Lecture 2:

Storage Rings

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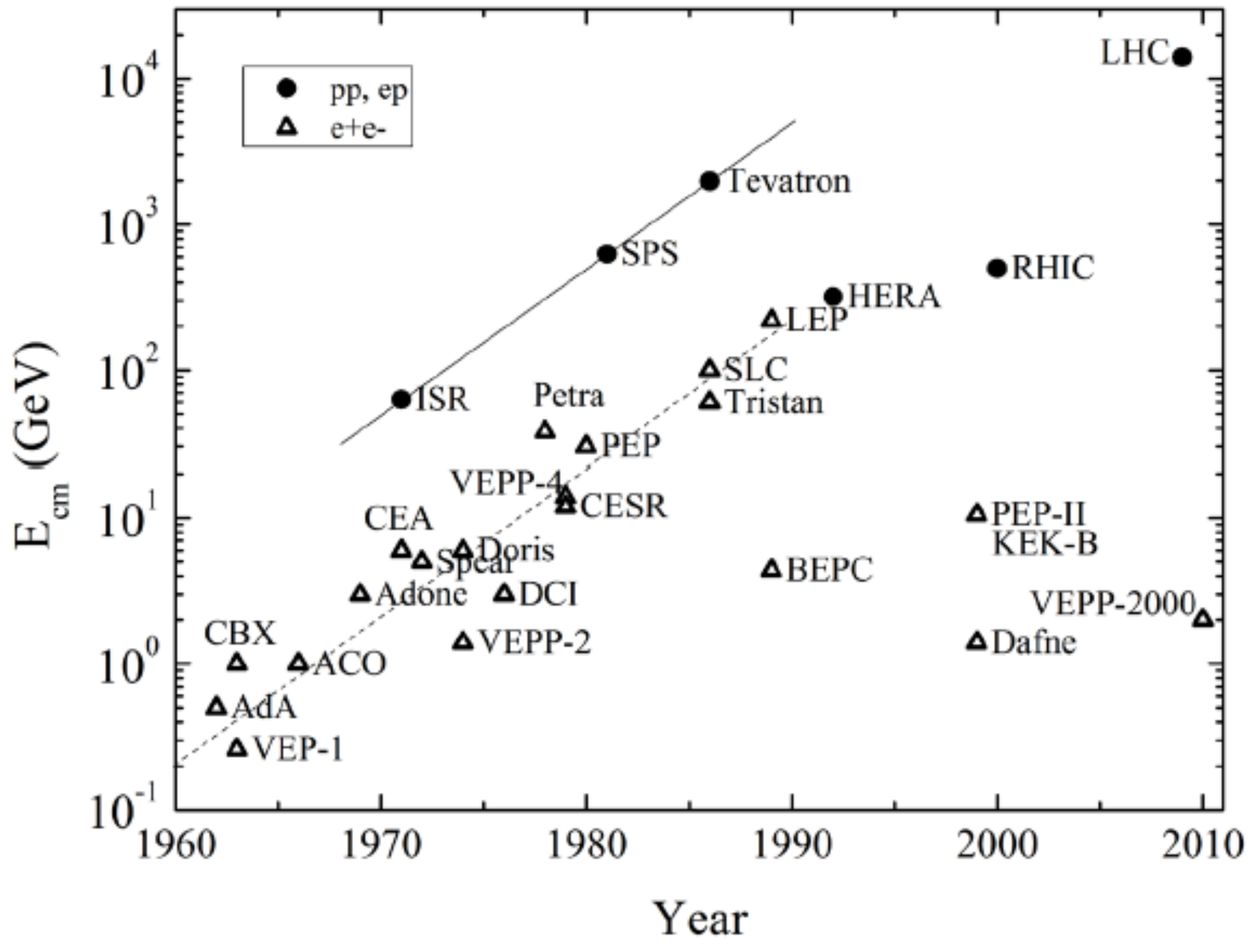
LHC

High-Energy Particle Colliders



V. Shiltsev,
Physics-Uspekhi, 2012

Third-Generation Light Sources

APS

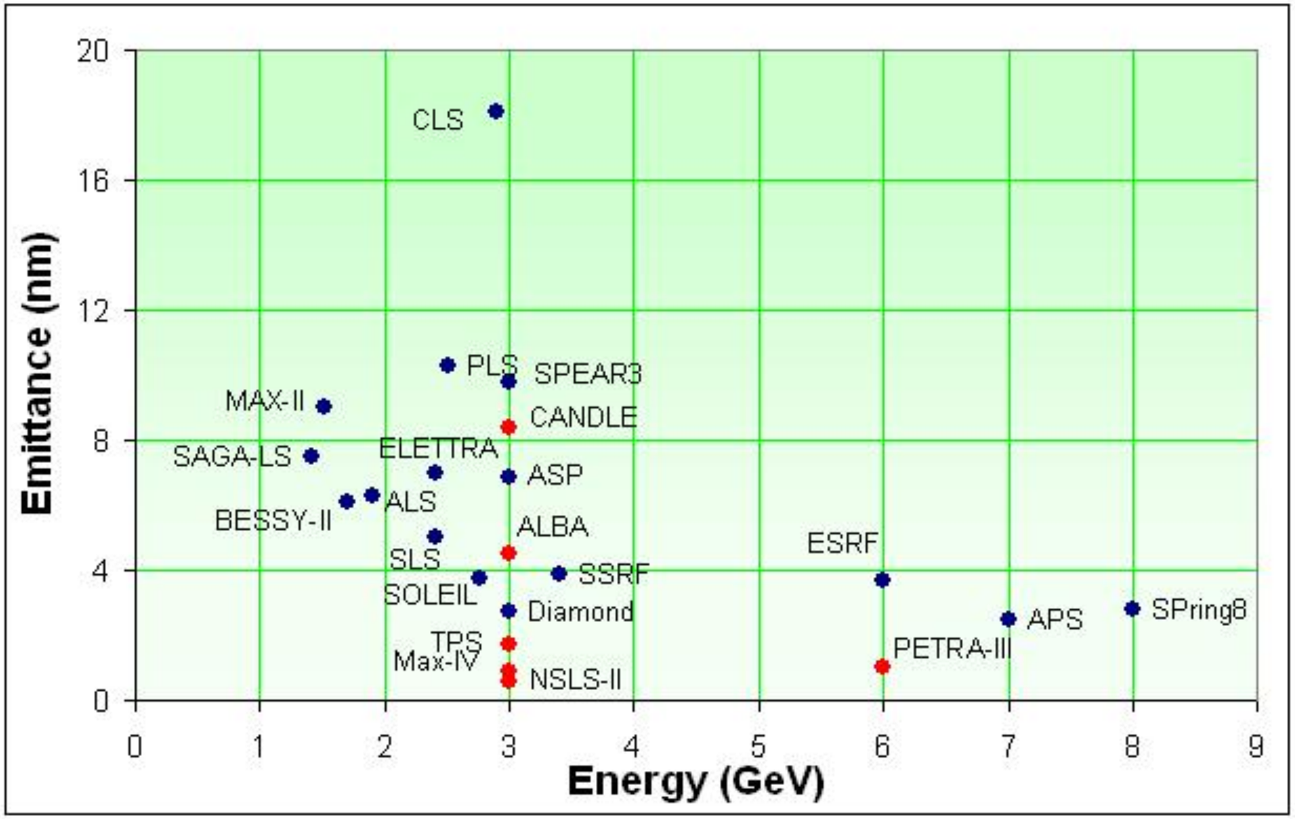
ALS

Credit: SSRF

SSRF

ESRF

Worldwide Electron Storage Rings



Courtesy of R. Bartolini, Low Emittance Rings Workshop, 2010, CERN

Physical Constants and Units

For electron:

Rest energy	$mc^2 = 0.51$	MeV
Classic radius	$r_e = \dfrac{e^2}{mc^2} = 2.82 \times 10^{-15}$	meter
Compton wavelength/2π	$\bar{\lambda}_e = \dfrac{\hbar}{mc} = r_e / \alpha = 3.86 \times 10^{-13}$	meter
Fine structure constant	$\alpha = \dfrac{e^2}{\hbar c} = \dfrac{1}{137}$	
Impedance of free space	$Z_0 = \dfrac{4\pi}{c} = 120\pi$	Ω
Alferov current	$I_A = \dfrac{ec}{r_e} = 17045$	A

Dynamics of Relativistic Particles

$$\boxed{\begin{aligned}\vec{E} &= -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\\ \vec{B} &= \nabla\times\vec{A}\end{aligned}}$$

Relative velocity $\beta = v/c,$

Lorentz factor $\gamma = \dfrac{1}{\sqrt{1-\beta^2}},$

Momentum $\vec{p} = \gamma m\vec{v},$

Energy $E = \gamma mc^2 = \sqrt{c^2p^2 + m^2c^4} = cp/\beta.$

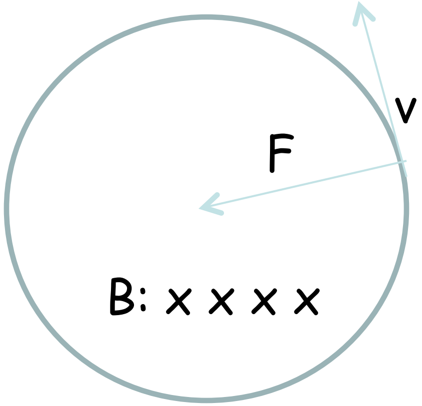
Equation of motion $\dfrac{d\vec{p}}{dt} = e(\vec{E} + \dfrac{\vec{v}}{c}\times\vec{B}),$ Lorentz force

Energy gain $\dfrac{dE}{dt} = e\vec{v}\cdot\vec{E}.$

in Uniform Magnetic Field

Equation of motion
$$\frac{d(\gamma m\vec{v})}{dt} = m\gamma\frac{d\vec{v}}{dt} = e\frac{\vec{v}}{c}\times\vec{B},$$

Assuming no velocity component in direction of B,



$$m\gamma\dot{v} = m\gamma\frac{v^2}{\rho} = evB/c$$

$$\Rightarrow \frac{pc}{e} = B\rho,$$

where ρ is the radius of the circular motion of the charged particle. This is the zeroth-order equation of circular accelerators. Bρ is called the magnetic rigidity.

1. Energy E=pc/β, so the higher energy the larger the ring.
2. Conversion: 1 GeV => 10/2.998 T-m.

LHC: 7 TeV => ρ=2.8 km and 27 km circumference, if B=8.36 T (p)

Hamiltonian of a Charged Particle in Electromagnetic field

The Hamiltonian is given by

$$H = e\phi + [m^2 c^4 + c^2 (\vec{p} - e\vec{A}/c)^2]^{1/2},$$

where p is the canonical momentum,

the one in the equation of motion

$$\vec{p} = \vec{P} + \frac{e}{c}\vec{A}.$$

The equation of motion is given by the Hamiltonian equation,

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}.$$

Here we have $(q_1, q_2, q_3) = (x,y,s)$ and $(p_1, p_2, p_3) = (p_x, p_y, p_s)$. They are a set of the first ordinary differential equations.

Hamiltonian Equation

Time t is the independent variable:

$$\frac{dx}{dt} = \frac{\partial H}{\partial p_x}, \frac{dp_x}{dt} = -\frac{\partial H}{\partial x},$$

$$\frac{dy}{dt} = \frac{\partial H}{\partial p_y}, \frac{dp_y}{dt} = -\frac{\partial H}{\partial y},$$

$$\frac{ds}{dt} = \frac{\partial H}{\partial p_s}, \frac{dp_s}{dt} = -\frac{\partial H}{\partial s}.$$

Path length s is the independent variable:

$$\frac{dx}{ds} = \frac{\partial \mathsf{H}}{\partial p_x}, \frac{dp_x}{ds} = -\frac{\partial \mathsf{H}}{\partial x},$$

$$\frac{dy}{ds} = \frac{\partial \mathsf{H}}{\partial p_y}, \frac{dp_y}{ds} = -\frac{\partial \mathsf{H}}{\partial y},$$

$$\frac{dt}{ds} = \frac{\partial \mathsf{H}}{\partial (-H)}, \frac{d(-H)}{ds} = -\frac{\partial \mathsf{H}}{\partial t}.$$

with

$$\mathsf{H} = -p_s(x, p_x, y, p_y, t, -H)$$

Derivation:

$$\frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = \frac{\partial H}{\partial p_x} / \frac{\partial H}{\partial p_s} = -\frac{\partial p_s}{\partial p_x} = \frac{\partial \mathsf{H}}{\partial p_x}$$

Scaled with Design Momentum

$$\frac{dx}{ds}=\frac{\partial \mathrm{H}\ /p_0}{\partial p_x/p_0},\frac{dp_x/p_0}{ds}=-\frac{\partial \mathrm{H}\ /p_0}{\partial x},$$

$$\frac{dy}{ds}=\frac{\partial \mathrm{H}\ /p_0}{\partial p_y/p_0},\frac{dp_y/p_0}{ds}=-\frac{\partial \mathrm{H}\ /p_0}{\partial y},$$

$$\frac{dt}{ds}=\frac{\partial \mathrm{H}\ /p_0}{\partial(-H/p_0)},\frac{d(-H/p_0)}{ds}=-\frac{\partial \mathrm{H}\ /p_0}{\partial t}.$$

with new scaled Hamiltonian

$$\mathrm{H}\ =-p_s(x,p_x,y,p_y,t,-H)/p_0$$

The form of the Hamiltonian equation is preserved.

Third Pair: Canonical Coordinate in Magnets

After scaling by p_0, a choice of the third pair of canonical coordinate is given by

$$\begin{array}{l} \dfrac{dt}{ds} = \dfrac{\partial H}{\partial(-E/p_0)}, \\ \dfrac{d(-E/p_0)}{ds} = 0. \end{array} \xrightarrow[E=cp/\beta]{} \begin{array}{l} \dfrac{d(vt)}{ds} = \dfrac{\partial H}{\partial(-p/p_0)}, \\ \dfrac{d(-p/p_0)}{ds} = 0. \end{array}$$

The third pair of canonical coordinate can be derived

$$\frac{d\delta}{ds} = 0,$$

$$\frac{d\ell}{ds} = -\frac{\partial H}{\partial \delta},$$

where l=vt and $\delta=(p-p_0)/p_0$.

Hamiltonian Using the Path Length *s* as Independent Variable in Rectangular Coordinate

The scaled Hamiltonian is suitable of quadrupole, sextupole, octopole, and skew quadrupole magnets is given by

$$H = -\frac{eA_s}{cp_0} - \sqrt{(1+\delta)^2 - p_x^2 - p_y^2},$$

where $\delta=(p-p_0)/p_0$ and p_0 is the reference momentum and A_s the component of the vector potential along the direction of propagation. For a storage ring, we choose $cp_0=eB\rho$ as shown previously.

Paraxial Approximation

$$\mathrm{H}_D = -\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}$$

$$= -(1+\delta)[1 - \frac{p_x^2}{(1+\delta)^2} - \frac{p_y^2}{(1+\delta)^2}]^{1/2}$$

paraxial approximation

$$\approx -(1+\delta)[1 - \frac{p_x^2}{2(1+\delta)^2} - \frac{p_y^2}{2(1+\delta)^2}]$$

$$= -(1+\delta) + \frac{p_x^2 + p_y^2}{2(1+\delta)}$$

Used in this course

The difference can be dealt with by symplectic integrators.

Hamiltonian and Transfer Map for a Drift

Use s as the independent variable, Hamiltonian in the paraxial approximation is given by

$$H_D = \frac{p_x^2 + p_y^2}{2(1+\delta)},$$

Solving the Hamiltonian equation, we obtain the transfer map of the drift:

$$x_f = x_i + \frac{p_{xi}}{1+\delta} L,$$

$$p_{xf} = p_{xi,}$$

$$y_f = y_i + \frac{p_{yi}}{1+\delta} L,$$

$$p_{yf} = p_{yi,}$$

$$\delta_f = \delta_{i,}$$

$$\ell_f = \ell_i + \frac{p_{xi}^2 + p_{yi}^2}{2(1+\delta)^2} L,$$

Where Λ is the length of the draft, subscript "i'" for the initial canonical coordinates and "f" for the final ones. One can show that it is indeed a symplectic map.

Symplecticity

A 6x6 matrix M is symplectic if it satisfies

$$\tilde{M} \cdot J \cdot M = J$$

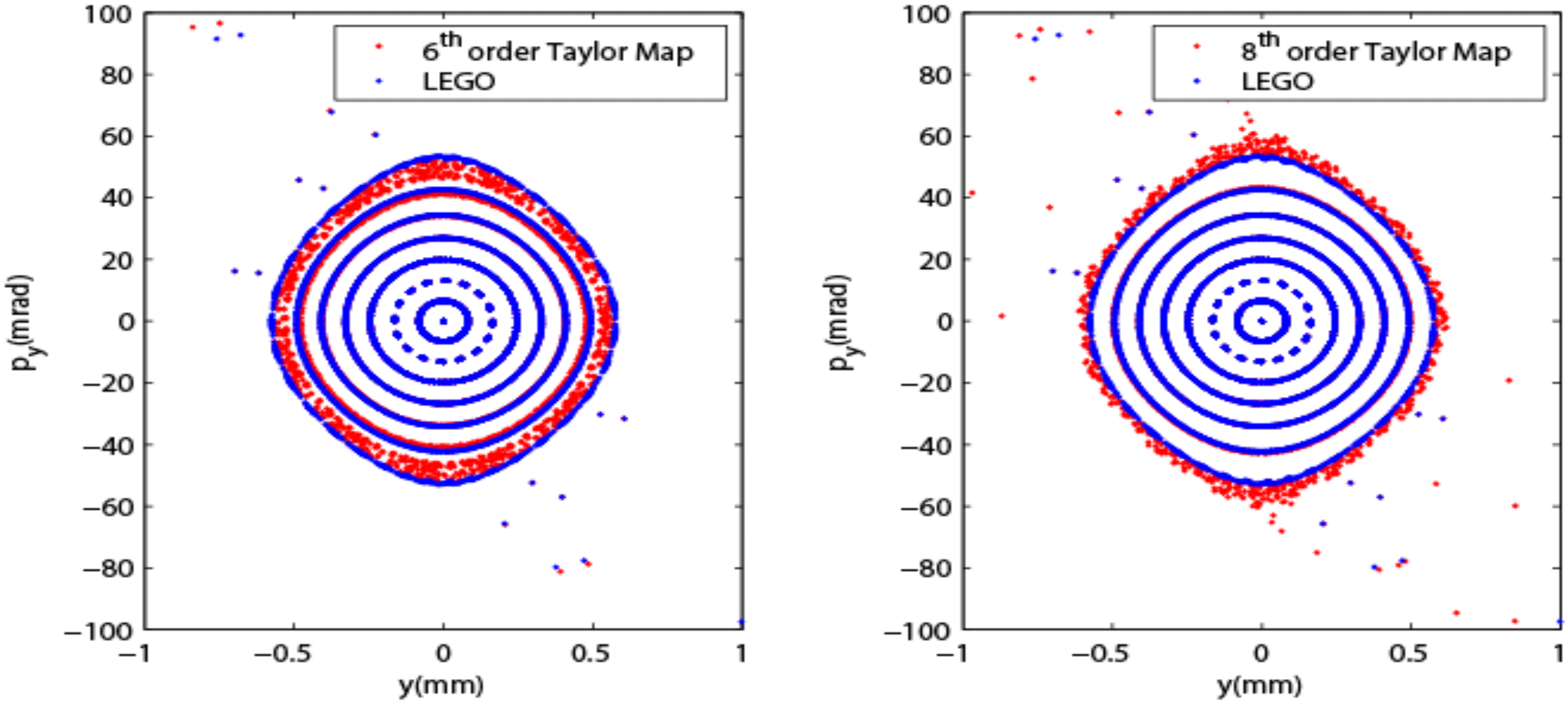
where

$$J = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

They form a group. It needs 21 independent parameters. A map is symplectic if its Jacob is sympectic.

Importance of Symplecticity

artificial damping or growth



Vector Potential of Magnets

A_x=A_y=0 and the component of vector potential along the propagating axis a

$$A_s = -Re[\sum_{n=1} \frac{1}{n}(b_n + ia_n)(x+iy)^n].$$

b_n and a_n for normal and skew components respectively. For a quadrupole magnet, we have

$$V_Q(x,y) = -\frac{A_s}{B\rho} = \frac{b_2}{2B\rho}(x^2 - y^2) = \frac{K_1}{2}(x^2 - y^2).$$

K_1 > 0, it focuses in x and defocuses in y. For a sextupole magnet, we have

$$V_S(x,y) = -\frac{A_s}{B\rho} = \frac{b_3}{3B\rho}(x^3 - 3xy^2) = \frac{K_2}{6}(x^3 - 3xy^2).$$

K_1, K_2 are the standard strengths for quadrupole and sextupole used in the program MAD.

Hamiltonian and Transfer Map for a Focusing Quadrupole Magnet

Use s as the independent variable, Hamiltonian in the paraxial approximation is given by

$$H_Q = \frac{1}{2(1+\delta)}(p_x^2 + p_y^2) + \frac{K_1}{2}(x^2 - y^2).$$

Solving the Hamiltonian equation, we obtain the transfer map of a focusing quadrupole:

$$\begin{aligned}
x_f &= x_i \cos(\kappa L) + \frac{p_{xi}}{\kappa(1+\delta)} \sin(\kappa L), \\
p_{xf} &= -\kappa(1+\delta) x_i \sin(\kappa L) + p_{xi} \cos(\kappa L), \\
y_f &= y_i \cosh(\kappa L) + \frac{p_{yi}}{\kappa(1+\delta)} \sinh(\kappa L), \\
p_{yf} &= \kappa(1+\delta) y_i \sinh(\kappa L) + p_{yi} \cosh(\kappa L), \\
\delta_f &= \delta_{i,} \\
\ell_f &= \ell_i + \Delta_Q(x_i, p_{xi}, y_i, p_{yi}, \delta_i, \ell_i),
\end{aligned}$$

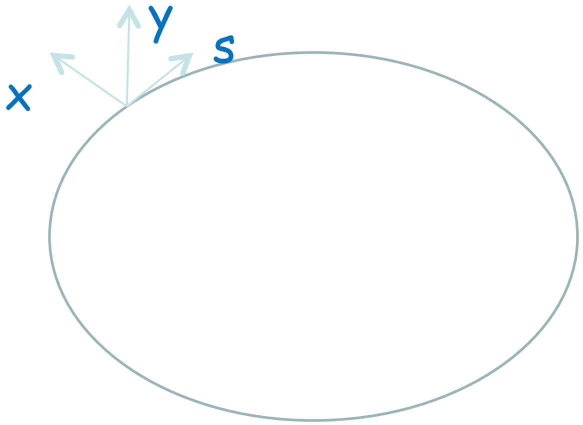
where Λ is the length of the quadrupole, $\kappa = \sqrt{K_1/(1+\delta)}$, the function Δ_Q in the path length can be found in ref. *Nucl. Inst. Meth. A645:168-174, 2011.*

Hamiltonian of Sector Bending Magnet

Similarly, the scaled Hamiltonian of a sector bending magnet can be derived using a curved coordinate system. Under the paraxial approximation, it is given by

$$H_D = \frac{x}{\rho} + \frac{x^2}{2\rho^2} - (1 + \frac{x}{\delta})\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}.$$

Here we have assumed that the magnetic field B matches with the bending radius r, namely $cp_0 = eB\rho$. The first term generates the dispersion and the second gives little focusing in the horizontal plane.



Sign convention:

s is the particle moving direction.
For a positive charge e, B_y is also positive.

Hamiltonian and Transfer Map for a Sector Bending Magnet

Use s as the independent variable, Hamiltonian in the paraxial approximation is given by

$$H_D = -(1+\frac{x}{\rho})\delta + \frac{x^2}{2\rho^2} + \frac{p_x^2 + p_y^2}{2(1+\delta)}.$$

Solving the Hamiltonian equation, we obtain the transfer map of a sector bend:

$$x_f = x_i \cos(\kappa L) + \frac{p_{xi}}{\kappa(1+\delta_i)} \sin(\kappa L) + \rho\delta_i(1-\cos(\kappa L)),$$

dispersion

$$p_{xf} = -\kappa(1+\delta)x_i \sin(\kappa L) + p_{xi}\cos(\kappa L) + \kappa(1+\delta_i)\rho\delta_i \sin(\kappa L),$$

$$y_f = y_i + \frac{p_{yi}}{(1+\delta_i)}L,$$

$$p_{yf} = p_{yi}$$

$$\delta_f = \delta_{i,}$$

Relative to the designed value

$$\ell_f = \ell_i + L(1-\frac{1}{\beta_0}) + \Delta_D(x_i, p_{xi}, y_i, p_{yi}, \delta_i, \rho, L),$$

where L is the length of the quadrupole, $\kappa = 1/(\rho\sqrt{(1+\delta)}),$ the function Δ_D in the path length can be found ref. *Nucl. Inst. Meth. A645:168-174, 2011.*

PEP-II Magnets

Low Energy Ring

- Positron energy 3.1 GeV
- Beam current 3 A
- 90^0 cell

Courtesy of T. Henderson, 1996

Dipole, 0.75 T

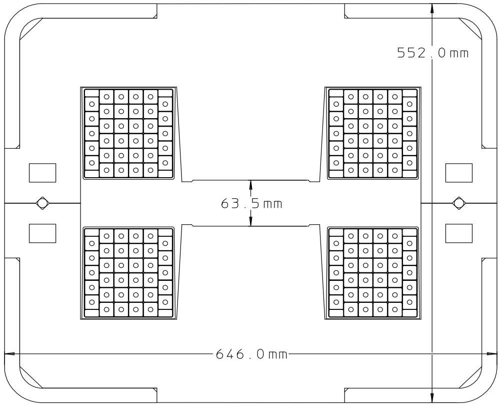


Figure 3: Dipole Layout.

Quadrupole, 4.5 T/m

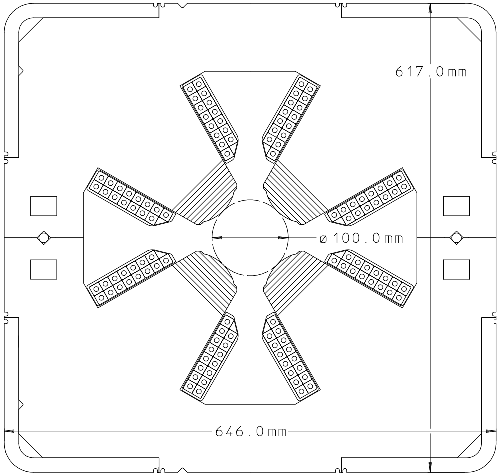


Figure 2: Quadrupole Layout with 15 turn per pole Coil.

Transporting Matrices

1. Drift with length L:

$$\begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2. Focusing quadrupole with length L and strength K

$$\begin{pmatrix} \cos(L\sqrt{K}) & \frac{1}{\sqrt{K}}\sin(L\sqrt{K}) & 0 & 0 & 0 & 0 \\ -\sqrt{K}\sin(L\sqrt{K}) & \cos(L\sqrt{K}) & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh(L\sqrt{K}) & \frac{1}{\sqrt{K}}\sinh(L\sqrt{K}) & 0 & 0 \\ 0 & 0 & \sqrt{K}\sinh(L\sqrt{K}) & \cosh(L\sqrt{K}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

3. Sector bend with radius ρ and length L

$$\begin{pmatrix} \cos\frac{L}{\rho} & \rho\sin\frac{L}{\rho} & 0 & 0 & \rho(1-\cos\frac{L}{\rho}) & 0 \\ -\frac{1}{\rho}\sin\frac{L}{\rho} & \cos\frac{L}{\rho} & 0 & 0 & \sin\frac{L}{\rho} & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \sin\frac{L}{\rho} & \rho(1-\cos\frac{L}{\rho}) & 0 & 0 & L-\rho\sin\frac{L}{\rho} & 1 \end{pmatrix}$$

Canonical coordinates used:

$z=(x, p_x, y, p_y, \delta, l)$
and $\delta=(p-p_0)/p_0$.

vt

Parameterization of Periodic Map

Given a transporting matrix of an ideal lattice,

$$M = \begin{pmatrix} M_{12} & M_{12} & 0 & 0 & D_x & 0 \\ M_{21} & M_{22} & 0 & 0 & D_{px} & 0 \\ 0 & 0 & M_{33} & M_{34} & 0 & 0 \\ 0 & 0 & M_{43} & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ M_{61} & M_{62} & 0 & 0 & M_{65} & 1 \end{pmatrix}$$

Solving the fixed point condition, we have the periodical dispersion,

$$\begin{pmatrix} \eta_x \\ \eta_{px} \end{pmatrix} = [\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}]^{-1} \begin{pmatrix} D_x \\ D_{px} \end{pmatrix}$$

Courant-Synder Parameters

One-turn matrix:

$$M_x = \begin{pmatrix} \cos\mu_x + \alpha_x \sin\mu_x & \beta_x \sin\mu_x \\ -\gamma_x \sin\mu_x & \cos\mu_x - \alpha_x \sin\mu_x \end{pmatrix}$$

Rotation matrix:

$$R_x = \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix}$$

There are related by a similarity transformation:

$$\mu_x = 2\pi\nu_x$$

betatron tune

$$M_x = A_x R_x A_x^{-1}$$

where A_x^{-1} is a transformations from the physical to the normalized coordinates:

$$A_x^{-1} = \begin{pmatrix} \frac{1}{\sqrt{\beta_x}} & 0 \\ \frac{\alpha_x}{\sqrt{\beta_x}} & \sqrt{\beta_x} \end{pmatrix}, A_x = \begin{pmatrix} \sqrt{\beta_x} & 0 \\ \frac{-\alpha_x}{\sqrt{\beta_x}} & \frac{1}{\sqrt{\beta_x}} \end{pmatrix}$$

All these matrices are symplectic. However, the transformation matrix A_x is not quite unique because of the commuting property of the rotational matrices.

Linear Normal Form

The one-turn matrix can be diagonalized in terms of blocks:

$$M = AFA^{-1}$$

where $A=A_\eta A_\beta$ and

$$F = \begin{pmatrix} \cos\mu_x & \sin\mu_x & 0 & 0 & 0 & 0 \\ -\sin\mu_x & \cos\mu_x & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\mu_y & \sin\mu_y & 0 & 0 \\ 0 & 0 & -\sin\mu_y & \cos\mu_y & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \end{pmatrix}$$

$$A_\eta = \begin{pmatrix} 1 & 0 & 0 & 0 & \eta_x & 0 \\ 0 & 1 & 0 & 0 & \eta_{px} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \eta_{px} & -\eta_x & 0 & 0 & 0 & 1 \end{pmatrix}$$

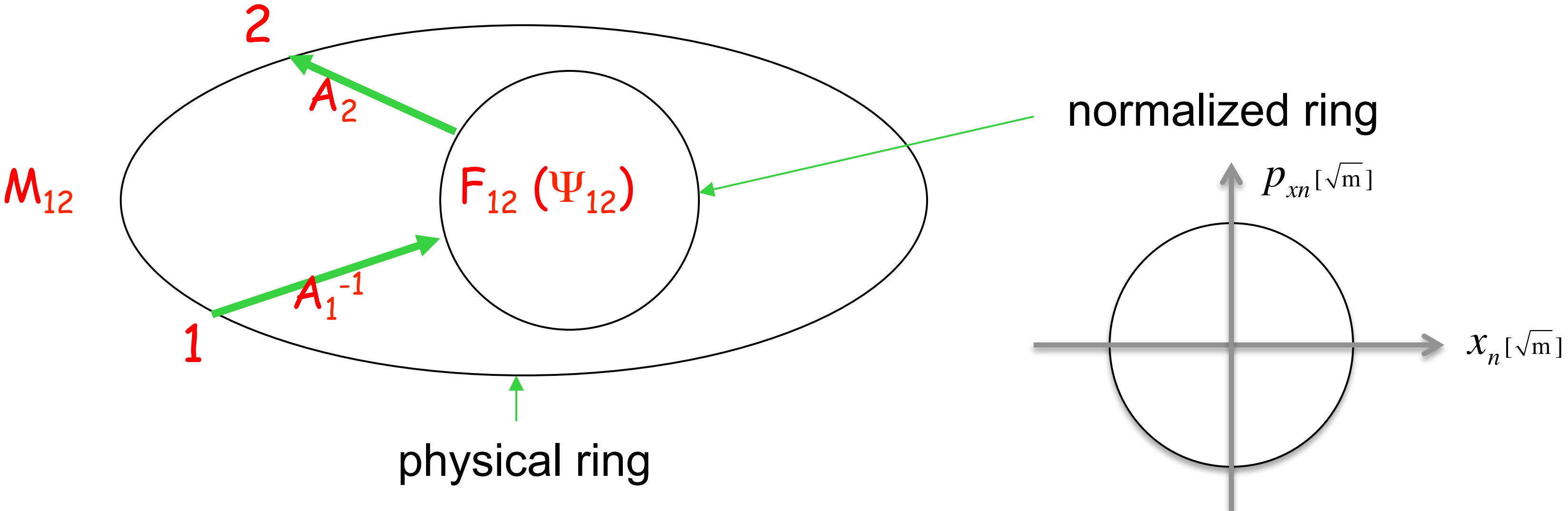
$$A_\beta = \begin{pmatrix} \sqrt{\beta_x} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\alpha_x}{\sqrt{\beta_x}} & \frac{1}{\sqrt{\beta_x}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\beta_y} & 0 & 0 & 0 \\ 0 & 0 & -\frac{\alpha_y}{\sqrt{\beta_y}} & \frac{1}{\sqrt{\beta_y}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

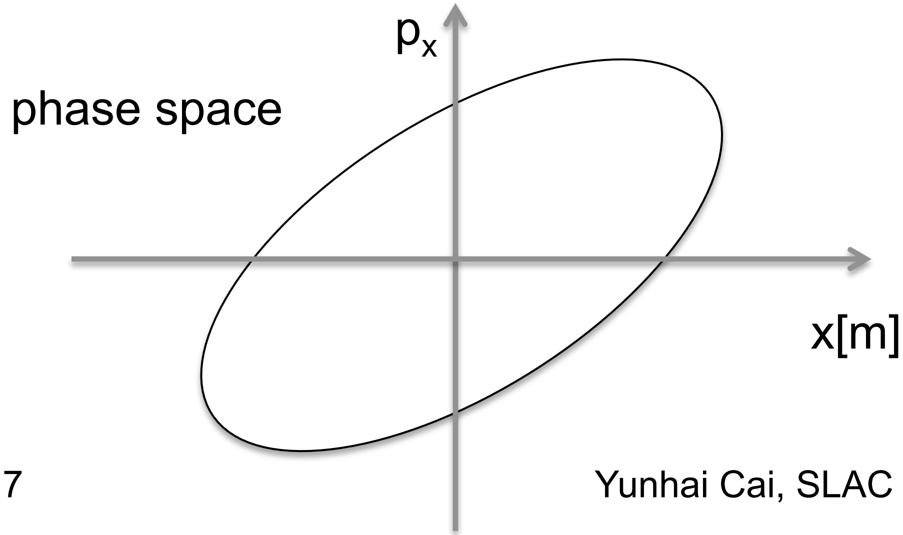
In particular,

curly H ↓

$$\lambda = M_{65} + (\beta_x\eta_{px}^2 + 2\alpha_x\eta_x\eta_{px} + \gamma_x\eta_x^2)\sin\mu_x$$

Visualization of Normal Form





Tune is an invariance.
But beta function changes from position to position.

Sextupoles

Quadrupoles introduce chromaticity. To correct the chromaticity, we have to introduce sextupoles. The sextupole potential relative to a dispersive orbit, $\eta_x\delta$ is given by,

$$V_S(x,y) = \frac{K_2}{6}[(x+\eta_x\delta)^3 - 3(x+\eta_x\delta)y^2]$$

$$= \frac{K_2}{6}[x^3 - 3xy^2] + \frac{K_2\eta_x\delta}{2}(x^2 - y^2) + \ldots$$

Price to pay

A chromatic quadrupole

For local chromatic correction of a quadrupole, one should set,

$K_2 = K_1 / \eta_x$

Large dispersion at sextupole position helps to reduce the nonlinearity.

Conversion between
(δ,l) and (E/p_0,t)

If there is a RF cavity in the system, it is easier to use E/p_0 and t as the third pair of canonical variable. On the other hand, it is easier to use (δ,l) in magnets as we have shown. So the conversion is necessary.

From (δ,l) to (E/p_0,t):

$$\frac{E}{p_0} = \sqrt{c^2(1+\delta)^2 + m^2c^4/p_0^2},$$

$$\gamma = \frac{E}{mc^2},$$

$$\beta = \sqrt{1-1/\gamma^2},$$

$$t = \frac{\ell}{c\beta}.$$

From (E/p_0,t) to (δ,l) :

$$\gamma = \frac{E}{mc^2},$$

$$\beta = \sqrt{1-1/\gamma^2},$$

$$\delta = \frac{\sqrt{E^2 - m^2c^4}}{cp_0} - 1,$$

$$\ell = c\beta t.$$

As long as there is no RF system in between, it is an inverse operation.

Conversion between
(δ,l) and (E/p_0,t)

If there is a RF cavity in the system, it is easier to use E/p_0 and t as the third pair of canonical variable. On the other hand, it is easier to use (δ,l) in magnets as we have shown. So the conversion is necessary.

From (δ,l) to (E/p_0,t):

$$u = \frac{mc^2}{p_0}\sqrt{1+(\frac{p_0}{mc})^2(1+\delta)^2},$$

$$t = \frac{m\ell}{(1+\delta)p_0}\sqrt{1+(\frac{p_0}{mc})^2(1+\delta)^2},$$

From (E/p_0,t) to (δ,l) :

$$\delta = \frac{mc}{p_0}\sqrt{(\frac{up_0}{mc^2})^2-1}-1,$$

$$\ell = \frac{mc^3 t}{up_0}\sqrt{(\frac{up_0}{mc^2})^2-1},$$

where u=E/p_0. It is easy to show that they are symplectic transformations.

Energy Gain in RF Cavity

From
$$\frac{dE}{dt} = e\vec{v}\cdot\vec{E},$$
$$\Rightarrow dE = eE_z dz,$$
$$\Rightarrow \Delta E = \int eE_z\, dz' = eV_{RF}(\underset{\uparrow}{-}t).$$

Arrival time (minus of time of flight)

With a proper choice of the RF cavity, we obtain

$$f_0 = c / C,$$
$$f_{RF} = hf_{0,}$$
$$\omega_{RF} = 2\pi f_{RF},$$

Energy gain:
$$E_f = E_i + eV_{RF}\sin(\omega_{RF}(-t_i) + \varphi_s).$$

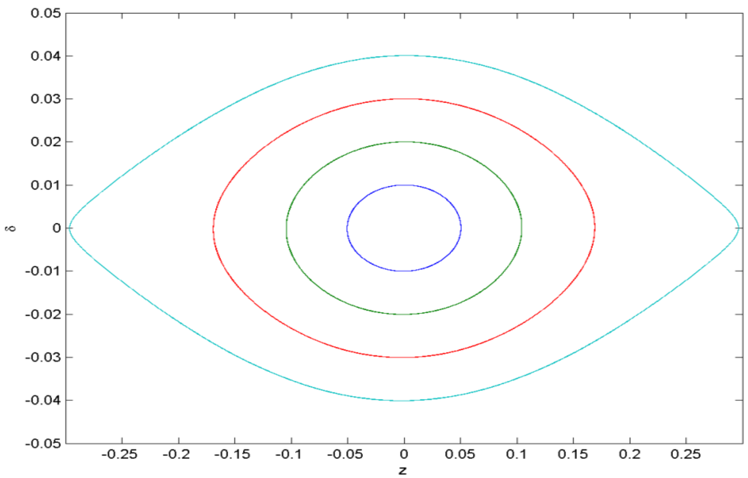
Clearly, it is a symplectic system.

RF Cavity and Synchrotron Oscillation

For a single RF in a ring, every turn we have

$$\begin{cases} \beta_{n+1}^2\delta_{n+1} = \beta_n^2\delta_n + \dfrac{eV_{RF}}{E_0}\sin(k_{RF}z_n + \varphi_s) \\ z_{n+1} = z_n - \eta C\delta_{n+1} \end{cases}$$

RF Bucket



$\eta=\alpha_p-1/\gamma^2$, is the “slip factor” and a the momentum compaction factor. Expand small z,

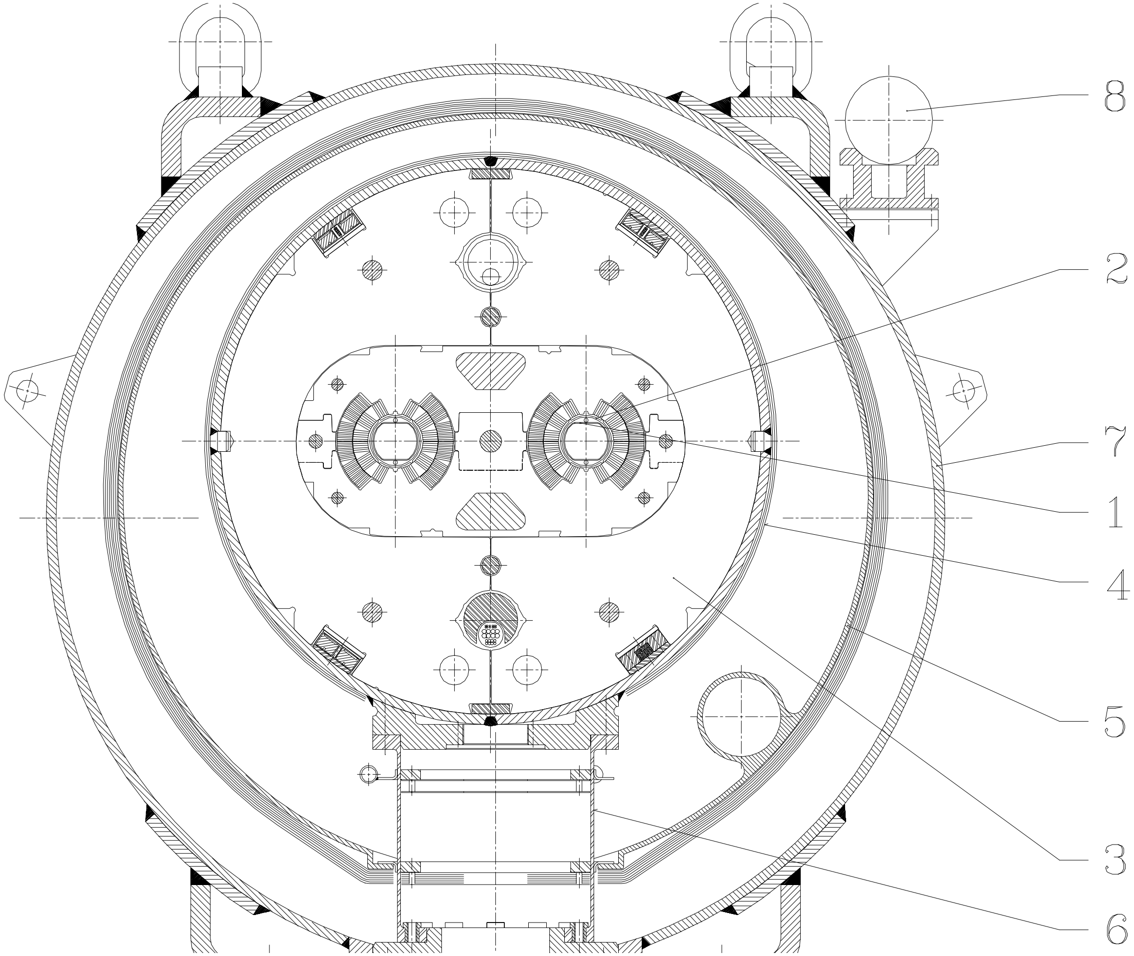
$$\begin{cases} \dot{\delta} = \dfrac{eV_{RF}k_{RF}}{\beta^2 T_0 E_0}\cos\varphi_s z \\ \dot{z} = -\dfrac{\eta C}{T_0}\delta \end{cases}$$

Synchrotron tune is given by

$$\upsilon_s = \sqrt{\frac{h\eta}{2\pi}\frac{eV_{RF}}{\beta^2 E_0}\cos\varphi_s},$$

where $\omega_s=\nu_s\omega_0$.

LHC Superconducting Dipoles



B: 8.36 T
T: 1.9 K
L: 14.2 m
A: 56 mm

Courtesy of D. Leroy, CERN

Luminosity

- Bunch luminosity

$$L_b = f_{rev}\frac{N_b^2}{4\pi\sigma_x\sigma_y}R_g$$

where R_g is a geometrical reduction from the hourglass effect and crossing angle. It is also a good indicator for dynamical effects at high bunch charge.

- Total luminosity:

$$L = n_b L_b$$

Beam-Beam Limit

- For round beams , the beam-beam parameter is given by

$$\xi = \frac{r_p N_b \beta^*}{4\pi\gamma\sigma^2}$$

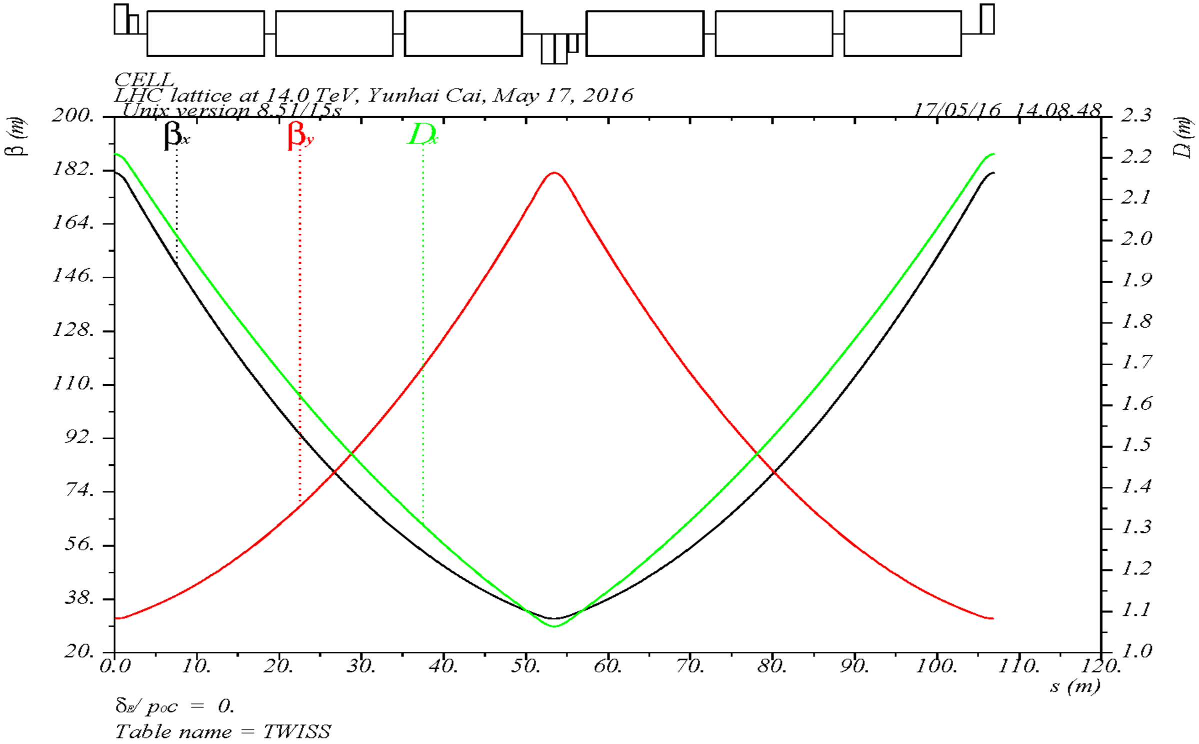
And the luminosity can be re-written as

$$L = \frac{cI\gamma\xi}{r_e r_p I_A \beta^*} R_g$$

where I_A=17045 A. We should expect twice of the luminosity increase from an energy doubler. A smaller β^* and larger ξ is also helpful. Taking an example of the LHC running at E_0=4 TeV in June 2012, we have I=0.37A, ξ=0.0075, R_g=0.82, β^*=0.6 m and the luminosity is $6.59\text{x}10^{33}$ $cm^{-2}s^{-1}$.

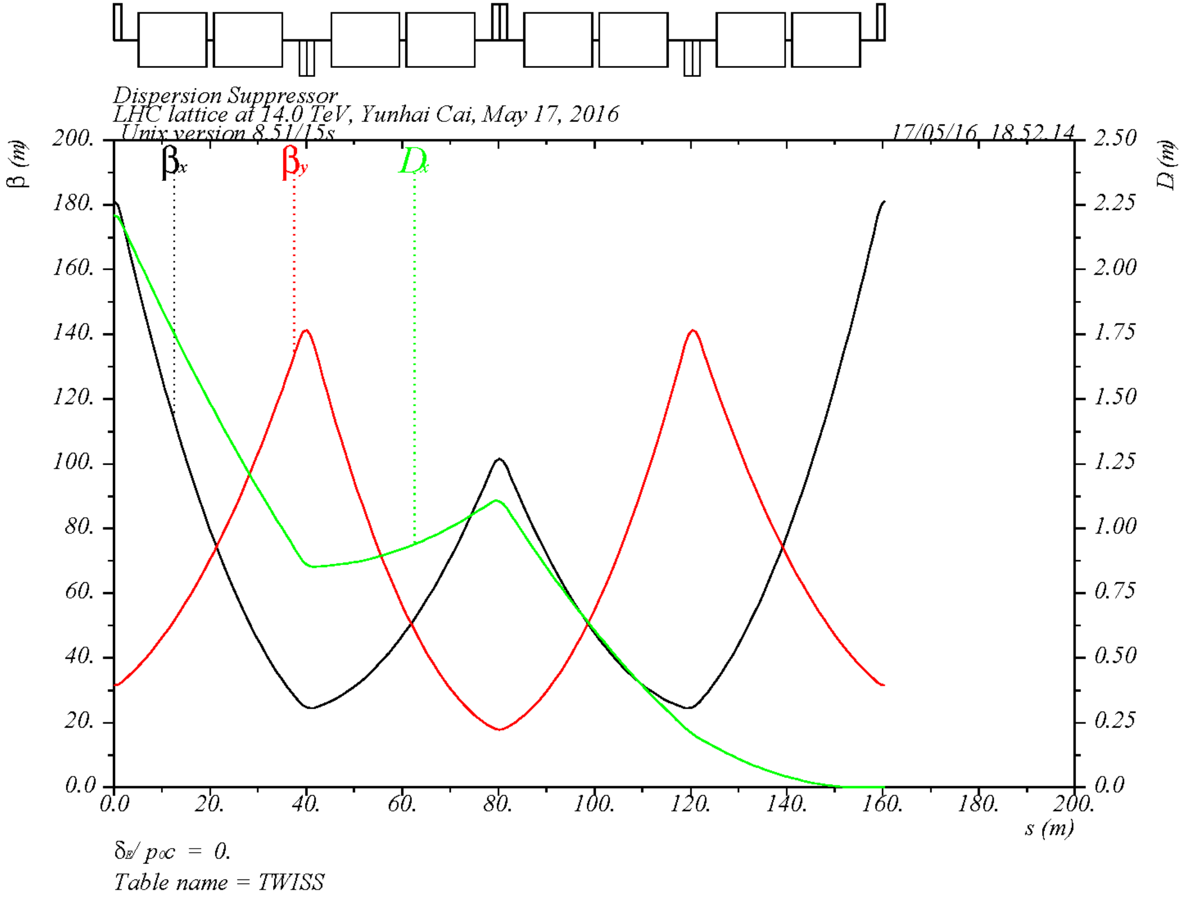
So what is the beam-beam parameter at 14 TeV of the beam energy?

LHC Cell to Double its Energy, with 60^0 Phase Advance

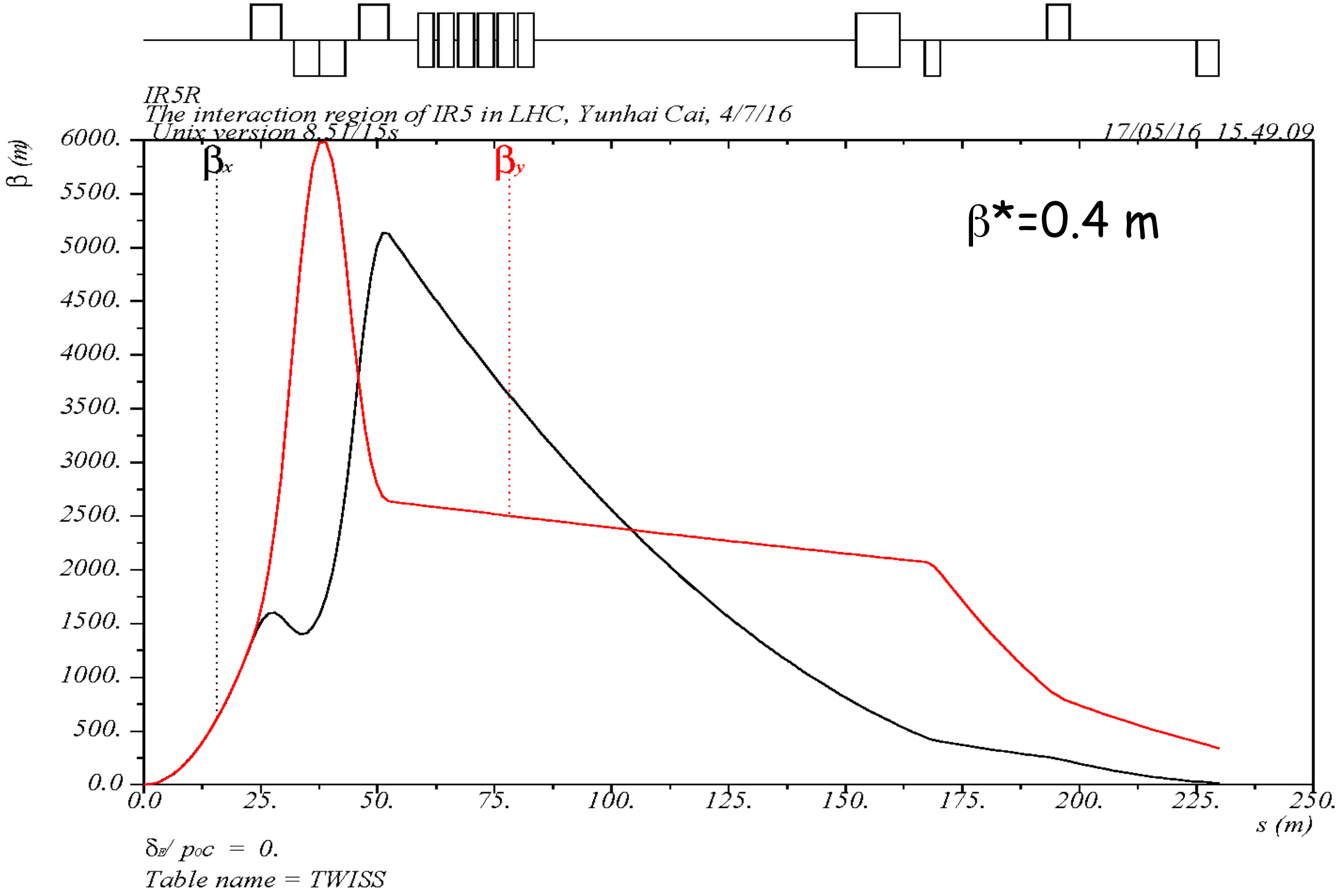


What are the optimal parameters?
Cell length? phase advance?

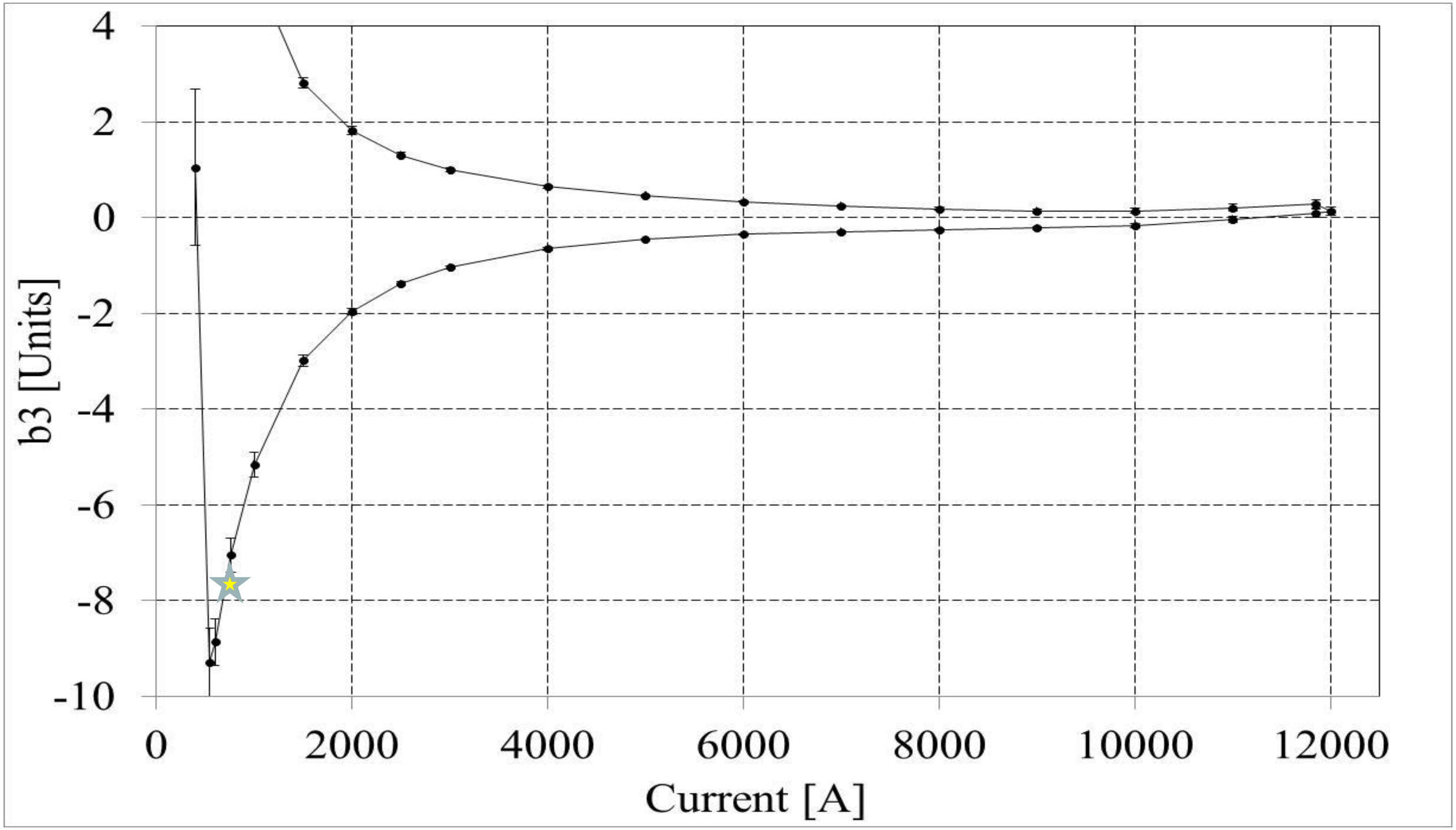
Dispersion Suppressor



Interaction Region



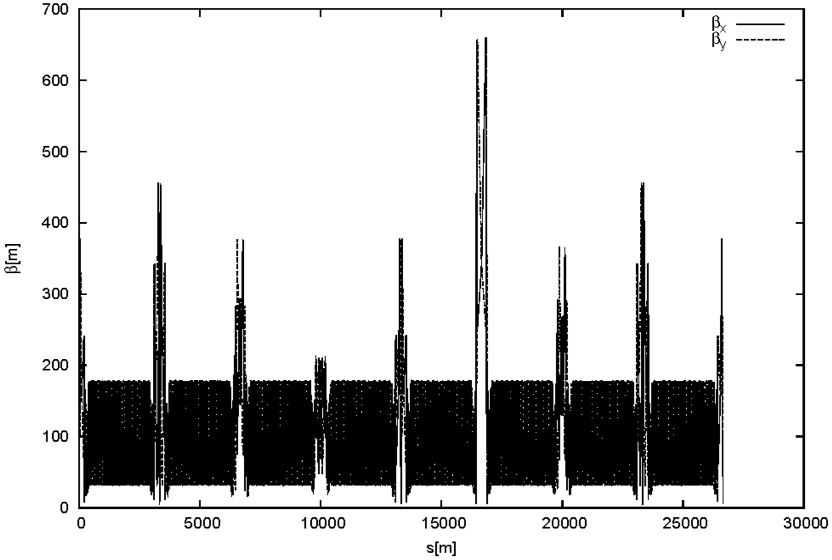
b_3 in the main LHC dipoles, and injection energy at 450 GeV



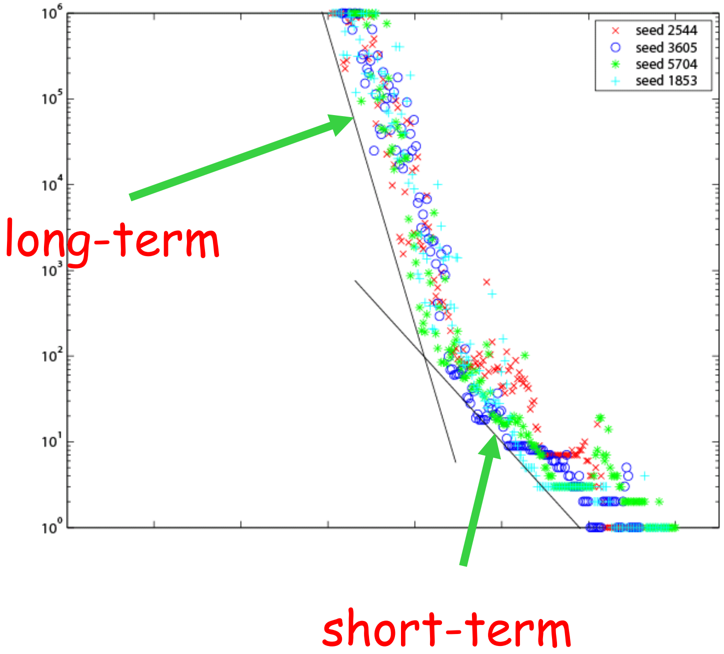
Courtesy of S. Izquierdo Bermudez, E. Todesco, D. Tommasini

Dynamic Aperture at Injection

LHC Lattice functions



Dynamic Aperture



long-term

short-term

- What is the best injection energy?
- What is the necessary field quality?
- What kind of corrector package?

Summary

1. Hamiltonian is fundamental for the beam dynamics in storage rings, including the linear optics.
2. To make the particle motion stable, we use harmonic oscillators in all three dimensions. In the longitudinal plane, the RF bucket makes its stability extremely robust. That why we can focus on the transverse dynamics.

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