RF Technology

S. Di Mitri (90min.)
Review of Relativistic Formulas

\[ \beta = \frac{\vec{v}}{c}, \quad c = 2.998 \times 10^8 \text{ m/s} \]

\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]

\[ |\beta| \approx 1 - \frac{1}{2\gamma^2} \quad \text{for } \gamma >> 1 \]

\[ \vec{p} = \beta \gamma m_c \]

\[ T = \frac{|\vec{p}|^2}{2m_e} \]

\[ E = T + m_e c^2 \]

\[ x' = \frac{p_x}{p_z} = \frac{|\vec{p}|}{p_z} \frac{x'}{\sqrt{x'^2 + y'^2 + 1}} \]

\[ y' = \frac{p_y}{p_z} = \frac{|\vec{p}|}{p_z} \frac{y'}{\sqrt{x'^2 + y'^2 + 1}} \]

Typically

\[ p_z \approx (10^3-10^5) p_x, p_y \]
RF Acceleration

\[ \vec{F} = \frac{d\vec{p}}{dt} = q(\vec{E} + \nu \times \vec{B}) \]

\[ \Delta T = \int \vec{F} ds = -\int_{0}^{L} eE_z dz \]

To accelerate charged particles, the RF wave must have electric fields along the direction of propagation of the particle and the wave itself. However, EM waves in free space only have electric field that is transverse to direction of propagation.

To get non-zero acceleration of charged particles co-propagating with the electromagnetic wave, we have to do the following:

1. Use a resonant cavity that has transverse magnetic (TM) modes. The TM_{010} mode has axial electric field to accelerate particles along the axial direction.

2. Load the cavity with disk-and-washers to slow the phase velocity of the RF wave to the speed of light \( c \), so that a charged particle traveling at speed slightly less than \( c \) will have non-zero acceleration accumulated over many RF cycles.
Standing Wave, Field Pattern

- Static picture of field pattern in a cell, as function of radius («pill-box» model):

- Dynamical picture of the on-axis accelerating mode $E_z$ along the structure. It is factorized in a temporal and spatial dependence. For the reference particle at $<z>=0$:

$$E_z \approx \hat{E}_z \cos(\omega_{RF} t + \phi_0) \cos(k_0 z) =$$

$$\hat{E}_z \cos(\omega_{RF} t + \phi_0) \equiv \hat{E}_z \cos(\phi_{RF})$$
Standing Wave, Single Kick Model

- $E_z$ is “resonating” with multiple reflections back and forth. **Loss-free propagation** can only occur if the field wavelength is an integer multiple of the iris separation (constructive interference between reflections):

$$\lambda_z = pd, \quad p = 1, 2, 3, \ldots; \quad k_z d = \frac{2\pi}{p}$$

- **Energy gain** in the approximation of single kick:

$$\Delta \gamma = -\int_0^{N_d} \frac{e}{m_e c^2} E_z dz = -\frac{e}{m_e c^2} \hat{E}_z N_d d \cos \phi_{RF} = \frac{e}{m_e c^2} V_0 \cos(\phi_{RF})$$
Resonant Cavity, RLC Circuit Model

\[ U = \frac{1}{2} CV^2 \quad P_d = \frac{V^2}{R} \]

\[ E_z^2 = \left( -\frac{dV}{dz} \right)^2 \approx \left( \frac{V}{L} \right)^2 = \frac{P_d r_s}{L} \approx r_s \frac{dP_d}{dz} \]

\[ \frac{r_s}{Q} = \frac{E_z^2}{\frac{dP_d}{dz}} \quad \frac{P_d}{\omega_{RF} U} = \frac{E_z^2}{\frac{dP_d}{dz}} \quad \frac{\frac{dP_d}{dz}}{\omega_{RF}} = \frac{E_z^2}{\omega_{RF} u} \]

\[ Q = \frac{\omega_{RF} U}{P_d} = \frac{\text{Energy stored in the cavity}}{\text{Energy dissipated in one RF period}} \]

\[ V = \sqrt{P_d r_s} \equiv \sqrt{P_d r_s L} \]

\[ E_z \]

Accelerating field

This ratio depends on the cavity geometry only.

- High Q, Quality Factor \( \sim \) high efficiency for energy storing
- High \( r_s \), Shunt Impedance per unit length \( \sim \) high accelerating field vs. dissipated power
Continuity Equation

\[ \frac{\partial u}{\partial t} + \frac{\partial P}{\partial z} + \frac{dP_d}{dz} + nevE_z = 0 \]

- Time-variation of the energy stored per unit length
- Energy flux per unit length
- Dissipated power per unit length
- Energy absorbed by the beam, per unit length ("beam loading")

- **In a SW**, we assume \( \frac{dP}{dz} = 0 \) and neglect \( nevE_z \). Thus we find:

\[ \frac{\partial u}{\partial t} = - \frac{dP_d}{dz} ; \quad \Rightarrow \quad \frac{dU}{dt} = - P_d = - \frac{\omega_{RF}}{Q} U ; \quad \Rightarrow \quad U = U_0 e^{-\frac{\omega_{RF} t}{Q}} \]

The energy stored in the structure decays exponentially with characteristic time \( Q/\omega_{RF} \).
Traveling-Wave Structure

In a traveling wave (TW) structure, the accelerating wave is co-propagating with the particles. Still parasitic power reflections have to avoided with “resonant” operating modes as in a SW.

\[
E_z \equiv \hat{E}_z \cos(\omega_{RF}t - k_z z + \phi_0) = \hat{E}_z(z)\cos(\phi_{RF})
\]

To keep particles and accelerating wavefront synchronous (phase-matched) all along the structure, we must have \(v_p \equiv \beta c\). This is ensured by the irises that “slow down” \(v_p\) acting as a capacitive load.

All previous formulas for SW are now valid with the prescription: \(P_d\) (dissipated) \(\rightarrow\) \(P\) (propagating).

Since \(P\) is now propagating along the structure, the filling time & group velocity are defined:

\[
v_g \equiv \frac{P}{dU/dz} \quad t_f \equiv \int_0^L \frac{dz}{v_g(z)}
\]
Constant Impedance (TW–CI)

\[
\frac{\partial u}{\partial t} + \frac{\partial P}{\partial z} + \frac{dP_d}{dz} + nevE_z = 0
\]

In a TW, we assume \( P_d = 0 \) and \( nevE_z = 0 \). Thus we find:

\[
\frac{\partial u}{\partial t} = - \frac{dP}{dz}; \quad \Rightarrow \quad P = -uv_g = - \frac{Qv_g}{\omega_{RF}} \frac{dP}{dz}; \quad \Rightarrow \quad \frac{dP}{P} = - \frac{\omega_{RF}}{Qv_g} dz
\]

From the \( r_s/Q \) ratio:

\[
\frac{r_s}{Q} = \frac{E_z^2}{\omega_{RF}u} \quad \Rightarrow E_z^2 = \frac{r_s}{Q} \omega_{RF}u = \text{const.} \times u(z)
\]

From the accelerating field:

\[
E_z^2 = -r_s \frac{dP}{dz} \quad \Rightarrow E_z(z) = \sqrt{2\tau} \frac{P_0r_s}{L} e^{-\frac{\tau z}{L}}
\]

N.B.: we are using the fact that \( v_g \) is constant all along the structure.

\[
\tau := \frac{\omega_{RF}L}{2Qv_g} \quad \text{Attenuation factor}
\]

\[
t_f := \int_0^L \frac{dz}{v_g(z)} = \frac{L}{v_g} = 2\tau \frac{Q}{\omega_{RF}}
\]
Constant Gradient (TW–CG)

In a TW, we assume \( P_d = 0 \) and \( n e v E_z = 0 \). Thus we find:

\[
\frac{\partial u}{\partial t} + \frac{\partial P}{\partial z} + \frac{dP_d}{dz} + n e v E_z = 0
\]

\[
\frac{\partial u}{\partial t} = - \frac{dP}{dz} = - \frac{E_z^2}{r_s} \equiv \text{const.}; \quad P(z) = P_0 - \frac{E_z^2}{r_s} z = P_0 + \frac{P(L) - P_0}{L} z;
\]

From the \( r_s/Q \) ratio:

\[
\frac{r_s}{\pi Q} = \frac{E_z^2}{\omega_{RF} u} \quad \Rightarrow \quad u = \frac{E_z^2 Q}{r_s \omega_{RF}} = \text{const.}
\]

\[
P(L) := P_0 e^{-2\tau} \quad v_g = \frac{P}{u} \propto z
\]

From the accelerating field:

\[
E_z(z) = -r_s \frac{dP}{dz} = \sqrt{\left(1 - e^{-2\tau}\right) \frac{P_0 r_s}{L}}
\]

N.B.: \( \tau \) is well-defined when \( v_g(L) \) is known.
Summary

- **SWs** have practically a higher acceleration efficiency, but longer filling time than **TWs**.

- **TW-CI** is simpler to build (all cells are identical). It usually provides a larger energy gain than a TW-CG, but with a higher peak field. This may induce discharges in the structure.

- **TW-CG** has cells with varying iris radius. It can provide the same energy gain than in a TW-CI but with a lower peak field. Commonly used in recent times.

- **N.B.**: the e.m. wave in a SW can be shown to be the superposition of 2 identical e.m. waves propagating through the structure in opposite directions. This implies that, for the same structure’s length $L$:

$$ R_{s,SW} = \frac{E_0^2 L}{dP_d} = \frac{E_0^2 L}{2 \frac{dP}{dz}} = \frac{R_{s,TW}}{2} $$
Micro- and Macro-Pulses

RF pulses

Macropulse separation = \( \frac{1}{\text{Repetition rate}} \)

RF pulse amplitude

Electron bunches

Fill time

Electron bunches

Bunch separation = \( \frac{n}{f} \)

\( n \) is the number of RF buckets between electron bunches
Normal- vs. Super-Conducting Tech

“Normal-Conducting” (NC): warm linacs, the thermal load threshold usually limits the maximum rep. rate.

For any target $E_z$, the power consumption is minimized by a high $r_s$. This is maximized through inner cells’ shaping. Example: $r_s \sim m \Omega$, $Q \sim 10^4$.

“Super-Conducting” (SC): He-liquid frozen structures that allow higher rep. rates than NC linacs.

The heat load is minimized by a large $Q$. This is made large with cell’s surface treatment. Example: $r_s \sim n \Omega$, $Q \sim 10^{10}$. 
RF Frequency

**Ansatz:** keep $L = V_0/E_z$ fixed, and scale all other dimensions as $f^{-1}$:

$$r_s = \frac{E_z L}{P_D} \propto \begin{cases} f^{1/2} & \text{for NC} \\ f^{-1} & \text{for SC} \end{cases}$$

$$Q = \frac{\omega_{RF} U}{\pi P_D} \propto \begin{cases} f^{-1/2} & \text{for NC} \\ f^{-2} & \text{for SC} \end{cases}$$

- **NC linacs** maximize $E_z$ through a high $r_s$, thus perform better at high $f$, but at the expense of small cells’ iris.

- **Single bunch interactions** with walls (short-range wakefields) become strong. They impose attention to the beam dynamics.

- **SC linacs** minimize the heat load with a large $Q$, thus perform better at low $f$, but at the expense of higher order field modes.

- **Inter-bunch interactions** with walls (long-range wakefields) become strong. They can be suppressed with mode dampers.

**Sort of available $f$ (GHz):** L-band (1.3), S-band (2.9), C-band (5.7) and X-band (11.4)
### Exercise, NC TW

<table>
<thead>
<tr>
<th>Parameter</th>
<th>S-band, NC</th>
<th>X-band, NC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working frequency, ( f ) [GHz]</td>
<td>2.998</td>
<td>11.424</td>
</tr>
<tr>
<td>Type</td>
<td>TW, Constant impedance</td>
<td>TW, Constant gradient</td>
</tr>
<tr>
<td>Phase Advance per Cell</td>
<td>( 2\pi/3 )</td>
<td>( 2\pi/3 )</td>
</tr>
<tr>
<td>Length, ( L ) [m]</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Average Iris Radius, ( a ) [mm]</td>
<td>10.0</td>
<td>3.5</td>
</tr>
<tr>
<td>Group Velocity, ( v_g ) [c]</td>
<td>0.01</td>
<td>0.01 (average)</td>
</tr>
<tr>
<td>Filling Time [(\mu s)]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Quality Factor, ( Q )</td>
<td>12000</td>
<td>8600</td>
</tr>
<tr>
<td>Attenuation Factor, ( \tau ) [neper]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Shunt Impedance, ( r_s ) [M(\Omega)/m]</td>
<td>55</td>
<td>80</td>
</tr>
<tr>
<td>Input Peak Power, ( P_{RF} ) [MW]</td>
<td>45</td>
<td>80</td>
</tr>
<tr>
<td>RF Pulse Duration [(\mu s)]</td>
<td>2.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Acc. Gradient, ( eV_0/L ) [MV/m]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Repetition Rate [Hz]</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

Design and parameters optimization really depends on what you take care more of....
Pulse Format (examples)

Normal-conducting RF Linac

Traveling-wave linac
Example: S-band SLAC, C-band SACLA
Water-cooled copper
Accelerating gradient $\sim 30$ MV/m
Maximum RF pulse $\sim 3$ $\mu$s
Fill time $< 1$ $\mu$s

Standing-wave linac
Example: L-band ISIR (Osaka)
Water-cooled copper
Accelerating gradient $\sim 20$ MV/m
Maximum RF pulse $\sim 30$ $\mu$s
Fill time $\sim 2$ $\mu$s

Super-conducting RF Linac
Example: L-band TESLA (DESY)
Liquid He cooled niobium
Accelerating gradient $\sim 15$ MV/m
Long pulse to continuous-wave (CW)
Fill time $\sim 500$ $\mu$s
Typical klystron power is **30 - 70 MW**, over a few µs.

SLED cavities (see next slide) allow to gain a **factor of 3-4 in peak power**, reducing the RF pulse length to sub-µs duration.

If the total power is enough to reach the maximum peak field tolerable by the cavity (before inducing discharges in the structure due to surface emission), the waveguide can be split into two lines to **supply multiple structures**.

The final RF pulse **duration** has to be longer than the structure **filling time**.
**SLED («Slac Energy Doubler via RF Pulse Compression»)**

- The SLED is a high Q-cavity which stores klystron energy during a large fraction of each RF pulse, and then discharges it rapidly into the accelerator. So, the output RF pulse is shortened, the peak power and the accelerating voltage both increase.

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**Comparison of Input Cavity Fields with and without Phase Modulation**

- **SLED with fast phase reversal**
- **SLED with phase modulation**
- **No SLED**

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**KLY-pulse**

- **SLED output**

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**Phase reversal**

- **KLY + SLED**

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**USPAS June 2015**

S. Di Mitri - Lecture_Tu5
RF Distribution and Linac Layout

1. **Target**: final $\langle E \rangle$

2. **Goals**: minimize $L$, minimize $P_{RF}$, operate at high rep. rate

3. **Means**: high $E_z$, high $r_s$, high $Q$.

4. **Trade-off**: RF power (cost of RF suppliers and electricity) vs. space (cost of accelerating structures and linac building).

5. **One possible strategy**:
   - fix the structure’s geometry ($f, L, r_s$);
   - balance the $P_{RF}$ per structure with the **number of klystrons**

\[
E_f \approx NGL = eNV_0 = eN \sqrt{P_{RF} r_s L}
\]

$M \equiv \text{number of structures per klystron}$
**Trends...**

- **Small** $M (<2:1)$ implies high $P_{RF}$ that is:
  - high $V_0$, small $N$
  - a *compact* linac but *many* klystrons

- **Large** $M (>4:1)$ implies low $P_{RF}$ that is:
  - low $V_0$, large $N$
  - *few* klystrons but a *long* linac

- This choice is limited by the *structure breakdown limit* at high accelerating gradients.
- Many klystrons and modulators may imply a tighter packing of RF components, thus more space needed in the “klystron gallery”.
- Well suited for *compact, NC* linacs at *low repetition rate*.

- This choice is limited by the *available space* for the linac.
- Many structures may imply a more complex cooling system.
- Well suited for *SC* linacs or *NC* linacs at *high repetition rate*. 
Examples

Assume $E_0 = 100\text{MeV}$, $E_f = 3\text{GeV}$, and a periodic linac layout.

**$M=2$:**

\[
(3000-100)\text{MeV}/138\text{MeV} = 21 \text{ KLYs}, 21 \text{ girders and a } 29 \text{ m long linac @10Hz}
\]

**$M=4$:**

\[
(3000-100)\text{MeV}/196\text{MeV} = 15 \text{ KLYs}, 30 \text{ girders and a } 42 \text{ m long linac @100Hz}
\]
**Estimate of Power Consumption**

The average power sent to the structure is \( \langle P_{RF,0} \rangle = P_{RF,0} \cdot \Delta t_{RF} \cdot \text{rep.rate} \)

Case \( M = 2 \): \( \langle P_{RF,0} \rangle = 120\text{MW} \cdot 0.7\mu\text{s} \cdot 10\text{Hz} = 0.84 \text{ kW / structure} \)

Case \( M = 4 \): \( \langle P_{RF,0} \rangle = 60\text{MW} \cdot 0.7\mu\text{s} \cdot 100\text{Hz} = 4.20 \text{ kW / structure} \)

We do not expect strong cooling requirements.

For such a structure, the attenuation factor is \( \tau \approx 1.5 \) neper. Being TW-CG, we evaluate that \( (1 - e^{-3}) = 5\% \) of input power goes to the load, while 95\% of that is used for acceleration. Thus, we have designed an efficient RF acceleration system.

i. Klystron power = \( 60\text{MW} \cdot 1.4\mu\text{s} \cdot 100\text{Hz} / 0.8 \) (efficiency) = 10.5 kW

ii. Modulator power = 10.5kW / 0.9 (efficiency) = 11.7 KW

iii. Using 1 klystron per 4 structures \( (M = 4) \), we require 11.7kW \cdot 15 \text{ RF stations} = 0.18 \text{ MW}

iv. We should add about 0.1 MW for the injector, 0.1 MW for the water cooling, 0.1 MW for the magnets and 0.1 MW for basic building power. In total we require 0.6 MW for a system this size.
Modeling in Tracking Codes ($\beta \to 1$)

In many tracking codes for ultra-relativistic particles, the following approximations are routinely adopted both for SW and TW structures:

1) cylindrically symmetric, spatially periodic RF structure;
2) the accelerating field radial dependence is ignored (particles are assumed to be traveling close to the electric axis: $E_z(r)=E_z(0)$);
3) higher harmonics of $E_z$ are neglected (negligible acceleration respect to the fundamental mode $n=0$);
4) the filling time is short enough respect to the RF pulse to ensure steady-state acceleration;
5) the transient time factor = 1;
6) the TW structure is a “constant-gradient” with attenuation factor = 0;
7) for any cell of finite length, acceleration is provided in the “drift-kick-drift” scheme (single energy kick in the cell’s center):

In this simplified picture, the total energy gain per structure is just:

$$\Delta \gamma(L) = \frac{|e|}{m_ec^2} V_0 \cos(\phi_{RF})$$