

# Lasers For e Beam Diagnostics

Triveni Rao, USPAS 2013, Duke  
University

## Advantages of using Laser

- Short pulse duration
- Large Bandwidth
- Large EM field
- Accurate phase information

## Methods:

- Generated photons
  - FEL
  - Compton Scattering
- Modify laser profile
  - Electro-optic effect

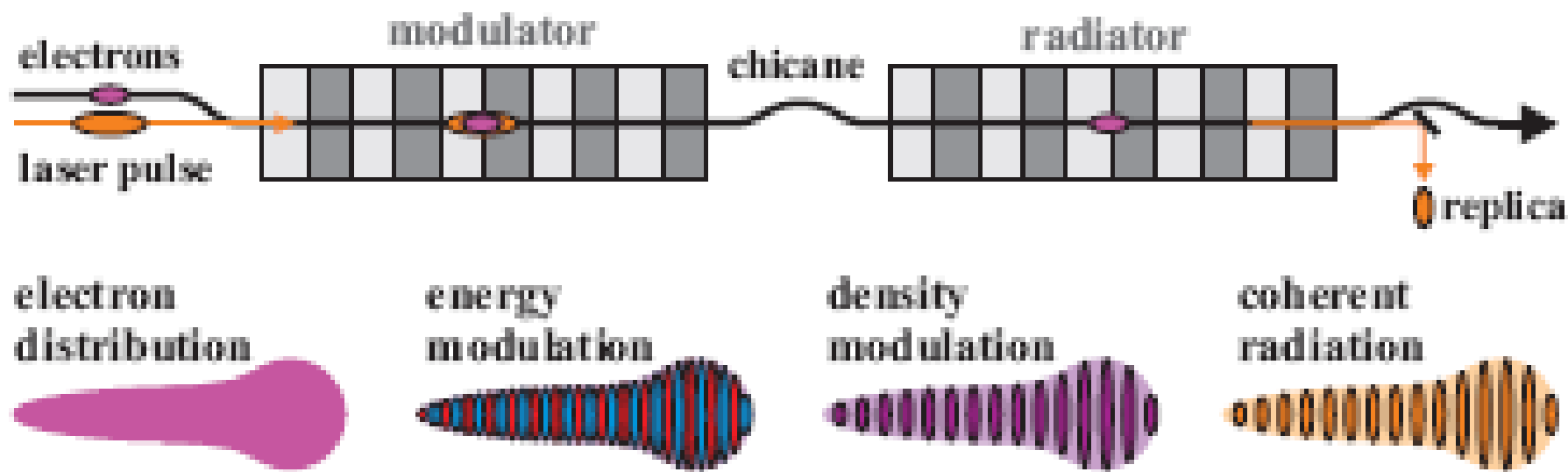
# Invasive Techniques

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A decorative graphic consisting of several concentric circles of varying sizes and colors (light blue, white, and dark blue) scattered across the bottom right portion of the slide.

# Source of radiation and use of radiation for electron diagnostics

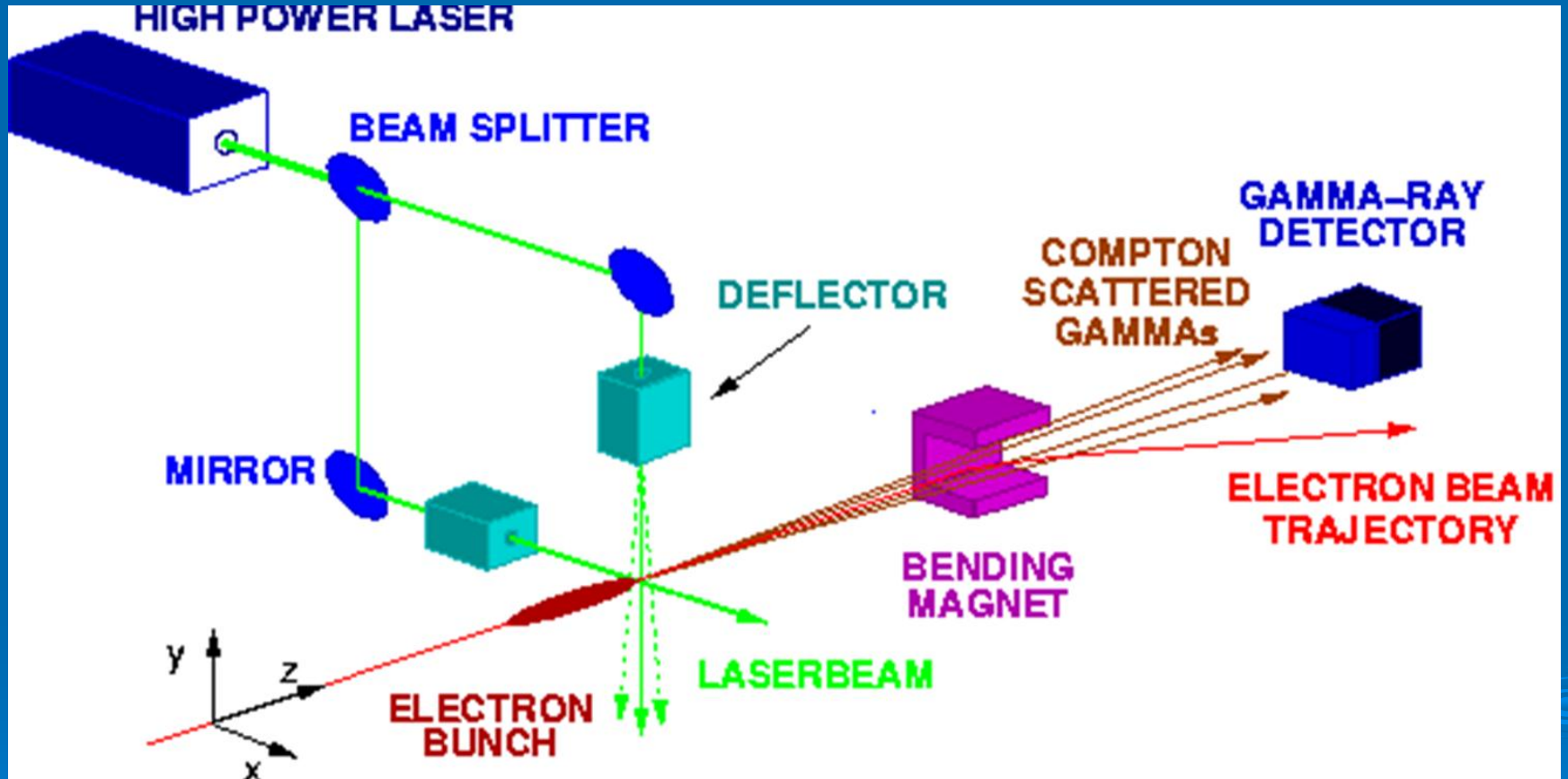
Slice emittance, longitudinal distribution of short (100 fs) electron bunch can not be measured by standard techniques → generate optical replica of the electron beam



Use standard optical technique to measure beam parameters

Courtesy: E. Saldin et al. Proc. Of PAC 07, P. 965  
Triveni Rao, USPAS 2013,  
Durham

# Laser wire scan



Courtesy:

<http://www.hep.ph.rhul.ac.uk/~kamps/lbbd/welcome.html#ScientificCase>

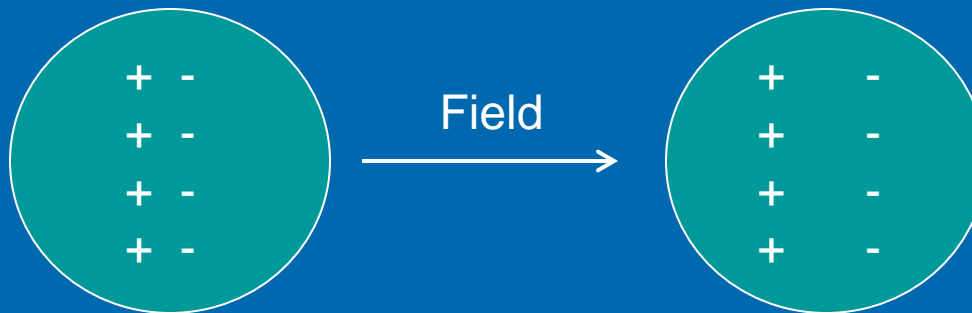
# Non-invasive Techniques

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A decorative graphic consisting of several sets of concentric circles in a lighter shade of blue, scattered across the bottom right portion of the slide.

# Electro optic effect as e- beam diagnostics

➤ What is electro-optic effect?



The electric polarization  $P$  can be written as

$$P_i = \epsilon_0 \sum_{j=1}^3 \chi_{ij}^{(1)} E_j + \epsilon_0 \sum_{j=1}^3 \sum_{k=1}^3 \chi_{ijk}^{(2)} E_j E_k + \epsilon_0 \sum_{j=1}^3 \sum_{k=1}^3 \sum_{l=1}^3 \chi_{ijkl}^{(3)} E_j E_k E_l + \dots$$

$\epsilon_0$  is vacuum permittivity,  $\chi^{(n)}$   $n^{\text{th}}$  order tensor electric susceptibility,  $i, j$ , and  $k$  are Cartesian indices,  $E_i, E_j$  can have different frequencies

The first term  $P = \epsilon_0 \chi^{(1)} : E$  applies to all linear optics and leads to optical index of refraction and dielectric constant .

For a **linear material**, terms with higher order  $\chi$  vanish and  $P = \epsilon_0 \chi^{(1)} : E$

The second term  $\epsilon_0 \sum_{j=1}^3 \sum_{k=1}^3 \chi_{ijk}^2 \vec{E}_j \vec{E}_k$  gives rise to optical mixing ( $\omega_1 + \omega_2$  and  $\omega_1 - \omega_2$ ) and second harmonic generation ( $\omega_1 = \omega_2$ ). When one of the fields varies very slowly compared to the other ( $\omega_1 \gg \omega_2$ ), then it is Pockel's effect where the input and output frequencies are the same and the index of refraction varies linearly with the applied slowly varying field.

$$\vec{P}_i(\omega_1) = \left( \chi_{ijk}^2 \vec{E}_k(\omega_2) \right) \vec{E}_j(\omega_1)$$



$\chi_{ijk}^2$  is a 3x3x3 third rank tensor. Since the input and output frequencies are the same ( $\omega_2 \sim 0$ ), it can be contracted notation for (ij) ie. 1=(11), 2=(22), 3=(33), 4=(23), 5=(13), 6= 12). This reduces the number of independent tensor elements from 27 to 18. The crystal symmetry may further reduce it since some of the tensors may be zero. In the contracted form, the 18 elements of  $\chi_{ijk}$  and the components of the polarization vector can be written as

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \epsilon_0 \begin{pmatrix} \chi_{11} \chi_{12} \chi_{13} \chi_{14} \chi_{15} \chi_{16} \\ \chi_{21} \chi_{22} \chi_{23} \chi_{24} \chi_{25} \chi_{26} \\ \chi_{31} \chi_{32} \chi_{33} \chi_{34} \chi_{35} \chi_{36} \end{pmatrix} \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_y E_z \\ 2E_x E_z \\ 2E_x E_y \end{pmatrix}$$

## How to relate to measurable quantity?

The index of refraction  $n(\omega)$  and the dielectric constant  $\epsilon(\omega)$  are related to the real part of the susceptibility by

$$n_{j,k}^2(\omega) = \frac{\epsilon_{j,k}(\omega)}{\epsilon_0} = 1 + \chi_{j,k}^1(\omega)$$

The susceptibility is related to the electro-optic coefficients  $r_{jk}$  through the optical impermeability  $B_{jk}$

$$B_{jk} = (1/\epsilon)_{jk} = (1/n^2)_{jk}$$

$$\Delta B_j = r_{jk} E_k = -\frac{2\Delta n_j}{n_j^3}$$

$$\Delta n_j = -\frac{1}{2} n_j^3 r_{jk} E_k$$

Phase difference and intensity modulation are

$$\delta = \frac{2\pi l \Delta n}{\lambda} \quad I = I_0 \sin^2\left(\frac{\delta}{2}\right)$$

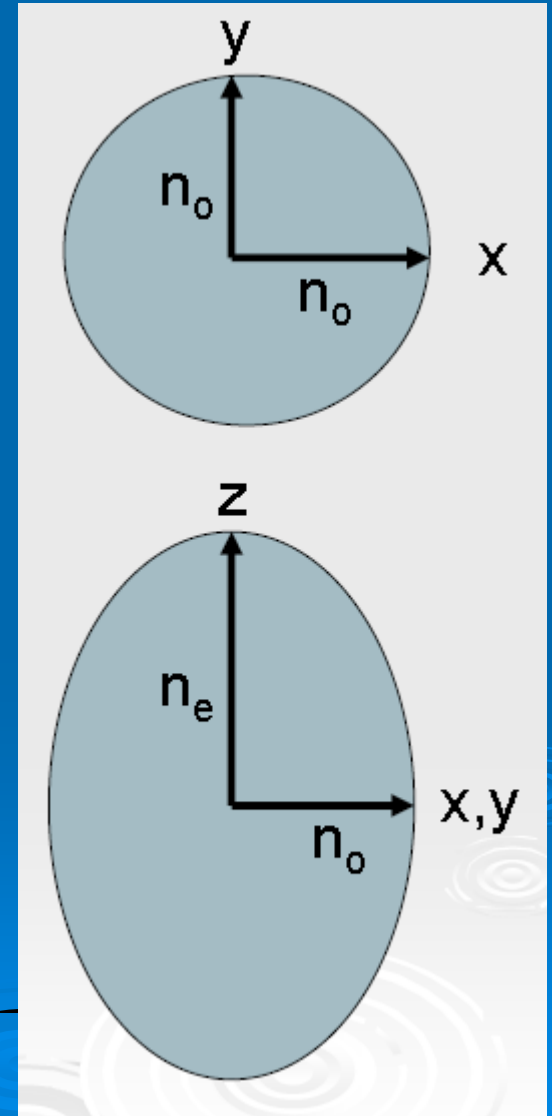
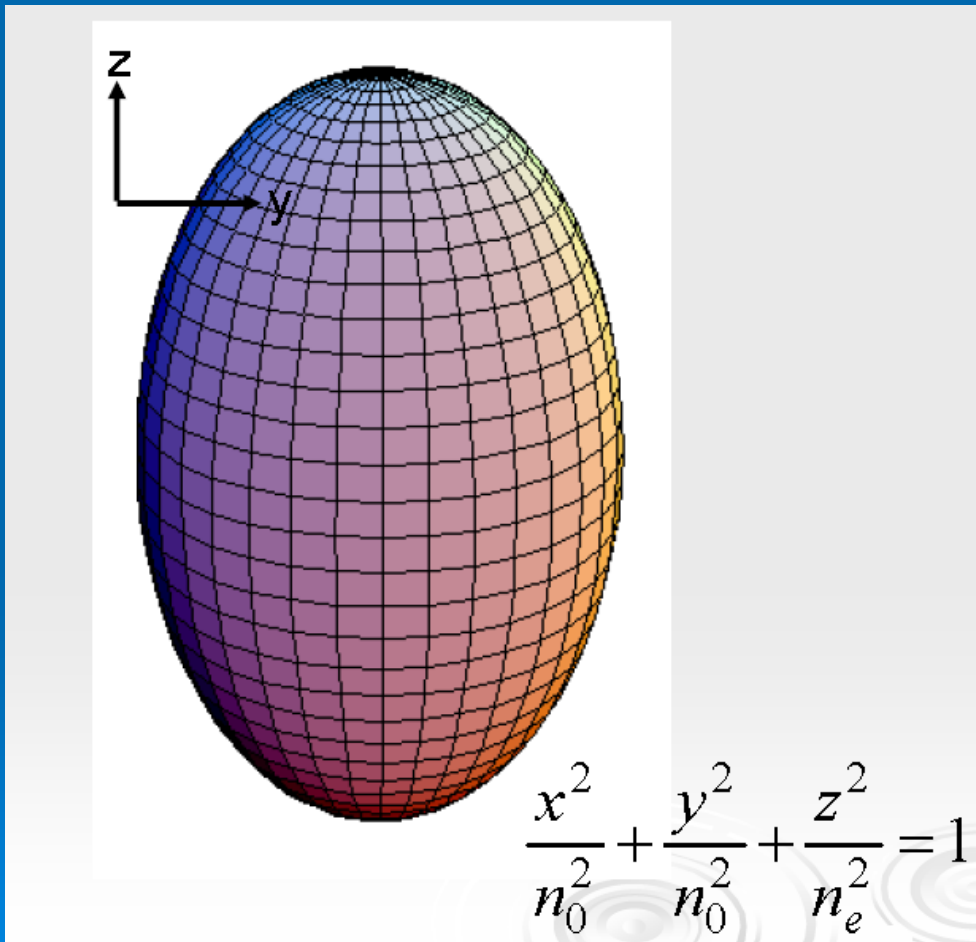
## Index of ellipsoid: general

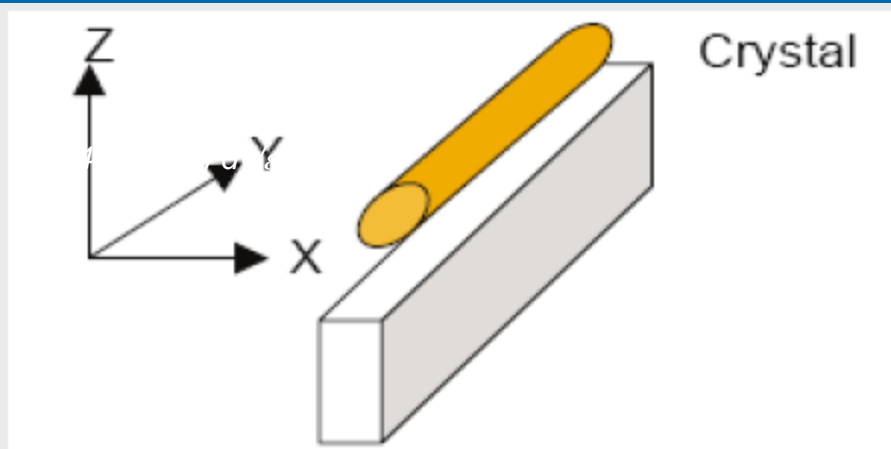
$$\frac{x^2}{N_1^2} + \frac{y^2}{N_2^2} + \frac{z^2}{N_3^2} + \frac{2yz}{N_4^2} + \frac{2xz}{N_5^2} + \frac{2xy}{N_6^2} = 1$$

Choose coordinate system appropriately

$$\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1$$

# Index Ellipsoid: Uniaxial crystal





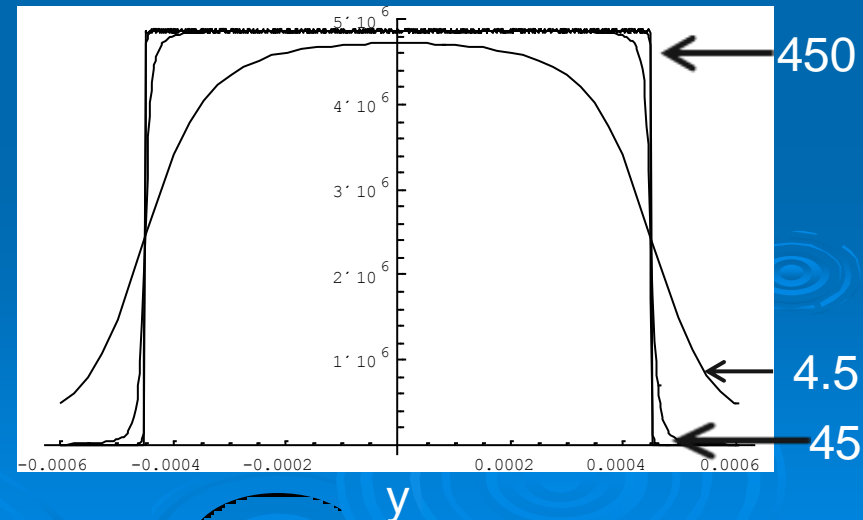
The electric field  $dE$  in the crystal with dielectric constant  $\epsilon$  at a distance  $r$  from the relativistic electron beam due to a charge  $\sigma dy$

$$d\vec{E} = (\gamma/4\pi\epsilon_0) \sigma(y) dy/\epsilon r^2 \vec{r}$$

The field decays rapidly along the  $y$  direction, past the extent of the beam

$$dE(x,t) = (\gamma/4\pi\epsilon_0) \sigma(y,t) dy/\epsilon r^2$$

$$dE(z,t) = (\gamma/4\pi\epsilon_0) \sigma(y,t) dy/\epsilon r^2$$



Time dependent field leads to time dependent change in index of refraction and index of ellipsoid

$$\Delta n_j = -\frac{1}{2} n_j^3 r_{jk} E_k \quad \frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1$$

$$x^2 \left( \frac{1}{n_1^2} + r_{1j} E_j \right) + y^2 \left( \frac{1}{n_2^2} + r_{2j} E_j \right) + z^2 \left( \frac{1}{n_3^2} + r_{3j} E_j \right) + 2yz(r_{4j} E_j) + 2xz(r_{5j} E_j) - 2xy(r_{6j} E_j) = 1$$

Redefine the principal axes  $X_i$ , calculate the refractive indices  $N_i$  and the phase difference  $\delta$  between the orthogonal components

$$\delta = \frac{2\pi}{\lambda} \int_0^L N_j - N_i dx_k$$

where  $x_k$  is the laser propagation direction,  $L$  is length of the crystal in that direction and  $N_{i,j}$  are refractive indices in directions orthogonal to propagation

$$I = I_o \sin^2(2\theta + 2\phi) \sin^2 \frac{\delta}{2}$$

where  $\theta$  is the angle made by the E vector of the laser beam with the y-z plane,  $\phi$  is the field dependent orientation of the new principal axis with the old one

# For Lithium Niobate

Rotation  $\phi$  of Principal axes

		Direction of propagation of Laser		
		x	y	z
Direction of propagation of Charge Bunch	x	$\phi \sim \frac{1}{2} \tan^{-1} \left( \frac{36.4 \times 10^{-12} E_y}{-0.016 + 3.4 \times 10^{-12} E_y - 21.1 \times 10^{-12} E_z} \right)$	$\phi \sim 0$	$\phi \sim 0$
	y	$\phi \sim 0$	$\phi \sim \frac{1}{2} \tan^{-1} \left( \frac{36.4 \times 10^{-12} E_x}{-0.016 - 21.1 \times 10^{-12} E_z} \right)$	$\phi \sim \pm \pi / 4$
	z	$\phi \sim \frac{1}{2} \tan^{-1} \left( \frac{36.4 \times 10^{-12} E_y}{-0.016 + 3.4 \times 10^{-12} E_y} \right)$	$\phi \sim \frac{1}{2} \tan^{-1} \left( \frac{36.4 \times 10^{-12} E_x}{-0.016 - 3.4 \times 10^{-12} E_y} \right)$	$\phi \sim \frac{1}{2} \tan^{-1} \left( \frac{E_x}{E_y} \right)$

$\phi$  Is independent of field for 4 cases, dependent on field along one direction in 1 case



# Total Retardation $\delta$ experienced by the laser beam for 3 possible directions of the laser and electron beams

		Direction of propagation of Laser		
		X	Y	Z
Direction of propagation of charge bunch	X	$-(1.19 \times 10^6)L$ $-(0.0014) \int_0^L E_z dx + (0.0003) \int_0^L E_y dx$ $-(2.97 \times 10^{-12}) \int_0^L E_y^2 dx + (1.95 \times 10^{-13}) \int_0^L E_z^2 dx$ $-(1.67 \times 10^{-14}) \int_0^L E_y E_z dx + \dots$	$-(1.19 \times 10^6)L$ $-(0.0014) \int_0^L E_z dy + (0.0003) \int_0^L E_y dy$ $+(1.95 \times 10^{-13}) \int_0^L E_z^2 dy - (1.67 \times 10^{-14}) \int_0^L E_y E_z dy$ $-(3.69 \times 10^{-15}) \int_0^L E_y^2 dy + \dots$	$\delta = -(0.0011) \int_0^L E_y dz$ $+(6.69 \times 10^{-14}) \int_0^L E_y E_z dz$ $-(3.51 \times 10^{-24}) \int_0^L E_y E_z^2 dz$ $-(9.13 \times 10^{-25}) \int_0^L E_y^3 dz + \dots$
	Y	$-(1.19 \times 10^6)L$ $-(0.0014) \int_0^L E_z dx$ $+(1.95 \times 10^{-13}) \int_0^L E_z^2 dx$ $-(2.51 \times 10^{-23}) \int_0^L E_z^3 dx + \dots$	$-(1.19 \times 10^6)L$ $-(0.0014) \int_0^L E_z dy$ $+(2.96 \times 10^{-12}) \int_0^L E_x^2 dy + (1.95 \times 10^{-13}) \int_0^L E_z^2 dy + \dots$	$\delta = -(0.0005) \int_0^L E_x dz$ $+(3.34 \times 10^{-14}) \int_0^L E_x E_z dz$ $-(1.76 \times 10^{-24}) \int_0^L E_x E_z^2 dz$ $-(1.14 \times 10^{-25}) \int_0^L E_x^3 dz + \dots$
	Z	$-(1.19 \times 10^6)L$ $+(0.0003) \int_0^L E_y dx$ $-(2.97 \times 10^{-12}) \int_0^L E_y^2 dx$ $-(5.71 \times 10^{-22}) \int_0^L E_y^3 dx + \dots$	$-(1.19 \times 10^6)L$ $-(0.0014) \int_0^L E_z dy + (0.0003) \int_0^L E_y dy$ $-(2.96 \times 10^{-12}) \int_0^L E_x^2 dy - (5.92 \times 10^{-12}) \int_0^L E_y E_x dy$ $-(3.69 \times 10^{-15}) \int_0^L E_y^2 dy + \dots$	$\delta = -(0.0005) \int_0^L E_x dz$ $-(0.0007) \int_0^L \frac{E_y^2}{E_x} dz$ $-(1.14 \times 10^{-25}) \int_0^L E_x^3 dz + \dots$

# Two specific cases

e along y axis, laser along x axis

$$\begin{aligned}\delta = & - (1.19 \times 10^6)L \\ & - (0.0014) \int_0^L E_z dx \\ & + (1.95 \times 10^{-13}) \int_0^L E_z^2 dx \\ & - (2.51 \times 10^{-23}) \int_0^L E_z^3 dx + \dots\end{aligned}$$

e along z axis, laser along x axis

$$\begin{aligned}\delta = & - (1.19 \times 10^6)L \\ & + (0.0003) \int_0^L E_y dx \\ & - (2.97 \times 10^{-12}) \int_0^L E_y^2 dx \\ & - (5.71 \times 10^{-22}) \int_0^L E_y^3 dx + \dots\end{aligned}$$

No cross filed terms, Large Static term, Large coefficient for case 1  
Need to compensate for the static term

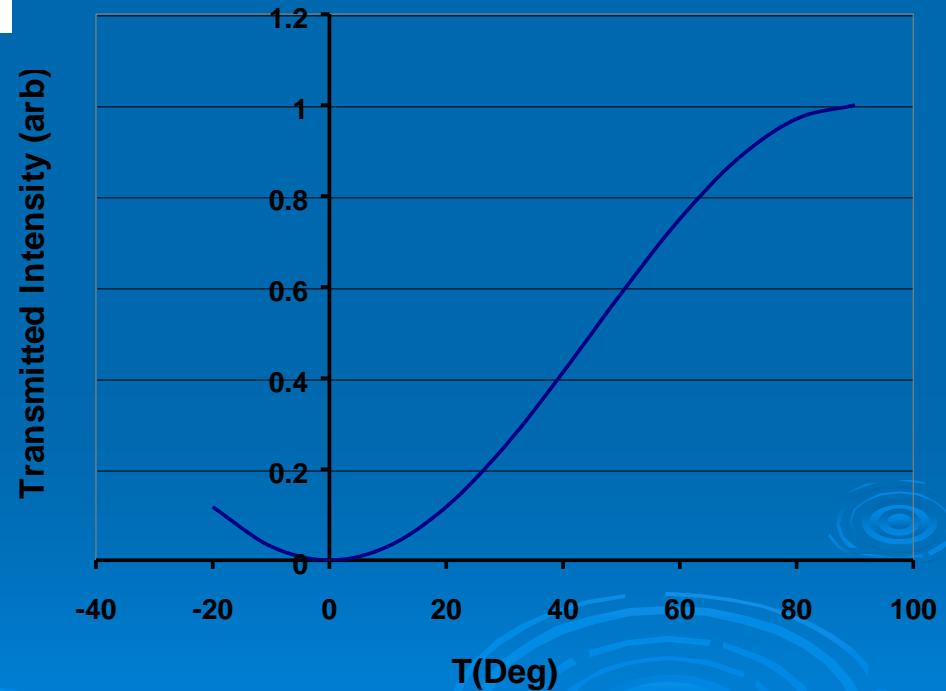
We showed that

$$I = I_o \sin^2(2\theta + 2\phi) \sin^2 \frac{\delta}{2}$$

If laser polarization is set at  $45^\circ$  to field free optical axis ( $\theta = 45^\circ$ )

$$I(t) = I_o [\eta + \sin^2(\delta_b + \delta(t))]$$

where  $\eta$  is the intensity extinction coefficient of the optical arrangement (fraction of the transmitted intensity in the absence of the crystal),  $\delta_b$  is the static retardation and  $\delta(t)$  is the time dependent component of the retardation.



Preset operation in linear regime

# For Measurements

- Select type of Crystal

  - E-O coefficient

  - Radiation damage

- Select orientation

  - Rotation

  - Cross & Static terms

- Select laser

  - Polarization

  - CW/Pulsed

  - Power

  - wavelength

- Select Detection scheme

  - Time

  - Spectrum

  - Spatial profile

- Select operating parameters

  - X'tal dimensions

  - Location

  - Laser transport

  - Optical system

  - Diagnostics

  - ...

# Measurement of Pulse duration in a single shot using CW laser and fast detector

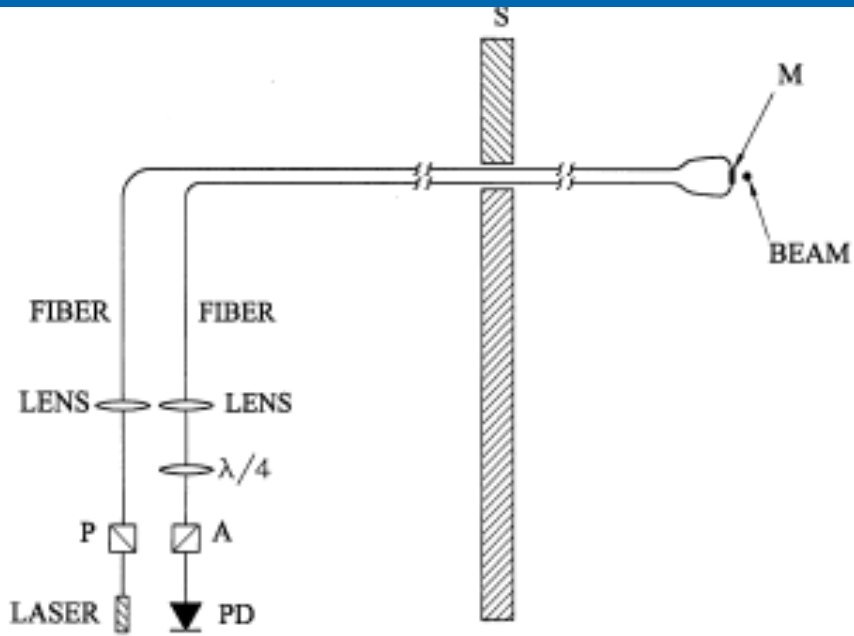
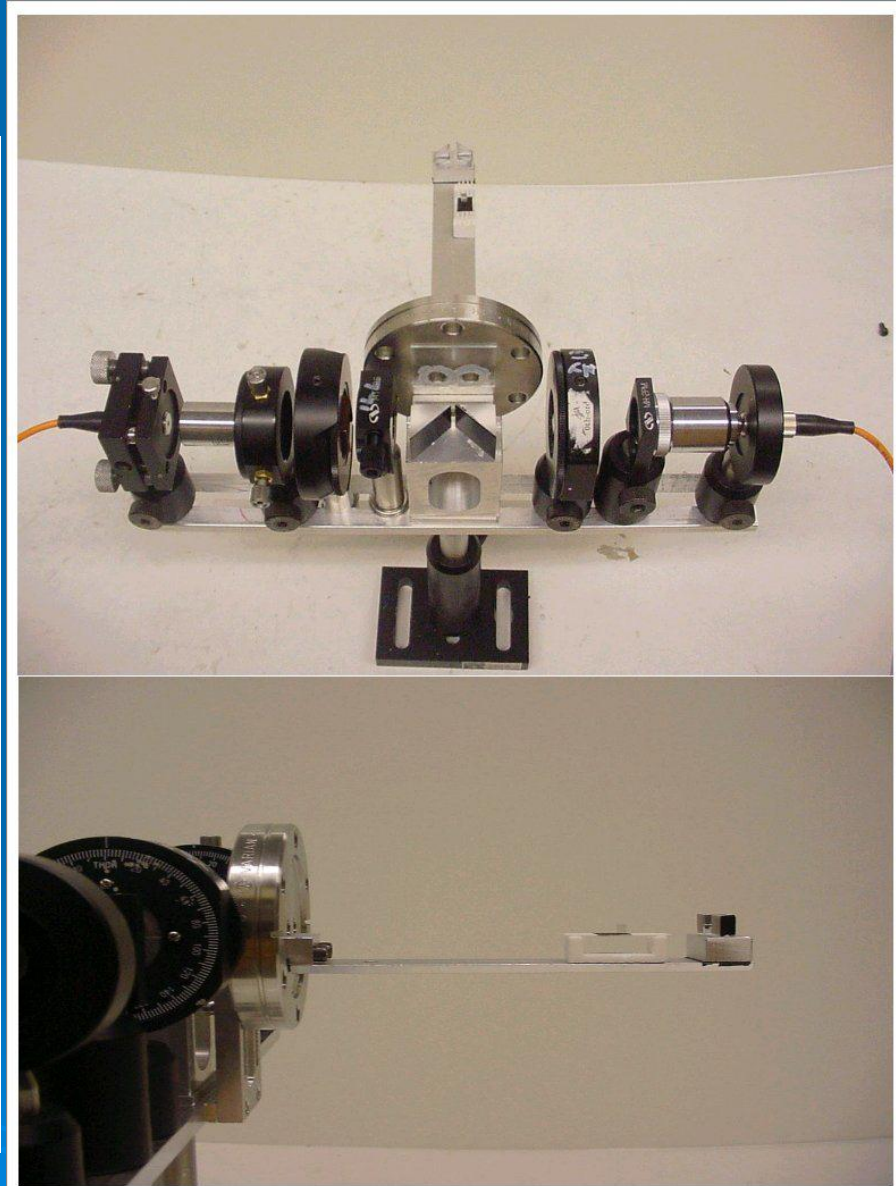
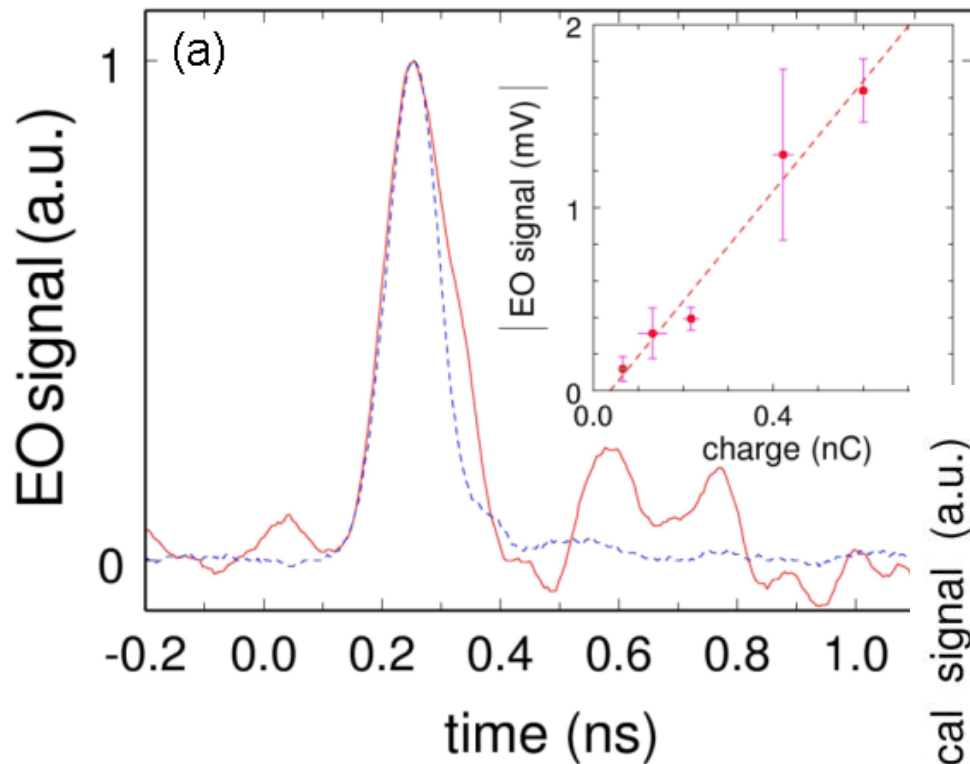
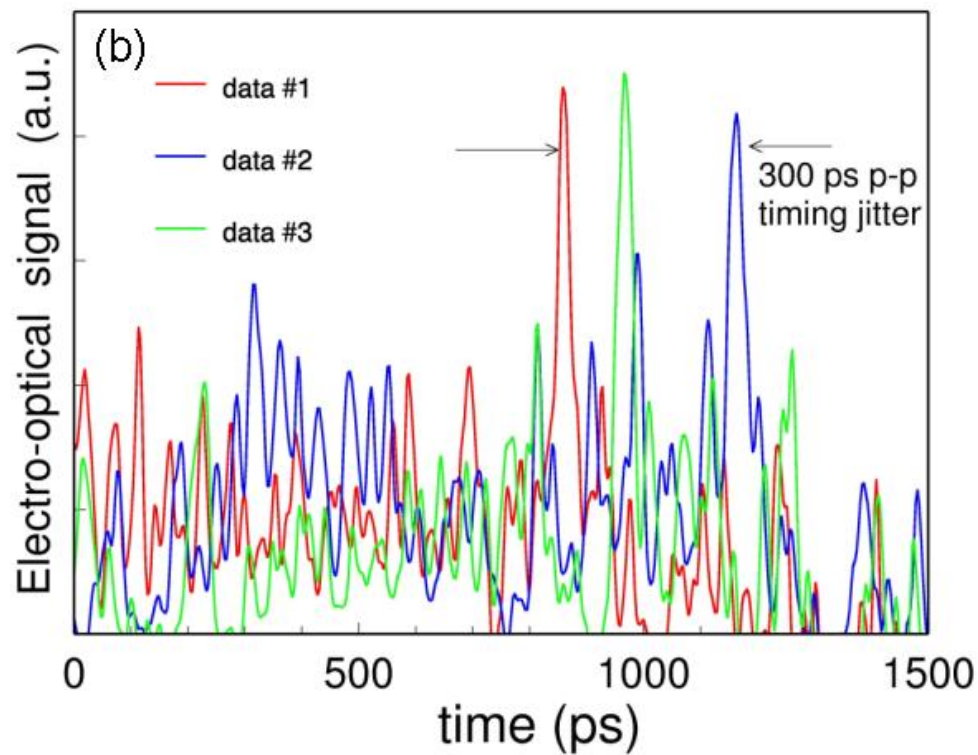


Fig. 1. The experimental setup for detecting a charged particle beam. The LiNbO<sub>3</sub> crystal (M) was located in vacuum several mm from the beam position which could be varied over several cm. The beam direction was perpendicular to the plane of the page. The positions of the polarization maintaining fibers, polarizer (P), lenses,  $\frac{1}{4}$  wave plate ( $\lambda/4$ ), analyzer (A), shield wall (S) and photodiode detector (PD) are schematically indicated.



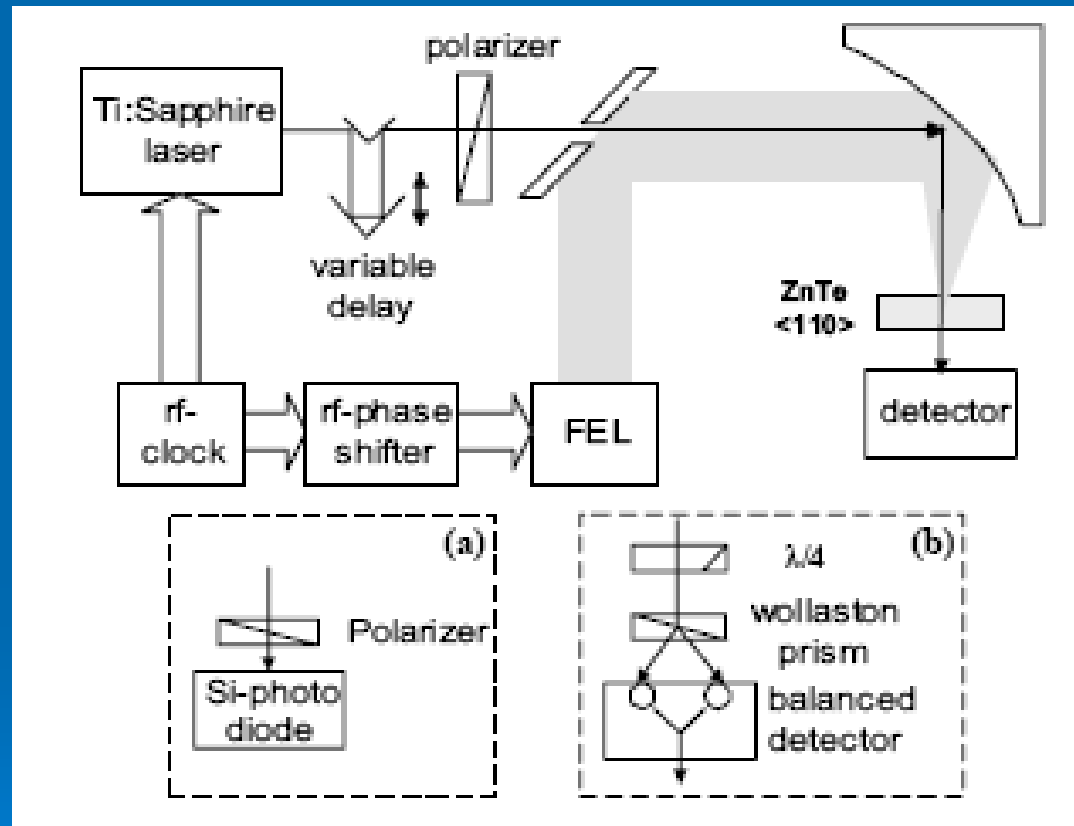


Using Streak Camera, 3 single shot traces superimposed



Using 7 GHz Oscilloscope:  
Resolution limited by the bandwidth  
of scope

# Measurement of pulse duration in multiple shots using ultrafast laser (12 fs) and photodiode



NIM A 475 (2001) 504-508

Triveni Rao, USPAS 2013, Duke University

# Converting temporal information to Spatial information: Information on pulse duration shape, distribution and timing

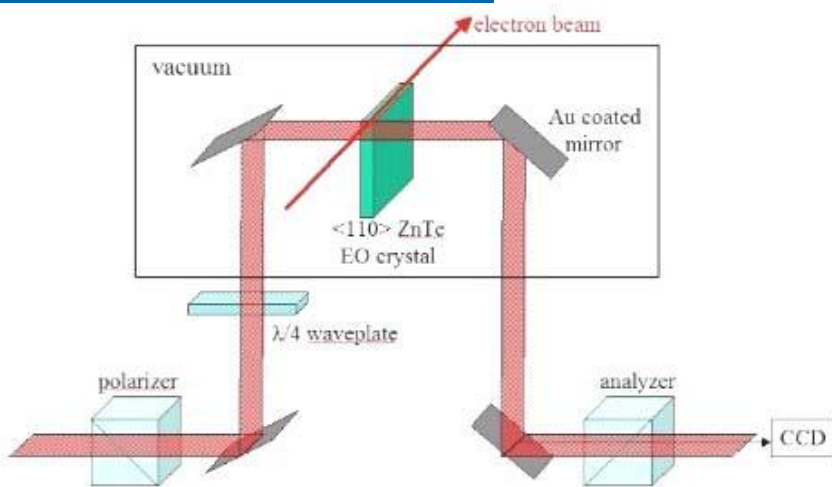
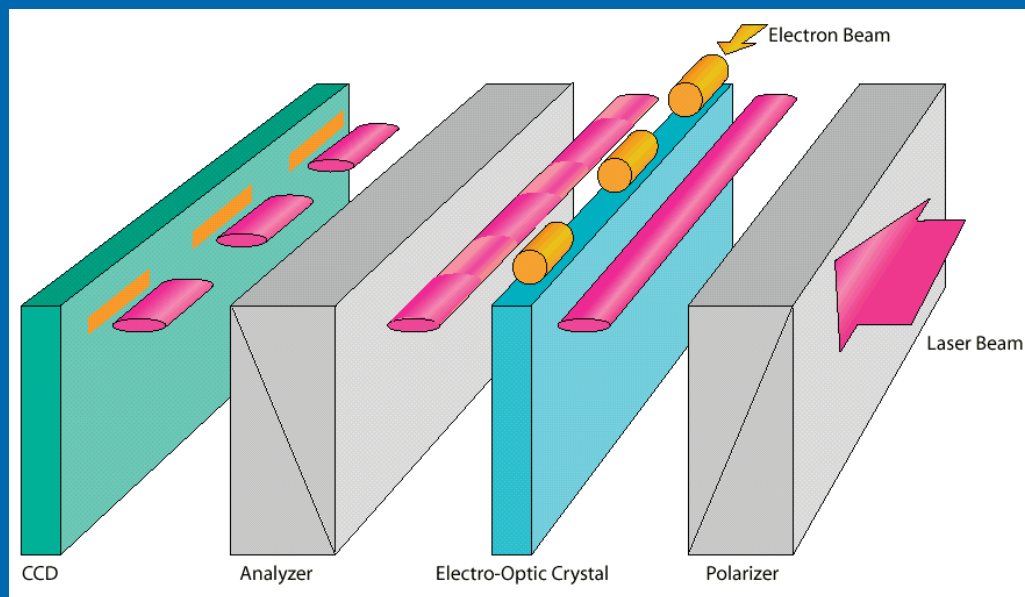


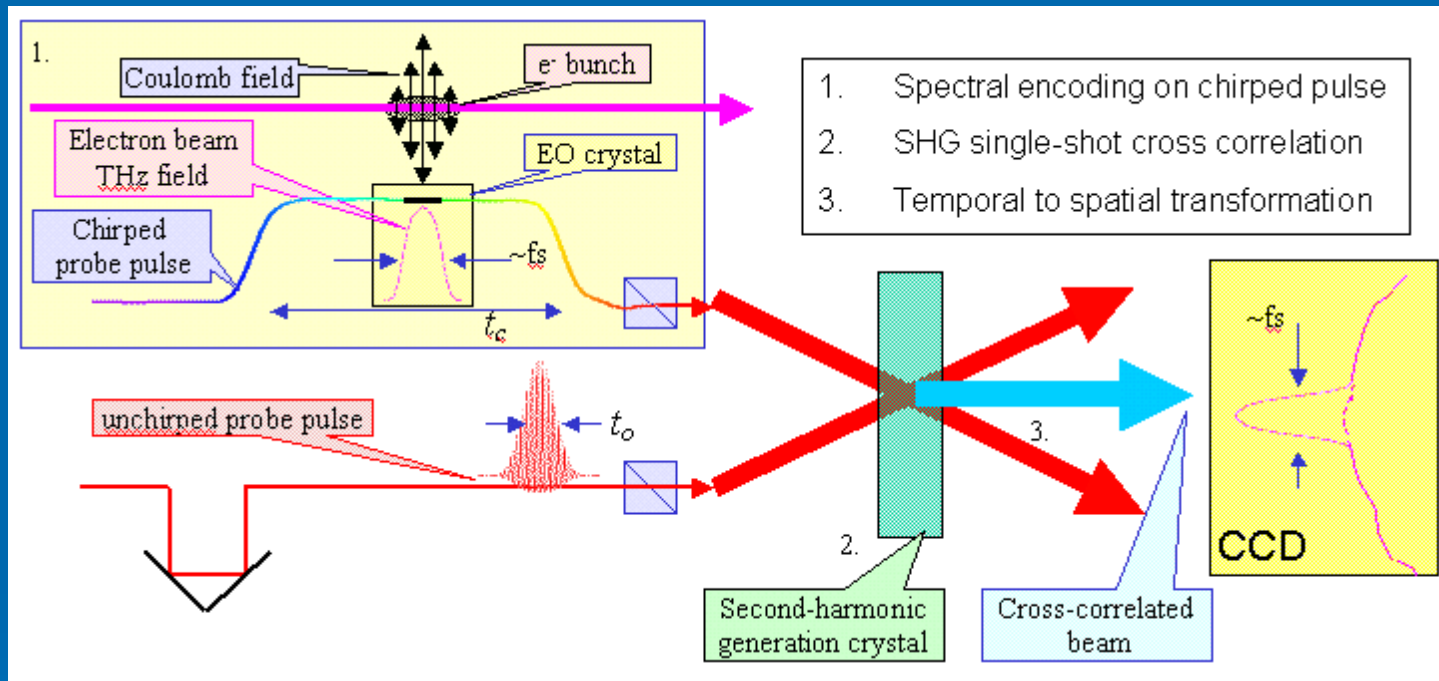
Fig. 1.3 Schematic of EO setup



Fig. 1.4 EO-flash detection module

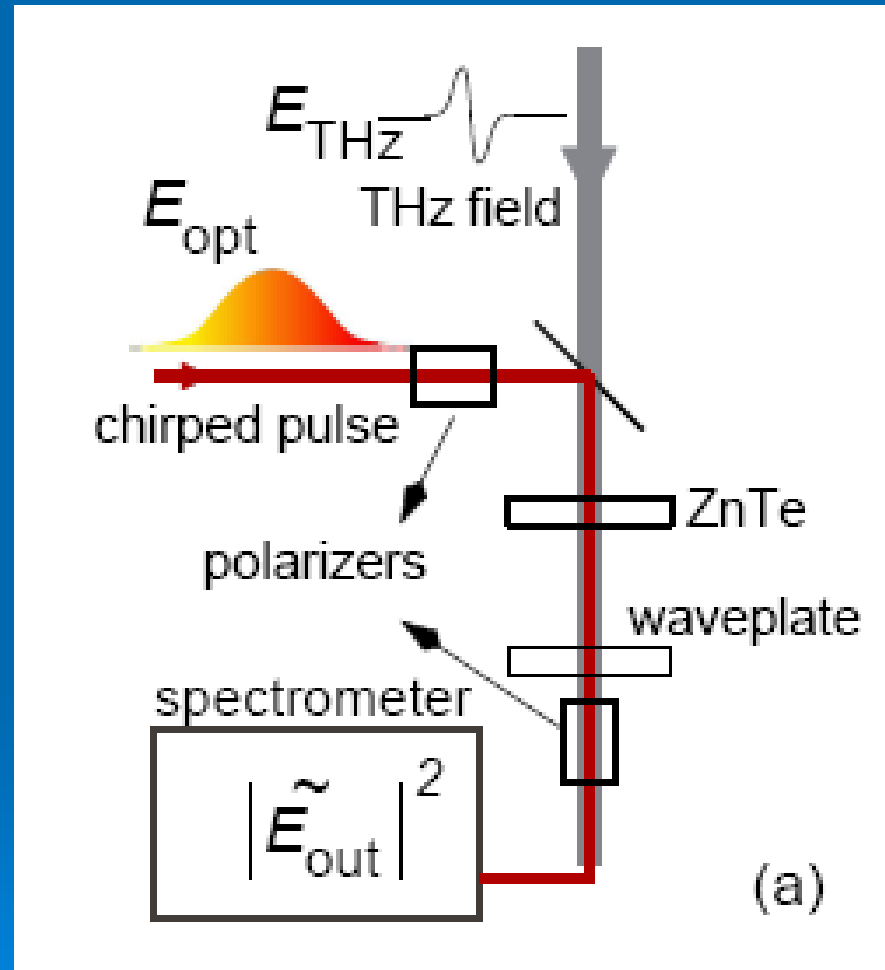


# Cross correlation between unchirped and chirped pulse-single shot, information on pulse duration, shape and timing



Replace CCD by spectrometer for spectral encoding of the e beam on chirped pulse

# Spectral encoding of electron beam on chirped pulse Single shot



# Limitations

- Crystal absorption and dispersion beyond 10 THz (100 fs) regime
- Short distance between e beam and crystal
- Edge effects in low relativistic regime