RF Linac for High-Gain FEL

Introduction

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Class Content

I. Introduction to XFEL
II. Introductory Beam Physics
III. Photoinjectors
IV. RF Linac
V. Transverse & Longitudinal Dynamics
VI. Bunch Compression

I. Introduction to XFEL

1. 3rd and 4th Generation Light Sources
2. Major components of an XFEL
3. Normal-conducting RF linac
4. Superconducting RF linac
5. FEL rho Parameter
6. Electron Beam Requirements

3^{rd} Generation Light Sources

Storage-ring-based 3^{rd} Generation Light Sources are the tools of discovery for

- Life Science (e.g., structures of protein microcrystals)
- Chemistry (e.g., detecting chemical species at surfaces)
- Materials Science (e.g., phase contrast imaging)
- Condensed Matter Physics (e.g., studying materials under pressure)

See XDL-2011 “Workshop on Science at the Hard X-ray Diffraction Limit”

3^{rd} Generation Light Sources are electron storage-ring facilities producing synchrotron radiation. Synchrotron radiation can be generated in undulators (alternating dipoles with $K < 1$), wigglers ($K>>1$) or single dipole magnets.

The brightness of 3GLS is $10^{23} – 10^{25}$ x-ray photons/(s-mm^2 -mrad^2-0.1%BW). Emittance in x is set by the balance between radiation damping and quantum excitation due to the random nature of photon emission. The y emittance is 1-2% the x emittance due to residual coupling between the x and y motions.

3GLS Example
Advanced Photon Source

4^{th} Generation Light Sources

Linac-based 4^{th} generation light sources (4GLS) enable new experiments in

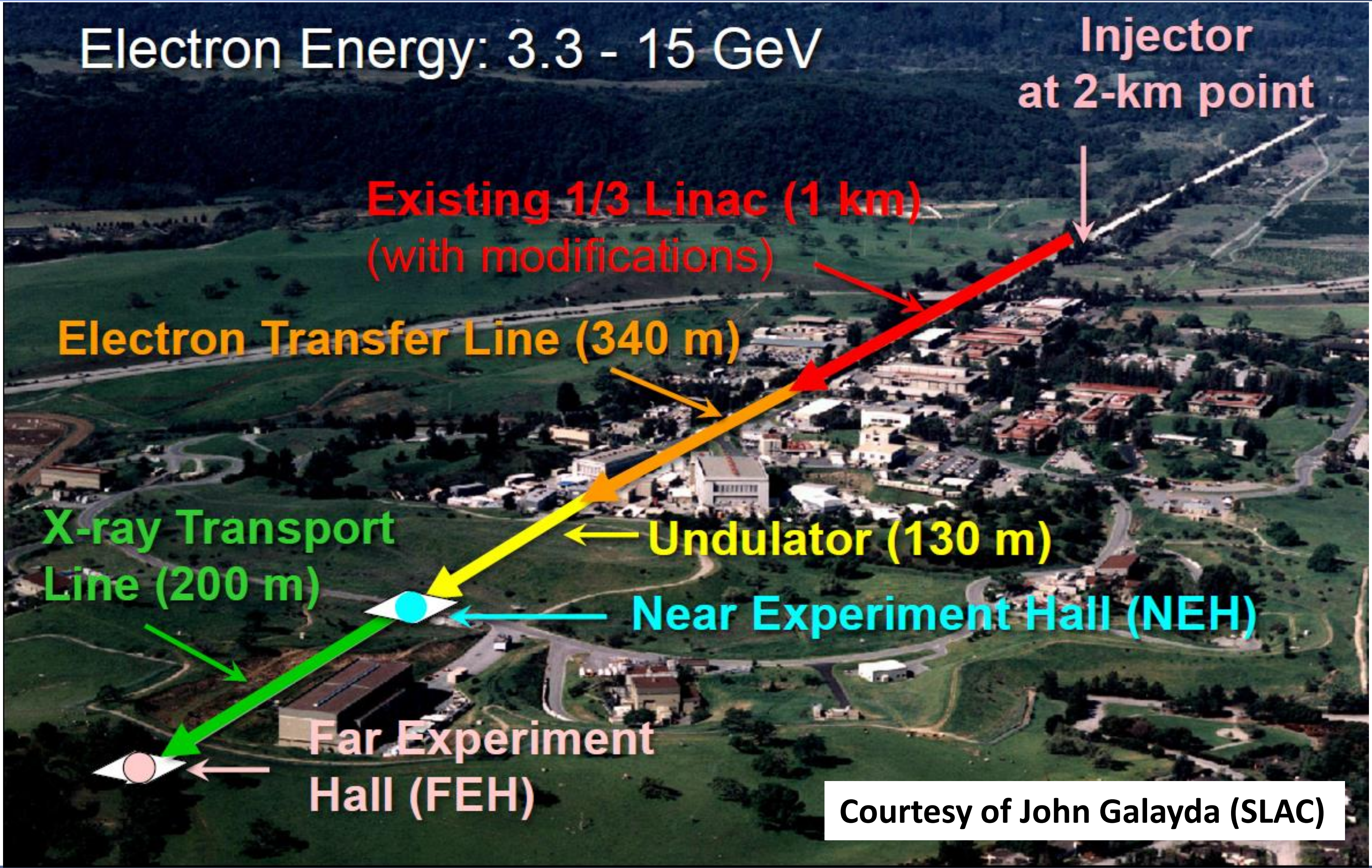
- Life Science (e.g., structures of protein nanocrystals)
- Chemistry (e.g., probing ultrafast dynamics of surface chemical reactions)
- Materials Science (e.g., 3D nanomorphology)
- Condensed Matter Physics (e.g., studying materials under extreme conditions)

For more applications, see “A Next Generation Light Source” LBNL CD0.

4GLS are RF linac-based X-ray FEL producing coherent, ultrafast x-ray pulses. 4GLS offer full transverse coherence, partial temporal coherence, 10^{10} - 10^{12} photons in fs pulses, and timing synchronization with an external laser.

The brightness of 4GLS is 10^{31} – 10^{33} x-ray photons/(s-mm^2 -mrad^2-0.1%BW). Emittance in x and y is less than $\lambda/4\pi$ so the x-ray beams have full transverse coherence. On the downside, each beamline of the 4GLS provides x-ray beams to only one or two users each time.

4GLS Example
Linac Coherent Light Source



Peak Brightness (Brilliance)

Peak brightness

$$B_{pk} = \frac{N_p}{(2\pi\varepsilon_x)(2\pi\varepsilon_y)\Delta t(\Delta\omega/\omega)}$$

N_p = number of photons

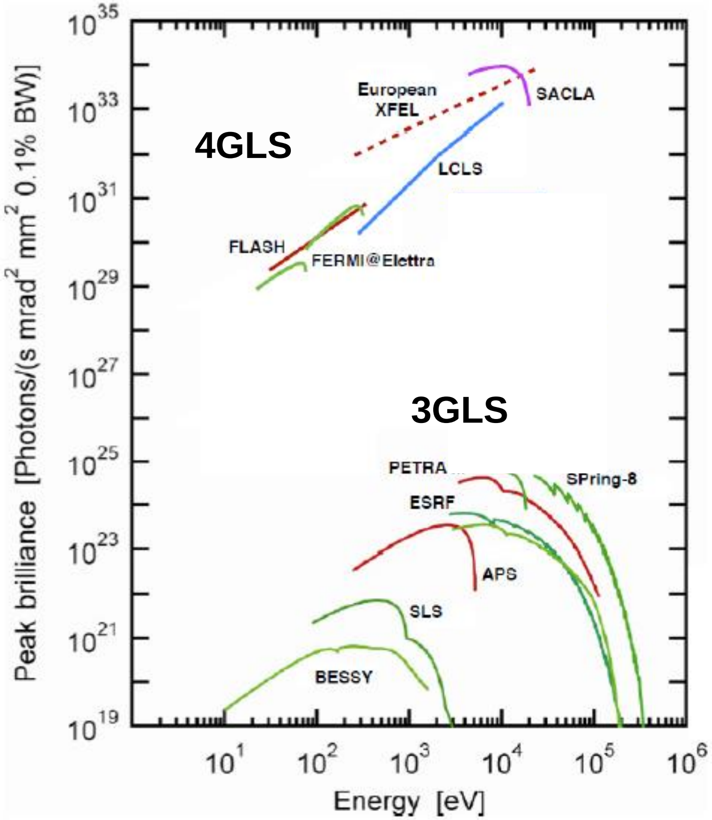
$\varepsilon_{x,y}$ = emittance in x, y

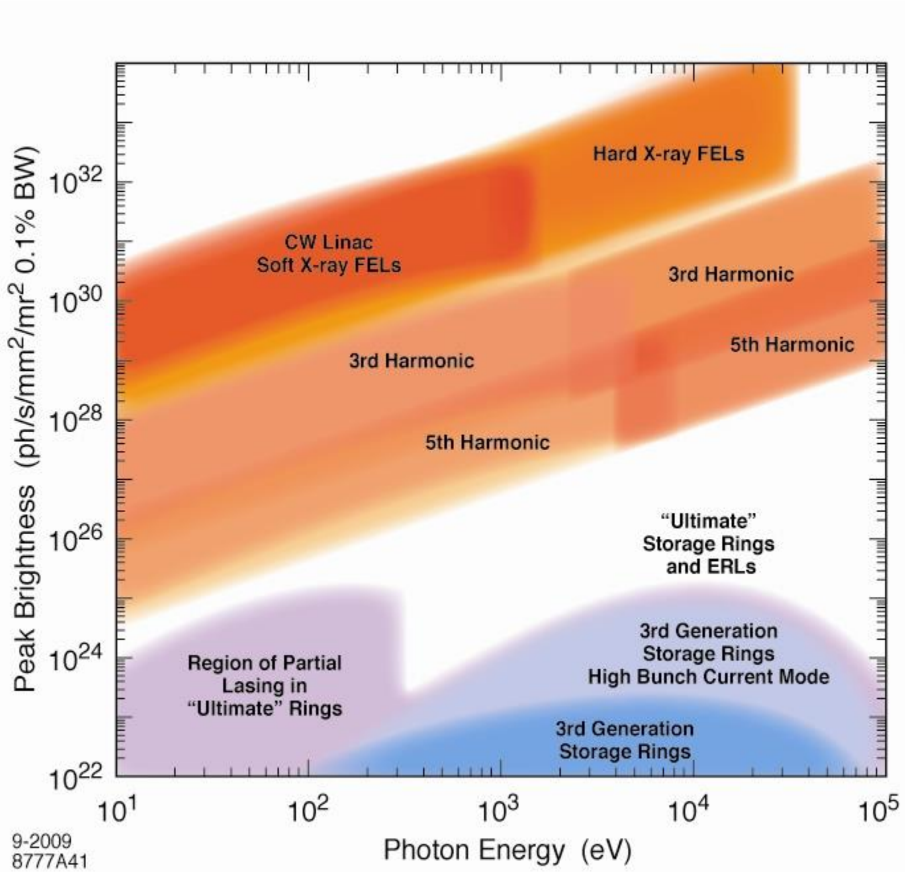
Δt = pulse length

$\Delta\omega/\omega$ = relative bandwidth (selected to be 0.1%)

FEL brightness is enhanced over SR brightness by the small emittance in x, the ultrashort pulse and by the FEL microbunching resulting in bunched beam emission that scales with N_e^2 (N_e = number of electrons/coherence length).

Brightness of FEL and SR





Peak brilliance of linac-based 4th generation light sources (XFEL) is ten orders of magnitude above that of the 3rd generation light sources and more than 20 orders of magnitude above Bremsstrahlung sources.

Review of relativistic formulas

Lorentz relativistic factor is equal to the ratio of the electron beam's total energy to electron rest mass energy (0.511 MeV).

$$\gamma = \frac{E_b}{m_e c^2} = \frac{E_k}{m_e c^2} + 1$$

β is the beam velocity relative to the speed of light, $\beta = \frac{\upsilon}{c}$.

Expressing β as a function of γ

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \longrightarrow \beta = \left(1 - \frac{1}{\gamma^2}\right)^{\frac{1}{2}}$$

$$\beta \approx 1 - \frac{1}{2\gamma^2}$$ for γ >> 1

XFEL emit tunable coherent x-rays

XFEL produce beams of coherent x-rays that are tunable by adjusting the electron beam energy, the undulator period (not common) or magnetic field.

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

where

$\lambda_u =$ undulator period

$\gamma =$ Lorentz relativistic factor of e- beam

K = dimensionless undulator field

$B_0 =$ on-axis magnetic field amplitude

$k_u = \frac{2\pi}{\lambda_u}$ undulator wavenumber

$$K = \frac{eB_0}{m_e c k_u}$$

$$K = 0.9336 \cdot B_0[T] \cdot \lambda_u[cm]$$

Current XFEL mostly operate in single-pass high-gain SASE

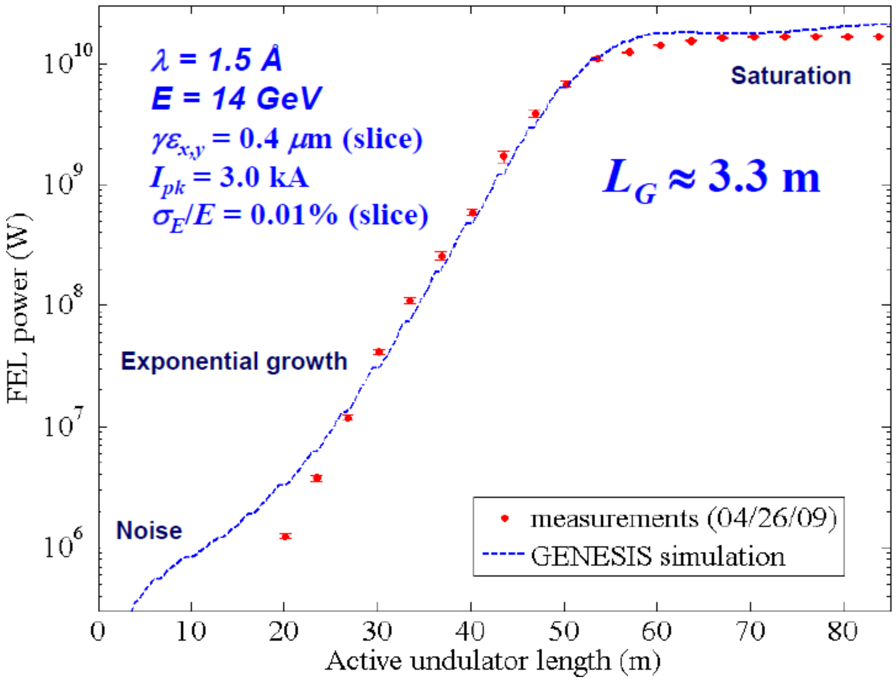
Self-Amplified Spontaneous Emission (SASE) starts up from noise and grows exponentially along the undulator length until the FEL power saturates at L ~ 20 gain lengths.

L_G = power gain length, undulator length over which power grows by *e* (2.7)

P_S = FEL power at saturation

$$P_S = \rho \frac{I_{pk} E_b}{e}$$

Courtesy of John Galayda (SLAC)

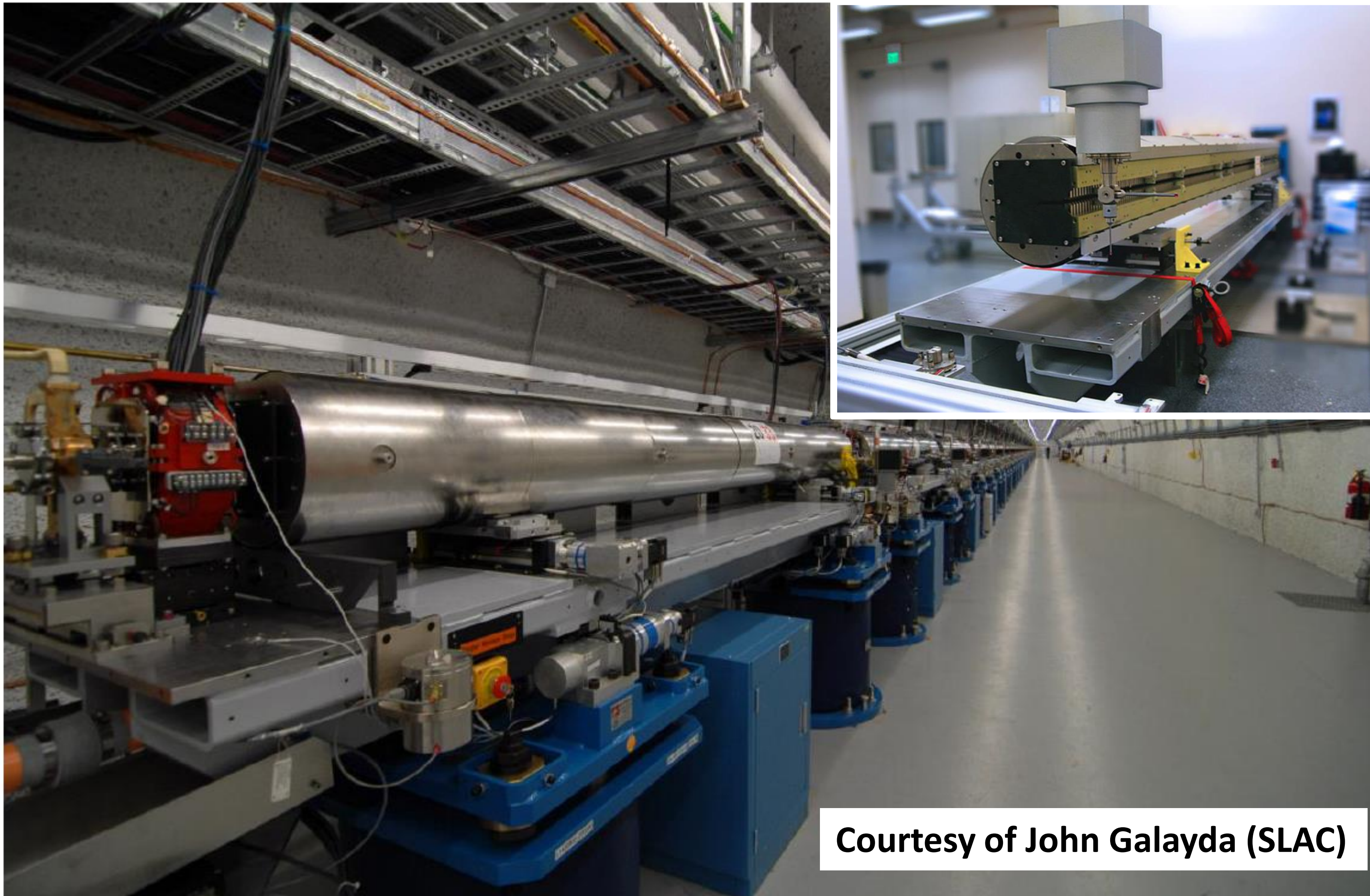


Linac Coherent Light Source first lasing

$\lambda_u = 3cm \qquad \rho \approx .0006$

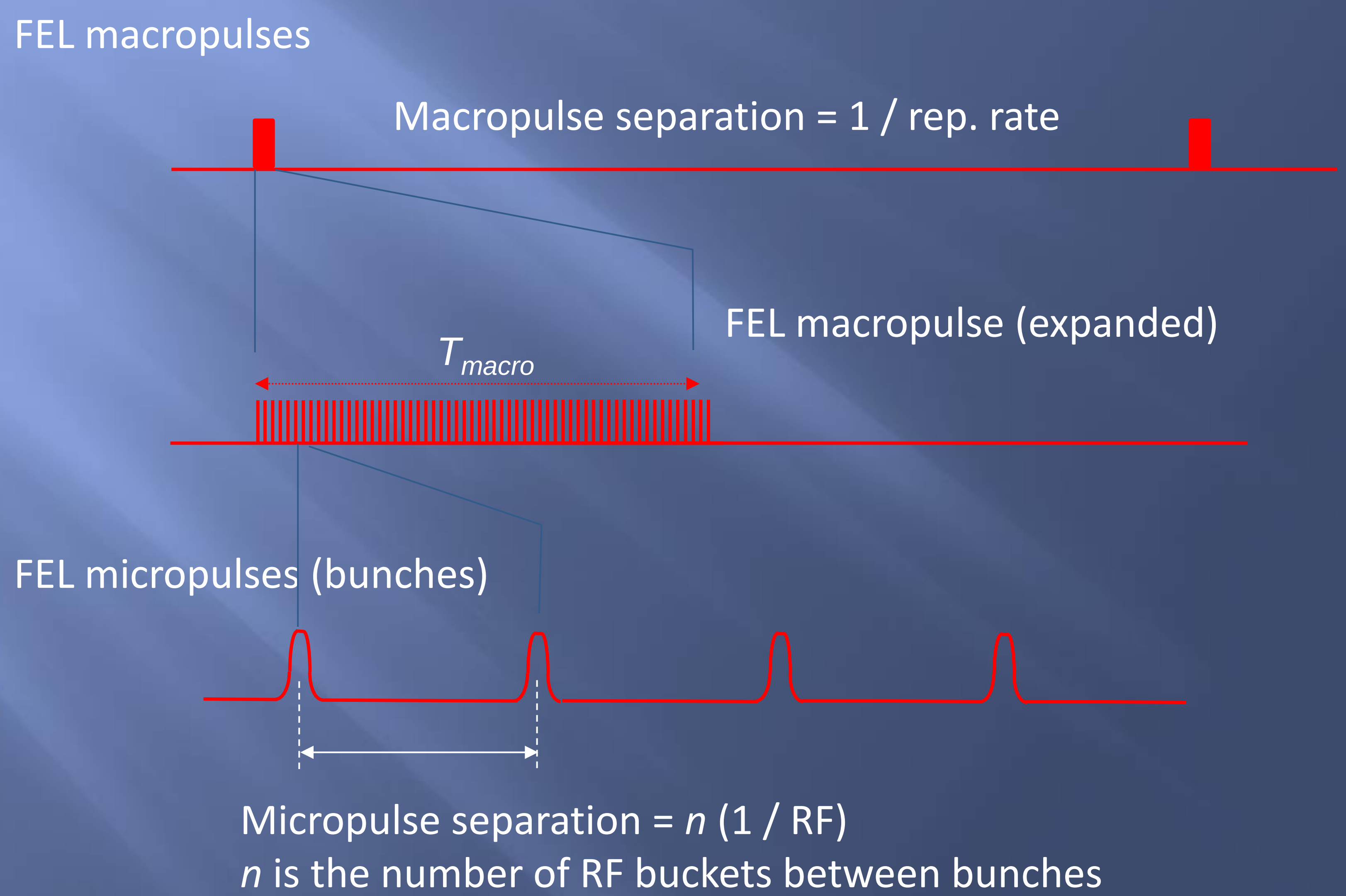
$L_G^{3D} = 3.3m \qquad P_S \approx 30GW$

LCLS-I Undulators



Courtesy of John Galayda (SLAC)

RF-linac FEL Pulse Format



Representative 4GLS Facilities

	FLASH European XFEL	LCLS-I	SACLA
Wavelength X-ray energy	450 –1 Å 0.3 –12 keV	25 –1.2 Å 0.5 – 10 keV	2.3 – 0.8 Å 5 – 15 keV
Beam energy	0.23 –17.5 GeV	3.3 –14 GeV	8 GeV
Linac type Frequency Linac length	SRF 1.3 GHz 2.1 km	NCRF 2.856 GHz 1 km	NCRF 5.712 GHz 0.4 km
Gun type, frequency Cathode	NCRF, 1.3 GHz Cs_2Te photocathode	NCRF, 2.856 GHz Cu photocathode	Pulsed DC gun CeB_6 thermionic
Bunch charge	130 – 1,000 pC	20 – 250 pC	200 pC
Bunch length	70 – 200 fs	5 - 500 fs	100 fs
rms emittance	0.4 – 1 μm	0.13 – 0.5 μm	0.6 μm
Bunches/macropulse Bunch spacing	2,700 222 ns	1	1
T_{macro} Repetition rate	600 μs 10	120	<60

Components of an RF-linac XFEL

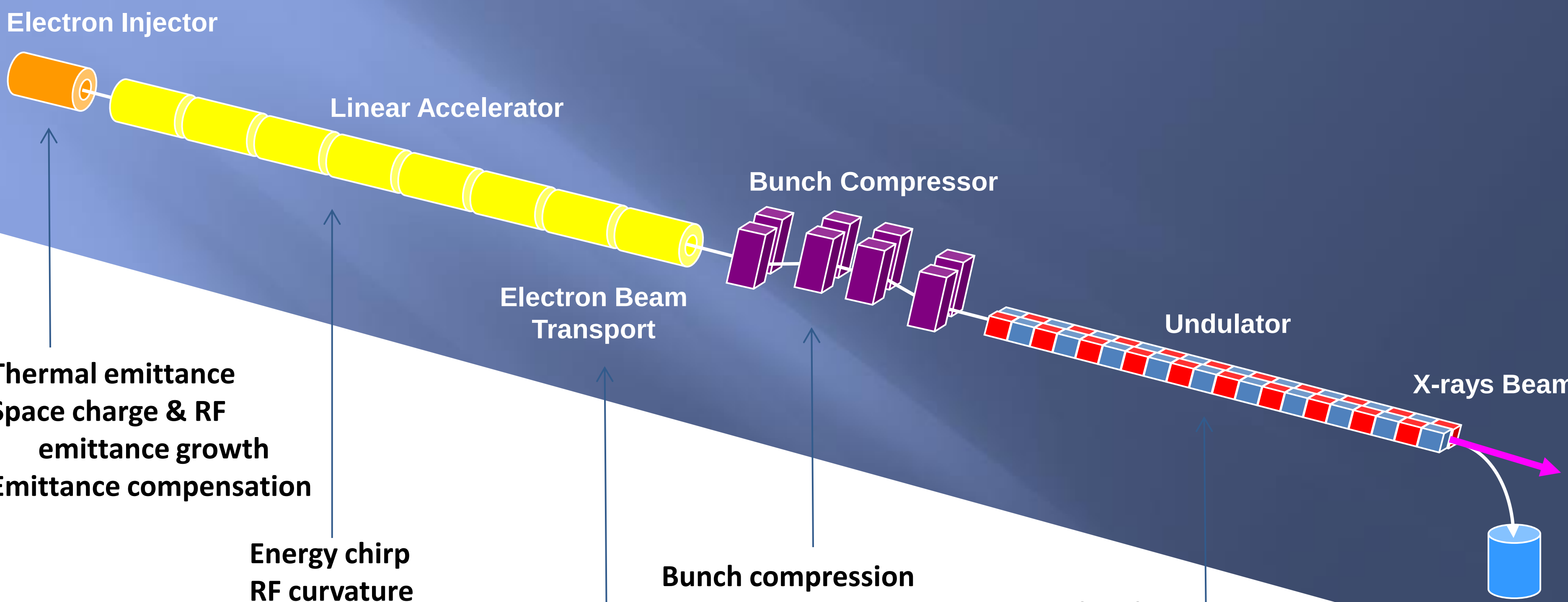
An RF-linac driven XFEL has the following major components:

- **PHOTOINJECTOR** to generate high-brightness electron beams
- **RF LINAC** to accelerate the beams to GeV energies
- **BEAM OPTICS** to transport the high-brightness electron beams
- **BUNCH COMPRESSORS** to produce the kA peak current
- Long undulators with electron beam focusing optics
- X-ray optics to transport the x-ray beams to experimental stations

We shall limit the scope of this course to the reviews of the first four topics. Even so, we can only cover the very basic since each of these four topics can easily require a one-week course at USPAS.

Our focus is the **BRIGHT ELECTRON BEAMS** to drive an x-ray FEL.

Important Beam Physics Challenges



Thermal emittance
Space charge & RF emittance growth
Emittance compensation

Energy chirp
RF curvature
Wake-fields

Bunch compression
Coherent synchrotron radiation (CSR)
Microbunching instabilities (μBI)

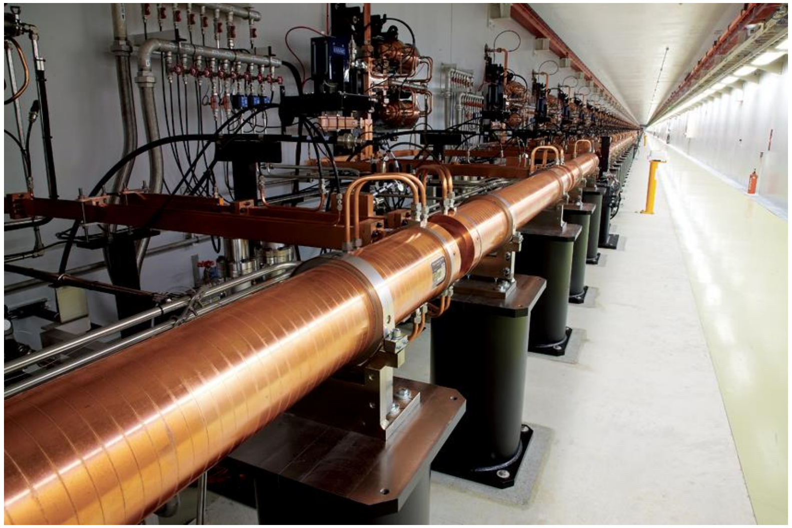
Brightness (emittance) preservation
Energy spread mitigation

Incoherent synchrotron radiation (ISR)
Resistive wake-fields
XFEL longitudinal coherence

Normal-conducting RF Linac

C-band klystron

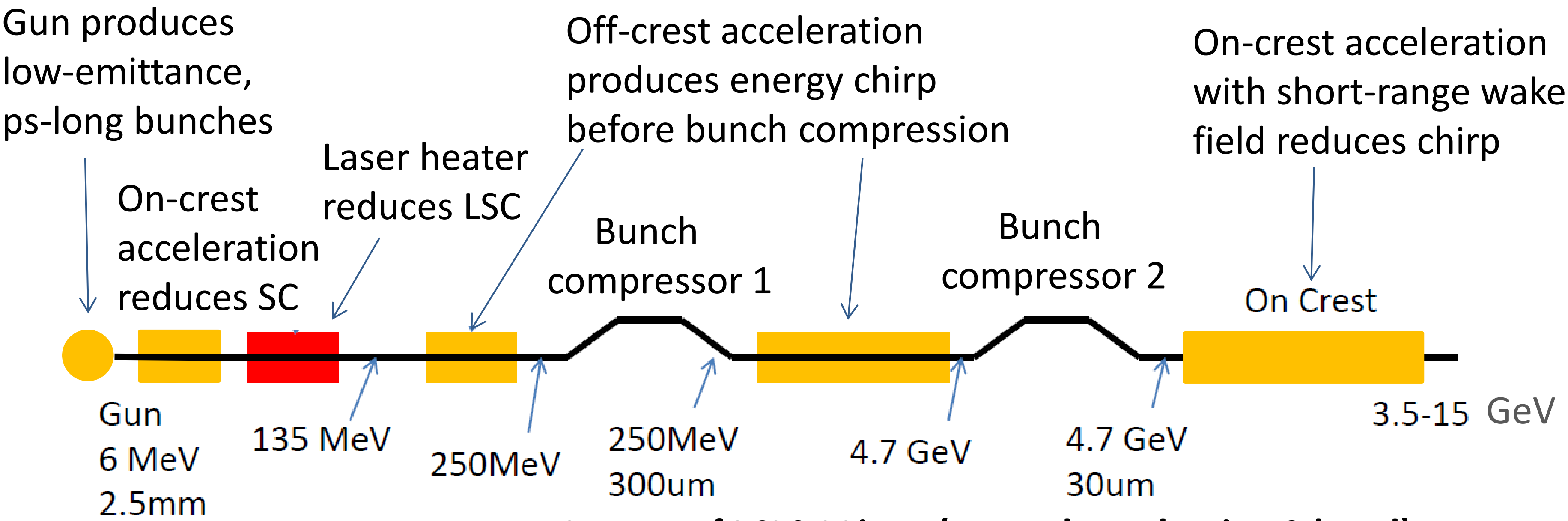
SACLA C-band linac



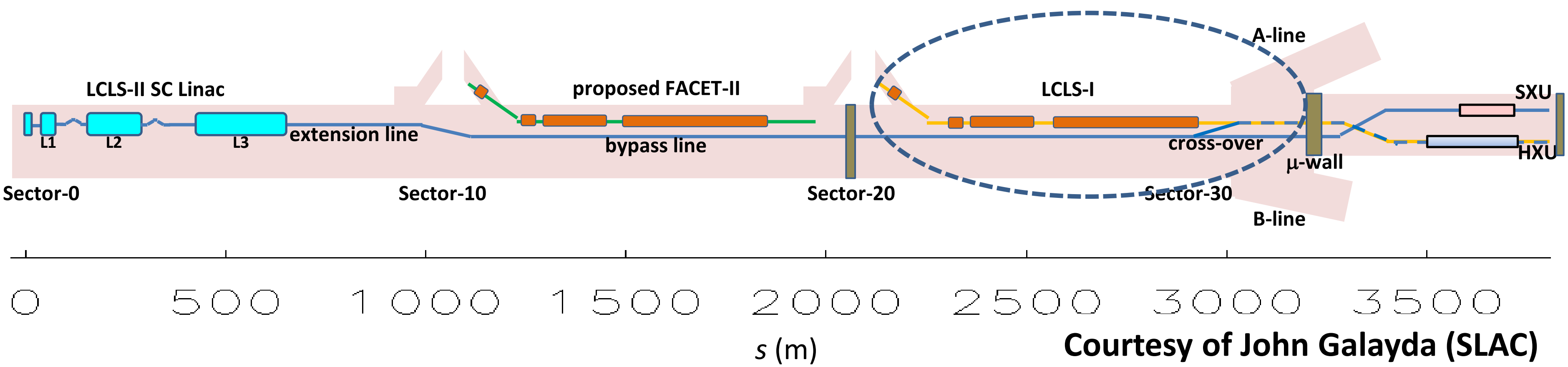
Typical operating conditions

- Water-cooled copper at room temperature
- Low duty factor (pulsed operation at low pulse repetition rate)
- Accelerating gradients limited by the available RF power
- Low RF efficiency (RF to beam power) due to surface ohmic losses
- Efficiency improves at higher frequency, limited by BBU and available klystrons
- Requiring high-power (MW) pulsed klystrons or RF compression.

LCLS-I: NC S-band Linac



Layout of LCLS-I Linac (normal-conducting S-band)

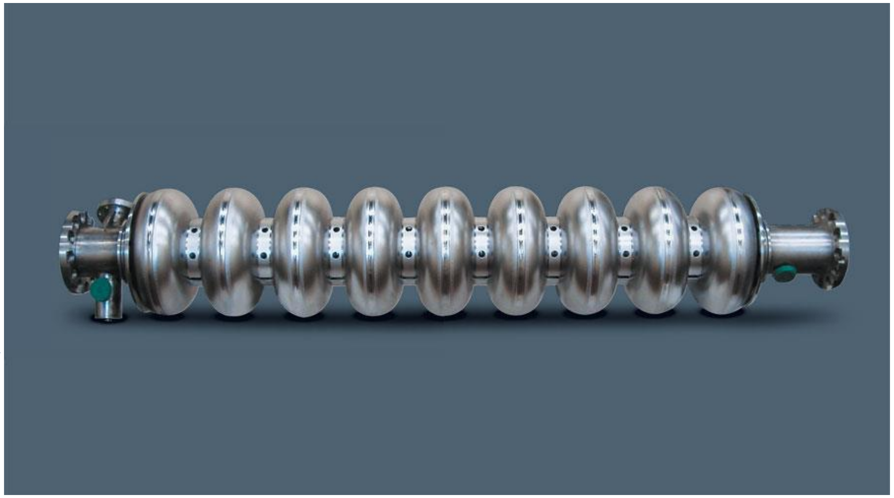


Courtesy of John Galayda (SLAC)

Superconducting RF Linac

L-band klystrons

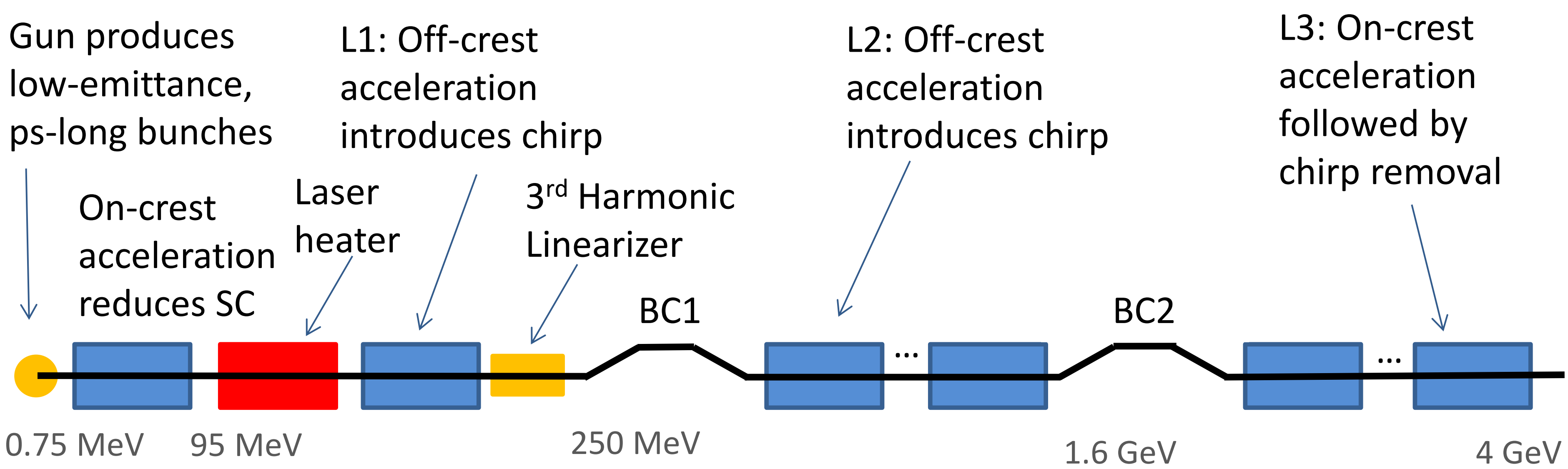
1.3-GHz SRF cavity



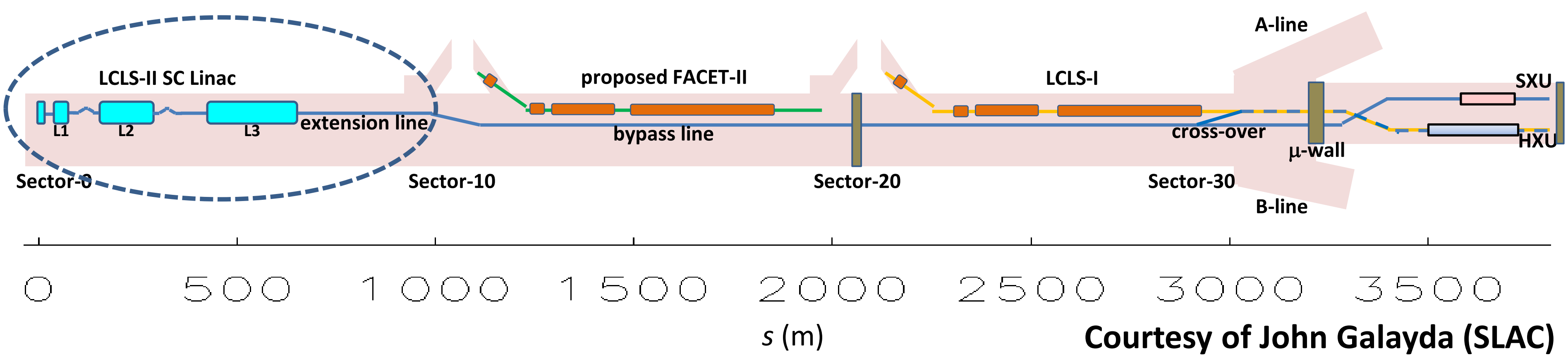
Typical operating conditions

- Niobium cooled with liquid helium at 2K (>500 MHz) or 4K (<500 MHz)
- High duty factor or continuous-wave (RF on all the time)
- Accelerating gradients set by RF dissipation which affects the cryoplant size
- Use little RF power (most of RF power goes into electron beams)
- RF loss due to surface resistance scales with f^2
- Requiring complex cryomodules, helium cryoplant and distribution system

LCLS-II: SC L-band Linac



Layout of LCLS-II Linac (mostly superconducting L-band)



Courtesy of John Galayda (SLAC)

FEL rho (Pierce) Parameter

The FEL ρ (Pierce) parameter determines the exponential gain and saturated efficiency of a high-gain FEL. The higher the beam energy (higher γ), the smaller ρ becomes. ρ scales with undulator K and period to the 2/3 power, and current density to the 1/3 power.

$$\rho = \frac{1}{2\gamma}\left(\frac{K \cdot JJ \cdot \lambda_u}{2\pi\sigma_b}\right)^{\frac{2}{3}}\left(\frac{I_{pk}}{I_A}\right)^{\frac{1}{3}}$$

- where

K = undulator parameter

$$JJ = J_0\left(\frac{K^2}{4+2K^2}\right) - J_1\left(\frac{K^2}{4+2K^2}\right)$$

$\lambda_u =$ undulator period

$I_{pk} =$ peak current

$\sigma_b =$ rms beam radius

$I_A =$ Alfven current (17 kA)

Power Gain Length

The FEL power gain length is the undulator length over which the FEL power grows by one e-folding (e = 2.7).

$$L_G = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$

A common mistake is to use the 1D ρ parameter to compute the gain length.

For XFEL, 3D effects (diffraction, emittance and energy spread) usually dominate the gain length calculations.

Typical causes of 3D effects that increase the gain length:

- Electron gun (e.g., poor choice of guns that lead to large emittance)
- Accelerators (e.g., advanced concepts that produce large energy spread)
- Bunch compression (e.g., CSR increases emittance and energy spread)
- Electron beam focusing (e.g., short beta function increases diffraction)

Optical Diffraction

The rms radius of a focused electron beam is determined by its un-normalized (geometric) emittance and the average β function of the focusing optics.

$$\sigma_b = \sqrt{\beta_{av}\varepsilon_u}$$

Optical diffraction is measured by the radiation Rayleigh length, which is given by the square of the electron beam's rms radius in the undulators divided by the photon beam's emittance, $\lambda/4\pi$.

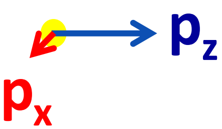
$$z_R = \frac{4\pi\sigma_b^2}{\lambda}$$

For diffraction 3D effect to be small, the gain length must be shorter than the Rayleigh length

$$L_G \leq z_R$$

Electron Beam Emittance

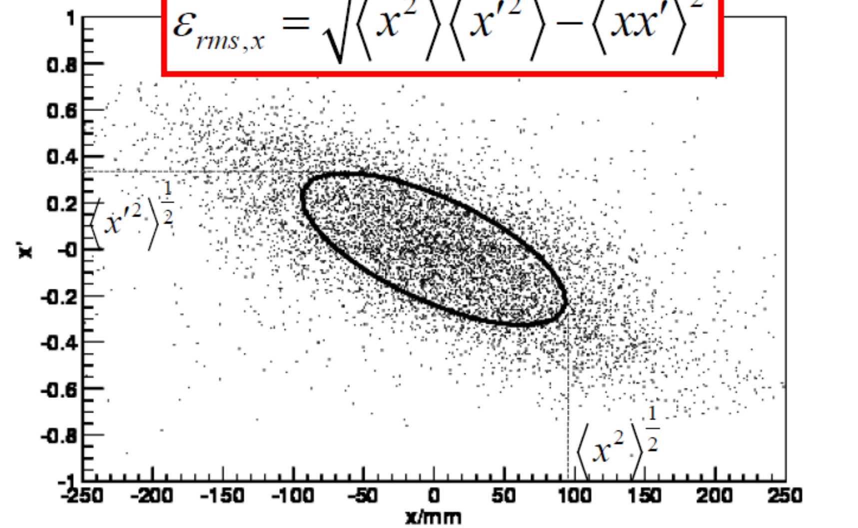
x and z momenta at low energy



$$x' = \frac{p_x}{p_z}$$

x and z momenta at high energy

$$\varepsilon_{rms,x} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$



Acceleration reduces *x′* by boosting p_z and reduces the geometric emittance

Photon beam emittance $\frac{\lambda}{4\pi}$

$$\varepsilon_u = \frac{\varepsilon_n}{\beta\gamma} \leq \frac{\lambda}{4\pi}$$

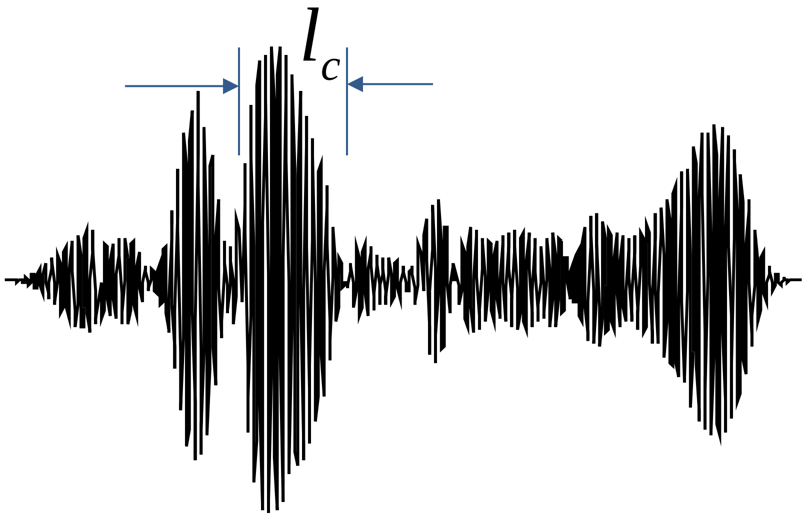
For emittance 3D effect to be small, the electron beam's un-normalized (geometric) emittance must be smaller than the photon beam emittance.

Required Energy Spread

Electrons must maintain the same axial velocity during the coherence length

$$l_c \approx N_c \lambda$$

$$N_c = \frac{1}{4\pi\rho}$$



where N_c = # of wavelengths in a coherence length

= √3 times # of periods in one gain length

For 3D effect due to energy spread to be small, the relative rms energy spread must be less than ρ

$$\frac{\sigma_\gamma}{\gamma} \leq \frac{1}{4\pi N_c} \longrightarrow \frac{\sigma_\gamma}{\gamma} \leq \rho$$

Conditions to minimize 3D Effects

Diffraction

$$L_G \le z_R \longrightarrow \frac{L_G \lambda}{4\pi} \le \frac{\beta_F \varepsilon_n}{\gamma}$$

Emittance

$$\varepsilon_u \le \frac{\lambda}{4\pi} \longrightarrow \varepsilon_n \le \frac{\gamma\lambda}{4\pi}$$

Energy spread

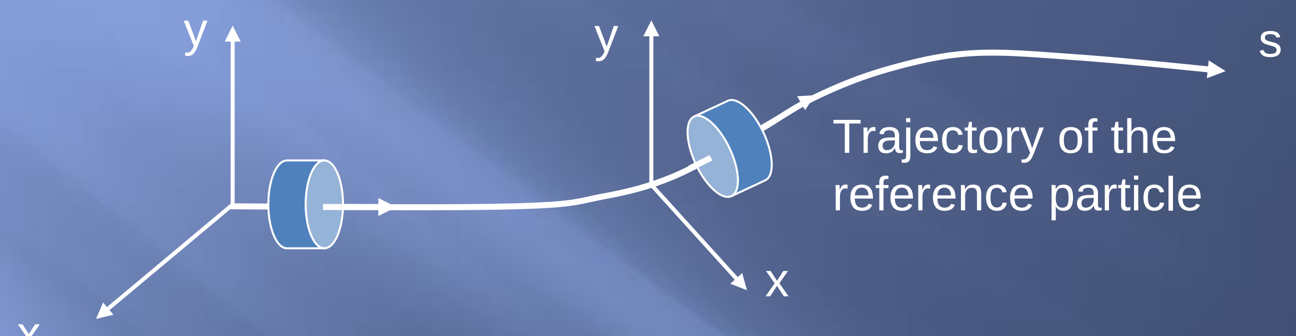
$$\frac{\sigma_\gamma}{\gamma} \le \frac{1}{4\pi N_c} \longrightarrow \frac{\sigma_\gamma}{\gamma} \le \rho$$

Violations of the above will lead to smaller 3D ρ, thus longer gain length.

II. Introductory Beam Physics

1. Lorentz Force
2. Beam Optics
3. Matrix Representations
4. Twiss (Courant-Snyder)
5. Beta Function
6. Emittance
7. Louisville Theorem

Beam Coordinates



The (x, y, s) coordinate system follows the reference electron, an ideal particle at the beam center.

A particle is characterized by

- its x(s) and y(s) which are deviations from the reference trajectory
- its slope x’(s) and y’(s) with respect to the reference trajectory
- its longitudinal position z where z = 0 is the bunch centroid along s
- its energy or momentum deviation $\Delta p/p_0$

Lorentz Force

Lorentz force determines the rate of change in the beam's energy and momentum

$$\mathbf{F} = -e\left(\mathbf{E} + \boldsymbol{\upsilon} \times \mathbf{B}\right)$$

Force caused by an electric field acting along the propagation direction changes the beam's energy

$$\Delta W = \int \mathbf{F} \cdot d\mathbf{s} = -\int e\mathbf{E} \cdot d\mathbf{s}$$

Force caused by a magnetic field perpendicular to the propagation direction changes the beam's momentum by Δp and thus change the direction by $\Delta p/p_0$

$$\Delta\mathbf{p} = \int \mathbf{F}dt = -\int e\left(\boldsymbol{\upsilon} \times \mathbf{B}\right)dt$$

Beam Optics

Dipoles

- Dipoles bend the beam and change its direction of propagation
- Dipoles also disperse the beam that has an energy spread
- Four dipoles (up, down, down, up) make a chicane bunch compressor. Chicanes are used to compress electron bunches (high peak current).

Quadrupoles

- Quadrupoles focus the beam in one plane and defocus in the other
- Three quads (triplet) are used to focus electron beams in both x and y.
- Focusing (F) and defocusing (D) quads are placed periodically to form a FODO lattice, where O denotes a drift or a non-focusing element.

Sextupoles

- Sextupoles are used to correct for chromatic aberrations or to linearize the longitudinal phase space affected by RF curvature.

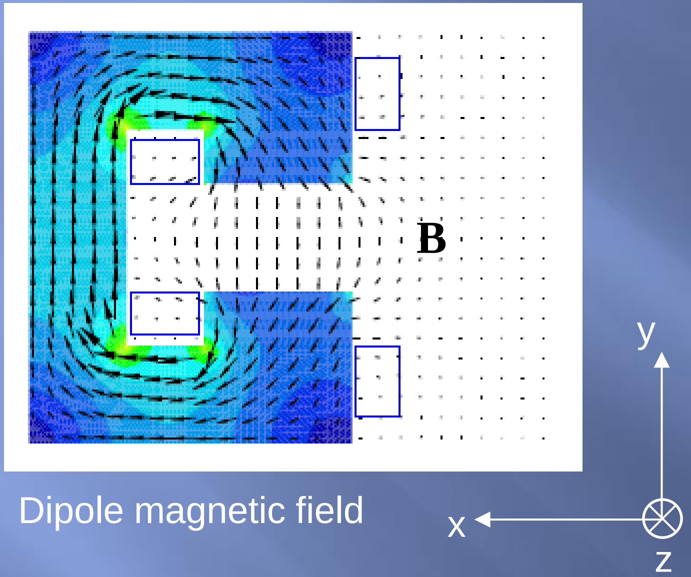
Matrix Representation

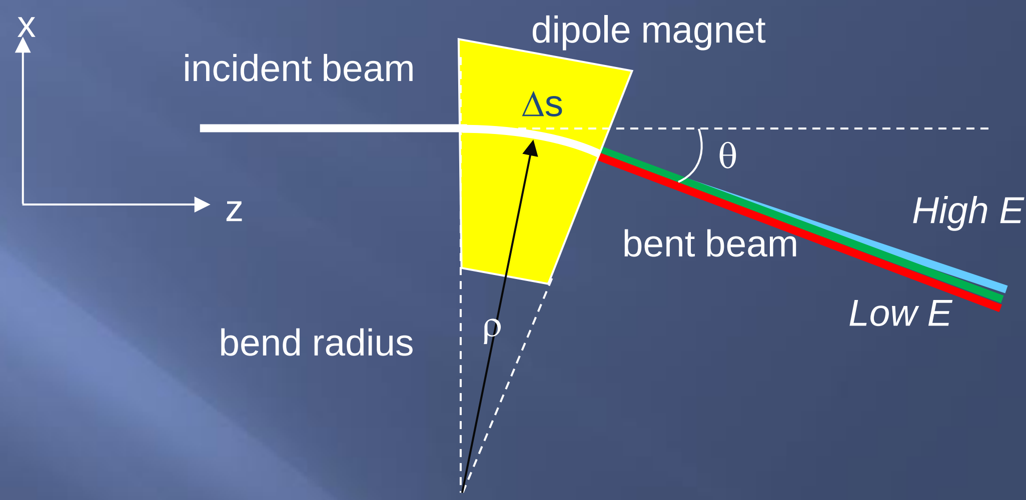
- Each particle is represented by a 1 x 6 vector.
- The beam transfer matrix maps the vector at location *n* into a new vector at location *n+1* (ignoring what happens in between).
- The R matrix is usually block diagonal with three 2x2 blocks (red borders) that are non-zero; the rest (shaded gray) are mostly zero.

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ ct \\ p \end{pmatrix}_{n+1} = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ ct \\ p \end{pmatrix}_{n}$$

- The matrix approach only deals with linear mapping of the beams. Higher order terms can be included to account for nonlinearities.

Dipole Magnet





Magnetic field $\mathbf{B} = B_0 \hat{\mathbf{y}}$

Bend radius $\dfrac{1}{\rho} = \dfrac{eB_0}{p_0}$

since $p_0 \approx \dfrac{E_b}{c}$ → $\dfrac{1}{\rho[m]} \approx \dfrac{0.3B_0[T]}{E_b[GeV]}$

Dispersion

$$\sigma_x = \eta_x \sigma_\delta$$

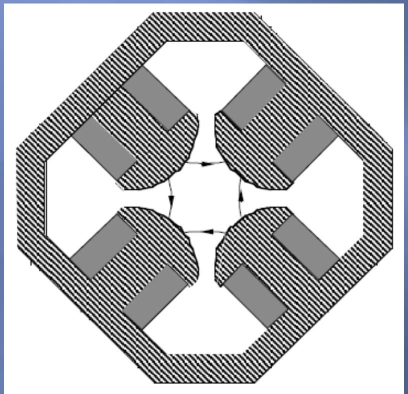
Relative energy deviation

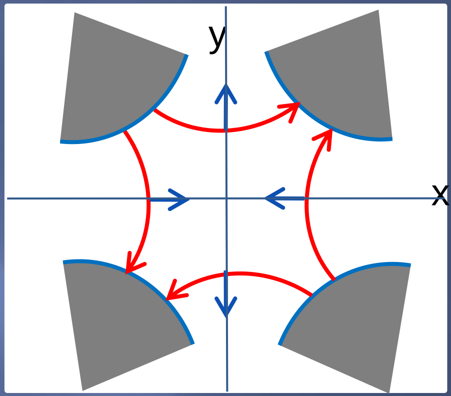
$$\sigma_\gamma = \left. \frac{E - E_0}{E_0} \right|_{rms}$$

Quadrupole Magnet

A quadrupole is a focusing element in one plane and defocusing in the other plane. By convention, an F quad focuses the beam in x and a D quad focuses the beam in y.

F quad





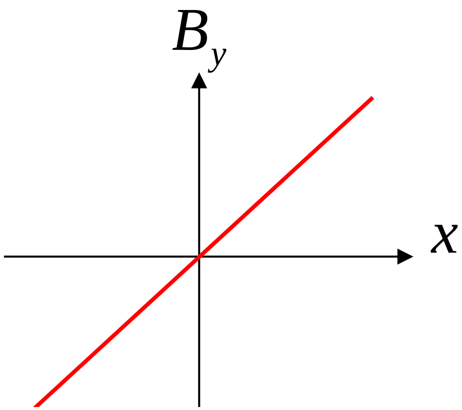
F quad magnetic field (red) and Lorentz force (blue) on electrons

Magnetic field
$$B_x = G_0 y$$
$$B_y = G_0 x$$

where G_0 : field gradient in T/m

Quad focusing strength $$K_x = \frac{e}{p_0}\frac{dB_y}{dx} = \frac{e}{p_0}G_0$$

Quad focal length $$f = \frac{1}{K \cdot l}$$

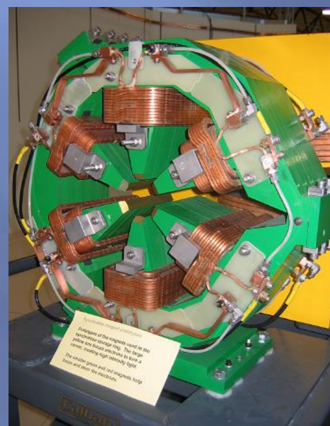


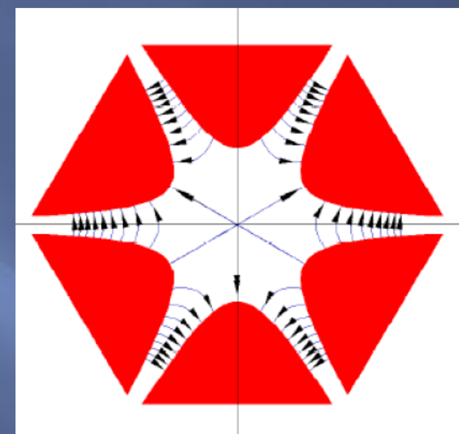
Magnetic field in y increases linearly with x, the deviation from center.

Sextupole Magnet

A sextupole is used to correct for chromatic aberrations or to linearize the longitudinal phase-space that has a quadratic energy-time correlation.

Sextupole

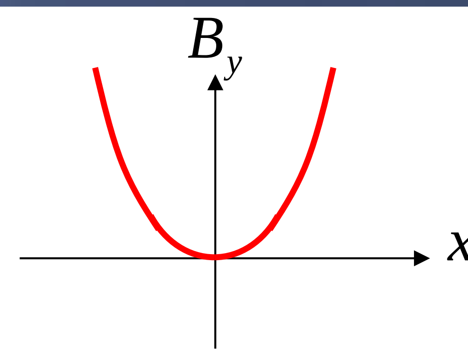




Sextupole field

Magnetic field
$$B_x = Cxy$$
$$B_y = \frac{1}{2}C\left(x^2 - y^2\right)$$

where C: sextupole strength in T/m^2



Magnetic field in y at y = 0 increases with x^2 where x is the distance from center.

Hill's Equation

Write a more general homogeneous differential equation to describe the quasi-harmonic motion of u (u can be either x or y). This is called Hill's equation (applicable for systems that are periodic in s).

$$u'' + K_u(s)u = 0$$

For the initial conditions

$u(0) = u_0$ $\quad\quad$ $u'(0) = u'_0$

the solution is

$$u(s) = C(s)u_0 + S(s)u'_0$$

Focusing quad $K > 0$

$$C(s) = \cos\left(\sqrt{K}s\right)$$

$$S(s) = \frac{1}{\sqrt{K}}\sin\left(\sqrt{K}s\right)$$

Defocusing quad $K < 0$

$$C(s) = \cosh\left(\sqrt{|K|}s\right)$$

$$S(s) = \frac{1}{\sqrt{|K|}}\sinh\left(\sqrt{|K|}s\right)$$

Matrix Representation in 2D

Solution to Hill's Equation

$$u(s) = C(s)u_0 + S(s)u'_0$$

Take the derivative

$$u'(s) = C'(s)u_0 + S'(s)u'_0$$

Rewrite both equations in the form of a transfer matrix

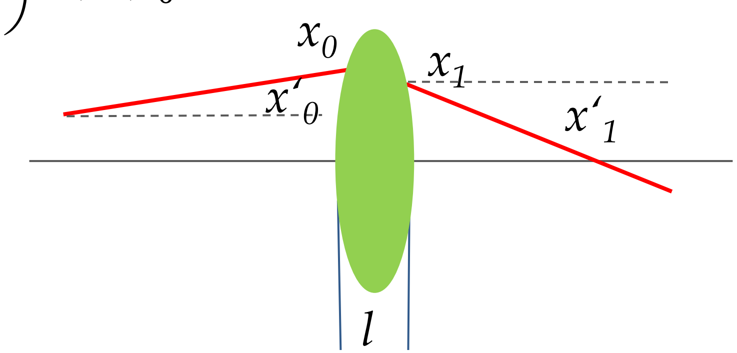
$$\begin{pmatrix} u(s) \\ u'(s) \end{pmatrix} = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix} \cdot \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$$

The transfer matrix is used in transport codes where one can map the beam from one location to another through *n* elements simply by concatenating (multiplying) the transfer matrices of these elements.

Transfer Matrix of a Focusing Quad in (x, x')

Consider the focusing quad (QF)

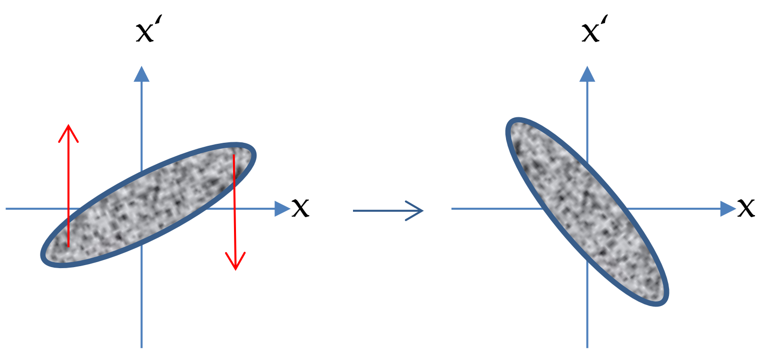
$$\begin{pmatrix} x \\ x' \end{pmatrix}_1 = \begin{pmatrix} \cos\left(\sqrt{K}l\right) & \frac{1}{\sqrt{K}}\sin\left(\sqrt{K}l\right) \\ -\sqrt{K}\sin\left(\sqrt{K}l\right) & \cos\left(\sqrt{K}l\right) \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_0$$



Effect of a quad on x and x'

<u>Thin lens approximation</u>: let the length of the quad go to zero while keeping the focal length constant at $f = (K \cdot l)^{-1}$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_1 = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_0$$



Effect of a quad on the x-x' space

Thin Lens Model of an F Quad

For an ideal quad and mono-energetic beams, all diagonal matrix elements are 1 and except R_{21} and R_{43}, all off-diagonal elements are 0.

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ ct \\ p \end{pmatrix}_{n+1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -\dfrac{1}{f} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{f} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ ct \\ p \end{pmatrix}_{n}$$

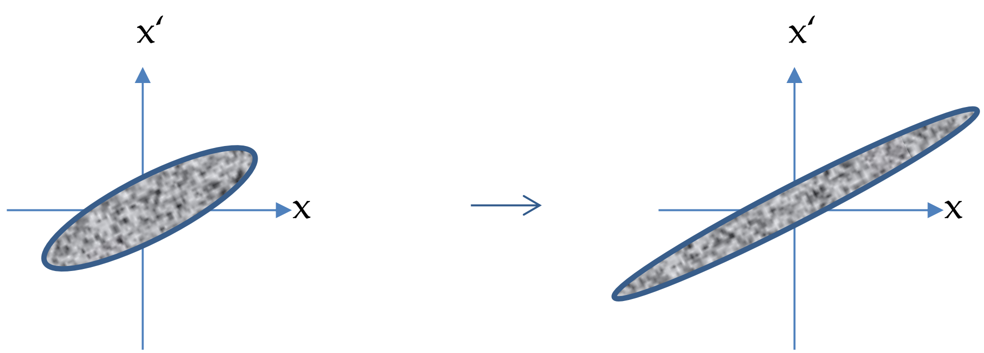
An inherent assumption is that the focal length does not depend on energy. This is obviously incorrect, since quads have energy-dependent focusing. To include chromatic effects, we must add off-axis elements into the matrix of the thick lens.

Thin Lens Model of a Drift

Consider a drift with distance L between points 1 and 2. The thin lens model for the drift is the limit of the transfer matrix when $K \to 0$.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \lim_{K\to 0} \begin{pmatrix} \cos\left(\sqrt{K}L\right) & \frac{1}{\sqrt{K}}\sin\left(\sqrt{K}L\right) \\ -\sqrt{K}\sin\left(\sqrt{K}L\right) & \cos\left(\sqrt{K}L\right) \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_1$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_1$$



Effect of a drift on the x-x’ space

Combining QF and Drift Matrices

Consider the combined map of a focusing quad with focal length *f* followed by a drift space with length L.

Transfer matrix of QF

$$\begin{pmatrix} x \\ x' \end{pmatrix}_1 = \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Transfer matrix of a drift length L

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1$$

$$\mathbf{x}_2 = \mathbf{M}_{Drift}\mathbf{M}_{Quad}\mathbf{x}_0$$

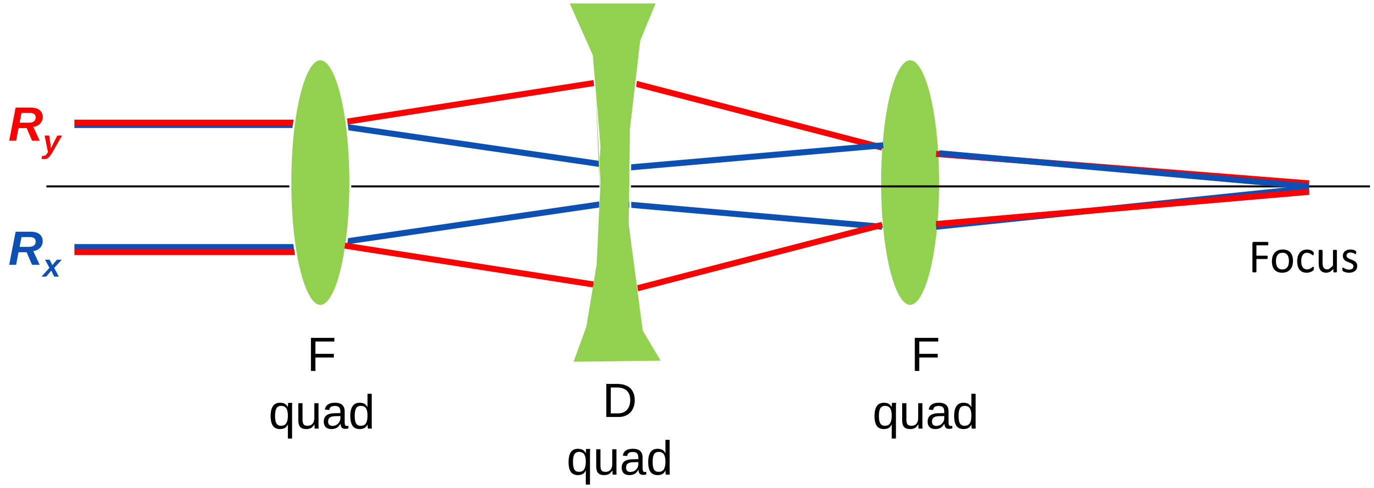
Note the order of matrix multiplication

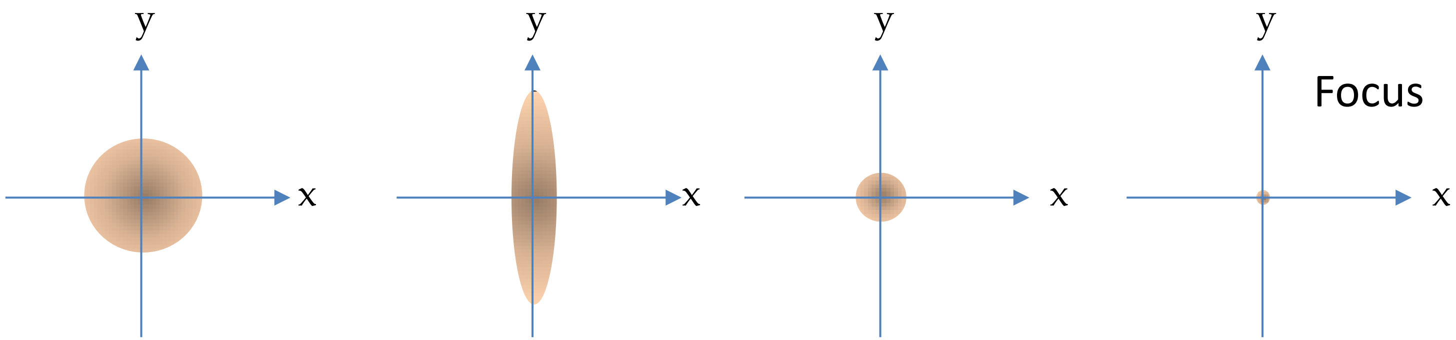
Combined transfer matrix

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \begin{pmatrix} 1-\dfrac{L}{f} & 1 \\ -\dfrac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Focusing with a Quad Triplet

A quad triplet consists of two F quads of equal focusing strength and a D quad with focusing strength twice that of the F quads.

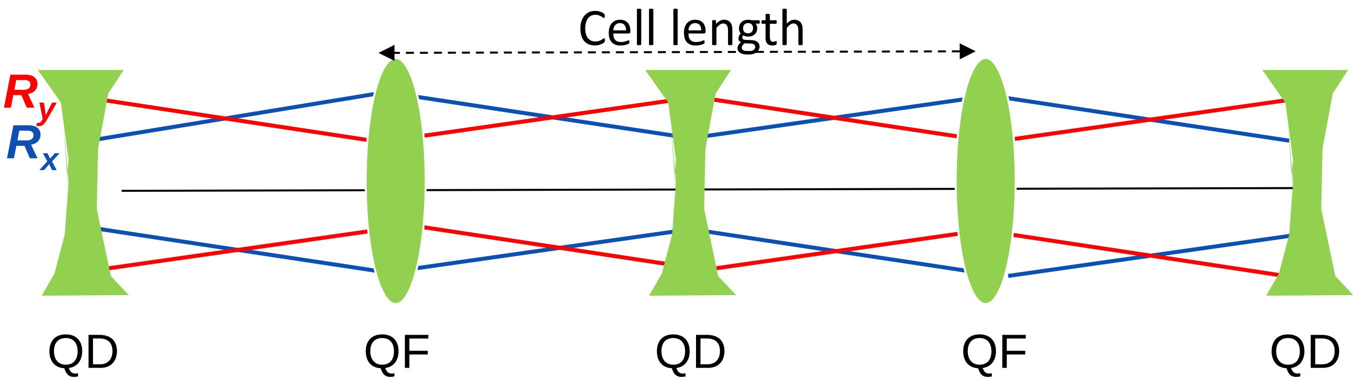




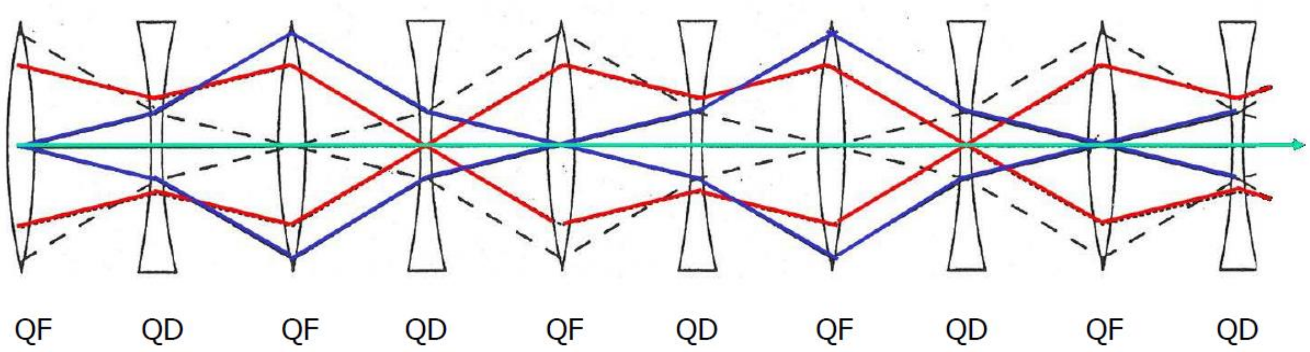
Evolution of electron beam's y-vs-x profile through a quad triplet

FODO Lattice

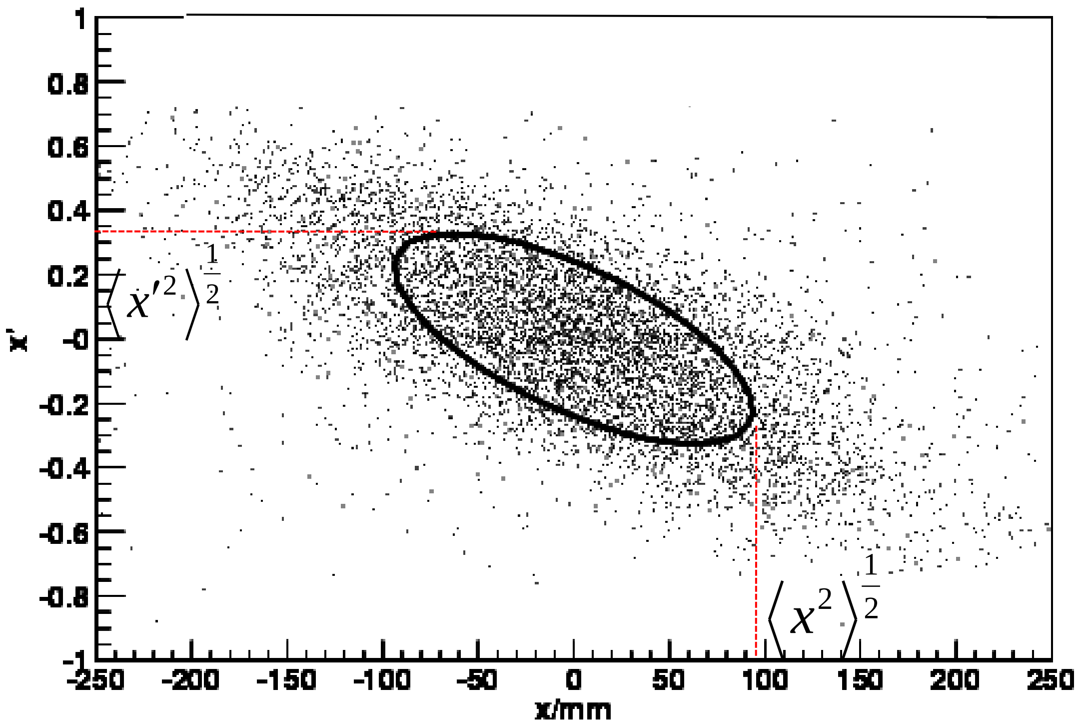
A FODO lattice consists of a periodic array of QF and QD of equal focusing strength separated by field-free drifts (e.g., accelerator sections or undulators) of equal length.



FODO lattice with 90° phase advance (one oscillation = 4 cells)



Particles in 2D Trace Space



Each particle is represented by a dot in the x-x' plot where x and x' the particle position and angle in x with respect to the principal trajectory. The above trace space corresponds to a converging beam.

Twiss Parameters

Beta function (beam size)

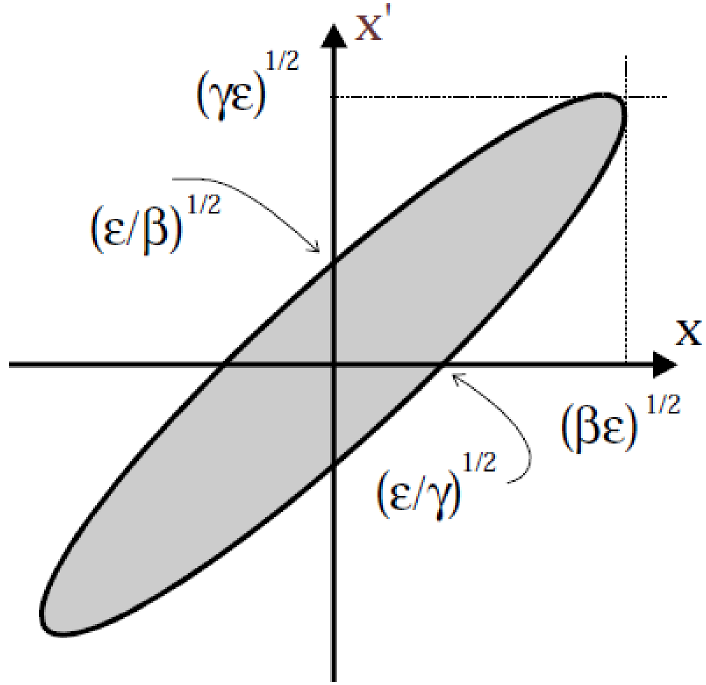
$$\beta = \frac{\langle x^2 \rangle}{\varepsilon}$$

Gamma function (beam divergence)

$$\gamma = \frac{\langle x'^2 \rangle}{\varepsilon}$$

Alpha function (phase-space angle)

$$\alpha = \frac{-\langle xx' \rangle}{\varepsilon}$$



Courant-Snyder invariant

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon$$

Only three of the four Twiss parameters (α, β, γ and ε) are independent.

rms Quantities

rms x^2 $$\langle x^2 \rangle = \int x^2 f(x, x', y, y') dx dx' dy dy'$$

rms x $$x_{rms} = \sqrt{\langle x^2 \rangle}$$

rms x'^2 $$\langle x'^2 \rangle = \int x'^2 f(x, x', y, y') dx dx' dy dy'$$

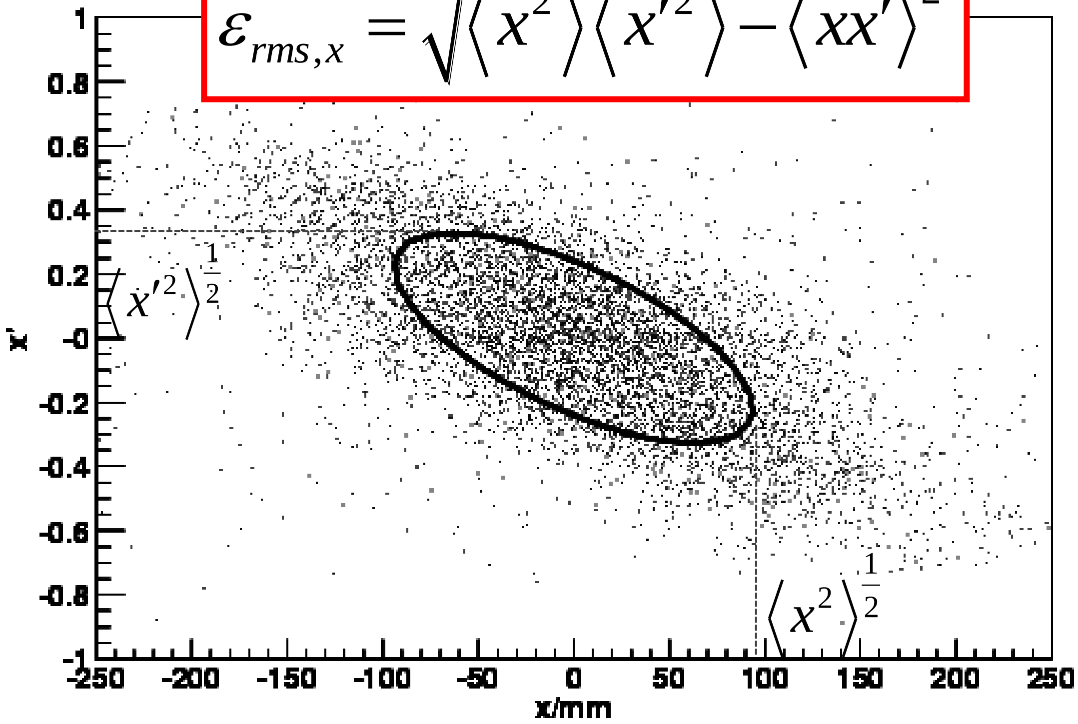
rms x’ $$x'_{rms} = \sqrt{\langle x'^2 \rangle}$$

Correlation term $$\langle x x' \rangle = \int x x' f(x, x', y, y') dx dx' dy dy'$$

The correlation term vanishes at the beam waist (upright x'-x ellipse).

Emittance

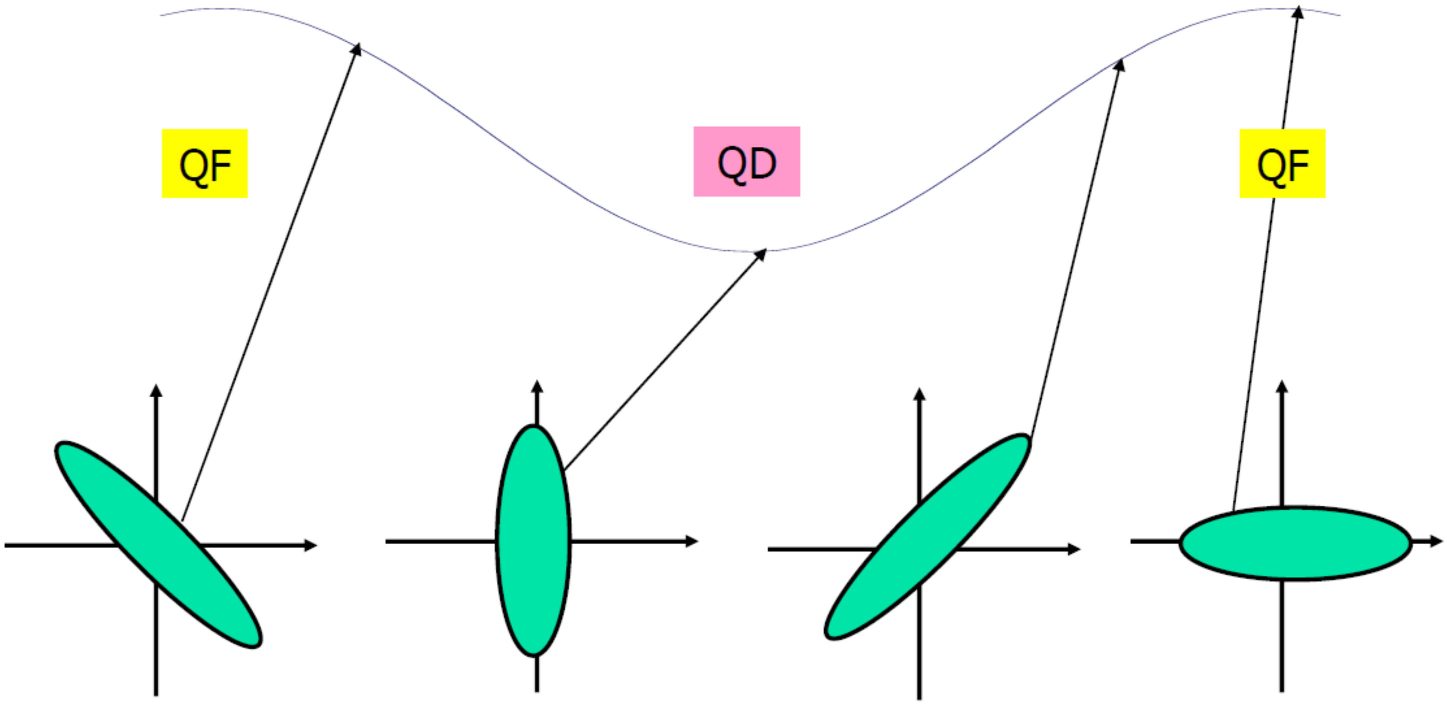
$$\varepsilon_{rms,x} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$



Beams are treated as a statistical distribution of particles in x'-x (also in y'-y and γ-ct) trace space . We can draw an ellipse around the particles such that 50% of the particles are found within the ellipse. The area of the ellipse is π times the rms beam emittance, which is defined in the equation above.

Emittance Conservation

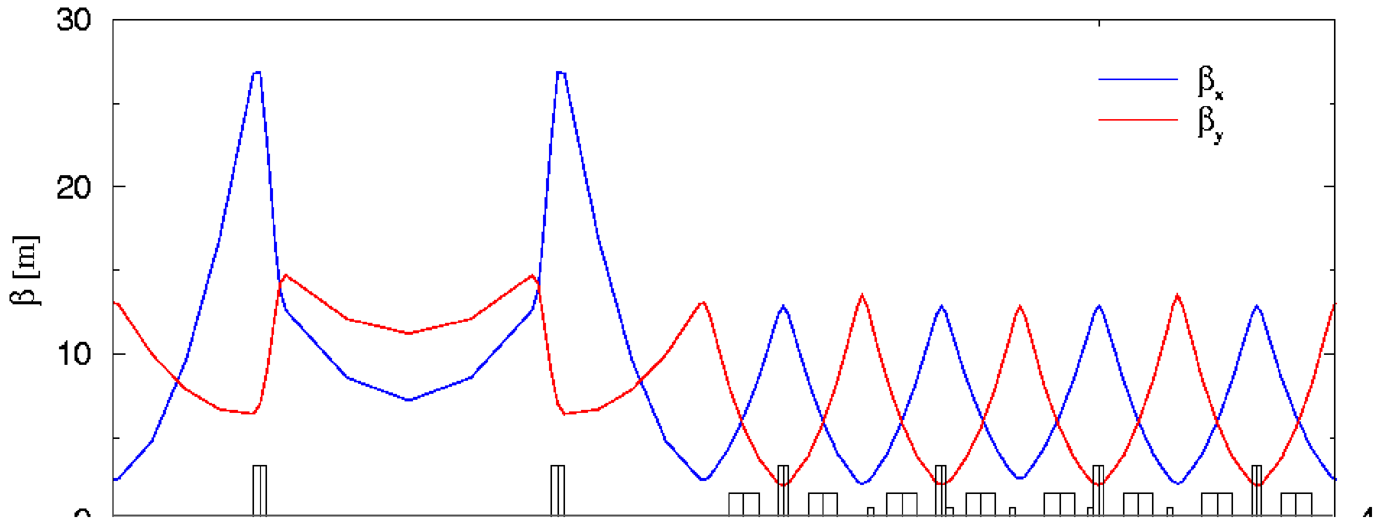
The phase-space ellipse along a FODO lattice changes but the ellipse area remains constant. The beam waist (ellipse is upright and σ_b is minimum) occurs at the QD's.



Liouville's theorem : In the absence of nonlinear forces or acceleration, the phase-space volume is constant. For the x-p_x phase space, the phase-space area (emittance) is conserved. Strictly speaking, this is only true for the slice emittance. The emittance of several slices projected on x-p_x can become smaller if their ellipses are lined up.

Beta Function

The beta function is used to describe the beam's size (square root of beta) and to identify the maximum amplitude of beam's envelope; beta also tells us where the beam is most sensitive to perturbations.



rms beam radii in x and y

$$\sigma_x = \sqrt{\beta_x \varepsilon_x} \qquad \sigma_y = \sqrt{\beta_y \varepsilon_y}$$

Summary

- 4GLS are x-ray FEL that produce tunable, fs coherent x-rays with peak brightness ten orders of magnitude higher than the 3GLS brightness.
- RF linac are used to drive the 4GLS because they produce electron beams with the requisite electron beam brightness for x-ray FEL.
- Both normal-conducting (copper) and superconducting (niobium) RF accelerators have been used for x-ray FEL.
- The important electron beam parameters are: beam energy, peak current, beam emittance, and beam energy spread.
- Transfer matrices are linear transformations (maps) of the 6D trace space of each particle in the beam from one location to another.