Storage Ring Dynamics: Longitudinal Motion

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Synchrotrons are based on Phase Stability

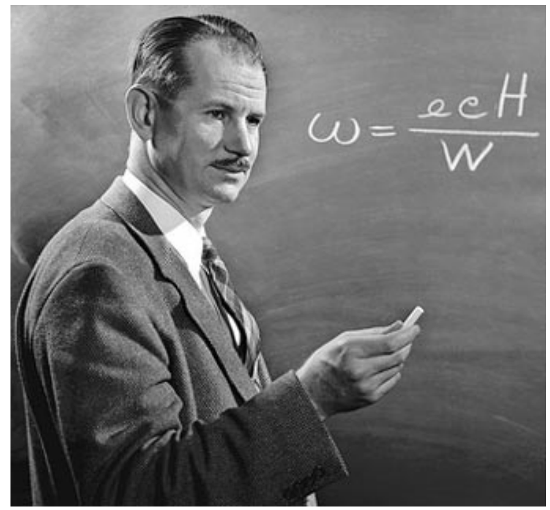
- Original concept of phase stability introduced independently by Veksler (1944) and MacMillan (1945)

The Synchrotron—A Proposed High Energy Particle Accelerator

EDWIN M. MCMILLAN
University of California, Berkeley, California
September, 5, 1945

ONE of the most successful methods for accelerating charged particles to very high energies involves the repeated application of an oscillating electric field, as in the cyclotron. If a very large number of individual accelerations is required, there may be difficulty in keeping the particles in step with the electric field. In the case of the cyclotron this difficulty appears when the relativistic mass change causes an appreciable variation in the angular velocity of the particles.

The device proposed here makes use of a "phase stability" possessed by certain orbits in a cyclotron. Consider



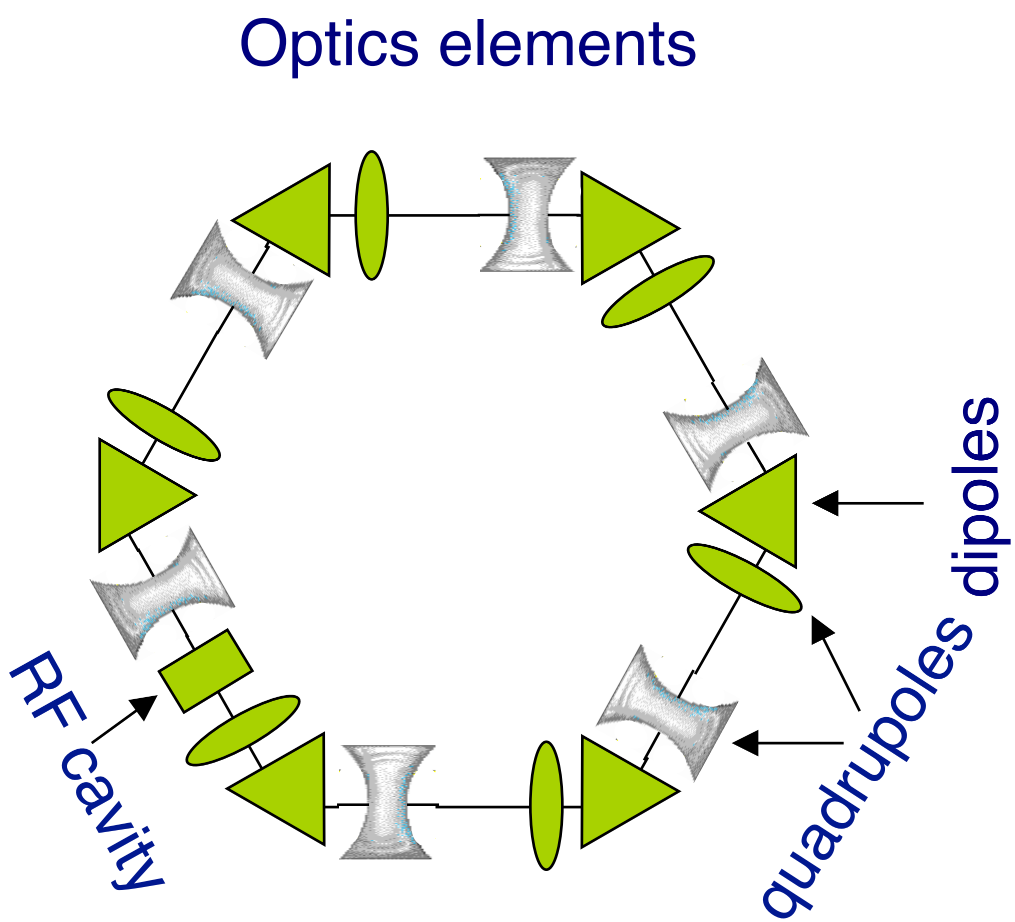
$$E_0 = (300cH)/(2\pi f), \qquad (1)$$

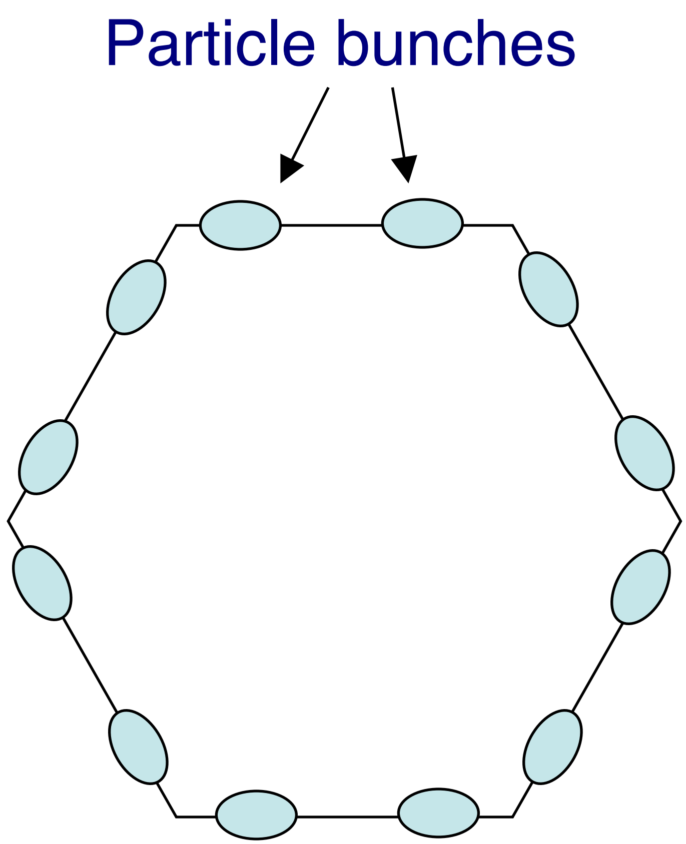
$$E = E_0[1-(d\phi)/(d\theta)], \qquad (2)$$

$$2\pi\frac{d}{d\theta}\left(E_0\frac{d\phi}{d\theta}\right)+V\sin\phi = \left[\frac{1}{f}\frac{dE_0}{dt}-\frac{300}{c}\frac{dF_0}{dt}+L\right]+\left[\frac{E_0}{f^2}\frac{df}{dt}\right]\frac{d\phi}{d\theta}, \qquad (3)$$

$$R = (E^2-E_r^2)^{\frac{1}{2}}/300H. \qquad (4)$$

In a particle storage rings, charged particles circulate around the ring in bunches for a large number of turns.





Particle Storage Rings

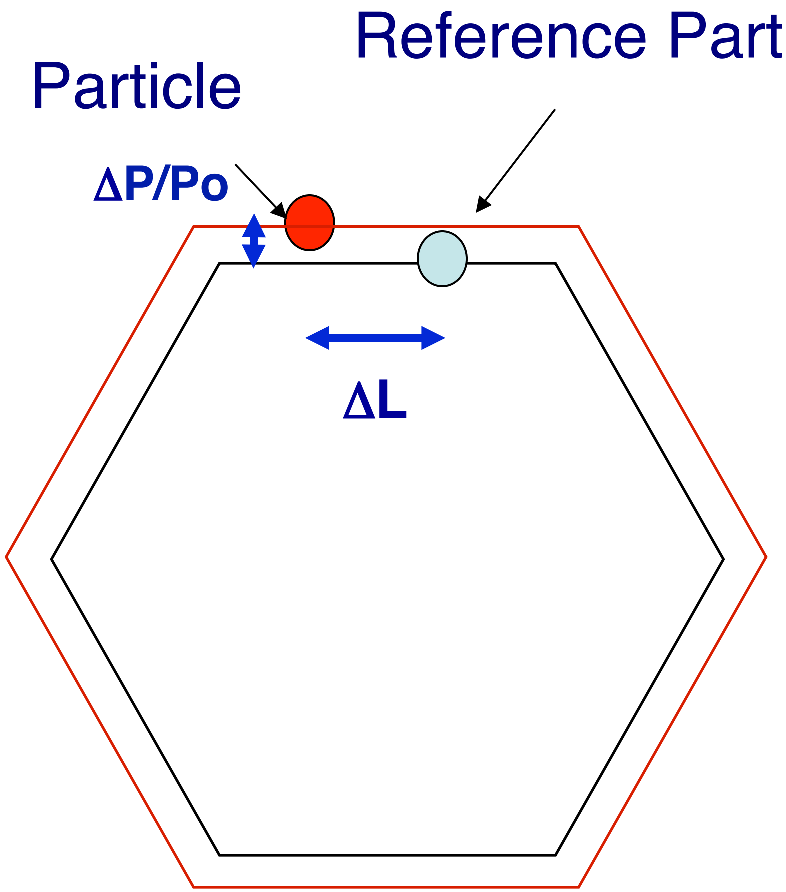
The longitudinal coordinates are

Length and Momentum

Lo → Revolution Length of the Reference Particle

Po → Momentum of the Reference Particle

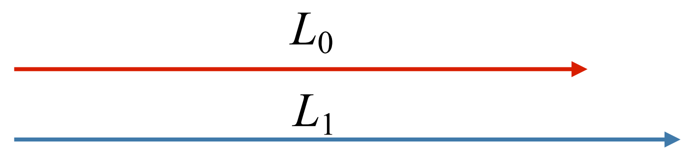
$\Delta L/Lo$, $\Delta P/Po$



Path Length Dependence on Velocity

Consider two particles with different momentum on parallel trajectories:

$$p_1 = p_0 + \Delta p$$



At a given instant *t*:

$$L_1 = \left(\beta_0 + \Delta\beta\right)ct \qquad L_0 = \beta_0\, ct$$

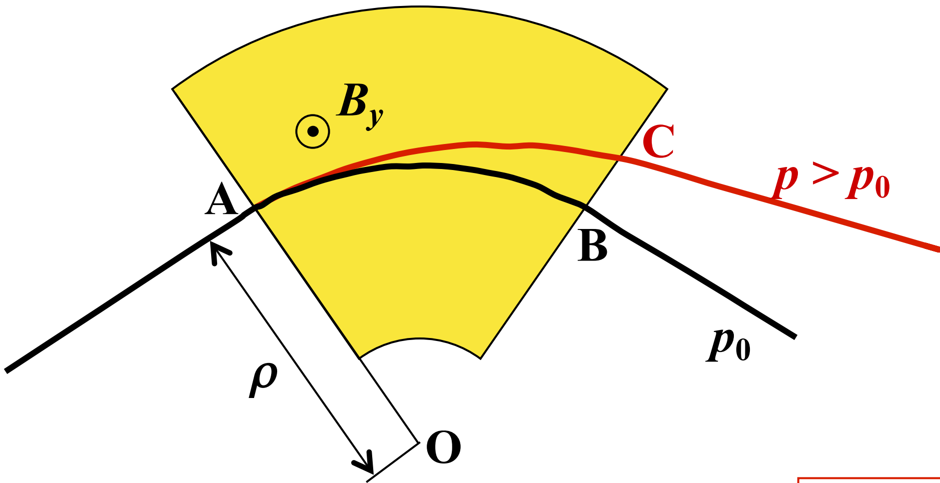
$$\boxed{\Rightarrow \frac{\Delta L}{L_0} = \frac{L_1 - L_0}{L_0} = \frac{\Delta\beta}{\beta_0}}$$

But: $$p = \beta\,\gamma\, m_0 c \quad \Rightarrow \quad \Delta p = m_0 c\,\Delta\left(\beta\,\gamma\right) = m_0 c \gamma^3 \Delta\beta$$

$$\Rightarrow \quad \frac{\Delta p}{p_0} = \gamma^2 \frac{\Delta\beta}{\beta} \qquad \boxed{\frac{\Delta L}{L_0} = \frac{1}{\gamma^2}\frac{\Delta p}{p_0}}$$

- **This path length dependence on momentum applies everywhere, also in straight trajectories.**
- **The effect quickly vanishes for relativistic particles.**
- **Higher momentum particles precede the ones with lower momentum.**

Path Length Dependence On Trajectory



$$\rho = \frac{p}{qB_z} = \frac{\beta\gamma m_0 c}{q B_z}$$

L_0 = Trajectory length between A and B

L = Trajectory length between A and C

$$\frac{L - L_0}{L_0} \propto \frac{p - p_0}{p_0} \Rightarrow \boxed{\frac{\Delta L}{L_0} = \alpha \frac{\Delta p}{p_0}}$$

where α is constant

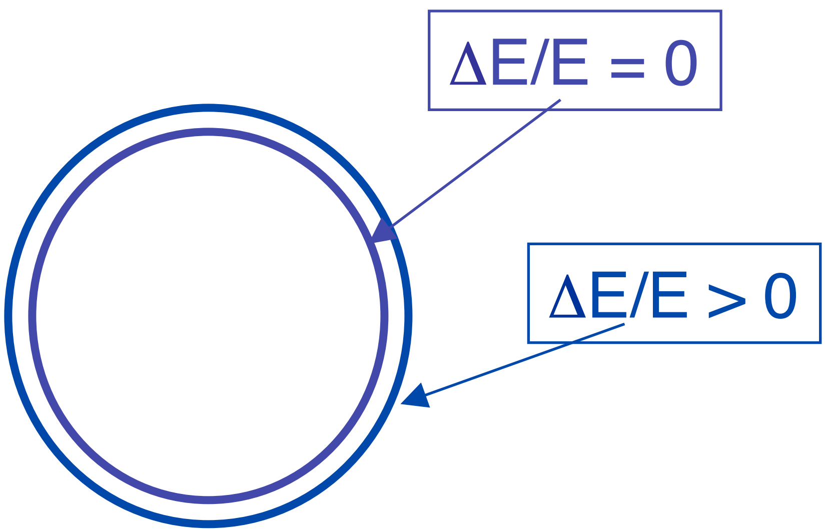
$$For \;\; \gamma >> 1 \;\; \Rightarrow \frac{\Delta L}{L_0} = \alpha \frac{\Delta p}{p_0} \cong \alpha \frac{\Delta E}{E_0}$$

In the example (sector bending magnet) $L > L_0$ so that $\alpha > 0$
Higher energy particles will leave the magnet later.

Momentum Compaction

Momentum compaction, α, is the change in the closed orbit length as a function of momentum.

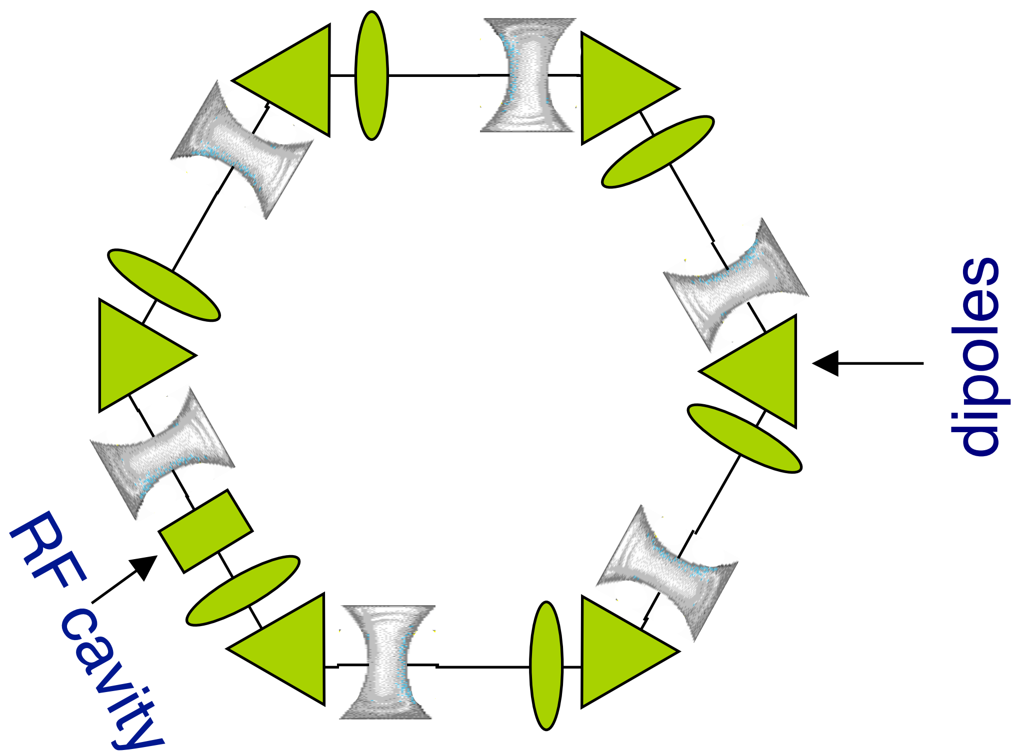
$$\frac{\Delta L}{L} = \alpha \frac{\Delta p}{p}$$



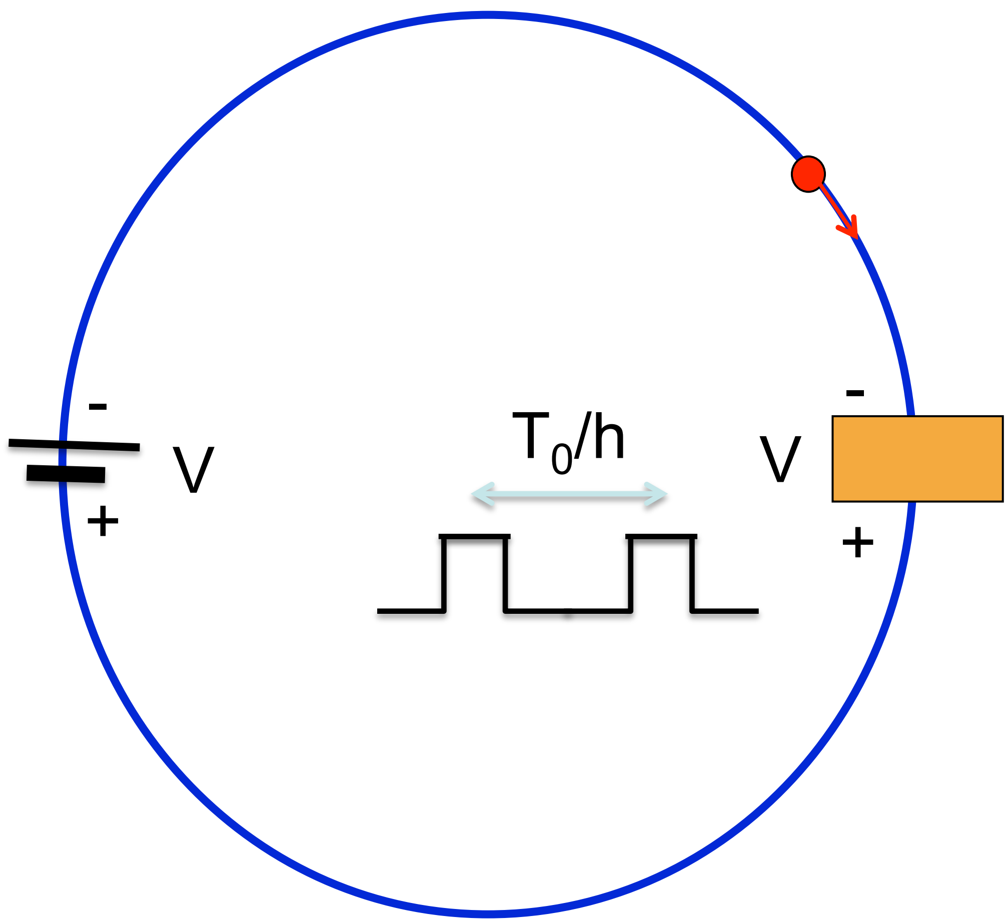
$$\alpha = \int_0^{L_0} \frac{D_x}{\rho} ds$$

Energy Gain and Loss

Lose energy in dipoles → Synchrotron Radiation
Gain Energy in the RF Cavity



Why do we need AC voltages to accelerate?



- Imagine a DC voltage across a gap.
  - No way to maintain DC voltage through vacuum chamber!
- Add a DC power supply or insulating gap.
  - Voltage cancelled for round trip around ring.
- Add switched DC voltage
  - Switch at a time period that is a sub-harmonic of the revolution period T_0. (I.e. switching frequency is harmonic of revolution frequency.

Cyclotron concept

- Voltage on "Ds" must reverse every half orbit

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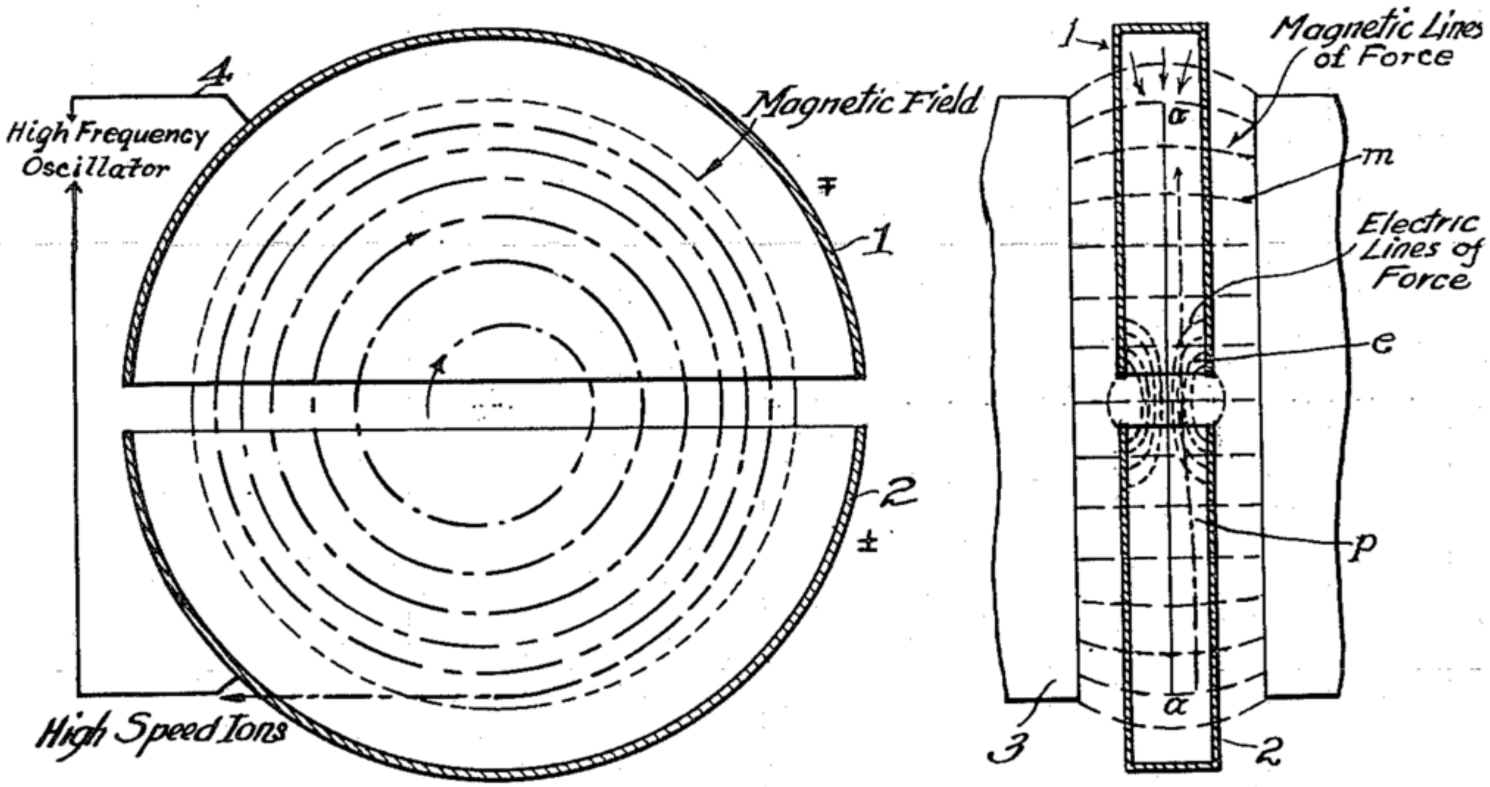

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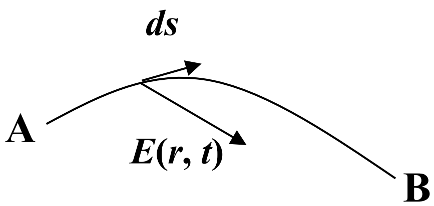
Energy Variation

• The energy gain for a particle that moves from A to B is given by:



$$\Delta E = q\int_0^L \bar{E}_F(\bar{r},t)\cdot d\bar{s} = qV$$

• We define as V the voltage gain for the particle.
V depends only on the particle trajectory and includes the contribution of every electric field present in the area (RF fields, space charge fields, fields due to the interaction with the vacuum chamber, …)

• The particle can also experience energy variations $U(E)$ that depend also on its energy, as for the case of the radiation emitted by a particle under acceleration (synchrotron radiation when the acceleration is transverse).

• The total energy variation will be given by the sum of the two terms:

$$\Delta E_T = qV + U(E)$$

Energy gain from RF voltage

Assuming a sinusoidal electric field $E_z = E_0 \cos(\omega_{RF} t + \phi_s)$ where the synchronous particle passes at the middle of the gap *g*, at time *t = 0*, the energy is

$$W(r,t) = q\int E_z dz = q\int_{-g/2}^{g/2} E_0 \cos(\omega_{RF}\frac{z}{v} + \phi_s)dz$$

And the energy gain is $\Delta W = qE_0 \int_{-q/2}^{g/2} \cos(\omega_{RF}\frac{z}{v})dz$

and finally $\Delta W = qV\frac{\sin\Theta/2}{\Theta/2} = qV\,\mathrm{T}$ with the transit time

factor defined as $\boxed{T = \frac{\sin(\omega g/2v)}{\omega g/2v}}$

It can be shown that in general $\mathrm{T} = \frac{\int_{-g/2}^{g/2} E(0,z)\cos\omega t(z)dz}{\int_{-g/2}^{g/2} E(0,z)dz}$

The Rate of Change of Energy

The energy variation for the reference particle is given by:

$$\Delta E_T(s_0) = qV(s_0) + U(E_0)$$

For particle with energy $E = E_0 + \Delta E$ and orbit position $s = s_0 + \Delta s$:

$$\Delta E_T(s) = qV(s_0 + \Delta s) + U(E_0 + \Delta E) \cong qV(s_0) + q\left.\frac{dV}{ds}\right|_{s_0}\Delta s + U(E_0) + \left.\frac{dU}{dE}\right|_{E_0}\Delta E$$

Where the last expression holds for the case where $\Delta s \ll L_0$ (reference orbit length) and $\Delta E \ll E_0$.

In this approximation we can express the average rate of change of the energy respect to the reference particle energy by:

$$\frac{\Delta E}{T_0} \cong \frac{\Delta E_T(s) - \Delta E_T(s_0)}{T_0}$$

$$\boxed{\frac{\Delta E}{T_0} \cong \frac{1}{T_0}\left(q\left.\frac{dV}{ds}\right|_{s_0}\Delta s + \left.\frac{dU}{dE}\right|_{E_0}\Delta E\right)}$$

$$T_0 = \frac{L_0}{\beta_0 c}$$

The Longitudinal Equation of Motion

We obtain the equations of motion for the longitudinal plane:

$$\frac{d^2 \Delta s}{dt^2} + 2\alpha_D \frac{d\Delta s}{dt} + \Omega^2 \Delta s = 0$$

$$\frac{\Delta E}{T_0} \cong \frac{1}{T_0}\left(q \left.\frac{dV}{ds}\right|_{s_0} \Delta s + \left.\frac{dU}{dE}\right|_{E_0} \Delta E \right)$$

$$\Delta s << L_0$$

$$\Delta E << E_0$$

Finally, by defining the quantities:

$$\Omega^2 = \alpha \frac{1}{p_0} \frac{q}{T_0} \left.\frac{dV}{ds}\right|_{s_0}$$

$$\alpha_D = -\frac{1}{2T_0} \left.\frac{dU}{dE}\right|_{E_0}$$

We will study the case of storage rings where *dV/ds* is mainly due to the RF system used for restoring the energy lost per turn by the beam.

The Damped Oscillator Equation

$$\frac{d^2 \Delta s}{dt^2} + 2\alpha_D \frac{d\Delta s}{dt} + \Omega^2 \Delta s = 0$$

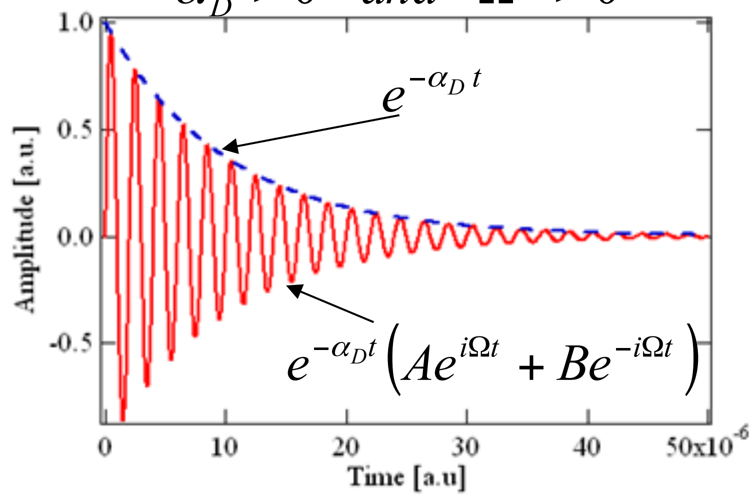
This expression is the well known damped harmonic oscillator equation, which has the general solution:

$$\Delta s(t) \cong e^{-\alpha_D t}\left(Ae^{i\Omega t} + Be^{-i\Omega t}\right)$$

$$\alpha_D > 0 \quad \Leftrightarrow \quad \text{damped oscillation}$$
$$\alpha_D < 0 \quad \Leftrightarrow \quad \text{anti-damped oscillation}$$

$$\Omega^2 > 0 \quad \Leftrightarrow \quad \text{stable oscillation}$$
$$\Omega^2 < 0 \quad \Leftrightarrow \quad \text{unstable motion}$$

$\alpha_D > 0 \quad and \quad \Omega^2 > 0$



The stable solution represents an oscillation with frequency $2\pi\Omega$ and with exponentially decreasing amplitude.

Damping in the Case of Storage Rings

- The case of damped oscillations is exactly what we want for storing particles in a storage ring.

$$\alpha_D > 0 \qquad \alpha_D = -\frac{1}{2T_0}\left.\frac{dU}{dE}\right|_{E_0} \quad \Rightarrow \quad \boxed{\left.\frac{dU}{dE}\right|_{E_0} < 0}$$

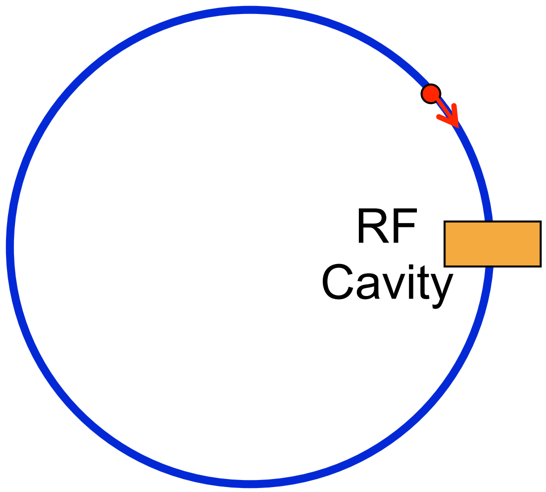
- The synchrotron radiation (SR) emitted when particles are on a curved trajectory satisfies the condition. The SR power scales as:

$$dU/dt = -P_{SR} \propto -(\beta\gamma)^4/\rho^2 = -\left(\gamma^2-1\right)^2/\rho^2 \qquad \rho \equiv trajectory\ radius$$

- Typically, synchrotron radiation damping is very efficient in electron storage rings and negligible in proton machines.
- The damping time $1/\alpha_D$ (~ ms for e$^-$, ~ 13 hours LHC at 7 TeV) is usually much larger than the period of the longitudinal oscillations $1/2\pi\Omega$ (~ μs). This implies that the damping term can be neglected when calculating the particle motion for $t << 1/\alpha_D$:

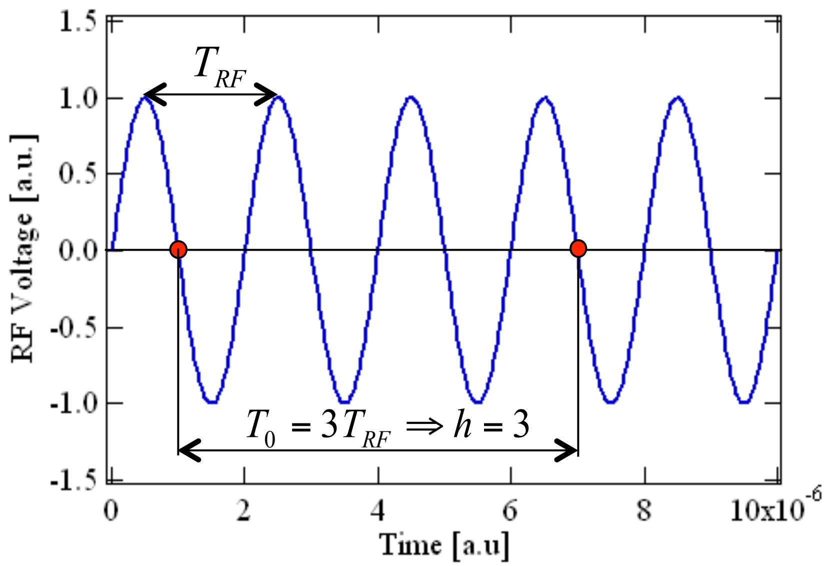
Synchronicity in Storage Rings

Let's consider a storage ring with reference trajectory of length L_0:



$$V_{RF}(t) = \hat{V}\sin(\omega_{RF} t)$$

$$T_0 = \frac{L_0}{\beta c} \qquad T_{RF} = \frac{1}{f_{RF}} = \frac{2\pi}{\omega_{RF}}$$



$$T_0 = hT_{RF} \Rightarrow f_0 = \frac{f_{RF}}{h}$$

Synchronicity Condition

The integer h is called the *harmonic number*

The Synchrotron Frequency and Tune

For our storage ring:

$$\Omega^2 = \alpha \frac{1}{p_0} \frac{q}{T_0} \frac{dV}{ds}\bigg|_{s_0}$$

$$s = \beta_0 c t$$

$$V_{RF}(t) = \hat{V}\sin(\omega_{RF} t) = \hat{V}\sin(h\omega_0 t)$$

$$\frac{dV}{ds}\bigg|_{s_0} = \frac{1}{\beta_0 c}\frac{dV}{dt}\bigg|_{t_0} = \frac{h\omega_0 \hat{V}}{\beta_0 c}\cos(\omega_{RF} t_0)$$

$$\boxed{\Omega^2 = \omega_0^2 \frac{q}{p_0}\frac{\alpha h \hat{V}}{2\pi\beta_0 c}\cos(\varphi_s) \quad \textit{synchrotron frequency}}$$

$$\boxed{\nu_S = \frac{\Omega}{\omega_0} \quad \textit{synchrotron tune}}$$

$$\boxed{\varphi_s = \omega_{RF}\, t_0 \equiv \textit{synchronous phase}}$$

The Synchrotron Frequency and Tune

If $\alpha_D << \Omega$ ➡ $\dfrac{d^2\Delta s}{dt^2} + \Omega^2 \Delta s = 0$ **Additionally:** $\Delta E(t) = -\dfrac{p_0}{\alpha}\dfrac{d\Delta s}{dt}$

➡ $\boxed{\Delta s = \Delta\hat{s}\cos(\Omega t + \psi)}$ $\boxed{\Delta E = \Delta\hat{s}\dfrac{p_0 \Omega}{\alpha}\sin(\Omega t + \psi)}$

A different set of variables:

$\boxed{Phase: \varphi = \phi - \phi_s}$ $\phi = \omega_{RF} t$ $s = \beta_0 c t$ ➡ $s = \beta_0 c \dfrac{\phi}{\omega_{RF}}$

$\boxed{Relative\ Momentum\ Deviation: \delta = \dfrac{\Delta p}{p_0}}$ $\Delta E = \beta_0 c \Delta p$

$\boxed{\varphi = \hat{\varphi}\cos(\Omega t + \psi)}$ $\boxed{\delta = \dfrac{\hat{\varphi}\Omega}{h\omega_0 \eta_C}\sin(\Omega t + \psi)}$ **Synchrotron Oscillations**

For $\Delta s << L_0$ and $\Delta E << E_0$.

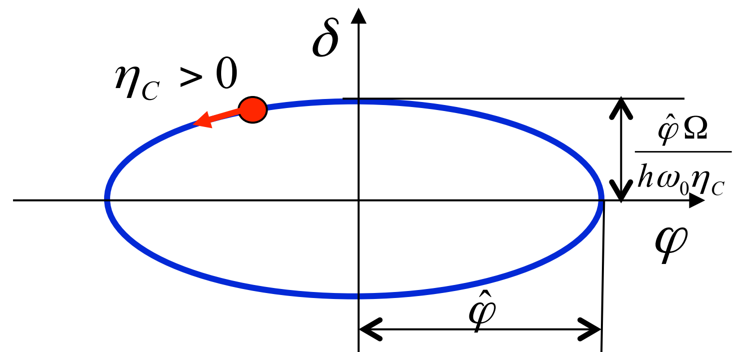
The Longitudinal Phase space

We just found:

$$\varphi = \hat{\varphi}\cos(\Omega t + \psi)$$

$$\delta = \frac{\hat{\varphi}\Omega}{h\omega_0\eta_C}\sin(\Omega t + \psi)$$

$$\frac{\varphi^2}{\hat{\varphi}^2} + \delta^2\left(\frac{h\omega_0\eta_C}{\hat{\varphi}\Omega}\right)^2 = 1$$

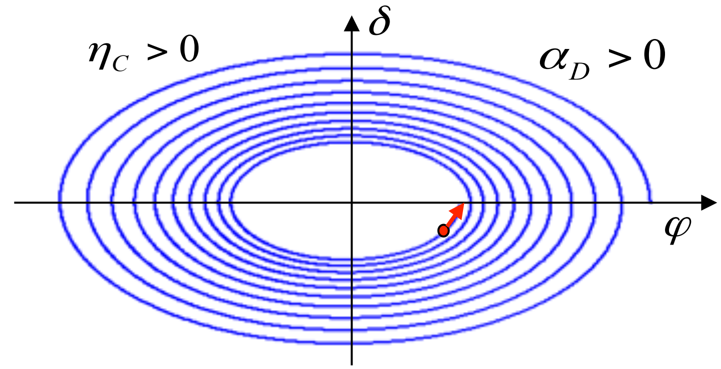


This equation represents an ellipse in the longitudinal phase space {φ, δ}

With damping:

$$\varphi = \hat{\varphi}e^{-\alpha_D t}\cos(\Omega t + \psi)$$

$$\delta = \frac{\hat{\varphi}\Omega}{h\omega_0\eta_C}e^{-\alpha_D t}\sin(\Omega t + \psi)$$



In rings with negligible synchrotron radiation (or with negligible non-Hamiltonian forces, the longitudinal emittance is conserved.

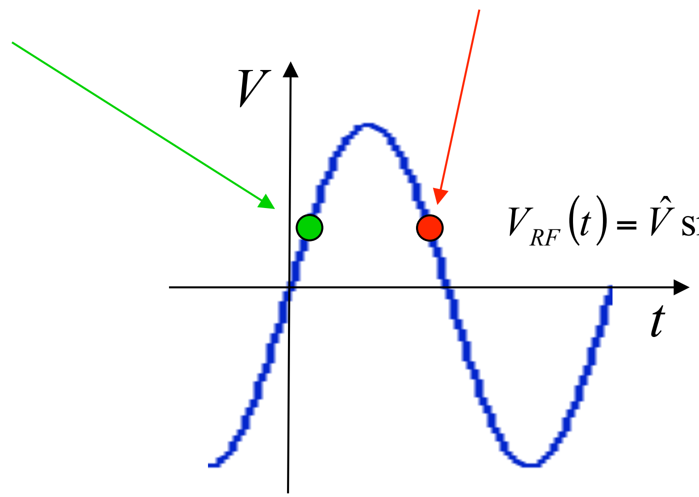
This is the case for heavy ion and for most proton machines.

Two synchronous phases → one stable one unstable

$$\sin\varphi_S = \frac{U_0}{q\hat{V}}$$

But

$$\frac{\Delta t}{T_0} = \frac{\Delta s}{L_0} = \alpha\frac{\Delta p}{p_0}$$



For positive charge particles:

$$For\ \alpha > 0 \quad \Rightarrow \quad \varphi_S^1\ \ stable, \quad \varphi_S^2\ \ unstable$$

$$For\ \alpha < 0 \quad \Rightarrow \quad \varphi_S^1\ \ unstable, \quad \varphi_S^2\ \ stable$$

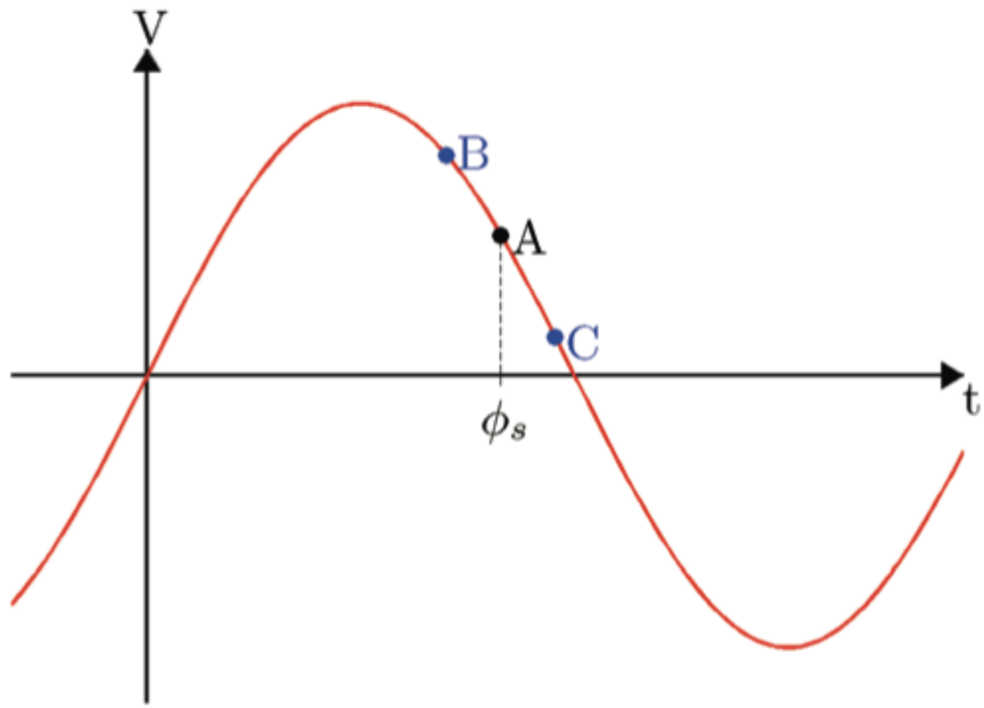
For negative charge particles all the phases are shifted by π.

We define as *transition energy* the energy at which α changes sign.

Crossing the transition energy during energy ramping requires a phase jump of ~ π

Phase Stability: Example

- A is the synchronous particle and arrives at the right time to receive the right energy gain.
- B arrives early and gains too much energy. Next turn it arrives later (for $\alpha>0$.)
- C arrives late and gains too little energy arrives earlier on the next turn.



Large Amplitude Oscillations

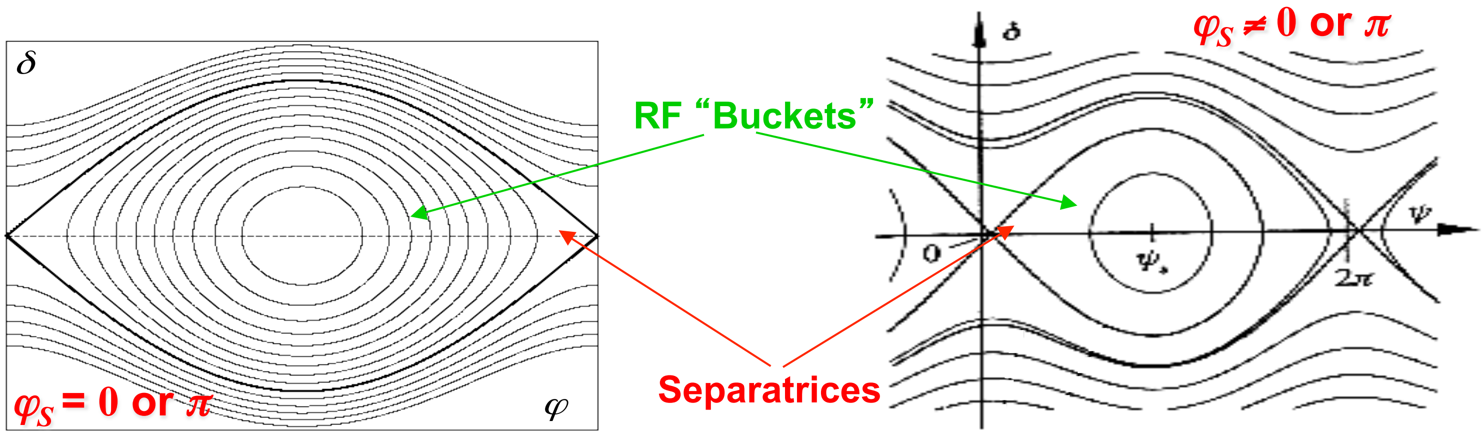
So far we have used the *small oscillation approximation* where:

$$\Delta E_T(\psi) = qV(\varphi_S+\varphi) = q\hat{V}\sin(\varphi_S+\varphi) \cong qV(\varphi_S) + q\left.\frac{dV}{d\varphi}\right|_{\varphi_S}\varphi = q\hat{V}\varphi_S + q\hat{V}\varphi$$

In the more general case of larger phase oscillations:

$$\Delta E_T(\psi) = qV(\varphi_S+\varphi) \cong q\hat{V}\sin(\varphi_S+\varphi)$$

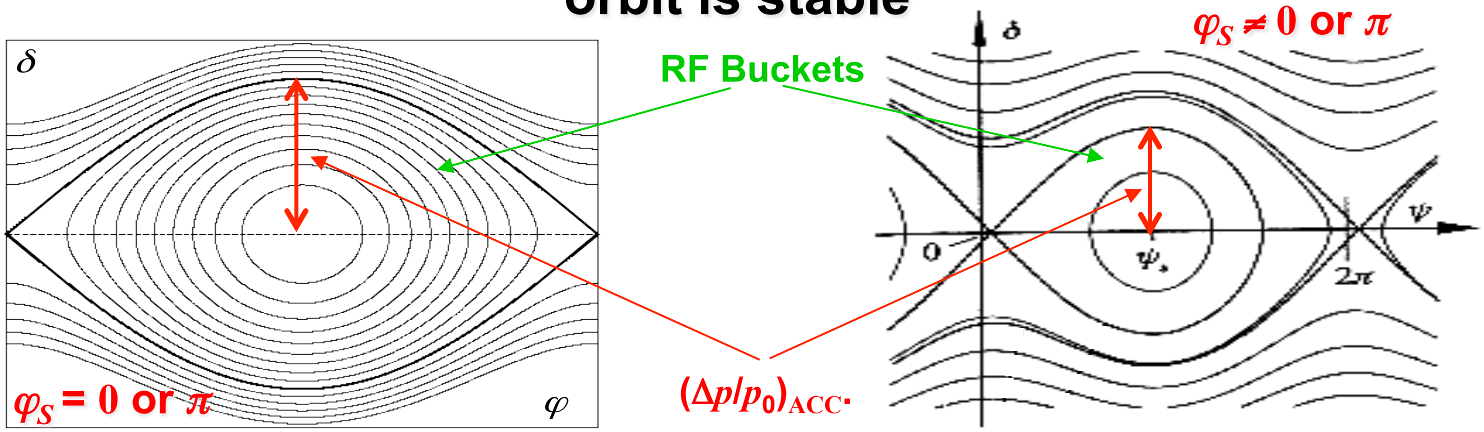
And by Numerical integration:



- **For larger amplitudes, trajectories in the phase space are not ellipsis anymore.**
- **Stable and unstable orbits exist. The two regions are separated by a special trajectory called *separatrix***
- **Larger amplitude orbits have smaller synchrotron frequencies**

Momentum Acceptance

The RF bucket is the area of the longitudinal phase space where a particle orbit is stable



The *momentum acceptance* is defined as the maximum momentum that a particle on a stable orbit can have.

$$\left(\frac{\Delta p}{p_0}\right)^2_{ACC} = \frac{2|q|\hat{V}}{\pi\, h|\eta_C|\beta\, c\, p_0}$$

$$\left(\frac{\Delta p}{p_0}\right)^2_{ACC} = \frac{F(Q)}{2Q}\frac{2|q|\hat{V}}{\pi\, h|\eta_C|\beta\, c\, p_0}$$

$$F(Q) = 2\left(\sqrt{Q^2 - 1} - \arccos\frac{1}{Q}\right)$$

$$Q = \frac{1}{\sin\varphi_s} = \frac{q\hat{V}}{U_0}$$

Over voltage factor

Bunch Length

• In electron storage rings, the statistical emission of synchrotron radiation photons generates gaussian bunches.

• The over voltage Q is usually large so that the core of the bunch "lives" in the small oscillation region of the bucket. The equation of motion in the phase space are elliptical:

$$\frac{\varphi^2}{\hat{\varphi}^2} + \delta^2 \left(\frac{h\omega_0 \eta_C}{\hat{\varphi}\Omega} \right)^2 = 1 \quad \Rightarrow \quad \hat{\varphi} = \frac{h\omega_0 \eta_C}{\Omega} \hat{\delta} \Rightarrow \Delta s = \frac{c\eta_C}{\Omega} \frac{\Delta p}{p_0}$$

• If σ_p/p_0 is the *rms relative momentum spread* of the gaussian distribution, then the *rms bunch length* is given by:

$$\boxed{\sigma_{\Delta S} = \frac{c\eta_C}{\Omega} \frac{\sigma_p}{p_0} = \sqrt{\frac{c^3}{2\pi q} \frac{p_0 \beta_0 \eta_C}{h f_0^2 \hat{V} \cos(\phi_S)}} \frac{\sigma_p}{p_0}}$$

• In the case of heavy ions and of most of protons machines, the whole RF bucket is usually filled with particles. The bunch length l is then proportional to the difference between the two extreme phases of the separatrix:

$$\boxed{l = (\varphi_2 - \varphi_1)\lambda_{RF}/2\pi}$$

Effects of the Synchrotron Radiation

• **A charged particle when accelerated radiates.**

• **In high energy storage rings transverse acceleration induces significant radiation (synchrotron radiation) while longitudinal acceleration generates negligible radiation ($1/\gamma^2$).**

$$\frac{dU}{dt} = -P_{SR} = -\frac{2\,c\,r_e}{3\left(m_0c^2\right)^3}\frac{E^4}{\rho^2}$$

$r_e \equiv$ *classical electron radius*

$\rho \equiv$ *trajectory curvature*

$$U_0 = \int_{finite\ \rho} P_{SR}\,dt \quad \textit{energy lost per turn}$$

$$\alpha_D = -\frac{1}{2T_0}\frac{dU}{dE}\bigg|_{E_0} = \frac{1}{2T_0}\frac{d}{dE}\left[\oint P_{SR}\left(E_0\right)dt\right]$$

α_{DX}, α_{DY} *damping in all planes*

$$\frac{\sigma_p}{p_0} \quad \textit{equilibrium momentum spread and emittances}$$

$\varepsilon_X, \varepsilon_Y$

• **Synchrotron radiation plays a major role in the dynamics of an electron storage ring**

Energy Lost per Turn

$$U_0 = \int_{finite\ \rho} P_{SR}\, dt \quad energy\ lost\ \ per\ turn$$

$$\frac{dU}{dt} = -P_{SR} = -\frac{2\, c\, r_e}{3\left(m_0 c^2\right)^3} \frac{E^4}{\rho^2}$$

- **For relativistic electrons:**

$$s = \beta c t \cong ct \Rightarrow dt = \frac{ds}{c}$$

$$\boxed{U_0 = \frac{1}{c} \int_{finite\ \rho} P_{SR}\, ds = \frac{2 r_e E_0^4}{3\left(m_0 c^2\right)^3} \int_{finite\ \rho} \frac{ds}{\rho^2}}$$

- **In the case of dipole magnets with constant radius *ρ* (*iso-magnetic* case):**

$$\boxed{U_0 = \frac{4\pi\, r_e}{3\left(m_0 c^2\right)^3} \frac{E_0^4}{\rho}}$$

- **The average radiated power is given by:**

$$\boxed{\left\langle P_{SR} \right\rangle = \frac{U_0}{T_0} = \frac{4\pi\, c\, r_e}{3\left(m_0 c^2\right)^3} \frac{E_0^4}{\rho\, L} \qquad L \equiv \ ring\ \ circumference}$$

Example: Bunch splitting (for the LHC)

motivation:

using existing accelerators, produce multiple high-current bunches

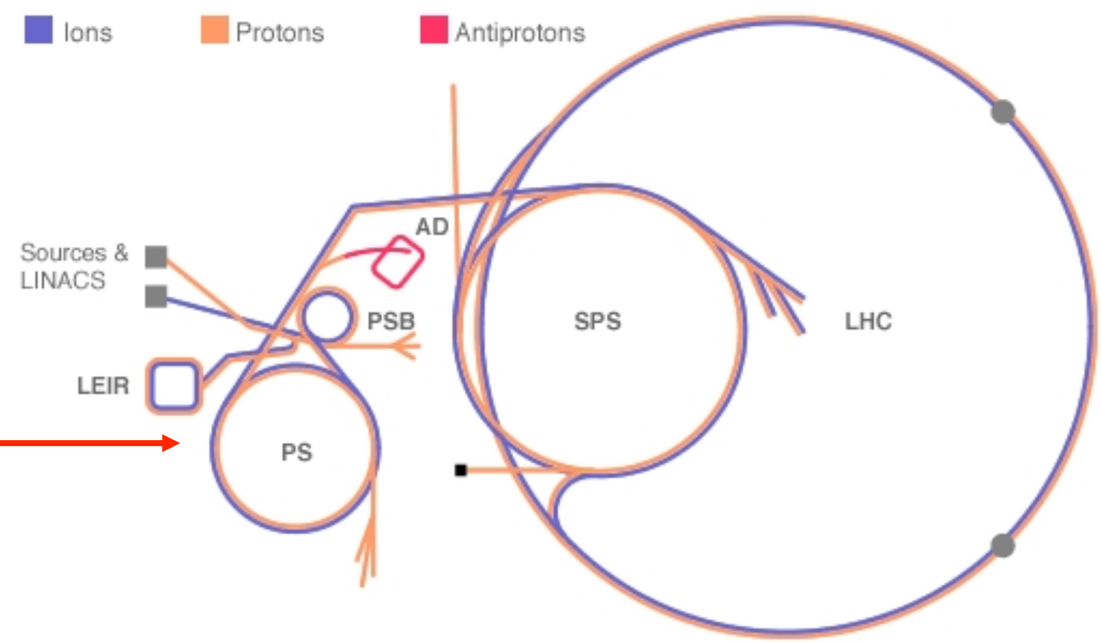
produce ~40 bunch trains of 72 bunches with 10^{11} protons and 25 ns bunch spacing (LHC)

history:

debunching of 6-7 high intensity bunches in the CERN PS + capture in higher-f rf system (microwave instability observed in the process leading to non-uniform beam distributions)

concept: application of higher-harmonic rf cavities

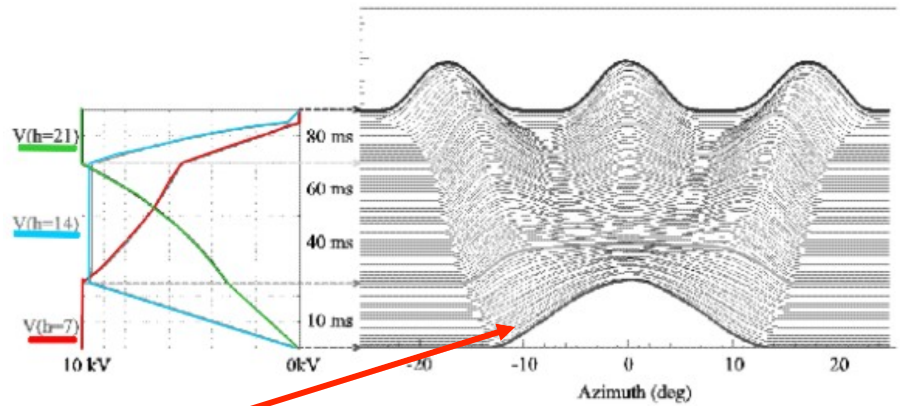
layout of the LHC including the preinjectors



one bunch from the PS booster gets split into twelve bunches in the CERN PS

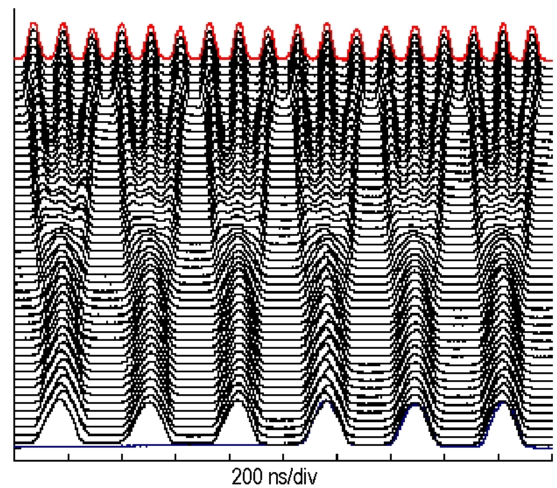
LHC Example: Split factor 1→ 3

example: simulation of bunch triple-splitting in the CERN PS (courtesy R. Garoby, 1999)



one of 6 bunches from the booster in the CERN PS

example: measurement of bunch triple-splitting in the CERN PS (courtesy R. Garoby, 2001)



Issues:

preservation of longitudinal beam emittance
- stability of initial conditions
- complicated (then) by B-field drift
- requires careful synchronization
- control of longitudinal coupled-bunch instabilities

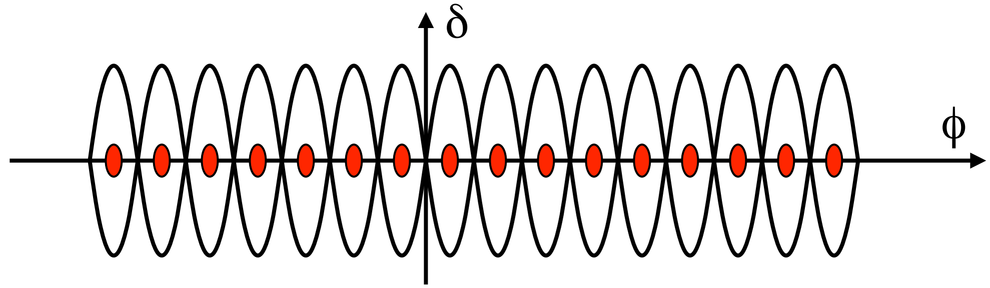
bunch intensity fluctuations
- stability of initial conditions

Example: Bunch coalescing

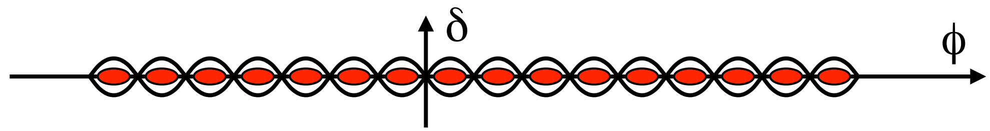
motivation: combine many bunches into 1 bunch for high peak intensity (and luminosity)

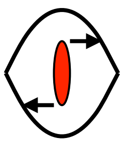
concept:

1) initial condition with multiple bunches in different high frequency rf buckets

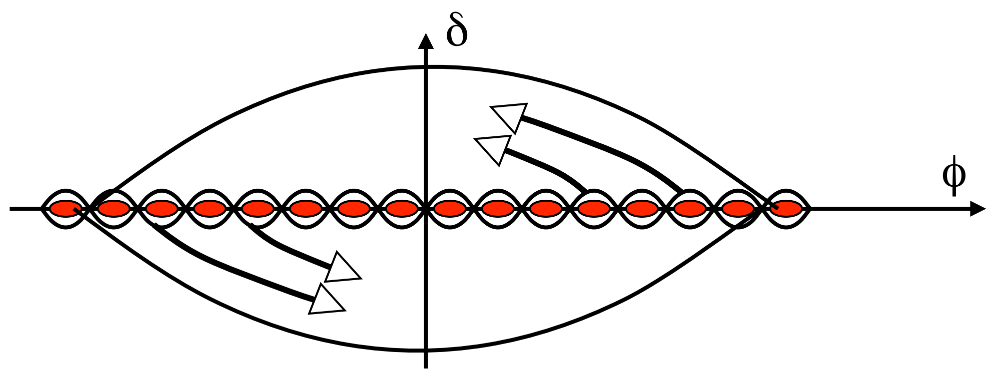


2) lower (vector sum) of cavity voltages



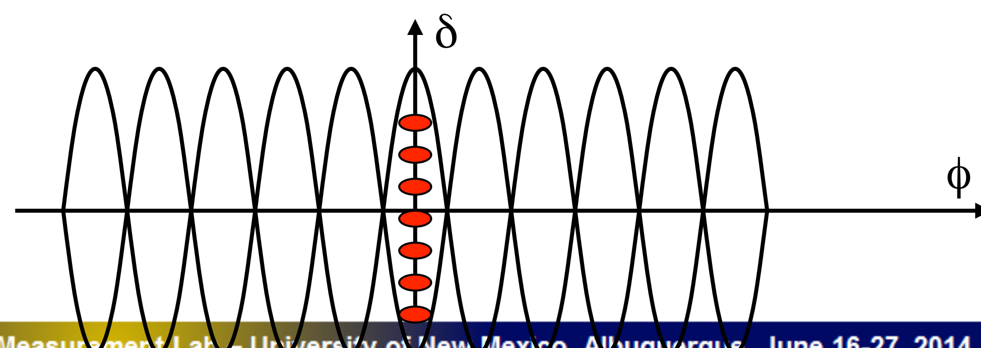


bunches “shear” due to longitudinal mismatch



3) turn on a subharmonic rf system

bunches rotate with new synchrotron frequency



4) restore initial rf (with appropriate phase), turn off the lower frequency rf system

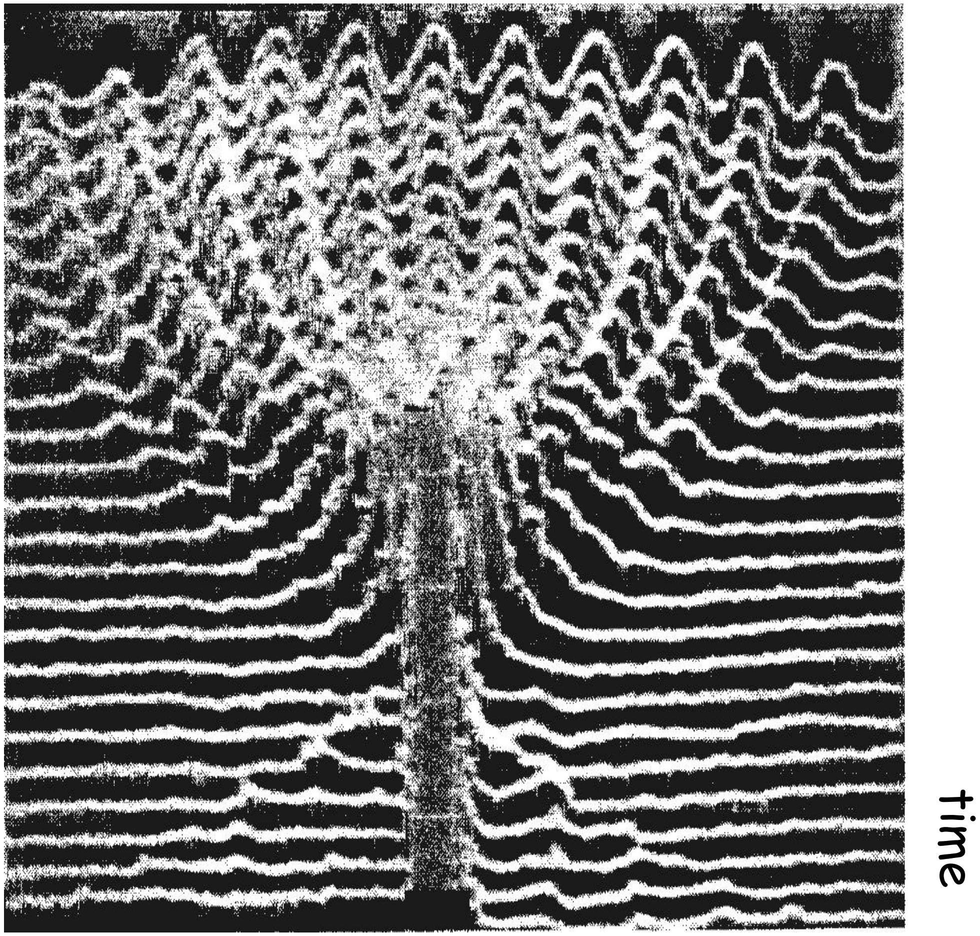
example: bunch coalescing in the Fermilab Main Ring (courtesy P. Martin, 1999)

initial condition: 11 bunches captured in 53 MHz rf buckets

“paraphrasing” – adiabatic reduction of the vector sum rf voltage by shift of the relative phases between rf cavities

application of higher voltage 2.5 MHz rf system (in practice, a 5 MHz rf system was used to help linearize the rotation)

capture of bunches in a single 53 MHz rf bucket



peak intensity monitor with successive traces spaced by 6.8 ms intervals

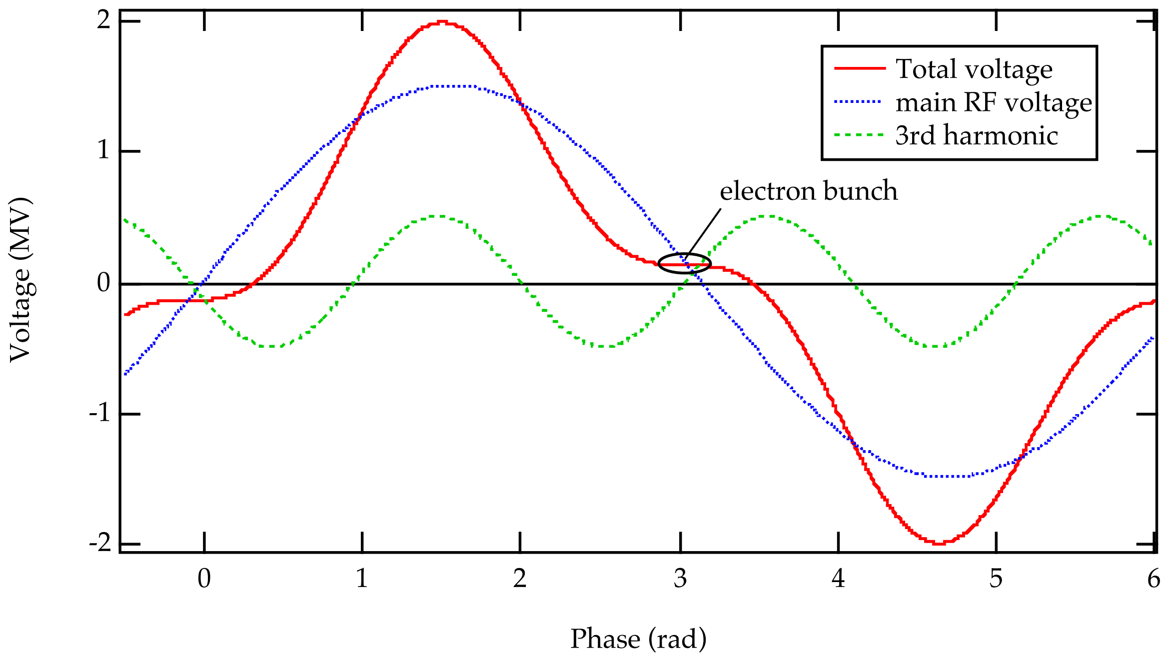
“snap coalescing” – fast change in voltage amplitude applied (instead of adiabatic voltage reduction)

observed advantage: avoidance of high-current beam instabilities during paraphrasing

observed disadvantage: reduced capture efficiency (~10%)

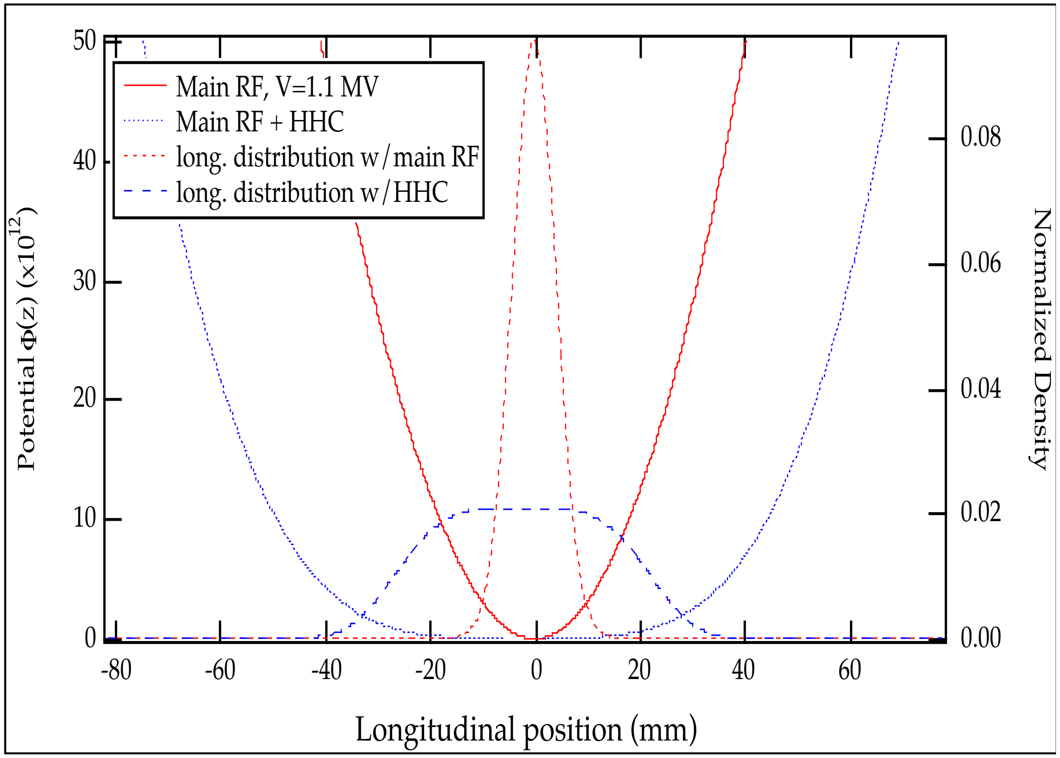
Example: Harmonic RF System

- The main sinusoidal RF gives a linear restoring force. Synchrotron motion is a simple damped harmonic oscillator.
- For a Gaussian beam energy distribution, the longitudinal bunch shape is Gaussian.
- We manipulate the bunch shape if higher harmonic RF is added.



Harmonic RF Systems

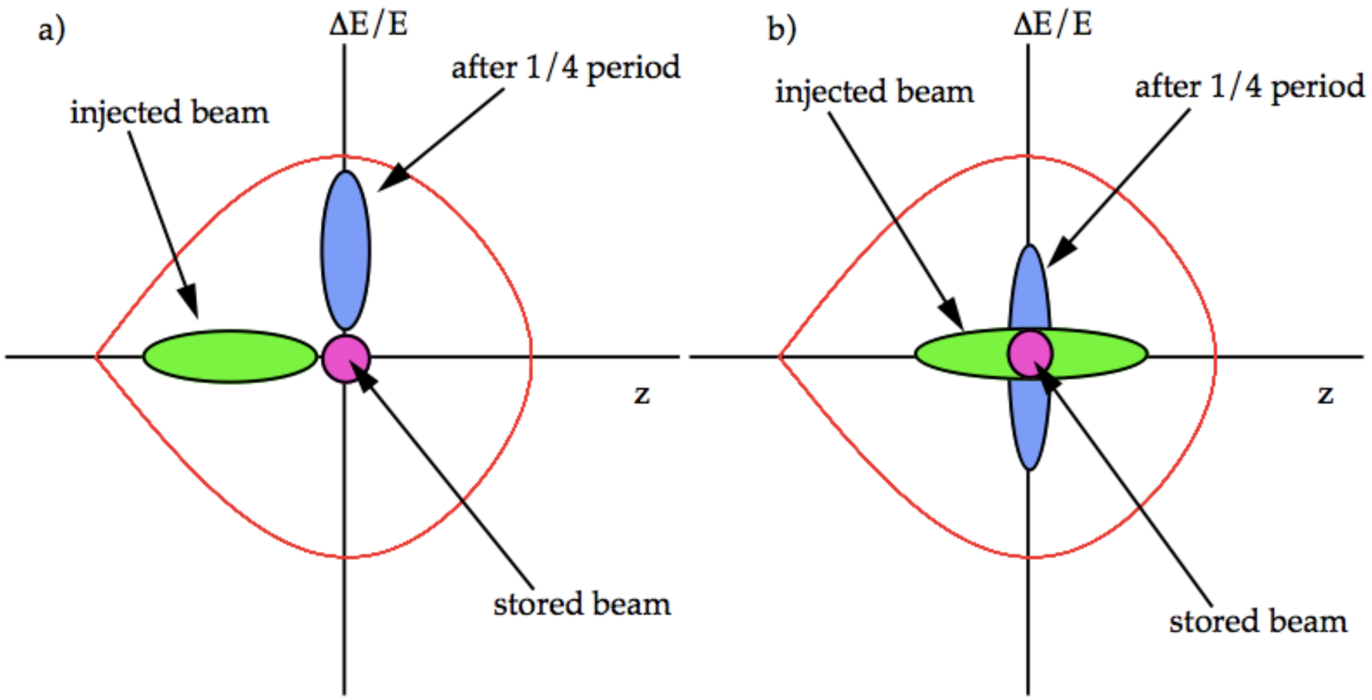
- The potential well can become substantially non-harmonic, with strong nonlinear detuning.
- Sometimes called “Landau” cavities because the synchrotron frequency spread provides “Landau damping” of coherent instabilities.



In this mode, the bunch is lengthened, and the peak current can be reduced by a factor of 2-4. This reduces the effect of Touschek scattering by the same factor. This is the **primary** means to improve beam lifetime in high brightness light sources.

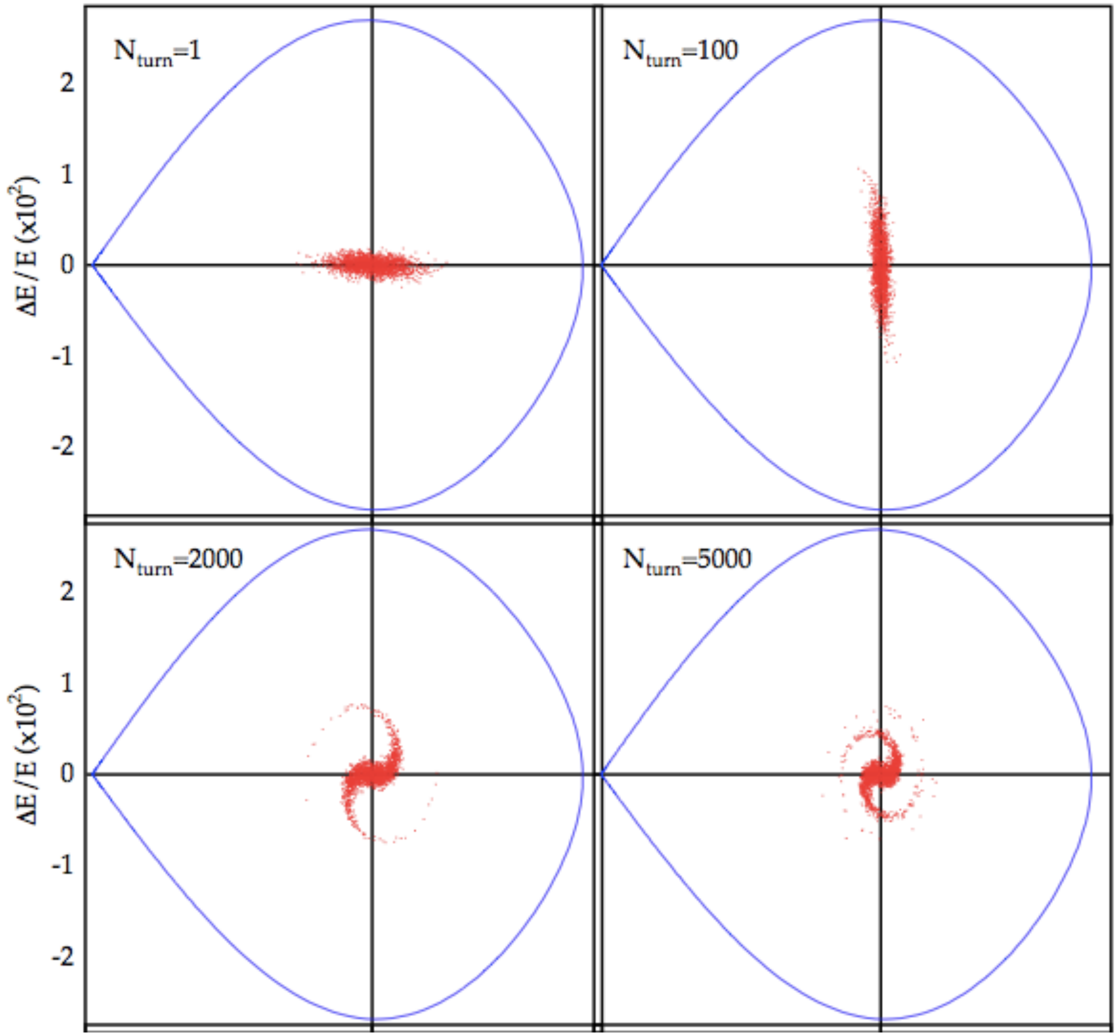
Example: Longitudinal injection transients

- During injection, the injected bunch shape and offset can be mismatched to the bucket, resulting in filamentation of the distribution. For an electron ring, the distribution eventually damps to the equilibrium.



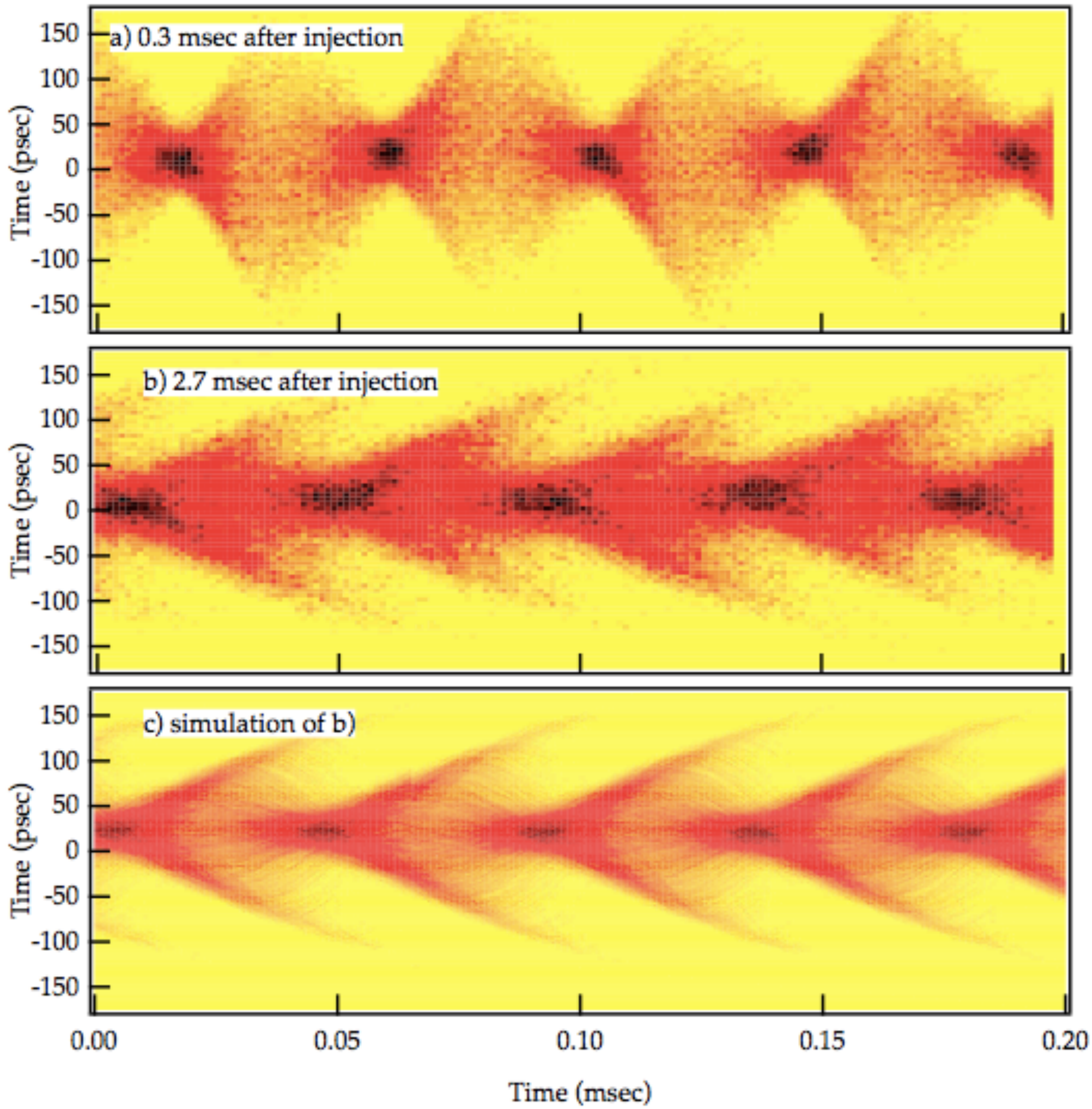
Example: Longitudinal injection transients

- For the mismatched bunch shape, the dependence of synchrotron tune on amplitude causes filamentation. For an electron ring, this eventually damps. For a hadron ring, the longitudinal phase space area is "increased".



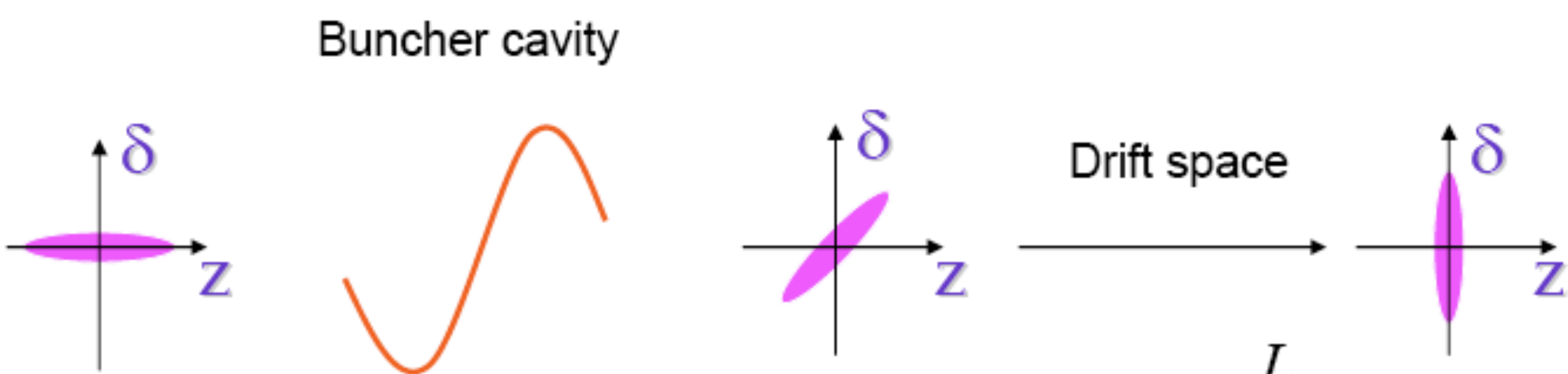
Example: Injection Transients Measurements

- Use a streak camera to record the longitudinal profile vs. time.
- a) observe quadrupole oscillations from bunch shape mismatch.
- b) filamentation



Ballistic bunch compression

- Usually used at very low energy, typically downstream of DC-gun
- Can be viewed as thin lens limit of velocity bunching
- Buncher imparts an energy chirp large enough to yield compression in a downstream drift

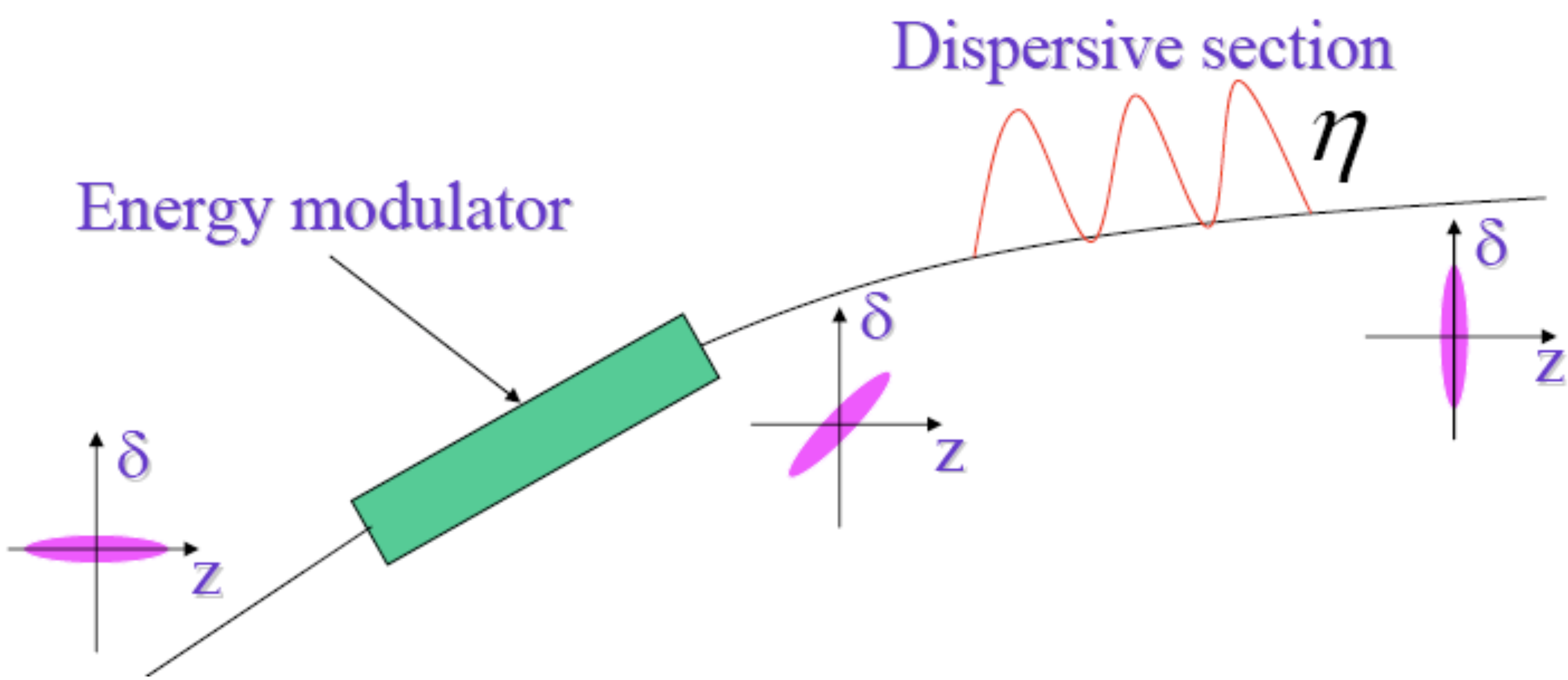


$$R_{56} = -\frac{L}{\gamma^2}$$

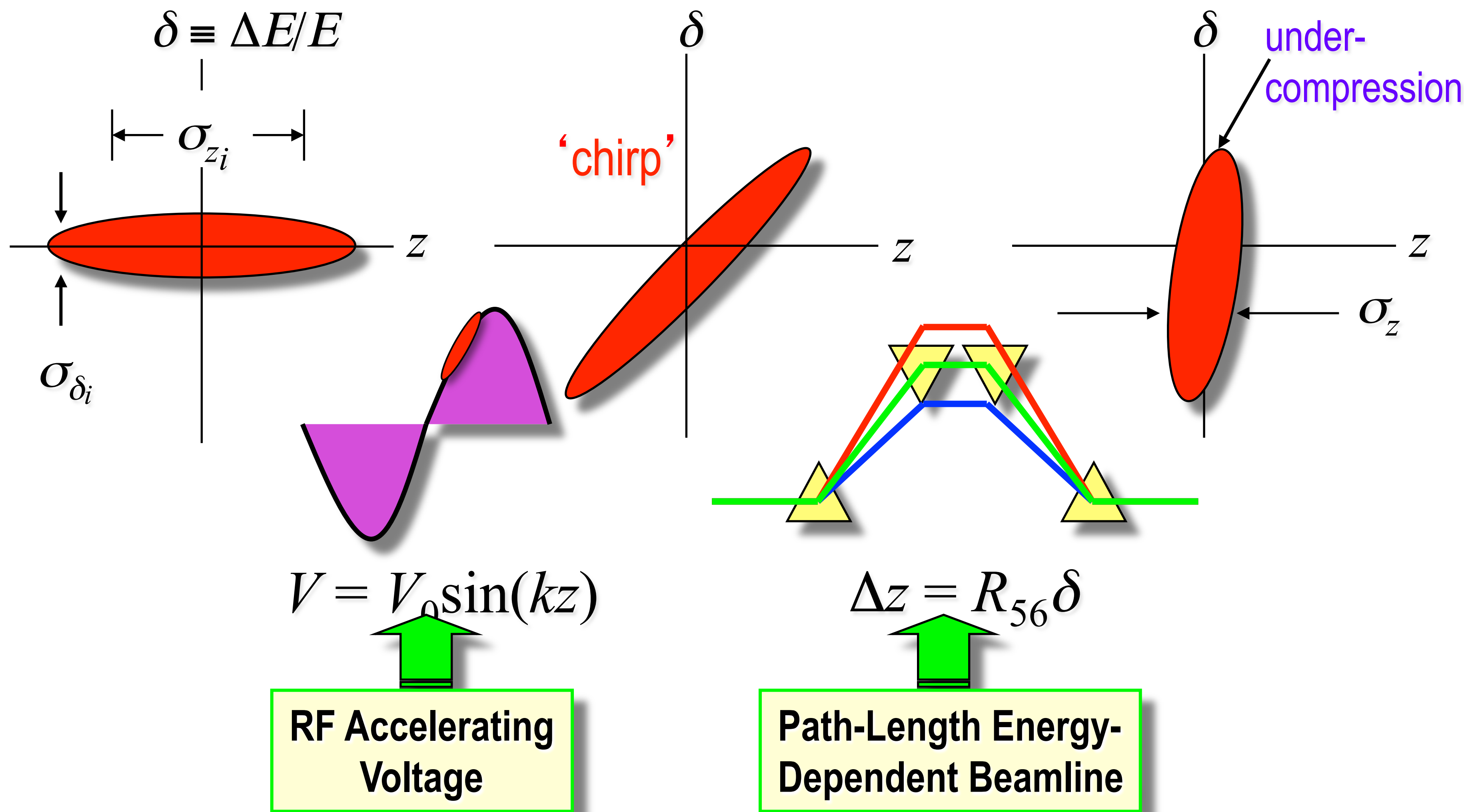
Linac longitudinal dynamics: Bunch compressors

Magnetic bunch compression

- Energy modulator: rf-structure, laser, wake-field
- Non-isochronous section
- In practice: multi-stage compression



Chicane Bunch Compression



To compress a bunch longitudinally, trajectory in dispersive region must be shorter for tail of the bunch than it is for the head.

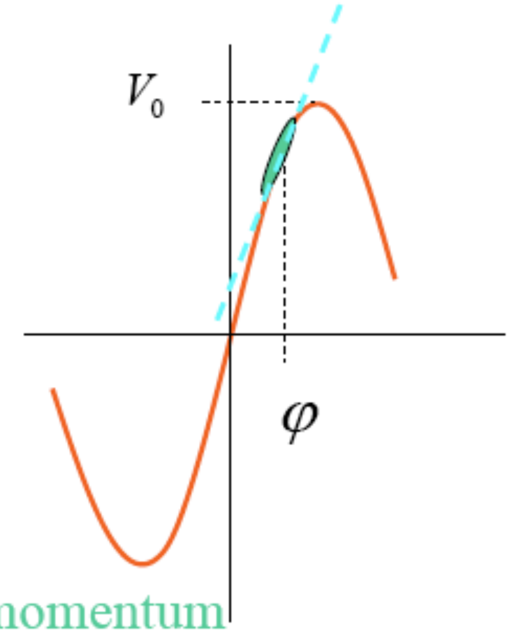
Linear Effects

- Energy time correlation:

$$E(z) = E_0 + eV_0 \cos(kz + \varphi)$$

$$\delta = \frac{eV_0}{E_0 + eV_0 \cos\varphi}\left[\cos(kz+\varphi) - \cos\varphi\right] = \kappa z + O(z^2)$$

chirp: $\kappa \equiv \frac{d\delta}{dz} = \frac{-keV_0}{E_0 + eV_0 \cos\varphi} \sin\varphi$



- Bunch compressor

1st order momentum compaction

$$z_f = z_i + R_{56}\delta_i$$

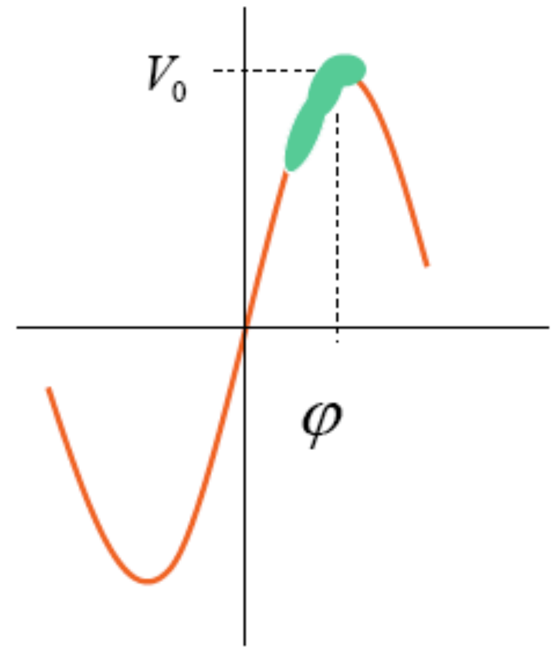
- Final bunch length and energy spread (1st order):

$$\sigma_{z,f} = \sqrt{(1+\kappa R_{56})^2 \sigma_{z,i}^2 + \underbrace{R_{56}^2 \sigma_{\delta,i}^2 E_0^2 / E^2}_{\text{Min bunch length}}}, \quad \sigma_{\delta,f} = \sqrt{\kappa^2 \sigma_{z,i}^2 + \sigma_{\delta,i}^2 E_0^2 / E^2}$$

Nonlinear effects

- Energy time correlation:

$$\delta = \kappa z + \mu z^2 + O(z^3)$$



- Bunch compressor

$$z_f = z_i + R_{56}\delta_i + T_{566}\delta_i^2$$

- Final bunch length is minimized if

$$0 = z_i(1 + \kappa R_{56}) + \underbrace{z_i^2(\mu R_{56} + \kappa^2 T_{566})}_{\text{Limit achievable minimum Bunch length}}$$

2nd order momentum compaction

Main issues

- How short can the bunch be compressed?
- Can low emittance be maintained?
- How large are the effects of space charge and coherent synchrotron radiation in bunch compression?

Summary

- Phase stability is necessary to maintain electrons on a stable orbit in a ring.
- Synchrotron oscillations can be modeled as simple damped harmonic oscillators:
  - Longitudinal focusing come from a time-varying accelerating field provided by an RF system.
  - Electrons with higher energy take a longer path length around the ring ($\alpha>0$)
  - Discrete photon emission excites oscillations
  - Energy dependence of SR gives radiation damping.