John Adams Institute for Accelerator Science

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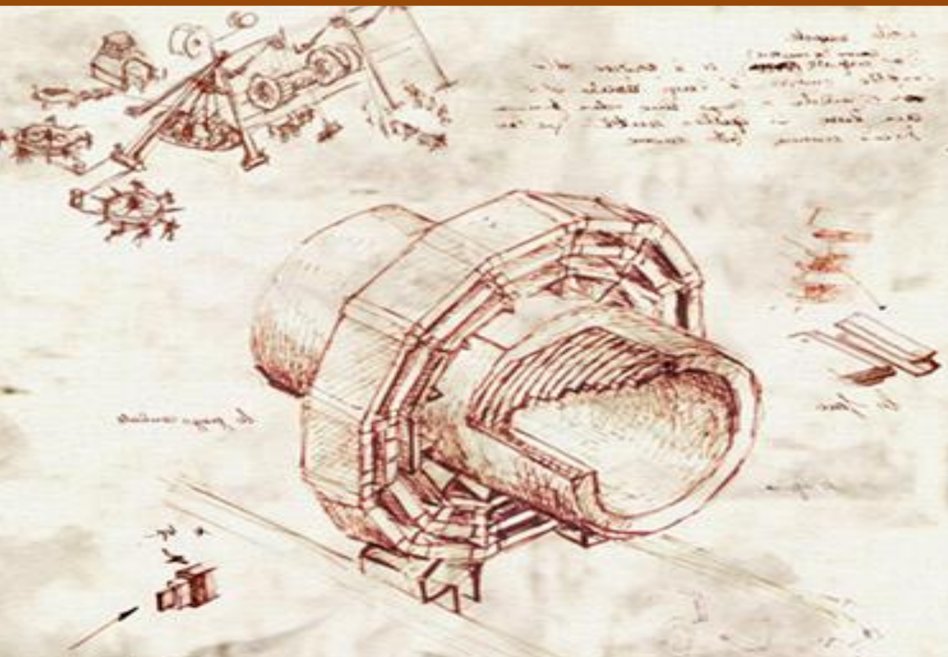
Unifying physics of accelerators, lasers and plasma



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John Adams Institute

Lecture 8: Free Electron Lasers

USPAS 2016

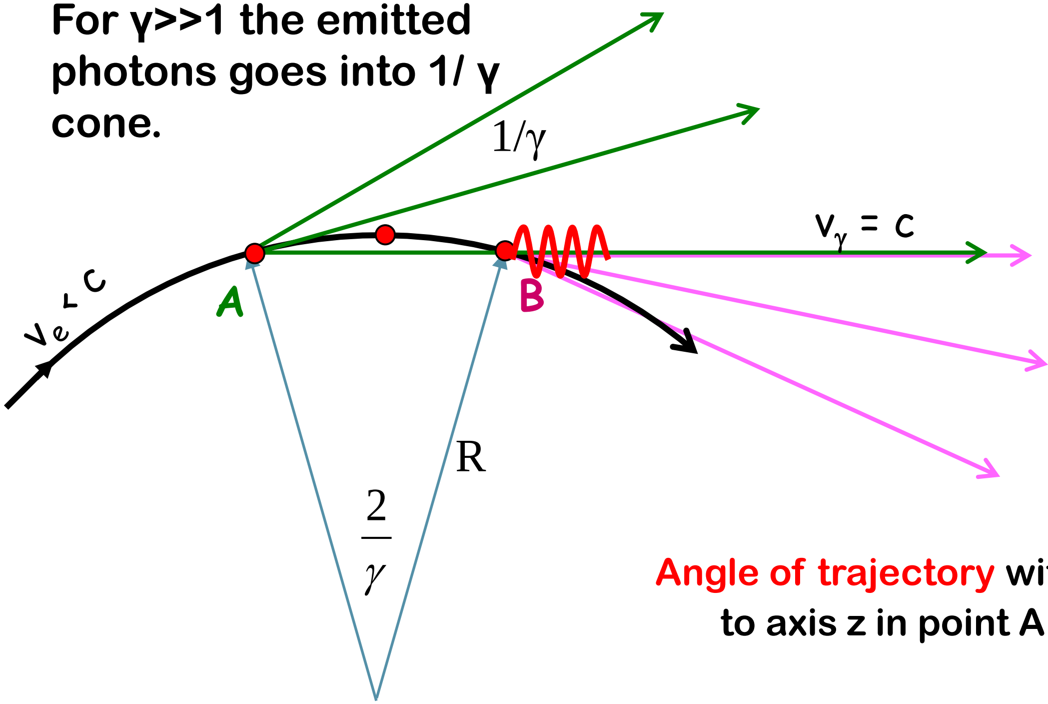
June 2016

Free Electron Lasers

- **Basic concept**
  - Recall bend, wiggler and undulator radiation
  - Undulator parameter
  - Micro-bunching
- **Types**
  - Oscillators
  - SASE
- **Examples**
  - FEL from Linac
  - FEL form LPWAs

Synchrotron radiation – from bends

For γ>>1 the emitted photons goes into 1/ γ cone.



Observer will see photons emitted during travel **along the arc 2R/γ**

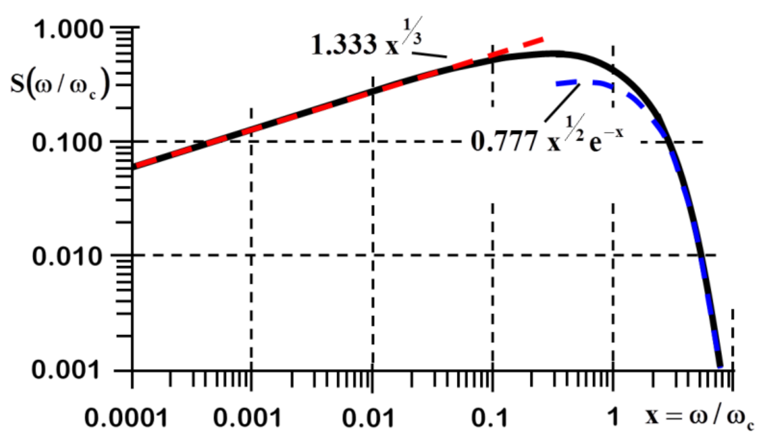
Take into account that photons travel with speed c, while particles with v.

This give us :

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{R}$$

Angle of trajectory with respect to axis z in point A or B is 1/γ

Detailed math predicts that SR spectrum from bends looks like this:

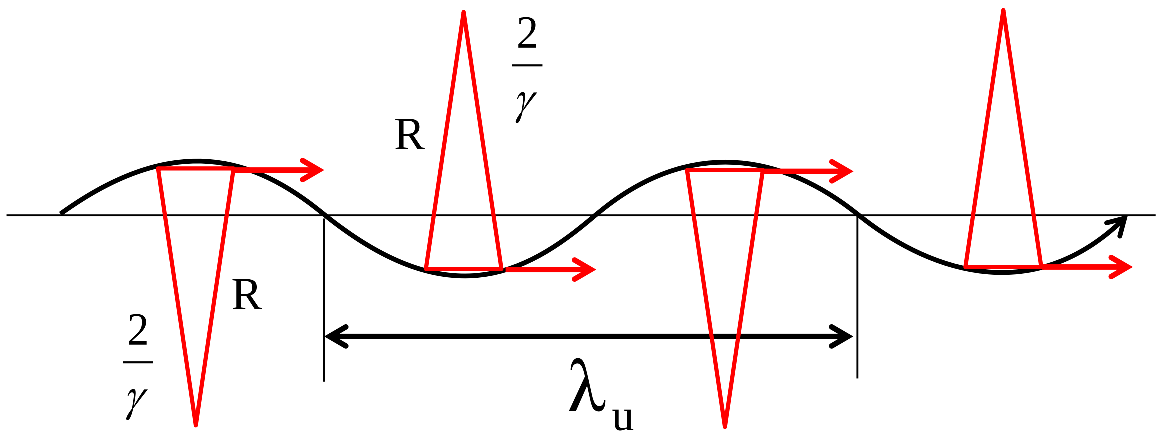


We also estimated:

$$\frac{dW}{dS} = \frac{2}{3}\frac{e^2\gamma^4}{R^2}$$

Recall L3: Radiation from sequence of bends

Assume that bends are arranged in sequence with +-+-+- polarity with period λ_u, so that trajectory wiggles:



Observer will see photons emitted during travel along the arc $2R/\gamma$

If $2R/\gamma \ll \lambda_u/2$, then radiation emitted at each wiggle is independent

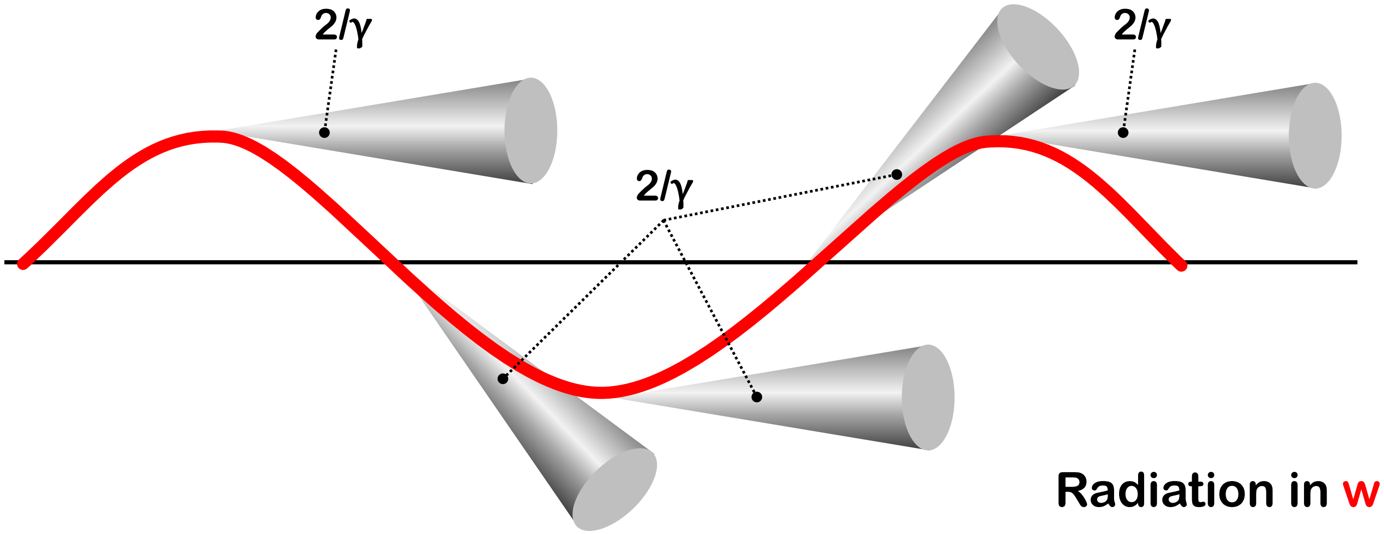
If $2R/\gamma \gg \lambda_u/2$ => regime where entire wiggling trajectory contribute to radiation

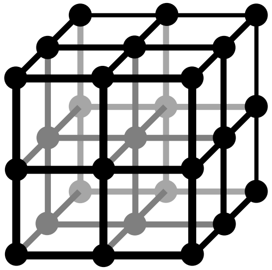
Define $K \sim \gamma \lambda_u / R$

$K \gg 1$ – wiggler regime

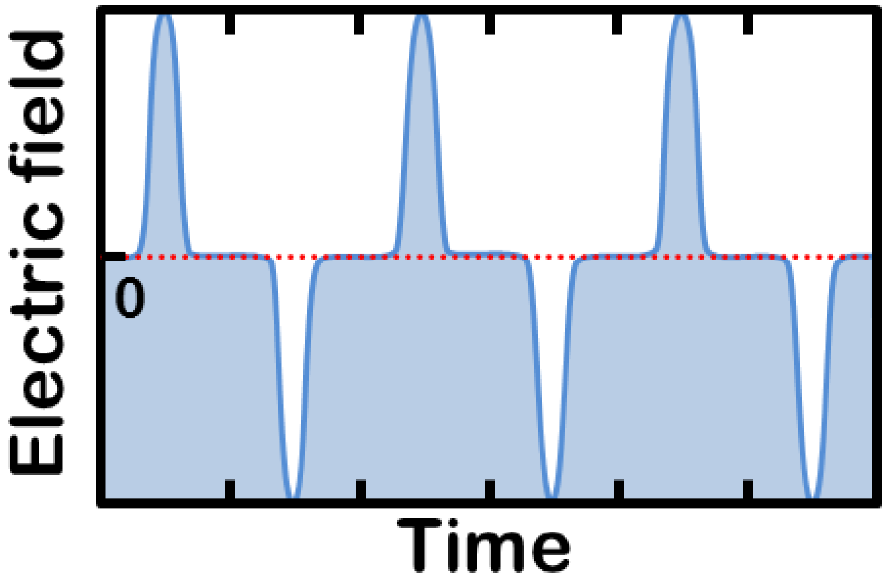
$K \ll 1$ – undulator regime

Compare spectra from wiggler and FEL



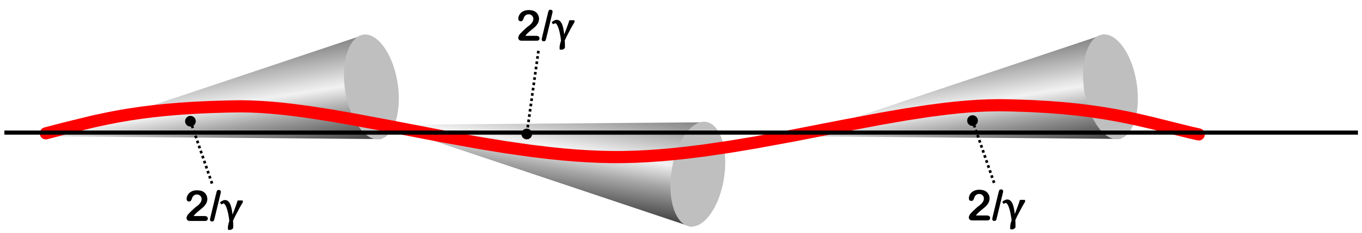


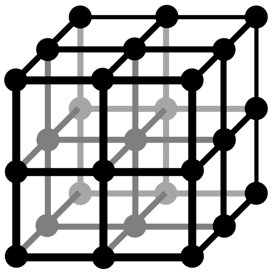
Radiation in wiggler, with K >> 1



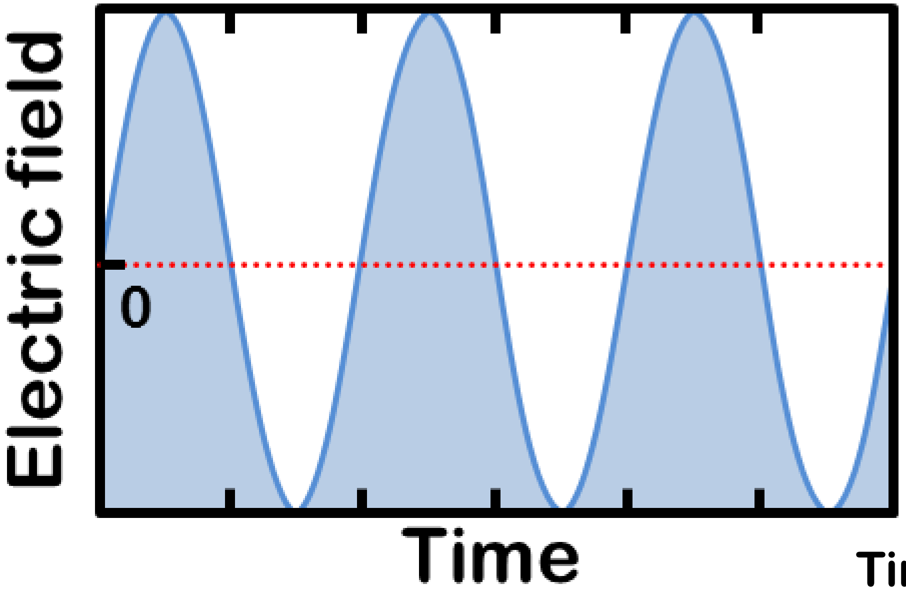
Time profile of radiation from wiggler

Compare spectra from wiggler and FEL



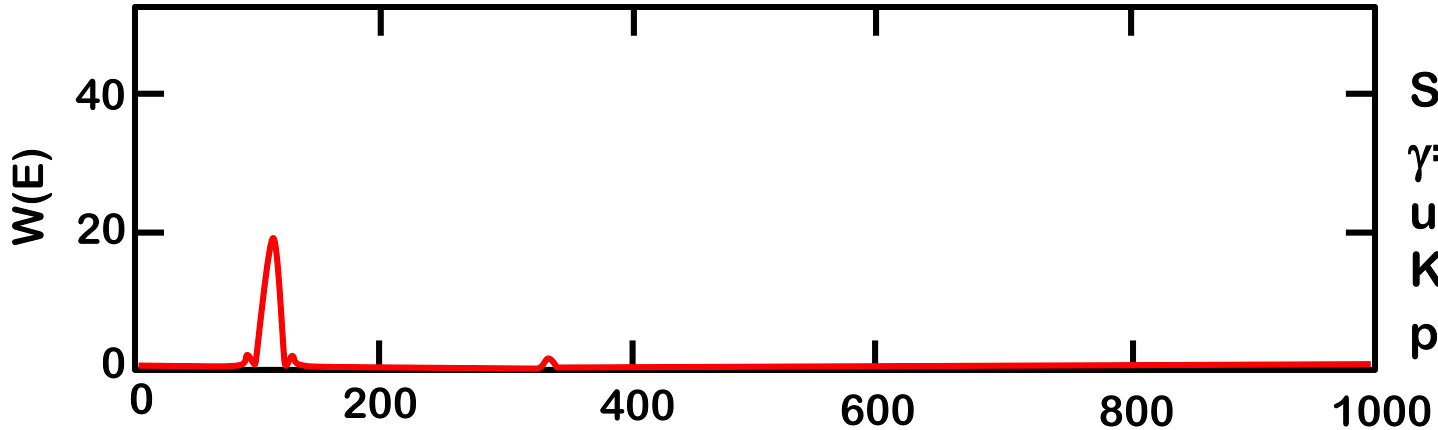


Radiation in undulator, with K << 1

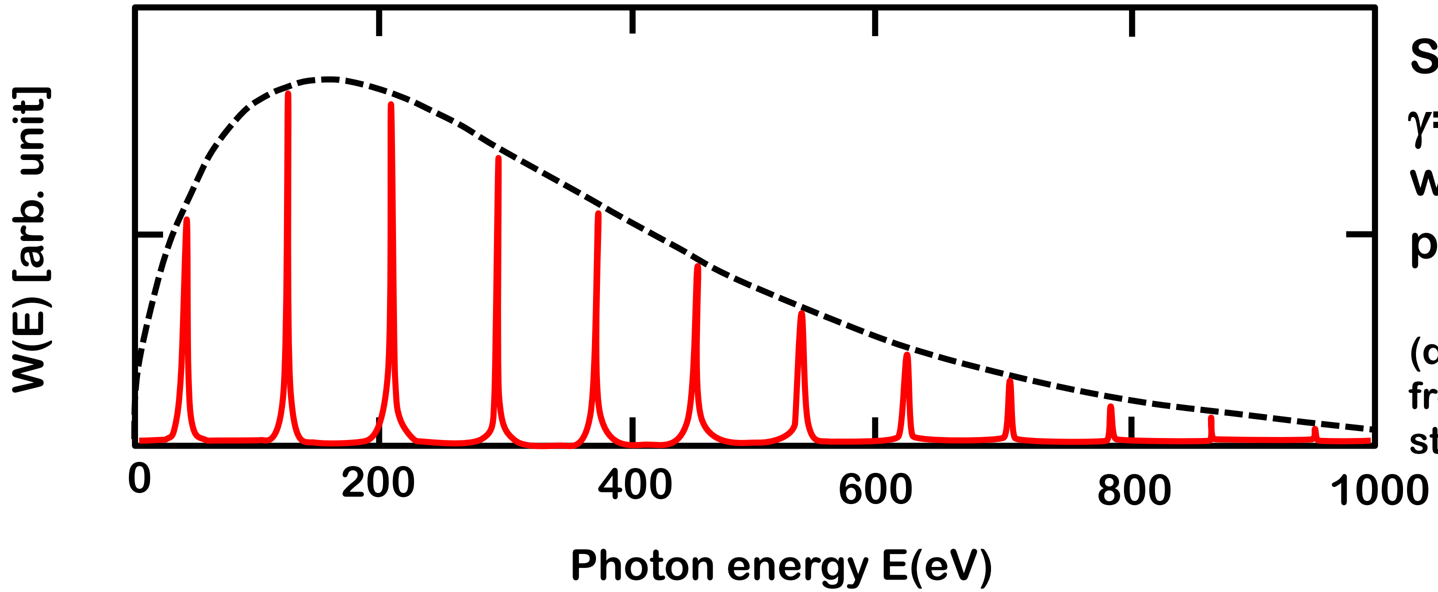


Time profile of radiation from undulator

Compare spectra from wiggler and FEL



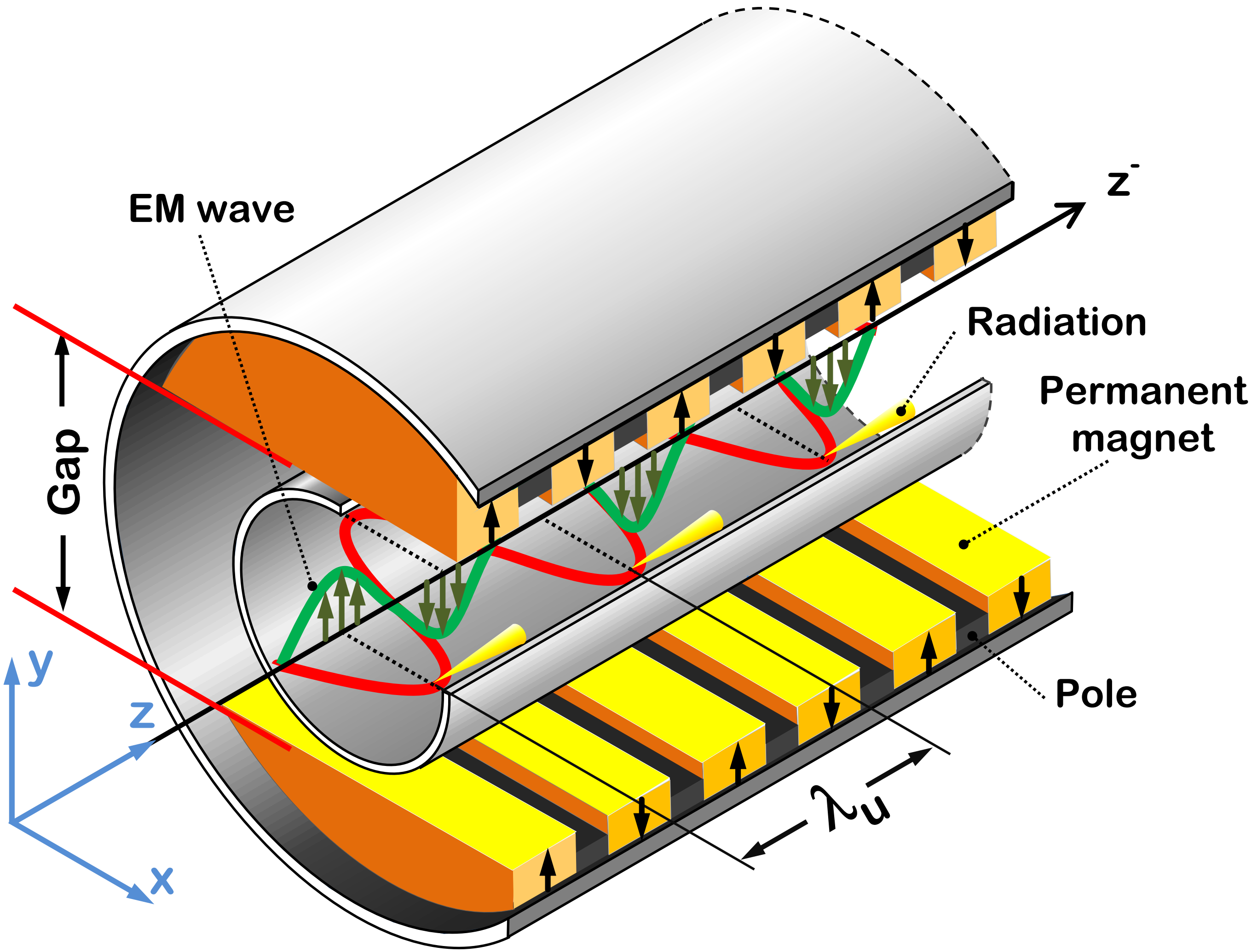
Spectrum for γ=1000 and undulator with K=0.5, with 10 periods of λ_u=2cm



Spectrum for γ=1000 and wiggler with K=2, with 10 periods of λ_u=2cm

(dashed line – spectrum from bends of same strength, arb.units)

Sequence of bends – wiggler or undulator



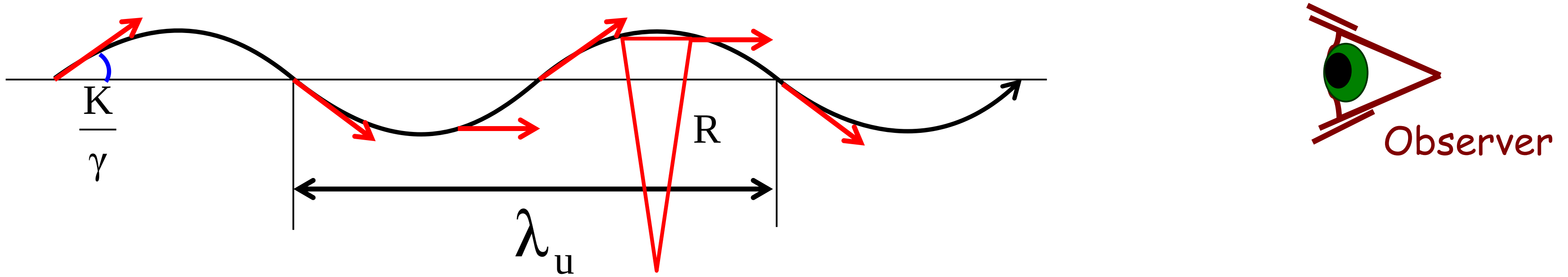
Sin-like Field:

$$B_y(z) = B_0 \sin(k_u z)$$

$$k_u = \frac{2\pi}{\lambda_u}$$

Radiation from sequence of bends

Let's parameterize the sin-like trajectory through sin-like field of +-+-+- bend sequence in such a way that the maximum angle of trajectory is equal to K/γ:



The trajectory thus parameterized as $$x = \frac{K}{\gamma}\frac{\lambda_u}{2\pi}\sin\left(\frac{2\pi z}{\lambda_u}\right)$$

If K <1, then the trajectory angle is always less than $1/\gamma$ and observer see the radiation without interruptions

Let's find the radius given by curvature of trajectory $$\frac{d^2x}{dz^2} = \frac{1}{R} \quad \Rightarrow \quad K = \frac{\lambda_u \gamma}{2\pi R}$$

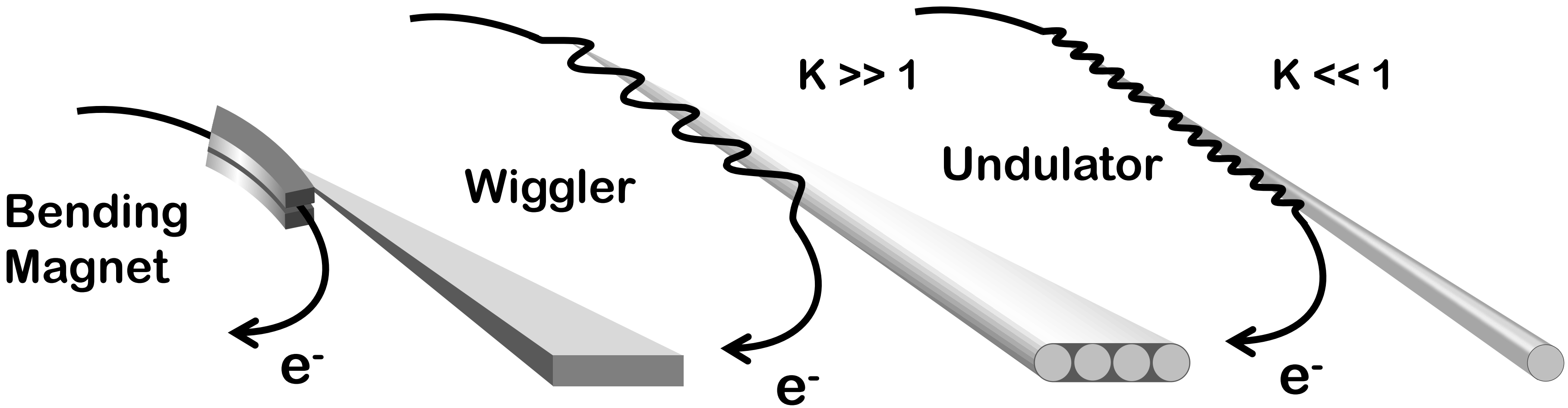
and radius given by magnetic field $$R = \frac{pc}{eB_0} \quad \Rightarrow \quad K = \frac{\lambda_u e B_0}{2\pi mc^2}$$

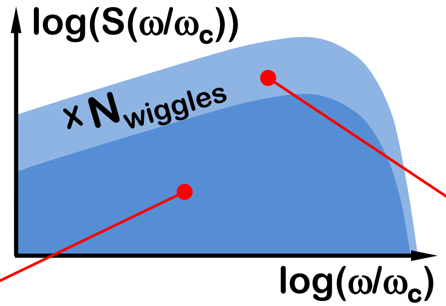
K << 1 – undulator regime

K >> 1 – wiggler regime

Wiggler and undulator radiation spectra

Parameter $K \sim \gamma \lambda_u / R$ defines different regimes of synchrotron radiation





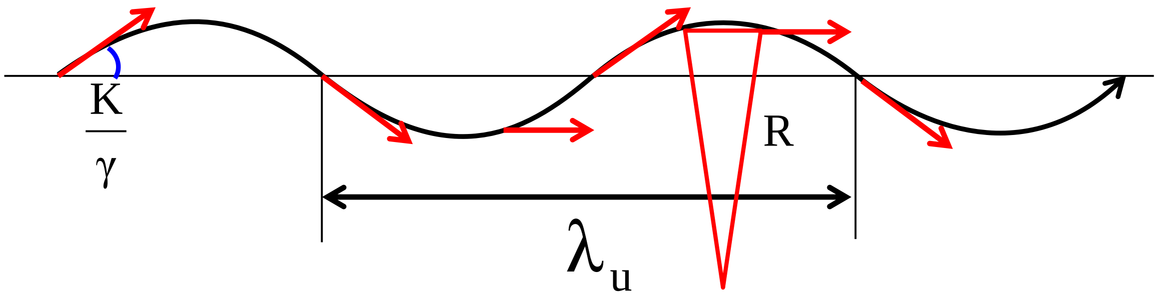
Spectrum from undulator is different

Spectrum from bend

Spectrum from wiggler similar to the one from bend (with internal structure)

Average longitudinal velocity in undulator

Let's look again at sin-like trajectory and find average longitudinal velocity



If the amplitude is zero, longitudinal velocity is $$v_{z0} = \beta c \approx c\left(1 - \frac{1}{2\gamma^2}\right)$$

We parameterized trajectory as $$x = \frac{K}{\gamma}\frac{\lambda_u}{2\pi}\sin\left(\frac{2\pi z}{\lambda_u}\right) \quad \Rightarrow \quad v_x = \beta c \frac{K}{\gamma}\sin()$$

In the second order, the longitudinal velocity is $$v_z \approx \beta c\left(1 - \frac{1}{2}\frac{v_x^2}{\beta^2 c^2}\right)$$

Thus, average longitudinal velocity is $$\langle v_z \rangle \approx c\left(1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)\right)$$

Additional longitudinal retardation due to transverse velocity

FEL basic concepts

In a storage ring the phase relationship between the radiation emitted by each electron is random and the spatial and temporal coherence of the radiation is limited.

The electrons emit radiation in an undulator incoherently

In a FEL the electron interact back with the radiation emitted in the undulator.

Under certain conditions this process can generate a microbunching of the beam.

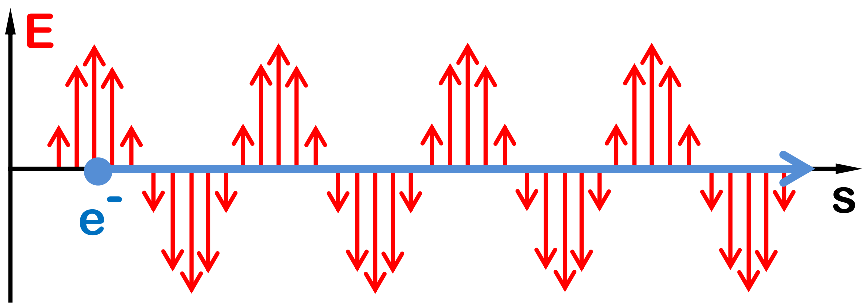
Microbunching happens mostly at the undulator resonant wavelength.

The electrons will now emit in phase with each other, coherently

The radiation power (and brilliance) will scale as N_e^2 not as N_e

FEL basics

Electron beam with only longitudinal velocity overlaid with EM wave



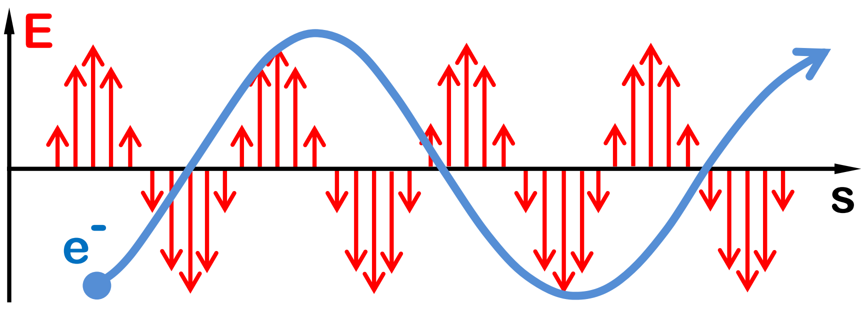
Energy exchange EM wave - electrons

$$\frac{dW}{dt} = \vec{\nabla} W \frac{d\vec{x}}{dt} = e\vec{E} \cdot \vec{v} = 0$$

If electrons have only longitudinal velocity,
no energy can be transferred between electrons and EM wave

FEL basics

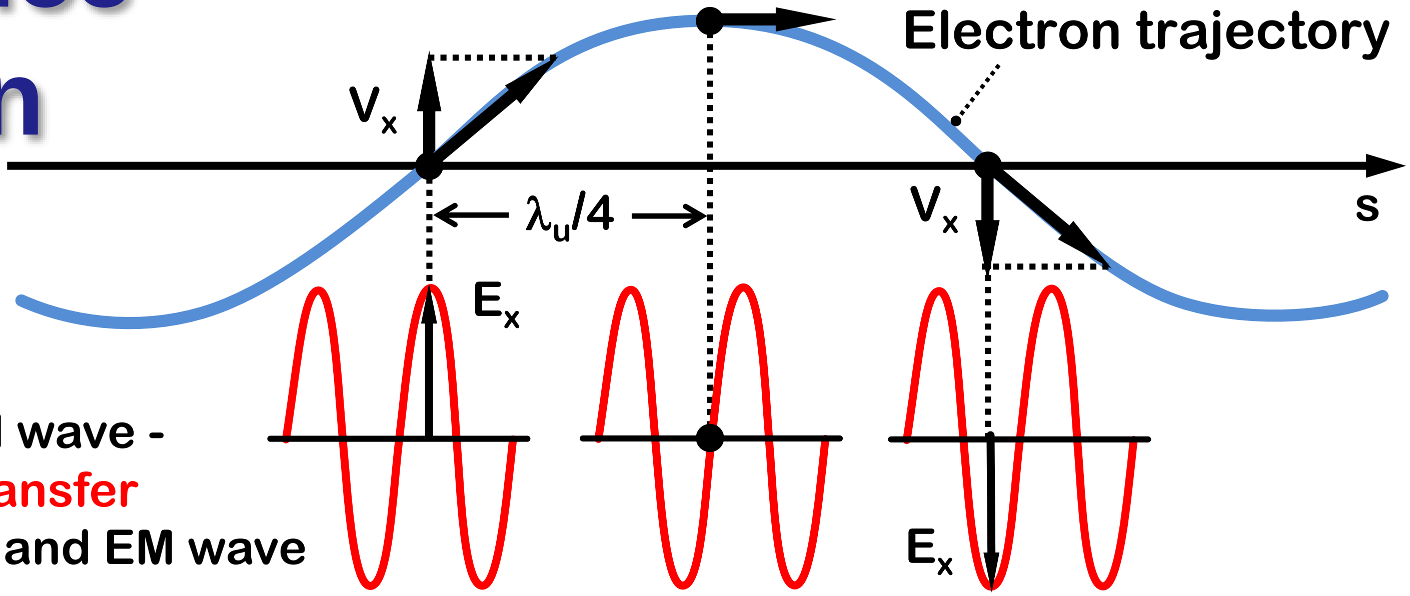
Electron beam with sin-like trajectory in undulator overlaid with EM wave



Energy exchange EM wave - electrons $\frac{dW}{dt} = e\,\vec{E} \cdot \vec{v} \neq 0$ **because** $\vec{v}_\perp \neq 0$

If electrons have transverse velocity,
energy can be transferred between electrons and EM wave

Resonance condition



For certain λ of EM wave - **resonant energy transfer** between electrons and EM wave

Condition for **resonant energy transfer** is that EM wave slips forward with respect to electron by λ /2 per half period of electron trajectory, i.e.

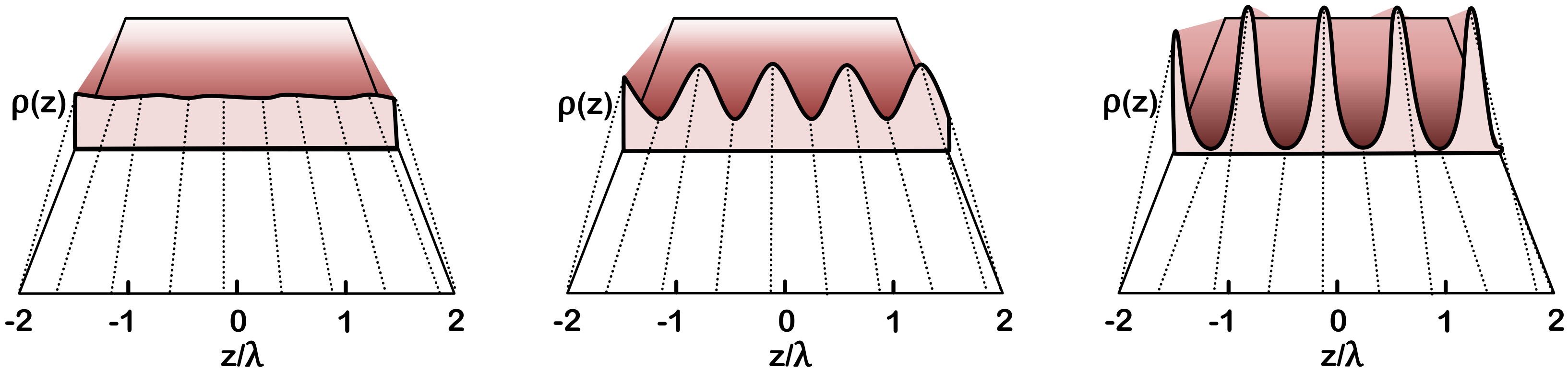
$$\lambda = \lambda_u \left(1 - \langle v_z \rangle / c\right)$$

Remember that in undulator $\langle v_z \rangle \approx c\left(1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)\right)$

This give us the **resonant EM wavelength:** $\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$

Slippage by 3 λ/2, 5λ /2, etc., also in resonance, leading to odd higher harmonics

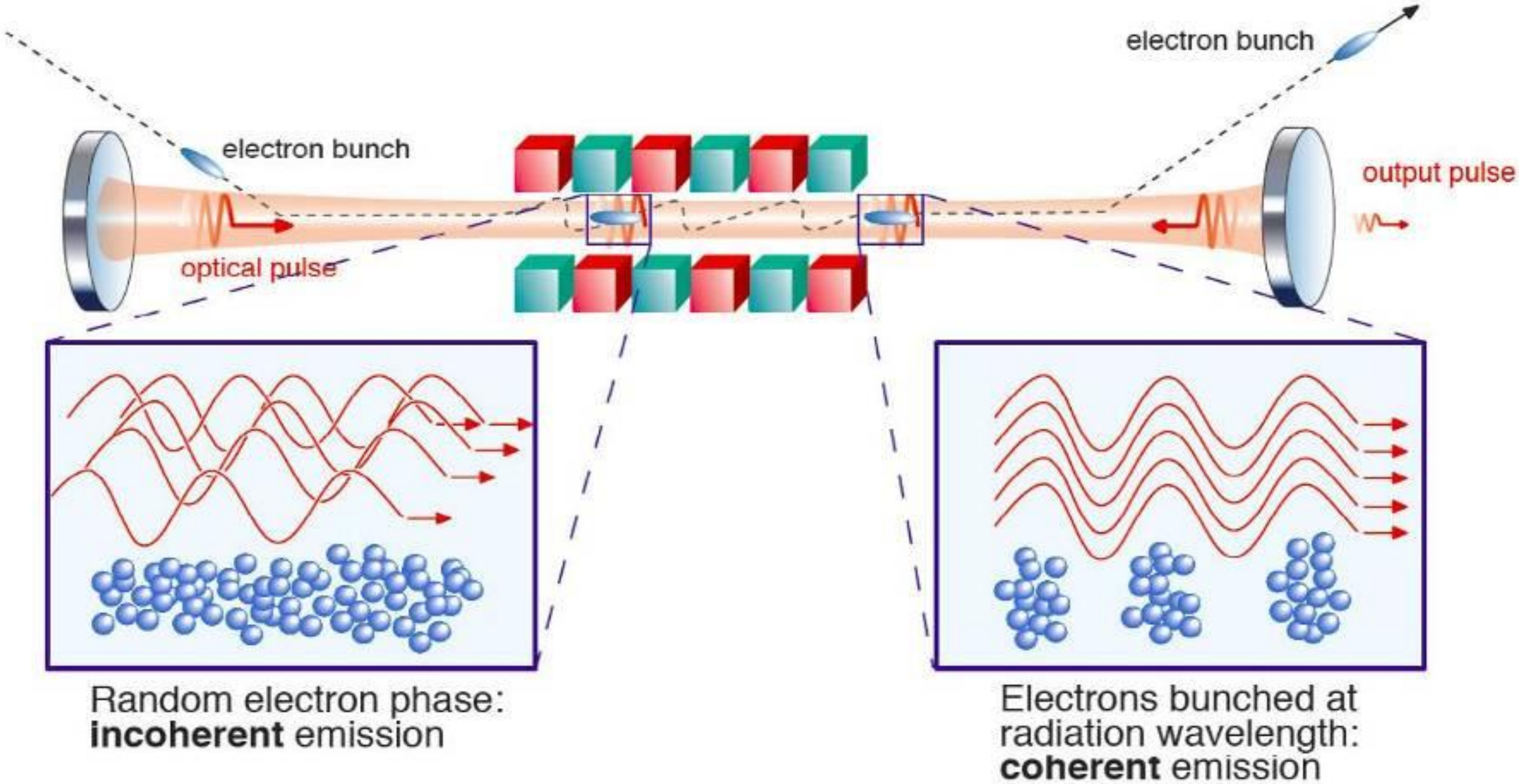
Micro-bunching



- Interaction with resonance EM wave => energy modulation
- Energy modulation => different path over sin-like trajectory
- Different path => density modulation along the bunch
- Initial EM wave can be external (seeding) of come from noise (Self Amplified Spontaneous Emission - SASE)
- Coherent emission of radiation of wavelength l with power P ~ N^2

FEL types – multi and single pass

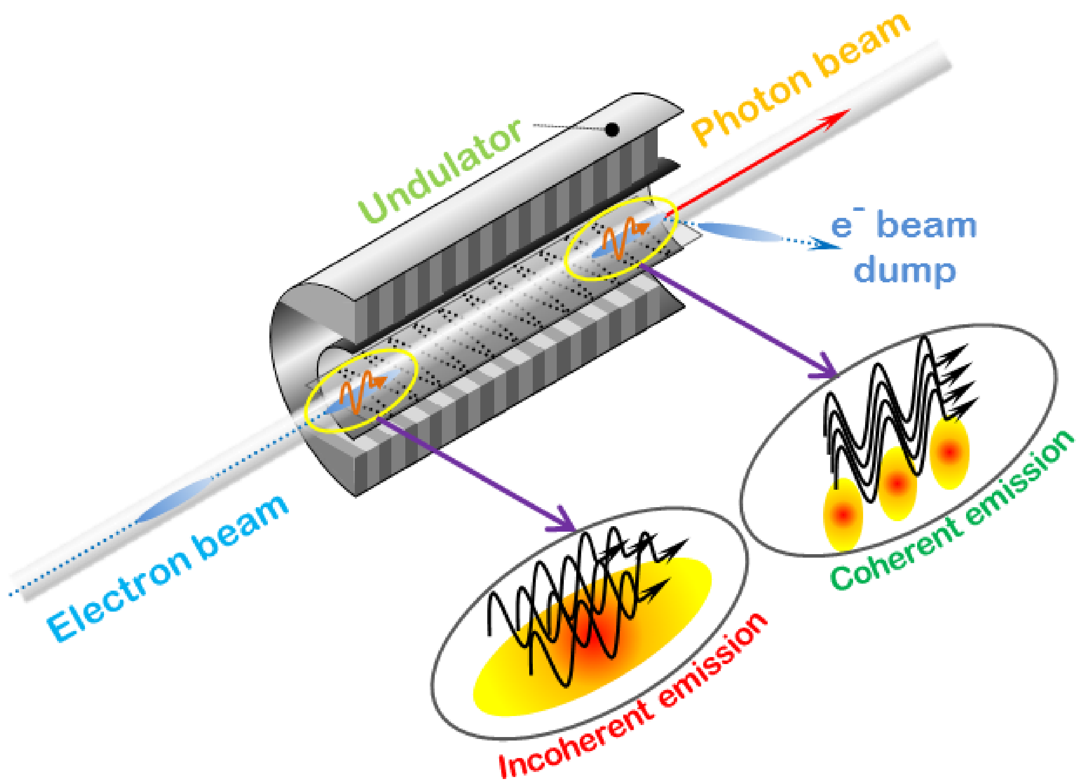
In single pass FEL the radiation is stored in a cavity.
The growth of radiation occurs over many bounces (low gain)



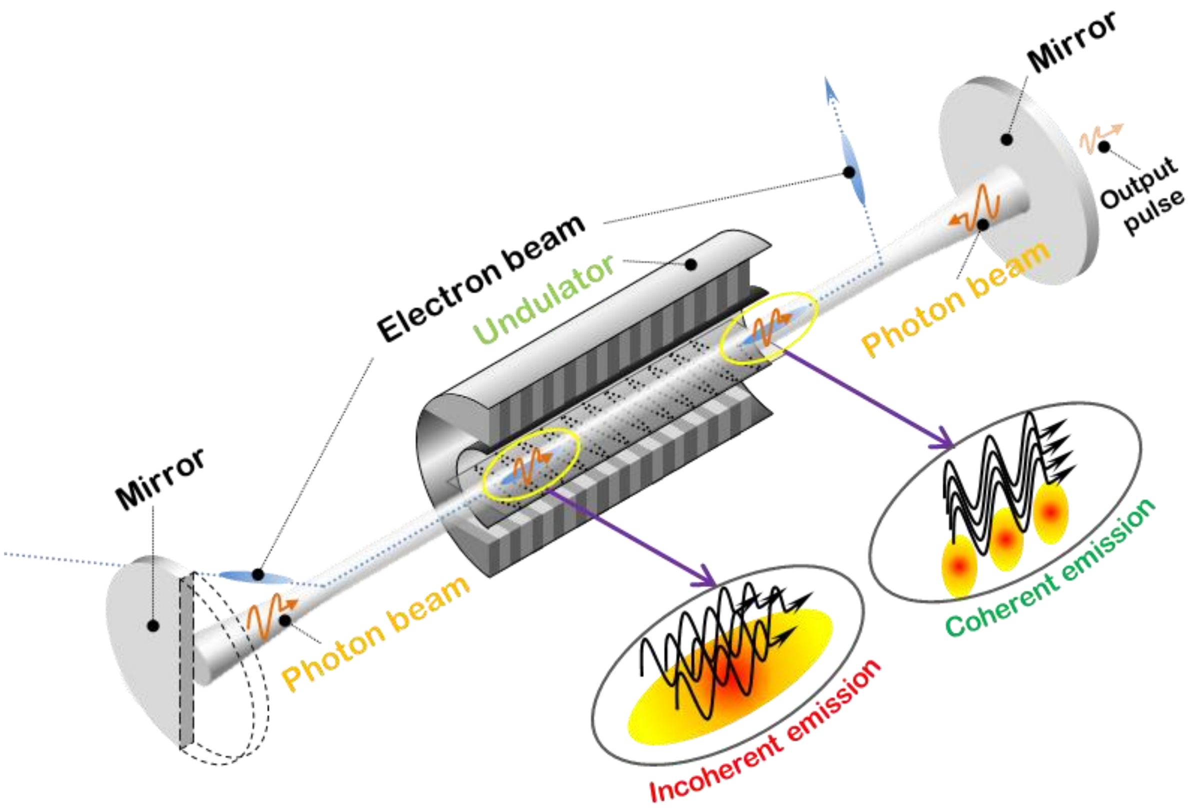
But for hard X-rays there are no good mirrors

FEL single pass

In single pass FEL the radiation grows within a single pass in the undulator (seeded or SASE)



FEL multi pass



FEL basic – details of micro-bunching (I)

How is the micro-bunching happening?

In certain conditions the interaction of the radiation emitted in an undulator, with the electron bunch itself, can be strong and generates a strong modulation of the energy of the electrons in the bunch. The equations of motion are

$$\frac{d\bar{p}}{dt} = e\bar{E} + \frac{e}{c}\bar{v} \times \bar{B} \qquad \bar{p} = m_e \gamma \bar{v}$$

$$\frac{dE}{dt} = e\bar{E} \cdot \bar{v} \qquad E = m_e c^2 \gamma$$

N.B. It is called laser but it can be explained entirely with classical electromagnetism

E and B are the magnetic field of the undulator and the undulator radiation

$$\bar{B} = B_0 (0, \quad \cos(k_u z), \quad 0)$$

$$\bar{E} = E_0 (\cos\alpha, \quad 0, \quad 0) \qquad \bar{B} = E_0 (0, \quad \cos\alpha, \quad 0) \qquad \alpha = kz - \omega t + \phi \qquad \omega = kc$$

Having simplified the undulator radiation with a plane wave, we can integrate them

FEL basic – details of micro-bunching (II)

The energy change of the electron occurs because of the coupling between

transverse (horizontal) oscillation of the electron in the undulator
and
transverse (horizontal) component of the electric field of the plane wave

$$\frac{dE}{dt} = e\overline{E} \cdot \overline{v} = eE_x v_x$$

unlike the RF cavities where the energy change occurs because of the coupling between

longitudinal velocity of the electron in the RF cavity
and
longitudinal component of the electric field in the RF cavity

$$\frac{dE}{dt} = e\overline{E} \cdot \overline{v} = eE_z v_z$$

FEL basic – details of micro-bunching (III)

Changing the independent variable form t to z, and integrating, the transverse velocity reads

$$\beta_x = -\frac{K}{\gamma}\sin k_u z - \frac{eE_0}{m_e \omega c \gamma}\sin\alpha$$

→ new term from the radiation (plane wave)

The energy change reads

$$\dot{\gamma} = -\frac{eE_0}{m_e c\gamma}\cos\alpha \cdot \left[\frac{eE_0}{m_e \omega c}\sin\alpha + K\sin k_u z\right]$$

These two equations make a system of first order differential equation for (z, γ)

We make the following assumptions

- small signal (keep first order only in E_0)
- small gain ($\Delta\gamma << \gamma$)
- radiation wavelength close to the fundamental undulator radiation wavelength
- averaging all quantities over one undulator period (to remove fast oscillations)

FEL basic – details of micro-bunching (IV)

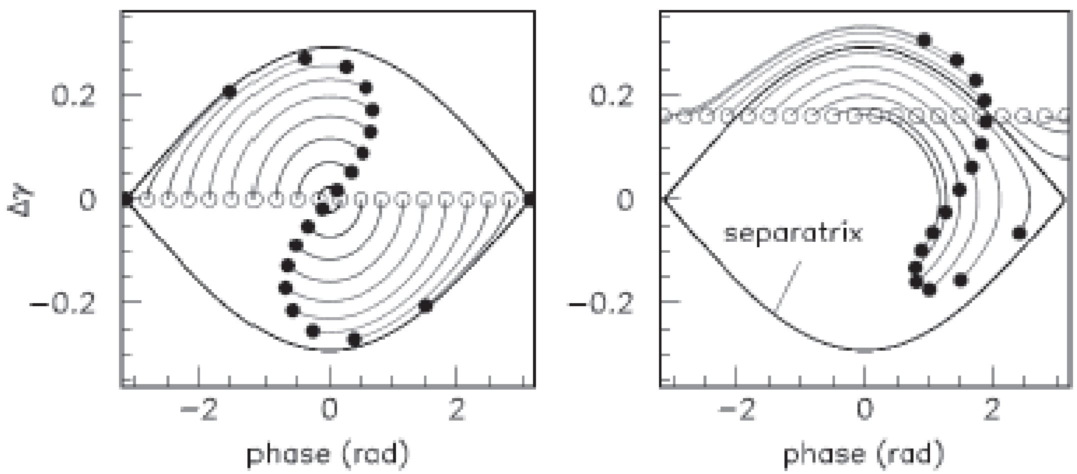
Introducing the variable

$$\zeta = k_u z + \alpha = (k + k_u)z - \omega t + \phi$$

the system of first order differential equations can be transformed in a second order differential equation

$$\ddot{\zeta} = -\frac{eE_0(k_u + k)[J_0(\xi) - J_1(\xi)](1 + K^2/2)K}{2m_e\gamma^4}\sin\zeta = \Omega^2 \sin\zeta$$

This is the so-called FEL-pendulum equation and describes the FEL interaction



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FEL basic – details of micro-bunching (IV)

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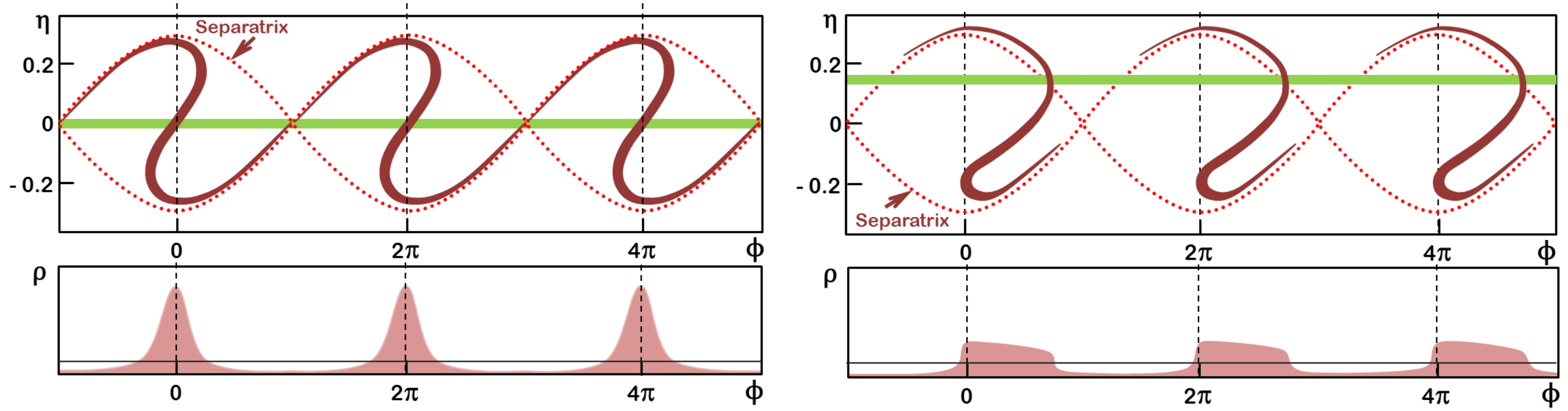


FEL basic – details of micro-bunching (V)

Each electron gain or loses energy depending on the relative phase ζ(0) between the transverse oscillation in the undulator and the phase of the radiation plane wave

$$\Delta\gamma = -\frac{eE_0K[J_0(\xi) - J_1(\xi)]L}{2m_e c^2 \beta_{z0}\gamma_0}\frac{\sin(\nu/2)}{\nu/2}\sin(\zeta(0)+\nu/2) + O(\Omega^2)$$

$$\nu = \left(k + k_u - \frac{\omega}{c\beta_0}\right)L$$

The average energy variation (over the initial phases ζ(0) of the electrons)

$$<\Delta\gamma>_\phi = \frac{eE_0K[J_0(\xi) - J_1(\xi)]\Omega^2}{8m_e c\gamma_0}\left(\frac{L}{c\beta_{z0}}\right)^3\frac{d}{d\nu}\left(\frac{\sin\nu/2}{\nu/2}\right)^2$$

The variation of the energy of the electrons correspond to a variation of the energy of the em wave.

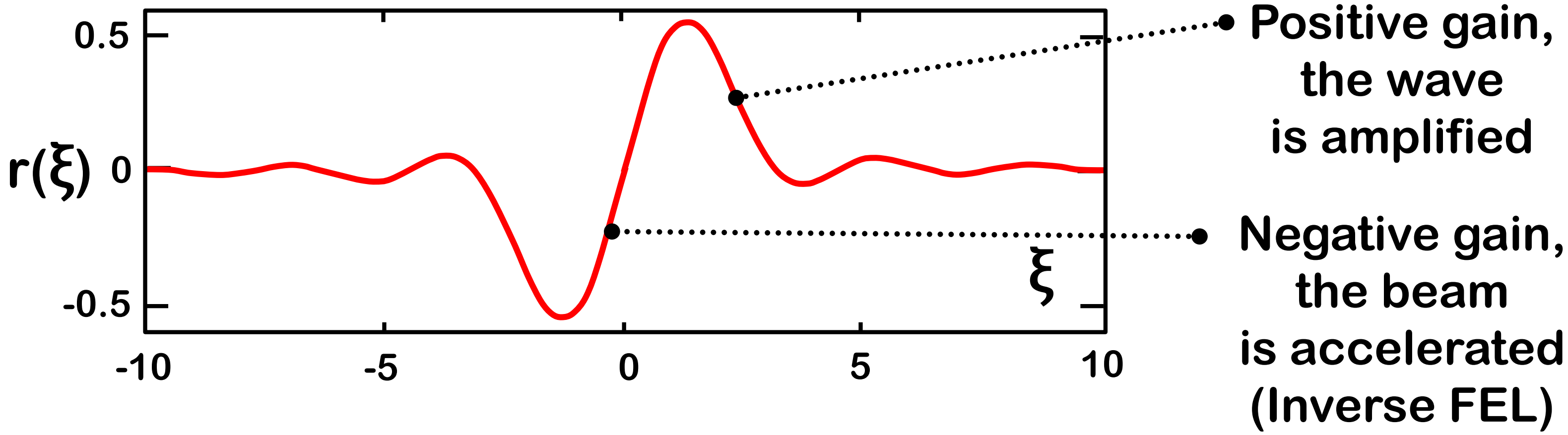
FEL small signal, small gain curve

We can define a gain as a relative change of the energy of the wave

$$G = \frac{\Delta E_{tot}}{W_0^L} = -m_e c^2 \frac{N}{W_0^L} < \Delta\gamma >_\phi$$

For a bunch with peak current I and transverse area $\Sigma_b = F\Sigma_L$

$$G = -\frac{\pi K^2 [J_0(\xi) - J_1(\xi)]^2 k_u L^3 (1+\beta_{z0})}{2\gamma^3 \beta_{z0}^3} \frac{F}{\Sigma_L} \frac{I}{I_0} \frac{d}{d\nu} \left(\frac{\sin \nu/2}{\nu/2} \right)^2$$



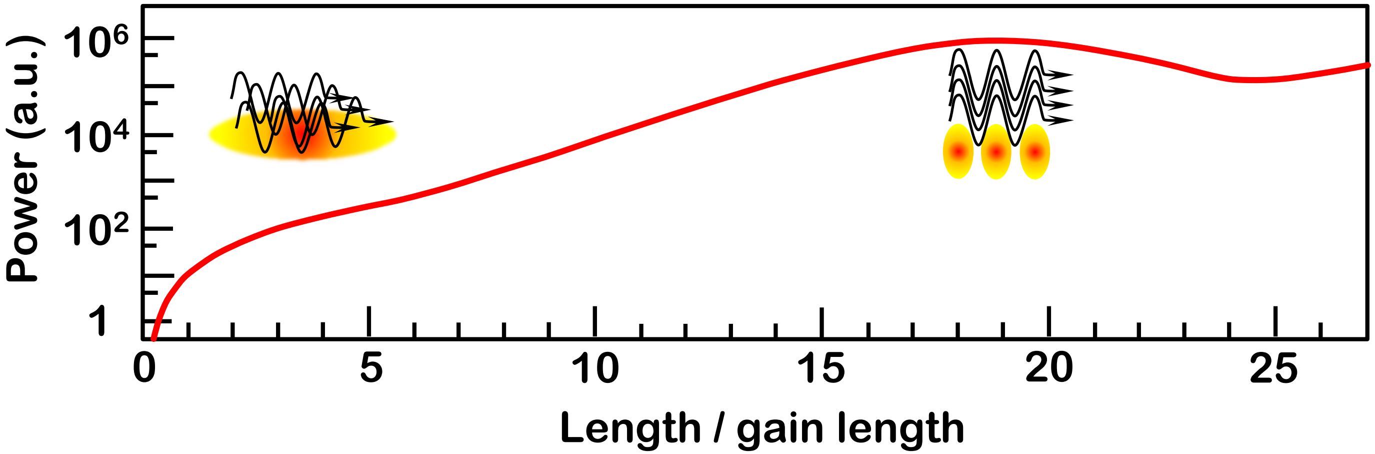
High-gain FELs

When the gain is so large that the wave amplitude changes within a single pass in the undulator, the previous approximations have to be revisited.

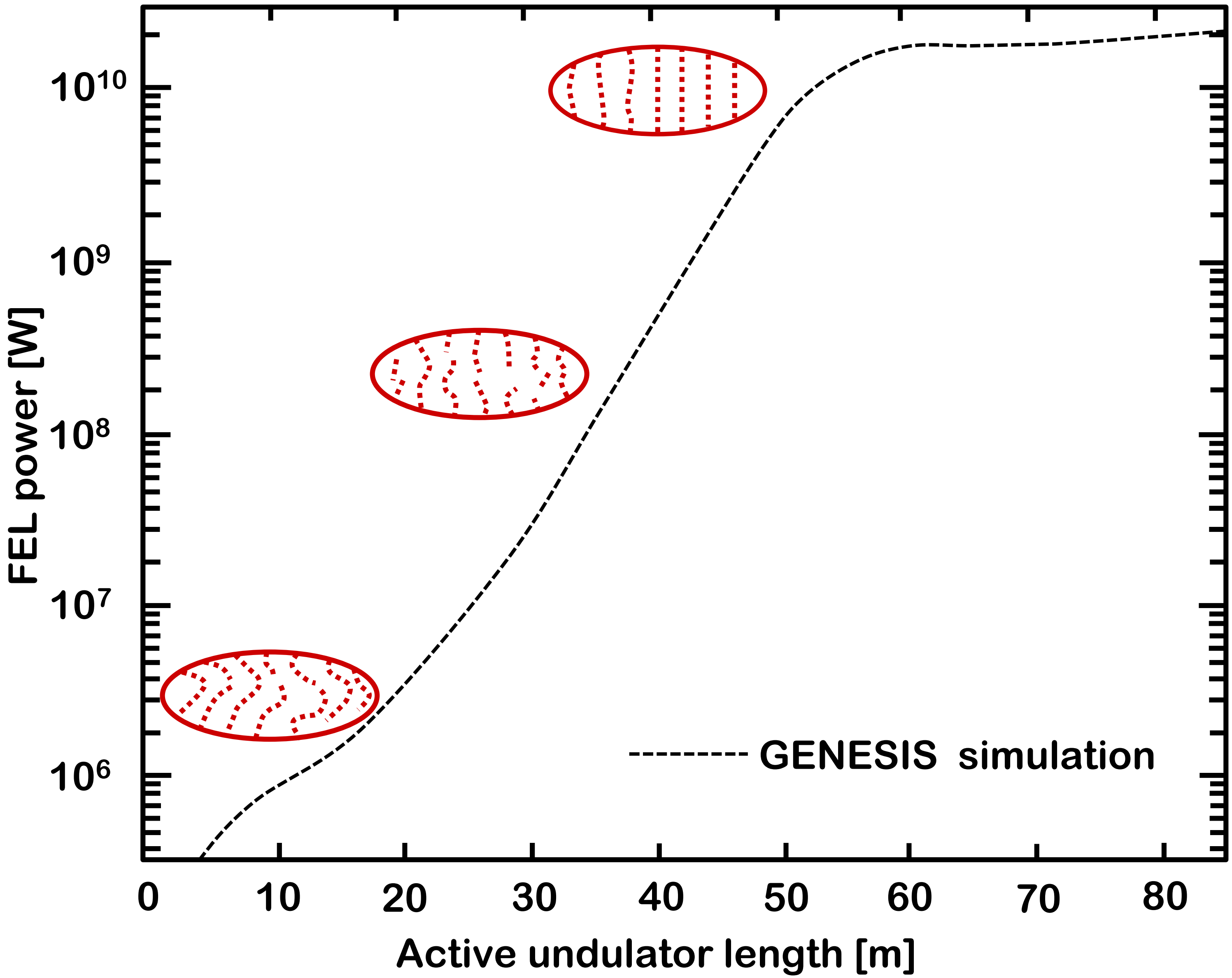
The wave amplitude must be described properly with the wave equation driven by the current density of the beam.

The result is an exponential growth of the radiation power until saturation is reached

$$P(s) \sim e^{s/L_g} \quad \text{with} \quad L_g = \frac{1}{\sqrt{3}}\left(\frac{4\gamma^3 m_e}{\mu_0 K^2 e^2 k_u n_e}\right)^{1/3}$$



LCLS simulations



FEL beam emittance requirements

Efficient lasing requires good overlap between electron and light beam

Emittance of synchrotron radiation photon beam is $\varepsilon = \frac{\lambda}{4\pi}$

Light beam: $w^2(s) = w_0^2 + \frac{\lambda^2}{\pi^2 w_0^2} s^2$

e- beam: $\sigma^2(s) = \sigma_0^2 + \frac{\varepsilon^2}{\sigma_0^2} s^2$

=> Requirement on emittance of electron beam $\varepsilon \leq \frac{\lambda}{4\pi}$

(we are talking here about geometrical emittance $\varepsilon = \frac{\varepsilon_N}{\gamma}$ **!)**

Need either a very bright e- source with very small ε_N or very high e- energy

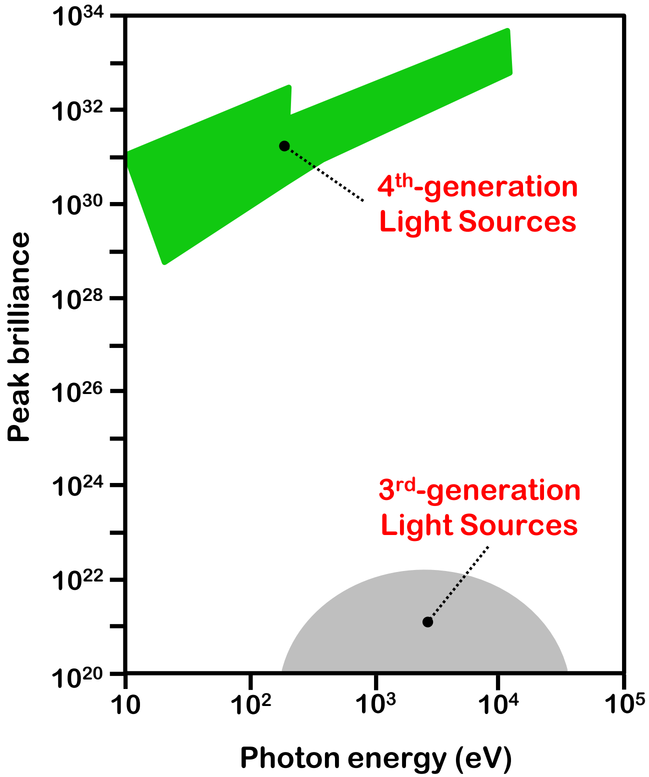
FEL and Laser comparison

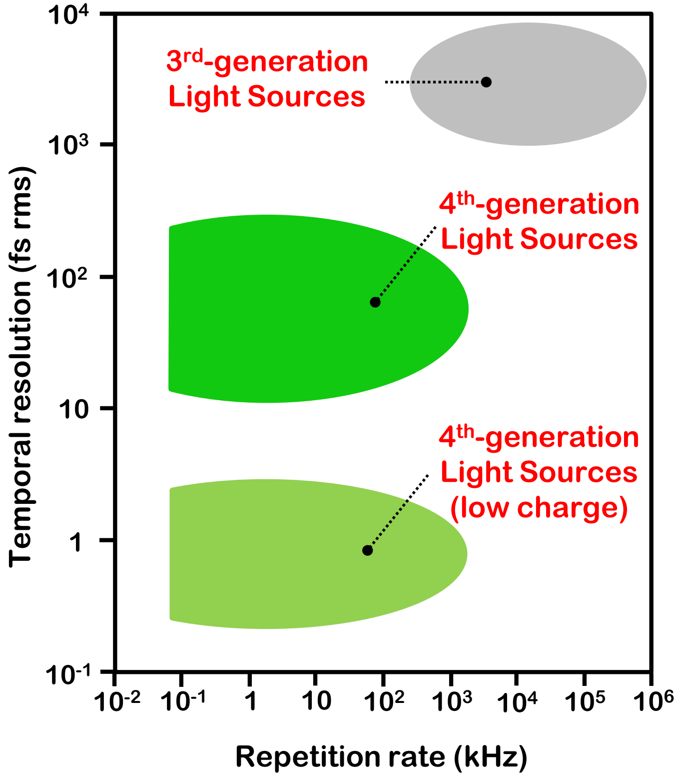
	LASER	FEL
Characteristics	Source of narrow, monochromatic and coherent light beams	
Configurations	Oscillator or amplifier	
First demonstration	1960	1977
Laser media	Solids, liquids, gases	Vacuum with electron beam in periodic magnetic field
Energy storage	Potential energy of electrons	Kinetic energy of electrons
Energy pump	Light or applied electric current	Electron accelerator
Theoretical basis	Quantum mechanics	Relativistic mechanics and electrodynamics
Wavelength definition	Energy levels of laser medium	Electron energy, magnetic field strength and period

FEL radiation properties

FELs provide peak brilliance 8 order of magnitudes larger than storage ring light sources

Average brilliance is 2-4 order of magnitude larger and radiation pulse lengths are of the order of 100s fs or less



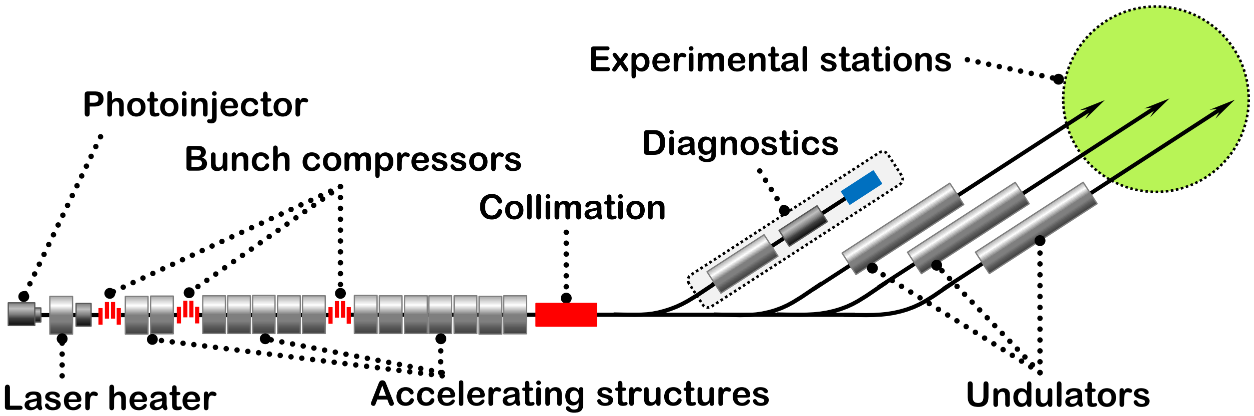


FEL amplifiers main components

An example taken from the UK New Light Source project (2010.)

High brightness electron gun operating at 1 kHz

2.25 GeV SC CW linac L- band with 50-200 pC



3 FELS covering the photon energy range 50 eV – 1 keV (50-300; 250-800; 430-1000)

- GW power level in 20 fs pulses
- laser HHG seeded for temporal coherence
- cascade harmonic FEL
- synchronised to conventional lasers and IR/THz sources for pump probe experiments

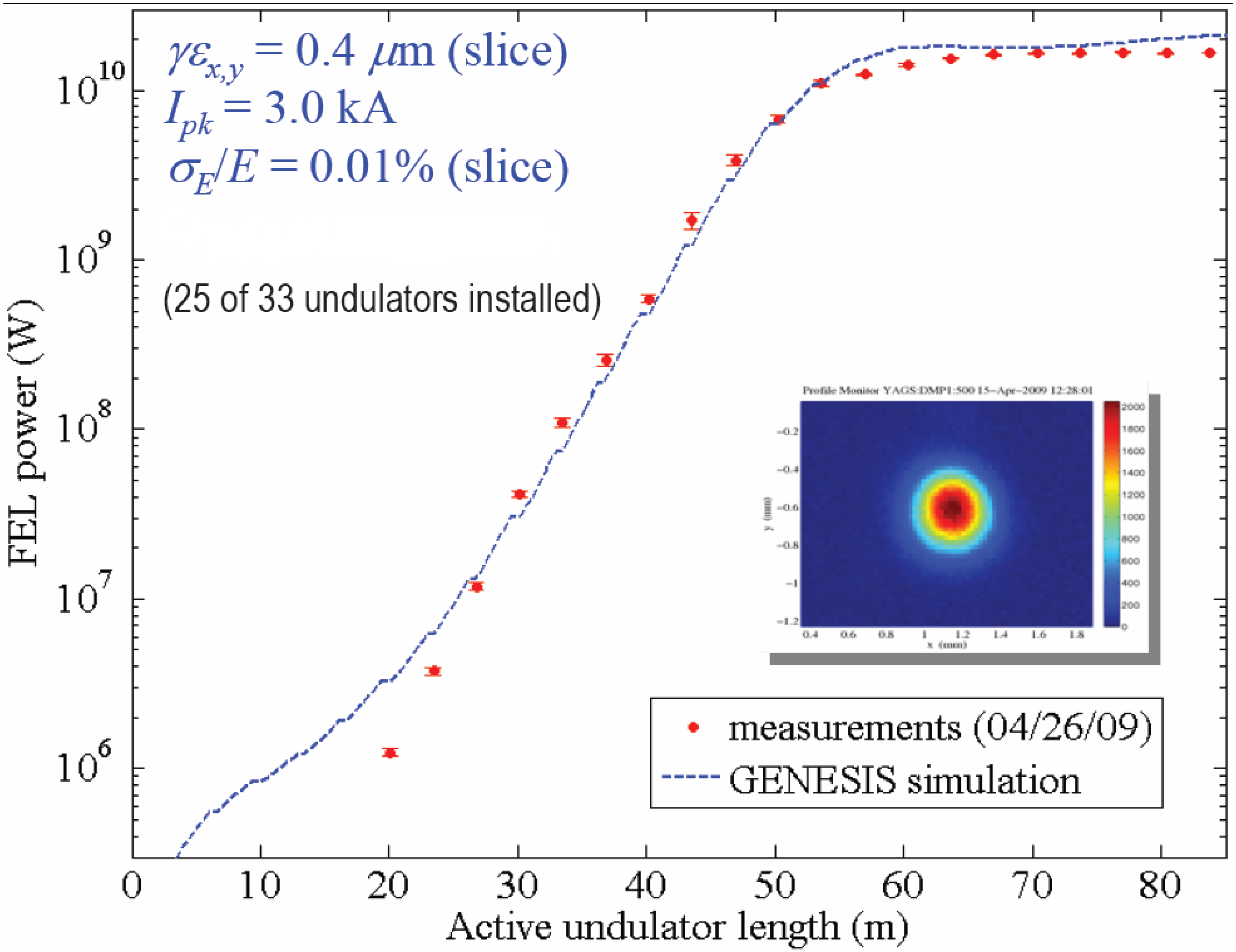
X-rays FELs

LCLS	0.15 nm	14 GeV	S-band	120 Hz	SASE
SACLA	0.1 nm	8 GeV	C-band	60 Hz	SASE
XFEL	0.1 nm	17.5 GeV	SC L-band	CW (10 Hz)	SASE
Swiss-FEL	0.1 nm	5.8 GeV	C-band	120 Hz	SASE

FLASH	47-6.5 nm	1 GeV	SC L-band	1MHz (5Hz)	SASE
FERMI	40-4 nm	1.2 GeV	NC S-band	50 Hz	seeded HGHG
SPARX	40-3 nm	1.5 GeV	NC S-band	100 Hz	SASE/seeded
Wisconsin	1 nm	2.2 GeV	SC/CW L-band	1 MHz	seeded HHG
LBNL	100-1 nm	2.5 GeV	SC/CW L-band	1 MHz	seeded
MAX-LAB	5-1 nm	3.0 GeV	NC S-band	200 Hz	SASE/seeded
Shanghai	10 nm	0.8-1.3 GeV	NC S-band	10 Hz	seeded HGHG
NLS	20-1 nm	2.2 GeV	SC/CW L-band	1-1000 kHz	seeded HHG

Swiss-FEL	10 nm	2.1 GeV	NC S-band	120 Hz	SASE/seeded
LCLS-II	0.25 nm	4 GeV	SC L-band	1 MHz	seeded

LCLS lasing at 1.5 Å (April 2009)



Accelerator Physics challenges

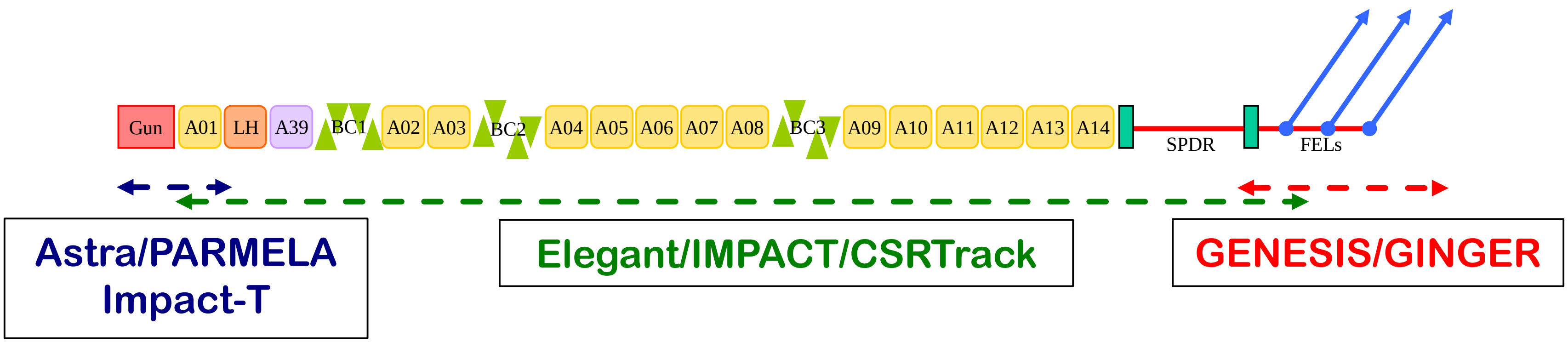
Soft X-ray are driven by high brightness electron beam

1 – 3 GeV	$\varepsilon_n \leq 1\ \mu m$
~ 1 kA	$\sigma_\gamma / \gamma \leq 10^{-4}$

This requires:

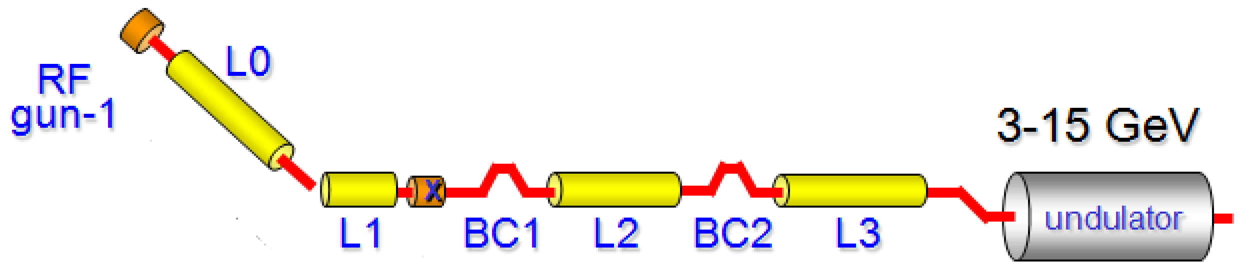
- **a low emittance gun (norm. emittance cannot be improved in the linac)**
- **acceleration and compression through the linac keeping the low emittance**

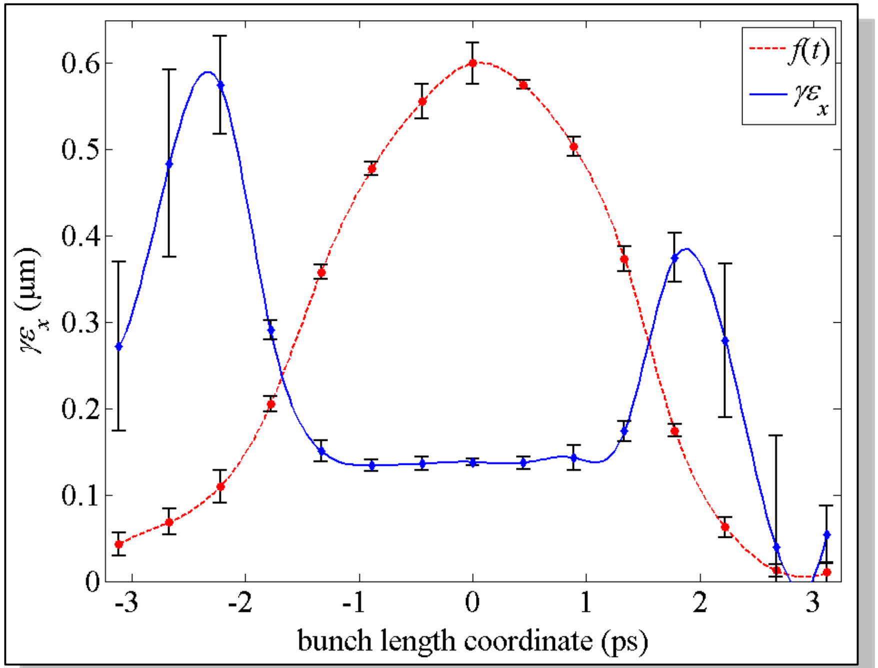
Optimisation validated by start-to-end simulation <u>Gun to FEL</u>



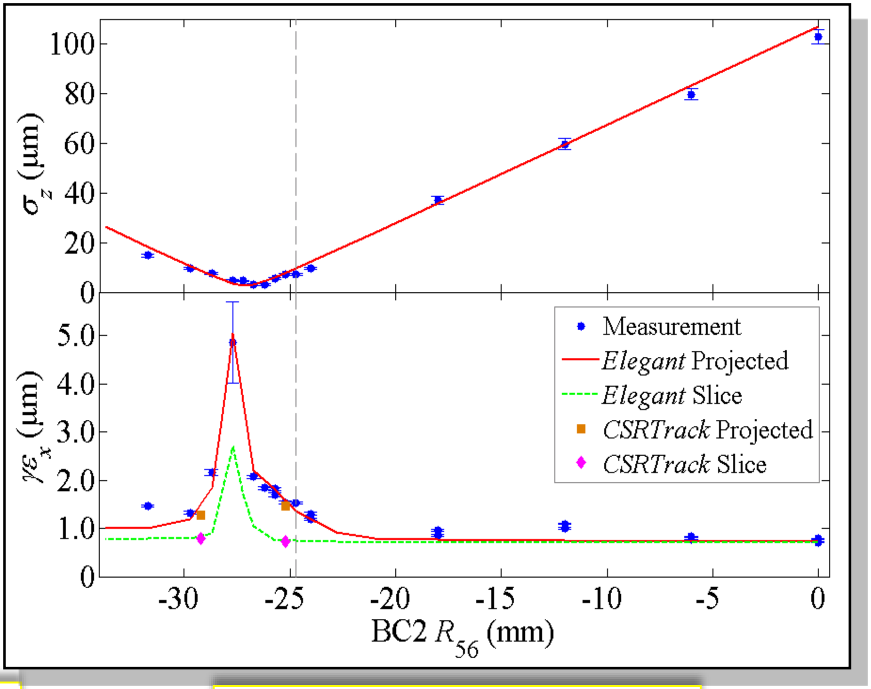
High brightness beam at LCLS

Managing collective effects with high brightness beams is a non trivial AP task





MEASURED SLICE EMITTANCE at 20 pC



CSR effects at BC2

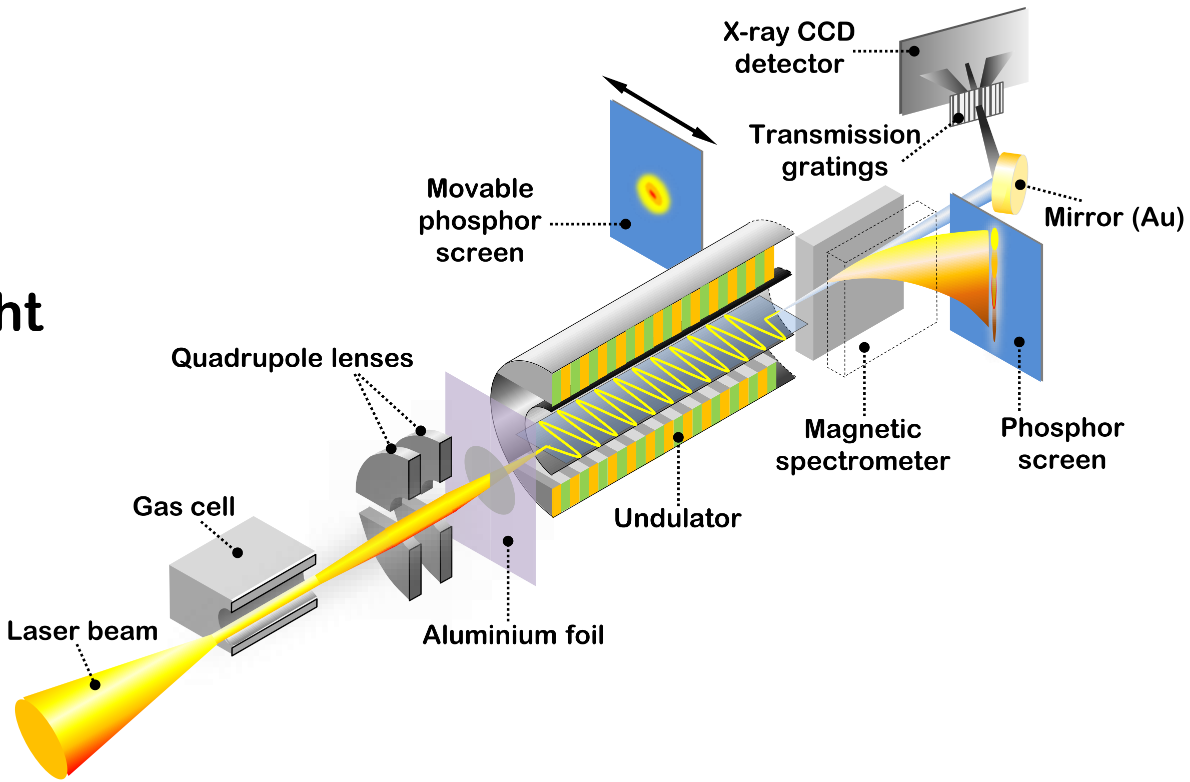
Beyond fourth generation light sources

The progress with laser plasma accelerators in the last years have open the possibility if using them for the generation for synchrotron radiation and even to drive a FELs

First observation of undulator radiation achieved in Soft X-ray

FEL type beam can be achieved with relatively modest improvements on what presently achieved and significant improvement on the stability of these beams

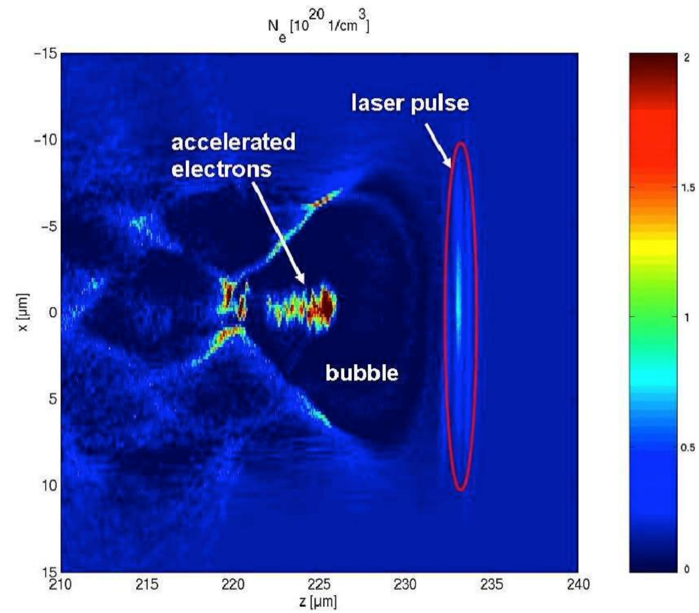
Layout of a compact light source driven by a LPWA



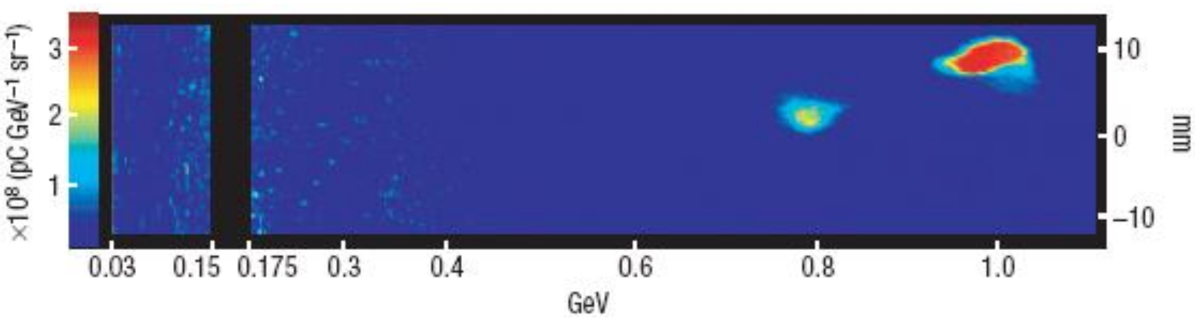
LBNL- Oxford experiment (2006)

Laser plasma wakefield accelerators demonstrated the possibility of generating GeV beam with promising electron beam qualities

Very large peak current makes up for poor energy spread in a possible FEL application



W. P. Leemans et al. *Nature Physics* **2** 696 (2006)



E = 1.0 +/-0.06 GeV
ΔE = 2.5% r.m.s
Δθ = 1.6 mrad r.m.s.
Q = 30 pC charge

Density 4.3 10^{18} cm^{-3}
Laser Power > 38 TW (73 fs) to 18 TW (40 fs)

Capillary: 310 μm
Laser: 40 TW
Density: 4.3 $\times 10^{18}$ cm^{-3}

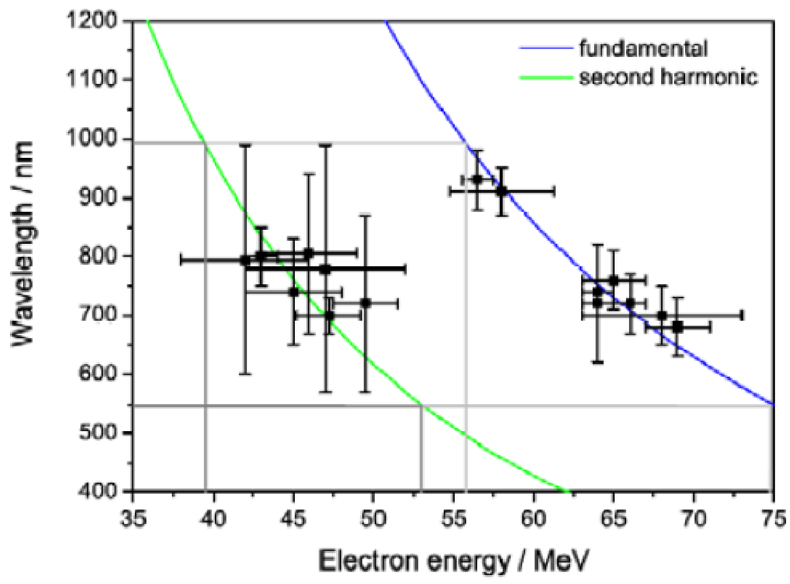
Undulator radiation from LPWA

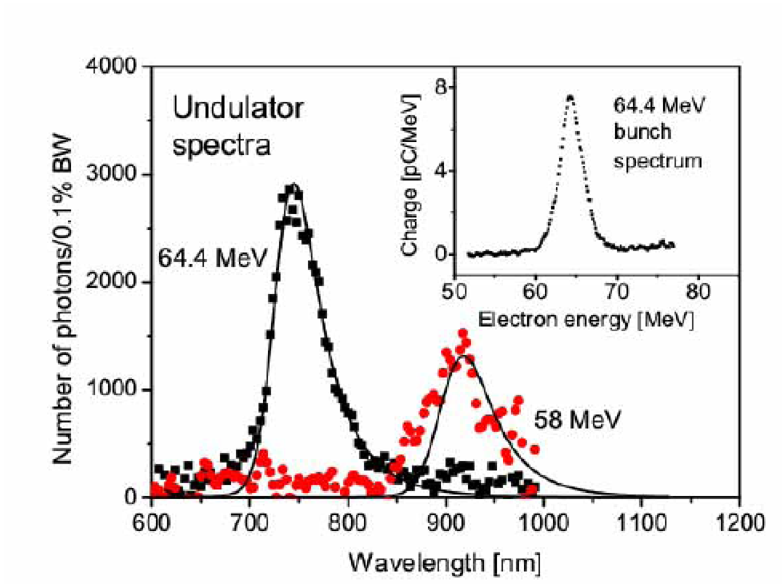
First combination of a laser-plasma wakefield accelerator, producing 55–75MeV electron bunches, with an undulator to generate visible synchrotron radiation

- Jena / Strathclyde / Stellenbosch experiment
- 55-70 MeV electrons
- VIS/IR synchrotron radiation

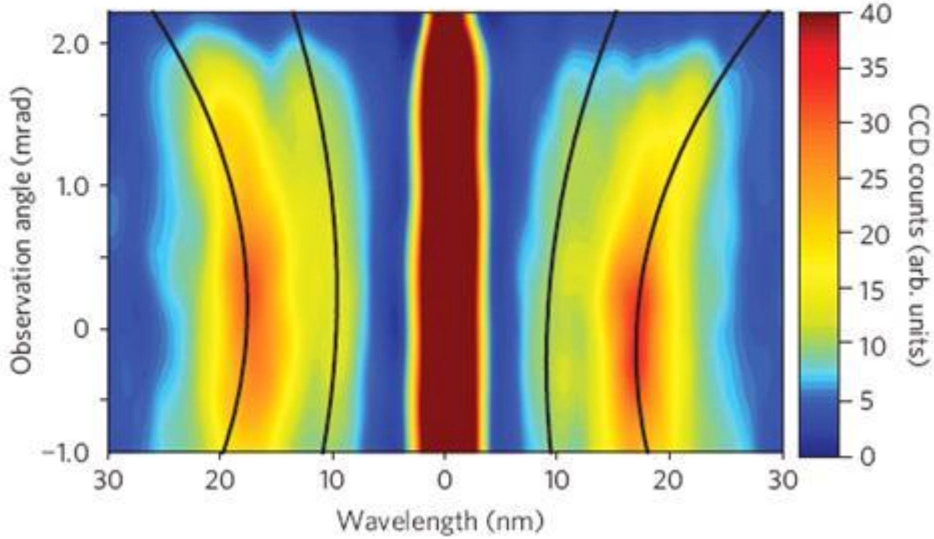
Schlenvoigt et al.,
Nature Phys. **4**, 130 (2008)

Gallacher et al.,
Phys. Plasmas **16**, 093102 (2009)

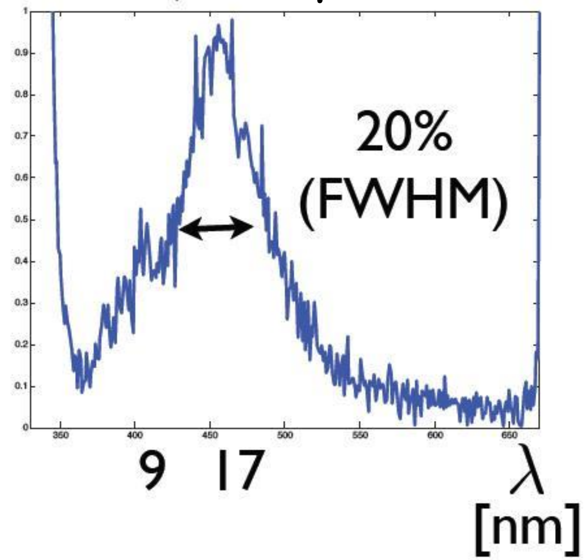




Undulator radiation Soft Xrays MPQ experiment

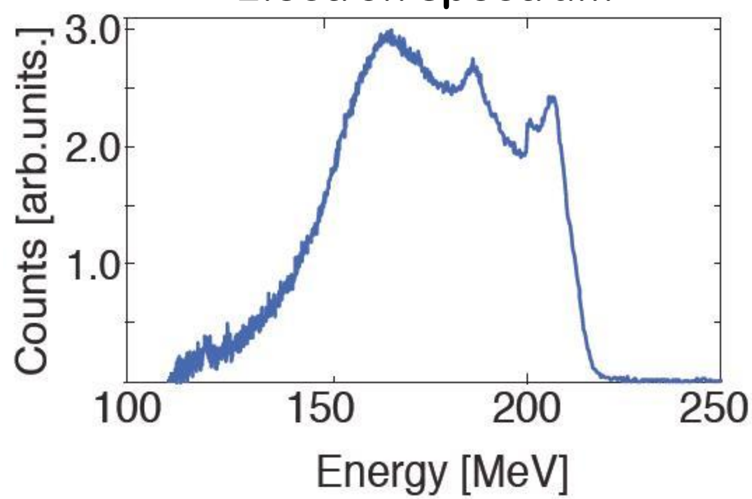


Radiation spectrum



$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \theta^2\gamma^2\right)$$

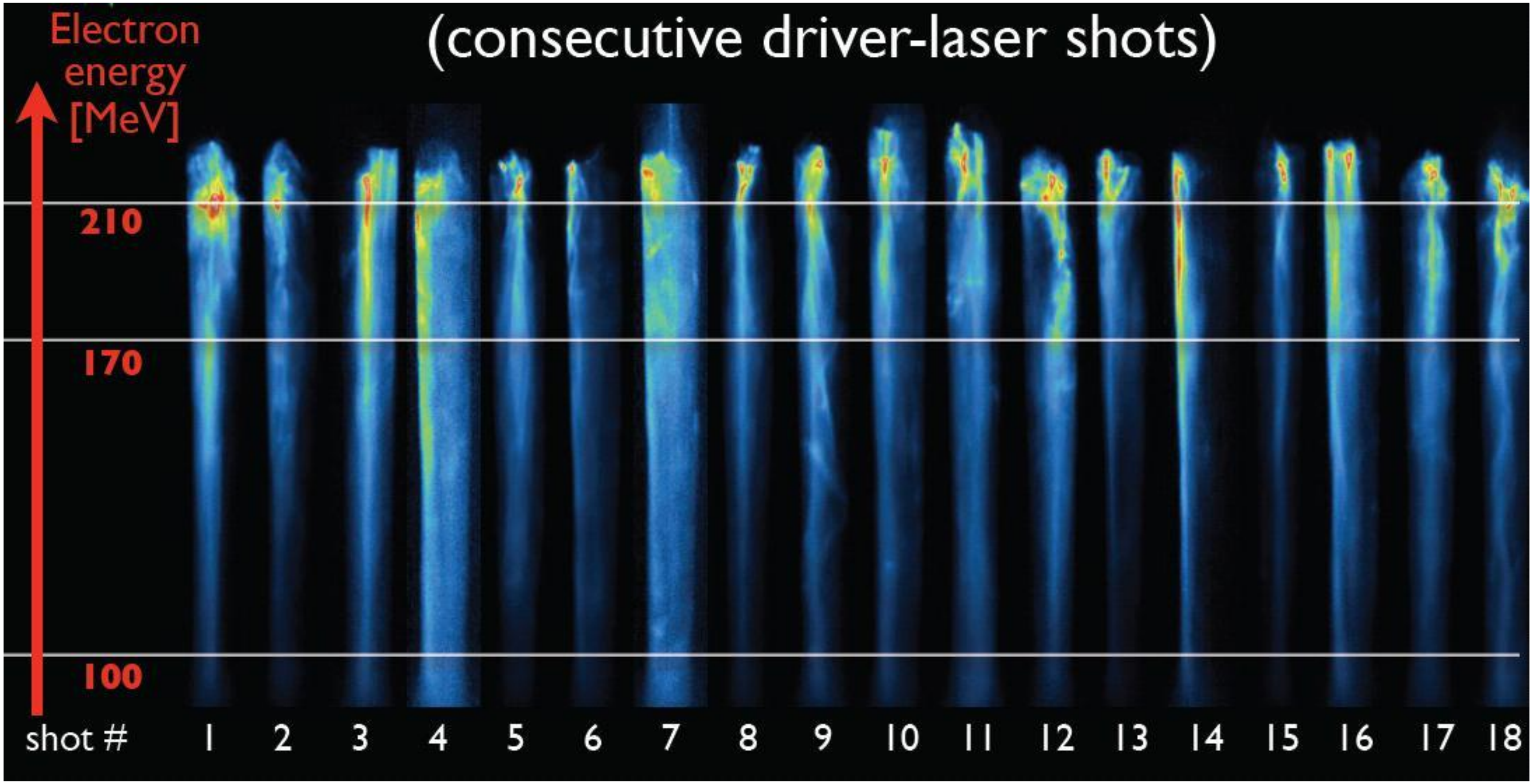
Electron spectrum



Spontaneous undulator radiation and off-axis dependence

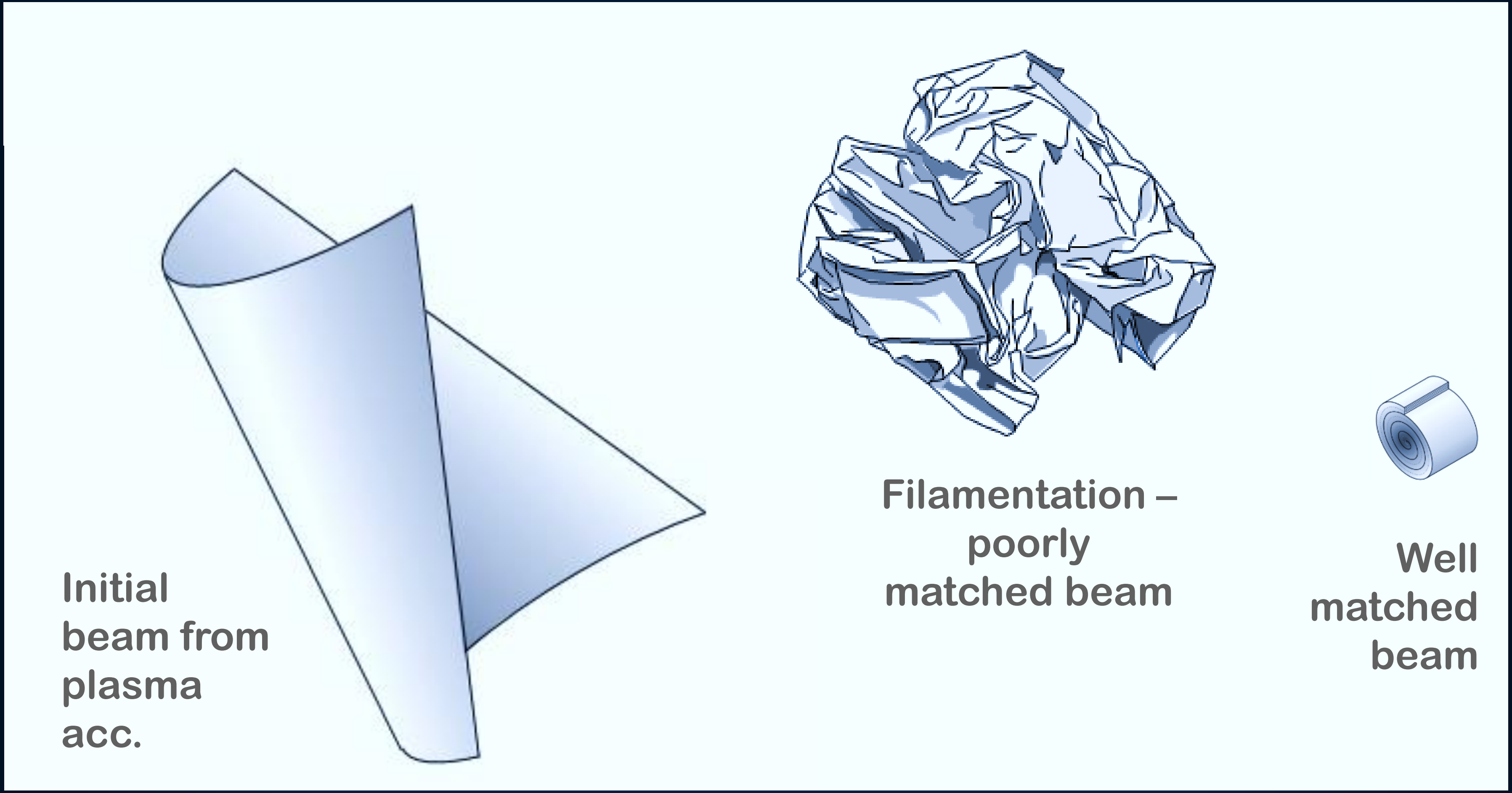
M. Fuchs et al, Nature Physics (2009)

Undulator radiation Soft Xrays – MPQ experiment



Stability of the electron beam quality is crucial for a successful FEL operation

Challenge of low-size, large divergence beams



For illustration of filamentation

3rd Gen SR sources and FEL - summary

- Third generation (storage rings) and FEL have complementary properties
  - SR are stable, serve many beamlines, approaching full transverse coherence with diffraction limited rings
    - Can have many tens of user beamlines
  - FEL have high brightness, short pulses, full transverse coherence
    - Can serve only a few beamlines at a time and very expensive
- New solutions are required to build more economic and compact radiation sources (table-top)
  - Laser plasma accelerators offer an interesting path
    - Require improvement in their beam quality – notably the energy spread from the actual few % to few 0.01 %

Summary of the lecture

- In this lecture we discussed
  - FEL Basic concept
    - Recall bend, wiggler and undulator radiation
    - Undulator parameter
    - Micro-bunching
    - Types
    - Oscillators
    - SASE
  - Examples
    - FEL from Linac
    - FEL form LPWAs