9
Introduction to superconducting magnets*

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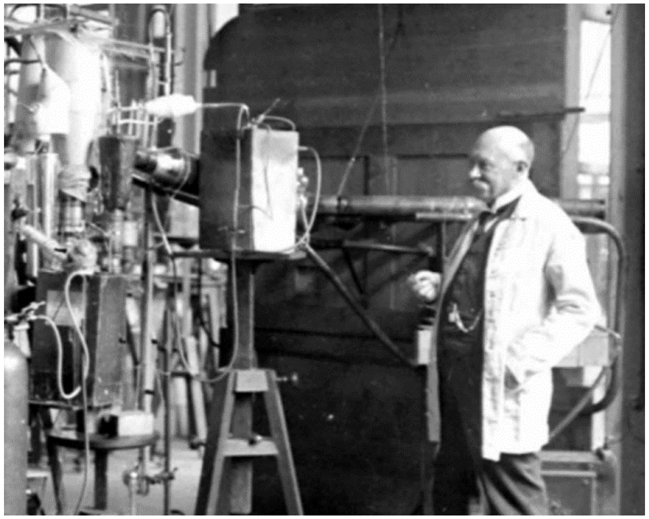
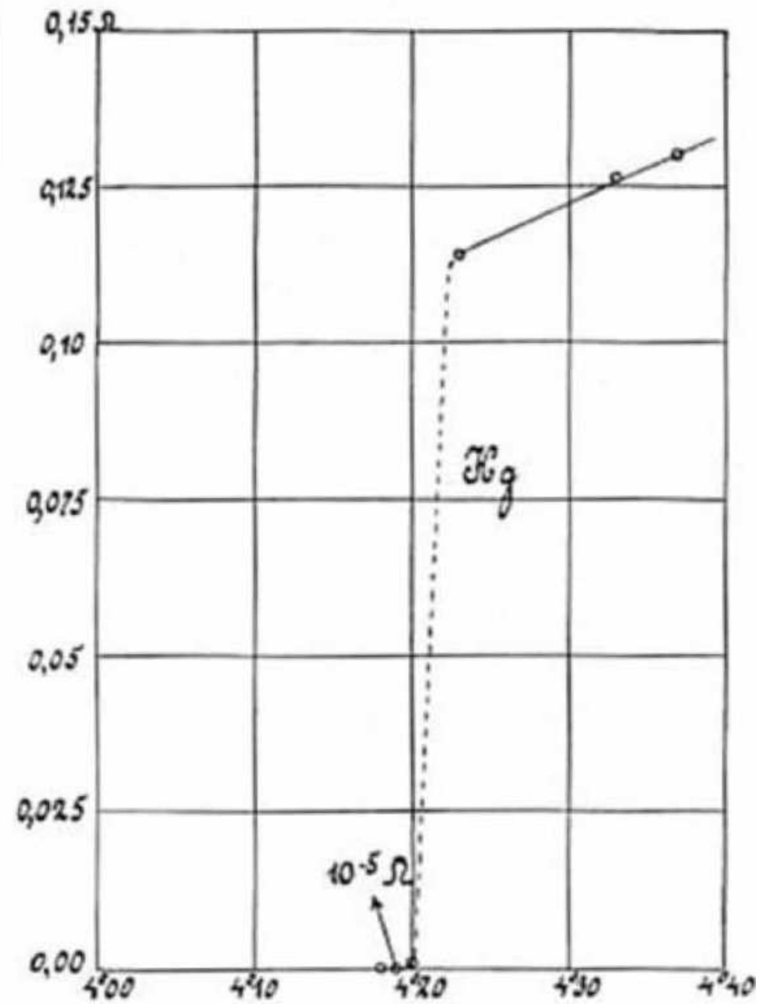
* From: “Superconducting Accelerator Magnets” by Paolo Ferracin, Ezio Todesco, Soren O. Prestemon and Helene Felice, January 2012

A Brief History of the Superconductivity



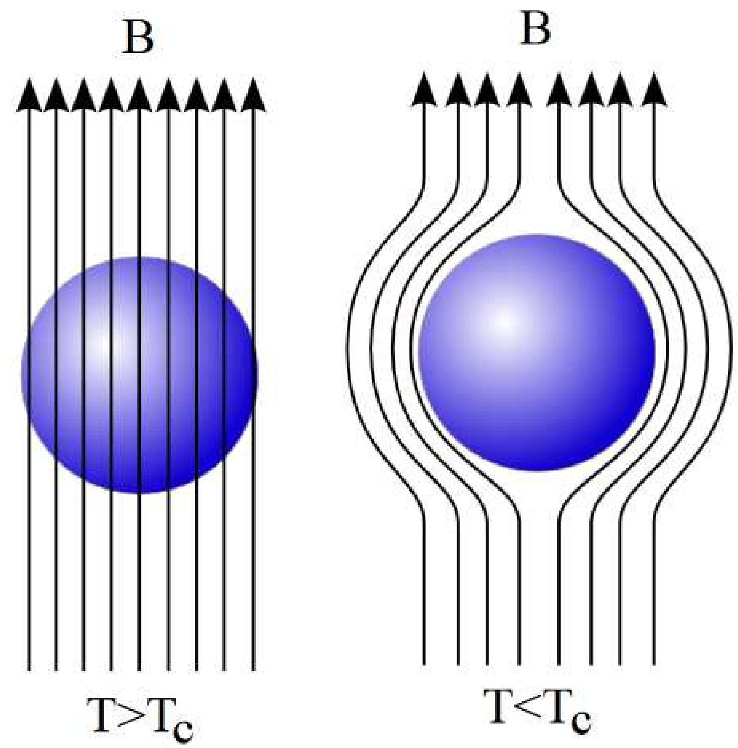
Heike Kamerlingh Onne



1908 – Successfully liquified helium (4.2 K)
1911 – Discovered the superconductivity while measuring the conductivity of Mercury as function of temperature
1913 – Nobel prize

A Brief History of the Superconductivity

1933 – Walther Meissner and Robert Ochsenfeld discover perfect diamagnetic property of supeconductors.
1935 – First theoretical works on SC by Heinz and Fritz London
1950 – Ginzburg and Landau proposed a macroscopic theory for SC.



Meissner effect

Why using SC magnets?

$$Br = \frac{P}{q} = \frac{\sqrt{K^2 + 2KE_o}}{qc}$$

Example: Lets calculate the magnetic rigidity for a 1 TeV proton:

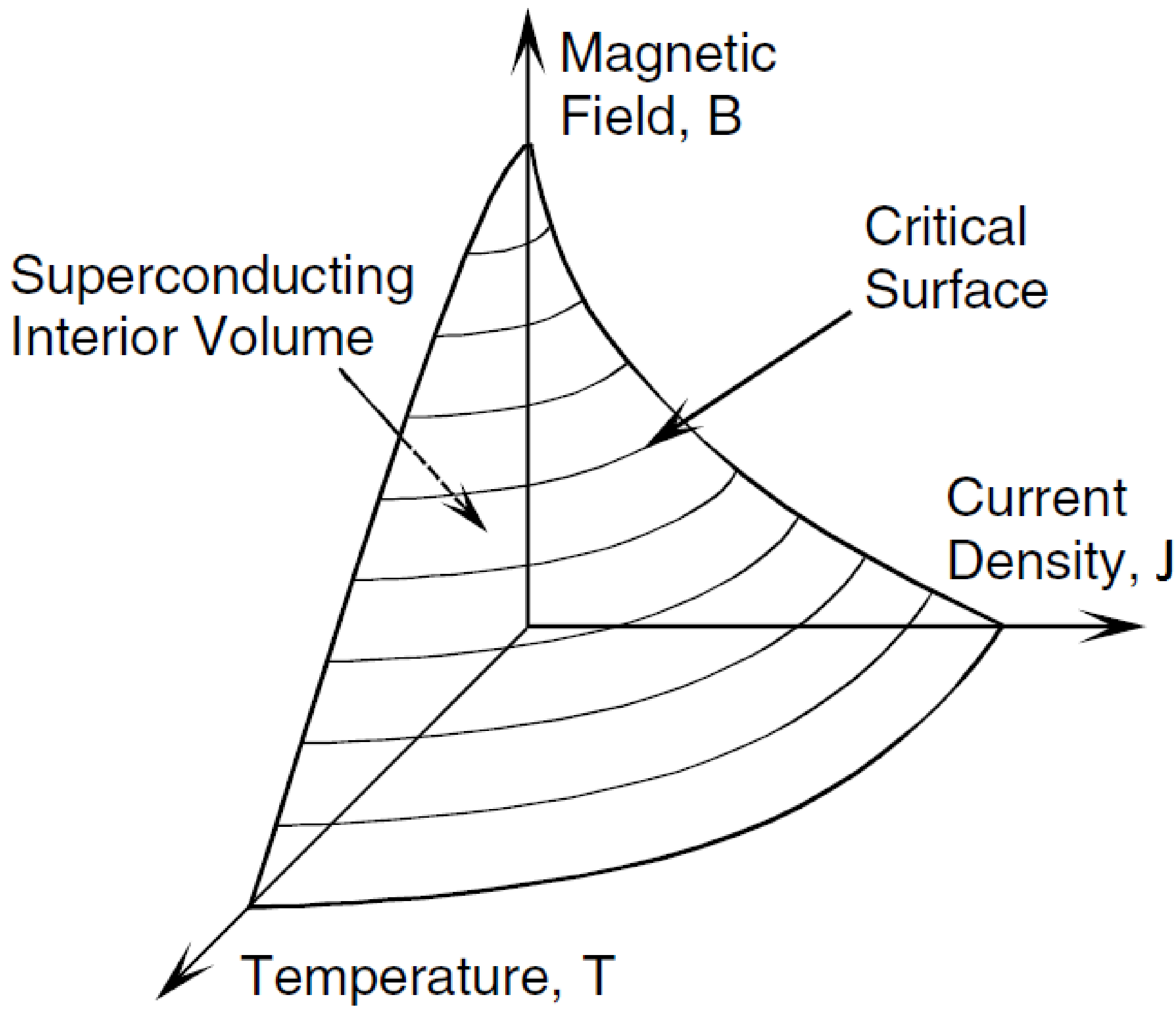
$$Br \approx \frac{1\ TeV}{c} \approx 3333\ T.m$$

Let us assume a maximum field of 1.5 T; the circumference of such machine will be:

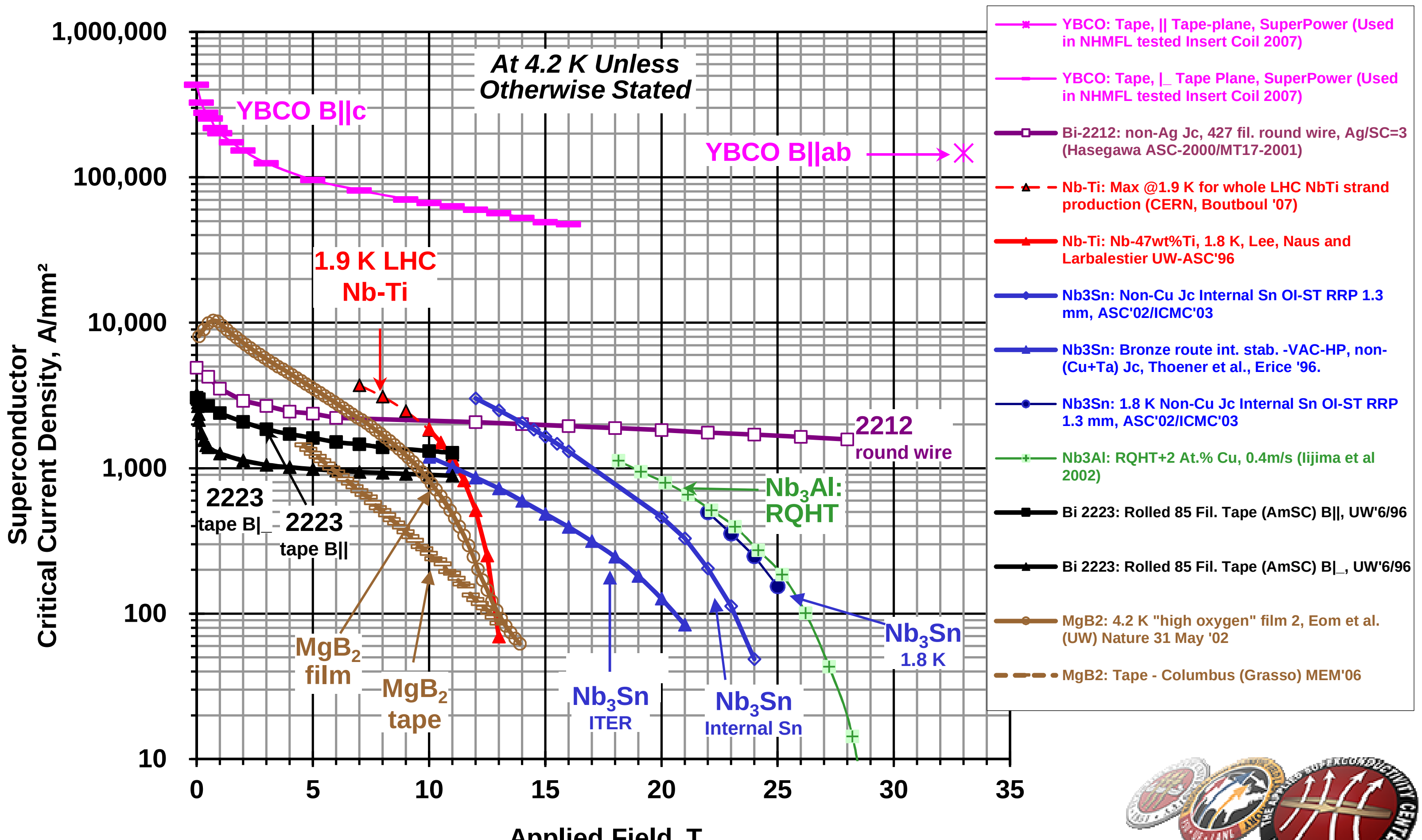
$$r = 2222\ m$$
$$C = 2\pi r \approx 14\ km$$

The Tevatron was the first machine to use large scale superconductor magnets with a 4.2 T in a 6.3 km circumference!

Critical surface



Critical surface for different SC materials



NbTi Parameterization

$$B_C(T) = B_{C0}\left[1-\left(\frac{T}{T_{C0}}\right)^{1.7}\right]$$ (Lubell's formula)

where B_{C0} is the critical field at zero temperature (B_{C0}~14.5 T)

$$\frac{J_c(B,T)}{J_{c\_ref}} = \frac{C}{B}\left(\frac{B}{B_c}\right)^{\alpha}\left(1-\frac{B}{B_c}\right)^{\beta}\left[1-\left(\frac{T}{T_{c0}}\right)^{1.7}\right]^{\gamma}$$ (Bottura's formula)

where J_{C_ref} is the critical current density at 4.2 K and 5 T (J_{C_ref} ~ 3000 A/mm^2);
C, α, β and γ are fitting parameters:

C ~ 31.4 T
α ~ 0.63
β ~ 1.0
γ ~ 2.3

Nb_3Sn Parameterization

$$J_c(B,T,\varepsilon) = \frac{C(\varepsilon)}{\sqrt{B}}\left(1-\frac{B}{B_c(T,\varepsilon)}\right)^2\left[1-\left(\frac{T}{T_{c0}(\varepsilon)}\right)^2\right]^2$$

(Summer's formula)

$$\frac{B_c(T,\varepsilon)}{B_{c0}} = \left[1-\left(\frac{T}{T_{c0}(\varepsilon)}\right)^2\right]\left\{1-0.31\left(\frac{T}{T_{c0}(\varepsilon)}\right)^2\left[1-1.77Ln\left(\frac{T}{T_{c0}(\varepsilon)}\right)\right]\right\}$$

where:

$$C(\varepsilon) = C_{0\_m}\left(1-\alpha|\varepsilon|^{1.7}\right)^{1/2}$$

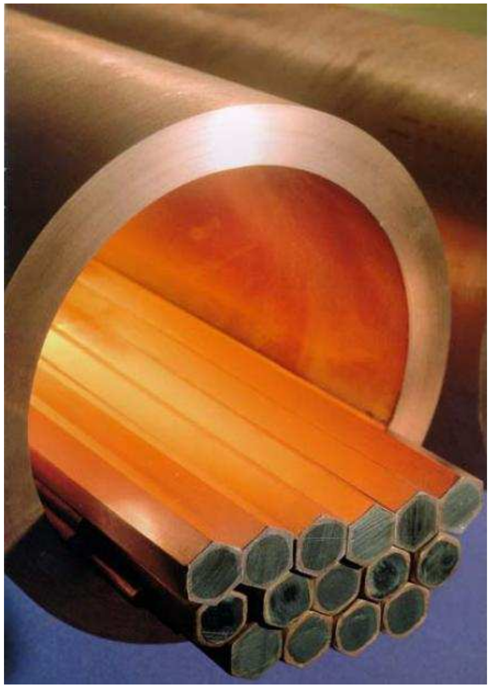
$$B_c(T,\varepsilon) = B_{c0\_m}\left(1-\alpha|\varepsilon|^{1.7}\right)$$

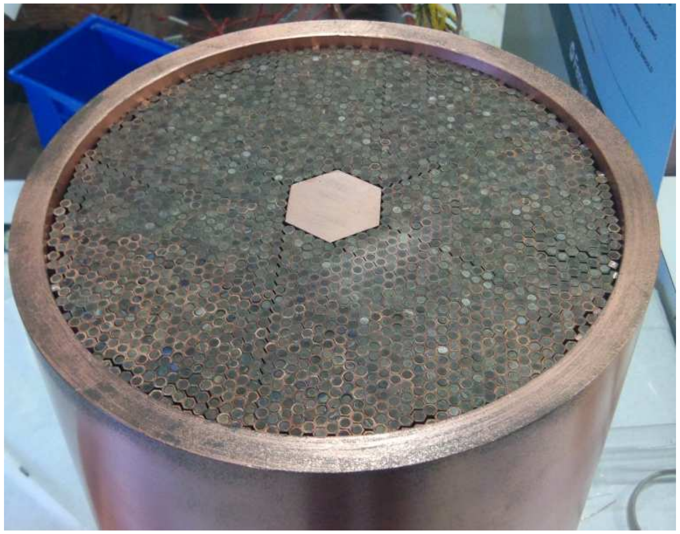
$$T_{c0}(\varepsilon) = T_{c0\_m}\left(1-\alpha|\varepsilon|^{1.7}\right)^{1/3}$$

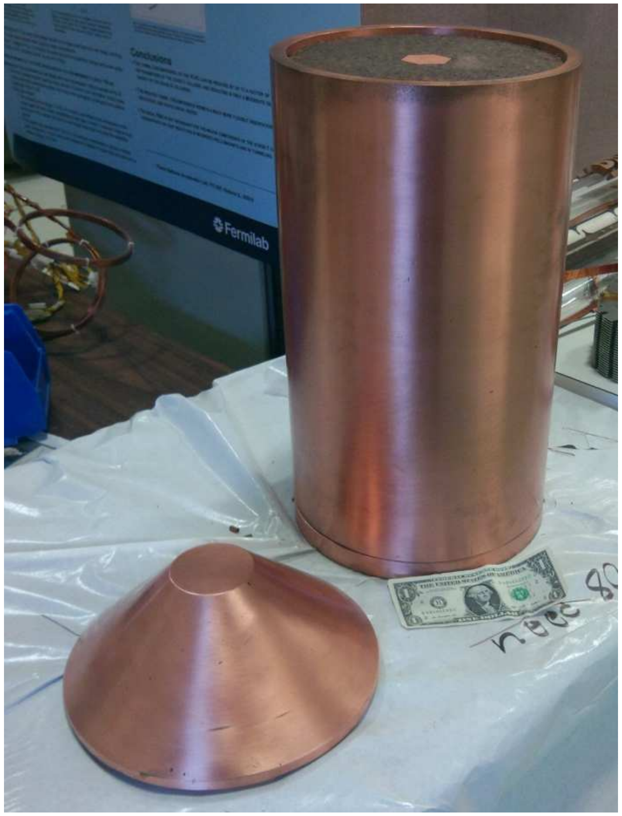
and:

α = 900
ε = -0.003
T_{c0_m} = 18K
C_{0_m} = 48500 $AT^{1/2}/mm^2$
(for J_c = 3000 A/mm^2 @ 4.2 K and 12 T)

Strand Fabrication

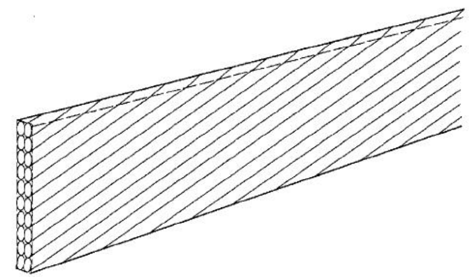






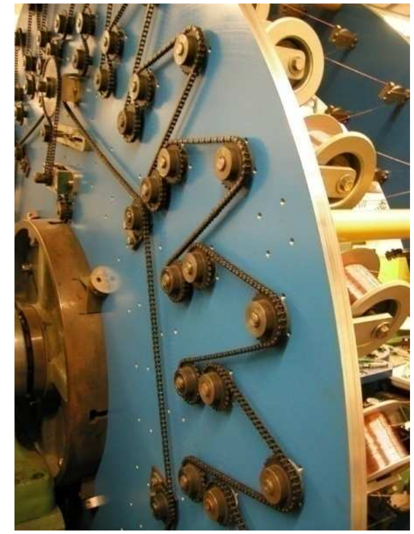
Superconducting cables

- Most of the superconducting coils for particle accelerators are wound from a multi-strand cable.
- The advantages of a multi-strand cable are:
  - reduction of the strand piece length;
  - reduction of number of turns
    - easy winding;
    - smaller coil inductance
      - less voltage required for power supply during ramp-up;
      - after a quench, faster current discharge and less coil voltage.
  - current redistribution in case of a defect or a quench in one strand.

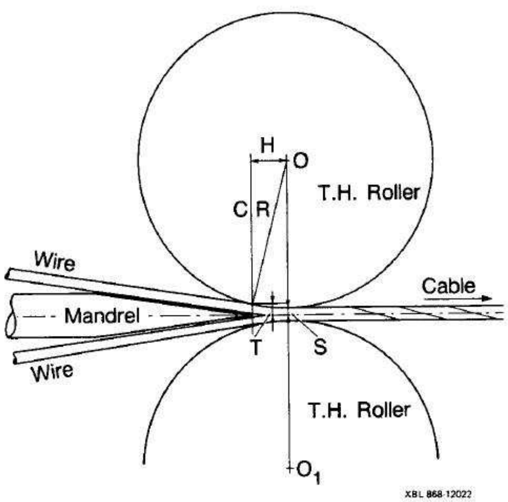


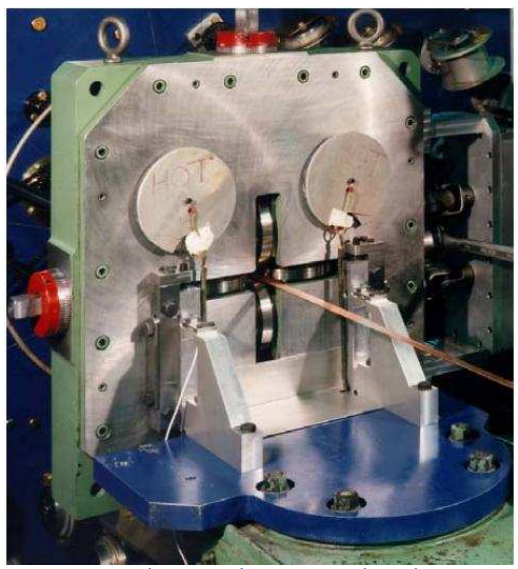
- The strands are twisted to
  - reduce interstrand coupling currents (see interfilament coupling currents)
    - Losses and field distortions
  - provide more mechanical stability
- The most commonly used multi-strand cables are the Rutherford cable and the cable-in-conduit.

Superconducting cables



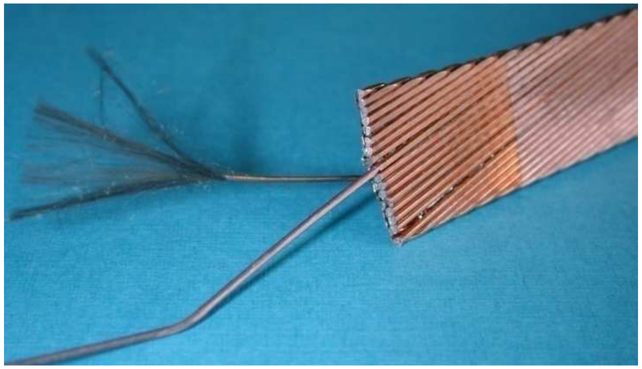
- Rutherford cables are fabricated by a cabling machine.
  - Strands are wound on spools mounted on a rotating drum.
  - Strands are twisted around a conical mandrel into an assembly of rolls (Turk's head). The rolls compact the cable and provide the final shape.





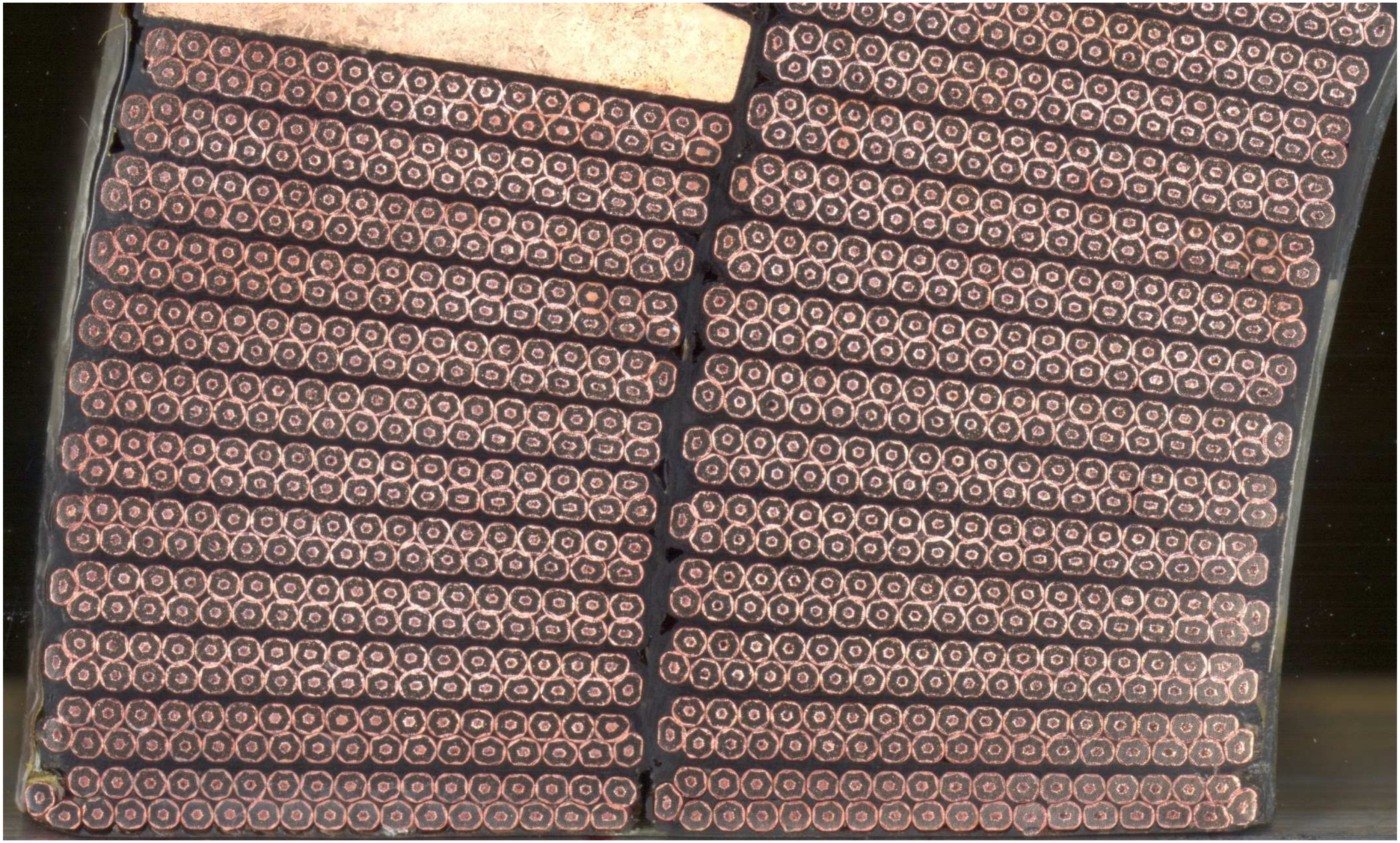
Superconducting cables

- The final shape of a Rutherford cable can be rectangular or trapezoidal.
- The cable design parameters are:
  – Number of wires N_{wire}
  – Wire diameter d_{wire}
  – Cable mid-thickness t_{cable}
  – Cable width w_{cable}
  – Pitch length p_{cable}
  – Pitch angle ψ_{cable} ($\tan\psi_{cable} = 2\, w_{cable} / p_{cable}$)
  – Cable compaction (or packing factor) k_{cable}



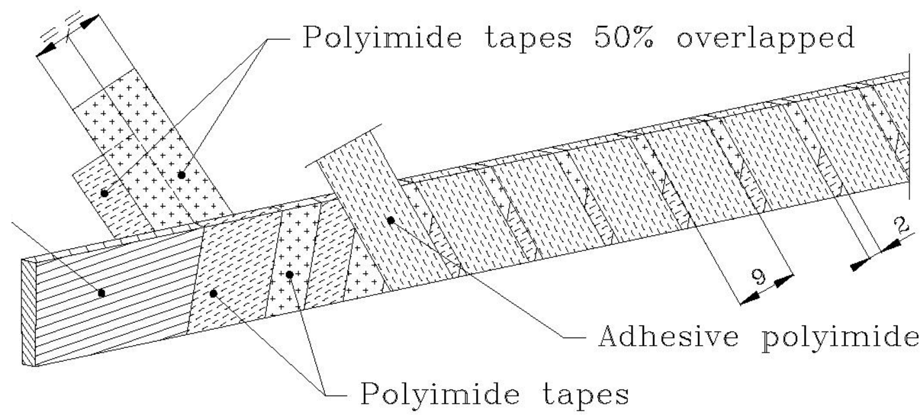
$$k_{cable} = \frac{N_{wire}\pi d_{wire}^2}{4 w_{cable} t_{cable} \cos\psi_{cable}}$$

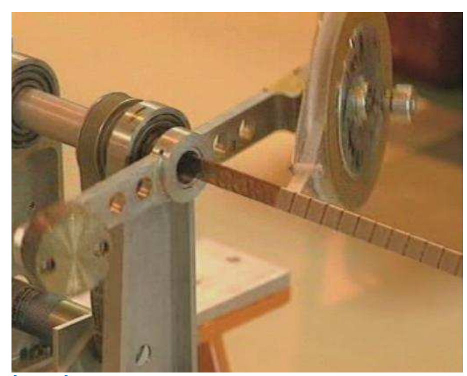
  – i.e the ratio of the sum of the cross-sectional area of the strands (in the direction parallel to the cable axis) to the cross-sectional area of the cable.
- Typical cable compaction: from 88% (Tevatron) to 92.3% (HERA).

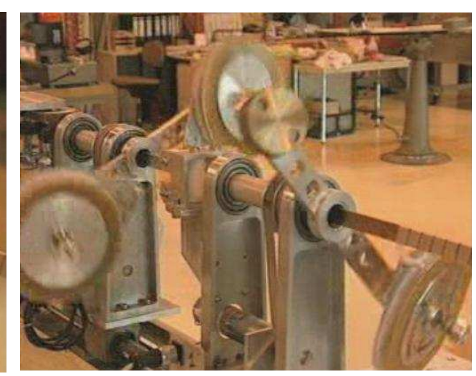


Cable insulation

- The cable insulation must feature
  - Good electrical properties to withstand high turn-to-turn voltage after a quench.
  - Good mechanical properties to withstand high pressure conditions
  - Porosity to allow penetration of helium (or epoxy)
  - Radiation hardness
- In NbTi magnets the most common insulation is a series of overlapped layers of polyimide (kapton).
- In the LHC case:
  - two polyimide layers 50.8 μm thick wrapped around the cable with a 50% overlap, with another adhesive polyimide tape 68.6 μm thick wrapped with a spacing of 2 mm.

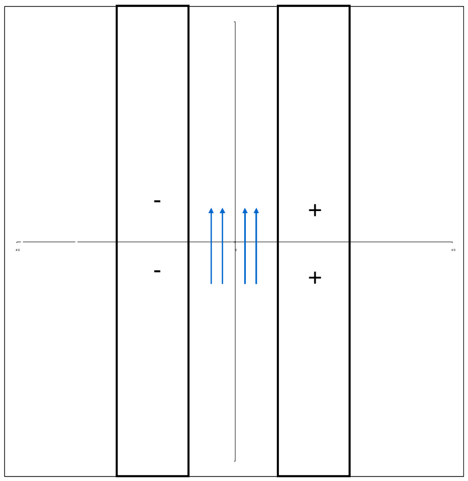




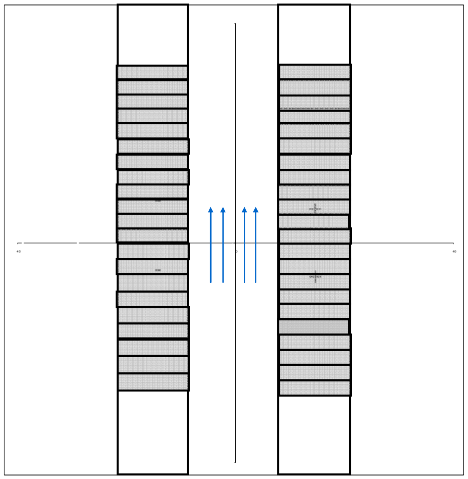


Superconducting Magnets Design
Perfect dipole

1 - Wall dipole (similar to the window frame magnet)



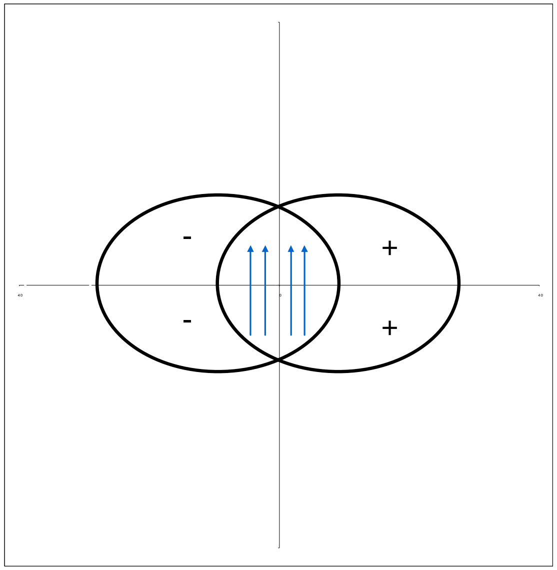
A wall-dipole, cross-section



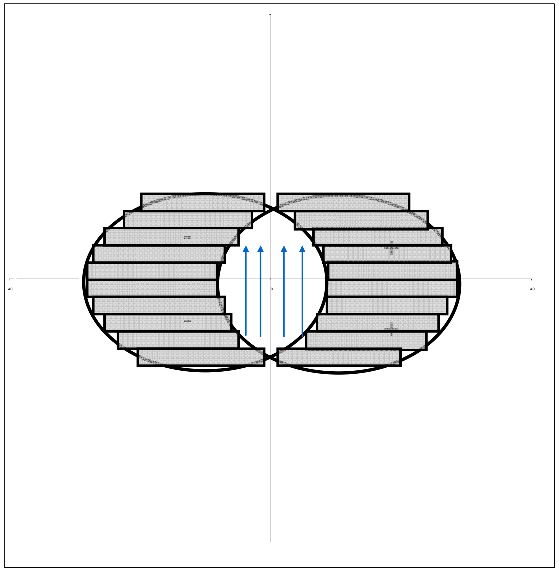
A practical winding with flat cables

Superconducting Magnets Design
Perfect dipole

2 - Intersecting ellipsis



Intersecting ellipses



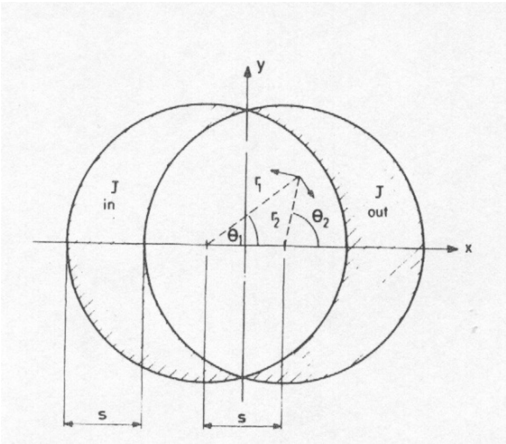
A practical (?) winding with flat cables

Intersecting Cylinders

within a cylinder carrying uniform current j0, the field is perpendicular to the radial direction and proportional to the distance to the center r:

$$B = -\frac{\mu_0 j_0 r}{2}$$

Combining the effect of the two cylinders



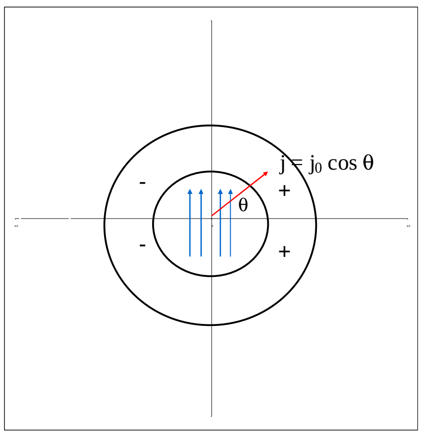
$$B_x = \frac{\mu_0 j_0 r}{2}\{-r_1 \sin\theta_1 + r_2 \sin\theta_2\} = 0$$

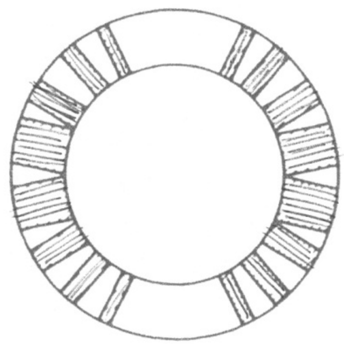
$$B_y = \frac{\mu_0 j_0 r}{2}\{-r_1 \cos\theta_1 + r_2 \cos\theta_2\} = -\frac{\mu_0 j_0}{2} s$$

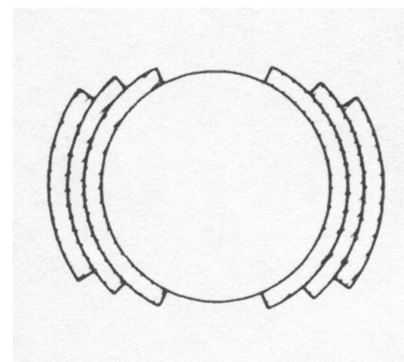
Similar proof for intersecting ellipses

Superconducting Magnets Design
Perfect dipole

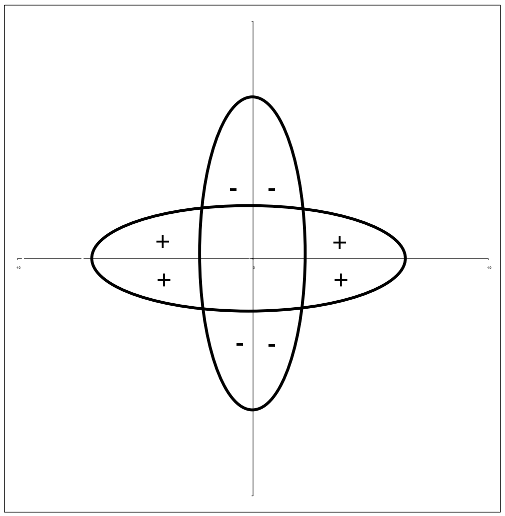
3 – Cos(θ) current distribution



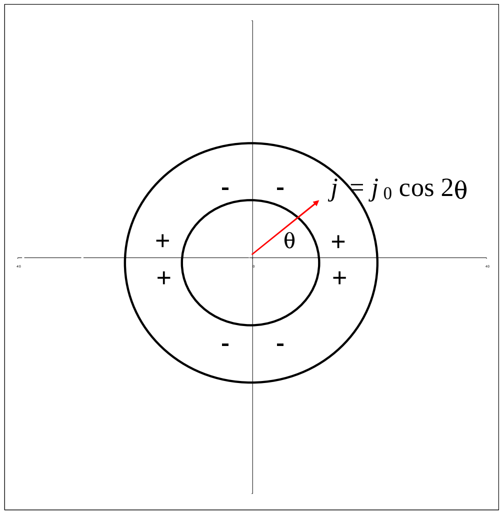




Superconducting Magnets Design
Perfect quadrupole

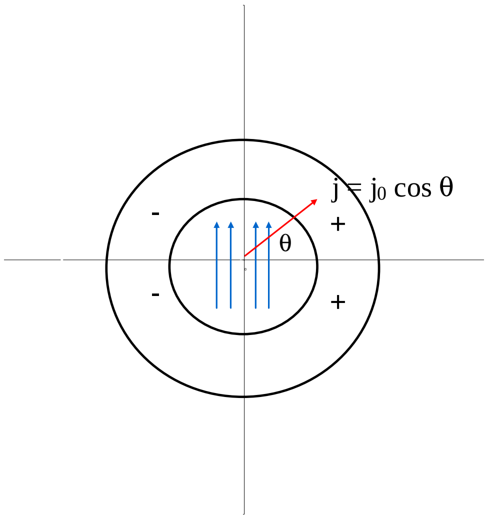


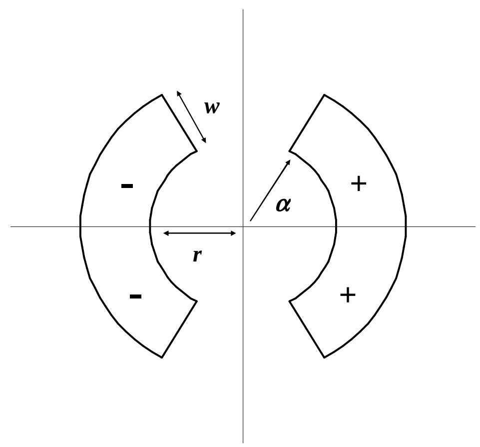
Quadrupole as two intersecting ellipses



Quadrupole as an ideal cos2θ

Dipole design using sector coils





$$B(z) = \sum_{n=1}^{\infty} C_n \left(\frac{z}{R_{ref}} \right)^{n-1}$$

$$C_n = -\frac{I\mu_0}{2\pi R_{ref}} \left(\frac{R_{ref}}{z_0} \right)^n$$

$$B_1 = -\frac{I\mu_0}{2\pi} \mathrm{Re}\left(\frac{1}{z_0} \right) = -\frac{I\mu_0}{2\pi} \frac{\cos\theta}{|z_0|} \qquad I \to j\rho d\rho d\theta$$

$$B_1 = -2\frac{j\mu_0}{2\pi} \int_{-\alpha}^{\alpha} \int_{r}^{r+w} \frac{\cos\theta}{\rho} \rho d\rho d\theta = -\frac{2j\mu_0}{\pi} w \sin\alpha$$

Multipoles of a dipole sector coil

$$C_n = -2\frac{j\mu_0 R_{ref}^{n-1}}{2\pi}\int_{-\alpha}^{\alpha}\int_{r}^{r+w}\frac{\exp(-in\theta)}{\rho^n}\rho d\rho d\theta = -\frac{j\mu_0 R_{ref}^{n-1}}{\pi}\int_{-\alpha}^{\alpha}\exp(-in\theta)d\theta\int_{r}^{r+w}\rho^{1-n}d\rho$$

for n = 2

$$B_2 = -\frac{j\mu_0 R_{ref}}{\pi}\sin(2\alpha)\log\left(1+\frac{w}{r}\right)$$

for n > 2

$$B_n = -\frac{j\mu_0 R_{ref}^{n-1}}{\pi}\frac{2\sin(\alpha n)}{n}\frac{(r+w)^{2-n}-r^{2-n}}{2-n}$$

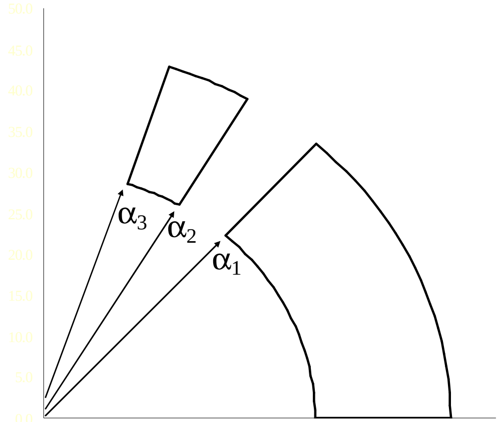
$$B_3 = \frac{\mu_0 j R_{ref}^2}{\pi}\frac{\sin(3\alpha)}{3}\left(\frac{1}{r}-\frac{1}{r+w}\right)$$

$$B_5 = \frac{\mu_0 j R_{ref}^4}{\pi}\frac{\sin(5\alpha)}{5}\left(\frac{1}{r^3}-\frac{1}{(r+w)^3}\right)$$

for α=π/3 (60°) B_3=0

for α=π/5 (36°) or for α=2π/5 (72°) B_5=0

Multi-sector dipole coil



$$B_3 = \frac{\mu_0 j R_{ref}^2}{\pi} \frac{\sin 3\alpha_3 - \sin 3\alpha_2 + \sin 3\alpha_1}{3} \left(\frac{1}{r} - \frac{1}{r+w} \right)$$

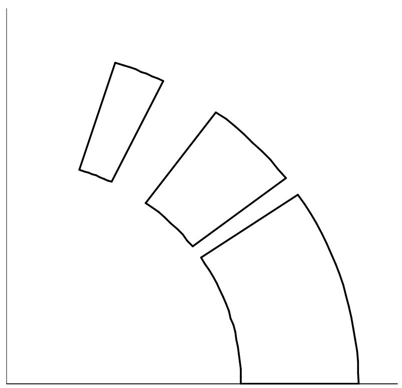
$$B_5 = \frac{\mu_0 j R_{ref}^4}{\pi} \frac{\sin 5\alpha_3 - \sin 5\alpha_2 + \sin 5\alpha_1}{5} \left(\frac{1}{r^3} - \frac{1}{(r+w)^3} \right)$$

$$\begin{cases} \sin(3\alpha_3) - \sin(3\alpha_2) + \sin(3\alpha_1) = 0 \\ \sin(5\alpha_3) - \sin(5\alpha_2) + \sin(5\alpha_1) = 0 \end{cases}$$

(48°,60°,72°) or (36°,44°,64°) are some of the possible solutions

[0°-43.2°, 52.2°-67.3°] sets also B_7 = 0 !

Multi-sector dipole coil



$(B_3$, B_5 and $B_7) = 0$

$$\sin(3\alpha_5) - \sin(3\alpha_4) + \sin(3\alpha_3) - \sin(3\alpha_2) + \sin(3\alpha_1) = 0$$

$$\sin(5\alpha_5) - \sin(5\alpha_4) + \sin(5\alpha_3) - \sin(5\alpha_2) + \sin(5\alpha_1) = 0$$

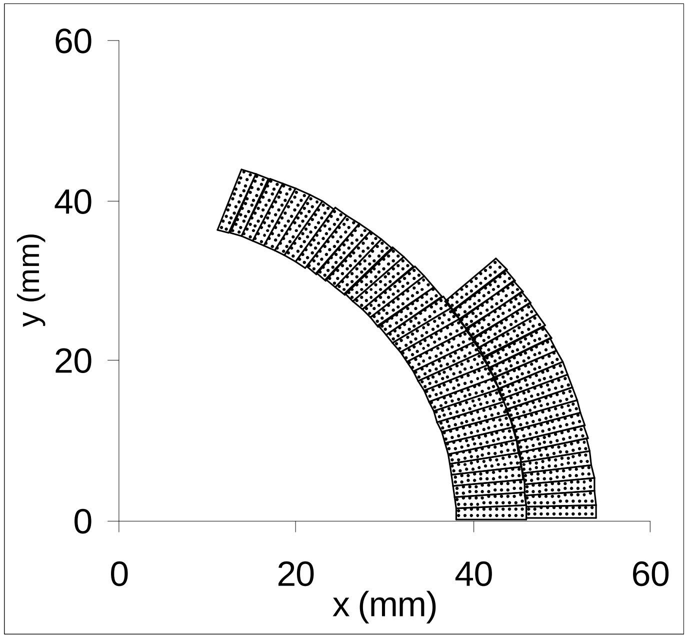
$$\sin(7\alpha_5) - \sin(7\alpha_4) + \sin(7\alpha_3) - \sin(7\alpha_2) + \sin(7\alpha_1) = 0$$

$$\sin(9\alpha_5) - \sin(9\alpha_4) + \sin(9\alpha_3) - \sin(9\alpha_2) + \sin(9\alpha_1) = 0$$

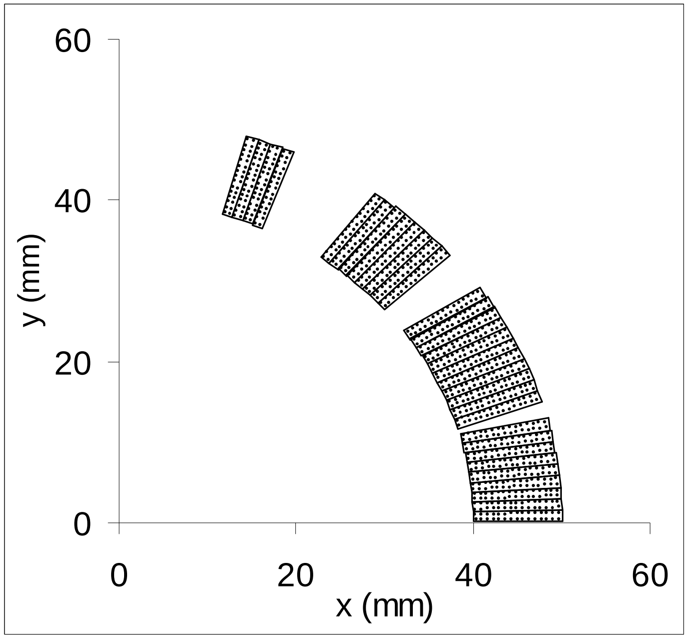
$$\sin(11\alpha_5) - \sin(11\alpha_4) + \sin(11\alpha_3) - \sin(11\alpha_2) + \sin(11\alpha_1) = 0$$

[0°-33.3°, 37.1°- 53.1°, 63.4°- 71.8°] sets (B3, B5, B7 , B9 and B11) = 0!

Examples



Tevatron main dipole - 1980

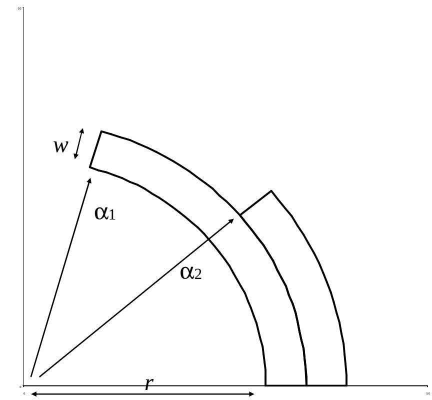


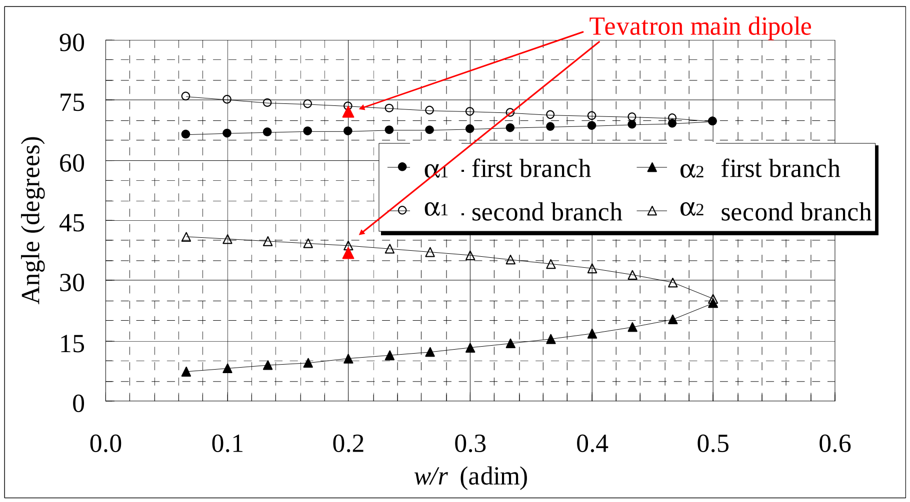
RHIC main dipole - 1995

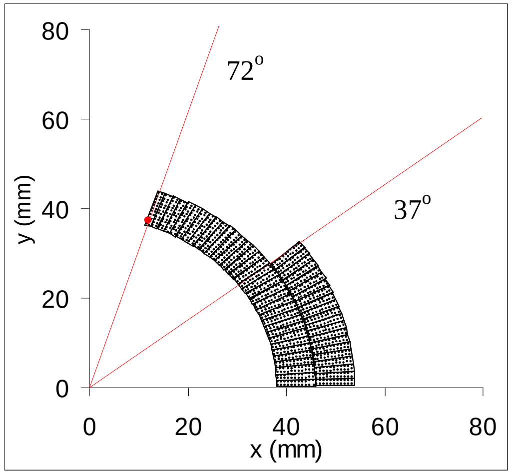
Two layer design

$$B_3 \propto \sin(3\alpha_1)\left(\frac{1}{r}-\frac{1}{r+w}\right)+\sin(3\alpha_2)\left(\frac{1}{r+w}-\frac{1}{r+2w}\right)$$

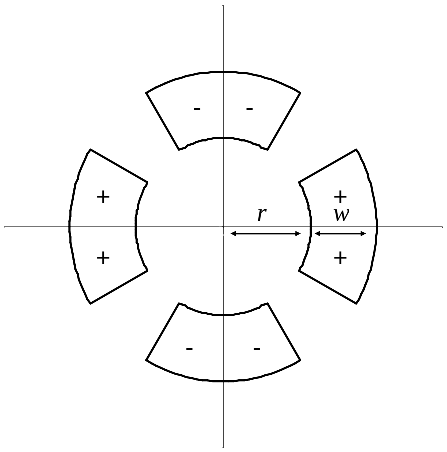
$$B_5 \propto \sin(5\alpha_1)\left(\frac{1}{r^3}-\frac{1}{(r+w)^3}\right)+\sin(5\alpha_2)\left(\frac{1}{(r+w)^3}-\frac{1}{(r+2w)^3}\right)$$







Quadrupole design using sector coils



$$B(z) = \sum_{n=1}^{\infty} C_n \left(\frac{z}{R_{ref}} \right)^{n-1}$$

$$C_n = -\frac{I\mu_0}{2\pi R_{ref}} \left(\frac{R_{ref}}{z_0} \right)^n$$

$$B_2 = -\frac{I\mu_0 R_{ref}}{2\pi} \mathrm{Re}\left(\frac{1}{z_0^2} \right) = -\frac{I\mu_0 R_{ref}}{2\pi} \frac{\cos 2\theta}{\left| z_0^2 \right|} \qquad I \rightarrow j\rho d\rho d\theta$$

$$B_2 = -8 \frac{j\mu_0 R_{ref}}{2\pi} \int_0^{\alpha} \int_r^{r+w} \frac{\cos 2\theta}{\rho^2} \rho d\rho d\theta = -\frac{4 j\mu_0 R_{ref}}{\pi} [\sin 2\alpha] \ln\left(1 + \frac{w}{r} \right)$$

Multipoles of a quadrupole sector coil

$$B_6 = \frac{\mu_0 j R_{ref}^5}{\pi} \frac{\sin(6\alpha)}{6} \left(\frac{1}{r^4} - \frac{1}{(r+w)^4} \right)$$

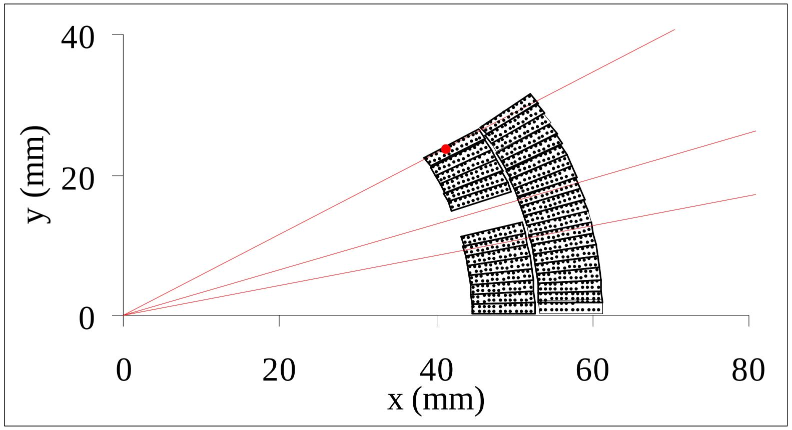
for $\alpha=\pi/6$ (30°) one has $B_6 = 0$

$$B_{10} = \frac{\mu_0 j R_{ref}^8}{\pi} \frac{\sin(10\alpha)}{10} \left(\frac{1}{r^8} - \frac{1}{(r+w)^8} \right)$$

for $\alpha=\pi/10$ (18°) or $\alpha=\pi/5$ (36°) one sets $B_{10} = 0$

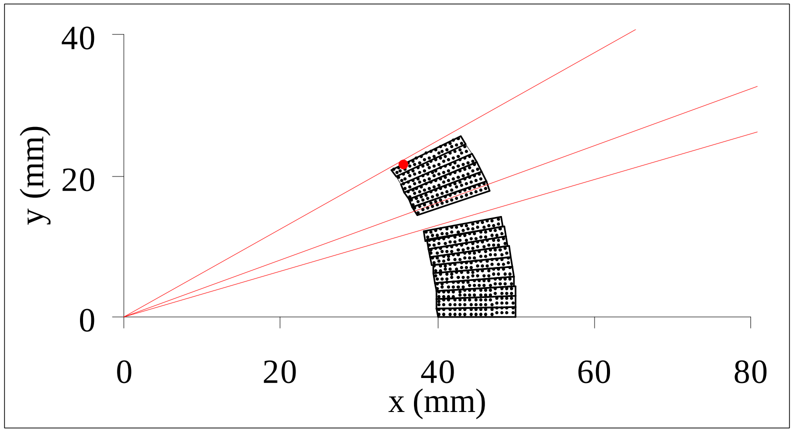
It follows the same philosophy of the Dipole design!

Examples



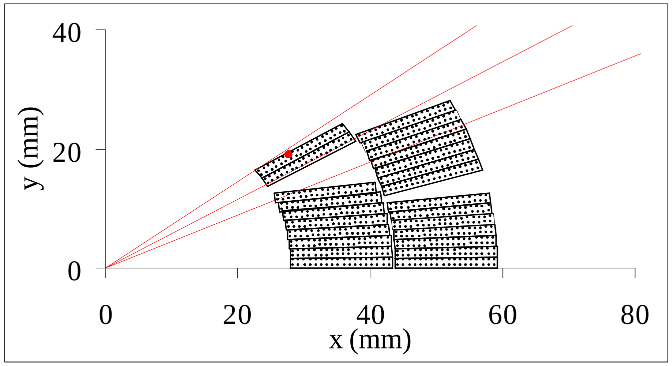
Tevatron main quadrupole

~[0°-12°, 18°-30°]



RHIC main quadrupole

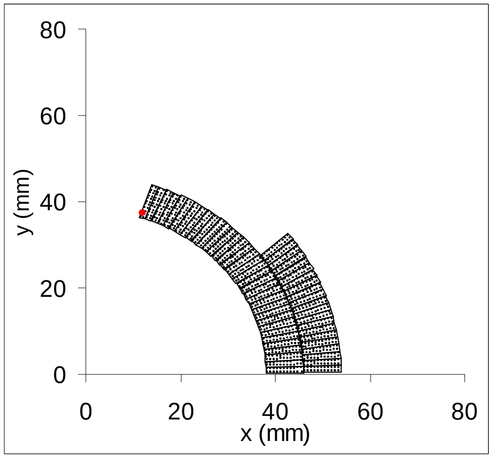
~[0°-18°, 22°-32°]



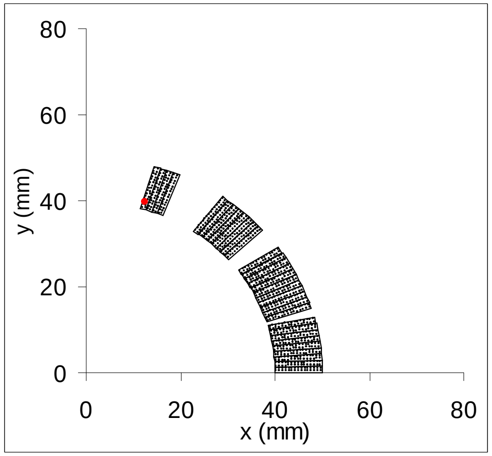
LHC main quadrupole

~[0°-24°, 30°-36°]

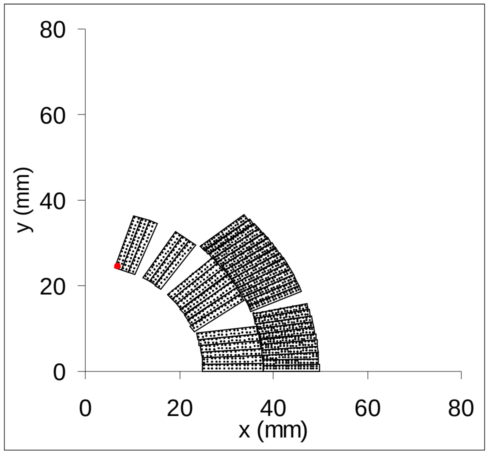
Peak field and bore field ratio (λ)



Tevatron main dipole –
location of the peak field

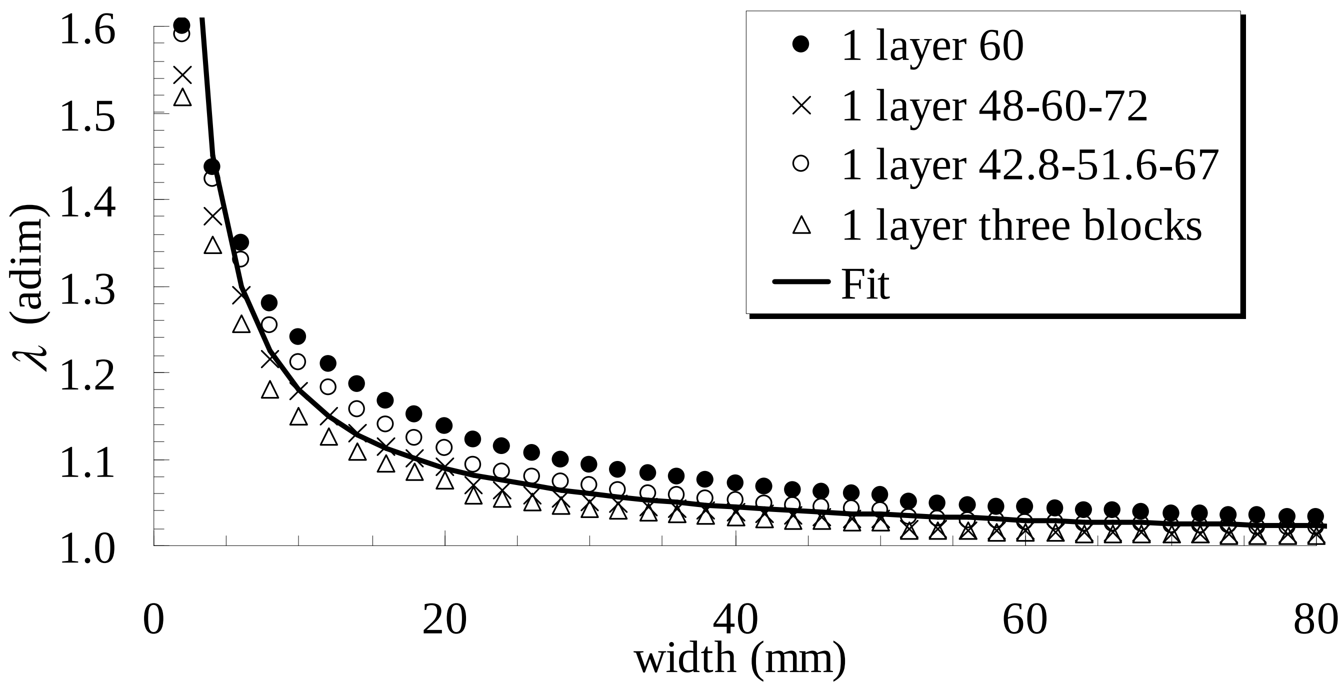


RHIC main dipole –
location of the peak field



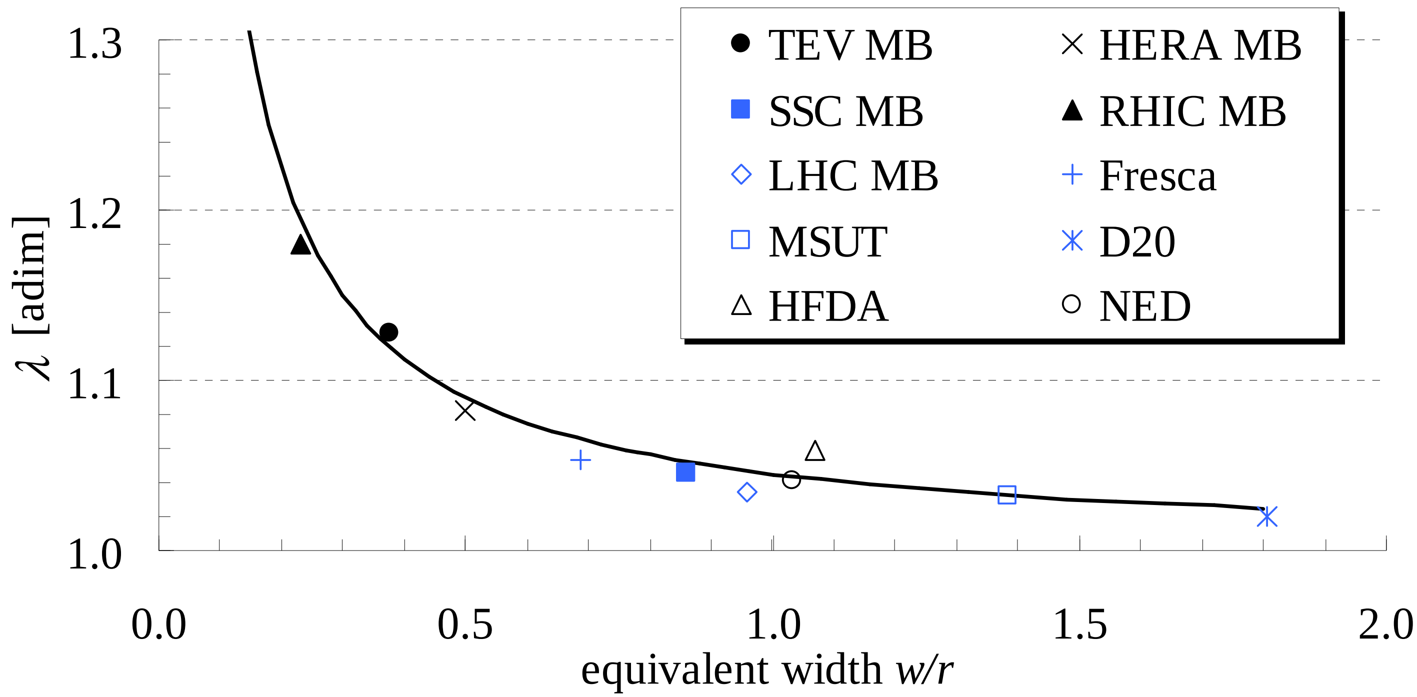
LHC main dipole –
location of the peak field

Peak field and bore field ratio (λ)



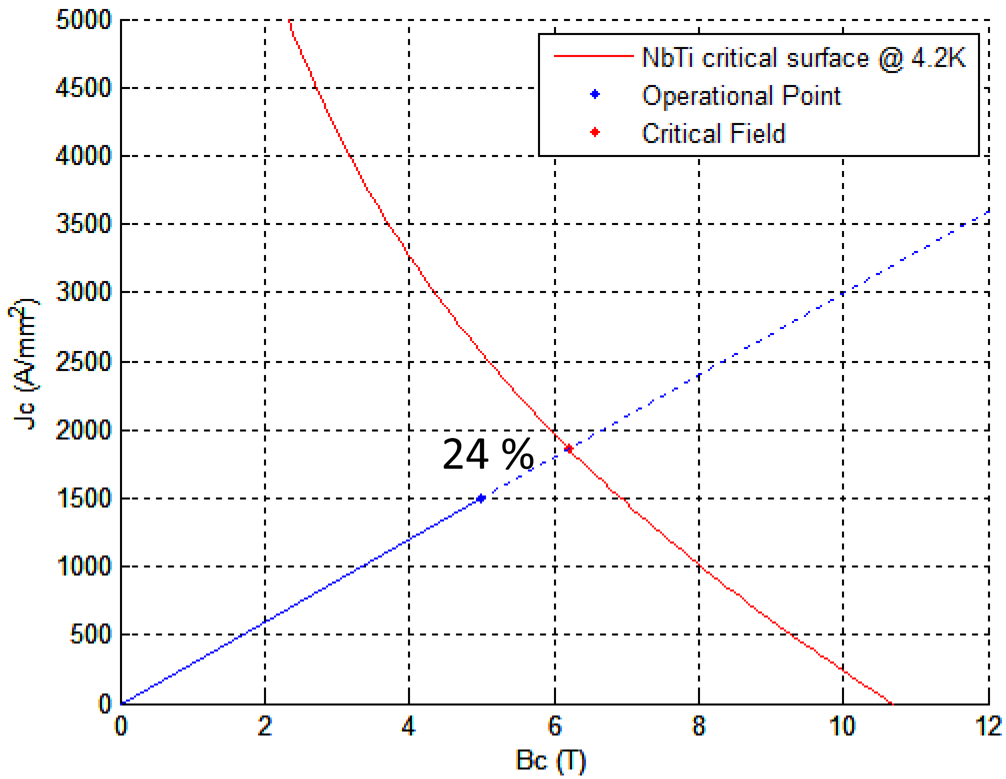
$$\lambda(w,r) \sim 1 + \frac{ar}{w} \qquad a \sim 0.045$$

Examples

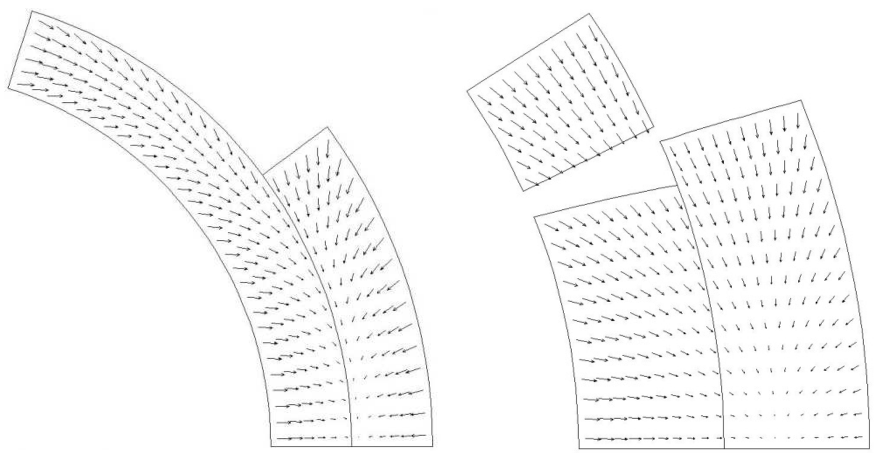


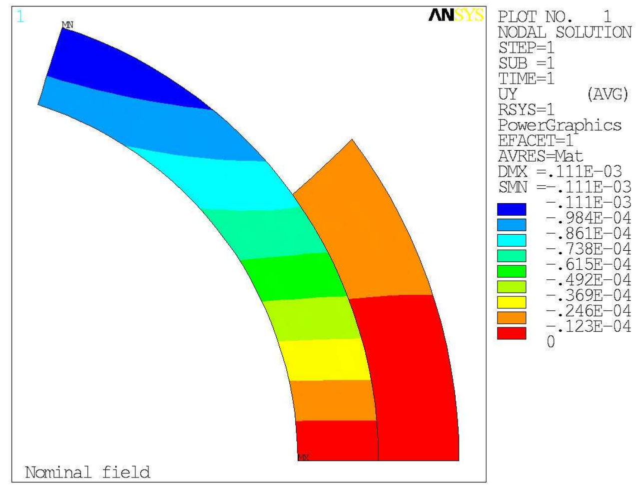
1.3
1.2
1.1
1.0
λ [adim]
0.0
0.5
1.0
1.5
2.0
equivalent width *w/r*
TEV MB
SSC MB
LHC MB
MSUT
HFDA
HERA MB
RHIC MB
Fresca
D20
NED

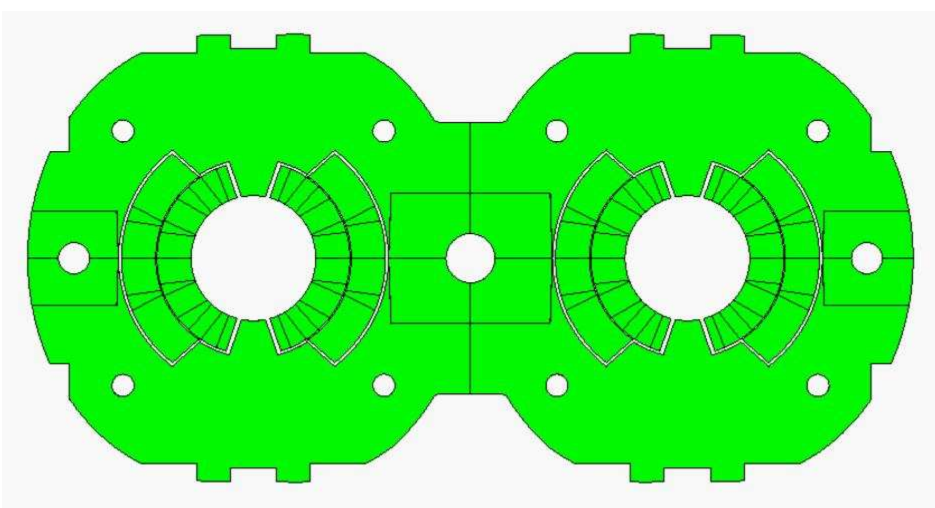
Operational Margin

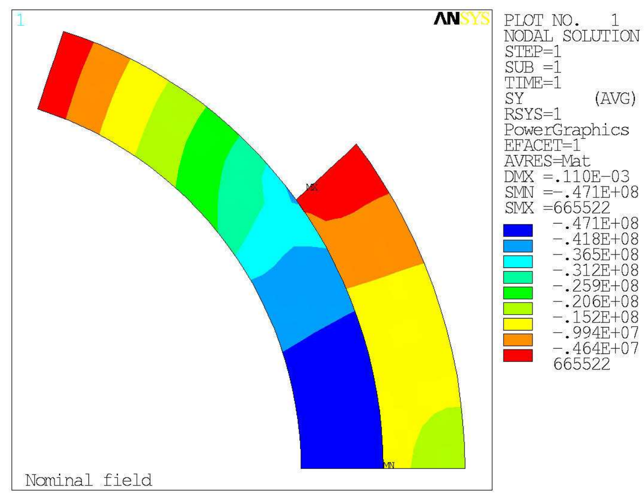


Lorentz Forces









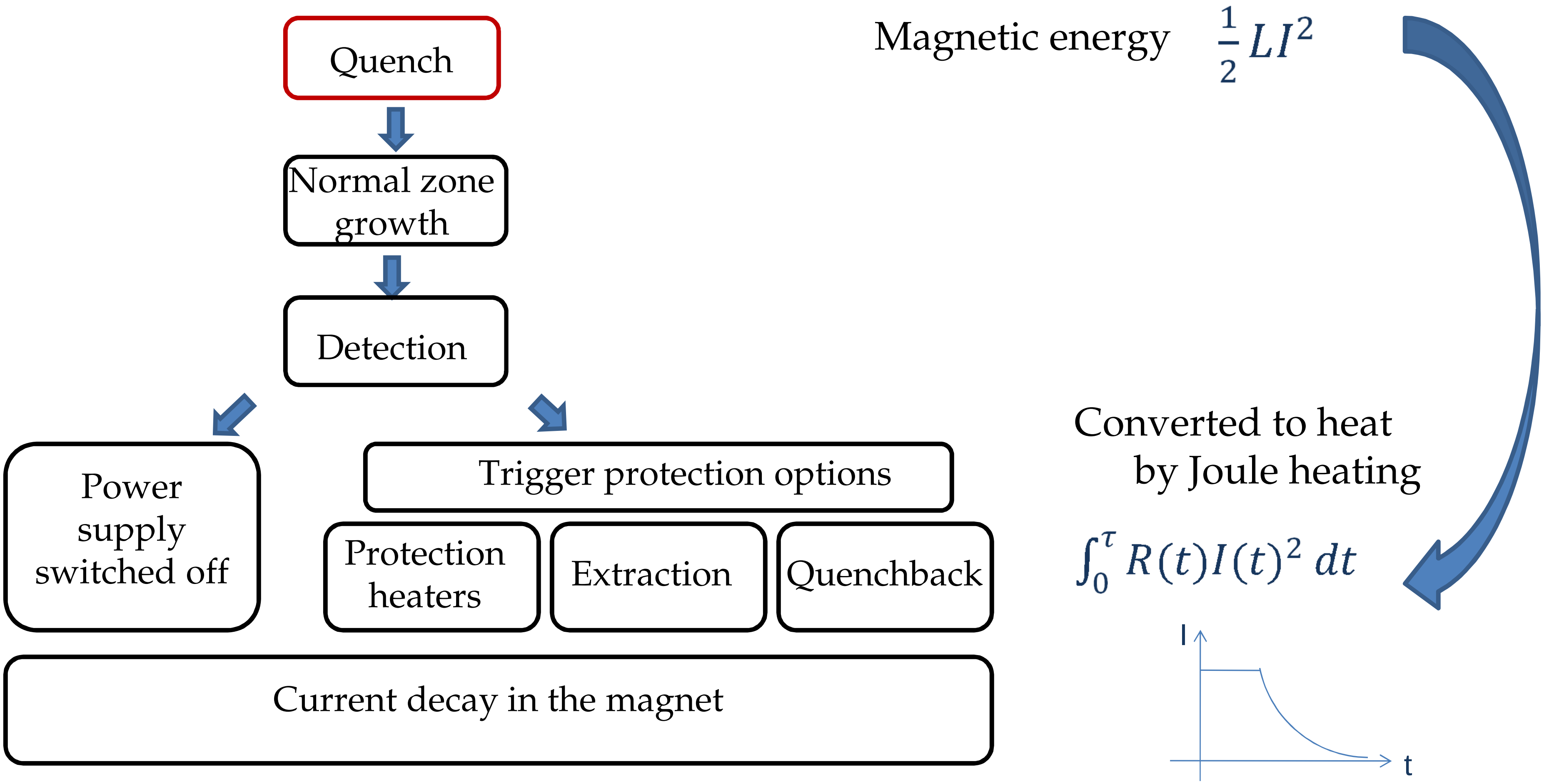
Quench protection

- A superconducting accelerator magnet has a large magnetic stored energy
- A quench produces a resistive zone
- Current is flowing through the magnet

Joule Heating
Voltages (R and L)

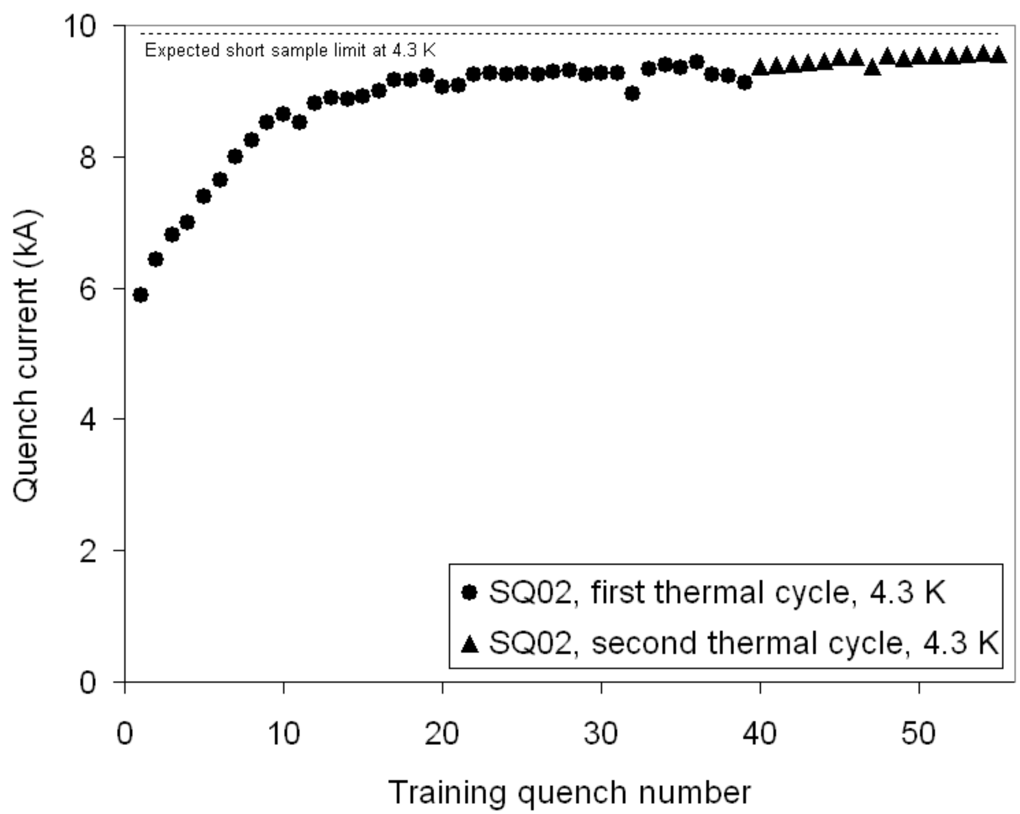
- The challenge of the protection is to provide a safe conversion of the magnetic energy to heat in order to minimize
  - Peak temperature ("hot spot") and temperature gradients in the magnet
  - Peak voltages
- The final goal being to avoid any magnet degradation
  - High temperature => damage to the insulation or stabilizer
  - Large temperature gradient => damage to the conductor due to differential thermal expansion of materials

General quench protection diagram



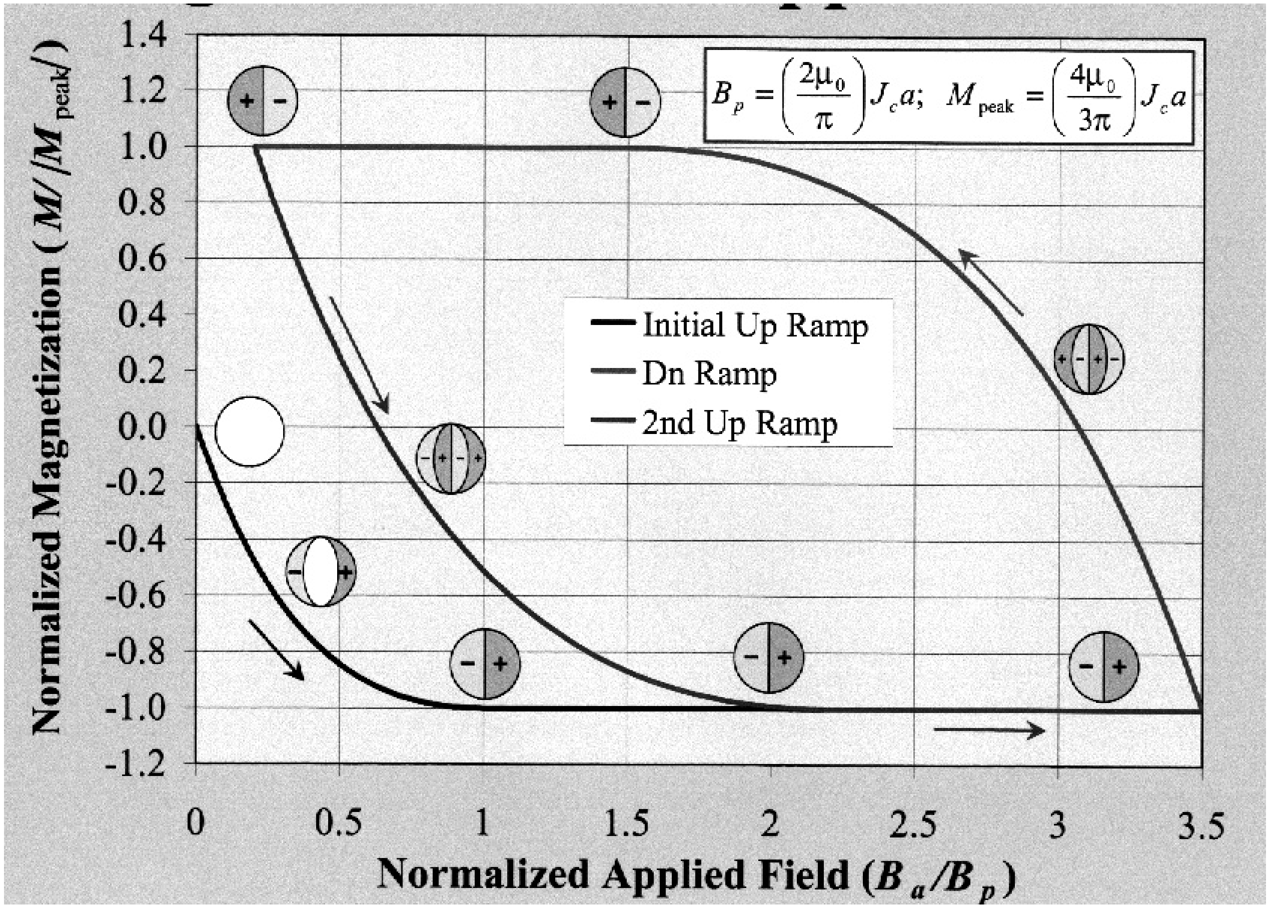
The faster this chain happens the safer is the magnet

Training

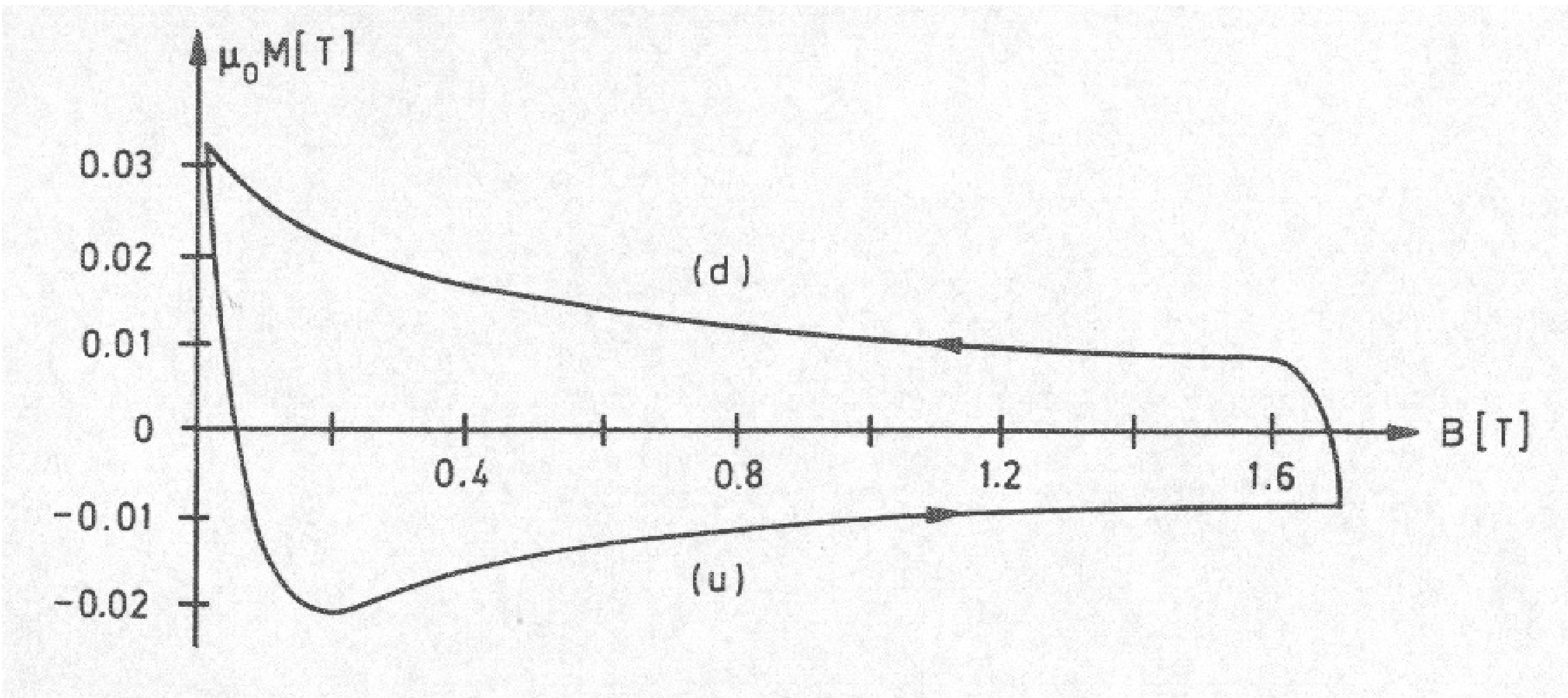


Expected short sample limit at 4.3 K
SQ02, first thermal cycle, 4.3 K
SQ02, second thermal cycle, 4.3 K
Quench current (kA)
Training quench number
0
2
4
6
8
10
0
10
20
30
40
50

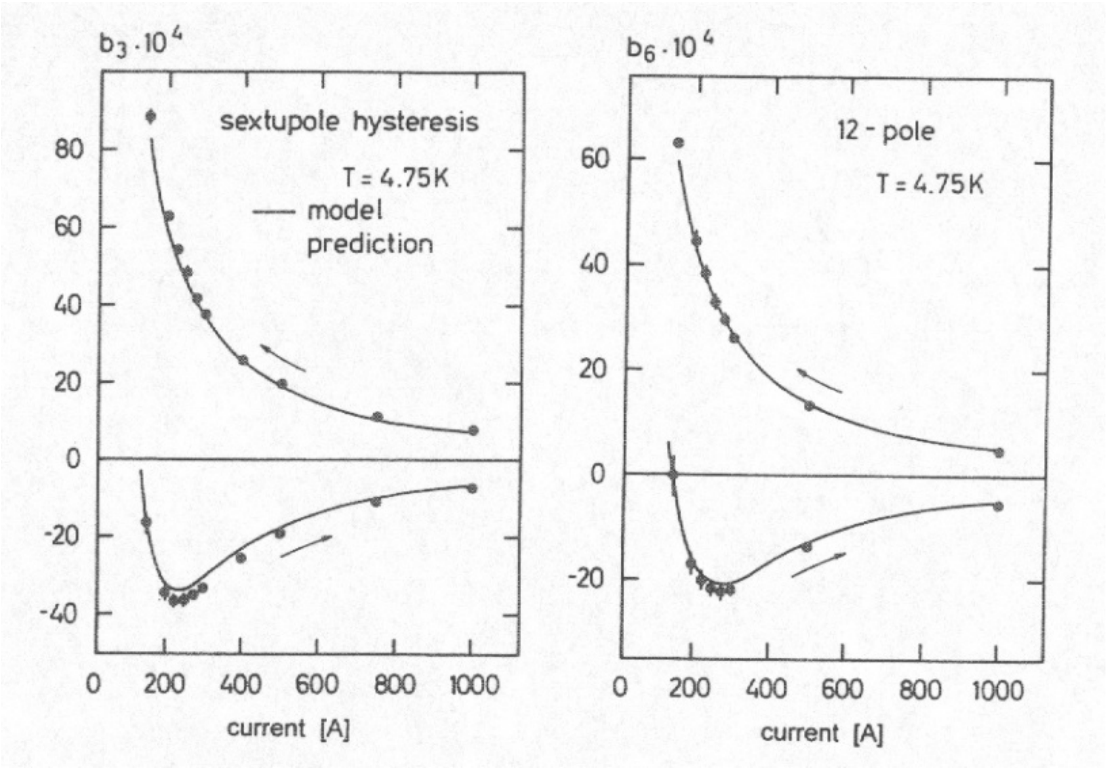
Magnetization



Magnetization



Magnetization



Summary

- Design and Fabrication of Superconducting Magnets belong to a different Universe

- Although the mathematical formulation for the field generation is shared, the design of superconducting magnets involves many other aspects:
  - Thermal considerations
  - Mechanical Analysis
  - Fabrication techniques
  - Quench Protection
  - Material Science

- If one is interested to learn more about superconducting magnet, one should attend to the Superconducting Accelerator Magnets USPAS course. The material for that course can be found at:

  http://etodesco.web.cern.ch/etodesco/uspas/uspas.html

Next…

Unusual design examples