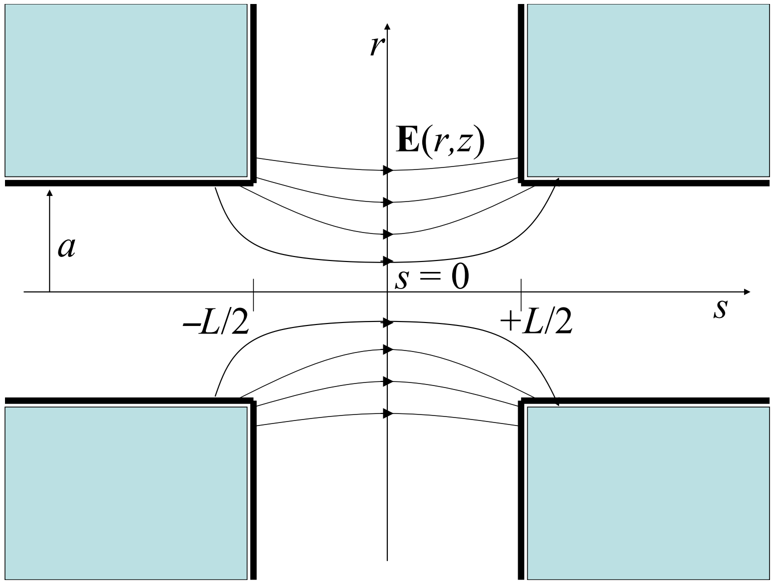
CONTROL ROOM ACCELERATOR PHYSICS

Day 4

Introduction to Acceleration

Outline

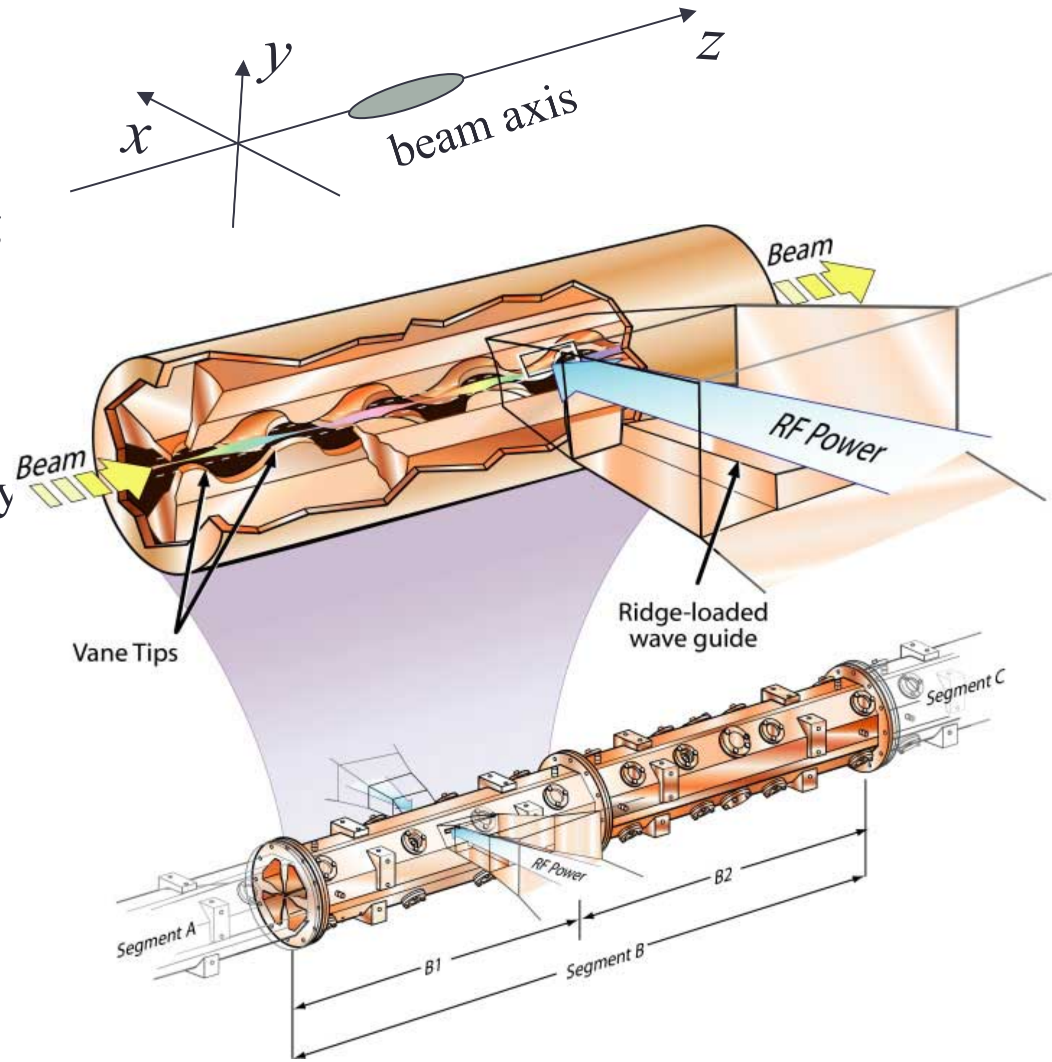
1. Introduction
2. Accelerating structures
3. Axial fields
4. The Panofsky equation



Basic Acceleration Principles

Three basic ingredients

1. Particle Beam
   1. Ensemble of self-interacting particles
2. Accelerating structure
   1. Resonant RF cavity specially shaped to provide strong fields on beam axis
3. RF Power
   1. RF energy converted to particle kinetic energy



Basic Accelerator Principles
We Use a Lot of RF Power

warm linac klystron galleries at SNS

SNS's electric bill for operations is about $1M per month

Basic Acceleration Principles

Lorentz Force and Work

- Lorentz Force Law
  - Describes force **F** on a particle in EM field

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)$$

F – force on particle

q – particle charge

v – particle velocity

E – electric field

B – magnetic field

- Energy Gain from EM field
  - Work *W* done on particle by field

$$W = \int \mathbf{F}\cdot d\mathbf{r}$$

$$= q\int \mathbf{E}\cdot d\mathbf{r} + q\int \cancelto{0}{\frac{d\mathbf{r}\times\mathbf{B}\cdot d\mathbf{r}}{dt}}$$

Only electric fields in the direction of propagation affect **energy gain**

•Accelerating structures create strong electric fields in the direction of propagation

Lorentz Force and Work
Summary

- Physics of beamlines
  - Magnetic fields cannot be used to accelerate particles
  - Acceleration occurs along the direction of electric field
  - Energy gain is independent of the particle velocity

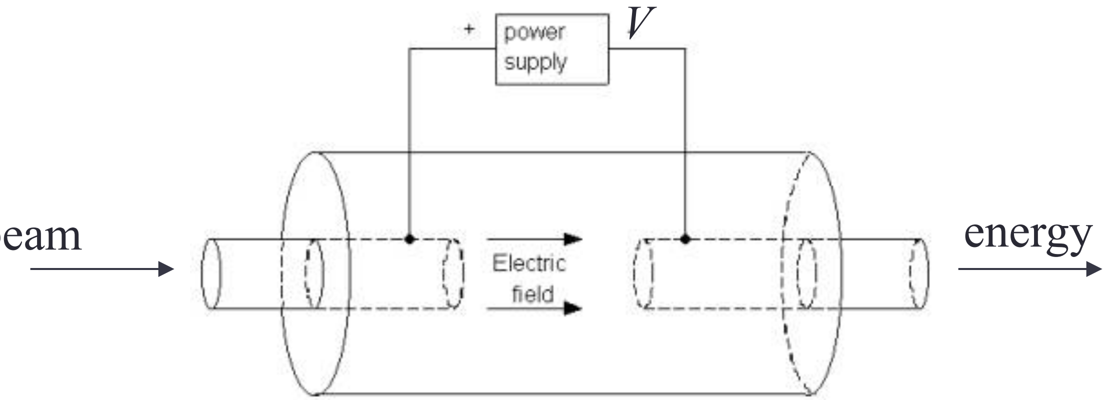
- Engineering beamlines
  - Longitudinal electric fields in the direction propagation are designed to be as large as possible
  - Magnetic fields are designed to bend particles for guidance and focusing along the beam path

Basic Accelerator Structures

Van der Graff Acceleration

- Static acceleration
  - Charged parallel plate capacitor with beam aperture
  - Particle falls through a potential V

particle beam



energy increased by V eV

- So why use RF?
  - For the SNS linac we would require a stack of plates with a total **floating** voltage of 1 GV (about 667 million D-cell batteries)

Acceleration to Higher Energies

- Terminal voltages of 20 MV provide sufficient beam energy for nuclear structure research, however particle physics, neutron production, and accelerator driven systems require beam energies > 1 GeV

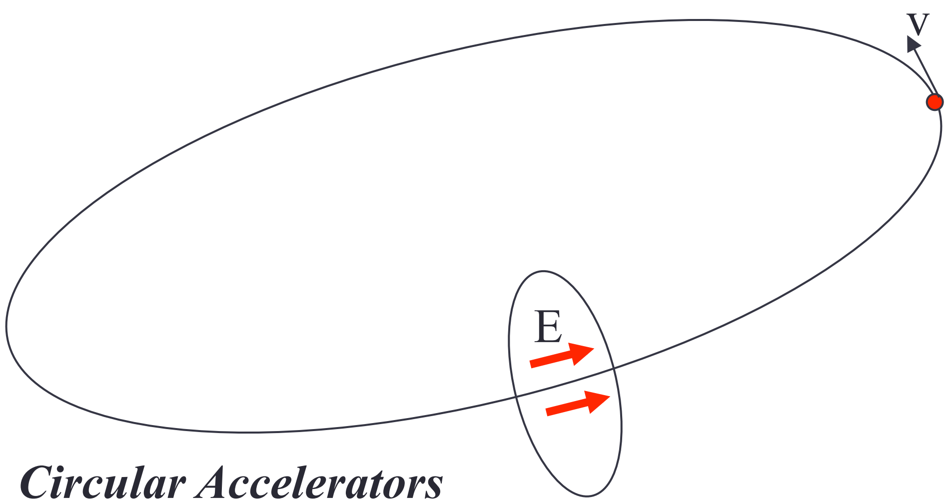
- How to attain higher beam energies?

- How to swing a child?
  1. Pull up to maximum height and let go: difficult and tiring (electrostatic accelerator)

  2. Repeatedly push in synchronism with the period of the motion

Acceleration by repeated application of RF accelerating fields

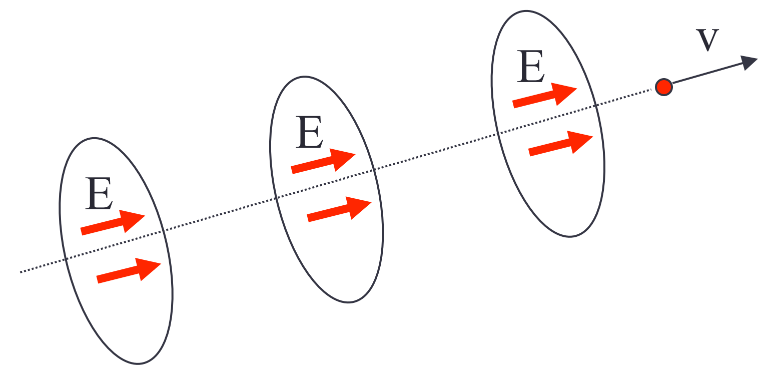
Two approaches for accelerating with time-varying fields



Circular Accelerators

Use one or a small number of RF cavities and make use of repeated passage through them: This approach leads to *circular accelerators*:

Cyclotrons, synchrotrons and their variants



Linear Accelerators

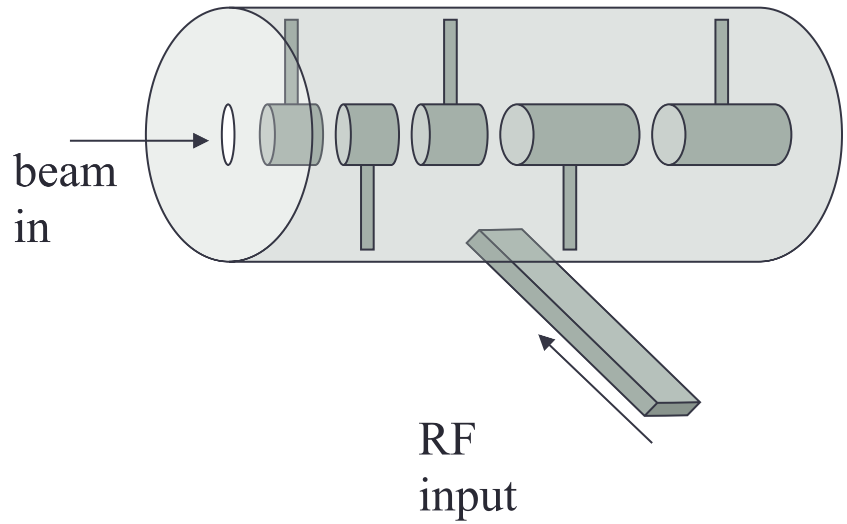
Use many cavities in which each particle only passes through once:

These are *linear accelerators*, or "*linacs*"

3. Linear Accelerating Structures
The Drift Tube Linac (DTL)

- DTL invented by Luis Alvarez in 1946 at Berkeley
  - Pillbox resonant cavity with grounded shielding tubes

- Beam
  - Beam injected after RF standing wave established
  - "Drift" tubes isolate beam while RF reverses in time

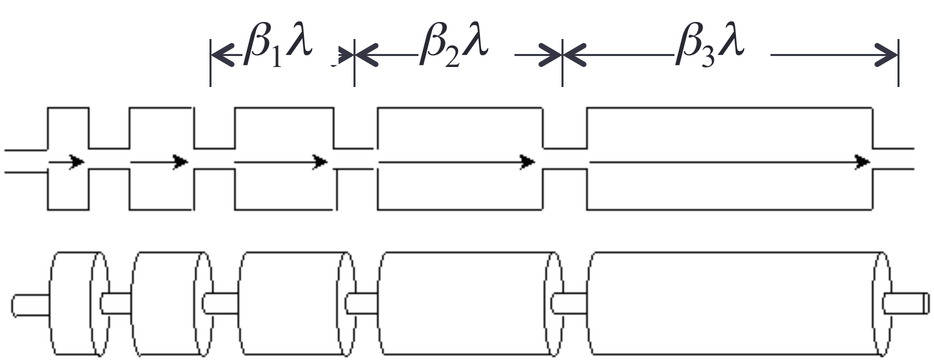


- RF Drive
  - Use fundamental TM_{010} mode (longitudinal E field)

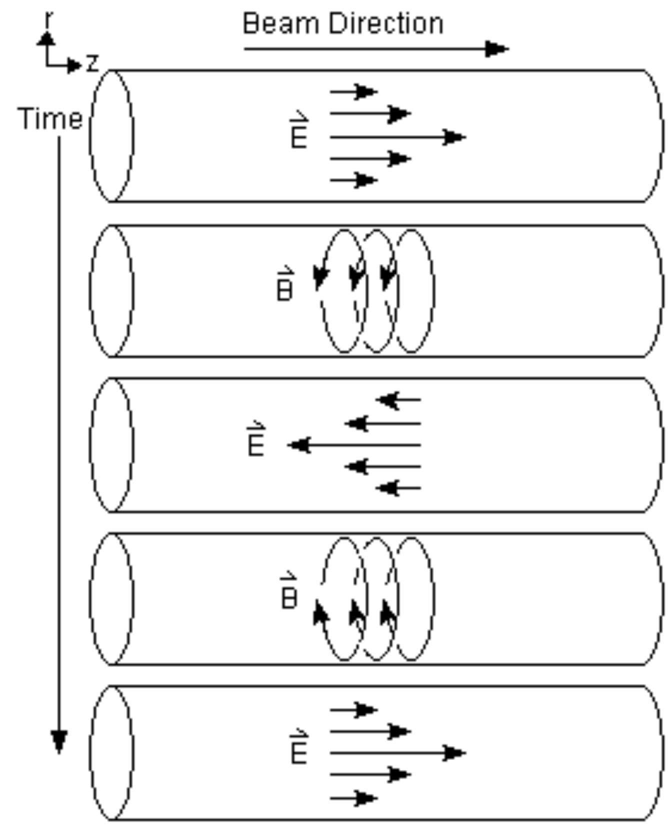
3. Accelerating Structures
Drift Tube Linac (cont.)

Without drift tubes no net acceleration could occur

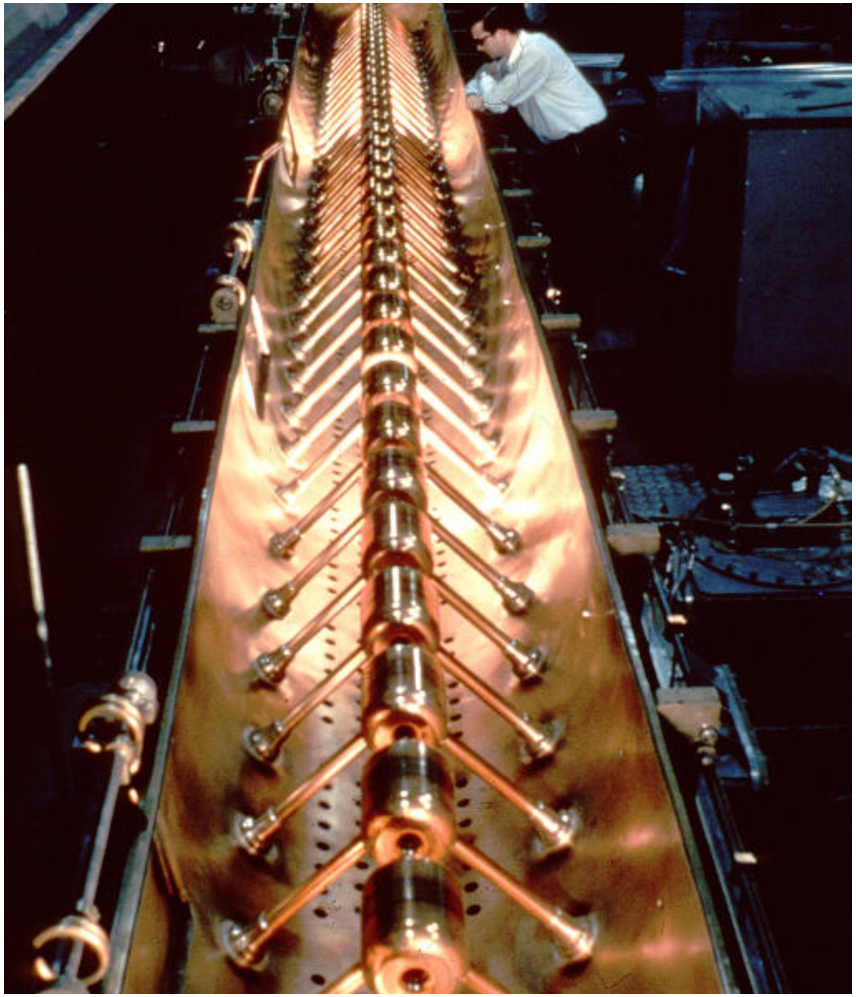
- Drift Tubes
  - Shield particle from negative RF cycle
  - Length = $\beta\lambda$, $\beta = v/c$
  - Must get longer as particle accelerates



Constant Frequency with an Increasing Acceleration Gap Length



3. Examples of Actual DTL Tanks



CERN DTL

Fermilab DTL Tank #2

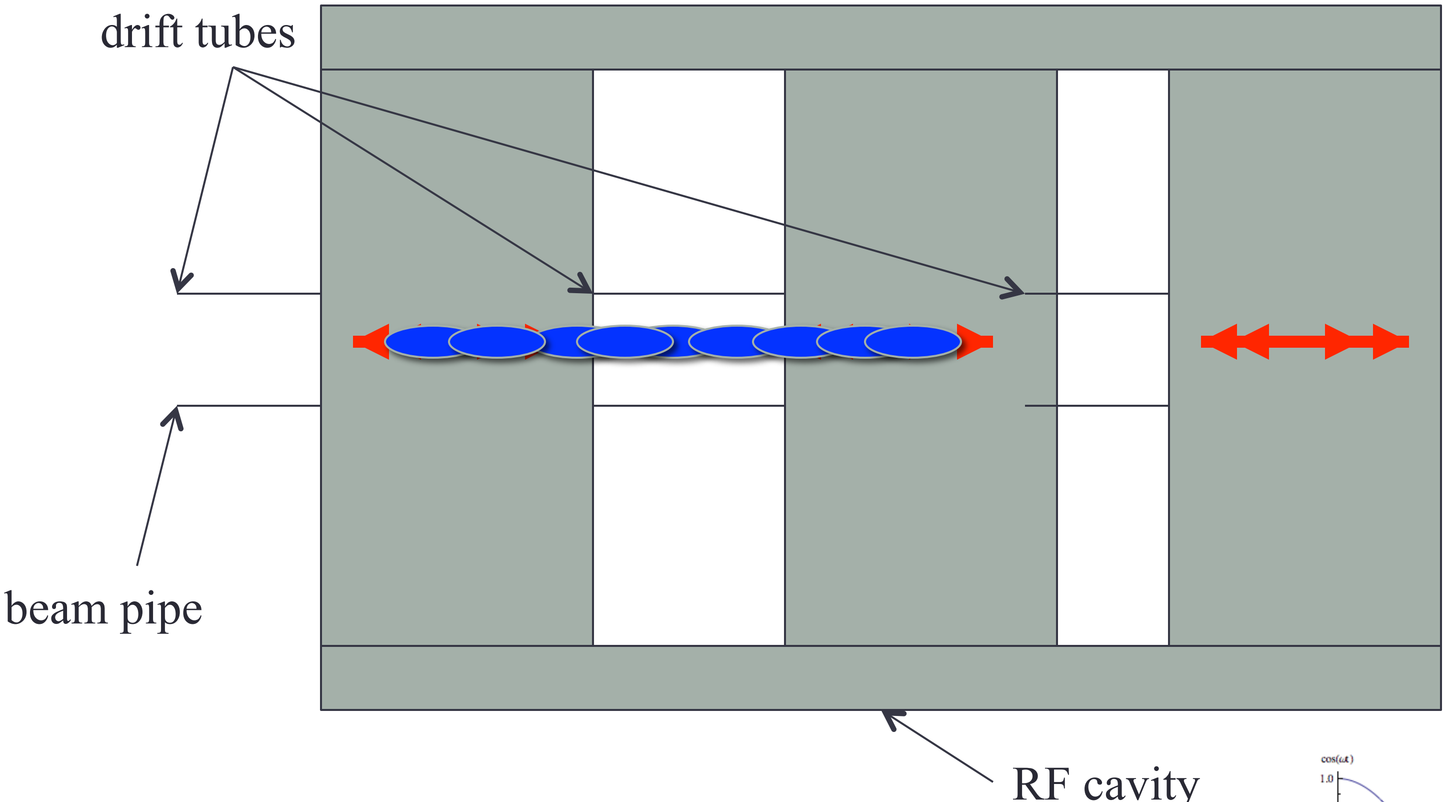
3. Accelerating Structures

SNS Facility - DTL Tanks and CCDTL Tanks

SNS DTL tanks 6 pre-install

SNS CCDTL tanks installed in tunnel

RF Cavity Excitation
Standing Waves



$\omega t=0$

$\omega t=\pi/4$

$\omega t=\pi/2$

$\omega t=3\pi/4$

$\omega t=\pi$

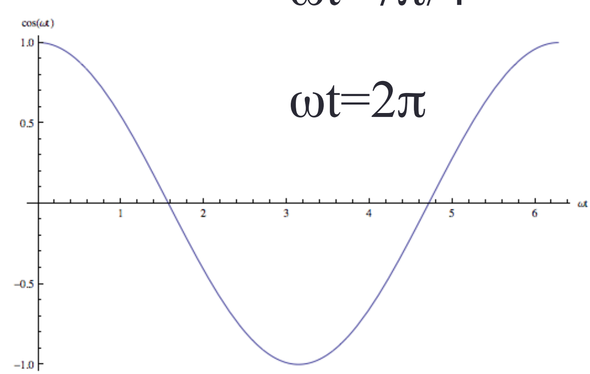
$\omega t=5\pi/4$

$\omega t=3\pi/2$

$\omega t=7\pi/4$

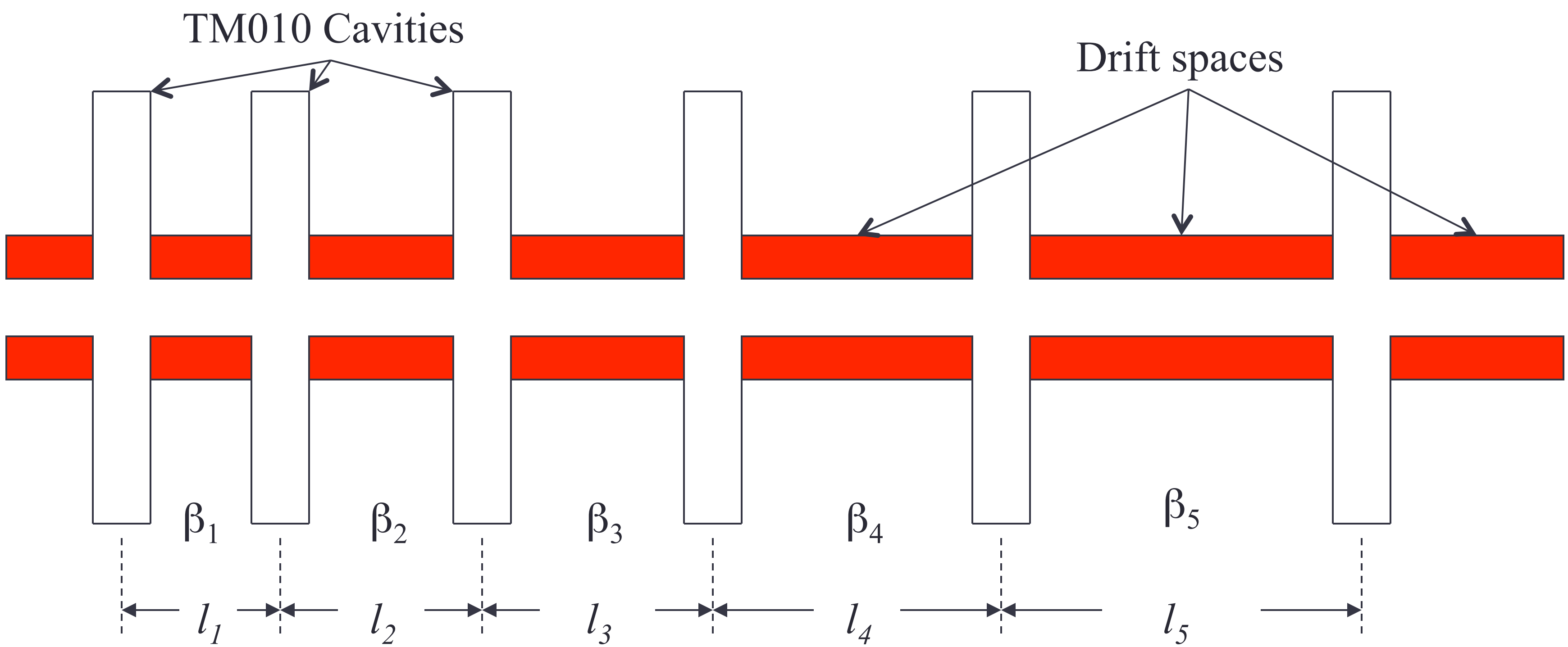
$\omega t=2\pi$

$$E_z(r,z;t) = E_z(r,z)\cos\omega t$$



DTL Phase Synchronism

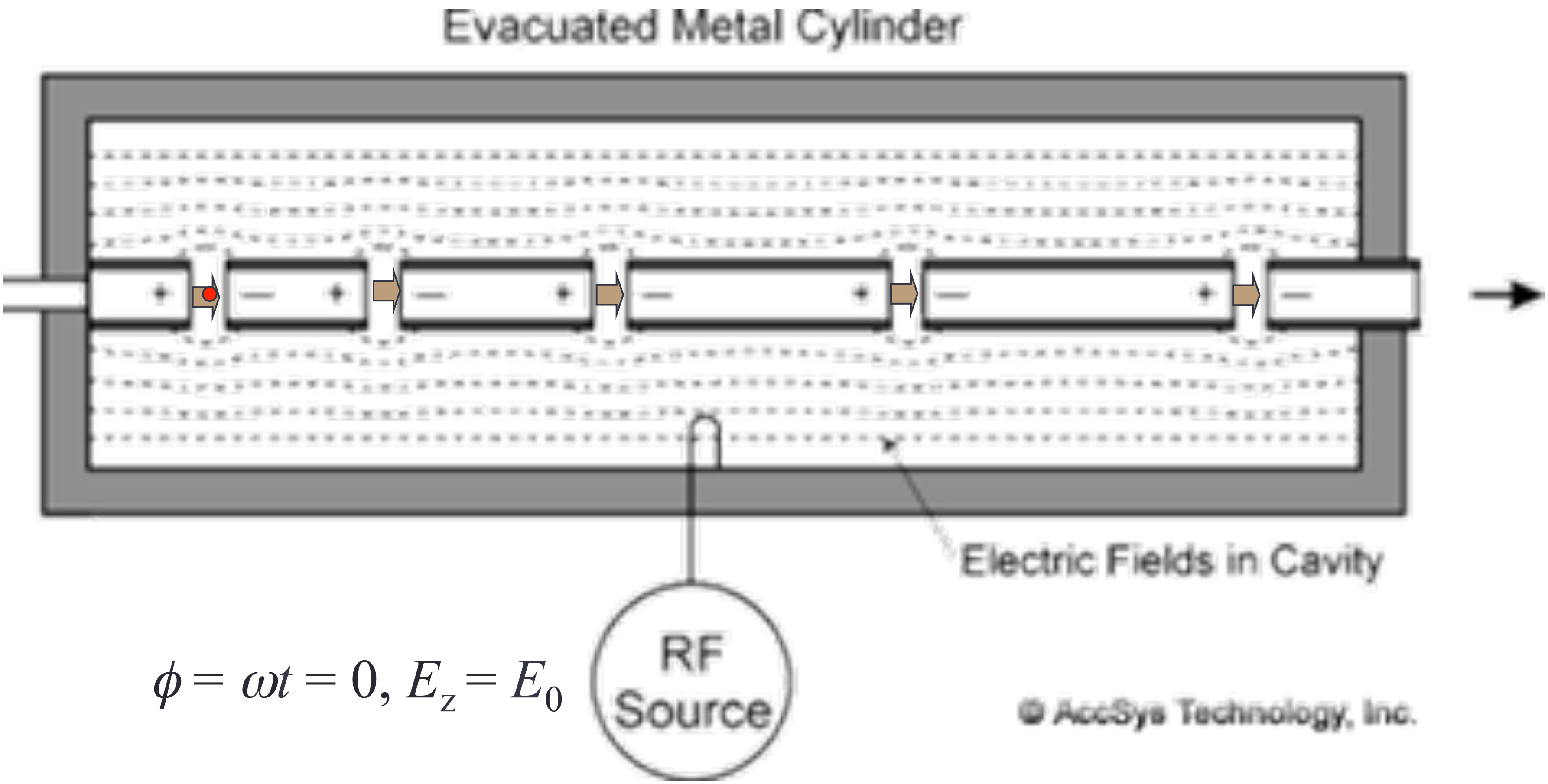
Synchronizing Particle Velocity and RF Phase



- Suppose we want a particle to arrive at the center of each gap at $\phi=0$. Then we space the drifts so that transit time Δt is equal to the RF period $1/f$.

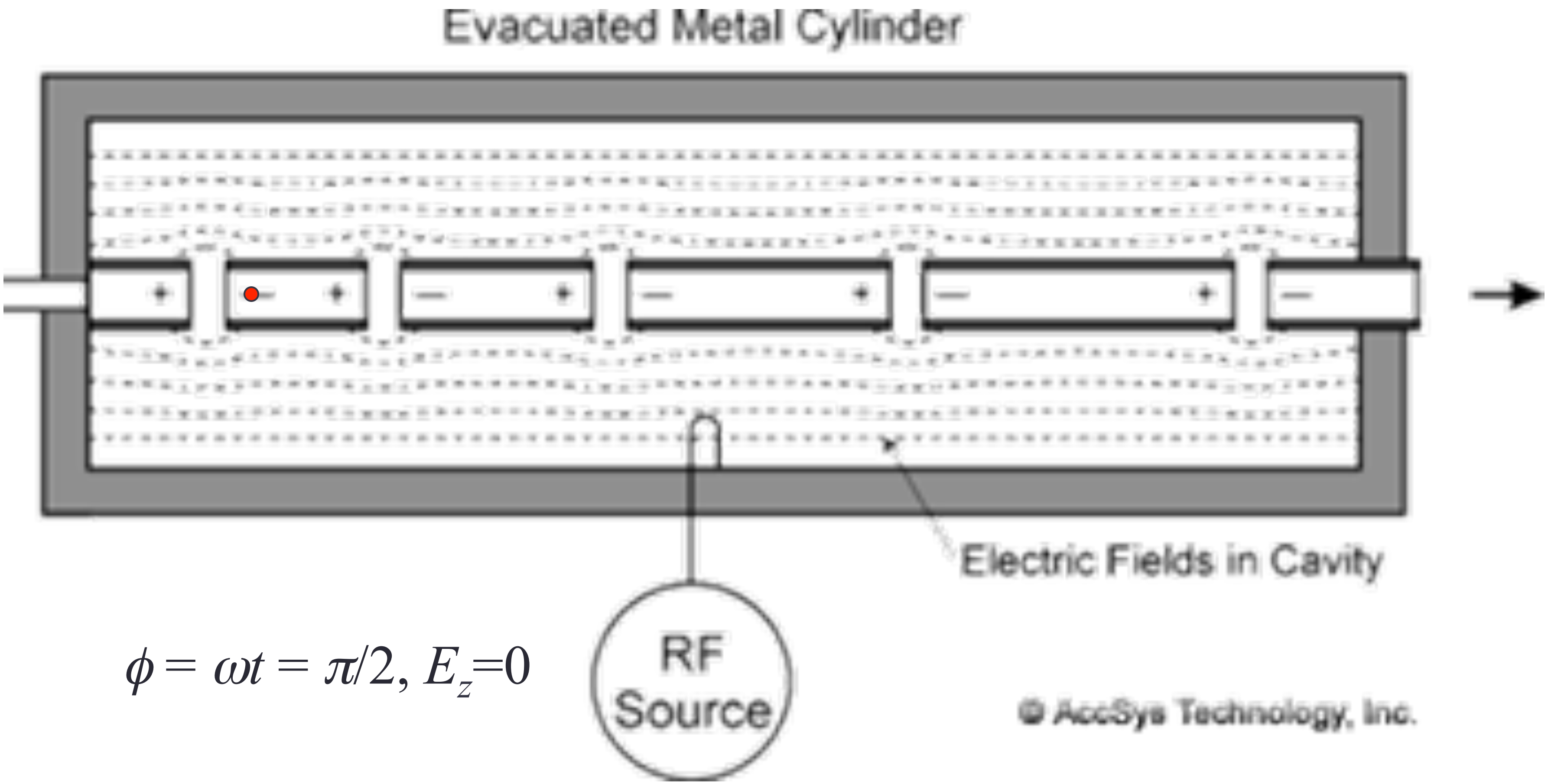
For example….

Zero-mode excitation of a Drift Tube Linac Tank



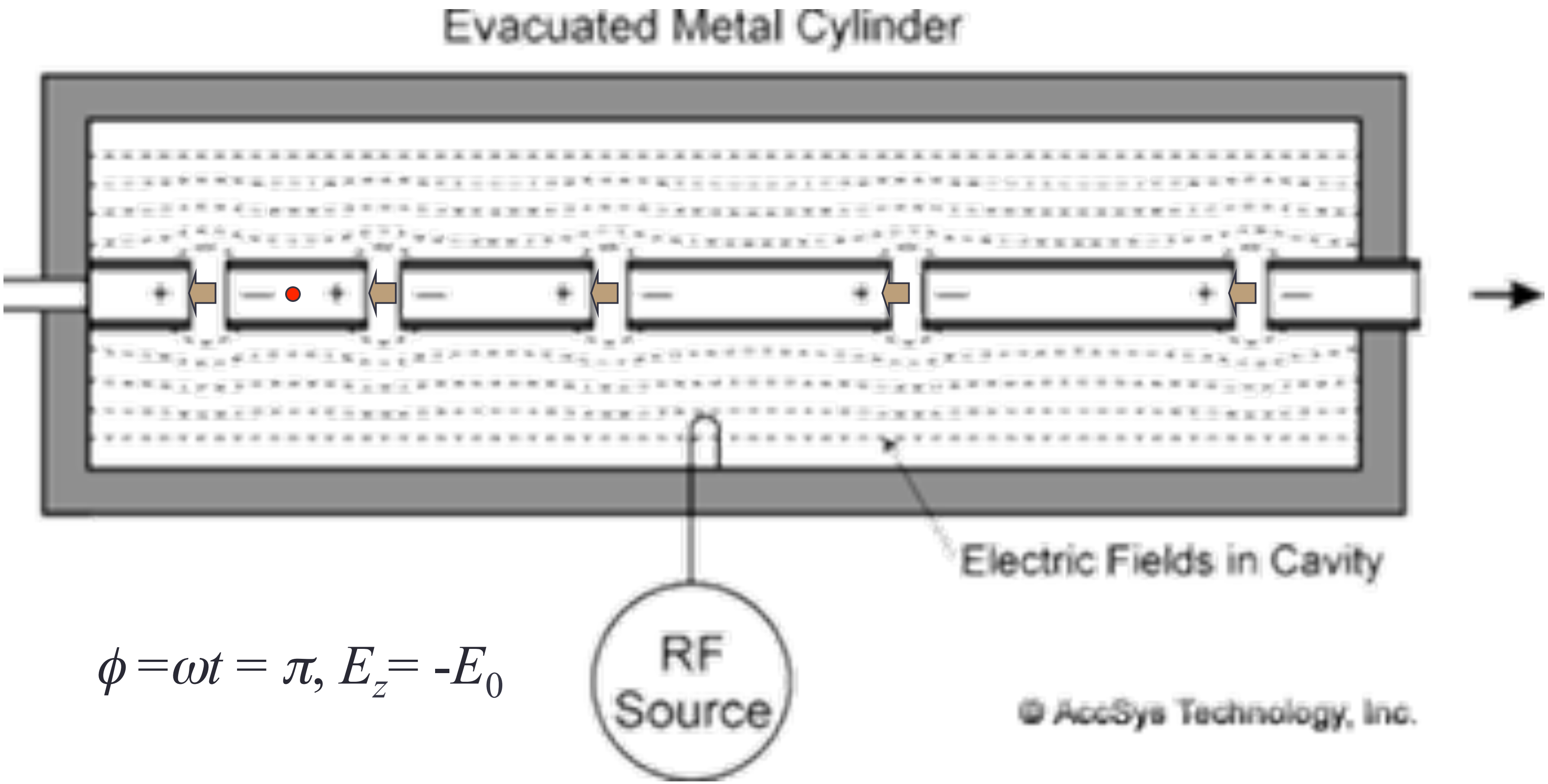
$$\phi = \omega t = 0,\ E_z = E_0$$

Zero-mode excitation of a Drift Tube Linac Tank



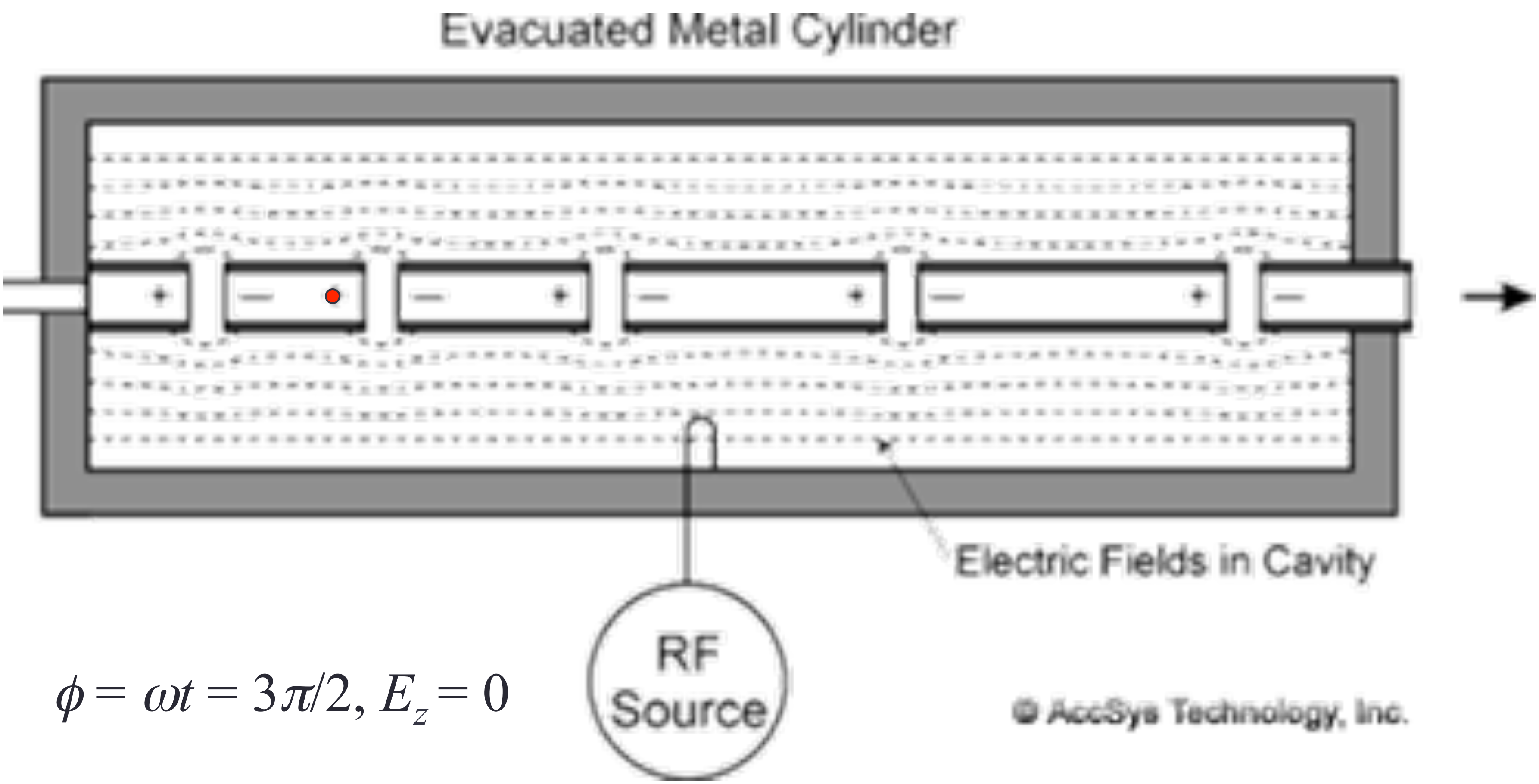
$\phi = \omega t = \pi/2,\ E_z = 0$

Zero-mode excitation of a Drift Tube Linac Tank



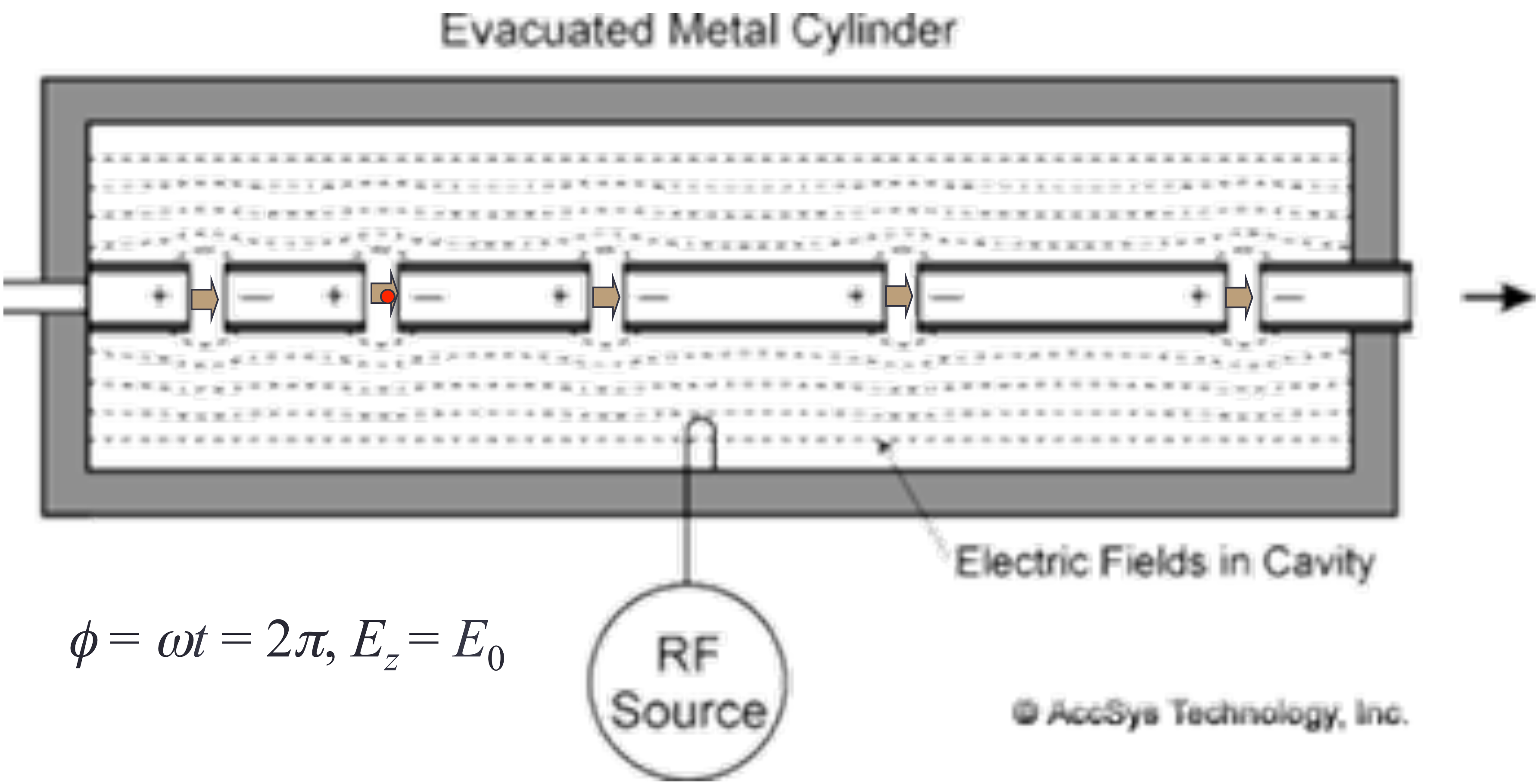
$\phi = \omega t = \pi,\ E_z = -E_0$

Zero-mode excitation of a Drift Tube Linac Tank



$\phi = \omega t = 3\pi/2,\ E_z = 0$

Zero-mode excitation of a Drift Tube Linac Tank



$$\phi = \omega t = 2\pi,\ E_z = E_0$$

Accelerating Structures

Control Room Application – Phase Synchronization

- In order for the (linear) accelerating structure to accelerate to full energy
  - The phase of the klystron must be synchronized to the arrival of the particle beam.
  - This is a common accelerator system task and building an application to do so is also common
  - The basic technique many such applications employ is to scan the phase of a single klystron driving a cavity then note the propagation time of the beam to some downstream location
    - The propagation time is smallest for the largest acceleration by the cavity
    - The klystron phase corresponding to the largest acceleration must be the zero reference phase of the cavity.
    - (Typically the cavity design phase is retarded by 20 to 30 degrees to provided longitudinal focusing)

Modeling Acceleration

The Panofsky Equation

In general, for an RF gap (say, between drift tubes) we can approximate the energy gain ΔW of a particle with the following assumptions:

- The RF structure (i.e., "tank") is cylindrically symmetric
- The velocity of the particle does not drastically increase in the gap
  - Proton or heavy ion
  - Relativistic electron
- The total gap electric potential is $V = \int E_z dz$ (across the gap)
- The "wave number" of the particle is $\bar{k} = 2\pi / \bar{\beta}\lambda$ where $\bar{\beta}$ is the "average" velocity through the gap
- The the following equation is known as the *Panofsky equation* …

Modeling Energy Gain

The Panofsky Equation and Interpretation

$$\Delta W = qV\left[T(\bar{k})\cos\varphi_0 - S(\bar{k})\sin\varphi_0\right]I_0(\bar{K}r) \quad \bar{K} = \frac{\omega}{c}\frac{1}{\bar{\beta}\bar{\gamma}}$$

- V is the potential drop across the entire gap
- ϕ_0 is the RF phase when the particle is in the gap center
- T is the Fourier cosine transform of E_z with a centered gap
  - In Accelerator vernacular, it is call the *transit time factor*
  - After normalization, the largest value is unity
  - Loosely represents lost energy from finite time gap transit

- S is the Fourier sine transform of E_z with a centered gap
  - It then represents the field error from mis-shapen electrodes
  - When ϕ_0 is 90 degrees these errors are most serious

- Bessel function $I_0(Kr)$ represents off-axis loss in energy

Acceleration

Summary

- Only electric fields in the direction of propagation can affect energy gains of the beam particles
- Accelerating structures are resonant RF cavities which are designed to have strong electric fields in the longitudinal direction
- Because of time-varying fields, particles must enter the acceleration gaps at the proper RF phase to experience acceleration
- The modeling equation for RF gaps is the Panofsky equation which relations energy gain to gap potential, particle velocity, and transit time factor

Supplemental Material

- Derivation of the Panofsky equation

Details of Energy Gain*

The Panofsky Equation and Transit Time Factor

Objective: compute the energy gain ΔW of particle through an RF gap

- To preserve Liouville's theorem there is also an associated phase jump $\Delta\phi$ for particle

- An analytic computation is difficult because
  - The fields are functions of time
  - The fields are functions of position
  - The particle's position is a function of time

- We can make simplifications and approximations to arrive at *Panofsky Equation*

*Advanced

Energy Gain

Cylindrical RF Gap

Energy gain through RF gap

- Given
  - Cylindrical Symmetry
  - Time-harmonic fields
  - $E_z(r,z)$ on axis
- Assumptions
  - Particle axial velocity v_z can be approximated constant through gap
  - No radical change in radial position r through gap

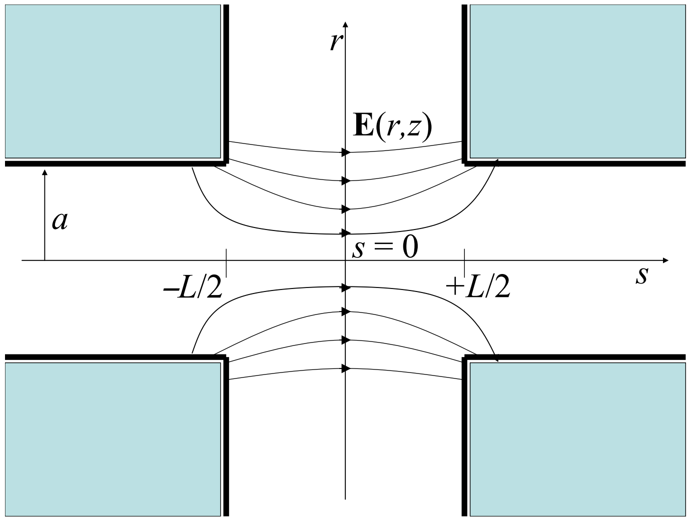


Figure : RF gap geometry

Electromagnetic Fields

Longitudinal Electric Fields

- Maxwell's Equations

$$\nabla \cdot \mathbf{E} = 0, \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t},$$

- Combining **E** creates vector wave equation

$$-\nabla^2 \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

- Assuming time-harmonic form $\cos(\omega t + \phi_0)$

$$\nabla^2 \mathbf{E} + \frac{\omega^2}{c^2}\mathbf{E} = 0$$

- Vector harmonic oscillator equation where

$$k^2 = \omega^2 / c^2$$

The general solution for the spatial component of the longitudinal electric field $E_z(r,z)$ to the above can be expressed

$$K^2 = \left| \frac{\omega^2}{c^2} - k^2 \right|$$

$$E_z(r,z) = \int_{-\infty}^{+\infty} \begin{Bmatrix} J_0(Kr) & \text{for} & k^2 < \omega^2/c^2 \\ I_0(Kr) & \text{for} & k^2 > \omega^2/c^2 \end{Bmatrix} \left[T(k)\cos kz + S(k)\sin kz \right] dk$$

Electromagnetic Fields

Longitudinal Electric Fields

$$E_z(r,z) = \int_{-\infty}^{+\infty} \begin{Bmatrix} J_0(Kr) \text{ for } k^2 < \omega^2/c^2 \\ I_0(Kr) \text{ for } k^2 > \omega^2/c^2 \end{Bmatrix} \left[T(k)\cos kz + S(k)\sin kz\right] dk$$

- Radial standing wave with dispersion relation $K^2 = \left|\dfrac{\omega^2}{c^2} - k^2\right|$
- Applying boundary conditions at on axis ($r = 0$) and by properties of Fourier transform

$$T(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E_z(0,z)\cos kz\, dz,$$ $T(k)$ is Fourier cosine transform of $E_z(0,z)$

$$S(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E_z(0,z)\sin kz\, dz,$$ $S(k)$ is Fourier sine transform of $E_z(0,z)$

(in the transform we assume E_z is from a single gap so that $E_z \rightarrow 0$ far from the gap)

Energy Gain

Computing Work with Longitudinal Electric Fields

- Putting it all together

$$\Delta W = \int_{-\infty}^{+\infty} q E_z\left(r, z, t(z)\right) dz$$

$$= q \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \begin{Bmatrix} J_0\left(Kr\right) & k^2 < \dfrac{\omega^2}{c^2} \\ I_0\left(Kr\right) & k^2 > \dfrac{\omega^2}{c^2} \end{Bmatrix} \left[T(k)\cos kz + S(k)\sin kz\right] \cos\left(\omega t(z) + \phi_0\right) dz\, dk$$

- Where we have interchanged the order of integration
- The dispersion relation is

$$K^2 = \left|\frac{\omega^2}{c^2} - k^2\right|$$

- The particle is at location z at time $t(z)$ where

$$t(z) = \int_0^z \frac{dz}{v_z(z)}$$

Energy Gain
The Velocity Assumption

- The particle is at location z at time $t(z)$ where $$t(z) = \int_0^z \frac{dz}{v_z(z)}$$
  - This expression is inconvenient since axial velocity is a function of energy W
  - Inconvenient or not $t(0) = 0 \;\Rightarrow\; \phi_0$ is the mid-gap RF phase
- **Assumption**: The axial velocity $v_z(t)$ through the gap can be approximated by a constant v_z
  - Typically valid for protons at most energies
  - Valid for electrons for energies > 511 MeV
- The value of v_z is usually taken to be the average of the initial and final velocities through the gap.

Energy Gain

The Velocity Assumption (cont)

- On average $t(z) = z/\bar{v}_z$
- Thus, $$\omega t + \phi_0 = \frac{2\pi f z}{\bar{v}_z} + \phi_0 = \frac{2\pi f z}{\bar{\beta} c} + \phi_0 = \frac{2\pi}{\bar{\beta}\lambda} z + \phi_0 = \bar{k} z + \phi_0$$
  - where $$\bar{\beta} \equiv \frac{\bar{v}_z}{c}$$ $$\bar{k} \equiv \frac{2\pi}{\bar{\beta}\lambda}$$
- Note $$\bar{k} \equiv \frac{2\pi}{\bar{\beta}\lambda} = \frac{2\pi c}{\bar{\beta}\lambda c} = \frac{\omega}{\bar{\beta} c} > \frac{\omega}{c}$$
- We can now compute the integration over z for ΔW

Energy Gain

Computing Work with Longitudinal Electric Fields

- Using the following facts

$$\cos(\bar{k}z + \phi_0) = \cos \bar{k}z \cos \phi_0 - \sin \bar{k}z \sin \phi_0$$

$$\int_{-\infty}^{+\infty} \cos k_1 z \cos k_2 z dz = 2\pi\delta(k_1 - k_2)$$

$$\int_{-\infty}^{+\infty} \sin k_1 z \sin k_2 z dz = 2\pi\delta(k_1 - k_2)$$

- We get

$$\Delta W = q2\pi \int_{-\infty}^{+\infty} \begin{Bmatrix} J_0(Kr) & k^2 < \dfrac{\omega^2}{c^2} \\ I_0(Kr) & k^2 > \dfrac{\omega^2}{c^2} \end{Bmatrix} \left[T(k)\cos\phi_0 - S(k)\sin\phi_0\right]\delta(k - \bar{k})\, dk$$

$$= q2\pi\left[T(\bar{k})\cos\phi_0 - S(\bar{k})\sin\phi_0\right] I_0(\bar{K}r)$$

$$\bar{K}^2 = \left|\frac{\omega^2}{c^2} - \bar{k}^2\right|$$

Where we have ignored the dependence of r upon z.

Energy Gain

Computing Work with Longitudinal Electric Fields

- One final adjustment: note that for the Fourier cosine transform

$$T(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E_z(0,z)\cos 0\, dz = \frac{V}{2\pi}$$

where V is the potential across the gap.
- This is the largest value for both $T(k)$ and $S(k)$ and we usually normalize to it.
  - That is, $T \rightarrow 2\pi T(k)/V$
    $S \rightarrow 2\pi T(k)/S$

Thus…

$$\Delta W = qV\left[T(\bar{k})\cos\phi_0 - S(\bar{k})\sin\phi_0\right] I_0(\bar{K}r)$$

$$\bar{K} = \left[\bar{k}^2 - \frac{\omega^2}{c^2}\right]^{1/2} = \frac{\omega}{c}\frac{1}{\bar{\beta}\bar{\gamma}}$$

- The above is referred to as the Panofsky equation

Energy Gain

Interpreting the Panofsky Equation

$$\Delta W = qV\left[T(\bar{k})\cos\phi_0 - S(\bar{k})\sin\phi_0\right]I_0(\bar{K}r) \qquad \bar{K} = \frac{\omega}{c}\frac{1}{\bar{\beta}\bar{\gamma}}$$

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