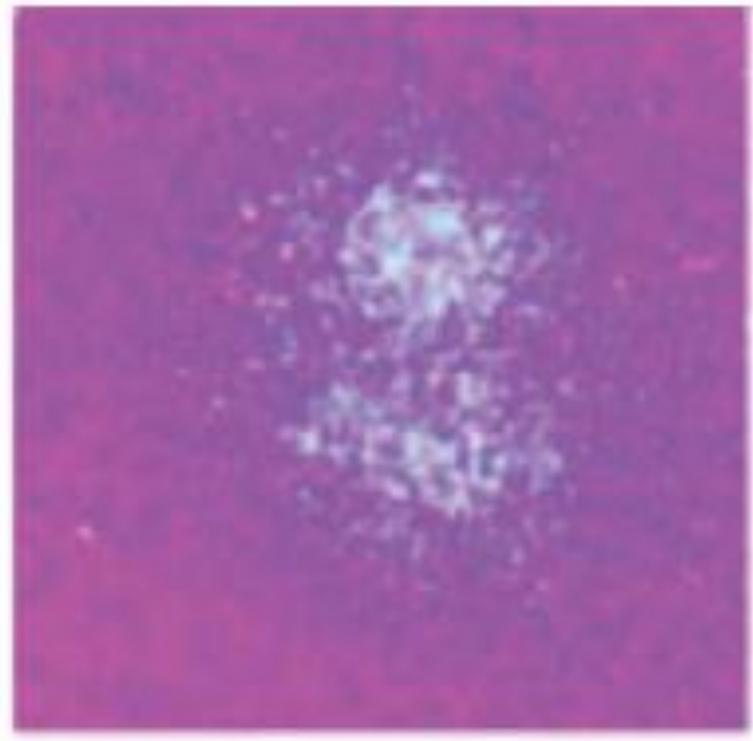


FIG. 2 (color). Simulated energy spectra of Thomson x rays. Solid lines: (black line) at the interaction point; (red line) on the detector after attenuation in the Be window and air; (blue line) filtered by a $10 \mu\text{m}$ Ag foil. Dashed lines show combined spectral transmission of the Be window with air (red line) and Ag foil (blue line). A green line shows the high-energy edge (6.5 keV) for the linear interpolation between



(a)



(b)

FIG. 3 (color). X-ray images observed on a luminescent screen: (a) without the Ag foil, and (b) with the $10\ \mu\text{m}$ Ag foil filter.

Linear Thomson Scattering: Laser propagating transverse to e⁻ Beam

Similar to counter propagating, but

- Wavelength of the emitted radiation is $\frac{1}{2}$
- Number of interacting electrons and photons is low
 - Low signal- may be as low as 10^{-3} per interaction
 - Need very high power laser
 - Need large electron density
 - Highly optimized laser transport

Laser Beam intensity:

Let the laser beam with Gaussian profile propagate along z. The intensity at (x, y, z) is

$$I_l(x, y, z) = I_{l,0} e^{-\left[\frac{(x-x_l)^2}{\omega^2(z-z_l)} + \frac{(y-y_l)^2}{\omega^2(z-z_l)}\right]} \left[\frac{\omega_0}{\omega(z - z_l)} \right]^2$$

$I_{l,0}$ is a constant and the waist ω_0 is at x_l, y_l, z_l

Total laser intensity is given by

$$I_{l,tot} = \pi \omega_0^2 I_{l,0}$$

Electron Beam Intensity

In the same coordinate system, the intensity of electron beam with a Gaussian distribution can be written as

$$I_e(x, y, z) = I_{b,0} e^{-\left[\frac{(x-x_b)^2}{\sigma_x^2} + \frac{(y-y_b)^2}{\sigma_y^2} + \frac{(z-z_b)^2}{\sigma_z^2} \right]}$$

$I_{b,0}$ is a constant, (x_b, y_b, z_b) is the center of the electron beam and σ_x , σ_y , and σ_z its size along x,y, and z respectively

Total electron beam intensity is

$$I_{b,tot} = \pi^{3/2} \sigma_x \sigma_y \sigma_z I_{b,0}$$

Courtesy
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Nicolas Delerue, University of Oxford
<http://nicolas.delerue.org>
<http://www-pnp.physics.ox.ac.uk/~delerue/>

Beams Overlap

$$N_{tot} = I_{l,0} I_{b,0} \times$$

$$\int \left[\frac{\omega_0}{\omega(z - z_l)} \right]^2 e^{-\frac{(x_l - x_b)^2}{\omega^2(z - z_l) + \sigma_x^2}} \sqrt{\pi \left[\frac{\sigma_x^2 \omega^2(z - z_l)}{\sigma_x^2 + \omega^2(z - z_l)} \right]}$$

$$\times e^{-\frac{(y_l - y_b)^2}{\omega^2(z - z_l) + \sigma_y^2}} \sqrt{\pi \left[\frac{\sigma_y^2 \omega^2(z - z_l)}{\sigma_y^2 + \omega^2(z - z_l)} \right]} \times$$

$$e^{-\left[\frac{(z - z_b)^2}{\sigma_z^2} \right]} dz$$

$$N_{tot} = \pi^{3/2} I_{l,0} I_{b,0} \sigma_x \sigma_y \sigma_z \omega_0^2 \times$$

$$e^{-\frac{(x_l - x_b)^2 + (y_l - y_b)^2}{[\omega^2(z_b - z_l) + \sigma_x^2] \times [\omega^2(z_b - z_l) + \sigma_y^2]}}$$

$$\sqrt{[\omega^2(z_b - z_l) + \sigma_x^2] \times [\omega^2(z_b - z_l) + \sigma_y^2]}$$

Laser Requirements:

Machine	sigma x microns	sigma y microns	Required Wavelength: microns
TESLA (Diagnostic section 250GeV)	120um	7um	0.67 um
JLC 500GeV	10	3	1.3 um
NLC 500GeV	10	1.25	0.22 um
NLC 1TeV	7.5	0.9	0.15 um

- Wavelength: Significantly smaller than the e spot size
Minimum laser spot size set by diffraction limit
- Power: Tens of MW
Efficiency $\sim 10^{-3}$
- Laser transport: Spot size diffraction limit
Rayleigh length $>$ other transverse dimension
- Damage threshold

Typical Issue

- The laser beam has a diameter of several millimeters.
- We want a wire size of only a [few] micrometers
- The laser light must be focused by wide aperture lens.
- No commercial lens seems to suits our needs
 => Custom design

Courtesy

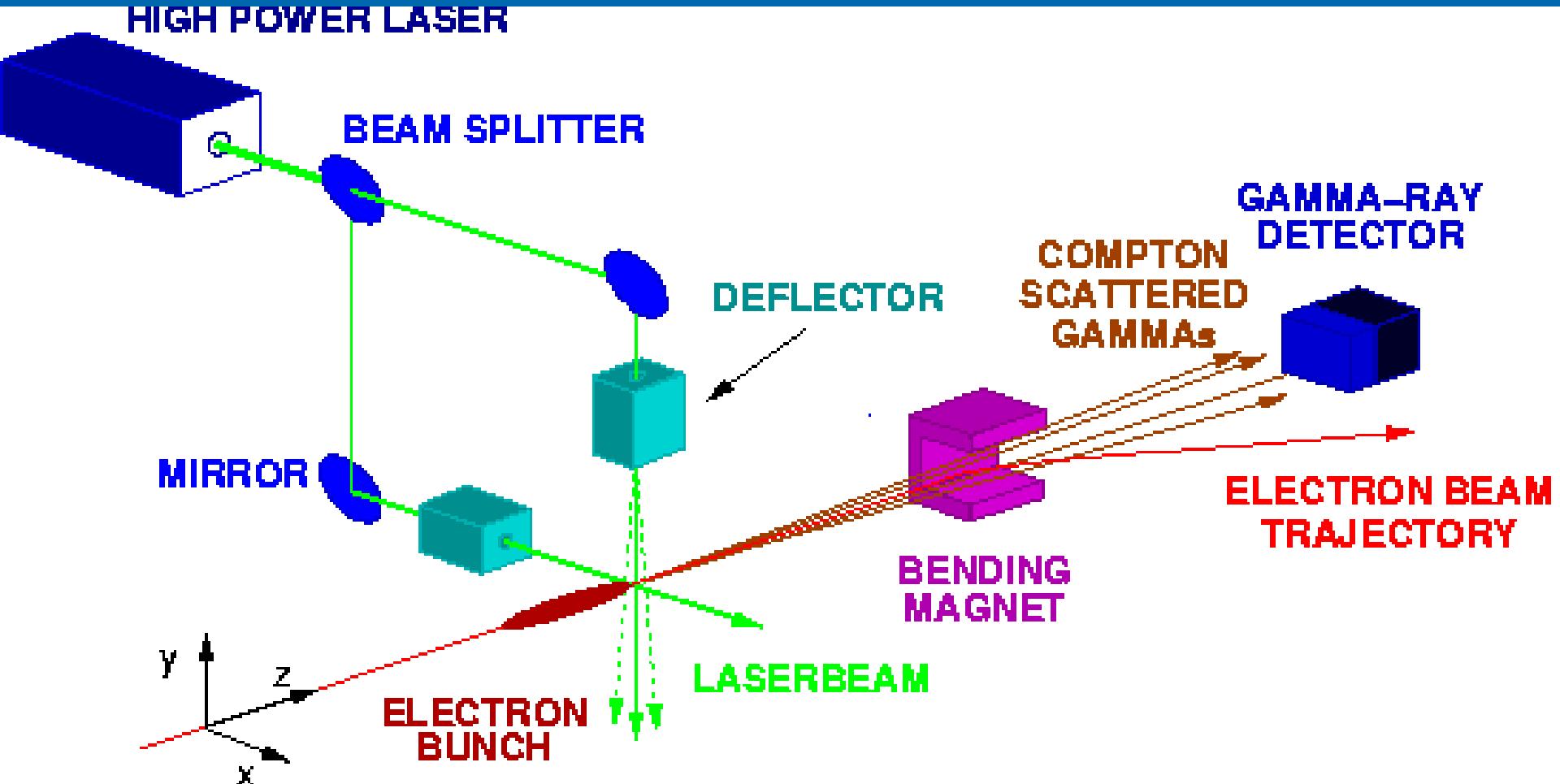
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<http://www-pnp.physics.ox.ac.uk/~delerue>

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- Goal: concentrate as much energy as possible in the smallest possible radius (gives the best performance).
- As the laser beam will be scanned across the lens, the size of the spot must remain constant over the scanning range.
- As the lens will be used with a high power laser, it must have no first order ghosts and as few second order ghosts as possible.
- To facilitate the alignment of the lens, aberrations must be kept as low as possible.
- Effect of a tilt of one element of the lens with respect to the others must be studied carefully

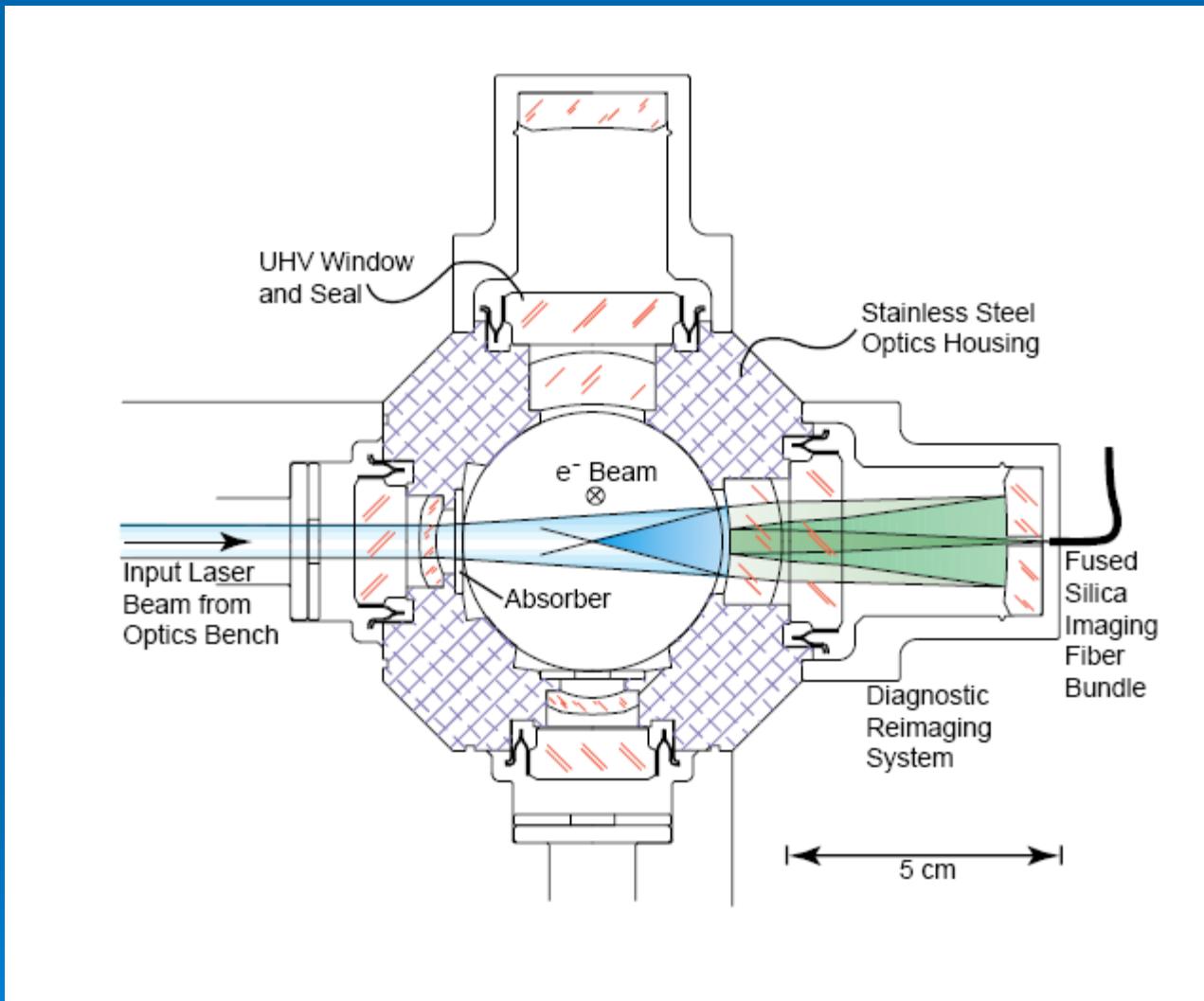
Laser Wire for e beam diagnostics



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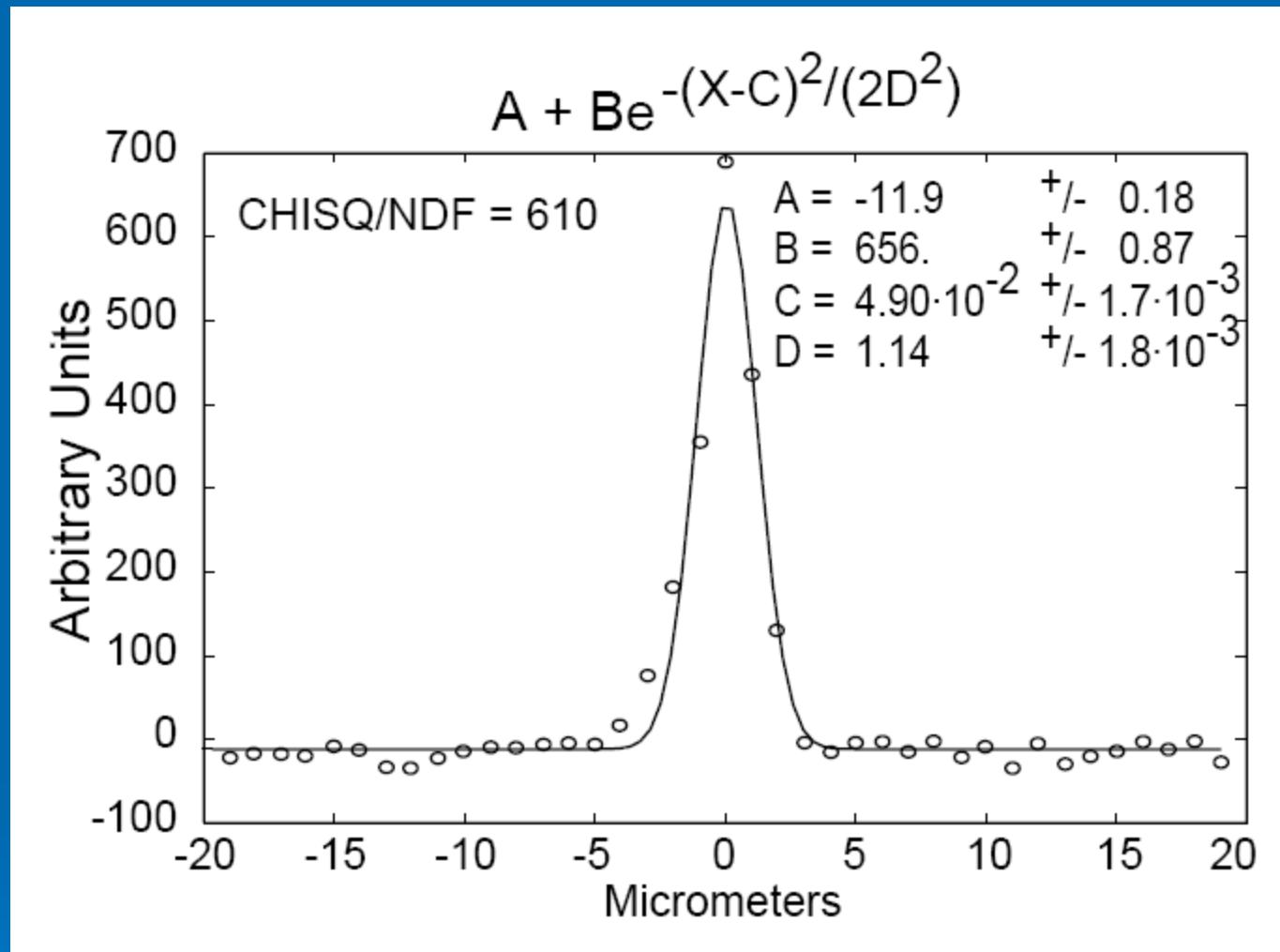
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Courtesy: <http://www.hep.ph.rhul.ac.uk/~kamps/lbbd/welcome.html#ScientificCase>



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Courtesy: P. Tenenbaum, T. Shintake, SLAC Pub 8057



Signal from Laser wire. $\lambda = 350$ nm, spot size $1\mu\text{m}$, power 10 MW