Beam Dynamics and Beam Losses -

Circular Machines

by
Verena Kain
CERN

Contents of this lecture

- Principles of transverse and longitudinal dynamics

- Beam loss mechanisms in circular machines

Why talking about beam loss?

Many origins of losses: Collisions, Beam Gas, Intra-Beam Scattering, Touschek, RF noise, collective effects, transition crossing, equipment failure,…

Beam losses cause:

- Impact on performance:
  - Luminosity, brightness,…
- Radio-activation leading to limitation of machine availability and maintainability
  - Hands-on maintainability requires activation of less than 1 mSv/h
- Down time: quenches in superconducting machines, damage to components

Beam Losses

Particles are lost on the vacuum chamber if their transverse trajectory amplitudes are larger than the dimension of the vacuum chamber.

Many mechanisms to create large amplitudes.

Important characteristic: how much beam loss in what time $\frac{\Delta N}{\Delta t}$

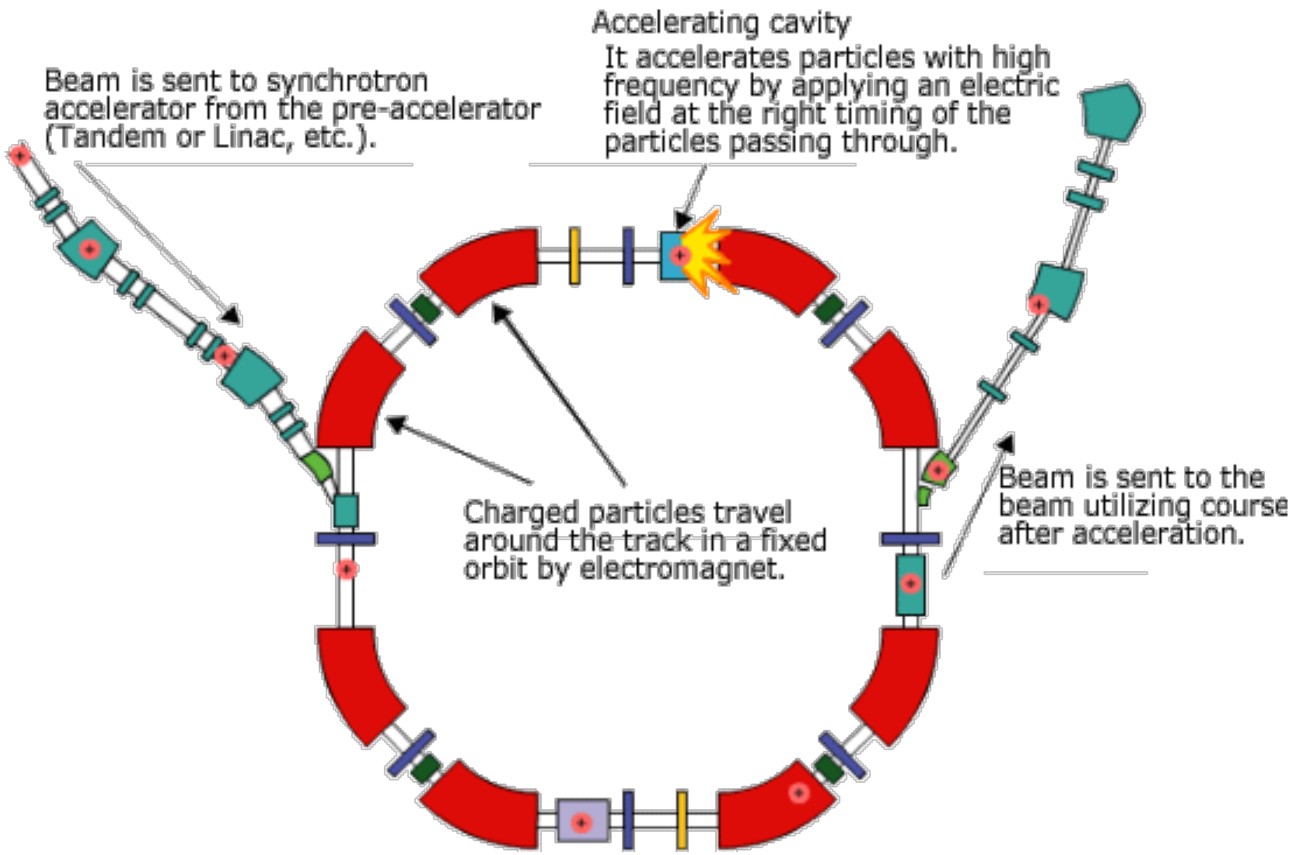
Beam Lifetime (τ): $N(t) = N_0 \cdot e^{-\frac{t}{\tau}}$

Requirements for design of machine protection reaction times, collimators and absorbers, instrumentation, etc:

- Minimum possible beam lifetimes? i.e. Fastest possible beam loss mechanisms for how much beam?
- Tolerable beam loss rate for accelerator components.

PRINCIPLES OF TRANSVERSE AND LONGITUDINAL BEAM DYNAMICS – SYNCHROTRONS

Synchrotrons – Basic Components



TRANSVERSE PLANE

Dipole magnets: guiding magnets

Usually use only magnetic fields for transverse control

$$\boxed{\vec{F} = q \cdot (\vec{E} + \vec{v} \times \vec{B})}$$ Lorentz Force

Circular accelerator: Lorentz Force = Centrifugal Force

$$F_L = qvB$$
$$F_{centr} = \frac{mv^2}{\rho}$$
$$\longrightarrow \frac{mv^2}{\rho} = qvB$$

$$\boxed{\frac{p}{q} = B\rho}$$

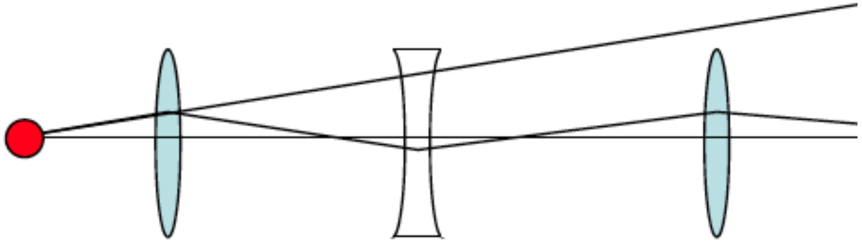
$B\rho$ Beam rigidity

Useful formula: $$\frac{1}{\rho[m]} \approx 0.3 \frac{B[T]}{p[GeV/c]}$$

Focusing is mandatory for stability

Define design trajectory with dipole magnets

Trajectories of particles in beam will deviate from design trajectory



➔ Focusing
- Particles should feel restoring force when deviating from design trajectory horizontally or vertically

Focusing with Quadrupole Magnets

Requirement: Lorentz force increases as a function of distance from design trajectory

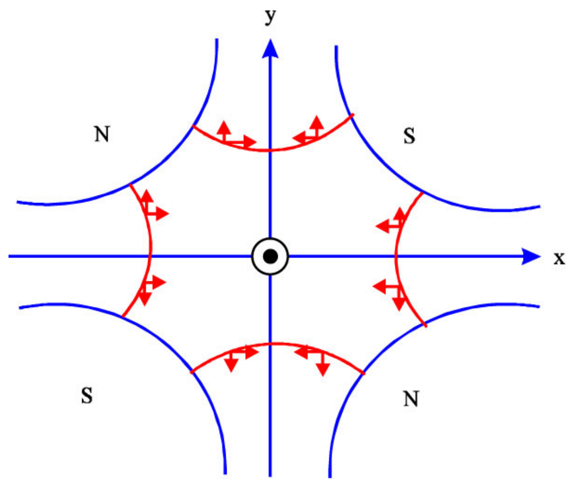
E.g. in the horizontal plane

$$F(x) = q \cdot v \cdot B(x)$$

We want a magnetic field that

$$B_y = g \cdot x \qquad B_x = g \cdot y$$

➔ Quadrupole magnet



The red arrows show the direction of the force on the particle

Gradient of quadrupole

$$g = \frac{2\mu_0 n I}{r^2} [\frac{T}{m}]$$

Normalized gradient, focusing strength

$$k = \frac{g}{p/e} [m^{-2}]$$

Towards the Equation of Motion

To calculate trajectories through dipoles and quadrupoles...

Taylor series expansion of B field:

$$B_y(x) = B_{y0} + \frac{\partial B_y}{\partial x} x + \frac{1}{2} \frac{\partial^2 B_y}{\partial x^2} x^2 + \frac{1}{3!} \frac{\partial^3 B_y}{\partial x^3} x^3 + \ldots$$

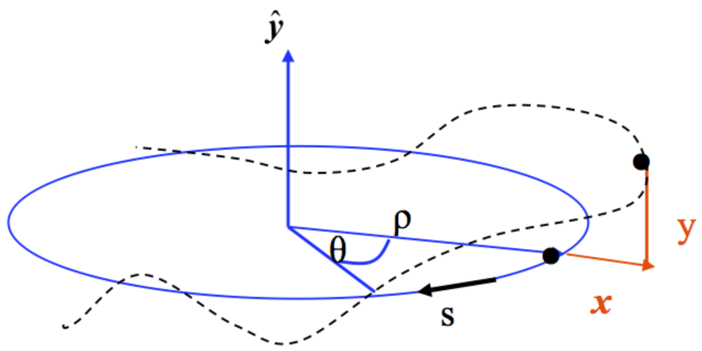
Normalize and keep only terms linear in x

$$\frac{B_y(x)}{p/e} = \frac{1}{\rho} + k\ x + \frac{1}{2} m x^2 + \frac{1}{3!} n x^3 + \ldots$$

$$\frac{B_y(x)}{p/e} \approx \frac{1}{\rho} + k\ x$$

Towards Equation of Motion

Use different coordinate system: Frenet-Serret rotating frame



The ideal particle stays on "design" trajectory. (x=0, y=0)
And: x,y << ρ

The design particle has momentum $p_0 = m_0 \gamma v$.

$\delta = \frac{p - p_0}{p_0} = \frac{\Delta p}{p}$ .... relative momentum offset of a particle

The Equation of Motion

All we have to do now is to write

$$F_r = m\ a_r = eB_y v$$

in the Frenet-Serret frame,

develop with x,y << ρ , and keeping only terms linear in x or y for magnetic field

after a bit of maths: <u>the equations of motion</u>

$$x'' + x(\frac{1}{\rho^2} - k) = 0$$

$$y'' + ky = 0$$

Assuming there are no vertical bends,
Quadrupole field changes sign between x and y

The Hill's Equation

We had...

$$x'' + (\frac{1}{\rho^2} - k)x = x'' + Kx = 0$$

Around the accelerator K will not be constant, but will depend on s

$$x''(s) + K(s)x(s) = 0$$ **Hill's equation**

Where

- restoring force ≠ const, K(s) depends on the position s
- K(s+L) = K(s) periodic function, where L is the "lattice period"

General solution of Hill's equation:

$$x(s) = \sqrt{\epsilon}\sqrt{\beta(s)}\cos(\psi(s) + \phi)$$

The Beta Function & Co

Solution of Hill's Equation is a quasi harmonic oscillation **(betatron oscillation)**: amplitude and phase depend on the position s in the ring.

$$x(s) = \sqrt{\epsilon}\sqrt{\beta(s)}\cos(\psi(s) + \phi)$$

integration constants: determined by initial conditions

The beta function is a periodic function determined by the focusing properties of the lattice: i.e. quadrupoles

$$\beta(s+L) = \beta(s)$$

The "phase advance" of the oscillation between the point 0 and point s in the lattice.

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

The transport matrix

Definition: $\alpha(s) = -\frac{1}{2}\beta'(s) \qquad \gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$

$$x(s) = \sqrt{\epsilon}\sqrt{\beta(s)}\cos(\psi(s)+\phi)$$

$$x'(s) = -\frac{\sqrt{\epsilon}}{\sqrt{\beta(s)}}\alpha(s)\cos(\psi(s)+\phi) + \sin(\psi(s)+\phi)$$

Let's assume for s(0) = s_0, $\psi(0) = 0$.

Defines ϕ from x_0 and x'_0, β_0 and α_0.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

Can compute the single particle trajectories between two locations if we know α, β at these positions!

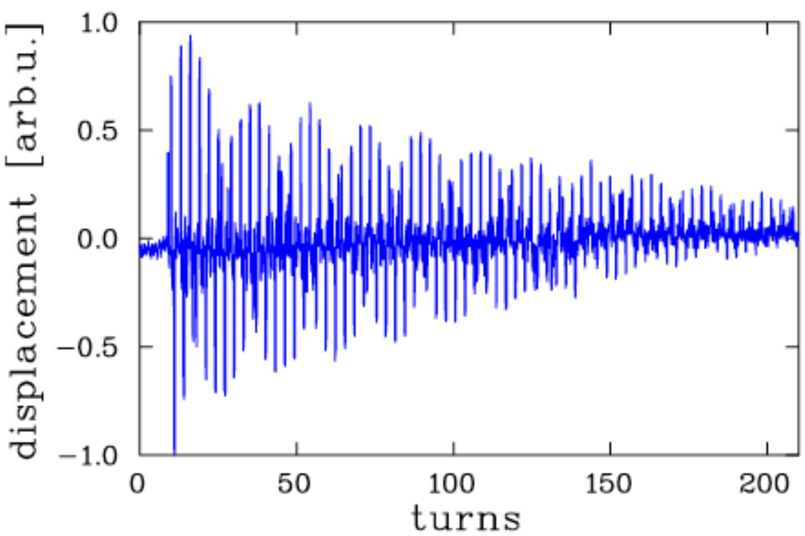
$$M = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos\psi + \alpha_0 \sin\psi) & \sqrt{\beta\beta_0}\sin\psi \\ \frac{(\alpha_0-\alpha)\cos\psi-(1+\alpha\alpha_0)\sin\psi}{\sqrt{\beta\beta_0}} & \sqrt{\frac{\beta_0}{\beta}}(\cos\psi - \alpha\sin\psi)) \end{pmatrix}$$

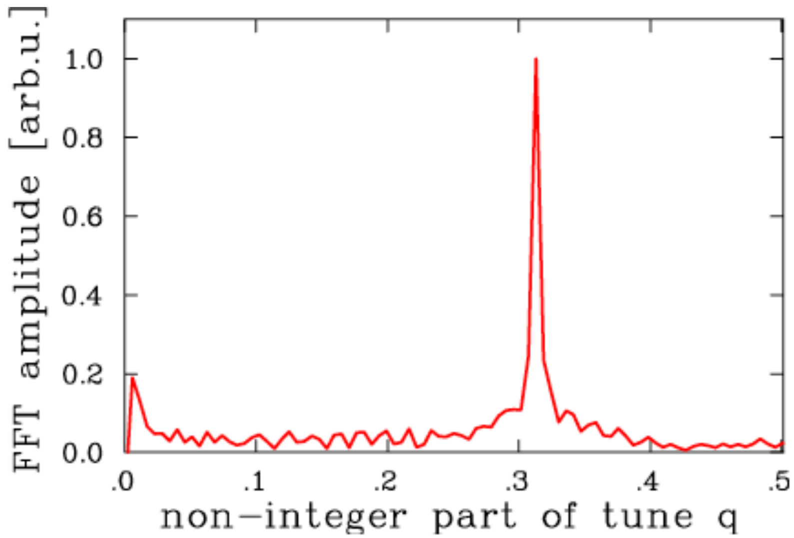
The Tune

The number of oscillations per turn is called "tune"

$$Q = \frac{\psi(L_{turn})}{2\pi} = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)}$$

With FFT of turn-by-turn data on beam position monitor get frequency of oscillation: tune





Phase-space ellipse

With:

$$x(s) = \sqrt{\epsilon}\sqrt{\beta(s)}\cos(\psi(s)+\phi)$$
$$x'(s) = -\frac{\sqrt{\epsilon}}{\sqrt{\beta(s)}}\alpha(s)\cos(\psi(s)+\phi)+\sin(\psi(s)+\phi)$$

One can solve for ϵ

$$\epsilon = \gamma(s)x(s)^2 + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$

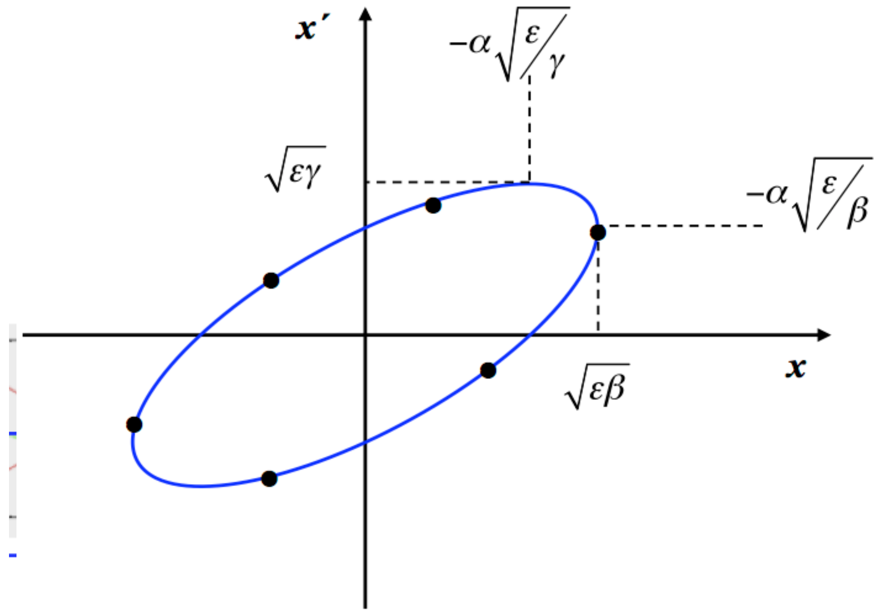
- is a constant of motion: Courant-Snyder invariant
- is parametric representation of an ellipse in the xx'space: phase-space
- Shape and orientation of ellipse are given by α, β and γ: the Twiss parameters

Phase-space ellipse

The area of the ellipse is constant (Liouville):

$$A = \pi \cdot \epsilon$$

The area of the ellipse is an intrinsic property of the beam and cannot be changed by the focusing properties.

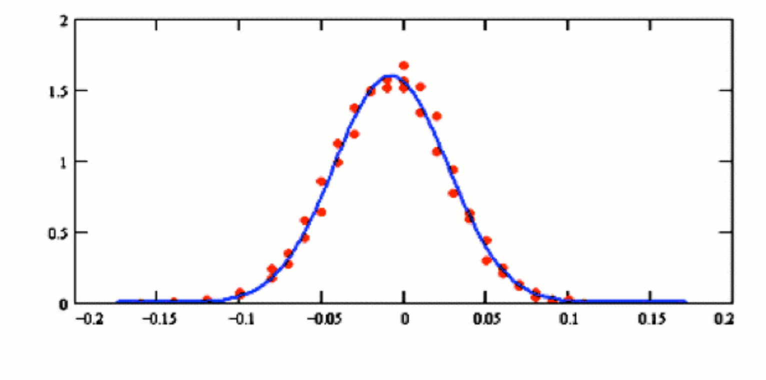


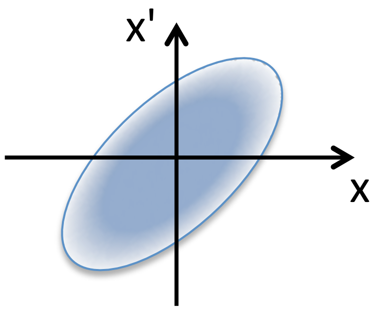
Emittance of an ensemble of particles

Typically particles in accelerator have Gaussian particle distribution in position and angle.

$$\rho(x) = \frac{N}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{x^2}{2\sigma_x^2}}$$

Transverse profile measurement and Gauss fit





Define beam emittance ε as ellipse with area in phase-space that contains 68.3 % of all particles. Such that

$$\sigma_x = \sqrt{\varepsilon \beta_x}$$

Liouville during Acceleration

Liouville's Theorem from Hamiltonian Mechanics:

canonical variables q, p

e.g. q = position = x

p = momentum = γmv = mc$\gamma\beta_x$

The theorem: $\int p\, dq = const$

We use x': $x' = \frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = \frac{\beta_x}{\beta}$ $\beta_x = v_x/c$

$$\int p\, dq = mc \int \gamma \beta_x dx = mc\gamma\beta \underbrace{\int x' dx}_{\varepsilon} = const$$

➔ $\varepsilon = \int x' dx \propto \frac{1}{\beta\gamma}$

The beam emittance shrinks during acceleration!!

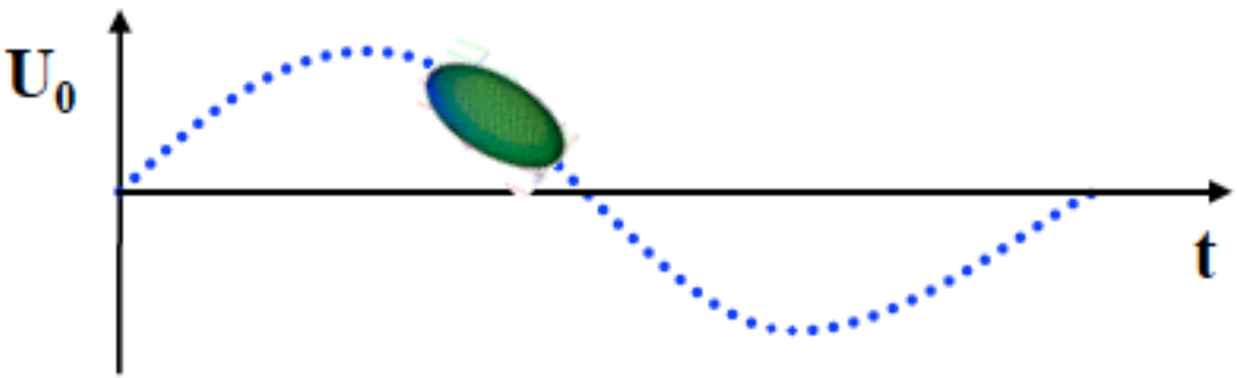
LONGITUDINAL PLANE

Acceleration

Using RF acceleration: multiple application of the same accelerating voltage.

...but accelerating voltage is changing with time while particles are going through the RF system.

➔ Longitudinal dynamics



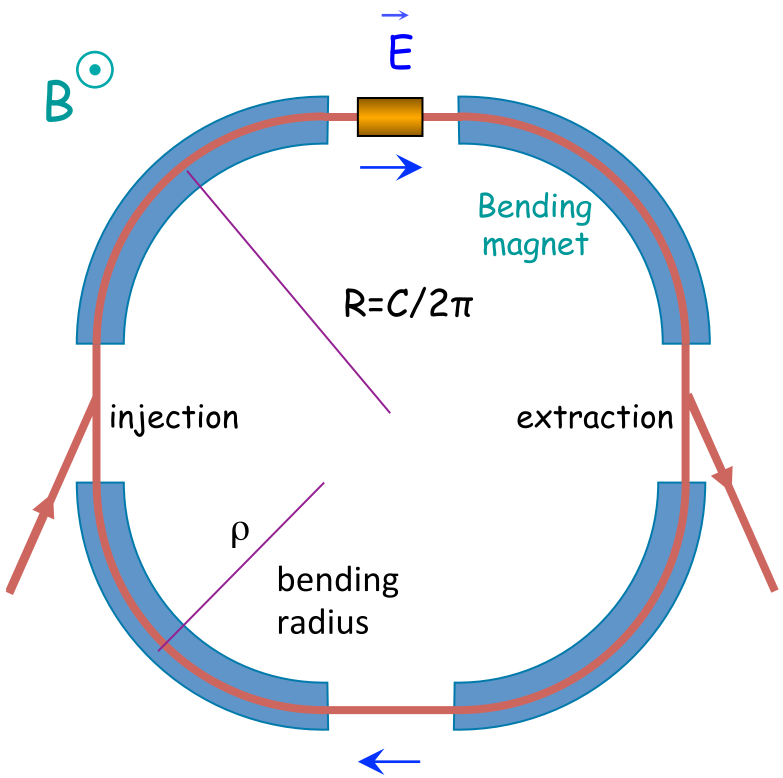
Not all particles arrive at the same time.

Not all particles will receive the same energy gain.

Not all particles will have the same energy.

Acceleration in a Synchrotron

Synchrotron: there is a synchronous RF phase of the RF field for which the energy gain fits the increase of the magnetic field



Energy gain per turn $eV \sin\phi = eV \sin\omega_{RF} t$

Reference particle, synchronous particle $\phi = \phi_s = const$

RF synchronism: the RF frequency must be locked to the revolution frequency $\omega_{RF} = h\omega_{rev}$

h...harmonic number

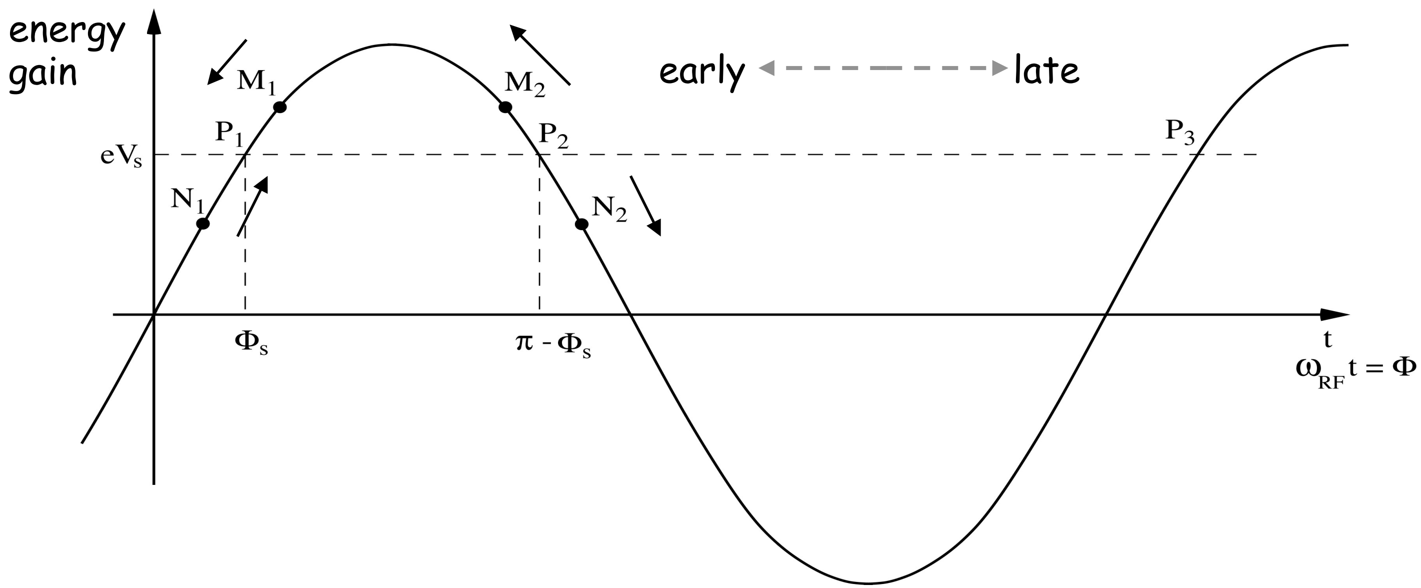
constant orbit, bending radius

variable magnetic field

Principle of Phase Stability

Assume the situation where energy increase is transferred into a velocity increase

Particles P_1, P_2 have the synchronous phase.



M_1 & N_1 will move towards P_1 => stable
M_2 & N_2 will go away from P_2 => unstable (and finally be lost)

RF voltage and phase during ramp

Energy gain per turn: $\Delta E = eV \sin \phi_s$

with $E^2 = E_0^2 + p^2c^2 \rightarrow \Delta E = v\Delta p$

and $v = \frac{2\pi R}{T_{turn}}$

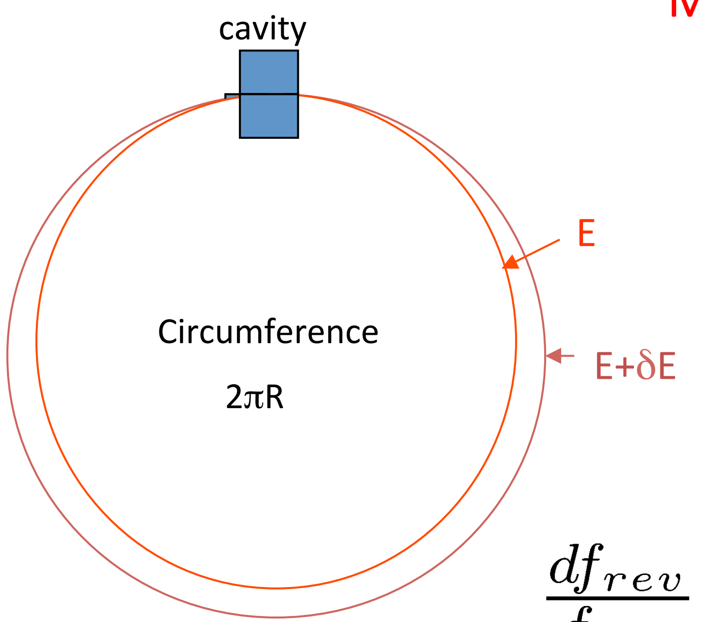
$$2\pi R \frac{dp}{dt} = q \cdot V \cdot \sin \phi_s$$

Stable phase and total supplied RF voltage change during acceleration.

The ramp in the LHC is slow. Takes > 15 minutes. Total energy gain per turn is only about 500 keV (ϕ close to 180°).

Some definitions

If a particle is slightly shifted in momentum, it will run on a different orbit with a different length.



Momentum compaction α: $\alpha = \frac{dL/L}{dp/p}$

The particle will also have a different velocity and hence a different revolution frequency: the slippage factor

$$\eta = \frac{df_{rev}/f_{rev}}{dp/p} = \frac{1}{\gamma^2} - \alpha$$

without prove

$$\frac{df_{rev}}{f_{rev}} = \left(\frac{1}{\gamma^2} - \alpha\right)\frac{dp}{p}$$

$$\gamma_{tr} = \frac{1}{\sqrt{\alpha}}$$

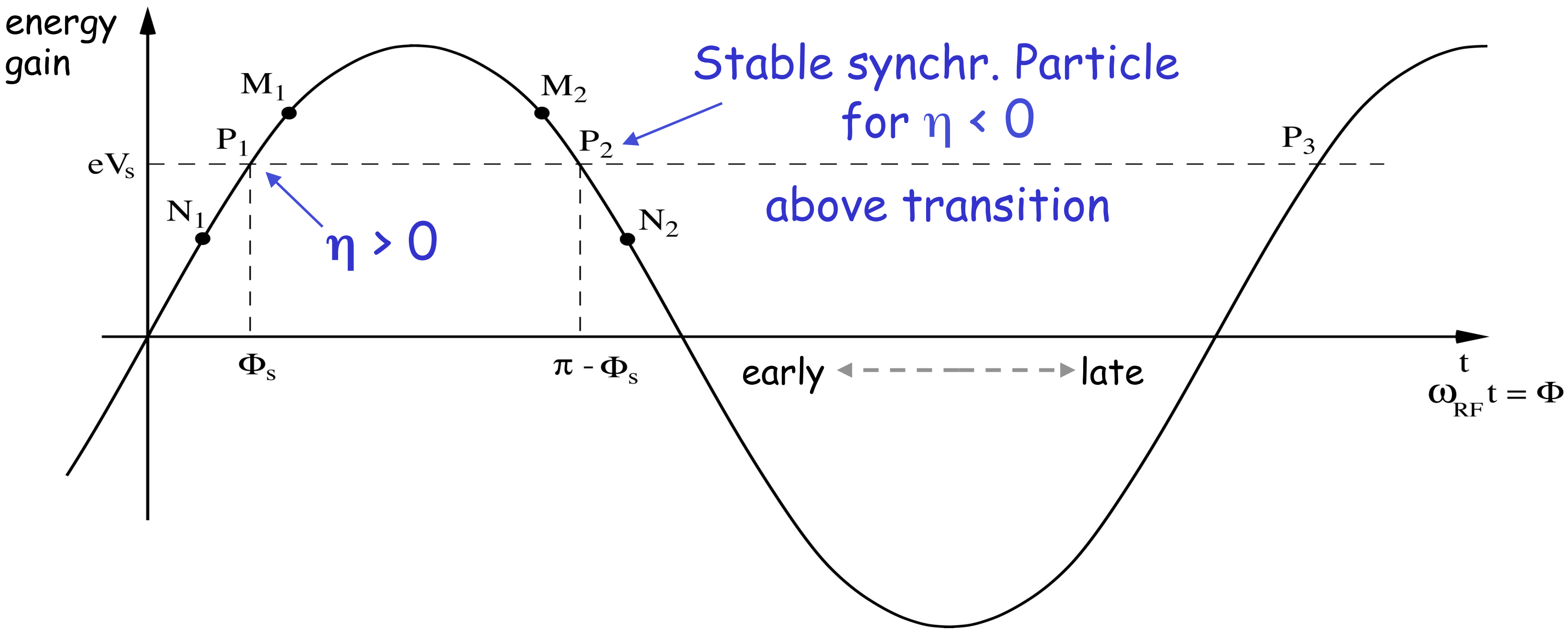
Momentum compaction defines transition energy.

Phase Stability in a Synchrotron

$$\frac{df_{rev}}{f_{rev}} = \eta \frac{dp}{p} = \left(\frac{1}{\gamma^2} - \alpha\right)\frac{dp}{p}$$

Below transition ($\eta > 0$): higher momentum, higher f_{rev}

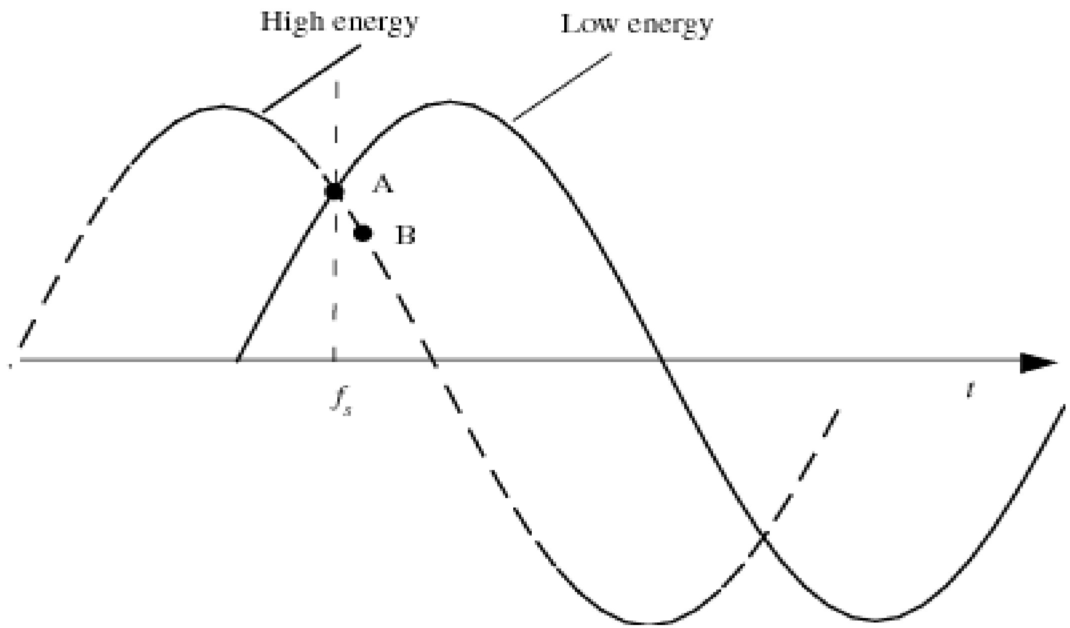
Above transition ($\eta < 0$): higher momentum, lower f_{rev}



Courtesy F. Tecker for drawings

Crossing Transition during Acceleration

Crossing transition during acceleration makes the previously stable synchronous phase unstable. The RF system needs to make a rapid change of the RF phase, a ‘phase jump’



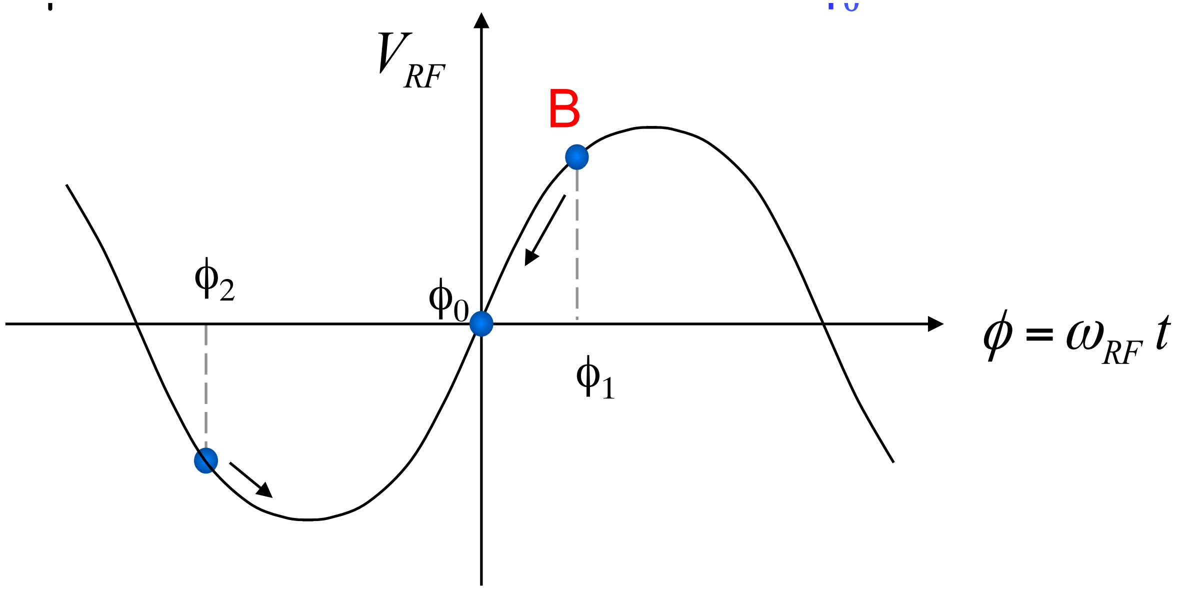
In the LHC transition energy γ_{tr} = 53 GeV. Injection energy is 450 GeV. The LHC is always above transition.

Synchrotron Oscillations

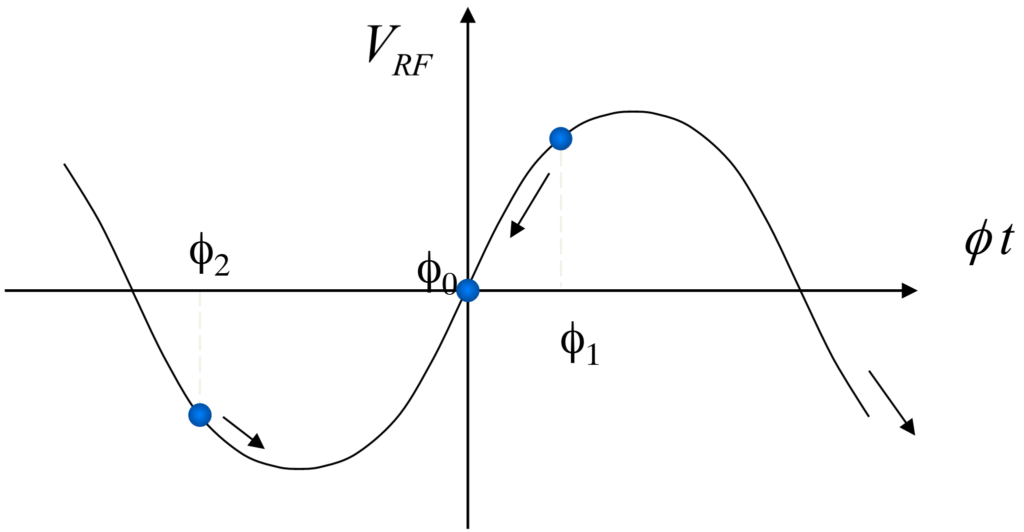
Like in the transverse plane the particles are performing an oscillation in longitudinal space.

Particles keep oscillating around the stable synchronous particle varying phase and dp/p.

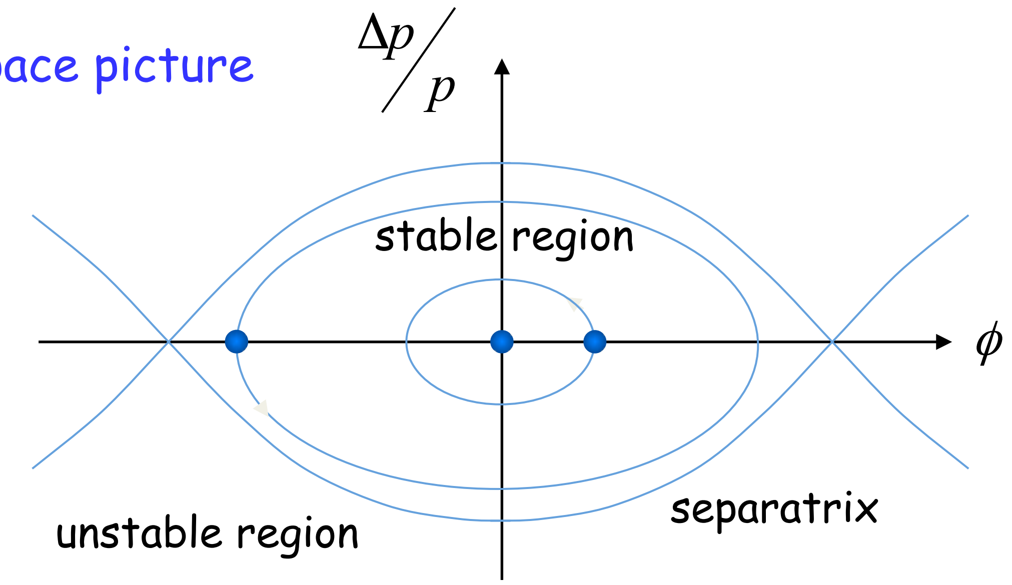
Assume: no acceleration, B = const, below transition $\eta > 0$
Stable phase = 0. B will oscillate around ϕ_0.



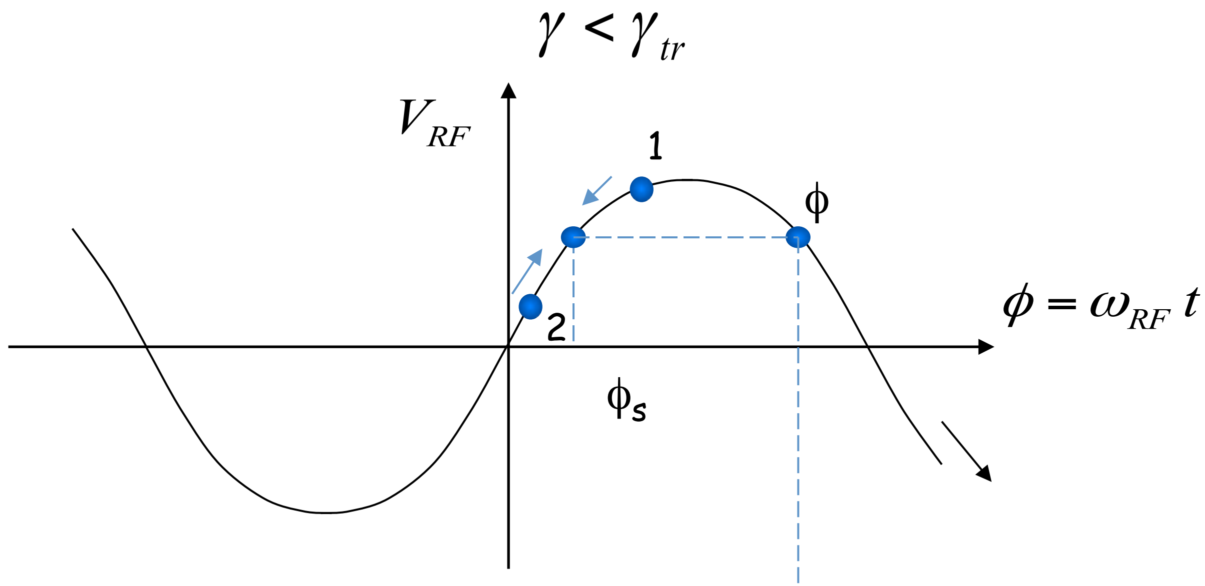
Synchrotron Oscillations – No acceleration



Phase space picture

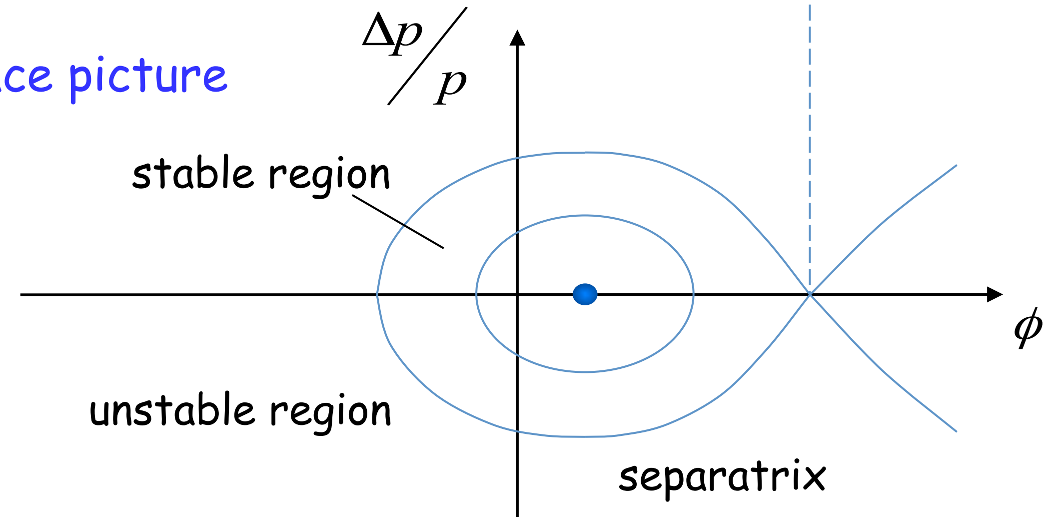


Synchrotron Oscillations – with Acceleration



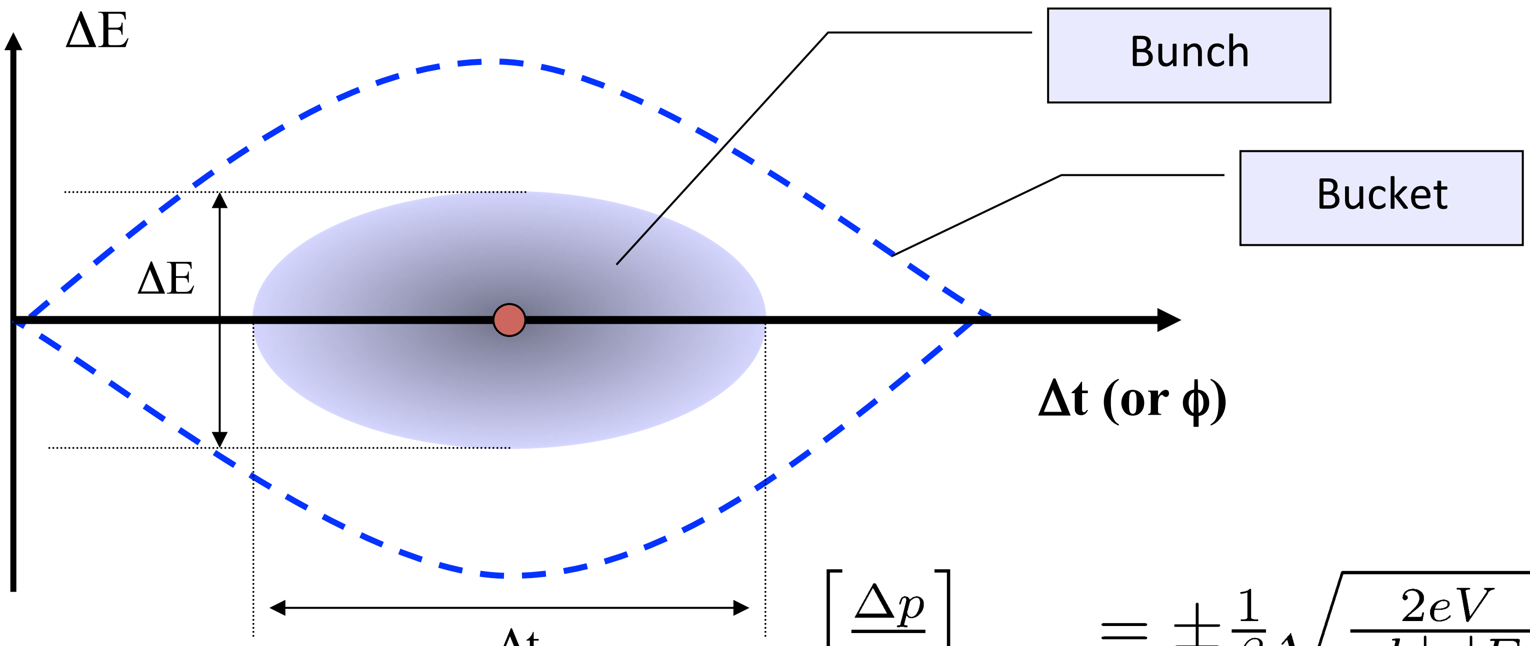
$$\phi_s < \phi < \pi - \phi_s$$

Phase space picture



Bucket & Bunch

The bunches of the beam fill usually a part of the bucket area.



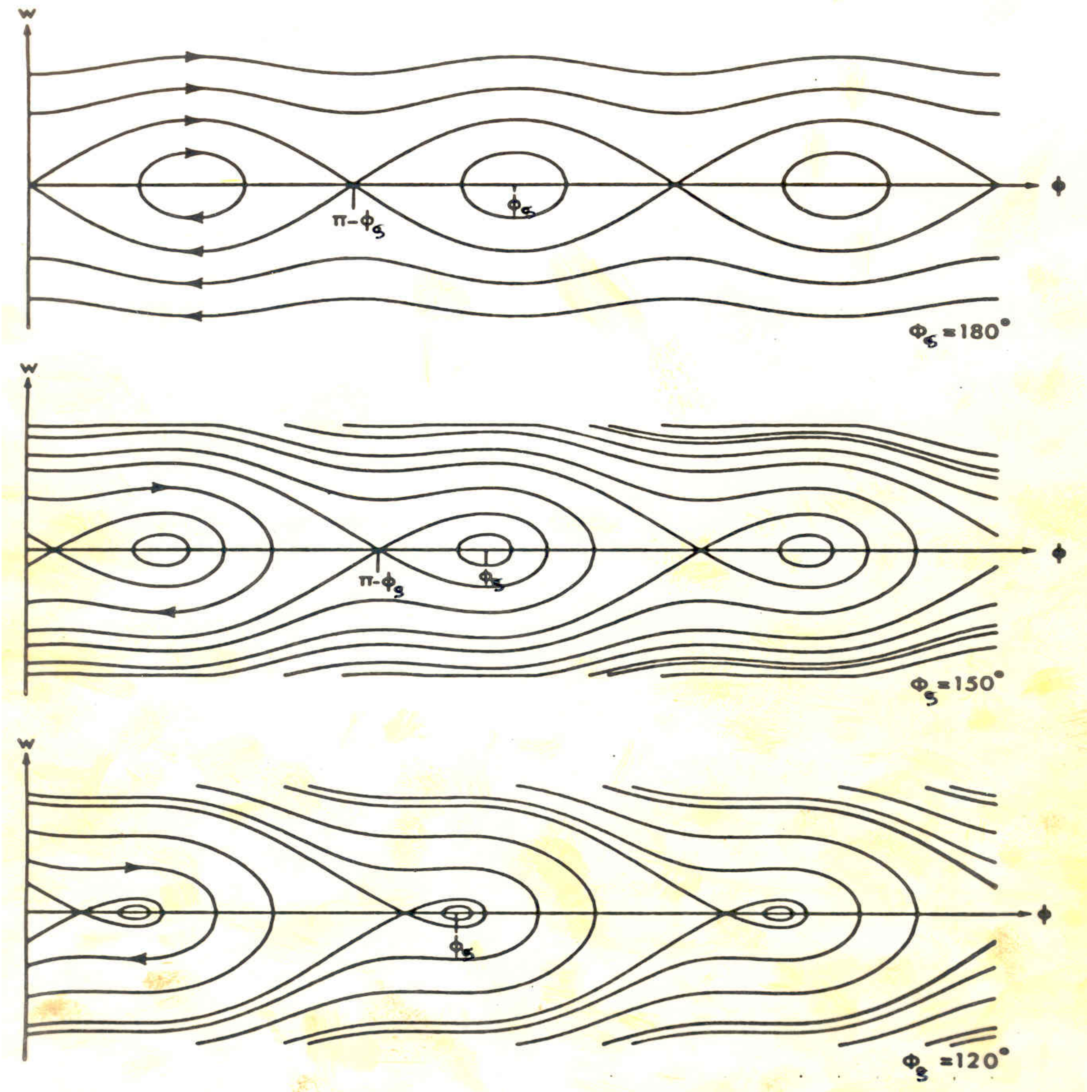
$$\left[\frac{\Delta p}{p}\right]_{max} = \pm\frac{1}{\beta}\sqrt{\frac{2eV}{\pi h|\eta|E_s}}$$

Bucket area = **longitudinal Acceptance** [eVs]

Bunch area = **longitudinal beam emittance** = **π.ΔE.Δt/4 [eVs]**

The ratio between these two is called filling factor.

RF Acceptance versus Synchronous Phase



RF acceptance plays an important role for capture at injection and the stored beam lifetime

The areas of stable motion (closed trajectories) are called "BUCKET".

As the synchronous phase gets closer to 90° the buckets gets smaller.

The number of circulating buckets is equal to "h".

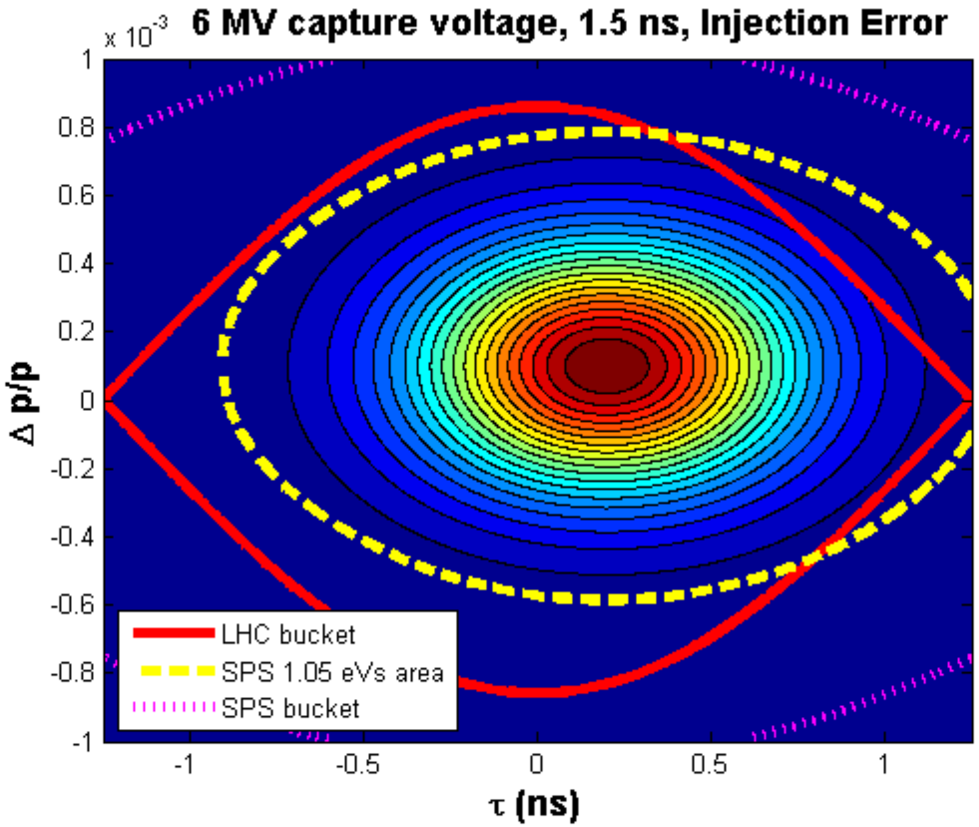
The phase extension of the bucket is maximum for ϕ_s =180° (or 0°) which correspond to no acceleration . The RF acceptance increases with the RF voltage.

Losses from capture at injection

Example SPS to LHC transfer:

Capture with 6 MV, in presence of 200 ps and 10^{-4} $\Delta p/p$ injection errors

Losses:
2.47 % if the bunch distribution is Gaussian with infinite tails
0.6 % if the distribution is a Gaussian truncated by the 1.05 eVs contour



Synchrotron motion

Synchronous particle: (p_s,ϕ_s)

Another particle P: (p,ϕ)

$$\Delta\phi = \phi - \phi_s$$

P will also have a different revolution frequency

$$\frac{d\Delta\phi}{dt} = -2\pi h \Delta f_{rev} \qquad \frac{d^2\Delta\phi}{dt^2} = -2\pi h \frac{d\Delta f_{rev}}{dt}$$

When crossing the cavity, the momentum increase will be different for the two particles:

$$2\pi R \frac{dp_s}{dt} = q \cdot V \cdot \sin\phi_s$$

$$2\pi R \frac{dp}{dt} = q \cdot V \cdot \sin\phi$$

$$2\pi R \frac{d\Delta p}{dt} = q \cdot V \cdot \sin\phi - q \cdot V \cdot \sin\phi_s$$

Synchrotron motion

Remember:

$$\eta = \frac{df_{rev}/f_{rev}}{dp/p} = \frac{\Delta f_{rev}/f_{rev}}{\Delta p/p_s}$$

Use η in:

$$\frac{d^2\Delta\phi}{dt^2} = -2\pi h \frac{d\Delta f_{rev}}{dt} = -\frac{2\pi\eta h f_{rev}}{p_s}\frac{d\Delta p}{dt}$$

With:

$$2\pi R \frac{d\Delta p}{dt} = q \cdot V \cdot \sin\phi - q \cdot V \cdot \sin\phi_s$$

We get a second-order non-linear differential equation describing the synchrotron motion:

$$\frac{d^2\Delta\phi}{dt^2} + \frac{\eta \cdot f_{RF}}{R \cdot p_s} q \cdot V (\sin\phi - \sin\phi_s) = 0$$

Synchrotron motion

For small amplitude oscillations, where we have small phase deviations from the synchronous particle:

And: $\sin\phi - \sin\phi_s = \sin(\phi_s + \Delta\phi) - \sin\phi_s \cong \cos\phi_s \Delta\phi$

We can linearize the equation from above

$$\frac{d^2\Delta\phi}{dt^2} + \left[\frac{\eta f_{RF}\cos\phi_s}{Rp_s} qV\right]\Delta\phi = 0$$

An **undamped resonanotor** with **resonant frequency Ω_s** called the synchrotron frequency.

$$\frac{d^2\Delta\phi}{dt^2} + \Omega_s^2 \Delta\phi = 0$$

$$\Omega_s = \sqrt{\frac{\eta f_{RF}\cos\phi_s}{Rp_s} qV}$$

Periodic motion is stable if $\eta.\cos(\phi_s) > 0$:

$\gamma \leq \gamma_{tr} \Rightarrow \eta \geq 0 \Rightarrow \cos\phi_s \geq 0 \Rightarrow \phi_s \in [0, \pi/2]$ Acceleration below transition

$\gamma \geq \gamma_{tr} \Rightarrow \eta \leq 0 \Rightarrow \cos\phi_s \leq 0 \Rightarrow \phi_s \in [\pi/2, \pi]$ Acceleration above transition

Back to transverse motion

We have heard that not all particles have exactly the same momentum.

In fact a bunch of particles contains a distribution of dp/p.

We have neglected this so far.

The typical momentum spread is in the order of $dp/p < 1.0 \times 10^{-3}$.

What does this do to the transverse motion of the particles?

Without going through the derivation: ➔ inhomogeneous differential equation

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = 0$$
$$y'' + ky = 0$$

➡

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = \frac{\Delta p}{p}\frac{1}{\rho}$$
$$y'' + ky = 0$$

Dispersion

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = \frac{\Delta p}{p}\frac{1}{\rho}$$

General solution:

$$x(s) = x_h(s) + x_i(s)$$

$$x_h''(s) + K(s) \cdot x_h(s) = 0 \qquad x_i''(s) + K(s) \cdot x_i(s) = \frac{1}{\rho} \cdot \frac{\Delta p}{p}$$

Define dispersion as:

$$D(s) = \frac{x_i(s)}{\Delta p/p}$$

Dispersion is the trajectory an ideal particle would have with $\Delta p/p = 1$.

The trajectory of any particle is the sum of X_β (s) plus dispersion × momentum offset.

D(s) is just another trajectory and will therefore be subject to the focusing properties of the lattice.

Dispersion

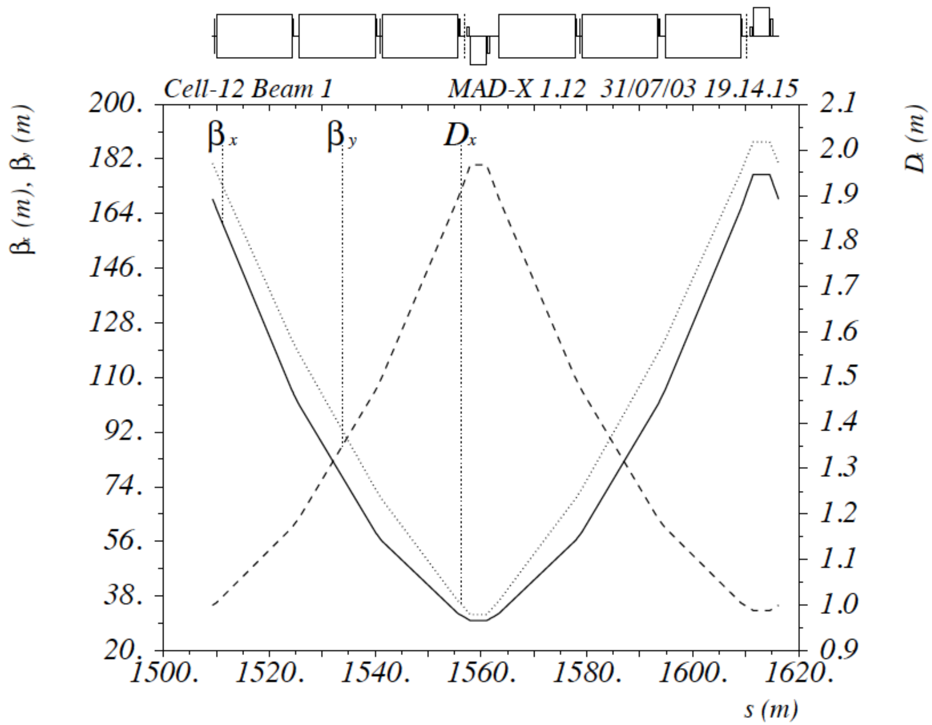
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0} \quad \Rightarrow \quad \begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0} + \frac{\Delta p}{p} \begin{pmatrix} D \\ D' \end{pmatrix}_{s_1}$$

Also has effect on beam size: momentum spread of beam

$$\sigma = \sqrt{\beta\varepsilon} \quad \Rightarrow \quad \sigma = \sqrt{\beta\varepsilon + D^2 (\frac{\Delta p}{p})^2}$$

Dispersion

Dispersion is created by dipole magnets and then focused by quadrupole magnets…



ξ: The contribution to the dispersion from dipoles between s_0 and s_1.

How does dispersion transform:

$$D(s_1) = m_{11_{s_0 \to s_1}} D(s_0) + m_{12_{s_0 \to s_1}} D'(s_0) + \xi$$

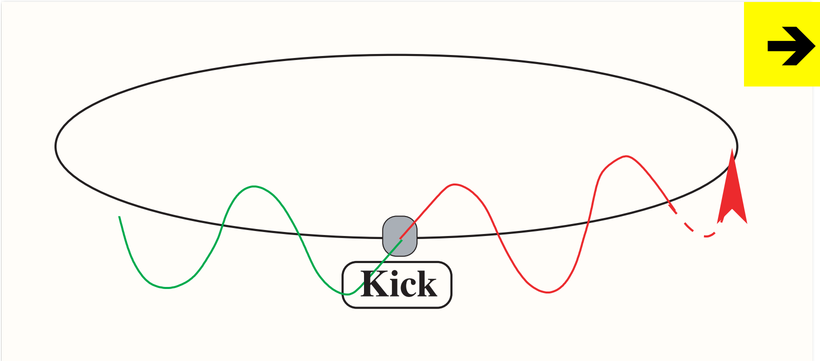
With $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$

BEAM LOSS MECHANISMS

The importance of the correct tune

The choice of phase advance per cell or tune and hence the focusing properties of the lattice have important implications.

Misalignment of quadrupoles or dipole field errors create orbit perturbations



➔ diverges for Q = N, where N is integer.

$$\begin{pmatrix} x \\ x' - \Delta x' \end{pmatrix} = M_{turn} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}$$

$$x(s) = \frac{\Delta x'}{2} \cdot \sqrt{\beta(s_0)\beta(s)} \frac{\cos(\pi Q - \psi_{s_0 \to s})}{\sin(\pi Q)}$$

Note:

1) The orbit amplitude will be larger at locations in the ring with larger β
2) The orbit amplitude will be larger if β_0 at the location of the kick is large

Gradient Error

The new one-turn matrix:

$$M_{turn_{dist}} = \begin{pmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -\gamma \sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 \\ -\Delta kl & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos 2\pi Q_0 + \alpha \sin 2\pi Q_0 & \beta \sin 2\pi Q_0 \\ -\gamma \sin 2\pi Q_0 & \cos 2\pi Q_0 - \alpha \sin 2\pi Q_0 \end{pmatrix}$$

With $Q = Q_0 + \Delta Q$, ΔQ small and $\mathrm{Trace}(M_{dist}) = \mathrm{Trace}(M_{error}.M_{turn})$:

$$\Delta Q = \frac{1}{4\pi} \beta \Delta k \cdot l$$

β at the error location

The quadrupole error leads to a tune change. The higher the β, the higher the effect.

And also a change of the beta functions.

Gradient Error

A gradient error also leads to changes of the beta functions: beta-beat

The relative beta function change:

$$\frac{\Delta\beta(s)}{\beta(s)} = -\frac{1}{2\sin 2\pi Q}\beta(s_0)\cos[2(\psi(s_0)-\psi(s))-2\pi Q]\cdot\Delta k\cdot l$$

➔ diverges for Q = N, N/2; where N is integer.

Non-linear imperfections

Non-linear equation of motion:

$$\frac{d^2x}{ds^2} + K(s) \cdot x = \frac{F_x}{v \cdot p}$$

The Lorentz force from the non-linear magnetic field

The magnetic field of multipole order n:

$$B_y(x,y) + i \cdot B_x(x,y) = (B_n(s) + iA_n(s)) \cdot (x + iy)^n$$

The normal and skew coefficients:

$$B_n(s) = \frac{1}{(n)!} \frac{\partial^n B_y}{\partial x^n} \qquad A_n(s) = \frac{1}{(n)!} \frac{\partial^n B_x}{\partial x^n}$$

Linear and Non-linear Imperfections - Resonances

Amplitudes grow for Q = N or N/2 in case of quadrupole error

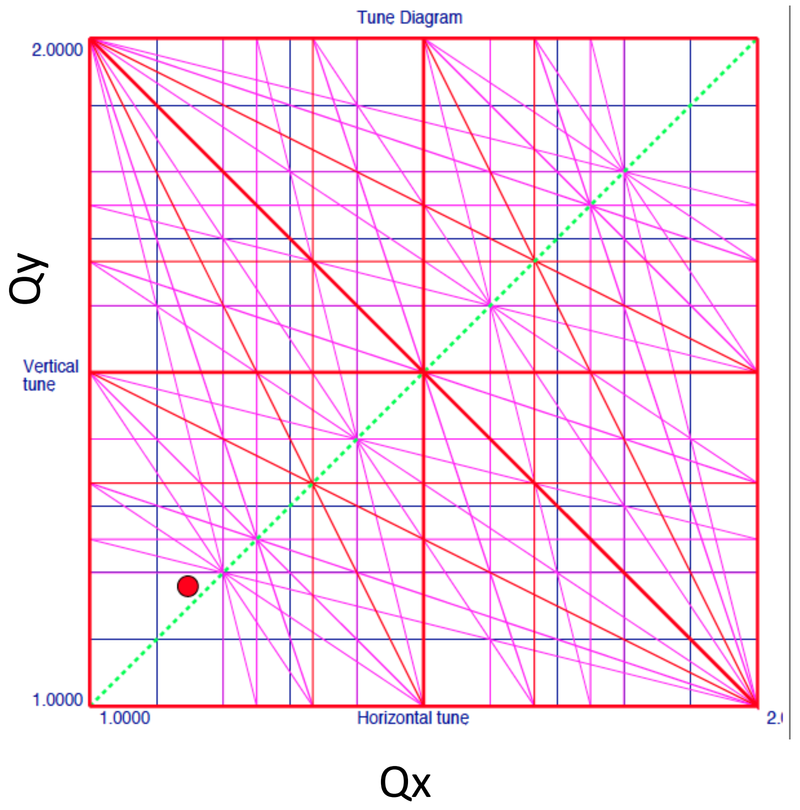
Sextupole perturbation: Q = N or N/3

Octupole perturbation: Q = N, N/2, N/4 etc.

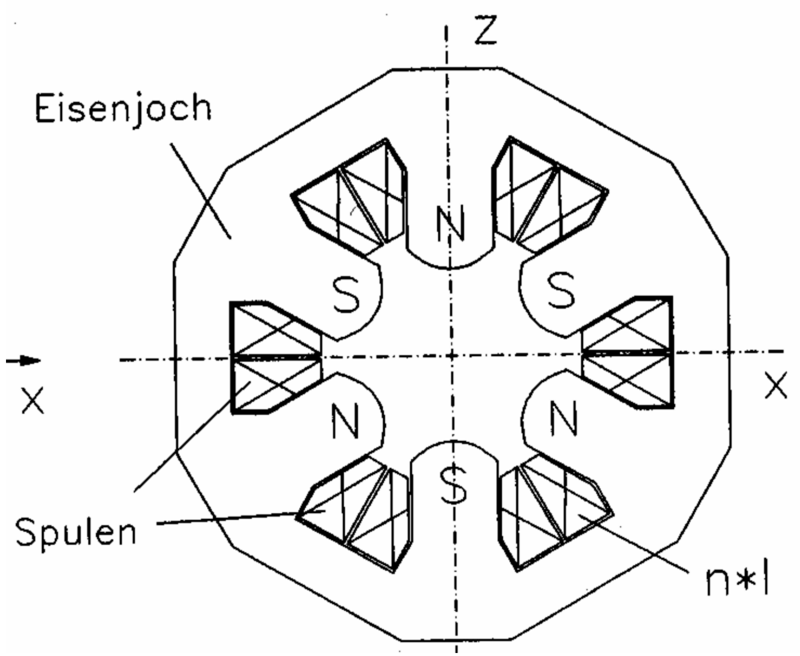
In general: avoid small integers n,m, N where

$$nQ_x + mQ_y = N$$

Working point has to be carefully chosen!!



Example: sextupole



$$B_x = \tilde{g}xy$$

$$B_y = \frac{1}{2}\tilde{g}(x^2 - y^2)$$

$$\Downarrow$$

$$\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \tilde{g}x$$ Linear rising "gradient"

The equations of motion become:

$$x'' + K_x(s) = -\frac{1}{2}m_{sext}(s)(x^2 - y^2)$$

$$y'' + K_y(s) = m_{sext}(s)xy$$

Effect of sextupoles on phase-space

Sextupoles create non-linear fields.

Depending on the tune the phase-space becomes more and more distorted. Motion becomes unstable close to the third order resonance.

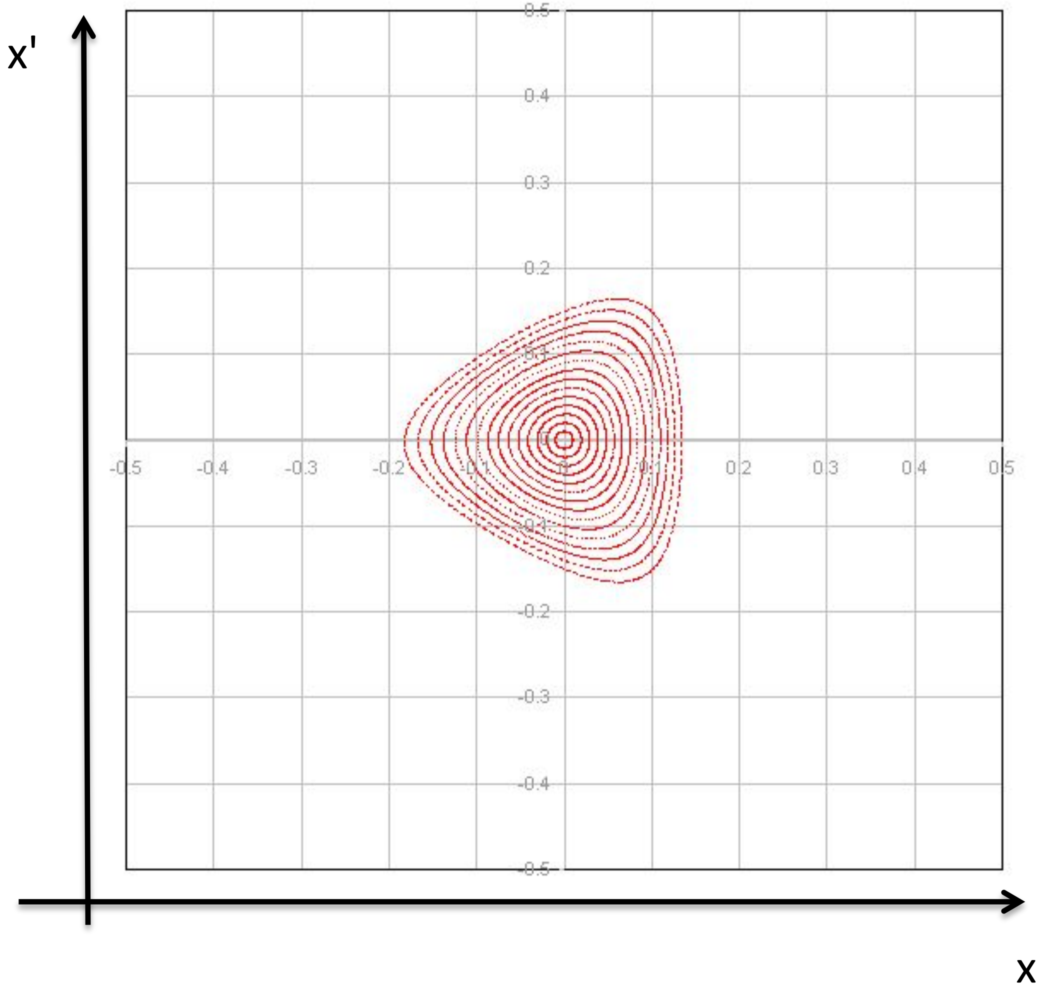
The sextupole kicks:

$$\Delta x' = -\frac{1}{2} m_{sext} l (x^2 - y^2)$$

$$\Delta y' = m_{sext} l x y$$

Amplitude of separatrix (unstable fixed points):

$$\propto \frac{Q - \frac{p}{3}}{m_{sext}}$$



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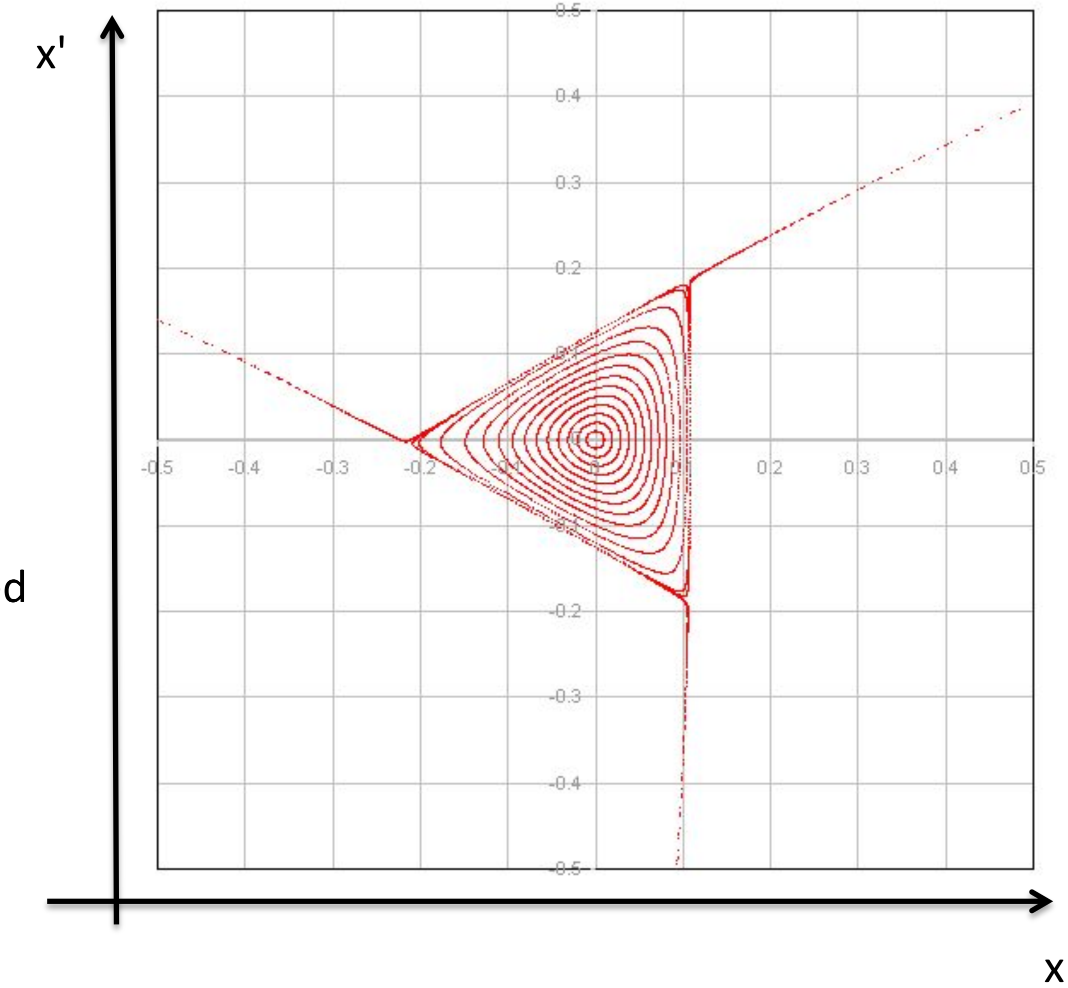


Chromaticity

The normalized quadrupole gradient is defined as

$$k = \frac{g}{p/e} \qquad\qquad p = p_0 + \Delta p$$

$$k = \frac{eg}{p_0 + \Delta p} \approx \frac{e}{p_0}\left(1 - \frac{\Delta p}{p_0}\right) g = k_0 + \Delta k$$

$$\Delta k = -\frac{\Delta p}{p_0} k_0$$

...a gradient error. Particles with different Δp/p will have different tunes.

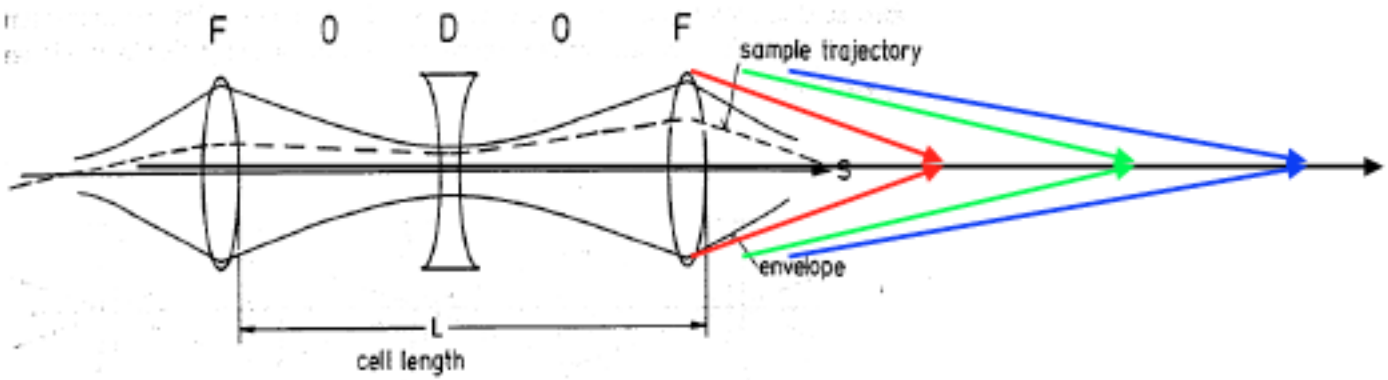


Figure 29: FODO cell

Chromaticity: Q'

The tune change for different Δp/p:

$$\Delta Q = \frac{1}{4\pi}\beta\Delta k \cdot l \quad \rightarrow \quad \Delta Q = -\frac{1}{4\pi}\frac{\Delta p}{p_0}k_0\beta l$$

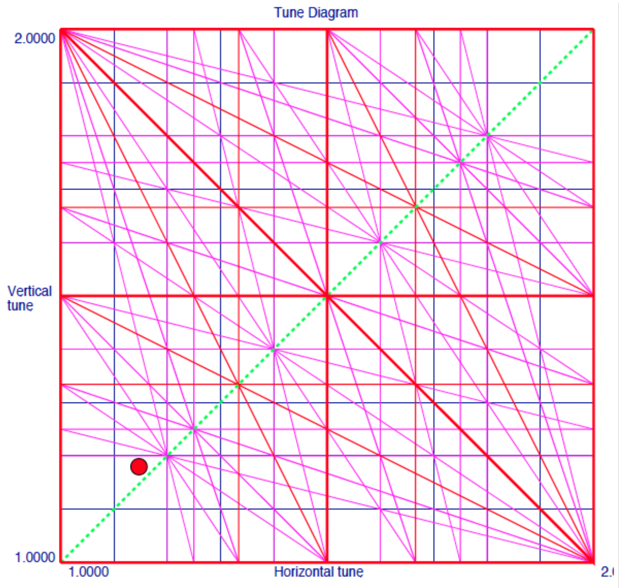
Definition of chromaticity:

$$\Delta Q = Q'\frac{\Delta p}{p} \qquad Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)ds$$

With the beam momentum spread indicates the size of the tune spot in the tune diagram.

Chromaticity is created by quadrupole fields in the horizontal and vertical plane.

Chromaticity: Q'



We cannot leave chromaticity uncorrected:

Example LHC
Q'= 250 [no units]
$\Delta p/p = +/- 0.2 \times 10^{-3}$

$\Delta Q = 0.256 \ldots 0.36$!!!! ➔ Particles will go across resonance lines and will be lost.

How to correct chromaticity?

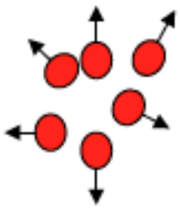
Sextupole fields at locations of dispersion:

1) Sort the particles according to momentum: $x_D(s) = D(s)\frac{\Delta p}{p}$
2) Magnetic field with linear rising "gradient" ➔ prop. to x^2

Collective Effects

Three categories: can cause beam instabilities, emittance blow-up, beam loss,...

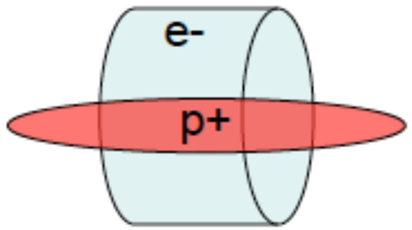
Beam-self: beam interacts with itself through space charge.



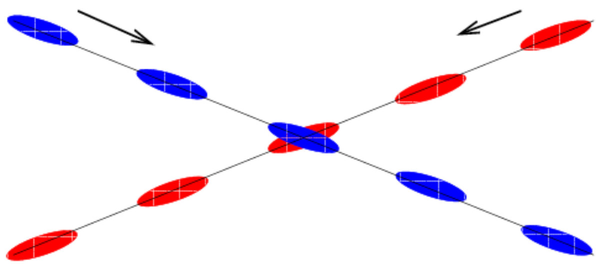
Tune spread $\propto \frac{1}{\beta^2\gamma^3}$

THE limitation in low energy machines

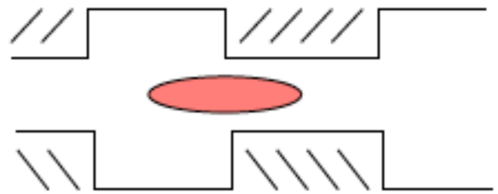
Beam-beam: colliding beams in colliders or ambient electron clouds (e-p instability).



Colliding beams. Tune spread/shift due to head-on collisions and long range collisions.



Beam-environment: beam interacts with machine (impedance-related instabilities).



Beam induces field in accelerator environment. Wake fields.
Wake fields can act back on trailing beam.

Fourier transform of Wake field is impedance.

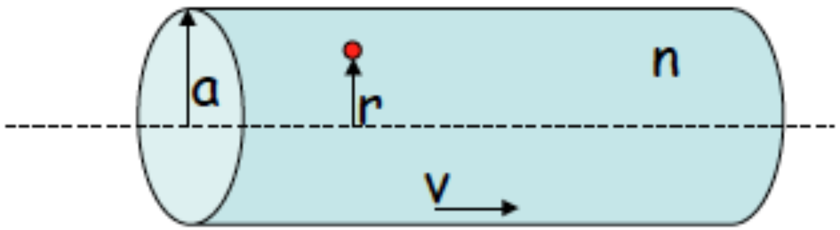
Can lead to component heating and/or instability.

Space-Charge Effect

The simplest and most fundamental of all collective effects

A simple approximation (direct space charge): beam as long cylinder.

Total force (E, B fields) on test particle in beam: uniformly charged cylinder of current I



$$F_r = F_E + F_B = \frac{eI}{2\pi c\beta\varepsilon_0\gamma^2 a^2} r$$

$$F_x = \frac{eI}{2\pi c\beta\varepsilon_0\gamma^2 a^2} x$$

$$x''(s) + K(s)x = \frac{F_{SC}}{m\gamma\beta^2 c^2}$$

Space charge → gradient error

→Defocusing

→ Tune shift

$$x''(s) + \left(K(s) - \frac{2r_0 I}{ea^2\beta^3\gamma^3 c}\right) x = 0$$

$$r_0 = \frac{e^2}{4\pi\varepsilon_0 m_0 c^2}$$ Classical particle radius

Space-Charge Effect

Tune shift from gradient error

$$\Delta Q_x = \frac{1}{4\pi}\int_0^{2\pi R}\beta(s)\Delta K_{SC}(s)ds = -\frac{r_0 R I}{e\beta^3\gamma^3 c}\left\langle \frac{\beta_x(s)}{a^2(s)} \right\rangle$$

For cylindrical beam

$$\Delta Q_x = -\frac{n r_0}{2\pi\varepsilon_x\beta^2\gamma^3}$$

$$\varepsilon_x = \frac{a^2}{\beta_x}$$

$$I = \frac{ne\beta c}{2\pi R}$$

$$\propto \frac{n}{\varepsilon_{x,y}}$$

$$\propto \frac{1}{\gamma^3}$$

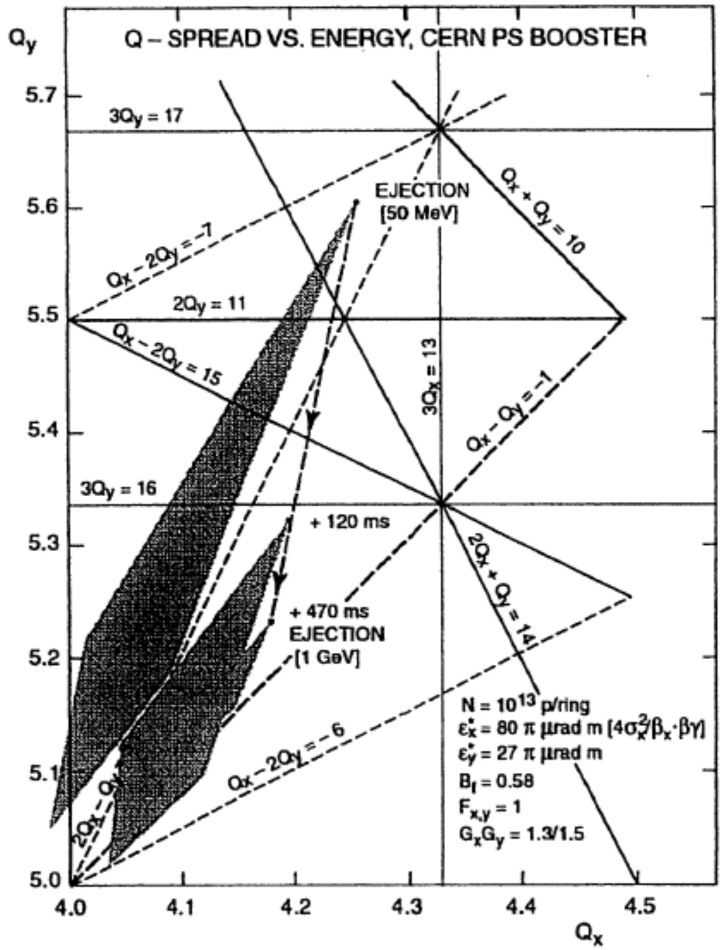
Not all particles in a beam will receive the same tune shift.

Variation in particle density, variation of space charge tune shift across bunch

➔ Tune spread

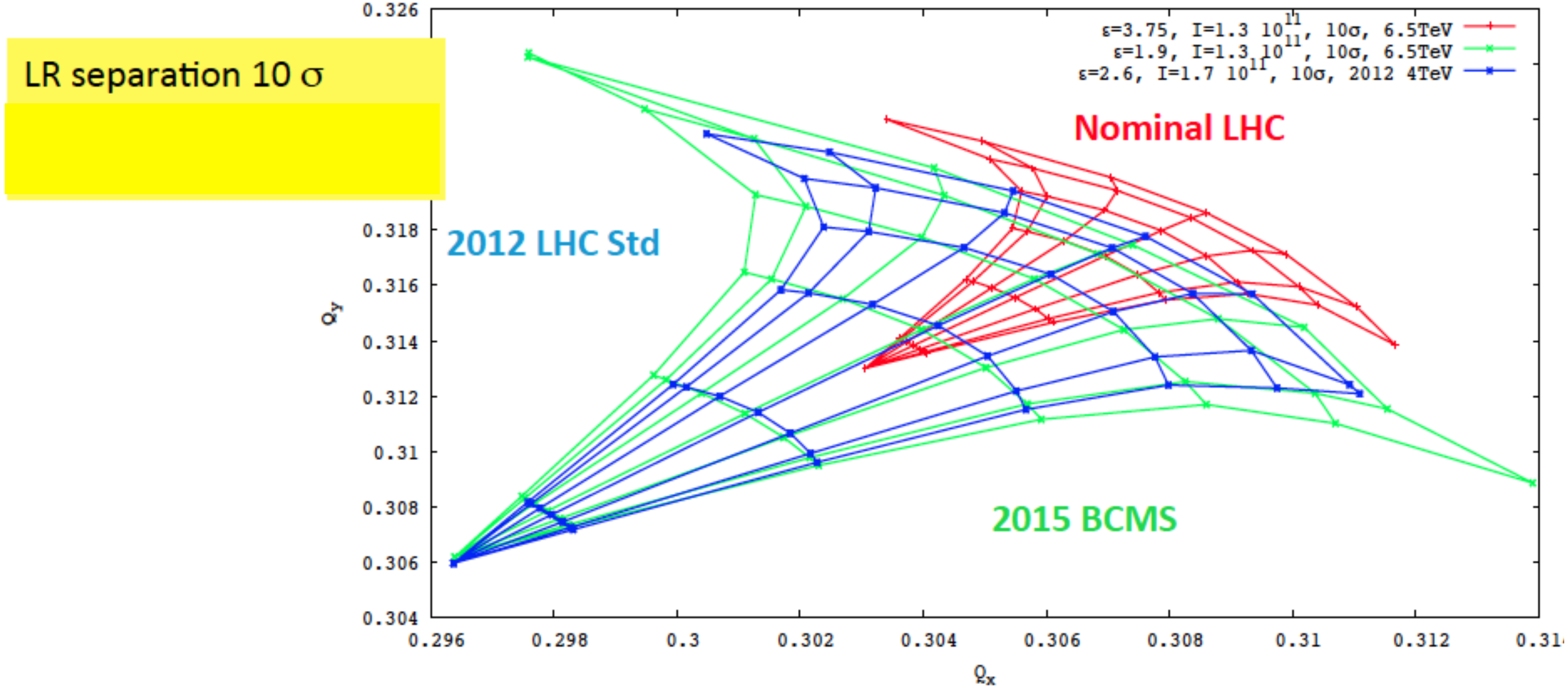
Space-Charge Effect

Space charge in the CERN PS booster for high intensity beams:



Not so easy to avoid resonances!!

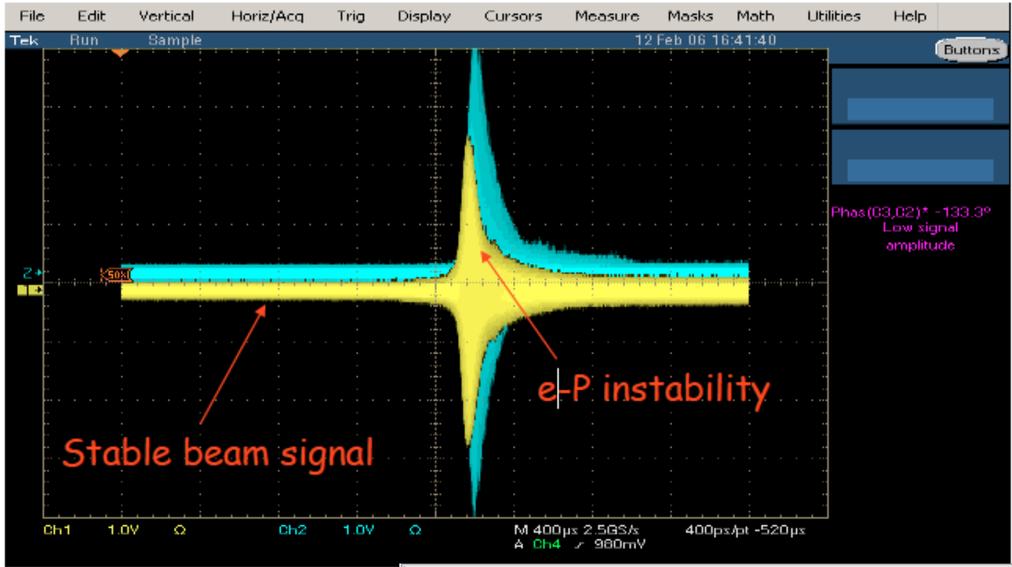
Beam-beam tune footprint



Courtesy T. Pieloni

Instability – what does it look like?

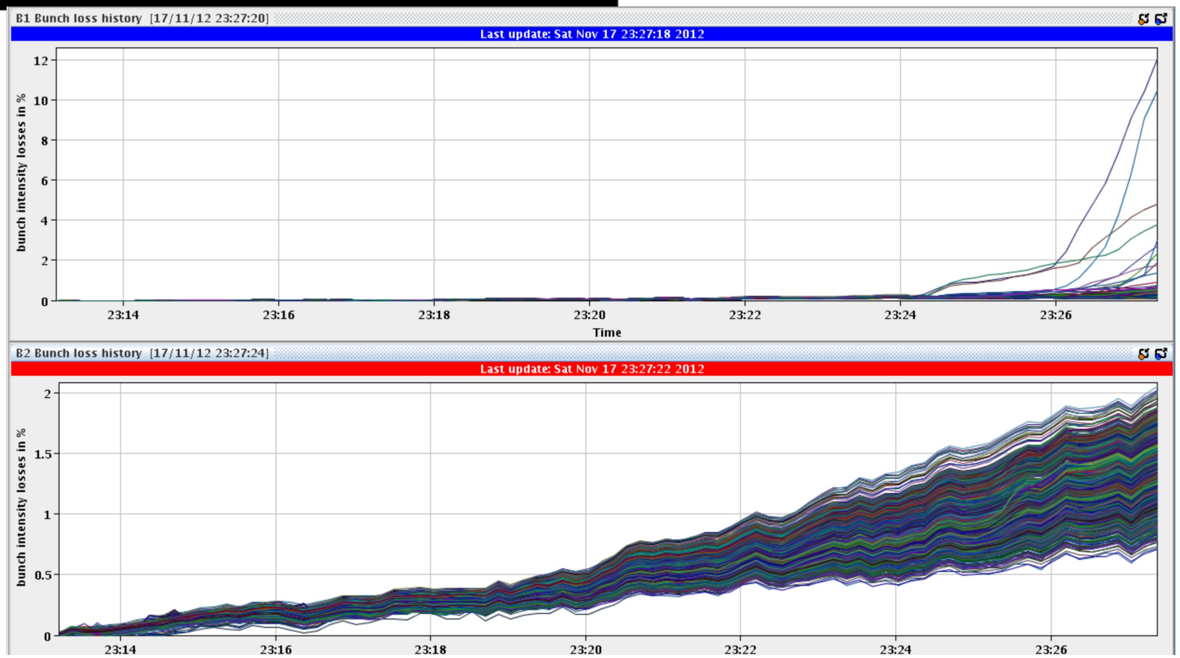
E.g. Fast rising coherent oscillation, beam losses, emittance blow-up



What to do?:

Octupoles, transverse damper, high chromaticity, head-on collisions,…

Increase tune spread



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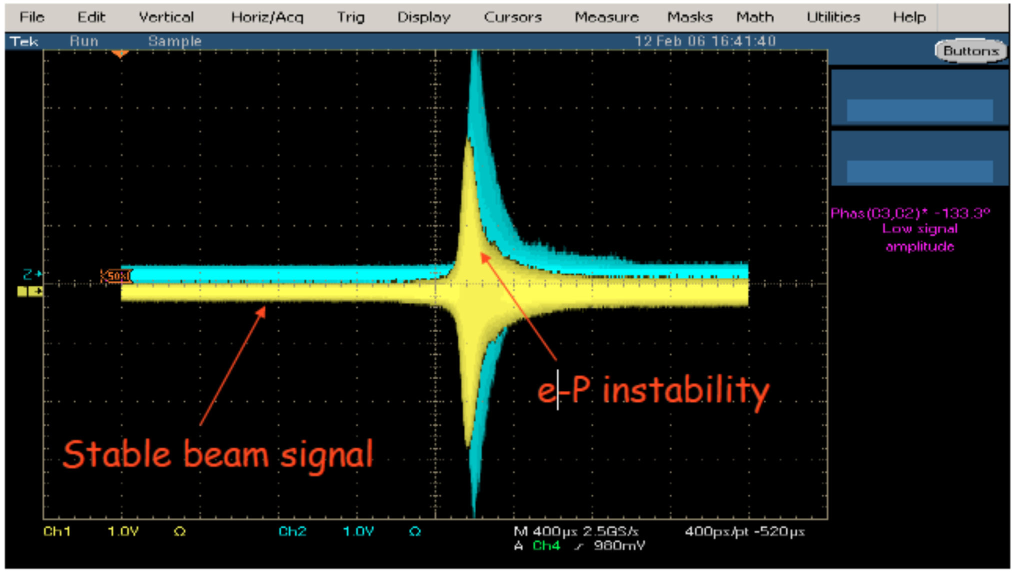

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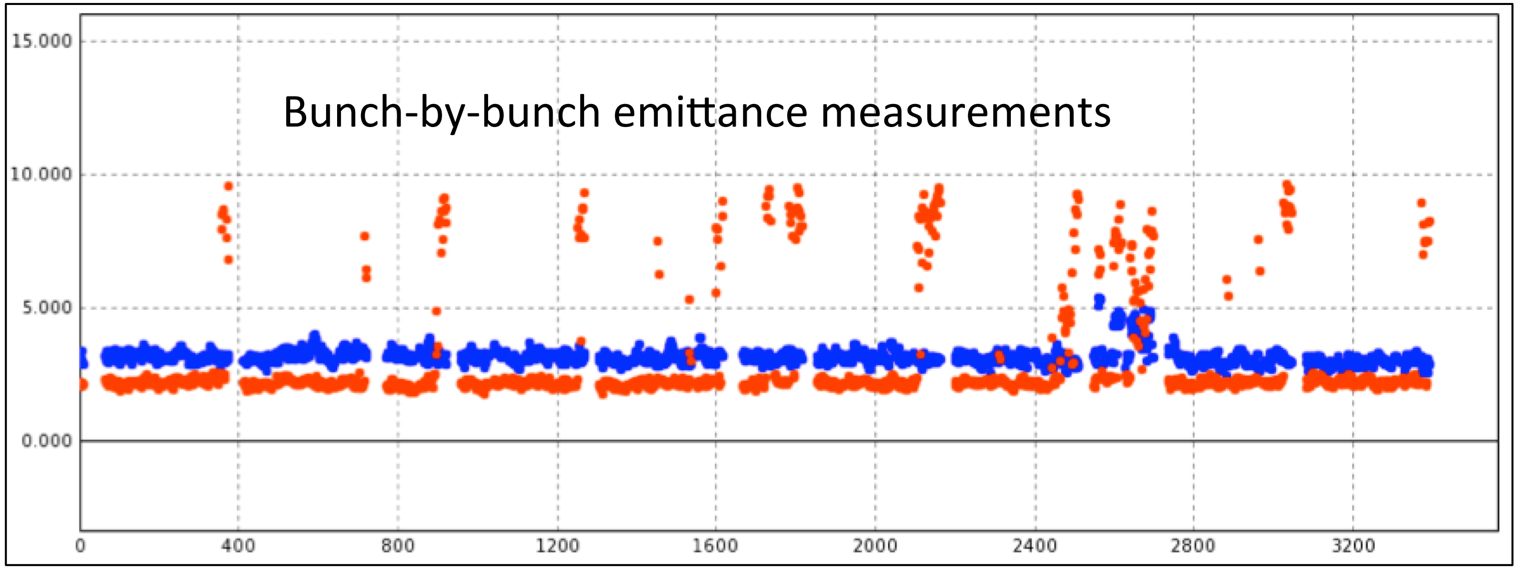


Mitigation of collective effects

Transverse feedback

Tune spread from chromaticity, octupoles, head-on beam-beam,... for Landau damping.

Landau damping:

- Coherent oscillation at frequency within beam frequency spread in general damped.

Transition revisited

Slip factor and transition energy

$$\eta = (\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2})$$

At transition energy $\eta = 0$ and no synchrotron oscillations (Ω_s=0)

$$\Omega_s = \sqrt{\frac{\eta f_{RF} \cos\phi_s}{R p_s} qV}$$

Close to transition the synchrotron frequency slows down.

Particles will not be able to catch up with rapid modification of bucket shape.

The bunch length becomes short and the momentum spread reaches maximum: losses due to limited momentum aperture.

Transition revisited

Loss of Landau damping due to loss of revolution frequency spread with $\eta = 0$

➔ rise to collective instabilities

Non-adiabatic synchrotron motion:

Non-adiabatic time

$$T_c = \left(\frac{\beta^2 E_{rest} \gamma_t^4}{4\pi f_0^2 \dot{\gamma} h V_0 |\cos\phi_s|} \right)^{1/3}$$

High acceleration rate $\dot{\gamma}$ is essential.

ACCIDENTAL BEAM LOSS – EQUIPMENT MALFUNCTIONING

Aperture Limitations and Obstacles

Due to misalignment of vacuum chamber/equipment

- Extraction equipment: septa on moveable girders
- Collimators
- Screens in the ring for circulating beam
- Bad orbit due to misaligned quadrupoles or orbit bumps

Orbit bumps can be there on purpose: e.g. extraction bumps, crossing angle bumps in colliders or by accident due to fake BPM readings

Generating debunched beam

Particles can escape from the bucket due: intrabeam scattering, beam-beam interactions, ...

Malfunctioning equipment:

- Noise of phase loop
- Problems with longitudinal blow-up
- Switch off of RF cavity: reduces acceptance of bucket

Acceptance of stationary bucket

$$\left[\frac{\Delta p}{p}\right]_{max} = \pm\frac{1}{\beta}\sqrt{\frac{2eV}{\pi h|\eta|E_s}}$$

Time for a particle to get lost once outside of bucket depends on:

- Energy loss per turn (synchrotron radiation, e-cloud, impedance,..)
- Momentum aperture (e.g. LHC $\Delta p/p = 3 \times 10^{-3}$): aperture vs. beta function and dispersion

Example LHC: time from separatrix to momentum aperture 7 TeV: ~60 s

Powering Failure of Dipole Magnet Circuits

Multiturn failures vs. very fast error kick ➔ closed orbit vs. trajectory

For "slow" failures (LHC ~ 50 turns): good estimate from CO formula

For a failure of a circuit with N dipoles in series

$$\Delta x_{CO}(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi Q}\left(\frac{I(t)}{I_0} - 1\right)\sum_i^N \theta_i \sqrt{\beta(s_i)}\cos(\psi(s) - \psi(s_i) + \pi Q)$$

Different possible failure cases – function I (t) needs to be established.

In most powering failure cases voltage will be set to zero and

$$I(t) = I_0 e^{-\frac{t}{\tau}} \qquad \tau = \frac{L}{R}$$

In case of a quench can be modelled as:

$$I(t) = I_0 e^{-\frac{t^2}{2\sigma^2}}$$

$$\sigma = 200ms$$

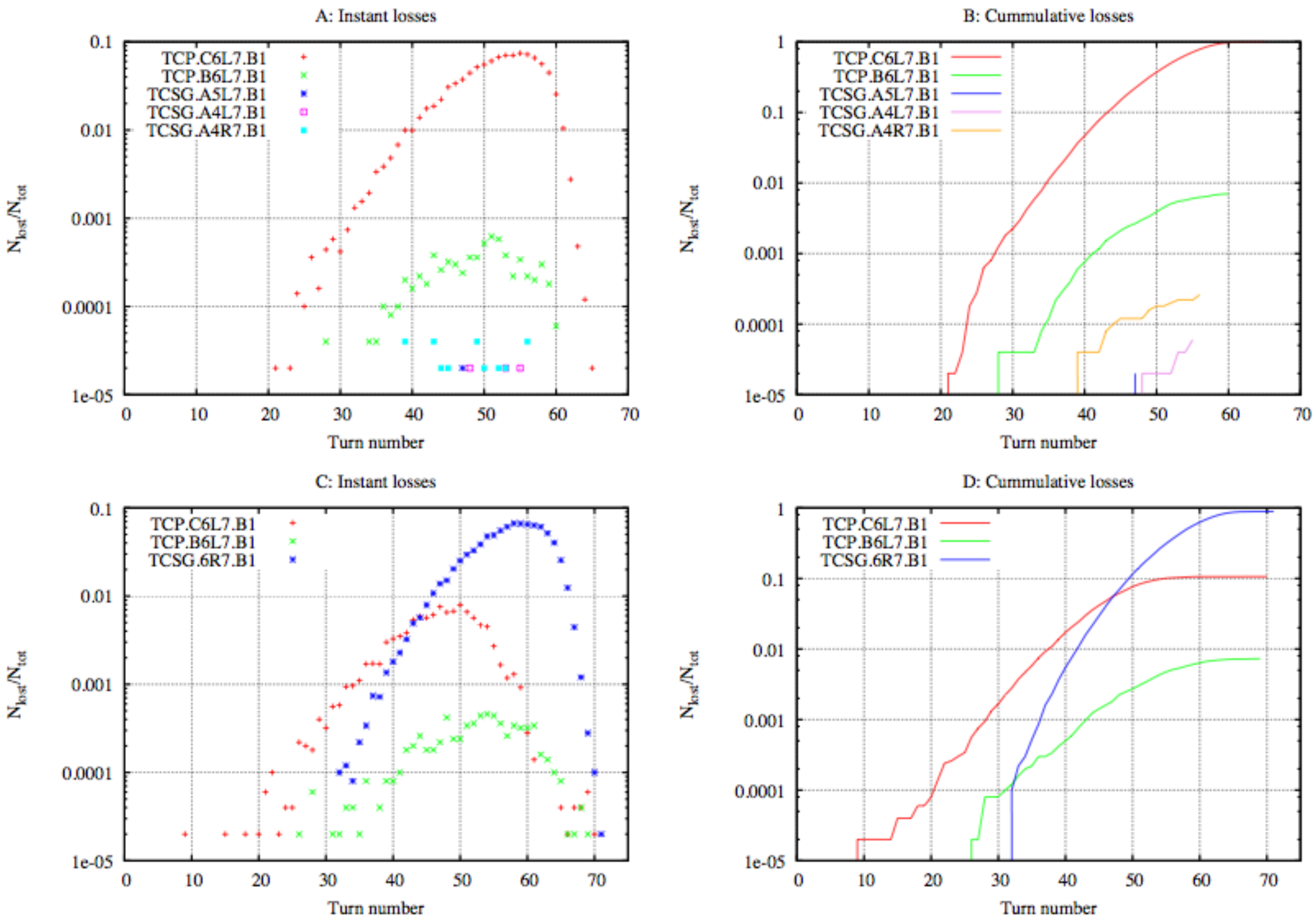
Example: Normal Conducting Separation Dipole D1 Failure in the LHC.

Loss rate at different collimators in the LHC: simulation results

Protection system reaction time: need to trigger before maximum sustainable loss rate reached.

Fast abort at 7 TeV

Powering failure at 450 GeV



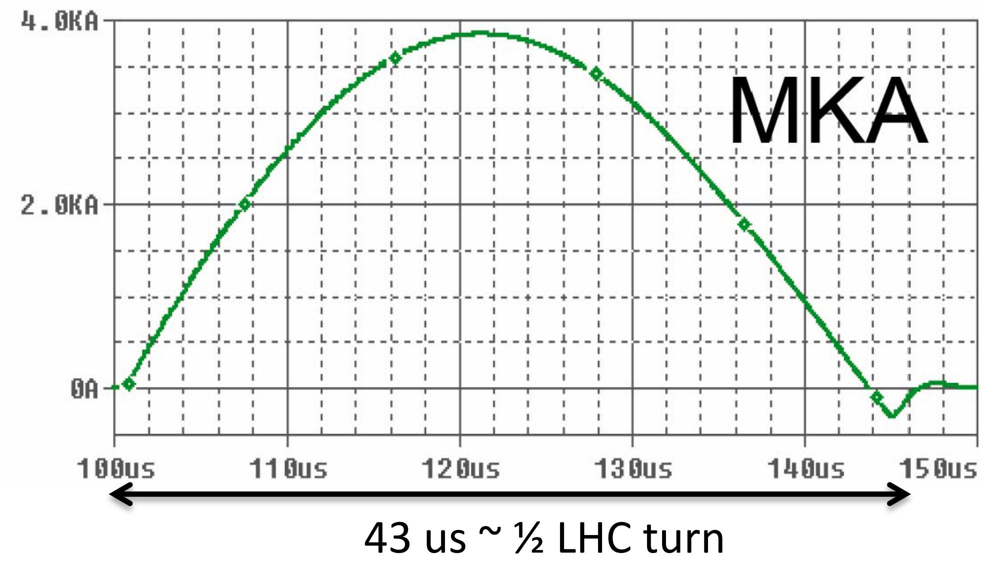
Fast transverse Kickers

Aperture kicker, tune kicker, crab cavities, transverse damper.

Can excite significant fractions of beam to high amplitudes within a single turn or a few turns.

High intensity/high energy accelerators:

Save mode of operation with limitations on strength and/or intensity have to be put in place.



Aperture kicker in the LHC

Voltage reduced for
5 σ oscillation at injection
1.4 σ at 7 TeV.

Operation only allowed with "safe beam" intensity.

Powering Failure of Quadrupole Magnet Circuits

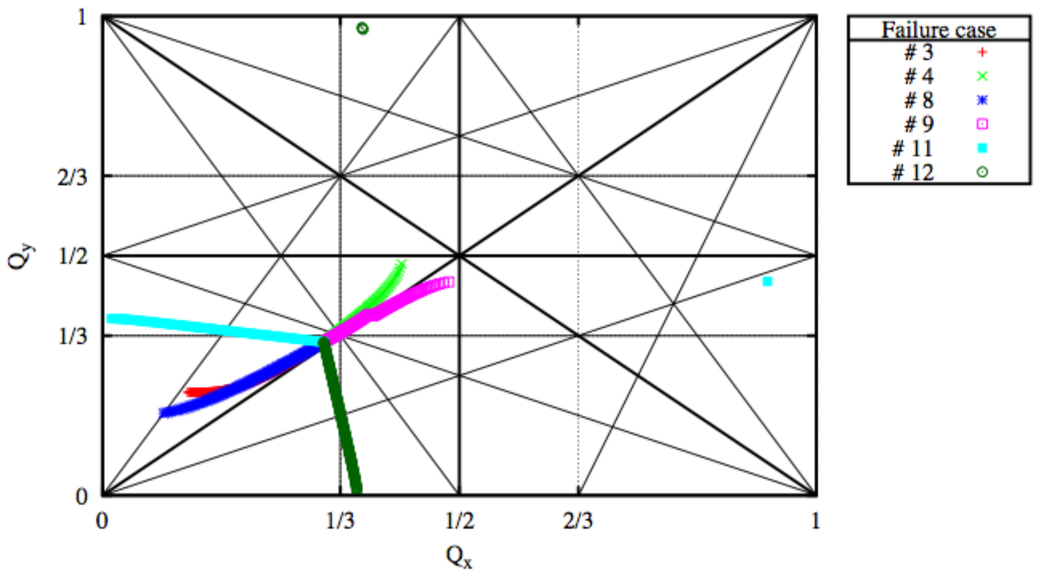
The effect on the beam for failure of higher multipoles than quadrupole in general slow.

Effect for quadrupole string connected to one power supply:

$$\Delta Q = \frac{1}{4\pi}\left(\frac{I(t)}{I_0} - 1\right)\sum_i k_i l_i \beta(s_i)$$

$$\frac{\Delta\beta(s)}{\beta(s)} = -\frac{1}{2\pi\sin(2\pi Q)}\left(\frac{I(t)}{I_0} - 1\right)\sum_i k_i l_i \beta(s_i)\cos(2(\psi(s) - \psi(s_i)) + 2\pi Q)$$

Crossing resonances



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