TM-CLASS CAVITY DESIGN

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Outline

• “Pill-box” cavity
• Cell shape design
  – EM design
  – Multipacting analysis
• Multi-cell cavity design
  – HOM analysis
  – Mechanical analysis
Simplest case: “Pill-box” cavity

Hollow right cylindrical enclosure

Operated in the $\text{T}M_{010}$ mode

\[
E_z = E_0 J_0 \left( \frac{2.405 \rho}{R} \right) e^{-i\omega t}
\]

\[
H_\phi = -i \frac{E_0}{\eta} J_1 \left( \frac{2.405 \rho}{R} \right) e^{-i\omega t}
\]

\[
\omega_{010} = \frac{2.405c}{R} \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}
\]
Modes in Pill Box Cavity

- E.m. field in the cavity is solution to the wave equation

\[
\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \begin{pmatrix} E \\ H \end{pmatrix} = 0 \\
\hat{n} \times E = 0, \quad \hat{n} \cdot H = 0
\]

- \( \text{TM}_{010} \)
  - Electric field is purely longitudinal
  - Electric and magnetic fields have no angular dependence
  - Frequency depends only on radius, independent on length

- \( \text{TM}_{0np} \)
  - Monopoles modes that can couple to the beam and exchange energy

- \( \text{TM}_{1np} \)
  - Dipole modes that can deflect the beam

- \( \text{TE} \) modes
  - No longitudinal E field
  - Can be used to deflect beam
The TM solution to the wave equation in cylindrical coordinates has the form:

\[ E_z \propto J_m(k_{mn}r) \cos m\theta \cos k_z z \]
\[ E_r \propto p J'_m(k_{mn}r) \cos m\theta \sin k_z z \]
\[ E_\theta \propto \frac{p}{r} J_m(k_{mn}r) \sin m\theta \sin k_z z \]
\[ B_z = 0 \]
\[ B_r \propto -i \frac{1}{r} J_m(k_{mn}r) \sin m\theta \cos k_z z \]
\[ B_\theta \propto -i J'_m(k_{mn}r) \cos m\theta \cos k_z z \]

$m, n, p$ are integers that describe the mode of the solution. The $J_m$ are Bessel functions of the first kind. $J'_m$ is the derivative of the Bessel function.

\[ k_z = \frac{\pi p}{L_{cav}}, \quad \frac{\omega^2}{c^2} = k_{mn}^2 + k_z^2 \quad J'_0(z) = -J_1(z), \quad J'_1(z) = J_0(z) - \frac{1}{z} J_1(z), \ldots \]

\[ k_{mn} = u_{mn}/R, \text{ where } u_{mn} \text{ is the } n\text{th root of } J_m(x) \]
TM mode indices $mnp$

The modes are described by three indices.

$m$ is the number of variation of field of the **azimuthal** variable $\phi$:

$m = 0, 1, 2, ...$

$n$ is the number of nulls in $E_z$ along the **radial** direction

$n = 1, 2, 3, ...$

$p$ is the number of nodes of $E_z$ along the **z-axis**.

$p = 0, 1, 2, ...$
**TM_{010} Mode in a Pill Box Cavity**

Energy content

\[ U = \frac{1}{2} \varepsilon_0 E_0^2 \pi J_1^2(x_{01})LR^2 \]

Power dissipation

\[ P = R_s \frac{E_0^2}{\eta^2} \pi J_1^2(x_{01})(R + L)R \]

Geometrical factor

\[ G = \eta \frac{x_{01}}{2} \frac{L}{R + L} \]

\[ x_{01} = 2.40483 \]

\[ J_1(x_{01}) = 0.51915 \]
TM\textsubscript{010} Mode in a Pill Box Cavity

Voltage Gain

\[ V_c = LE_0 \frac{\sin \left( \frac{\pi L}{\lambda} \right)}{\pi L/\lambda} \]

Gradient

\[ E_{acc} = \frac{V_c}{L} \]

Shunt impedance

\[ R_a = \frac{4\eta^2 L^2}{\pi^3 R_s J_1^2(x_01)R(R + L)} \]

\[ R_a/Q_0 \text{ is independent of } R_s \]
Real Cavities

Beam tubes reduce the electric field on axis

- Gradient decreases
- Peak fields increase
- R/Q decreases
Real Cavities

TM010

$f = 1323 \text{ MHz}$
Single Cell Cavities

Electric field high at iris

Magnetic field high at equator
Single Cell Cavities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Cornell SC 500 MHz</th>
<th>Pillbox</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>270 Ω</td>
<td>257 Ω</td>
</tr>
<tr>
<td>$R_a/Q_0$</td>
<td>88 ohm/cell</td>
<td>196 Ω/cell</td>
</tr>
<tr>
<td>$E_{pk}/E_{acc}$</td>
<td>2.5</td>
<td>1.6</td>
</tr>
<tr>
<td>$H_{pk}/E_{acc}$</td>
<td>52 Oe/MV/m</td>
<td>30.5 Oe/(MV/m)</td>
</tr>
</tbody>
</table>
Cell Shape Design

- What is the purpose of the cavity?
- What EM parameters should be optimized to meet the design specs?

The “perfect” shape does not exist, it all depends on your application
Example: CEBAF Upgrade

- “High Gradient” shape: lowest $E_p/E_{acc}$
- “Low Loss” shape: lowest cryogenic losses, maximize $G(R/Q)$
**CEBAF Upgrade Shape Comparison**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>OC-shape</th>
<th>HG-Shape</th>
<th>LL-Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_{eq}$</td>
<td>[mm]</td>
<td>187.03</td>
<td>180.50</td>
<td>174.00</td>
</tr>
<tr>
<td>$\Omega_{iris}$</td>
<td>[mm]</td>
<td>70.00</td>
<td>61.40</td>
<td>53.00</td>
</tr>
<tr>
<td>$k_{ee}$</td>
<td>[%]</td>
<td>3.29</td>
<td>1.72</td>
<td>1.49</td>
</tr>
<tr>
<td>$E_{peak}/E_{acc}$</td>
<td></td>
<td>2.56</td>
<td>1.89</td>
<td>2.17</td>
</tr>
<tr>
<td>$B_{peak}/E_{acc}$</td>
<td>[mT·(MV/m)$^{-2}$]</td>
<td>4.56</td>
<td>4.26</td>
<td>3.74</td>
</tr>
<tr>
<td>Lorentz factor* $k_L$</td>
<td>[Hz·(MV/m)$^{-2}$]</td>
<td>-1.35</td>
<td>-1.1</td>
<td>-1.2</td>
</tr>
<tr>
<td>$R/Q$</td>
<td>[Ω]</td>
<td>96.5</td>
<td>111.9</td>
<td>128.8</td>
</tr>
<tr>
<td>$r/q = (R/Q)/\text{length}$</td>
<td>[Ω/m]</td>
<td>965</td>
<td>1119</td>
<td>1288</td>
</tr>
<tr>
<td>$G$</td>
<td>[Ω]</td>
<td>273.8</td>
<td>265.5</td>
<td>280.3</td>
</tr>
<tr>
<td>$R/Q\times G$</td>
<td>[Ω*Ω]</td>
<td>26421</td>
<td>29709</td>
<td>36102</td>
</tr>
</tbody>
</table>

**CEBAF Upgrade: cryo-budget limit of 30W/cavity. Higher energy gain can be obtained using LL-shape.**
New Trend in TM-Cavity Design

- The **field emission is not a hard limit** in the performance of sc cavities if the surface preparation is done in the right way.

- Unlikely this, **magnetic flux on the wall** limits performance of a sc cavity ($Q_0$ decreases or/and quench). Hard limit $\sim 180$ mT for Nb.

\[ B_{peak} / E_{acc} \text{ should be low} \]

1. Cavities may operate at higher gradients.

2. Cavities may operate at lower cryogenic load.

\[ \frac{P_{dissipated}}{V_{acc}^2} \equiv \frac{R_s}{G \cdot (R/Q)} \]
New Shapes for ILC

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{\text{iris}}$ [mm]</td>
<td>35</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td>$k_{\text{cc}}$ [%]</td>
<td>1.9</td>
<td>1.52</td>
<td>1.8</td>
</tr>
<tr>
<td>$E_{\text{peak}}/E_{\text{acc}}$</td>
<td>1.98</td>
<td>2.36</td>
<td>2.21</td>
</tr>
<tr>
<td>$B_{\text{peak}}/E_{\text{acc}}$ [mT/(MV/m)]</td>
<td>4.15</td>
<td>3.61</td>
<td>3.76</td>
</tr>
<tr>
<td>$R/Q$ [Ω]</td>
<td>113.8</td>
<td>133.7</td>
<td>126.8</td>
</tr>
<tr>
<td>$G$ [Ω]</td>
<td>271</td>
<td>284</td>
<td>277</td>
</tr>
<tr>
<td>$R/Q^<em>G$ [Ω</em>Ω]</td>
<td>30840</td>
<td>37970</td>
<td>35123</td>
</tr>
</tbody>
</table>
RF Simulation Codes for Cavity Design

The solution to 2D (or 3D) Helmholtz equation can be analytically found only for very few geometries (pillbox, spherical resonators or rectangular resonator).

\[
(\nabla^2 + \omega^2 \varepsilon \mu) A = 0
\]

We need numerical methods:
- **Approximating operator** (Finite Difference Methods)
- **Approximating function** (Finite Element Methods)

- 2D is fast and allows to define geometry of a cylindrical symmetric body (inner and end-cells) of the cavity.

- 3D is much more time consuming but necessary for modeling of full equipped cavity with FPC and HOM couplers and if needed to model fabrication errors. Also coupling strength for FPC and damping of HOMs can be modeled only 3D.
SUPERFISH

- Free, 2D finite-difference code to design cylindrically symmetric structures (monopole modes only)
- Use symmetry planes to reduce number of mesh points

http://laacg1.lanl.gov/laacg/services/download_sf.phtml
CST Microwave Studio

- Expensive, 3D finite-element code, used to design complex RF structure. [http://www.cst.com/Content/Products/MWS/Overview.aspx](http://www.cst.com/Content/Products/MWS/Overview.aspx)

- Runs on PC
- Perfect Boundary Approximation

Hexahedral mesh
Omega3P

- SLAC, 3D code, high-order Parallel Finite Element (PFE) method
- Runs on Linux
- Tetrahedral conformal mesh
- High order finite elements (basis order $p = 1$)
- Separate software for user interface (CuBit)
Cell Shape Parametrization

- Full parametric model of the cavity in terms of 7 meaningful geometrical parameters:
  - Ellipse ratio at the equator \((R=B/A)\) ruled by mechanics
  - Ellipse ratio at the iris \((r=b/a)\)
  - Side wall inclination \((\alpha)\) and position \((d)\)
  - \(E_{\text{peak}}\) vs. \(B_{\text{peak}}\) tradeoff and coupling \(k_{cc}\)
  - Cavity iris radius \(R_{\text{iris}}\) coupling \(k_{cc}\)
  - Half-cell Length \(L/2 = \lambda \beta / 4\)
  - Cavity radius \(D\) used for frequency tuning

- Behavior of all e.m. and mechanical properties has been found as a function of the above parameters
**BuildCavity**: parametric tool for the analysis of the cavity shape on the EM parameters:

- All RF computations are handled by SUPERFISH
- Inner cell tuning is performed through the cell diameter, all the characteristic cell parameters stay constant: R, r, α, d, L, R_{iris}
- End cell tuning is performed through the wall angle inclination, α, or distance, d. R, L and R_{iris} are independently settable.
- Multicell cavity is then built to minimize the field unflatness, compute the effective β and the final cavity performances.
- A proper file to transfer the cavity geometry to ANSYS is then generated.

### Inner cell data

- L = 56.8 mm
- R = 1
- r = 1.7
- α = 7°
- d = 11 mm
- R_{iris} = 43 mm
Parameter Choices

- Choose the cavity frequency  ➔ Equator diameter $D$
Beam Acceleration: cell length

Accelerating mode in a multi-cell structure

Cell length, $L = \lambda \beta / 2$

Synchrotron acceleration and max of $(R/Q)_{acc} \leftrightarrow L_{active} = NL_{cell} = Nc\beta/(2f)$ and the injection takes place at an optimum phase $\phi_{opt}$ which ensures that particles will arrive at the mid-plane of the first cell when $E_{acc}$ reaches its maximum (+q passing to the right) or minimum (-q passing to the right).
One Big “Knob”: $R_{\text{iris}}$

Why for a smaller aperture ($R_{\text{iris}}$)?

- $(R/Q)$ is bigger
- $E_{\text{peak}}/E_{\text{acc}}$, $B_{\text{peak}}/E_{\text{acc}}$ is lower

$E_{\text{acc}}$ is higher at the same stored energy in the cell

$R_{\text{iris}} = 40 \text{ mm}$  $R_{\text{iris}} = 20 \text{ mm}$

$E(z)$ for small and big iris radius
More on $R_{\text{iris}}$

We know that a smaller aperture makes:

- $(R/Q)$ higher
- $B_{\text{peak}}/E_{\text{acc}}, E_{\text{peak}}/E_{\text{acc}}$ lower

\[ (+) \]

but unfortunately a smaller aperture makes:

- HOMs impedances ($k_\perp, k_\parallel$) higher
- cell-to-cell coupling ($k_{cc}$) weaker

\[ (-) \]
“Rule of thumb” for Optimizing Peak Surface Fields

Add “electric volume” at the iris to reduce $E_{\text{peak}}$

Add “magnetic volume” at the equator to reduce $B_{\text{peak}}$
Pushing the Design: Reentrant cavity

- 3 independent parameters: A, B, a
- potential issue with cavity forming and cleaning
RF Tests of New Cavity Shapes: LL

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{tr}$ [MHz]</td>
<td>1286.6</td>
</tr>
<tr>
<td>$E_{peak}/E_{acc}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_{peak}/E_{acc}$ [mT/(MV/m)]</td>
<td>3.71</td>
</tr>
<tr>
<td>$R/Q$ [Ω]</td>
<td>130.0</td>
</tr>
<tr>
<td>$G$ [Ω]</td>
<td>279</td>
</tr>
<tr>
<td>$\varnothing_{iris}$ [mm]</td>
<td>61</td>
</tr>
</tbody>
</table>

9-cell LL cavity was tested at JLab up to $E_{acc}=36$MV/m
RF Tests of New Cavity Shapes: RE

<table>
<thead>
<tr>
<th></th>
<th>RE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_\pi$ [MHz]</td>
<td>1278.6</td>
</tr>
<tr>
<td>$E_{\text{peak}}/E_{\text{acc}}$</td>
<td>-</td>
</tr>
<tr>
<td>$B_{\text{peak}}/E_{\text{acc}}$ [mT/(MV/m)]</td>
<td>3.79</td>
</tr>
<tr>
<td>$R/Q$ [Ω]</td>
<td>126.0</td>
</tr>
<tr>
<td>$G$ [Ω]</td>
<td>278</td>
</tr>
<tr>
<td>$\varnothing_{\text{iris}}$ [mm]</td>
<td>68</td>
</tr>
</tbody>
</table>

9-cell RE cavity was tested at Cornell up to $E_{\text{acc}} = 28\text{MV/m}$
Want more?...Half-Reentrant Cavity

Parameters of the two proposed half-reentrant inner cells compared to the proposed Low Loss ILC geometry:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>High-(k_{cc}) HR</th>
<th>Low-(k_{cc}) HR</th>
<th>Low Loss ILC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (MHz)</td>
<td>1300</td>
<td>1300</td>
<td>1300</td>
</tr>
<tr>
<td>Wall angle (°)</td>
<td>6</td>
<td>6</td>
<td>0.165</td>
</tr>
<tr>
<td>(E_{peak}/E_{acc}) (--)</td>
<td>2.40</td>
<td>2.38</td>
<td>2.36</td>
</tr>
<tr>
<td>(B_{peak}/E_{acc}) (\text{MT}/\text{MV/m})</td>
<td>3.78</td>
<td>3.60</td>
<td>3.61</td>
</tr>
<tr>
<td>(R/Q) (Ω)</td>
<td>123</td>
<td>135</td>
<td>134</td>
</tr>
<tr>
<td>(G) (Ω)</td>
<td>283</td>
<td>283</td>
<td>284</td>
</tr>
<tr>
<td>((R/Q) \cdot G) (Ω^2)</td>
<td>34,673</td>
<td>38,021</td>
<td>37,970</td>
</tr>
<tr>
<td>(k_{cc}) (%)</td>
<td>2.09</td>
<td>1.51</td>
<td>1.52</td>
</tr>
<tr>
<td>(r_{f}) (cm)</td>
<td>3.34</td>
<td>2.97</td>
<td>3.00</td>
</tr>
</tbody>
</table>
End-Cell Design

The geometry of end-cells differs from the geometry of inner cells due to the attached beam tubes.

Their function is multi-folded and their geometry must fulfill three requirements:

• field flatness and frequency of the accelerating mode
• field strength of the accelerating mode at FPC location enabling operation with matched $Q_{ext}$
• fields strength of dangerous HOMs ensuring their required damping by means of HOM couplers or/and beam line absorbers.

All three make design of the end-cells more difficult than inner cells.
Example: SNS MB cavity

Optimization done with BuildCavity

- $R_{\text{iris}}$ set to 65 mm to have enough field at the power coupler antenna
- $d$ set 1 mm lower than the in-cell
- optimization of $r = b/a$ at iris
- $\alpha$ set to 10 deg to have the necessary stiffening
- Slater compensation (increase of the magnetic volume) of the cut-off tube ($\downarrow f$), $d$ reduction ($\downarrow f$), $\alpha$ and $R_{\text{iris}}$ increase ($\uparrow f$) by increasing the equator radius $\Rightarrow$ 4 dies
- the frequency of end cell + tube is about 40 kHz lower than the in-cell’s due to the asymmetry

$$\frac{\Delta f}{f_0} = -\frac{1}{4U} \iiint \left( \mu_0 H^2 - \varepsilon_0 E^2 \right) dV$$
More Examples of End Cell Optimizations

• Same $R_{eq}$ as inner cell, use $L_e$ as parameter to adjust the frequency
• Adjust parameters $A_e$, $B_e$, $a_e$, $b_e$ and $\alpha$ to minimize either $E_{peak}/E_{acc}$ or losses

• By adding more parameters ($a_t$, $b_t$, $c$, $R_{bp}$) it is possible to optimize the propagation of unwanted HOM, without increasing $E_{peak}/E_{acc}$ or losses for the fundamental mode.
Multicell Cavities

Single-cell is attractive from the RF-point of view:
- Easier to manage HOM damping
- No field flatness problem.
- Input coupler transfers less power
- Easy for cleaning and preparation

- But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.

A multi-cell structure is less expensive and offers higher real-estate gradient but:

- Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells
- Other problems arise: HOM trapping...
Coupling between cells

The normalized difference between these frequencies is a measure of the energy flow via the coupling region.

\[ k_{cc} = \frac{\omega_\pi - \omega_0}{\omega_\pi + \omega_0} \]

Symmetry plane for the H field

Symmetry plane for the E field which is an additional solution
Equivalent Circuit

Mode frequencies:
\[
\frac{\omega_m^2}{\omega_0^2} = 1 + 2k \left(1 - \cos \frac{\pi m}{N}\right)
\]

\[
\frac{\omega_N - \omega_{N-1}}{\omega_0} \sim k \left(1 - \cos \frac{\pi}{N}\right) \approx \frac{k}{2} \left(\frac{\pi}{N}\right)^2
\]

Voltages in cells:
\[
V_j^m = \sin \left(\pi m \frac{2j-1}{2N}\right)
\]

\[
k = \frac{C}{C_k}, \quad C_b = C_k / 2
\]
Pass-Band Modes Frequencies

9-cell cavity

\[ \omega_0 (1 + 4k)^{1/2} \]
Cell Excitations in Pass-Band Modes

9 Cell, Mode 1

9 Cell, Mode 4

9 Cell, Mode 7

9 Cell, Mode 2

9 Cell, Mode 5

9 Cell, Mode 8

9 Cell, Mode 3

9 Cell, Mode 6

9 Cell, Mode 9
Field Flatness

Geometrical differences between cells causes a mixing of the eigenmodes.

Sensitivity to mechanical deformation depends on mode spacing.

Field flatness factor for elliptical cavities with arbitrary $\beta=\nu/c$:

$$a_{ff} = \frac{N^2}{k_{cc} \cdot \beta}$$
Pros and Cons of Multicells

- Cost of accelerators are lower (less auxiliaries: LHe vessels, tuners, fundamental power couplers, control electronics)
- Higher real-estate gradient (better fill factor)
- Field flatness vs. N
- HOM trapping vs. N
- Power capability of fundamental power couplers vs. N
- Chemical treatment and final preparation become more complicated
- The worst performing cell limits whole multi-cell structure
**β<1 Cavities: Transit Time Factor**

LINAC design is made assuming “ideal” cavity with sinusoidal field on-axis

\[
E_{z,\text{ideal}}(z) = E_0 \sin\left(\frac{2\pi}{\lambda \beta_g} z\right) \quad \beta_g = \frac{2L}{\lambda}
\]

The “real” $\beta$, $\beta_c$, of a multi-cell cavity is defined as the $\beta_g$ of an ideal cavity that has an the energy gain curve for the synchronous particle closest to the one of the real cavity

\[
\delta W(\beta, \beta_c) = q \int_0^{L_{nw}} E_z(z) \cos\left[\frac{\omega}{\beta_c} (z - z_c)\right] \, dz
\]

\[
T^{(N)}(\beta, \beta_c) = \frac{\delta W(\beta, \beta_c)}{\delta W(\beta = \beta_c, \beta_c)}
\]

\[
\delta W = q \Delta V = q E_{acc} L_{att} T^{(N)}(\beta, \beta_c) \cos \phi_S
\]
$\beta_g = 0.61$ Cavity for SNS

Effective $\beta$ that matches the TTF curve = 0.630

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Inner cell</th>
<th>End Cell Left</th>
<th>End Group (coupler)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ [cm]</td>
<td>5.68</td>
<td>5.68</td>
<td>5.68</td>
</tr>
<tr>
<td>$R_{iris}$ [cm]</td>
<td>4.3</td>
<td>4.3</td>
<td>4.3</td>
</tr>
<tr>
<td>$D$ [cm]</td>
<td>16.376</td>
<td>16.376</td>
<td>16.698</td>
</tr>
<tr>
<td>$d$ [cm]</td>
<td>1.1</td>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>$r$</td>
<td>1.7</td>
<td>1.5</td>
<td>1.7</td>
</tr>
<tr>
<td>$R$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\alpha$ [deg]</td>
<td>7.0</td>
<td>8.36</td>
<td>7.0</td>
</tr>
</tbody>
</table>

KL70 = -2.9 [Hz/(MV/m)$^2$]  \hspace{1cm} KL80 = -3.4 [Hz/(MV/m)$^2$]

Nb thickness = 3.8 mm
\[ \beta_g = 0.81 \text{ Cavity for SNS} \]

Effective \( \beta \) that matches the TTF curve = 0.832

- \( E_p/E_{acc} \) = 2.19 (2.14 inner cell)
- \( B_p/E_{acc} \) [mT/(MV/m)] = 4.72 (4.58 inner cell)
- \( R/Q \) [\( \Omega \)] = 484.8
- \( G \) [\( \Omega \)] = 233
- \( k \) [%] = 1.52
- \( Q_{BCS} @ 2 \text{ K} \) [10\(^9\)] = 36.2
- Frequency [MHz] = 805.004
- Field Flatness [%] = 1.1

Geometrical Parameters

<table>
<thead>
<tr>
<th></th>
<th>Inner cell</th>
<th>End Cell Left</th>
<th>End Group (coupler)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Left</td>
<td>Right</td>
</tr>
<tr>
<td>( L ) [cm]</td>
<td>7.55</td>
<td>7.55</td>
<td>7.55</td>
</tr>
<tr>
<td>( R_{iris} ) [cm]</td>
<td>4.88</td>
<td>4.88</td>
<td>4.88</td>
</tr>
<tr>
<td>( D ) [cm]</td>
<td>16.415</td>
<td>16.415</td>
<td>16.611</td>
</tr>
<tr>
<td>( d ) [cm]</td>
<td>1.5</td>
<td>1.3</td>
<td>1.5</td>
</tr>
<tr>
<td>( r )</td>
<td>1.8</td>
<td>1.6</td>
<td>1.8</td>
</tr>
<tr>
<td>( R )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \alpha ) [deg]</td>
<td>7.0</td>
<td>10.072</td>
<td>7.0</td>
</tr>
</tbody>
</table>

\( KL70 = -0.7 \text{ [Hz/(MV/m)^2]} \) \hspace{1cm} \( KL80 = -0.8 \text{ [Hz/(MV/m)^2]} \)

Nb thickness = 3.8 mm-
Once the cavity shape has been designed, multipacting simulations have to be done:

- get the fields on the contour
- electrons are launched from given initial sites at given phases of the RF field
- for a fixed field level the electron trajectories are calculated by integrating the equations of motion, until the electrons hit the wall
- record the location, phase, and impact energy
- the number of secondary electrons is determined, given the SEY function
- the trajectory calculation is continued if the field phase is such as secondary electrons leave the wall
- after a given number of impacts N the No. of free electrons and their avg. impact energy and the No. of secondary electrons is calculated

**Counter function**: field levels at which resonant conditions are satisfied

At field levels where **Enhanced counter function** > No. initial electrons: Multipacting
MultiPac

- 2D code, has its own FEM field solver
- Runs on Linux
- MATLAB user interface
FishPact

- 2D code, uses SUPERFISH to compute surface fields
- Runs on PC
Track3P

- 3D code, uses Omega3P for field solver
- Runs on Supercomputer, user interface not fully developed

Example: Multipacting found in beam pipe step of LL cavity
Example: Multipacting in SNS HOM Coupler

- SNS SCRF cavity experienced RF heating at HOM coupler
- 3D MP simulations showed MP barriers closed to measurements
- Similar analysis are carried out for ILC ICHIRO and crab cavity

Field level in HOM couplers

<table>
<thead>
<tr>
<th>max E at HOM notch gap (MV/m)</th>
<th>HOM2 coupler</th>
<th>HOM1 coupler</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

HOM coupler notch gap (mm) 0.48 - 0.54

Track3P
Expt. MP bands

Field Level (MV/m) 0 - 20
Mechanical Analysis

The mechanical design of a cavity follows its RF design:
- Lorentz Force Detuning
- Mechanical Resonances
- Structural stability under different load conditions

Lorentz Force Detuning

\[ P = \frac{\mu_0 H_s^2 - \varepsilon_0 E_s^2}{4} \]

E and H at \( E_{acc} = 25 \text{ MV/m} \) in TESLA inner-cup

\[ 50 \text{ MV/m} \]

\[ 92 \text{ kA/m} \]
Stiffening

Surface deformation without and with stiffening ring

No stiffening ring
Wall thickness 3mm

Stiffening ring at r=54mm
Wall thickness 3mm

Surface deformation due to Lorentz force in TESLA coil, without stiff ring

$10^{-4} m$

$3 \times 10^{-5} m$

Essential for the operation of a pulsed accelerator

$\Delta f = k_L (E_{acc})^2$

$k_L = -1 \text{ Hz/(MV/m)}^2$
Optimal stiffening ring position

The Lorentz forces coefficients for 15 different stiffening ring positions are evaluated automatically with ANSYS, preparing the geometry and reading the fields from the SFO output from SUPERFISH.

**Fixed cell length L**

$$R_{\text{stiff}} = 80 \text{ mm}$$

**Reference data**

- L = 56.8 mm
- R = 1
- r = 1.7
- d = 11 mm
- k = 1.5%
- $$R_{\text{iris}} = 43 \text{ mm}$$
- Nb thick. = 3.8 mm

Displacements [mm]
**$K_L$ for different boundary conditions**

- The estimate for $K_L$ strongly depends on the cell boundaries. We compute it for 3 different cases:
  - Fixed cell length
  - Free cell length
  - Helium Vessel/Tuning System (= 3 tubes with diameter 30 mm and thickness 2 mm)
Mode Analysis

Calculate mechanical resonances of a multi-cell cavity as they modulate frequency of the accelerating mode. Sources of their excitation: vacuum pumps, ground vibrations...

<table>
<thead>
<tr>
<th>Mode</th>
<th>Test Data</th>
<th>FE Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>31</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>38</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>53</td>
<td>48</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>72</td>
</tr>
<tr>
<td>6</td>
<td>82</td>
<td>83</td>
</tr>
<tr>
<td>7</td>
<td>125</td>
<td>124</td>
</tr>
</tbody>
</table>

Frequency (Hz) | PSD (g²/Hz)
--- | ---
1.0E-16 | 0
1.0E-15 | 10
1.0E-14 | 20
1.0E-13 | 30
1.0E-12 | 40
13 | vertical/equator
31 | axial/equator
38 | tangential/equator
70 | axial/flange
82 | 125
53 |
Typical Cavity Mechanical Design Requirements

- Minimize/prevent microphonics
- Withstand loss of vacuum accident up to 5 atm
- Withstand cool down at 1.65 atm
- Adhere to intent of ASME B&P Code
  - Allowable Stress ($S_m$) = $2/3$ Yield Stress
  - Primary Membrane Stress ($P_m$) $\leq (S_m)$
  - $P_m +$ Bending $\leq 1.5*S_m$
  - $P_m +$ Bending $+$ Secondary Stress $\leq 3*S_m$
- Allowable Stresses
  - Warm Niobium = 4,667 psi
  - Cold Niobium = 53,333 psi
Mechanical analysis tools

- ANSYS: FEM multiphysics solver

Peak von Mises stress in cold cavity with 5 atm pressure and 2 mm tuning displacement, calculated on CEBAF LL Upgrade cavity
Cavities, large and small...
500 MHz, Single-cell
Pulsed LINACs (ILC, XFEL)

- High gradient ($\geq 25$ MV/m)
- Moderate HOM damping ($Q_{\text{ext}} = 10^4 - 10^5$)
- High peak (> 250 kW), low average RF power (~ 5 kW)

ILC: 21,000 cavities!
CW Low-Current LINACs (CEBAF, ELBE)

- Moderate to low (8 – 20 MV/m)
- Relaxed HOM damping requirements
- Low average RF power (5 – 13 kW)
CW High-Current ERLs

• Moderate gradient ($\geq 15 - 20$ MV/m)

• Strong HOM damping ($Q_{\text{ext}} = 10^2 - 10^4$)

• Low average RF power (few kW)
CW High-Current Injectors for ERLs

- Moderate to low gradient (5 - 15 MV/m)
- Strong HOM damping ($Q_{ext} = 10^2 – 10^4$)
- High average RF power (50 - 500 kW)
CW High-Current Storage Rings

- Relatively low gradient (5 - 9 MV/m)
- Strong HOM damping ($Q_{\text{ext}} \sim 10^2$)
- High average RF power (up to 390 kW)
Summary

Cavity DESIGN: be creative and have fun with it!