# RF FUNDAMENTALS
# MICROPHONICS

**Jean Delayen**

**Center for Accelerator Science**
**Old Dominion University**
**and**
**Thomas Jefferson National Accelerator Facility**

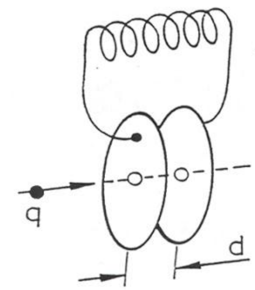
# Equivalent Circuit for an rf Cavity

**Simple LC circuit representing an accelerating resonator**

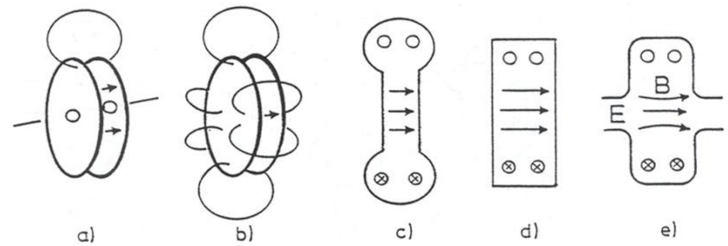
**Metamorphosis of the LC circuit into an accelerating cavity**

**Chain of weakly coupled pillbox cavities representing an accelerating cavity**

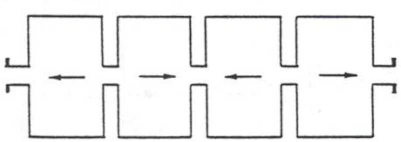
**Chain of coupled pendula as its mechanical analogue**



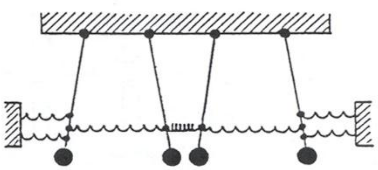

Simple lumped L-C circuit repesenting an accelerating resonator.
$\omega_o^2 = 1/LC$




Metamorphosis of the L-C circuit of Fig. 1 into an accelerating cavity (after R.P.Feynman[33]). Fig. 5d shows the cylindrical "pillbox cavity" and Fig. 5e a slightly modified pillbox cavity with beam holes (typical $\beta$ between 0.5 and 1.0). Fig. 5c resembles a low $\beta$ version of the pillbox variety ($0.2<\beta<0.5$).

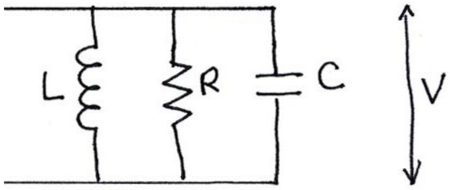


Chain of weakly-coupled pillbox cavities representing an accelerating module



Chain of coupled pendula as a mechanical analogue to Fig. 6a

# Parallel Circuit Model of an Electromagnetic Mode



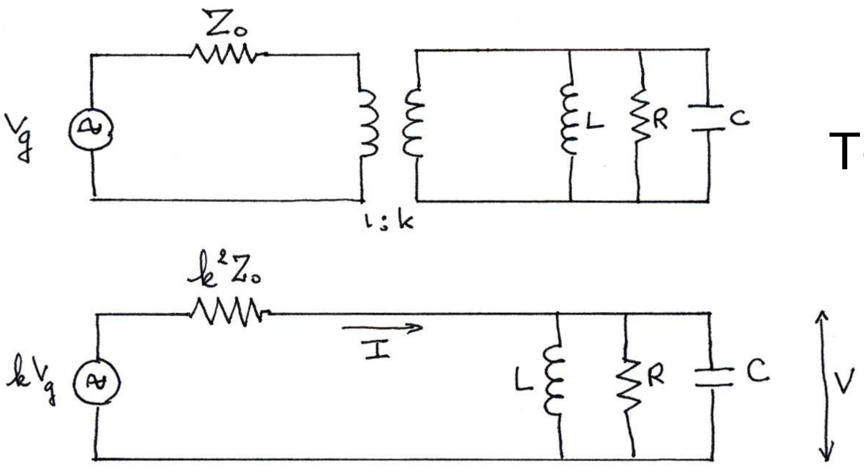

- Power dissipated in resistor R: $$P_{diss} = \frac{1}{2}\frac{V_c^2}{R}$$
- Shunt impedance: $$R_{sh} \equiv \frac{V_c^2}{P_{diss}} \qquad \Rightarrow \quad R_{sh} = 2R$$
- Quality factor of resonator:

$$Q_0 \equiv \frac{\omega_0 U}{P_{diss}} = \omega_0 CR = \frac{R}{L\omega_c} = R\left(\frac{C}{L}\right)^{1/2}$$

$$\tilde{Z} = R\left[1 + iQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]^{-1}$$

$$\omega \approx \omega_0 \ , \qquad \tilde{Z} \approx R\left[1 + 2iQ_0\left(\frac{\omega - \omega_0}{\omega_0}\right)\right]^{-1}$$

# 1-Port System




Total impedance: $k^2 Z_0 + \dfrac{R}{1 + 2i\dfrac{Q_0}{\omega_0}\Delta\omega}$

$$I = \frac{kV_g}{k^2 Z_0 + \dfrac{R}{1 + 2i\dfrac{Q_0}{\omega_0}\Delta\omega}}$$

$$V = kV_g \frac{R}{R + k^2 Z_0\left(1 + 2i\dfrac{Q_0}{\omega_0}\Delta\omega\right)}$$

# 1-Port System

Energy content $U = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q_0}{\omega R} V^2$

$$= \frac{1}{2} \frac{Q_0}{\omega R} \; k^2 V_g^2 \; \frac{R^2}{\left(R + k^2 Z_0\right)^2 + 4k^4 Z_0^2 Q_0^2 \left(\frac{\Delta\omega}{\omega_0}\right)^2}$$

Incident power: $P_{inc} = \frac{V_g^2}{8Z_0}$

Define coupling coefficient: $\beta = \frac{R}{k_0^2 Z_0}$

$$\frac{U}{P_{inc}} = \frac{Q_0}{\omega_0} \frac{4\beta}{(1+\beta)^2} \frac{1}{1 + \left(\frac{2Q_0}{1+\beta}\right)^2 \left(\frac{\Delta\omega}{\omega_0}\right)^2}$$

# 1-Port System

Power dissipated

$$P_{diss} = \frac{\omega U}{Q_0} = P_{inc} \frac{4\beta}{(1+\beta)^2} \frac{1}{1+\left(\frac{2Q_0}{1+\beta}\right)^2 \left(\frac{\Delta\omega}{\omega_0}\right)^2}$$

Optimal coupling: $\frac{U}{P_{inc}}$ maximum or $P_{diss} = P_{inc}$

$\Rightarrow \Delta\omega = 0,\quad \beta = 1$ : critical coupling

Reflected power

$$P_{ref} = P_{inc} - P_{diss} = P_{inc} \left[ 1 - \frac{4\beta}{(1+\beta)^2} \frac{1}{1+\left(\frac{2Q_0}{1+\beta} \frac{\Delta\omega}{\omega_0}\right)^2} \right]$$

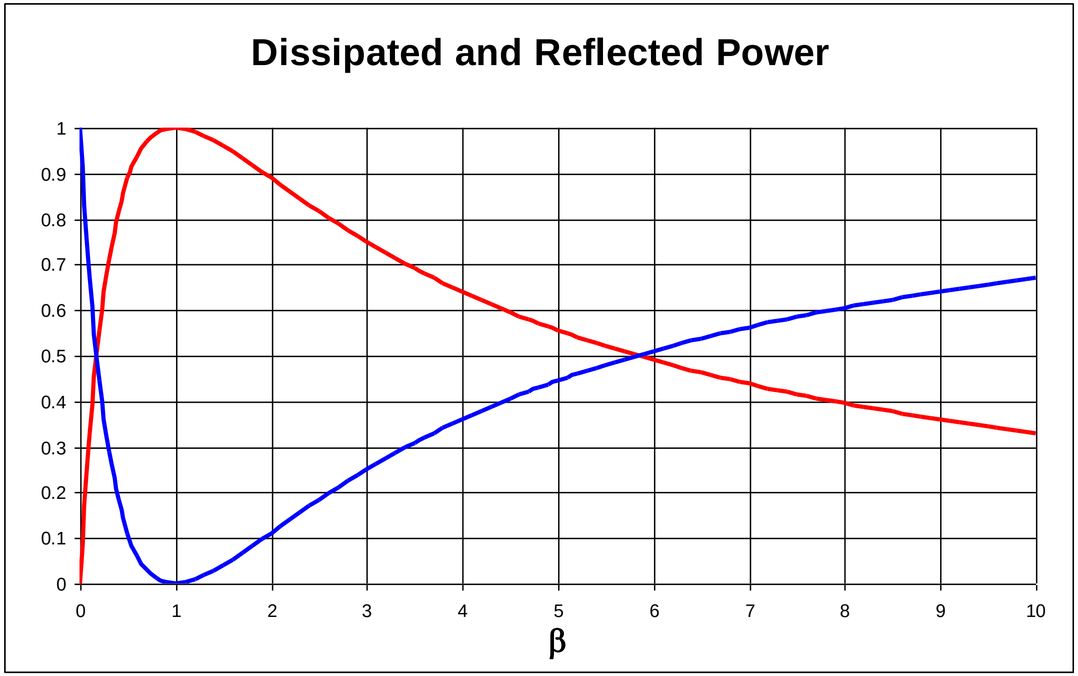
# 1-Port System

At resonance

$$U = \frac{Q_0}{\omega_0} \frac{4\beta}{(1+\beta)^2} P_{inc}$$

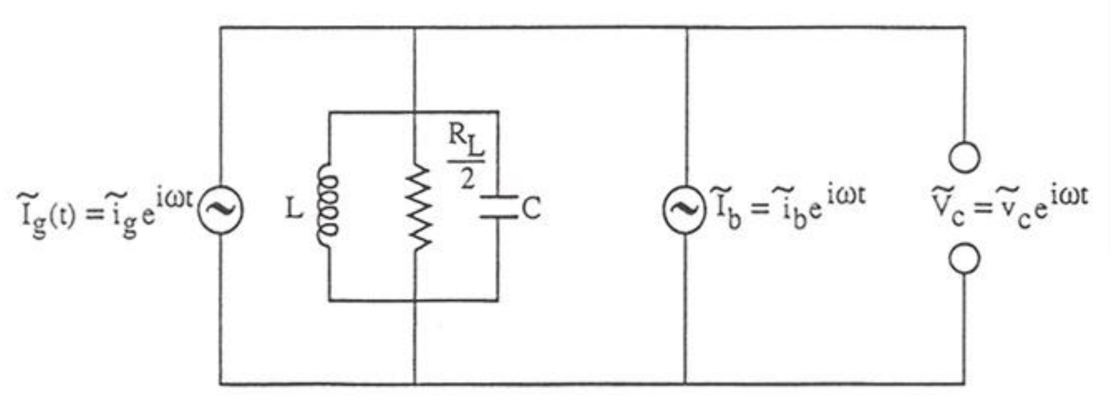
$$P_{diss} = \frac{4\beta}{(1+\beta)^2} P_{inc}$$

$$P_{ref} = \left(\frac{1-\beta}{1+\beta}\right)^2 P_{inc}$$



# Equivalent Circuit for a Cavity with Beam

- Beam in the rf cavity is represented by a current generator.
- Equivalent circuit:




$$R_L = \frac{R_{sh}}{(1+\beta)}$$

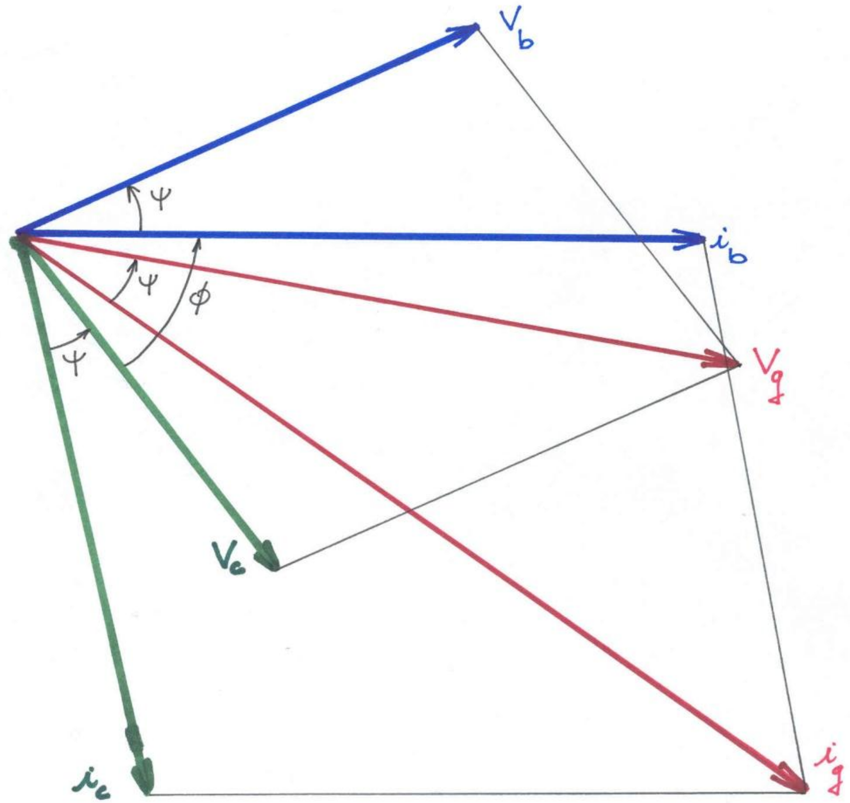
$\tilde{i}_b$ produces $\tilde{V}_b$ with phase $\psi$ (detuning angle)

$\tilde{i}_g$ produces $\tilde{V}_g$ with phase $\psi$

$$\tilde{V}_c = \tilde{V}_g - \tilde{V}_b$$

$$\tan \psi = -2\frac{Q_0}{1+\beta}\frac{\Delta\omega}{\omega_0}$$

# Equivalent Circuit for a Cavity with Beam




$$V_g = (P_g R_{sh})^{1/2} \frac{2\beta^{1/2}}{1+\beta} \cos\psi$$

$$V_b = \frac{i_b R_{sh}}{2(1+\beta)} \cos\psi$$

$$i_b = 2 i_0 \frac{\sin\frac{\theta_b}{2}}{\frac{\theta_b}{2}}$$

$i_b$: beam rf current

$i_0$: beam dc current

$\theta_b$: beam bunch length

# Equivalent Circuit for a Cavity with Beam

$$\boxed{P_g = \frac{V_c^2}{R_{sh}} \frac{1}{4\beta} \left\{ \left(1+\beta+b\right)^2 + \left[ (1+\beta)\tan\psi - b\tan\phi \right]^2 \right\}}$$

$$b = \frac{\text{Power absorbed by the beam}}{\text{Power dissipated in the cavity}} = \frac{R_{sh} i_0 \cos\phi}{V_c}$$

**Minimize $P_g$:**

$$(1+\beta_{opt})\tan\psi_{opt} = b \ \tan\phi$$

$$\beta_{opt} = |1+b|$$

$$P_g^{opt} = \frac{V_c^2}{R_{sh}} \frac{|1+b| + (1+b)}{2}$$

# Frequency Control

**Energy gain** $$W = qV\cos\phi$$

**Energy gain error** $$\frac{\delta W}{W} = \frac{\delta V}{V} - \delta\phi \tan\phi$$

**The fluctuations in cavity field amplitude and phase come mostly from the fluctuations in cavity frequency**

**Need for fast frequency control**

**Minimization of rf power requires matching of average cavity frequency to reference frequency**

**Need for slow frequency tuners**

# Some Definitions

- **Ponderomotive effects: changes in frequency caused by the electromagnetic field (radiation pressure)**
  - **Static Lorentz detuning (cw operation)**
  - **Dynamic Lorentz detuning (pulsed operation)**

- **Microphonics: changes in frequency caused by connections to the external world**
  - **Vibrations**
  - **Pressure fluctuations**

**Note: The two are not completely independent.**

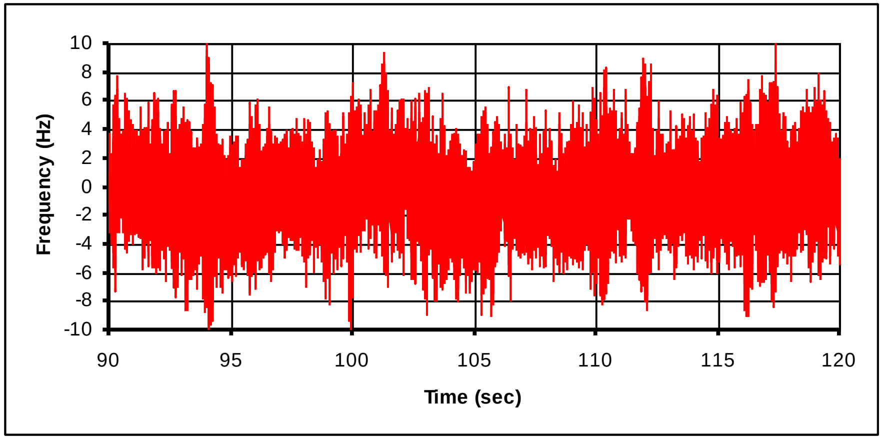
**When phase and amplitude feedbacks are active, ponderomotive effects can change the response to external disturbances**

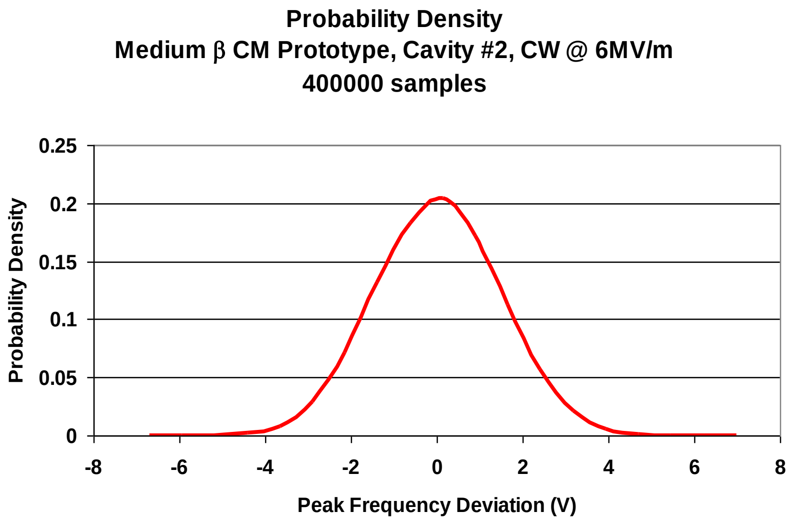
# Cavity with Beam and Microphonics

- The detuning is now $\tan\psi = -2Q_L \dfrac{\delta\omega_0 \pm \delta\omega_m}{\omega_0}$ $\quad\quad \tan\psi_0 = -2Q_L \dfrac{\delta\omega_0}{\omega_0}$

  where $\delta\omega_0$ is the static detuning (controllable)

  and $\delta\omega_m$ is the random dynamic detuning (uncontrollable)



# $Q_{ext}$ Optimization with Microphonics

Condition for optimum coupling:

$$\beta_{opt} = \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_{sh}} \left[ (b+1) + \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \right]$$

In the absence of beam (b=0):

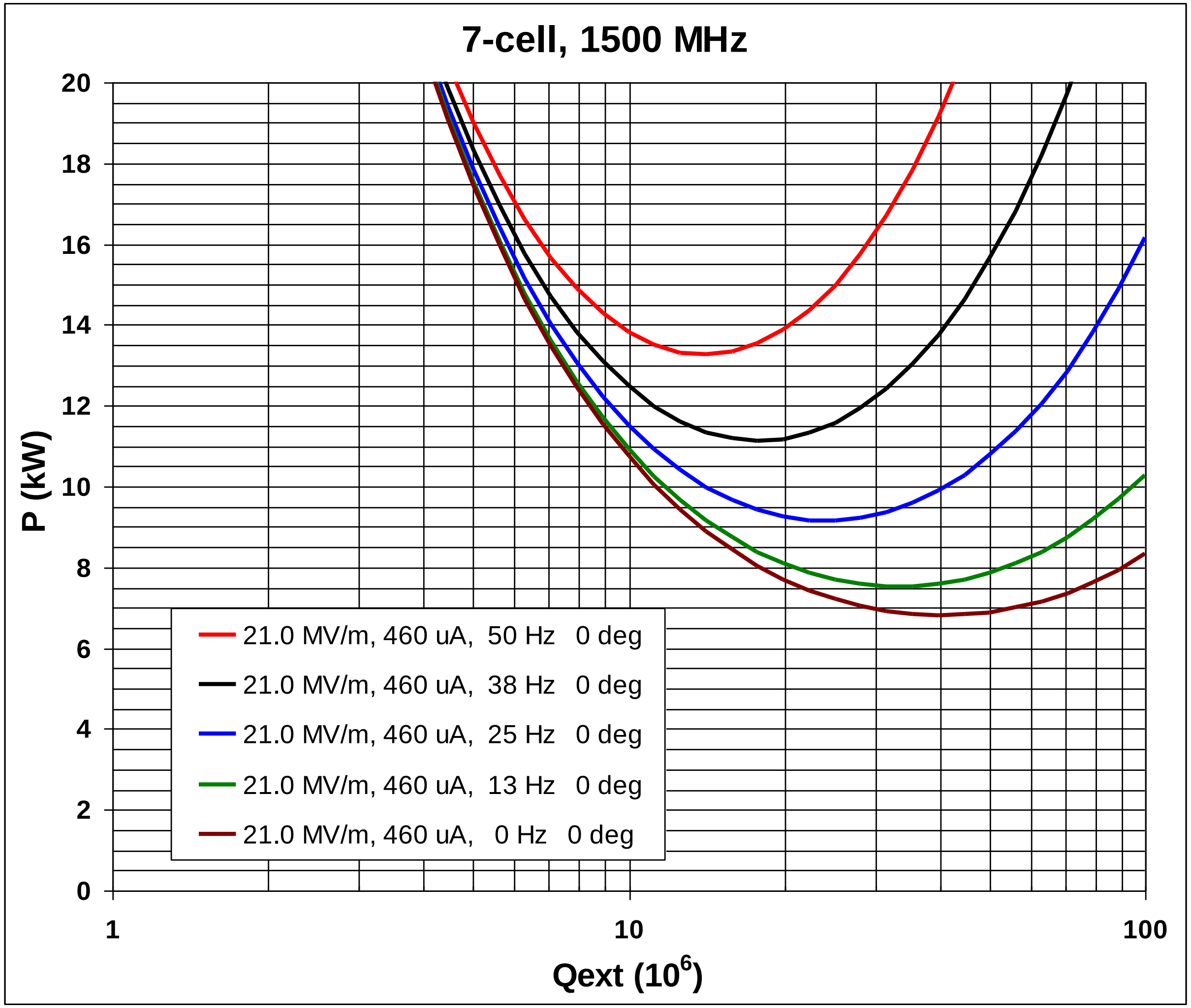
$$\beta_{opt} = \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_{sh}} \left[ 1 + \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \right]$$

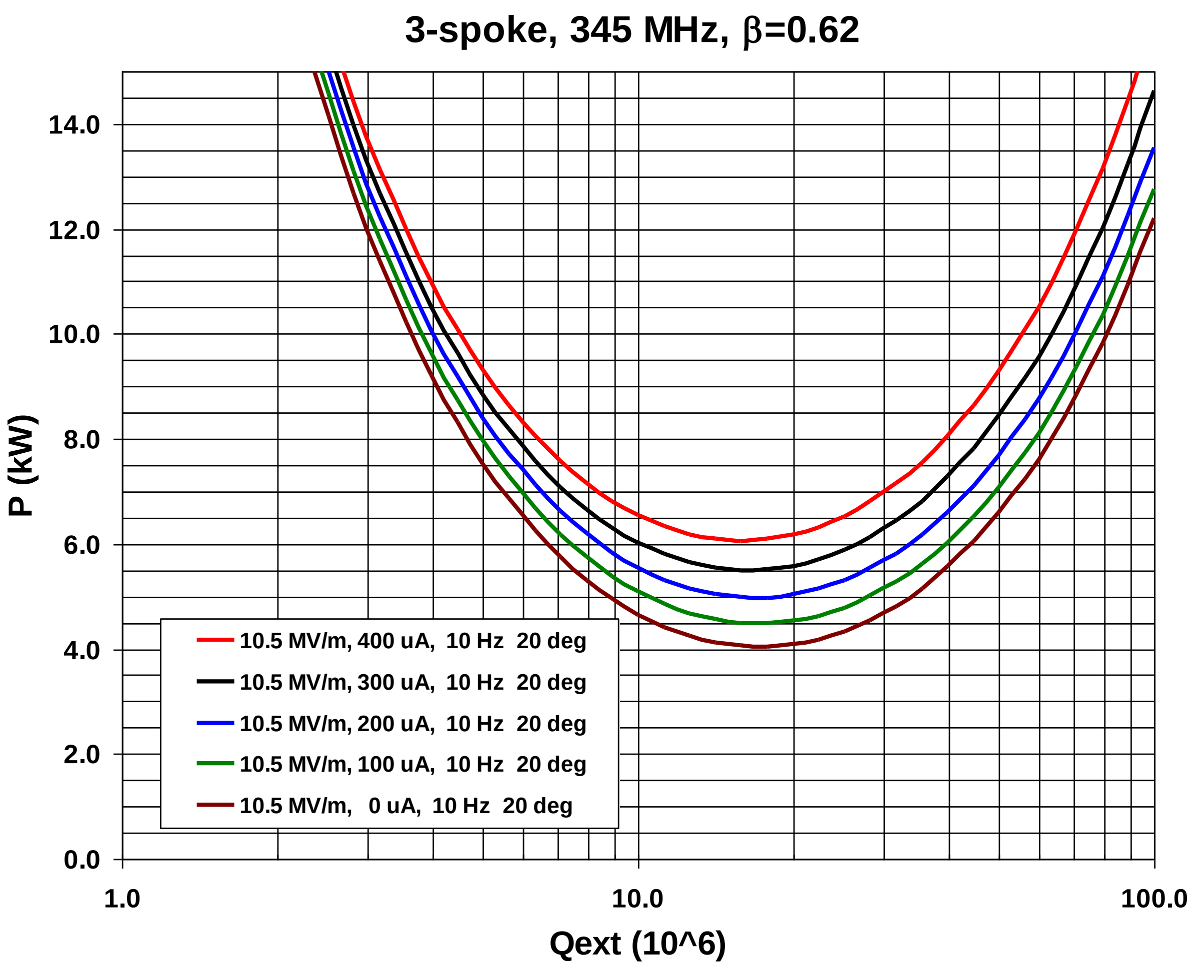
$\simeq U\ \delta\omega_m$ If $\delta\omega_m$ is very large

# Example



7-cell, 1500 MHz
P (kW)
Qext (10^6)
0
2
4
6
8
10
12
14
16
18
20
1
10
100
21.0 MV/m, 460 uA, 50 Hz 0 deg
21.0 MV/m, 460 uA, 38 Hz 0 deg
21.0 MV/m, 460 uA, 25 Hz 0 deg
21.0 MV/m, 460 uA, 13 Hz 0 deg
21.0 MV/m, 460 uA, 0 Hz 0 deg

# Example



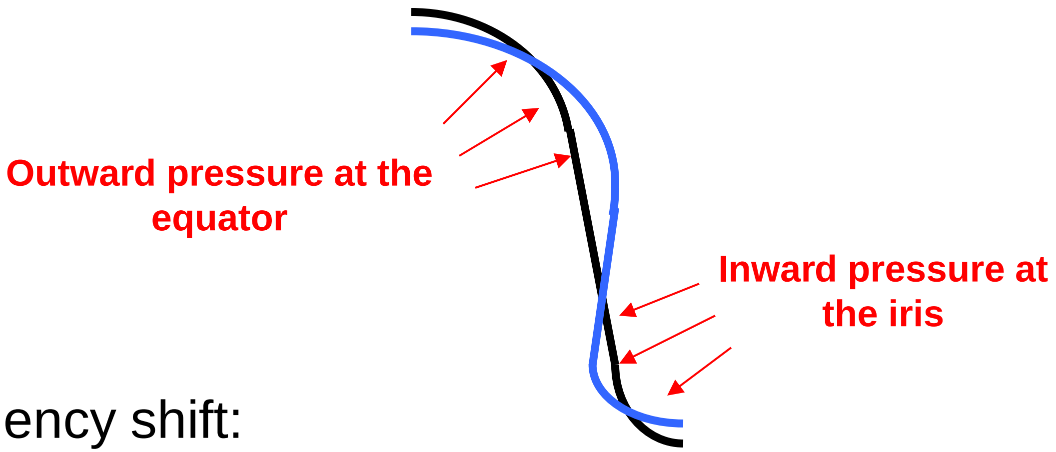
3-spoke, 345 MHz, β=0.62
P (kW)
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
1.0
10.0
100.0
Qext (10^6)
10.5 MV/m, 400 uA, 10 Hz 20 deg
10.5 MV/m, 300 uA, 10 Hz 20 deg
10.5 MV/m, 200 uA, 10 Hz 20 deg
10.5 MV/m, 100 uA, 10 Hz 20 deg
10.5 MV/m, 0 uA, 10 Hz 20 deg

# Lorentz Detuning

**Pressure deforms the cavity wall:**

RF power produces radiation pressure:

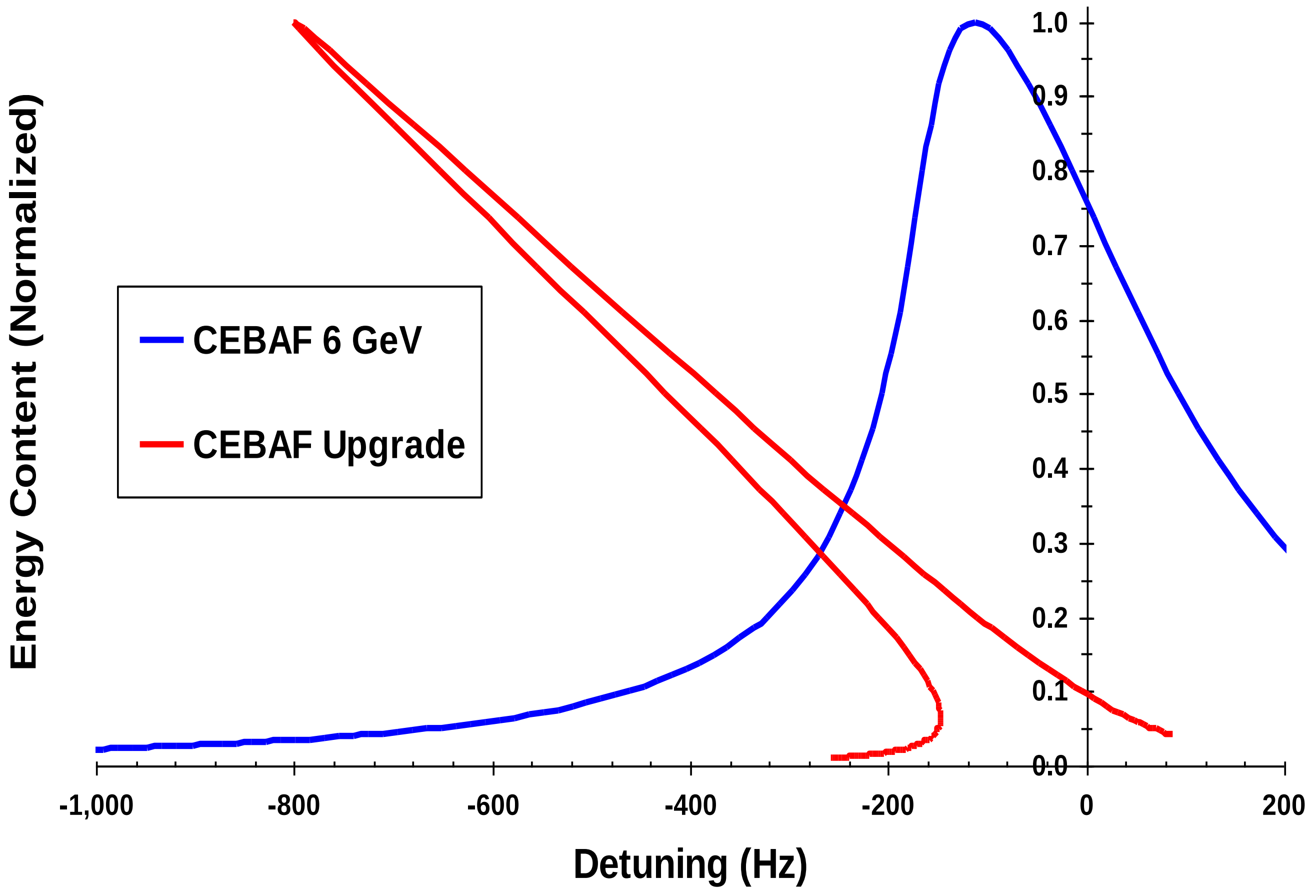
$$P = \frac{\mu_0 H^2 - \varepsilon_0 E^2}{4}$$




Deformation produces a frequency shift:

$$\Delta f = -k_L E_{acc}^2$$

# Lorentz Detuning



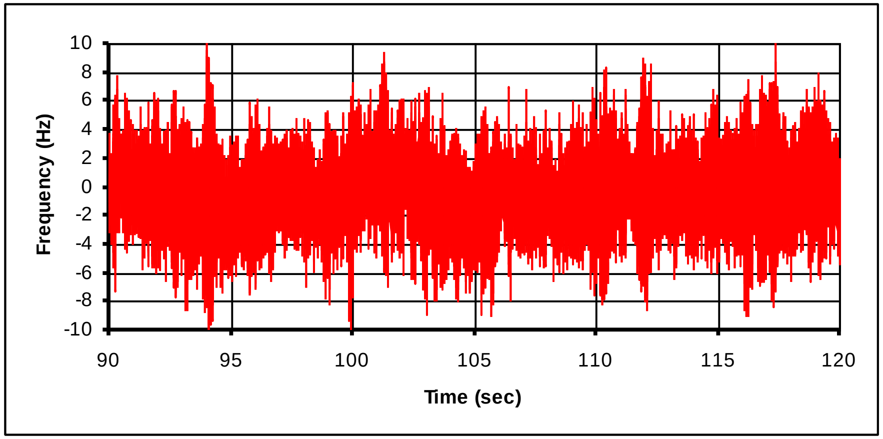
# Microphonics

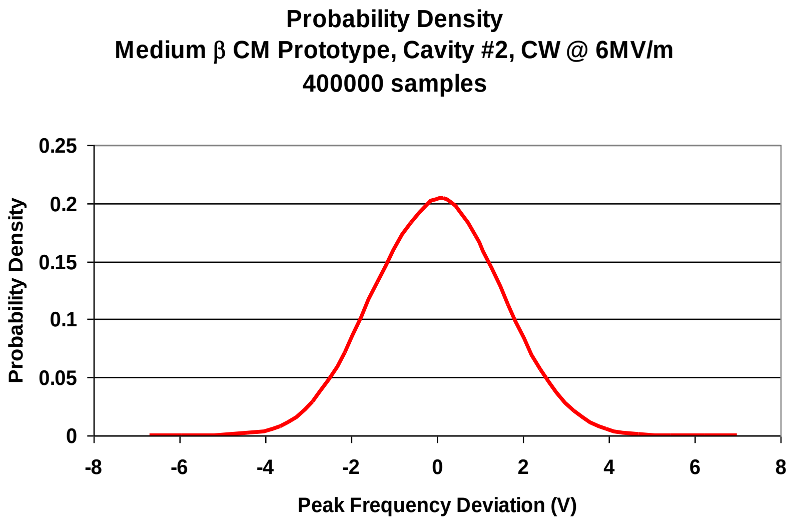
- **Total detuning**

$$\delta\omega_0 + \delta\omega_m$$

where $\delta\omega_0$ is the static detuning (controllable)

and $\delta\omega_m$ is the random dynamic detuning (uncontrollable)



# Ponderomotive Effects

- **Adiabatic theorem applied to harmonic oscillators (Boltzmann-Ehrenfest)**

If $\varepsilon = \frac{1}{\omega^2}\frac{d\omega}{dt} \ll 1$, then $\frac{U}{\omega}$ is an adiabatic invariant to all orders

$$\Delta\left(\frac{U}{\omega}\right) \Big/ \left(\frac{U}{\omega}\right) \sim o(e^{-d/\varepsilon}) \quad \Rightarrow \quad \boxed{\frac{\Delta\omega}{\omega} = \frac{\Delta U}{U}} \qquad \text{(Slater)}$$

Quantum mechanical picture: the number of photons is constant: $U = N\hbar\omega$

$$U = \int_V dv \left[ \frac{\mu_0}{4} H^2(\vec{r}) + \frac{\varepsilon_0}{4} E^2(\vec{r}) \right] \text{ (energy content)}$$

$$\Delta U = -\int_S dS\, \vec{n}(\vec{r}) \cdot \vec{\xi}(\vec{r}) \left[ \frac{\mu_0}{4} H^2(\vec{r}) - \frac{\varepsilon_0}{4} E^2(\vec{r}) \right] \text{ (work done by radiation pressure)}$$

# Ponderomotive Effects

$$\frac{\Delta\omega}{\omega} = -\frac{\int_S dS\, \vec{n}(\vec{r})\cdot\vec{\xi}(\vec{r})\left[\frac{\mu_0}{4}H^2(\vec{r}) - \frac{\varepsilon_0}{4}E^2(\vec{r})\right]}{\int_V dv\left[\frac{\mu_0}{4}H^2(\vec{r}) + \frac{\varepsilon_0}{4}E^2(\vec{r})\right]}$$

Expand wall displacements and forces in normal modes of vibration $\phi_\mu(\vec{r})$ of the resonator

$$\int_S dS\, \phi_\mu(\vec{r})\, \phi_\nu(\vec{r}) = \delta_{\mu\nu}$$

$$\xi(\vec{r}) = \sum_\mu q_\mu \phi_\mu(\vec{r}) \qquad q_\mu = \int_S \xi(\vec{r})\, \phi_\mu(\vec{r})\, dS$$

$$F(\vec{r}) = \sum_\mu F_\mu \phi_\mu(\vec{r}) \qquad F_\mu = \int_S F(\vec{r})\, \phi_\mu(\vec{r})\, dS$$

# Ponderomotive Effects

Equation of motion of mechanical mode $\mu$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\mu} - \frac{\partial L}{\partial q_\mu} + \frac{\partial \Phi}{\partial \dot{q}_\mu} = F_\mu \qquad L = T - U \qquad \text{(Euler-Lagrange)}$$

$$U = \frac{1}{2}\sum_\mu c_\mu q_\mu^2 \quad \text{(elastic potential energy)} \qquad c_\mu\text{: elastic constant}$$

$$T = \frac{1}{2}\sum_\mu c_\mu \frac{\dot{q}_\mu^2}{\Omega_\mu^2} \quad \text{(kinetic energy)} \qquad \Omega_\mu\text{: frequency}$$

$$\Phi = \sum_\mu \frac{c_\mu}{\tau_\mu}\frac{\dot{q}_\mu^2}{\Omega_\mu^2} \quad \text{(power loss)} \qquad \tau_\mu\text{: decay time}$$

$$\boxed{\ddot{q}_\mu + \frac{2}{\tau_\mu}\dot{q}_\mu + \Omega_\mu^2 q_\mu = \frac{\Omega_\mu^2}{c_\mu}F_\mu}$$

# Ponderomotive Effects

The frequency shift $\Delta\omega_\mu$ caused by the mechanical mode $\mu$ is proportional to $q_\mu$

$$\boxed{\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\Delta\dot{\omega}_\mu + \Omega_\mu^2\Delta\omega_\mu = -\frac{\omega_0}{c_\mu}\left(\frac{F_\mu}{U}\right)^2 \Omega_\mu^2 U = -k_\mu \Omega_\mu^2 V^2}$$

Total frequency shift: $\Delta\omega(t) = \sum_\mu \Delta\omega_\mu(t)$

Static frequency shift: $\Delta\omega_0 = \sum_\mu \Delta\omega_{\mu 0} = -V^2 \sum_\mu k_\mu$

Static Lorentz coefficient: $k = \sum_\mu k_\mu$

# Ponderomotive Effects – Mechanical Modes

$$\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\Delta\dot{\omega}_\mu + \Omega_\mu^{\ 2}\Delta\omega_\mu = -\Omega_\mu^2 k_\mu V_0^{\ 2} + \cancel{n(t)}$$

**Fluctuations around steady state:**

$$\Delta\omega_\mu = \Delta\omega_{\mu o} + \delta\omega_\mu$$

$$V = V_0(1+\delta v)$$

**Linearized equation of motion for mechanical mode:**

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\delta\dot{\omega}_\mu + \Omega_\mu^2\delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_0^{\ 2}\delta v$$

**The mechanical mode is driven by fluctuations in the electromagnetic mode amplitude.**

**Variations in the mechanical mode amplitude causes a variation of the electromagnetic mode frequency, which can cause a variation of its amplitude.**

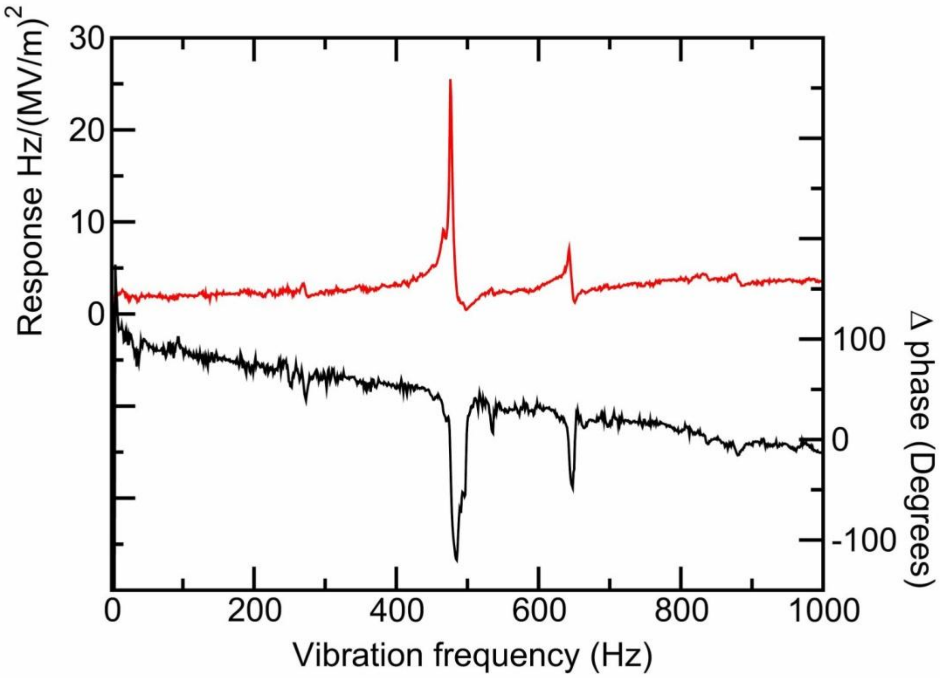
**→Closed feedback system between electromagnetic and mechanical modes, that can lead to instabilities.**

# Lorentz Transfer Function

$$\delta\ddot{\omega}_{\mu} + \frac{2}{\tau_{\mu}}\delta\dot{\omega}_{\mu} + \Omega_{\mu}^{2}\delta\omega_{\mu} = -2\Omega_{\mu}^{2}k_{\mu}V_{0}^{2}\delta v$$

$$\delta\omega_{\mu}(\omega) = \frac{-2\Omega_{\mu}^{2}k_{\mu}V_{0}^{2}}{\left(\Omega_{\mu}^{2} - \omega^{2}\right) + \frac{2}{\tau_{\mu}}i\omega}\delta v(\omega)$$

**TEM-class cavities**
**ANL, single-spoke, 354 MHz, β=0.4**



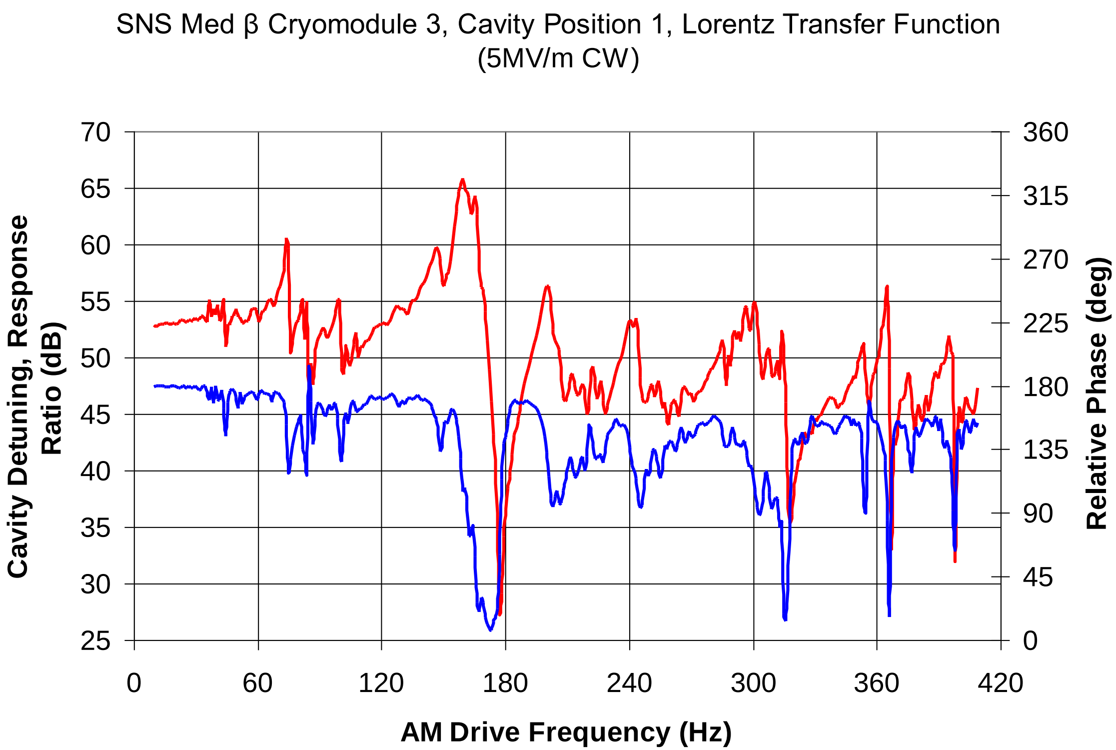

**simple spectrum with few modes**

# Lorentz Transfer Function

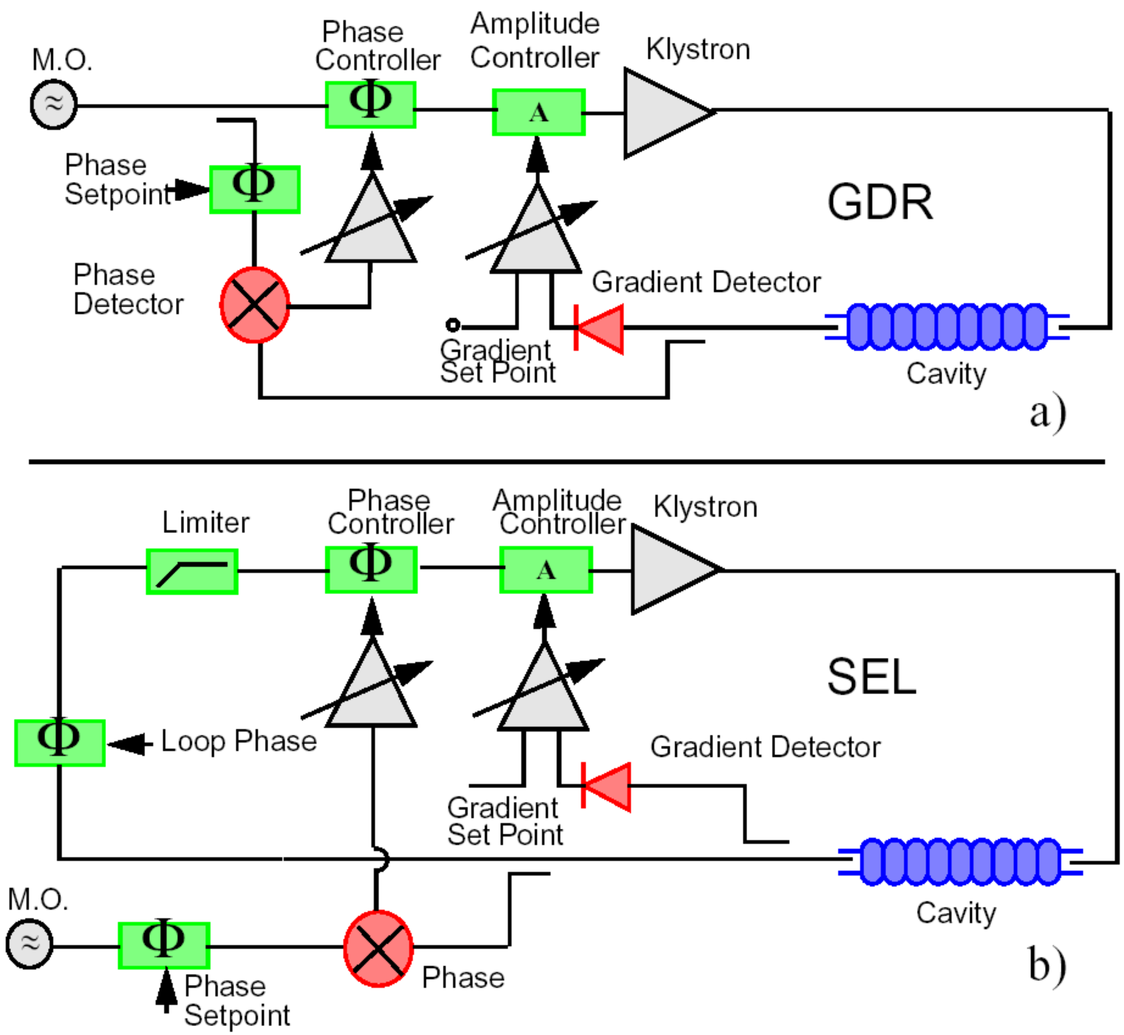
**TM-class cavities (Jlab, 6-cell elliptical, 805 MHz, β=0.61)**

**Rich frequency spectrum from low to high frequencies**

**Large variations between cavities**



# GDR and SEL


M.O.
Phase Controller
Amplitude Controller
Klystron
Phase Setpoint
GDR
Phase Detector
Gradient Detector
Gradient Set Point
Cavity
a)
Limiter
Phase Controller
Amplitude Controller
Klystron
SEL
Loop Phase
Gradient Detector
Gradient Set Point
M.O.
Cavity
Phase
Phase Setpoint
b)

# Generator-Driven Resonator

- **In a generator-driven resonator the coupling between the electromagnetic and mechanical modes can lead to two ponderomotive instabilities**

- **Monotonic instability : Jump phenomenon where the amplitudes of the electromagnetic and mechanical modes increase or decrease exponentially until limited by non-linear effects**

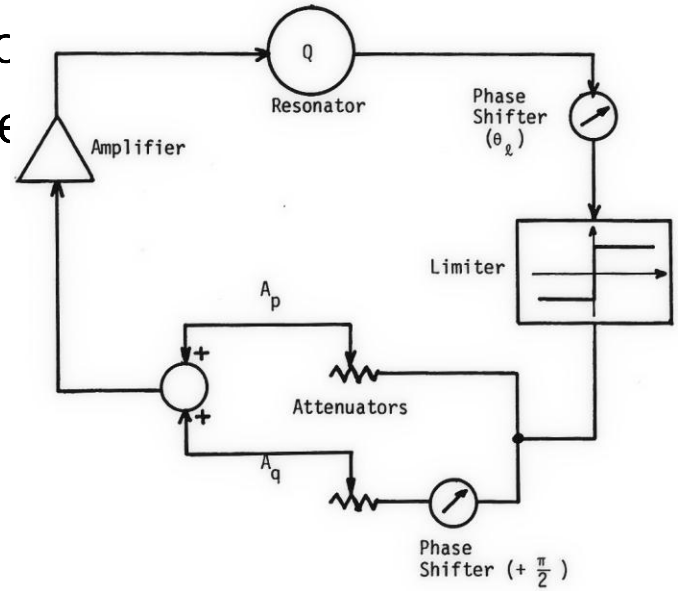
- **Oscillatory instability : The amplitudes of both modes oscillate and increase at an exponential rate until limited by non-linear effects**

# Self-Excited Loop-Principle of Stabilization

Controlling the external phase shift $\theta_l$ can cc

fluctuations in the cavity frequency $\omega_c$ so the

external frequency reference $\omega_r$.

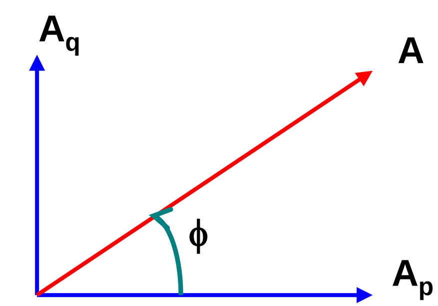



$$\omega = \omega_c + \frac{\omega_c}{2Q}\tan\theta_l$$

Instead of introducing an additional external

this is usually done by adding a signal in quadrature

→ The cavity field amplitude is unaffected by the phase stabilization even in the absence of amplitude feedback.



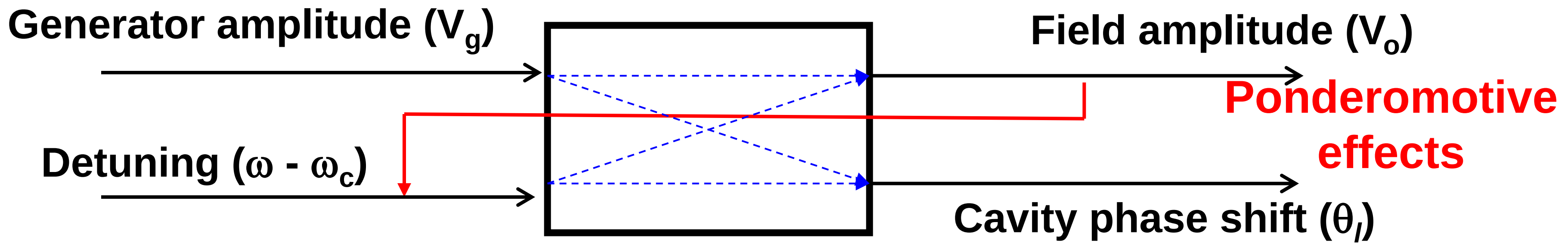
# Self-Excited Loop

- Resonators operated in self-excited loops in the absence of feedback are free of ponderomotive instabilities. An SEL is equivalent to the ideal VCO.
  – Amplitude is stable
  – Frequency of the loop tracks the frequency of the cavity

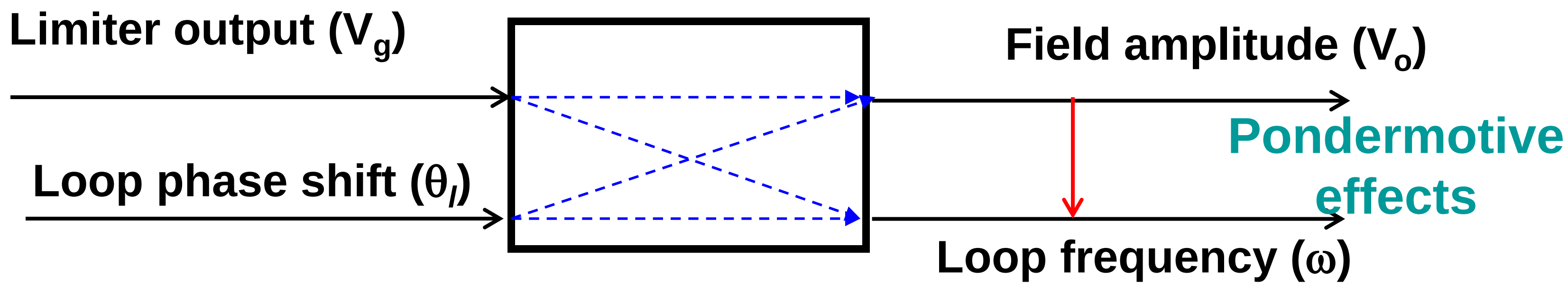
- Phase stabilization can reintroduce instabilities, but they are easily controlled with small amount of amplitude feedback

# Input-Output Variables

- **Generator - driven cavity**

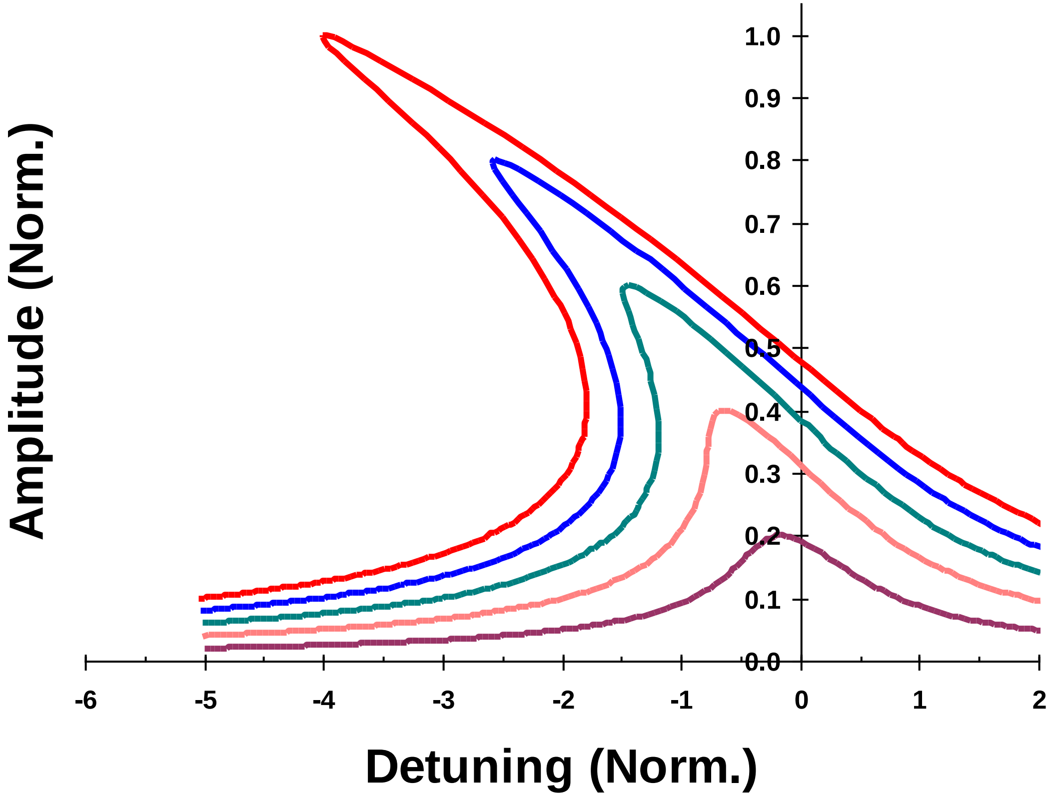



- **Cavity in a self-excited loop**



# Lorentz Detuning

**During transient operation (rise time and decay time) the loop frequency automatically tracts the resonator frequency. Lorentz detuning has no effect and is automatically compensated**



# Microphonics

- Microphonics: changes in frequency caused by connections to the external world
  - Vibrations
  - Pressure fluctuations

When phase and amplitude feedbacks are active, ponderomotive effects can change the response to external disturbances

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\delta\dot{\omega}_\mu + \Omega_\mu{}^2\delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_0{}^2\delta v + n(t)$$

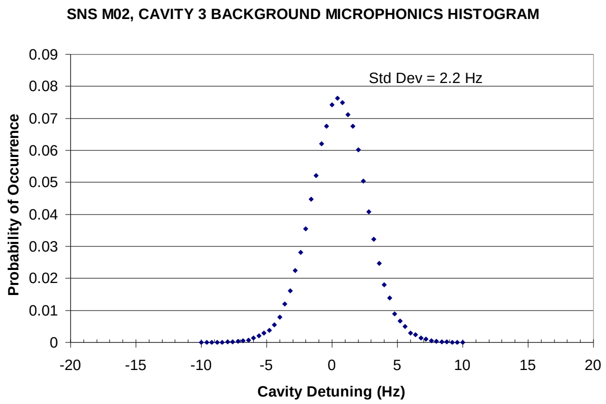
# Microphonics

Two extreme classes of driving terms:

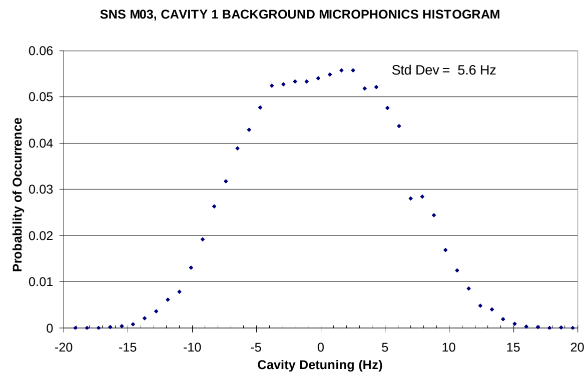
- Deterministic, monochromatic
  - Constant, well defined frequency
  - Constant amplitude
- Stochastic
  - Broadband (compared to bandwidth of mechanical mode)
  - Will be modeled by gaussian stationary white noise process

# Microphonics (probability density)

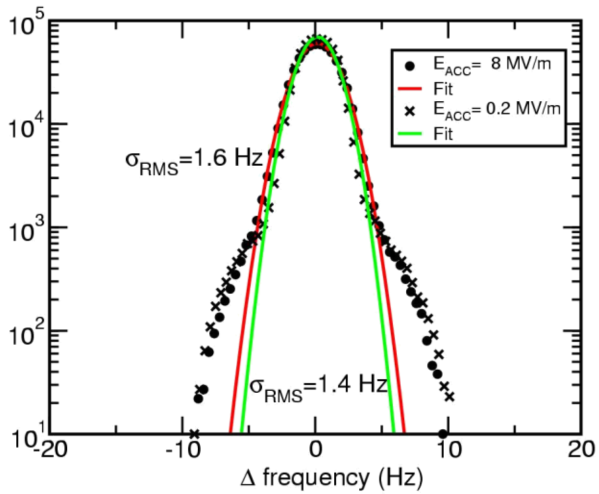
| Single gaussian | Bimodal | Multi-gaussian |
|---|---|---|
| Noise driven | Single-frequency driven | Non-stationary noise |




805 MHz TM




805 MHz TM

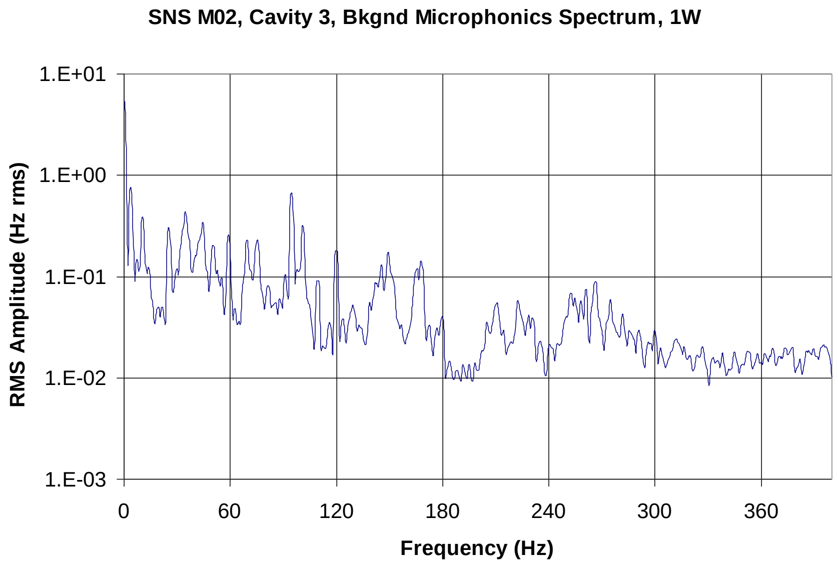


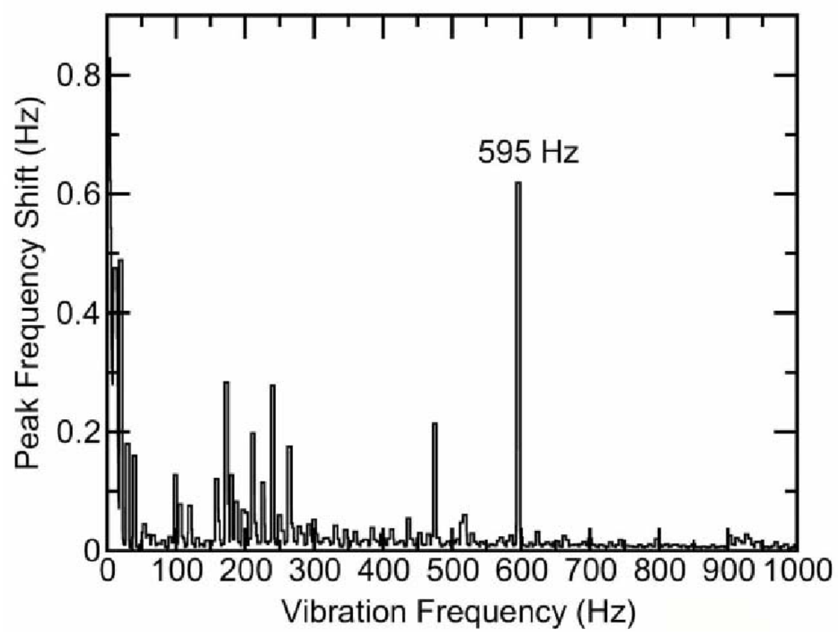

172 MHz TEM

# Microphonics (frequency spectrum)

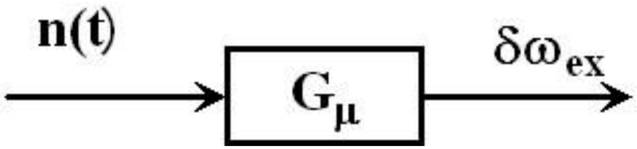
**TM-class cavities (JLab, 6-cell elliptical, 805 MHz, β=0.61)**
- **Rich frequency spectrum from low to high frequencies**
- **Large variations between cavities**

**TEM-class cavities (ANL, single-spoke, 354 MHz, β=0.4)**
- **Dominated by low frequency (<10 Hz) from pressure fluctuations**
- **Few high frequency mechanical modes that contribute little to microphonics level.**



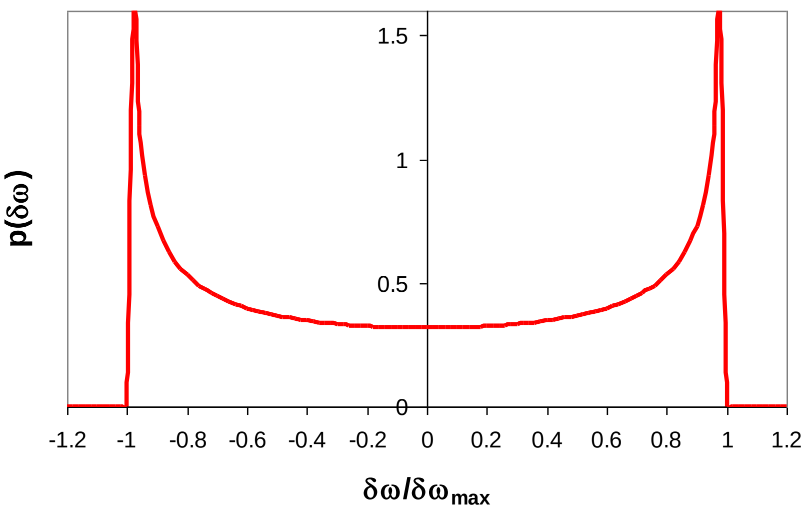



# Probability Density (histogram)



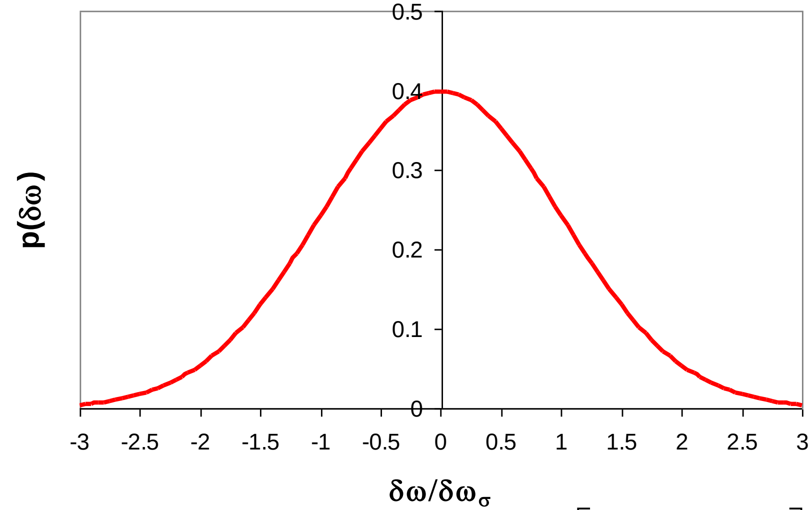

**Harmonic oscillator ($\Omega_\mu$,$\tau_\mu$) driven by:**

**Single frequency, constant amplitude**




$$p(\delta\omega) = \frac{1}{\pi\sqrt{\delta\omega_{max}^2 - \delta\omega^2}}$$

**White noise, gaussian**



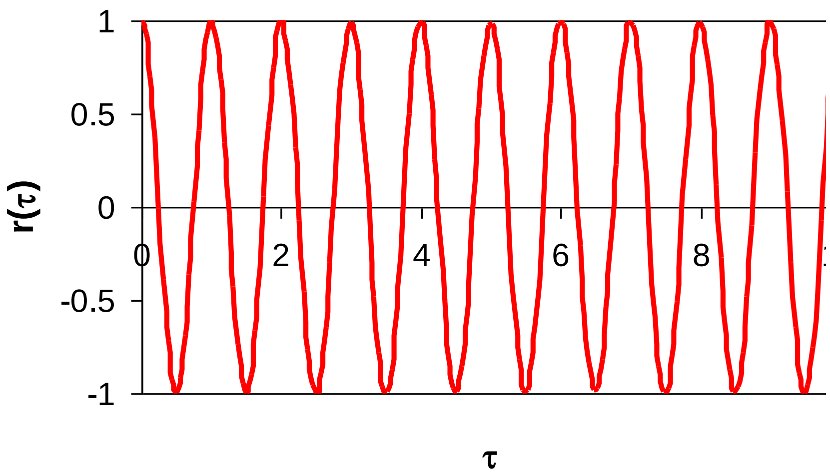

$$p(\delta\omega) = \frac{1}{\sigma_\omega\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\delta\omega}{\sigma_\omega}\right)^2\right]$$

# Autocorrelation Function

$$R_x(\tau) = \langle x(t)\,x(t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)\,x(t+\tau)\,dt$$

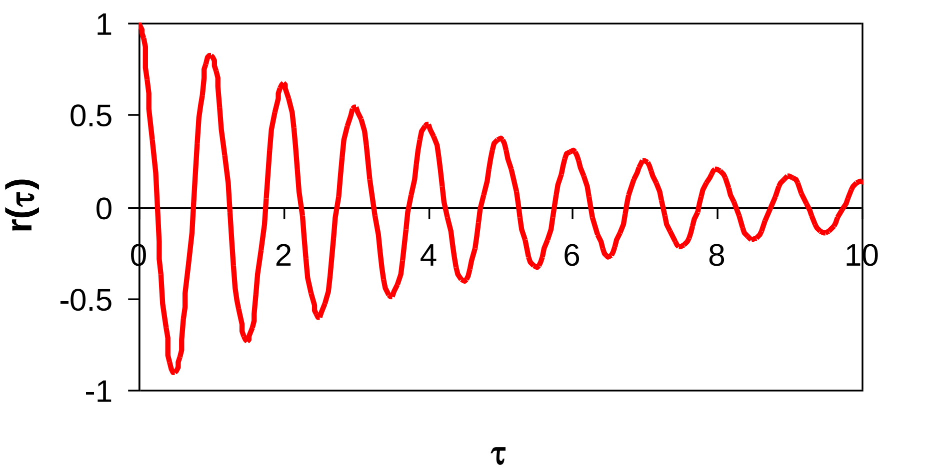
**Harmonic oscillator ($\Omega_\mu$,$\tau_\mu$) driven by:**

**Single frequency, constant amplitude**




$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\omega_d \tau)$$

**White noise, gaussian**




$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\Omega_\mu \tau) e^{-|\tau/\tau_\mu|}$$

# Stationary Stochastic Processes

$x(t)$: stationary random variable

Autocorrelation function: $$R_x(\tau) = \langle x(t)\,x(t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)\,x(t+\tau)\,dt$$

Spectral Density $S_x(\omega)$: Amount of power between $\omega$ and $d\omega$

$S_x(\omega)$ and $R_x(\tau)$ are related through the Fourier Transform (Wiener-Khintchine)

$$S_x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_x(\tau)\,e^{-i\omega\tau}\,d\tau \qquad R_x(\tau) = \int_{-\infty}^{\infty} S_x(\omega)\,e^{i\omega\tau}\,d\omega$$

Mean square value:

$$\langle x^2\rangle = R_x(0) = \int_{-\infty}^{\infty} S_x(\omega)\,d\omega$$

# Stationary Stochastic Processes

For a stationary random process driving a linear system

$$x(t) \longrightarrow \boxed{T(i\omega)} \longrightarrow y(t)$$

$$\left\langle y^2 \right\rangle = R_y(0) = \int_{-\infty}^{+\infty} S_y(\omega)\, d\omega \qquad \left\langle x^2 \right\rangle = R_x(0) = \int_{-\infty}^{+\infty} S_x(\omega)\, d\omega$$

$R_y(\tau) \quad \left[R_x(\tau)\right]$: auto correlation function of $y(t) \quad \left[x(t)\right]$

$S_y(\omega) \quad \left[S_x(\omega)\right]$: spectral density of $y(t) \quad \left[x(t)\right]$

$$\boxed{S_y(\omega) = S_x(\omega)\, \left|T(i\omega)\right|^2}$$

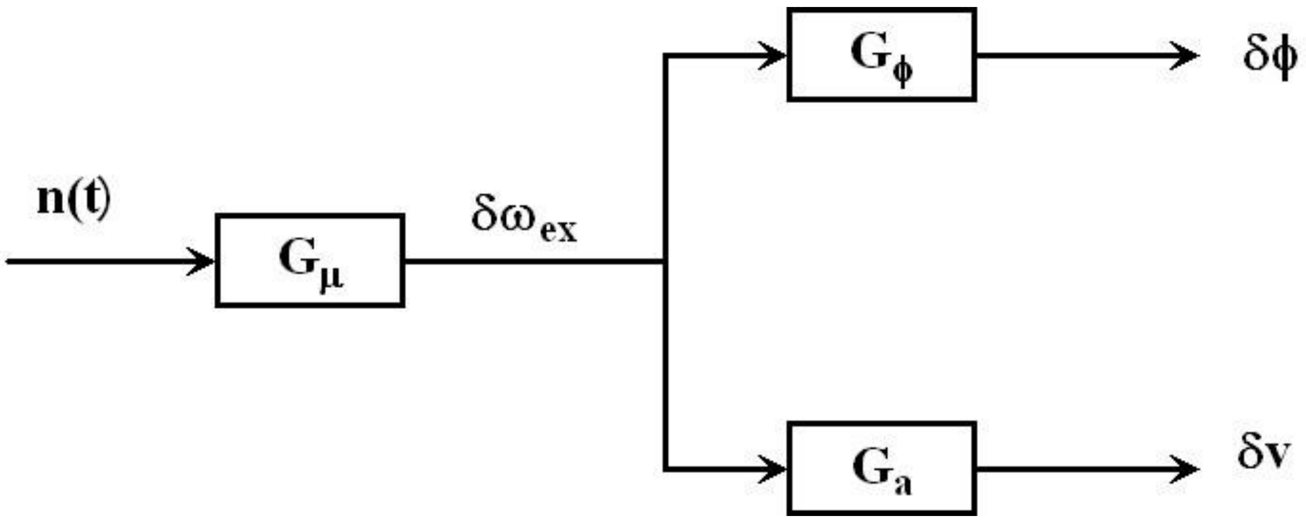
$$\boxed{\left\langle y^2 \right\rangle = \int_{-\infty}^{+\infty} S_x(\omega)\, \left|T(i\omega)\right|^2\, d\omega}$$

# Performance of Control System

Residual phase and amplitude errors caused by microphonics

Can also be done for beam current amplitude and phase fluctuations

Assume a single mechanical oscillator of frequency $\Omega_\mu$ and decay time $\tau_\mu$ excited by white noise of spectral density $A^2$



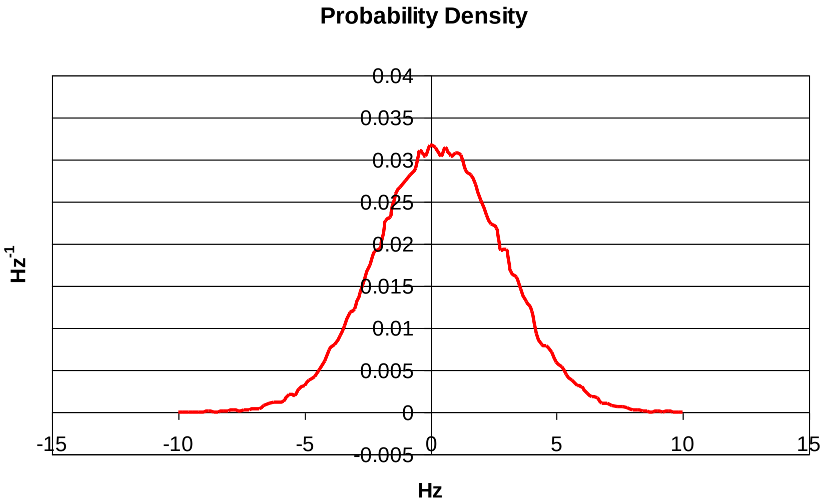
# Performance of Control System

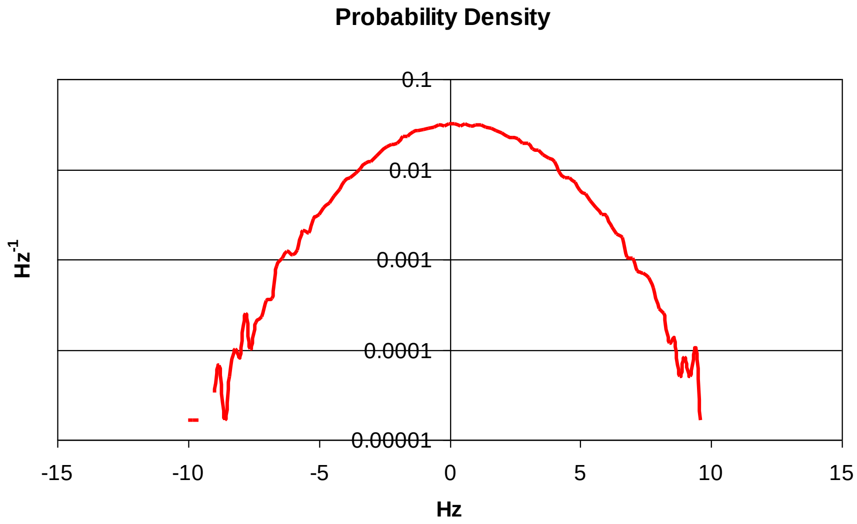
$$<\delta\omega_{ex}^2>= A^2\int_{-\infty}^{+\infty}\left|G_\mu(i\omega)\right|^2 d\omega = A^2\int_{-\infty}^{+\infty}\frac{d\omega}{\left|-\omega^2+\frac{2}{\tau_\mu}i\omega+\Omega_\mu^2\right|^2} = A^2\frac{\pi\tau_\mu}{2\Omega_\mu^2}$$

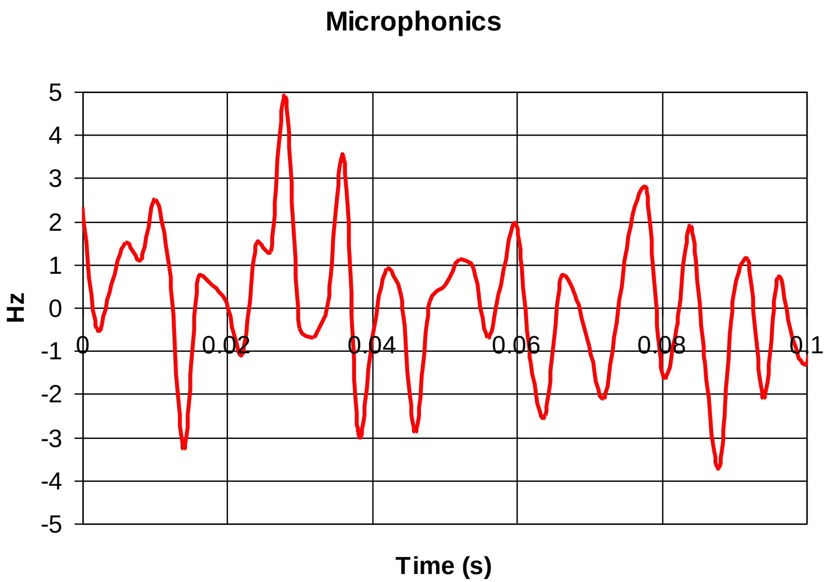
$$<\delta v^2>= A^2\int_{-\infty}^{+\infty}\left|G_\mu(i\omega)\,G_a(i\omega)\right|^2 d\omega = <\delta\omega_{ex}^2>\frac{2\Omega_\mu^2}{\pi\tau_\mu}\int_{-\infty}^{+\infty}\left|\frac{G_a(i\omega)}{-\omega^2+\frac{2}{\tau_\mu}i\omega+\Omega_\mu^2}\right|^2 d\omega$$

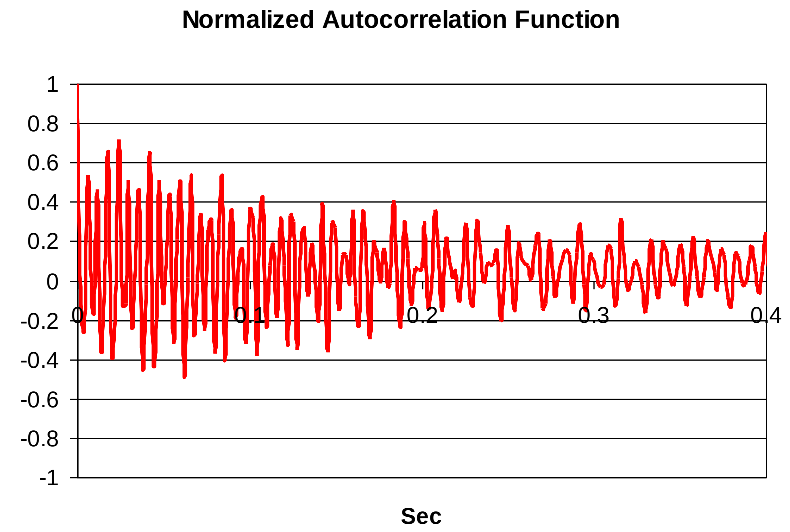
$$<\delta\varphi^2>= A^2\int_{-\infty}^{+\infty}\left|G_\mu(i\omega)\,G_\varphi(i\omega)\right|^2 d\omega = <\delta\omega_{ex}^2>\frac{2\Omega_\mu^2}{\pi\tau_\mu}\int_{-\infty}^{+\infty}\left|\frac{G_\varphi(i\omega)}{-\omega^2+\frac{2}{\tau_\mu}i\omega+\Omega_\mu^2}\right|^2 d\omega$$

# The Real World

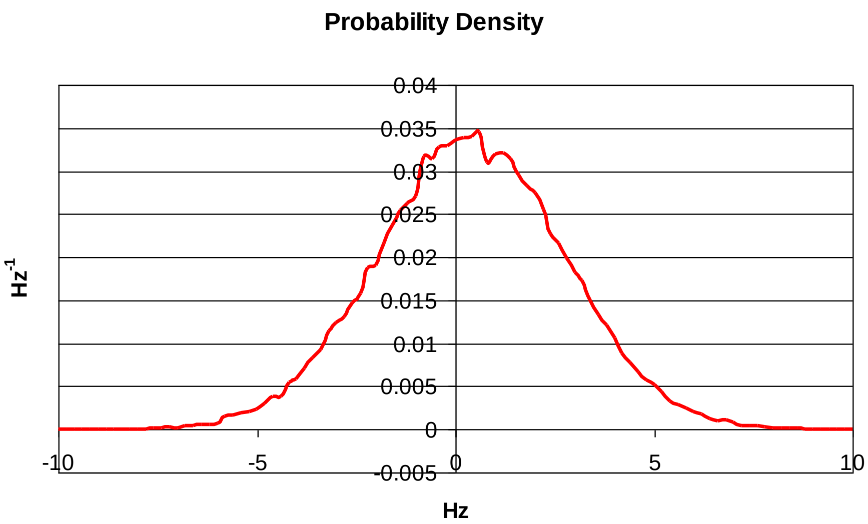


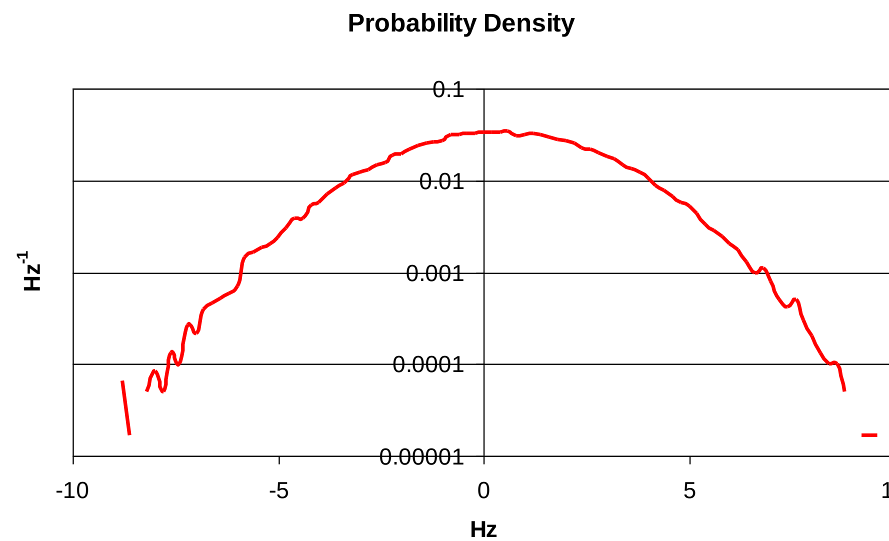



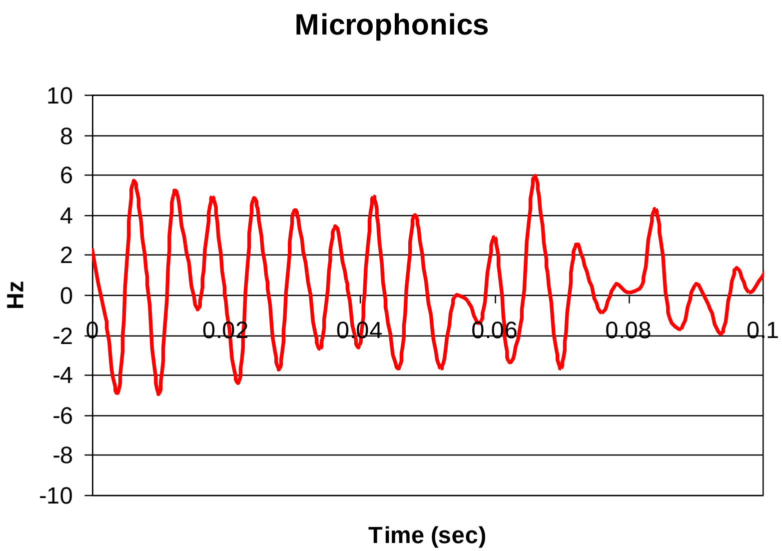



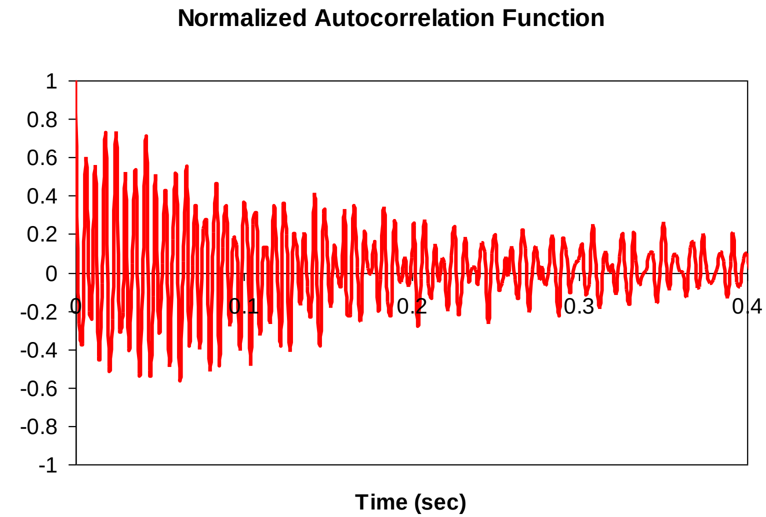



# The Real World

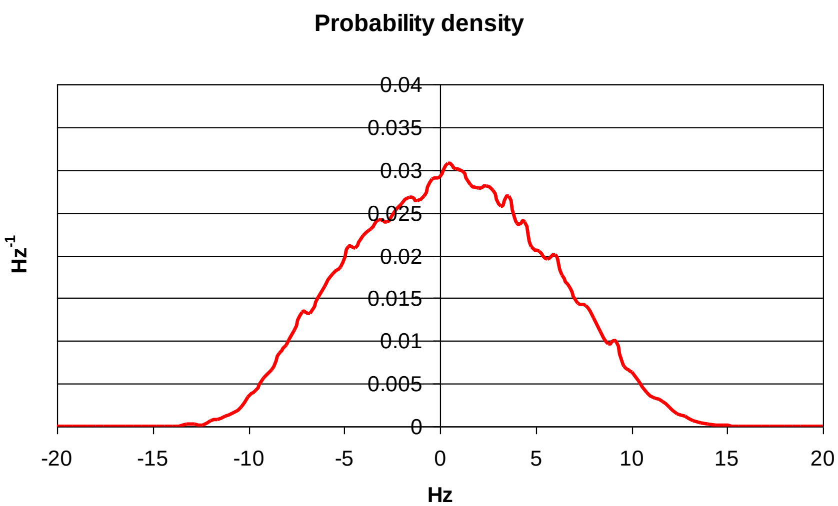


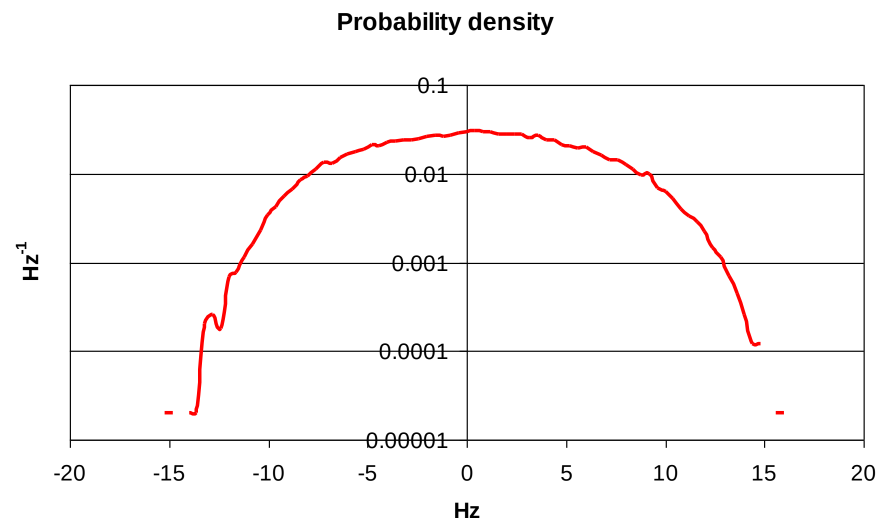



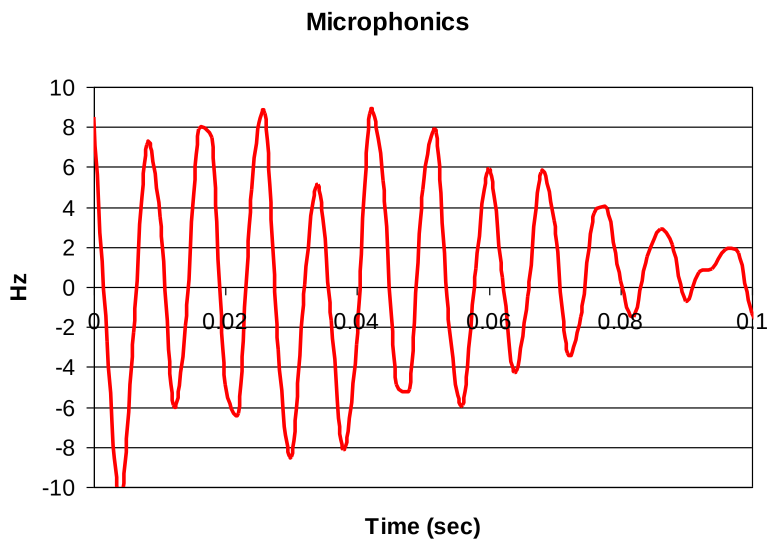



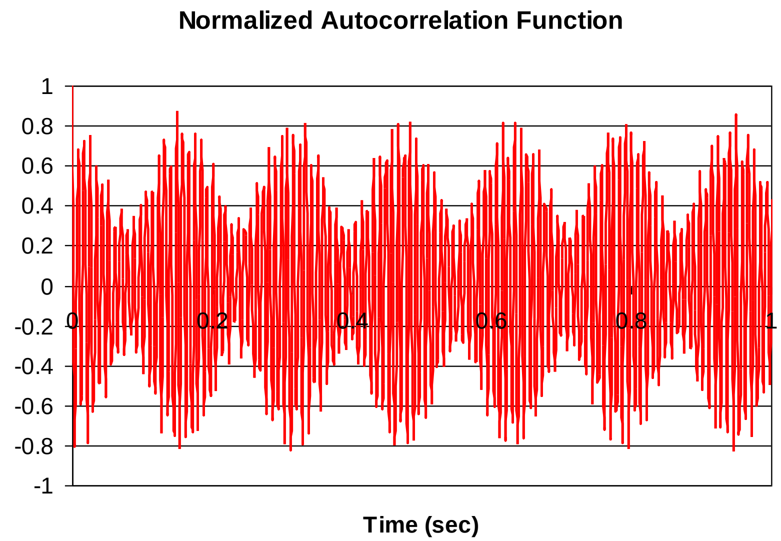



# The Real World



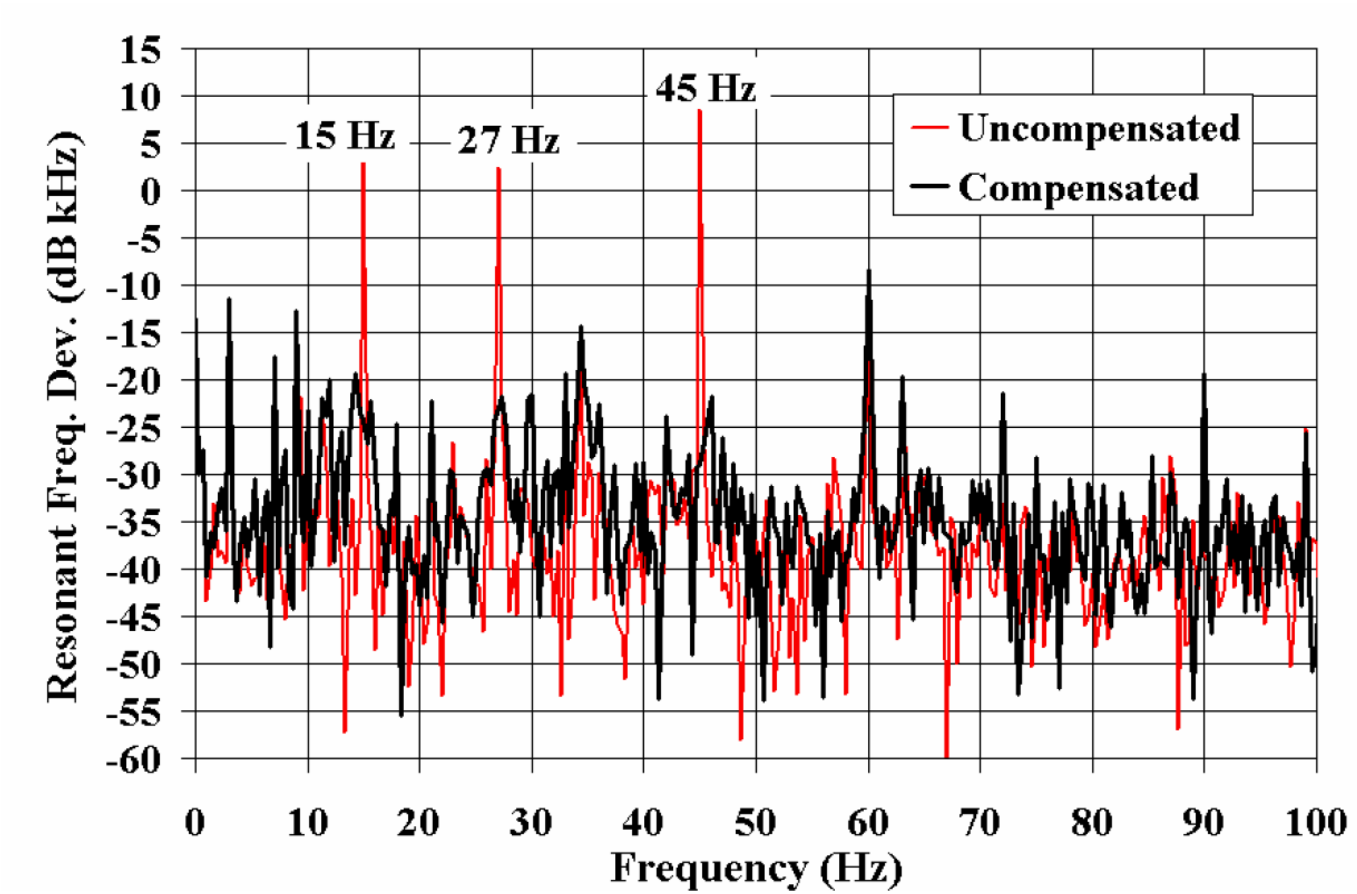



# Piezo Control of Microphonics

**FNAL, 3-cell 3.9 GHz**



# Piezo control of microphonics

**MSU, 6-cell elliptical 805 MHz, $\beta$=0.49**

**Adaptive feedforward compensation**

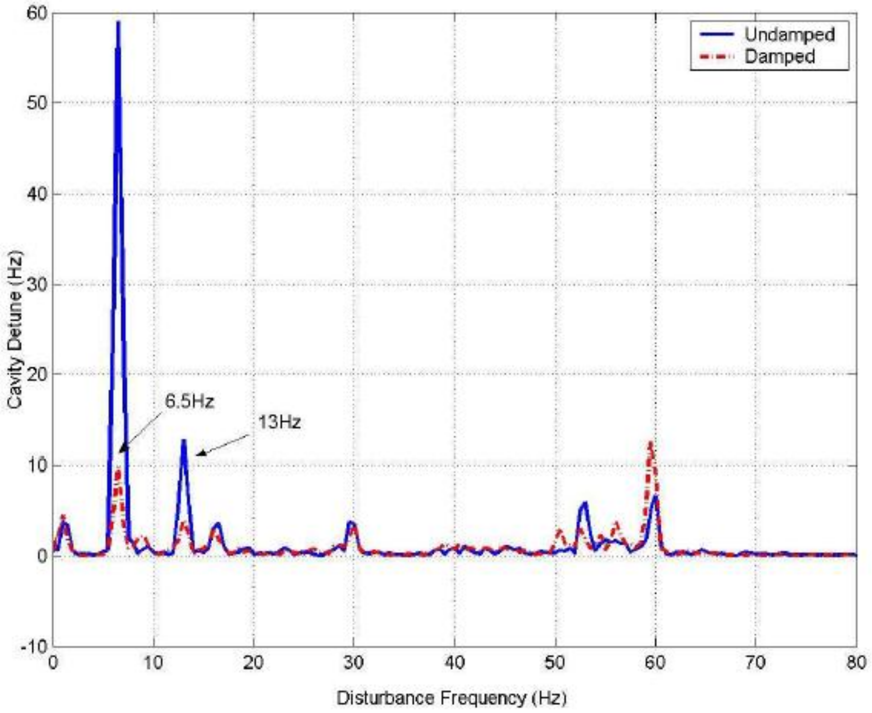



Figure 2. Active damping of helium oscillations at 2K.

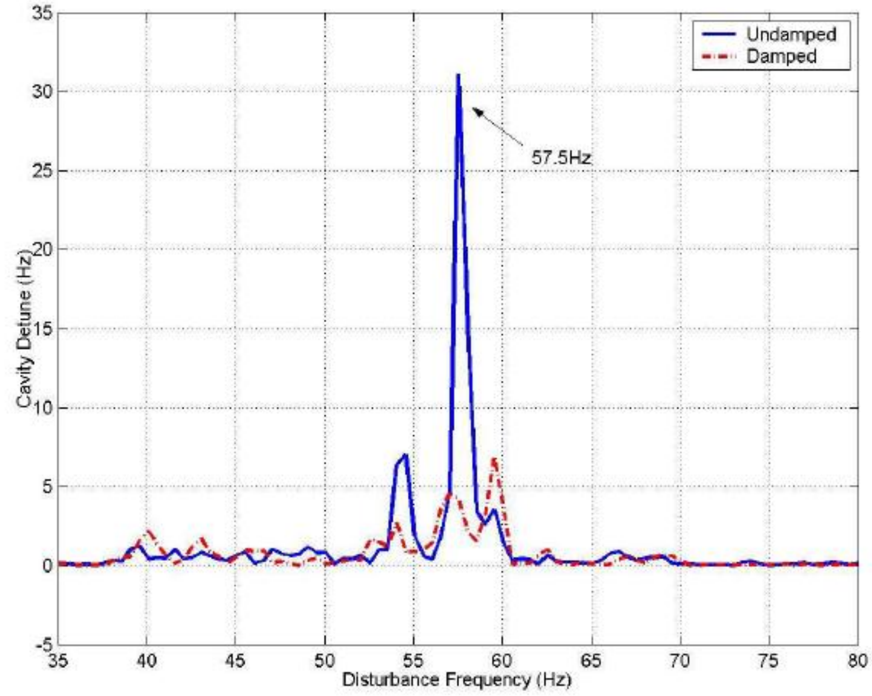


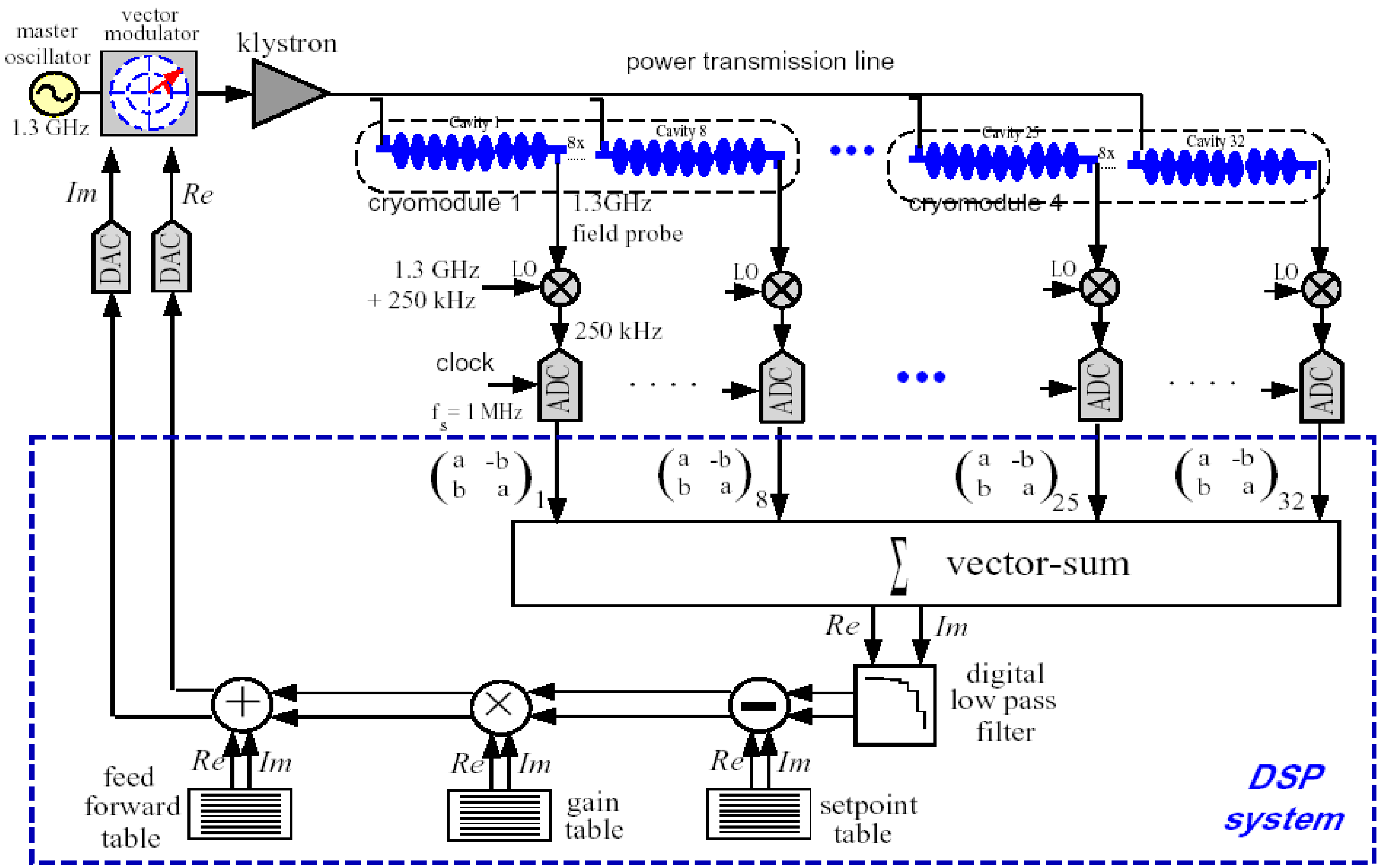

Figure 3. Active damping of external vibration at 2K.

# SEL and GDR

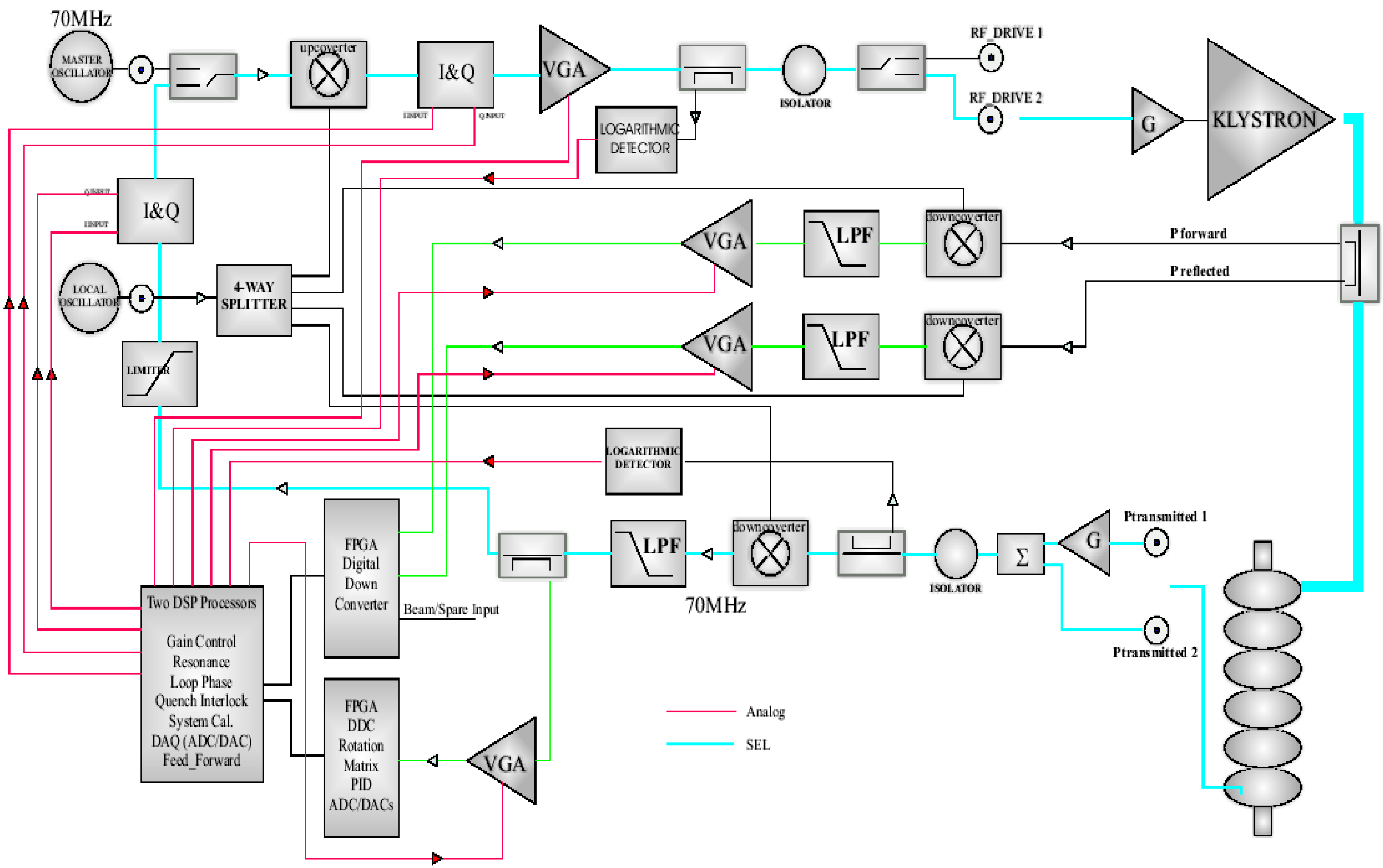
- SEL are best suited for high gradient, high-loaded Q cavities operated cw.
  - Well behaved with respect to ponderomotive instabilities
  - Unaffected by Lorentz detuning at power up
  - Able to run independently of external rf source
  - Rise time can be random and slow (starts from noise)

- GDR are best suited for low-Q cavities operated for short pulse length.
  - Fast predictable rise time
  - Power up can be hampered by Lorentz detuning

# TESLA Control System

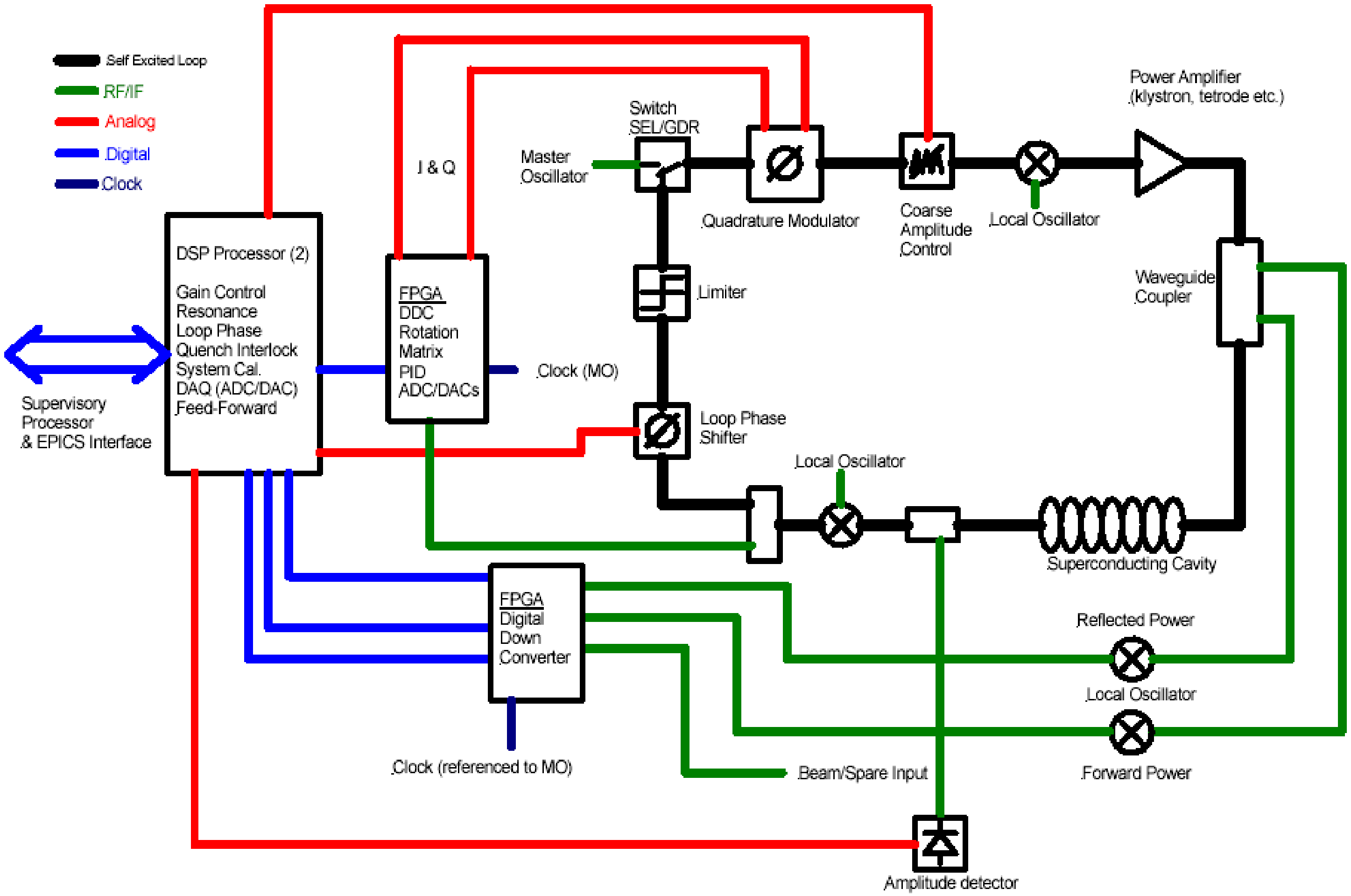


# Low level rf control development



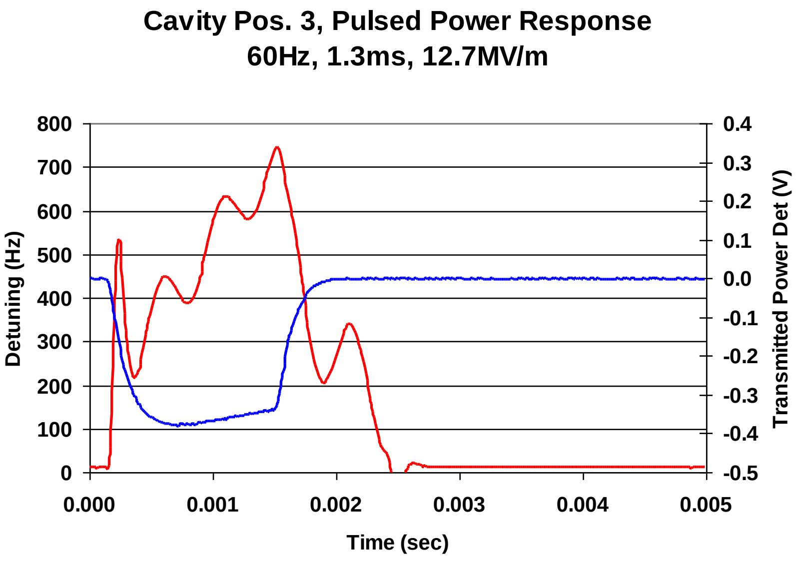

**Concept for a LLRF control system**

# Basic LLRF Block Diagram



# Pulsed Operation

- Under pulsed operation Lorentz detuning can have a complicated dynamic behavior



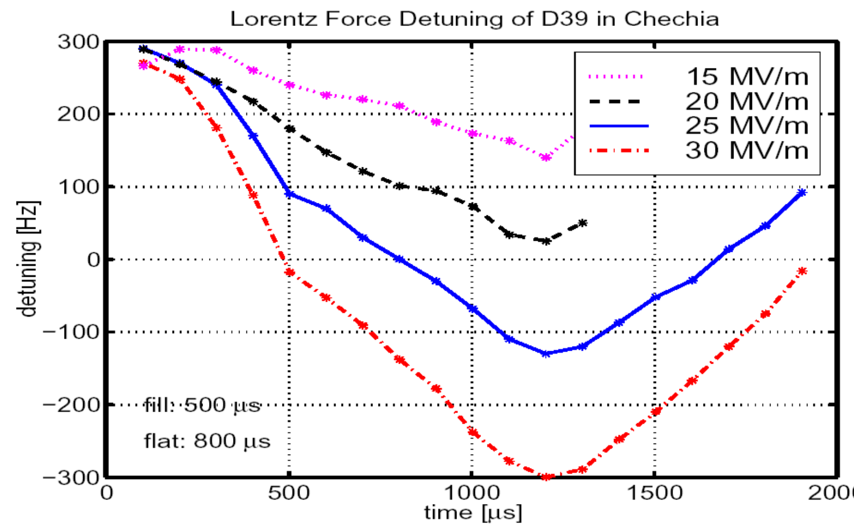
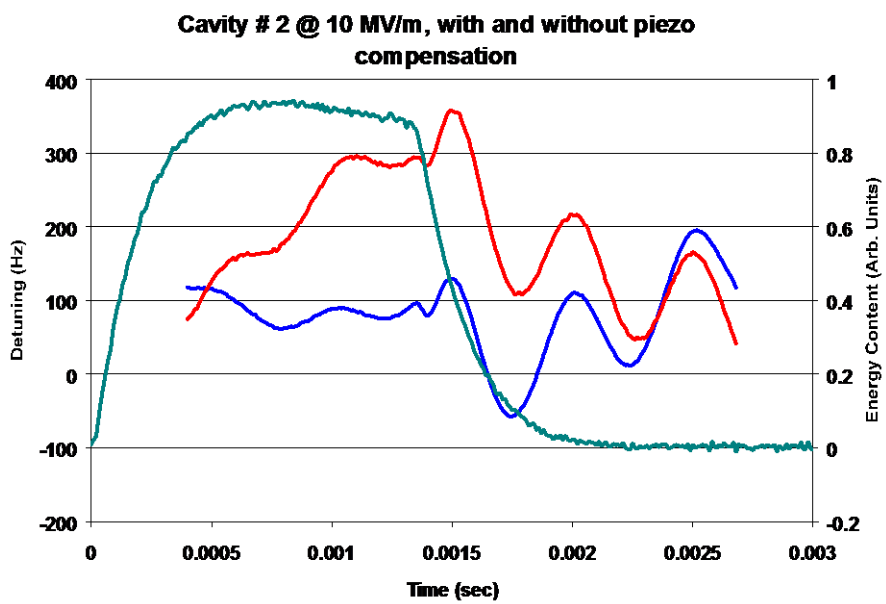




Fig. 2: Lorentz force detuning measured for a TESLA cavity at different gradients.

# Pulsed Operation

- Fast piezoelectric tuners can be used to compensate the dynamic Lorentz detuning



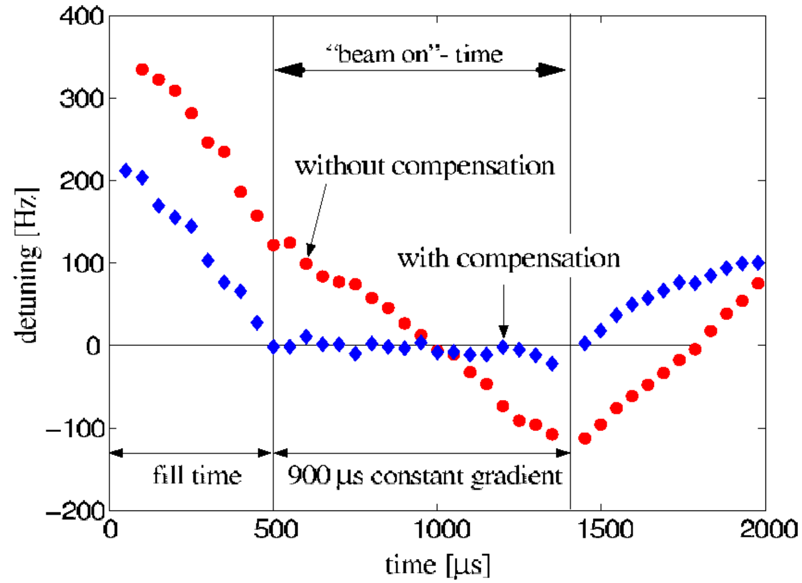




Figure 2. Lorentz force compensation at the TTF