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Introduction to superconducting magnets*

Mauricio Lopes – FNAL

* From: “Superconducting Accelerator Magnets” by Paolo Ferracin, Ezio Todesco, Soren O. Prestemon and Helene Felice, January 2012
A Brief History of the Superconductivity

Heike Kamerlingh Onne

1908 – Successfully liquified helium (4.2 K)
1911 – Discovered the superconductivity while measuring the conductivity of Mercury as function of temperature
1913 – Nobel prize
A Brief History of the Superconductivity

1933 – Walther Meissner and Robert Ochsenfeld discover perfect diamagnetic property of superconductors.
1935 – First theoretical works on SC by Heinz and Fritz London
1950 – Ginzburg and Landau proposed a macroscopic theory for SC.

Meissner effect
Why using SC magnets?

\[ Br = \frac{P}{q} = \frac{\sqrt{K^2 + 2KE_0}}{qc} \]

Example: Let's calculate the magnetic rigidity for a 1 TeV proton:

\[ Br \approx \frac{1 \text{ TeV}}{c} \approx 3333 \, \text{T.m} \]

Let us assume a maximum field of 1.5 T; the circumference of such machine will be:

\[ r = 2222 \, m \]
\[ C = 2\pi r \approx 14 \, km \]

The Tevatron was the first machine to use large scale superconductor magnets with a 4.2 T in a 6.3 km circumference!
Critical surface

- Magnetic Field, B
- Superconducting Interior Volume
- Critical Surface
- Current Density, J
- Temperature, T
Critical surface for different SC materials

![Graph showing critical current density vs. applied field for various superconductor materials.](image-url)

- **YBCO B||c**: Used in NHMFL tested Insert Coil 2007
- **YBCO B||ab**: Used in NHMFL tested Insert Coil 2007
- **Bi-2212**: 427 fil. round wire, Ag/SC=3 (Hasegawa ASC-2000/MT17-2001)
- **Nb-Ti**: Max @1.9 K for whole LHC NbTi strand production (CERN, Boutilou '07)
- **Nb-Ti**: Nb+7%Ti, 1.8 K, Lee, Naus and Larbalestier UW-ASC'96
- **Nb3Sn**: Non-Cu Jc Internal Sn OI-ST RRP 1.3 mm, ASC'02/ICMC'03
- **Nb3Sn**: Bronze route int. stab. -VAC-HP, non- (Cu+Ta) Jc, Thoener et al., Erice '96.
- **Nb3Sn**: 1.8 K Non-Cu Jc Internal Sn OI-ST RRP 1.3 mm, ASC'02/ICMC'03
- **MgB2**: 4.2 K “high oxygen” film 2, Eom et al. (UW) Nature 31 May '02
- **MgB2**: Tape - Columbus (Grasso) MEM'06

At 4.2 K Unless Otherwise Stated
NbTi Parameterization

\[ B_c(T) = B_{c0} \left[ 1 - \left( \frac{T}{T_{c0}} \right)^{1.7} \right] \]  
(Lubell’s formula)

where \( B_{c0} \) is the critical field at zero temperature (\( B_{c0} \approx 14.5 \, \text{T} \))

\[ \frac{J_c(B,T)}{J_{c\_ref}} = C \left( \frac{B}{B_c} \right)^\alpha \left( 1 - \frac{B}{B_c} \right)^\beta \left[ 1 - \left( \frac{T}{T_{c0}} \right)^{1.7} \right] ^\gamma \]  
(Bottura’s formula)

where \( J_{c\_ref} \) is the critical current density at 4.2 K and 5 T (\( J_{c\_ref} \approx 3000 \, \text{A/mm}^2 \)); \( C, \alpha, \beta \) and \( \gamma \) are fitting parameters:

- \( C \approx 31.4 \, \text{T} \)
- \( \alpha \approx 0.63 \)
- \( \beta \approx 1.0 \)
- \( \gamma \approx 2.3 \)
\[ J_c(B, T, \varepsilon) = \frac{C(\varepsilon)}{\sqrt{B}} \left( 1 - \frac{B}{B_c(T, \varepsilon)} \right)^2 \left[ 1 - \left( \frac{T}{T_{c0}(\varepsilon)} \right)^2 \right]^2 \] (Summer's formula)

\[ \frac{B_c(T, \varepsilon)}{B_{c0}} = \left[ 1 - \left( \frac{T}{T_{c0}(\varepsilon)} \right)^2 \right] \left[ 1 - 0.31 \left( \frac{T}{T_{c0}(\varepsilon)} \right)^2 \right] \left[ 1 - 1.77 Ln \left( \frac{T}{T_{c0}(\varepsilon)} \right) \right] \]

where:

\[ C(\varepsilon) = C_{0\_m} \left( 1 - \alpha |\varepsilon|^{1.7} \right)^{1/2} \]

\[ B_c(T, \varepsilon) = B_{c0\_m} \left( 1 - \alpha |\varepsilon|^{1.7} \right) \]

\[ T_{c0}(\varepsilon) = T_{c0\_m} \left( 1 - \alpha |\varepsilon|^{1.7} \right)^{1/3} \]

and:

\[ \alpha = 900 \]

\[ \varepsilon = -0.003 \]

\[ T_{c0\_m} = 18K \]

\[ C_{0\_m} = 48500 \text{ A/mm}^2 \]

(for \( J_c = 3000 \text{ A/mm}^2 @ 4.2 \text{ K and 12 T} \)
Strand Fabrication
Superconducting cables

- Most of the superconducting coils for particle accelerators are wound from a multi-strand cable.
- The advantages of a multi-strand cable are:
  - reduction of the strand piece length;
  - reduction of number of turns
    - easy winding;
    - smaller coil inductance
      - less voltage required for power supply during ramp-up;
      - after a quench, faster current discharge and less coil voltage.
  - current redistribution in case of a defect or a quench in one strand.
- The strands are twisted to
  - reduce interstrand coupling currents (see interfilament coupling currents)
    - Losses and field distortions
      - provide more mechanical stability
- The most commonly used multi-strand cables are the Rutherford cable and the cable-in-conduit.
Superconducting cables

- Rutherford cables are fabricated by a cabling machine.
  - Strands are wound on spools mounted on a rotating drum.
  - Strands are twisted around a conical mandrel into an assembly of rolls (Turk’s head). The rolls compact the cable and provide the final shape.
Superconducting cables

- The final shape of a Rutherford cable can be rectangular or trapezoidal.
- The cable design parameters are:
  - Number of wires $N_{wire}$
  - Wire diameter $d_{wire}$
  - Cable mid-thickness $t_{cable}$
  - Cable width $w_{cable}$
  - Pitch length $p_{cable}$
  - Pitch angle $\psi_{cable}$ ($\tan \psi_{cable} = \frac{2 w_{cable}}{p_{cable}}$)
  - Cable compaction (or packing factor) $k_{cable}$

\[
k_{cable} = \frac{N_{wire} \pi d_{wire}^2}{4 w_{cable} t_{cable} \cos \psi_{cable}}
\]

- i.e the ratio of the sum of the cross-sectional area of the strands (in the direction parallel to the cable axis) to the cross-sectional area of the cable.
- Typical cable compaction: from 88% (Tevatron) to 92.3% (HERA).
Cable insulation

• The cable insulation must feature
  – Good electrical properties to withstand high turn-to-turn voltage after a quench.
  – Good mechanical properties to withstand high pressure conditions
  – Porosity to allow penetration of helium (or epoxy)
  – Radiation hardness
• In NbTi magnets the most common insulation is a series of overlapped layers of polyimide (kapton).
• In the LHC case:
  – two polyimide layers 50.8 µm thick wrapped around the cable with a 50% overlap, with another adhesive polyimide tape 68.6 µm thick wrapped with a spacing of 2 mm.
Superconducting Magnets Design

Perfect dipole

1 - Wall dipole (similar to the window frame magnet)

A wall-dipole, cross-section

A practical winding with flat cables
Superconducting Magnets Design
Perfect dipole

2 - Intersecting ellipses

Intersecting ellipses

A practical (?) winding with flat cables
within a cylinder carrying uniform current \( j_0 \), the field is perpendicular to the radial direction and proportional to the distance to the center \( r \):

\[
B = -\frac{\mu_0 j_0 r}{2}
\]

Combining the effect of the two cylinders

\[
B_x = \frac{\mu_0 j_0 r}{2} \{- r_1 \sin \theta_1 + r_2 \sin \theta_2 \} = 0
\]

\[
B_y = \frac{\mu_0 j_0 r}{2} \{- r_1 \cos \theta_1 + r_2 \cos \theta_2 \} = -\frac{\mu_0 j_0}{2} s
\]

Similar proof for intersecting ellipses
Superconducting Magnets Design
Perfect dipole

$3 - \cos(\theta)$ current distribution
Superconducting Magnets Design
Perfect quadrupole

Quadrupole as two intersecting ellipses

Quadrupole as an ideal $\cos 2\theta$

$j = j_0 \cos 2\theta$
Dipole design using sector coils

\[ B(z) = \sum_{n=1}^{\infty} C_n \left( \frac{z}{R_{\text{ref}}} \right)^{n-1} \]

\[ C_n = -\frac{I\mu_0}{2\pi R_{\text{ref}}} \left( \frac{R_{\text{ref}}}{z_0} \right)^n \]

\[ B_1 = -\frac{I\mu_0}{2\pi} \text{Re} \left( \frac{1}{z_0} \right) = -\frac{I\mu_0}{2\pi} \frac{\cos \theta}{|z_0|} \quad I \rightarrow j\rho d\rho d\theta \]

\[ B_1 = -2\frac{j\mu_0}{2\pi} \int_{-\alpha}^{\alpha} \int_{r}^{r+w} \frac{\cos \theta}{\rho} \rho d\rho d\theta = -\frac{2j\mu_0}{\pi} w \sin \alpha \]
Multipoles of a dipole sector coil

\[ C_n = -2 \frac{j \mu_0 R_{ref}^{n-1}}{2\pi} \int_{-\alpha}^{\alpha} \int_r^{r+w} \frac{\exp(-in\theta)}{\rho^n} \rho d\rho d\theta = -\frac{j \mu_0 R_{ref}^{n-1}}{\pi} \int_{-\alpha}^{\alpha} \exp(-in\theta) d\theta \int_r^{r+w} \rho^{1-n} d\rho \]

for \( n = 2 \)

\[ B_2 = -\frac{j \mu_0 R_{ref}}{\pi} \sin(2\alpha) \log \left( 1 + \frac{w}{r} \right) \]

for \( n > 2 \)

\[ B_n = -\frac{j \mu_0 R_{ref}^{n-1}}{\pi} \frac{2 \sin(n\alpha) (r + w)^{2-n} - r^{2-n}}{n \ 2-n} \]

\[ B_3 = \frac{\mu_0 j R_{ref}^2}{\pi} \frac{\sin(3\alpha)}{3} \left( \frac{1}{r} - \frac{1}{r+w} \right) \]

\[ B_5 = \frac{\mu_0 j R_{ref}^4}{\pi} \frac{\sin(5\alpha)}{5} \left( \frac{1}{r^3} - \frac{1}{(r+w)^3} \right) \]

for \( \alpha = \pi/3 \ (60^\circ) \ B_3 = 0 \)

for \( \alpha = \pi/5 \ (36^\circ) \) or for \( \alpha = 2\pi/5 \ (72^\circ) \ B_5 = 0 \)
Multi-sector dipole coil

\[
B_3 = \frac{\mu_0 j R_{ref}^2}{\pi} \sin 3\alpha_3 - \sin 3\alpha_2 + \sin 3\alpha_1 \left( \frac{1}{r} - \frac{1}{r + w} \right) \\
B_5 = \frac{\mu_0 j R_{ref}^4}{\pi} \sin 5\alpha_3 - \sin 5\alpha_2 + \sin 5\alpha_1 \left( \frac{1}{r^3} - \frac{1}{(r + w)^3} \right)
\]

\[
\begin{align*}
\sin(3\alpha_3) - \sin(3\alpha_2) + \sin(3\alpha_1) &= 0 \\
\sin(5\alpha_3) - \sin(5\alpha_2) + \sin(5\alpha_1) &= 0
\end{align*}
\]

(48°,60°,72°) or (36°,44°,64°) are some of the possible solutions

\[0°-43.2°, 52.2°-67.3°\] sets also \(B_7 = 0\)!
Multi-sector dipole coil

\[ \sin(3\alpha_5) - \sin(3\alpha_4) + \sin(3\alpha_3) - \sin(3\alpha_2) + \sin(3\alpha_1) = 0 \]
\[ \sin(5\alpha_5) - \sin(5\alpha_4) + \sin(5\alpha_3) - \sin(5\alpha_2) + \sin(5\alpha_1) = 0 \]
\[ \sin(7\alpha_5) - \sin(7\alpha_4) + \sin(7\alpha_3) - \sin(7\alpha_2) + \sin(7\alpha_1) = 0 \]
\[ \sin(9\alpha_5) - \sin(9\alpha_4) + \sin(9\alpha_3) - \sin(9\alpha_2) + \sin(9\alpha_1) = 0 \]
\[ \sin(11\alpha_5) - \sin(11\alpha_4) + \sin(11\alpha_3) - \sin(11\alpha_2) + \sin(11\alpha_1) = 0 \]

\([0^\circ-33.3^\circ, 37.1^\circ-53.1^\circ, 63.4^\circ-71.8^\circ]\) sets \((B_3, B_5, B_7, B_9 \text{ and } B_{11}) = 0! \)
Examples

Tevatron main dipole - 1980

RHIC main dipole - 1995
two layer design

\[ B_3 \propto \sin(3\alpha_1) \left( \frac{1}{r} - \frac{1}{r + w} \right) + \sin(3\alpha_2) \left( \frac{1}{r + w} - \frac{1}{r + 2w} \right) \]

\[ B_5 \propto \sin(5\alpha_1) \left( \frac{1}{r^3} - \frac{1}{(r + w)^3} \right) + \sin(5\alpha_2) \left( \frac{1}{(r + w)^3} - \frac{1}{(r + 2w)^3} \right) \]
Quadrupole design using sector coils

\[ B(z) = \sum_{n=1}^{\infty} C_n \left( \frac{z}{R_{\text{ref}}} \right)^{n-1} \]

\[ C_n = -\frac{I\mu_0}{2\pi R_{\text{ref}}} \left( \frac{R_{\text{ref}}}{z_0} \right)^n \]

\[ B_2 = -\frac{I\mu_0 R_{\text{ref}}}{2\pi} \text{Re} \left( \frac{1}{z_0^2} \right) = -\frac{I\mu_0 R_{\text{ref}}}{2\pi} \frac{\cos 2\theta}{|z_0^2|} \quad I \rightarrow j\rho d\rho d\theta \]

\[ B_2 = -8 \frac{j\mu_0 R_{\text{ref}}}{2\pi} \int_0^r \int_0^{r+w} \frac{\cos 2\theta}{\rho^2} \rho d\rho d\theta = -\frac{4 j\mu_0 R_{\text{ref}}}{\pi} \left[ \sin 2\alpha \ln \left( 1 + \frac{w}{r} \right) \right] \]
Multipoles of a quadrupole sector coil

\[ B_6 = \frac{\mu_0 j R_{\text{ref}}^5}{\pi} \sin(6\alpha) \left( \frac{1}{r^4} - \frac{1}{(r+w)^4} \right) \]

for \( \alpha = \pi/6 \) (30°) one has \( B_6 = 0 \)

\[ B_{10} = \frac{\mu_0 j R_{\text{ref}}^8}{\pi} \sin(10\alpha) \left( \frac{1}{r^8} - \frac{1}{(r+w)^8} \right) \]

for \( \alpha = \pi/10 \) (18°) or \( \alpha = \pi/5 \) (36°) one sets \( B_{10} = 0 \)

It follows the same philosophy of the Dipole design!
Examples

Tevatron main quadrupole

~[0°-12°, 18°-30°]

RHIC main quadrupole

~[0°-18°, 22°-32°]

LHC main quadrupole

~[0°-24°, 30°-36°]
Peak field and bore field ratio ($\lambda$)

Tevatron main dipole – location of the peak field

RHIC main dipole – location of the peak field

LHC main dipole – location of the peak field
Peak field and bore field ratio ($\lambda$)

\[
\lambda(w, r) \sim 1 + \frac{ar}{w} \\
a \sim 0.045
\]
Examples

\[ \lambda \text{ [adim]} \]

- TEV MB
- HERA MB
- SSC MB
- RHIC MB
- LHC MB
- Fresca
- MSUT
- D20
- HFDA
- NED

Equivalent width \( w/r \): 0.0 0.5 1.0 1.5 2.0
Operational Margin

![Graph showing the operational margin](image)

- NbTi critical surface @ 4.2K
- Operational Point
- Critical Field

24%
Lorentz Forces
Quench protection

- A superconducting accelerator magnet has a large magnetic stored energy
- A quench produces a resistive zone
- Current is flowing through the magnet

Joule Heating Voltages (R and L)

The challenge of the protection is to provide a safe conversion of the magnetic energy to heat in order to minimize

- Peak temperature ("hot spot") and temperature gradients in the magnet
- Peak voltages

The final goal being to avoid any magnet degradation

- High temperature => damage to the insulation or stabilizer
- Large temperature gradient => damage to the conductor due to differential thermal expansion of materials
General quench protection diagram

Quench

Normal zone growth

Detection

Power supply switched off

Trigger protection options

Protection heaters

Extraction

Quenchback

Current decay in the magnet

Magnetic energy
$$\frac{1}{2} LI^2$$

Converted to heat by Joule heating

$$\int_0^T R(t)I(t)^2 \, dt$$

The faster this chain happens the safer is the magnet
Training

![Plot showing quench current (kA) vs. training quench number. The plot includes data points for SQ02, first thermal cycle at 4.3 K and SQ02, second thermal cycle at 4.3 K. The expected short sample limit at 4.3 K is indicated by a dashed line.]
Magnetization

\[ B_p = \left( \frac{2 \mu_0}{\pi} \right) J_c a; \quad M_{\text{peak}} = \left( \frac{4 \mu_0}{3\pi} \right) J_c a \]

Normalized Magnetization \( M / M_{\text{peak}} \)

- Initial Up Ramp
- Down Ramp
- 2nd Up Ramp

Normalized Applied Field \( B_a / B_p \)
Magnetization
Magnetization
Summary

• Design and Fabrication of Superconducting Magnets belong to a different Universe

• Although the mathematical formulation for the field generation is shared, the design of superconducting magnets involves many other aspects:
  o Thermal considerations
  o Mechanical Analysis
  o Fabrication techniques
  o Quench Protection
  o Material Science

• If one is interested to learn more about superconducting magnet, one should attend to the Superconducting Accelerator Magnets USPAS course. The material for that course can be found at:
  http://etodesco.web.cern.ch/etodesco/uspas/uspas.html
Unusual design examples