MATLAB Physics - III

- You should be reading the example scripts as templates for your projects.
- You should run the demos/scripts yourselves.
- We are available to answer questions by e-mail or during the homework sessions.
- Homework and "project".
  - You should be thinking about your project.
  - Pick a topic that interets you.

Third Category of Topics

- General topics of the interaction of particles (charged) with electric and magnetic fields.
- Hadron specialists ~ ignore radiation (but LHC …)
- Electron specialists worry about SR radiation (but muon collider …)
- Final topic will be simple beam design (GUI) of a quadrupole doublet.

Driven, Damped SHO

- **Set up for general second order ODE with a driving harmonic force. Symbolic math.**

A general second order inhomogeneous differential equation appears in Eq. 5.14. It can be simplified by expressing time in units of the undriven and undamped circular frequency ω_0, or τ. When the natural frequency is defined to be one, there remain three parameters defining the equation, a damping factor b, a driving amplitude C and the ratio of the driving frequency to the natural frequency k.

$$md^2y/d^2t + mady/dt + by = A\sin(\omega\tau)$$

$$d^2y/d^2\tau + bdy/d\tau + y = C\sin(kt) \qquad 5.14$$

$$\tau = \omega_o t, \omega_o^2 = b/m, k = \omega/\omega_o$$

Symbolic Solutions ?

- The second order ODE has a symbolic solution.
- In general try for a symbolic solution first using ”solve” or “dsolve”, “int”, “diff”
- If that does not work, use numerical “ode45” or “quad” or “gradient”
- The free SHO frequency is shifted by damping.

$$\omega_b / \omega_o = \sqrt{1-(b/2)^2} + ib/2 \qquad 5.15$$

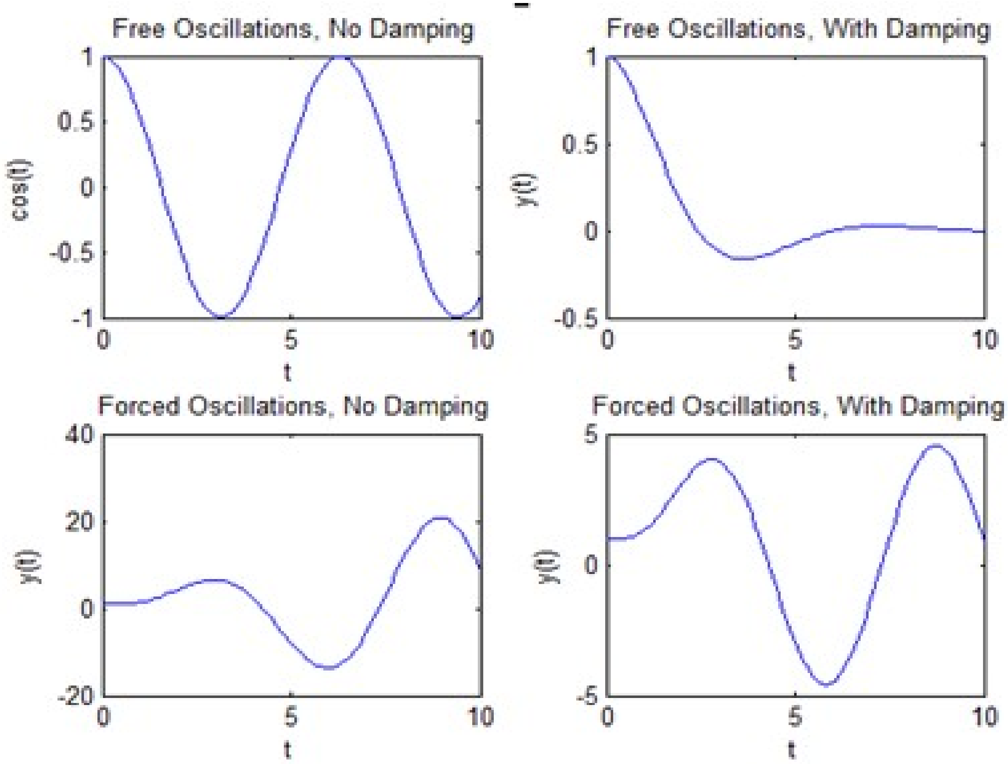
Printout for Driven/Damped SHO

- Symbolic solutions. Run "Damped_Forced_SHO"

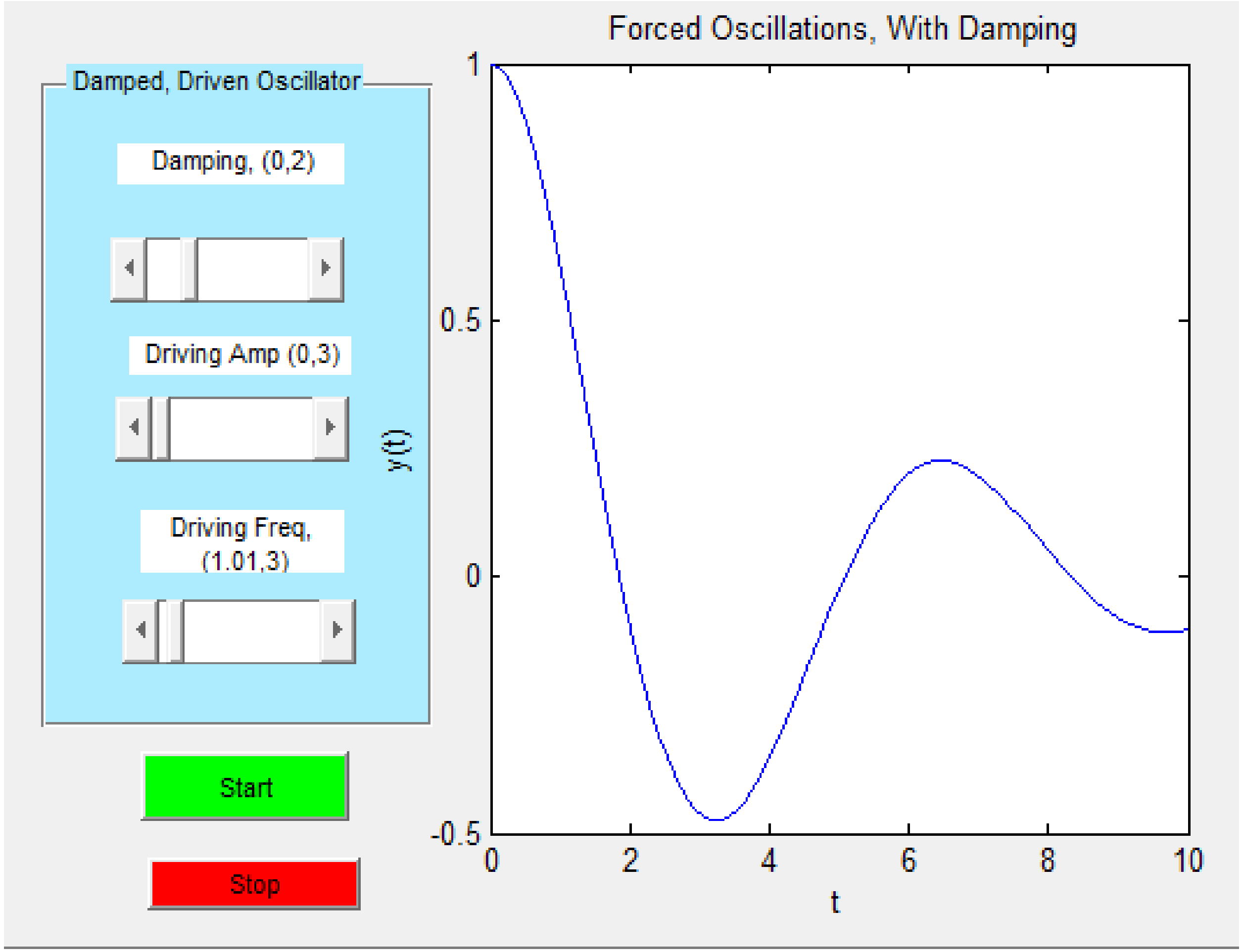
```
Simple Harmonic, No Damping, No Driving

  C2 cos(t) + C3 sin(t)
Simple Harmonic, Damped, No Driving

      /     / b    #1 \ \                      /     / b    #1 \ \
  exp| - t | - - -- | | (b + #1)    exp| - t | - + -- | | (b - #1)
      \     \ 2    2  / /                      \     \ 2    2  / /
  ------------------------------ - ------------------------------
                2 #1                              2 #1

  where

                              1/2
     #1 == ((b - 2) (b + 2))
Enter Damping b : 1
Simple Harmonic, No Damping, Driving

                 /   c k        c k    \          / c cos(t (k - 1))   c cos(t (k + 1)) \
  cos(t) + sin(t) | ------- - ------- | - sin(t) | ---------------- + ---------------- | -
                 \ 2 k - 2   2 k + 2 /          \     2 k - 2            2 k + 2      /

           / c sin(t (k - 1))   c sin(t (k + 1)) \
     cos(t) | ---------------- - ---------------- |
           \     2 k - 2            2 k + 2      /
Enter amplitude c and frequency k as [ , , ]): [5,1.1]
Simple Harmonic, Damped and Driven
Enter damping b, amplitude c and driving frequency k as [ , , ]): [1 5 1.1]
```

Command Line Results

- Command line script is the default – a few GUI wrappers are available.



GUI for Damped, Driven SHO



Use the sliders to change the damping, and the driving amplitude and frequency

Motion in E and B Fields (NR)

- Drift – velocity separator, v = E/B. Use in momentum selected beam to physically select masses; pions, kaons, protons – low momentum beams.

$$d^2\bar{x} / d^2t = q / m[\vec{E} + (d\bar{x} / dt)x\vec{B}]$$

Run "ExB_ODE_NR"

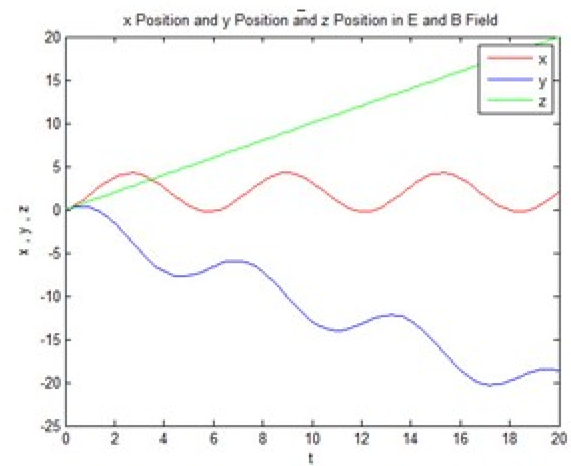


Figure 3.26: The three position components as a function of time. The basic circular motion of the x and y positions is evident.

SR – Electric, Magnetic Field

- **Uniform E is solvable simply. In B, frequency depends on energy (SR).**

-

$$dP / dt = qE$$
$$P = qEt$$
$$\beta = P / \varepsilon = at / \sqrt{(at)^2 + 1}$$
$$a = qE / m$$
$$z = a[\sqrt{(at)^2 + 1} - 1]$$

$$d\vec{P} / dt = q(\vec{E} + \vec{v}x\vec{B})$$
$$\vec{\beta} = \vec{P} / \varepsilon = \vec{P} / \sqrt{\vec{P}^2 + M^2}$$
$$d\vec{P} / dt = q(\vec{E} + \vec{P}x\vec{B} / \varepsilon)$$
$$d\vec{x} / dt = c\vec{P} / \varepsilon$$

Doppler, Cerenkov

- For a particle passing through a medium, there is a Doppler shift. If v > c in the medium, Cerenkov radiation. Velocity selection-> particle ID. Run “Doppler_Cerenkov”

$$\omega / \omega_o = 1 - v\cos\theta / v_s$$

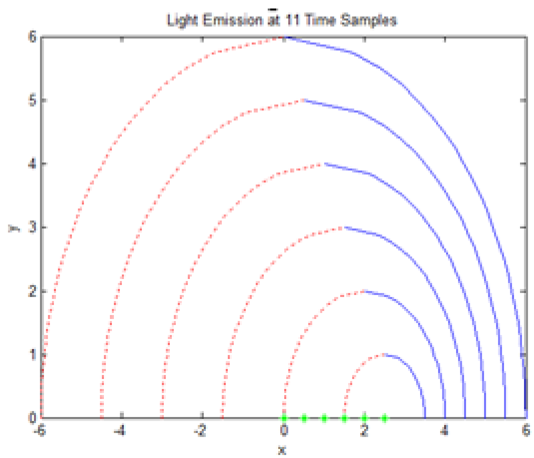


Figure 4.15: Outgoing waves in the case where $v/v_s = 0.5$. The regions of wavelength compression and expansion are seen in the forward and backward positions. The emission points are green*.

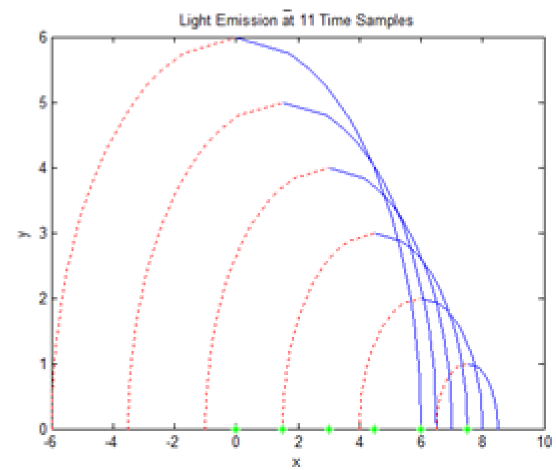


Figure 4.16: Outgoing waves in the case where $v/v_s = 1.5$. The regions of wavelength compression and expansion are seen in the forward and backward positions. The emission points are green *.

Biot-Savert for a Current Loop

- **Integral is elliptics. Either expand or do the intergal over dI numerically. Simpler numerically ?**

$$R : R(\cos\psi\ \hat{i} + \sin\psi\ \hat{j})$$

$$d\vec{I} = R(-\sin\psi\ \hat{i} + \cos\psi\ \hat{j})$$

$$\vec{r} = ((x - R\cos\psi)\hat{i} + (y - R\sin\psi\ \hat{j}) + z\hat{k}$$

$$d\vec{B} = (d\vec{I}x\vec{r}) / r^3$$

$$source\ at\ R,\ field\ at\ r = (x, y, z)$$

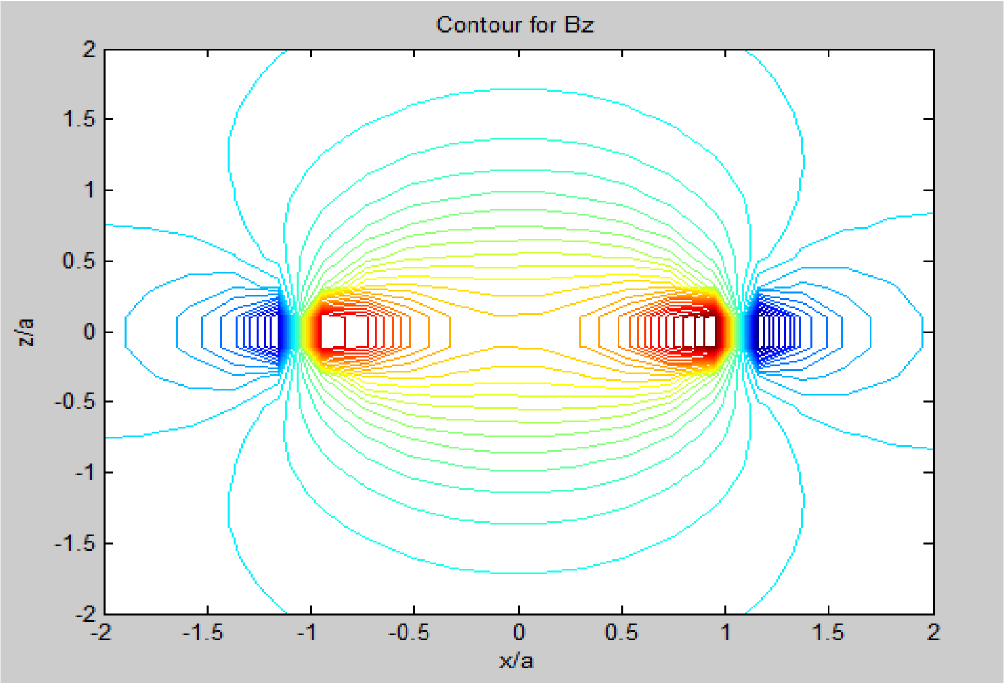
$$d\vec{B} = \hat{i}(z\cos\psi) + \hat{j}(z\sin\psi) + \hat{k}(y\sin\psi - x\cos\psi + R^2)$$

$$d\vec{B} = d\vec{B} / [(r^2 + R^2) - 2 * R * (x\cos\psi - y\sin\psi)]^{3/2}$$

Current Loop – Biot - Savert

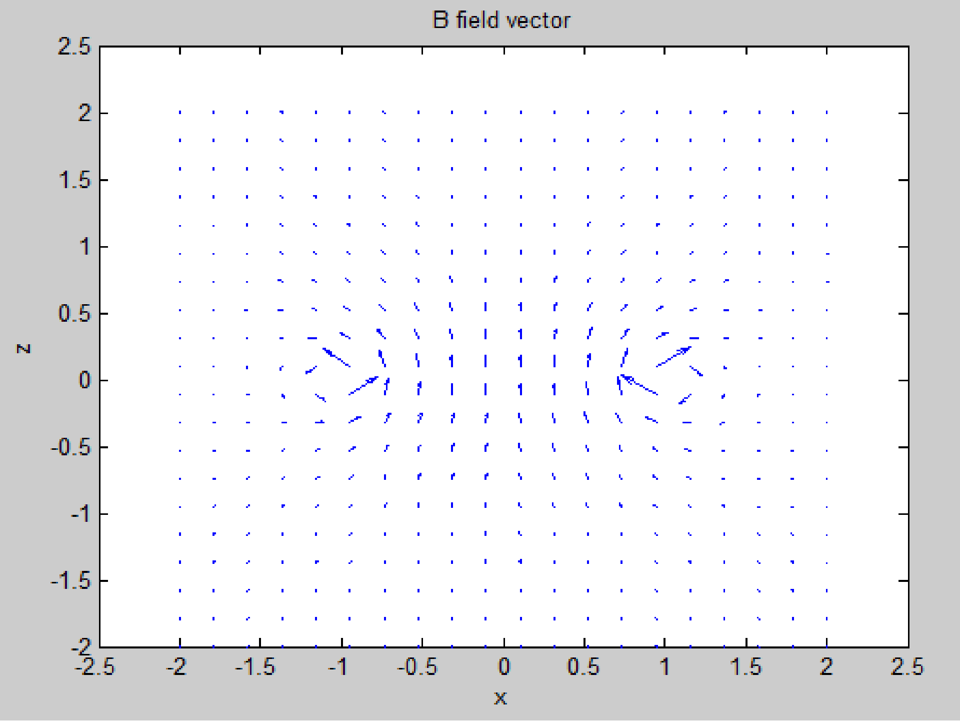
- **Do the integration for Biot-Savert numerically.**
- **Check on axis limit. Run "Current_Loop"**

$$B_z = 2\pi\ R^2 / (z^2 + R)^{3/2}$$



Vector Field

- Check limit – at x = y = 0, Bx=By=0. Use "quiver"



2 Current Loops

- Helmholtz coil
  ~ uniform B field
- Prototype for a dipole magnet
- Run "Helmholtz_Coil"

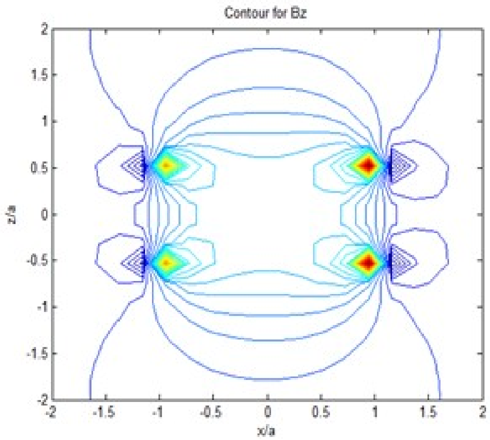
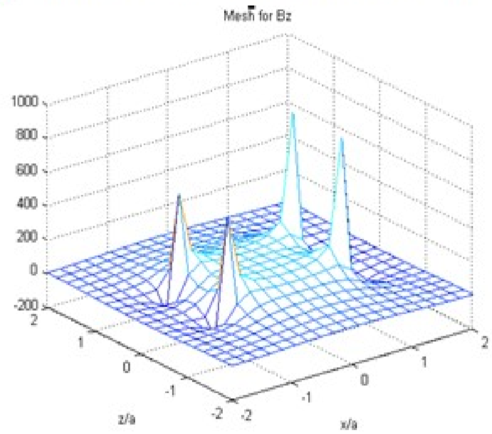
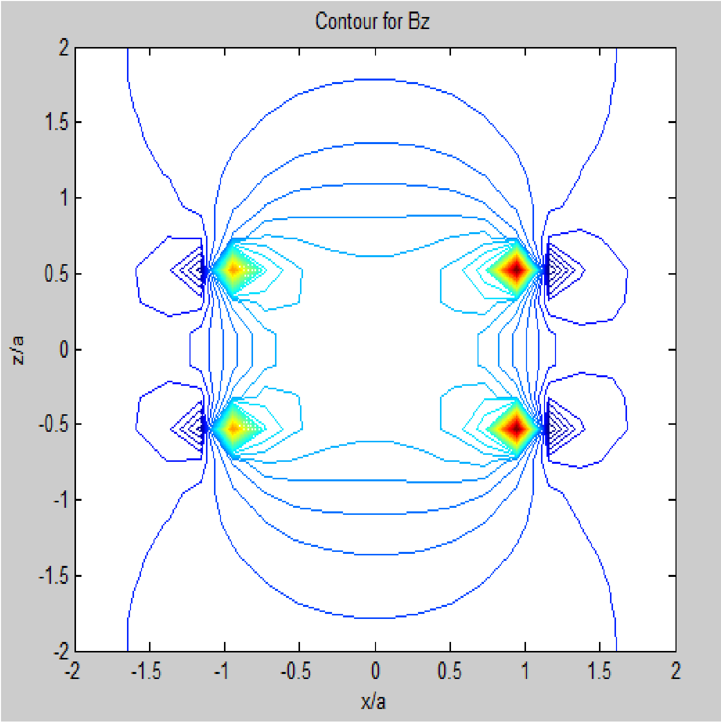


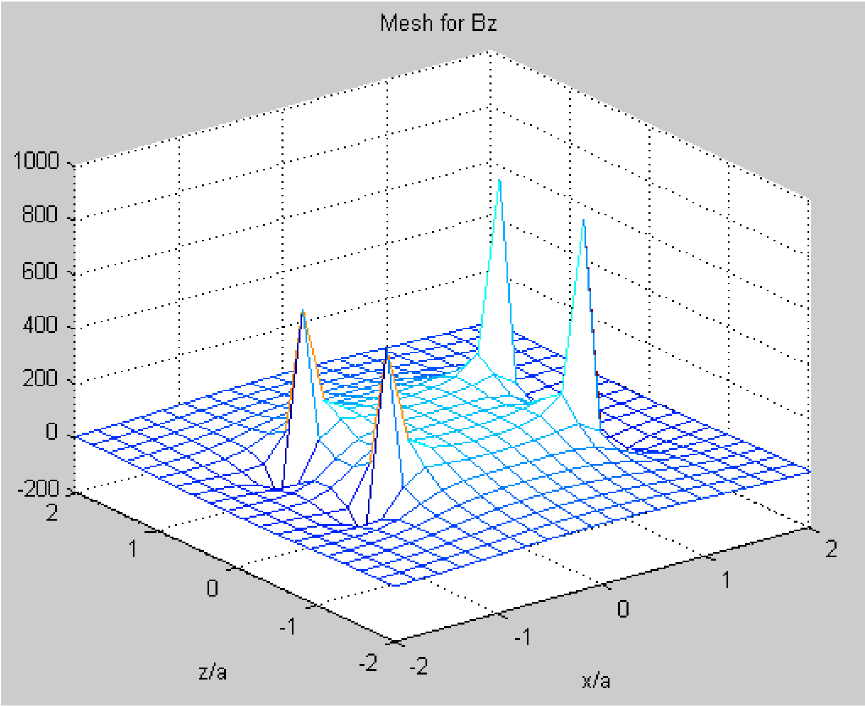
Figure 3.8: Contourplot for Bz due to two current loops separated by a distance equal tc their radius.



Helmholtz Coil

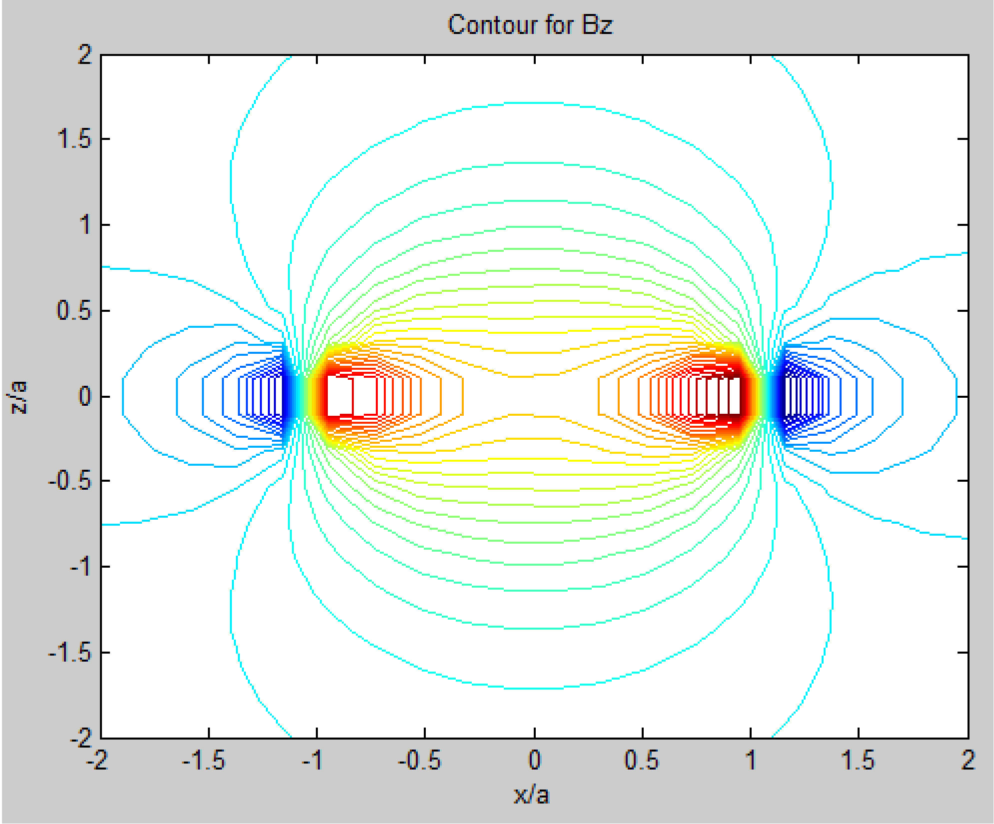
- Add fields due to 2 loops - ~ uniform B, 2d=1
- Check d = 0 limits?



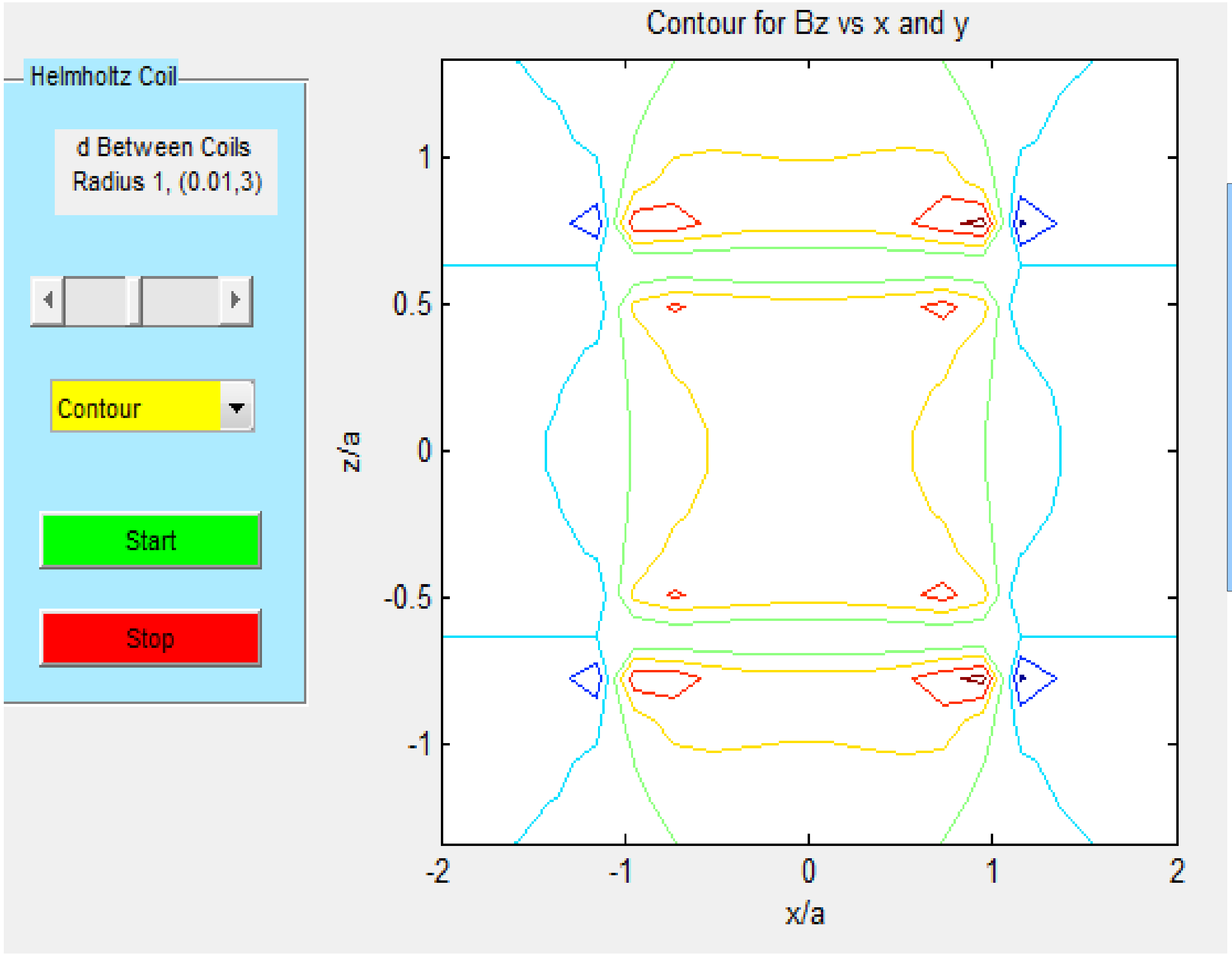


d = 0 Limit?

- **Limiting contour**



GUI for Helmholtz Coil



Only 1 parameter - distance between loops

Cyclotron

- Use B to contain the beam and E to accelerate when crossing the “dees”. Run “Cyclotron”
- Frequency is not energy dependent (NR). $ → ramp B

$$\omega = qB / m$$

$$r = v_T / \omega$$

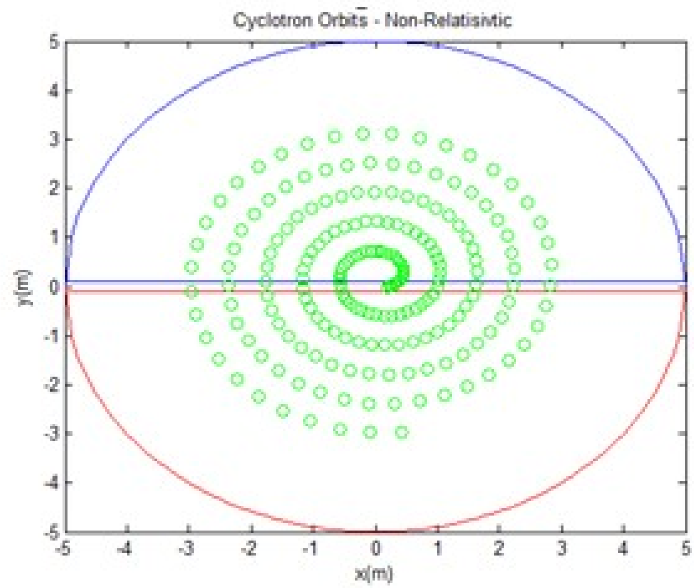


Figure 3.28: End of the movie for a charged particle in a cyclotron with 10 half revolutions and with an energy kick of 0.3 at each crossing of the “dees”.

Dipole radiation

- NR particle radiates as a dipole in angles. Radiative fields go as 1/r.
- There are static like fields near the source
- Near and far zones depend on kr.

$$E_r r^3 = d(2z/r)(1-ikr)e^{ikr}$$

$$E_\theta r^3 = d(x/r)[1-ikr-(kr)^2]e^{ikr}$$

Far Zone and Radiation

- **Dipole (NR) radiation. Run "Dipole_Power"**

Dipole electromagnetic radiation is explored in the regime where the radius and inverse wave vector, r and 1/k, are much larger than the size of the dipole in the script "Dipole_Power". The velocity c is taken to be one. The expression for the dipole power angular distribution is shown in Eq. 3.12. The dipole angular distribution is the sin squared of the polar angle of k with respect to the dipole direction. There is a wave outgoing at the speed of light which falls as a radiated energy as inverse of radius, so that the power crossing a sphere of radius r is independent of the size of r. This is the basic characteristic of a radiation field.

$$dP / d\Omega = k \sin^2 \theta \sqrt{1 + (1/kr)^2 [\cos(k(t-r)) + \tan^{-1}(kr)]} / r \qquad 3.12$$

Dipole Radiation

- **Movie of system behavior – exact for distance from dipole itself large.**

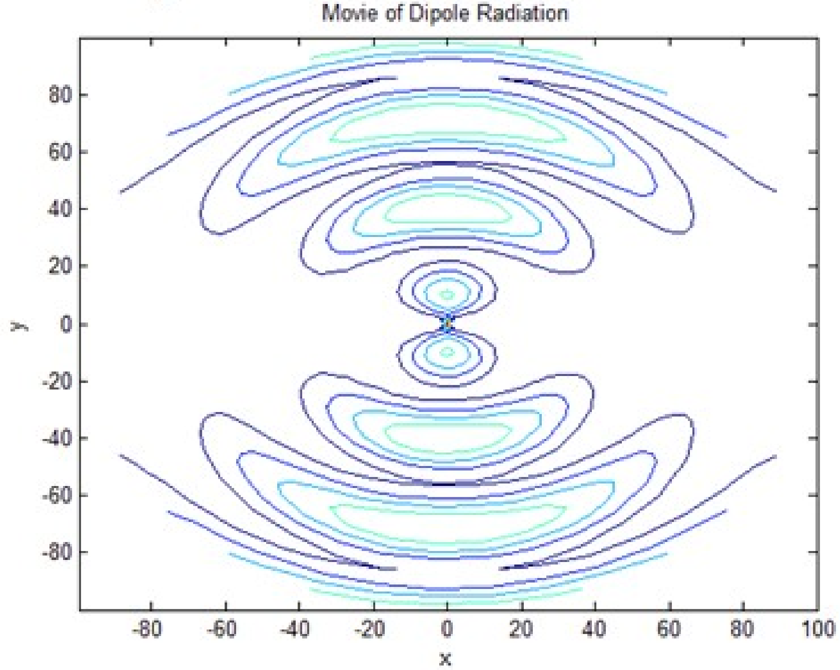
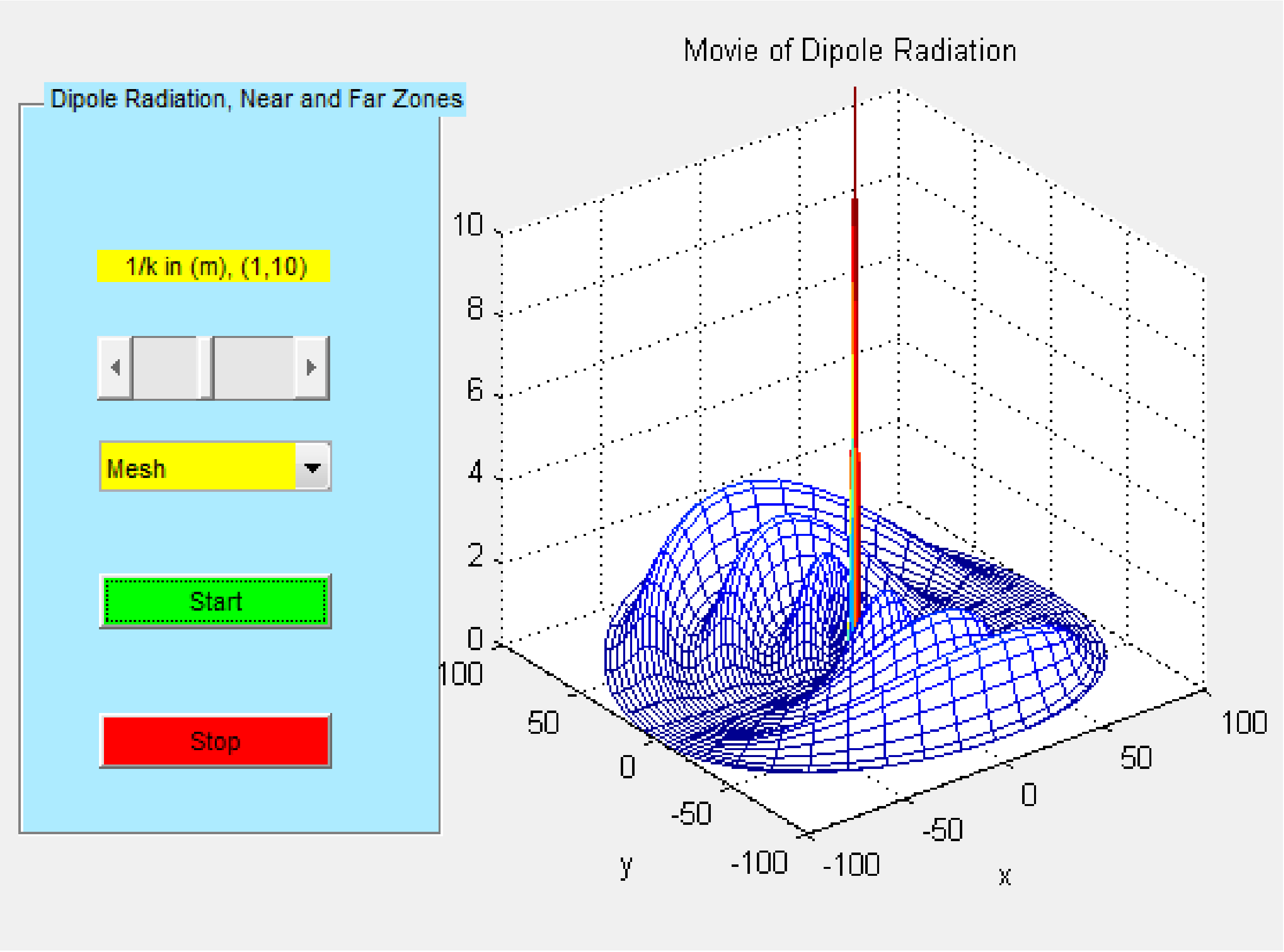


Figure 3.46: Radiated angular pattern for a dipole when r and 1/k are greater than the size of the dipole itself.

GUI for Dipole Radiation



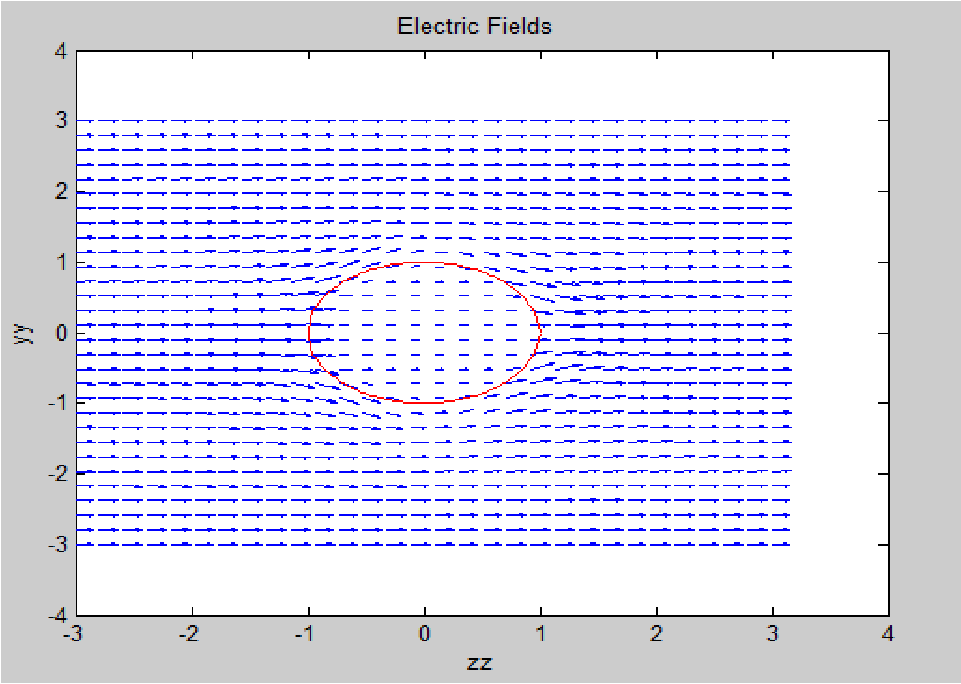
Drop Menu for mesh and contour display.

Shielding

- Stray E and B fields often need to be shielded against
- E shielding uses conductors
- B shielding uses materials with high magnetic permiability
- Limits? To be checked
- Variables to choose are the shielding thickness and conductivity/mu value.

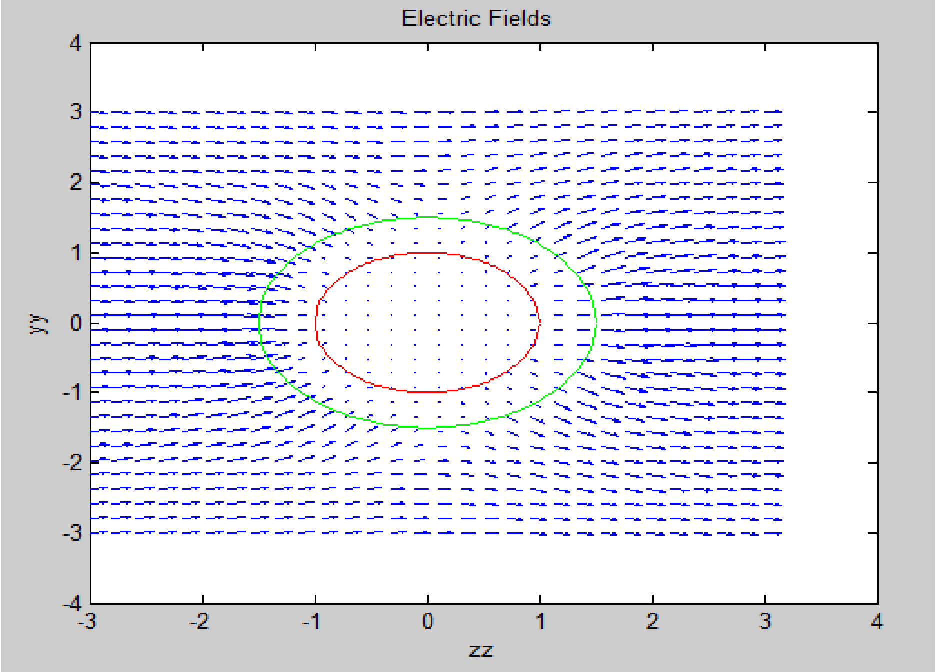
Dielectric Sphere in E Field

- Normal D is continuous, not E. Static field.
- Run "Dielectric"



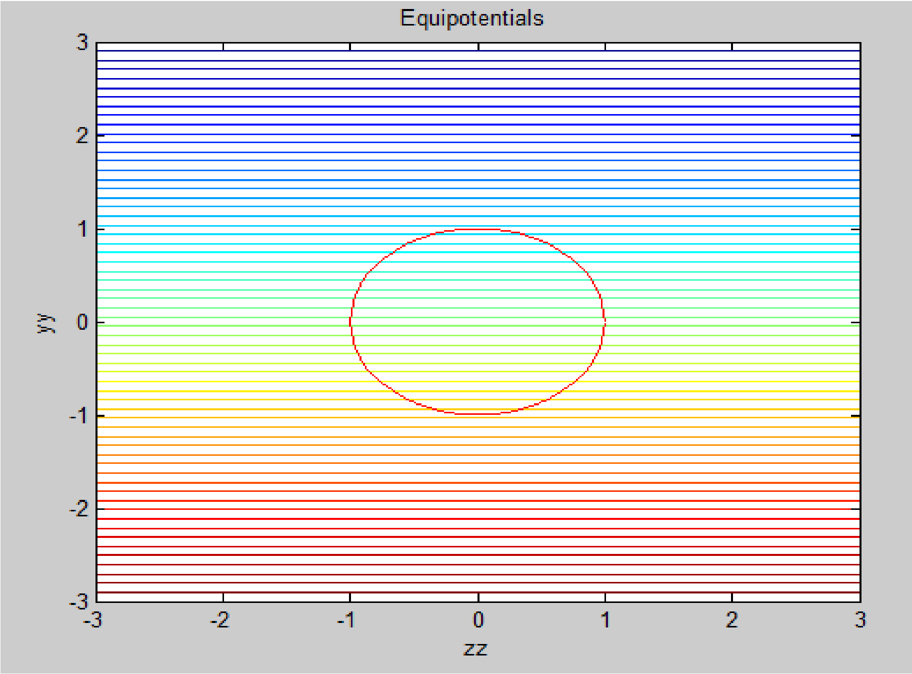
E Shield with K ?

- Like the B shield with high u material. Note BC at inner and outer cylinder surfaces. Vary k.
- Run “Dielectric_Sphere”



Limits ?

- K = 1, vacuum. Check limits for physical reasonableness
- K-> Inf, conductor



Skin Depth

- **As in QM, photon can "tunnel" into a conductor by a small amount. Frequency dependent - > r.f. "plumbing".**

The conductivity σ relates the current density, J, and the electric field, E, in the microscopic form of Ohm's law. The wave in the conductor has a complex wave vector, k, which means that there is an exponential penetration of the wave into the conductor by a characteristic distance d which is proportional to the inverse of the imaginary component of the wave vector k. The form for k in Eq. 3.17 is closely related to the previous discussion of dispersion, with σ playing the role of the parameter δ in Eq. 3.14.

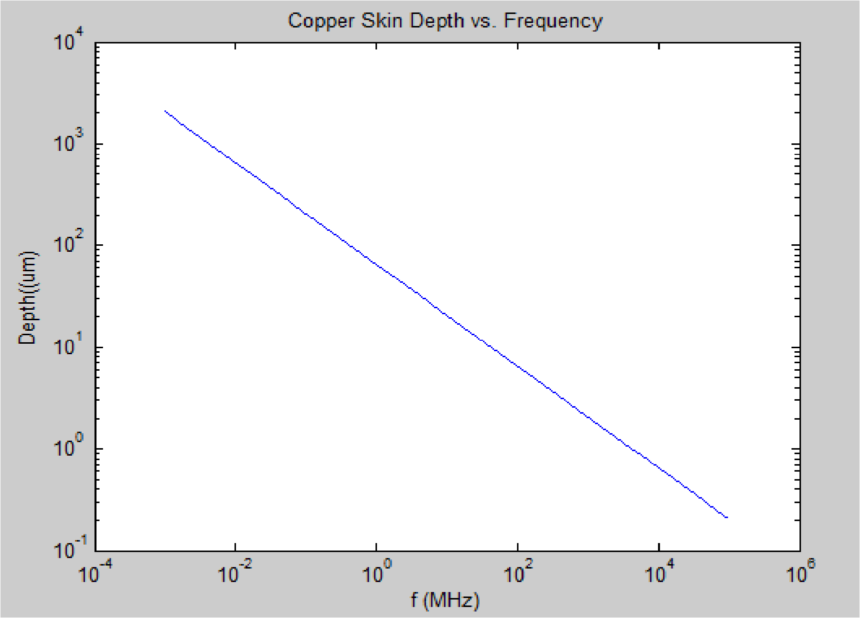
$$\vec{J} = \sigma \vec{E}$$

$$k^2 = (\omega / c)^2 [1 + i(4\pi\sigma / \omega)] \qquad 3.17$$

$$d \sim c / \sqrt{2\pi\omega\sigma}$$

Skin Depth

- Perfect conductor shields static fields (e.g. image). In VB=CB e are free to move to respond to E fields.
- Oscillating fields penetrate a conductor by a "Skin depth" ~ 100 um for 1 MHz. Run "Skin_Depth"



B Shield Calculation

- A la Jackson – b.c at outer radius and inner radius. Induced fields ~ rcosθ and ~ 1/r^2.
- Coefficients from b.c.
- Limits?
- Variables to choose are the shielding thickness and mu value.

```
for i = 1:length(zz);
    for j = 1:length(yy)
        r = sqrt(zz(i) .^2 + yy(j) .^2);
        ct = zz(i) ./r;
        if r > b
            phib(i,j) = -r .*ct + (alf .*ct) ./(r .^2);
        end
        if  r < 1
            phib(i,j) = (del .*r .*ct);
        end
        if r < b & r > 1
            phib(i,j) = (bet .*r .*ct) + (gam .*ct) ./(r .^2);
        end
    end
```

Magnetic Shielding

- Recall K → Inf for a conductor. For B fields, u → Inf (“mu metal”) - saturation? There are no free magnetic charges – just magnetic dipoles that can align. Once all aligned?
- Ratio 1.1, pick u. Run “Magnetic_Shield”

$$\Phi_{out} = -B_o r\cos\theta + (\alpha / r^2)\cos\theta$$
$$\Phi_{in} = \delta r\cos\theta$$
$$\Phi_{\mu} = \beta r\cos\theta + (\gamma / r^2)\cos\theta \qquad (3.4)$$

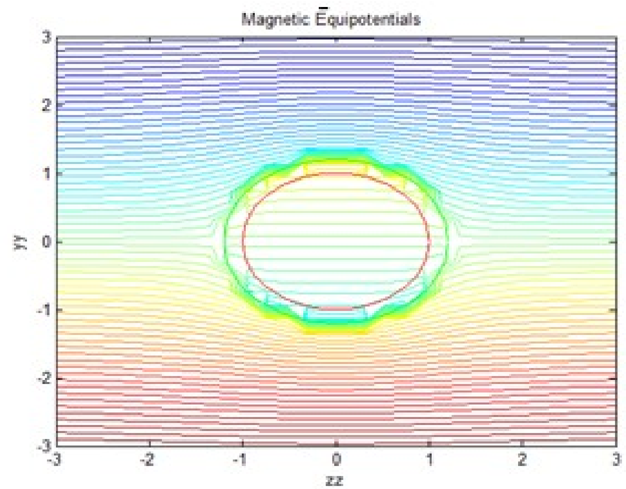


Figure 3.10 Potential for a metallic sphere immersed in a uniform magnetic field oriented along the z axis for b/a = 1.2 and $\mu = 10$.

Quadrupole

- **Dipole (e.g. Helmholtz) has ~ uniform field over a volume. Quadrupole has a B gradient which increases with distance from the origin. Lorentz force is toward the origin (F) in one plane and away (D) in the other plane.**

$$\Phi = (dB/dr)xy$$
$$B_x = -(dB/dr)y$$
$$B_y = -(dB/dr)x$$

$$k = a(dB/dr)/p$$
$$\phi = \sqrt{k}L$$
$$\begin{bmatrix} x \\ dx/dz \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi/\sqrt{k} \\ -\sqrt{k}\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x_o \\ (dx/dz)_o \end{bmatrix} \qquad (4.9)$$

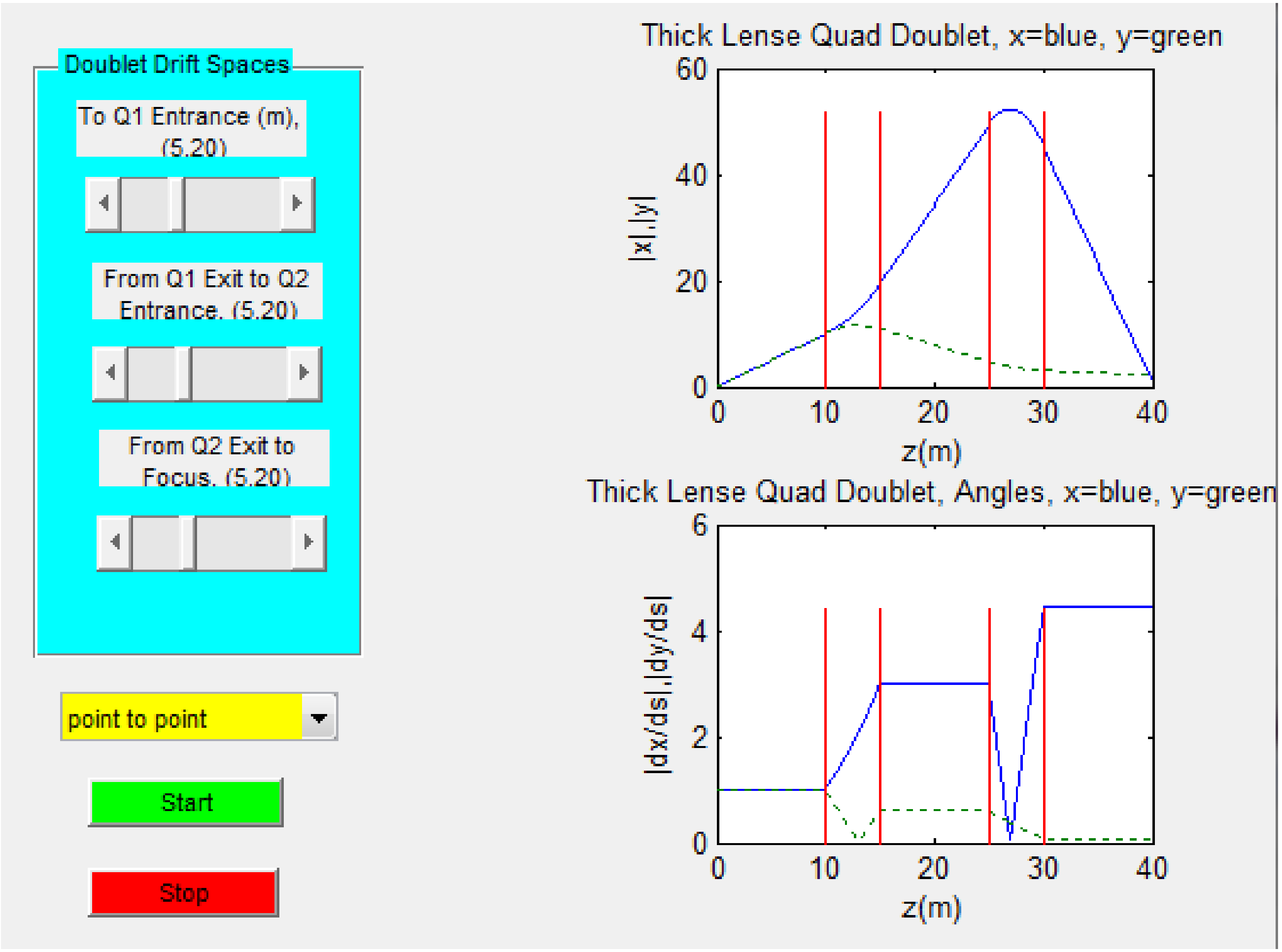
Thin Lense

- Beamline as a series of matrices acting on a vector x, dx/ds,y,dy/ds
- Dipole is unit matrix ignoring dp/p captured by the beam.
- “Drift” has straight line behavior – no forces
- Quadrupole has sin,cos,sinh,cosh matrix elements; nonlinear-> use fminsearch
- Use thin lens to solve; starting values needed

Doublet

- Simplest system which can provide a focus for both x and y motion (EM is not like classical lense optics),
- Thin lense provides solutions (quadratic equations) – 2 equations in 2 unknowns – the F and D focal lengths.
- Options are point to point (M12=0), point to parallel (M22=0) and parallel to point (M11=0). Run “Quad_Doublet”

GUI for Quad Doublet



Why no parallel to parallel option ?

Menu for focus condition

Sliders for The 3 drift distances