Lecture 5:

Electron Rings

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SLC Damping Rings (SLAC)

Radiation Damping

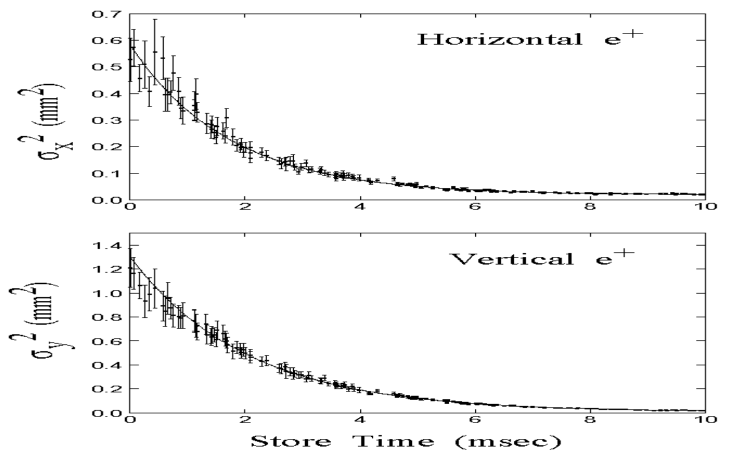


Figure 2. Sample of data for the positron damping ring. The vertical scale represents the real size of the bunch.

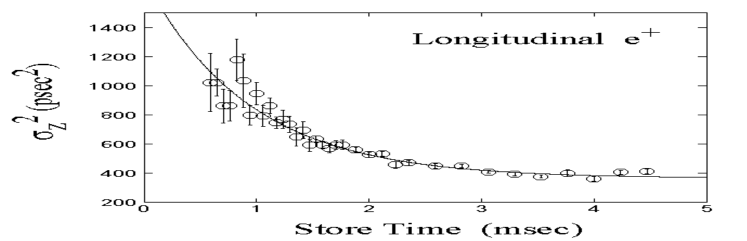
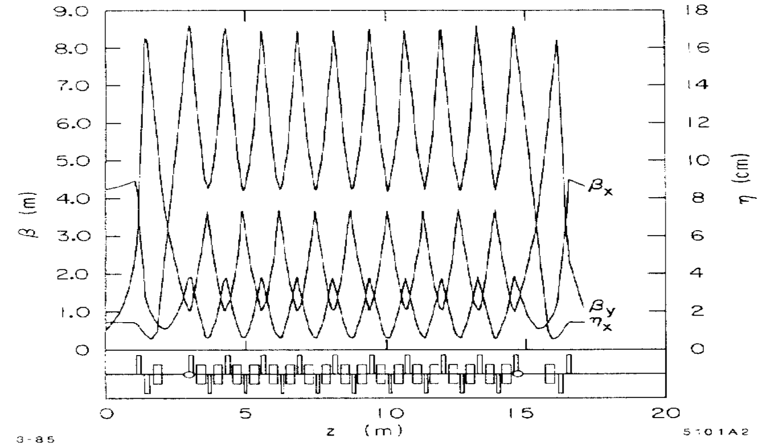


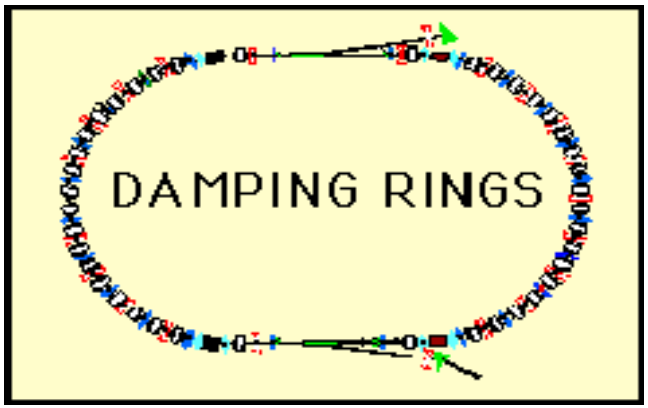
Figure 3. Longitudinal damping time data. The origin of the horizontal axis represents injection time.

C. Simopoulos and R.L. Holtzaple (1996)

FODO Lattice (1985)



Layout

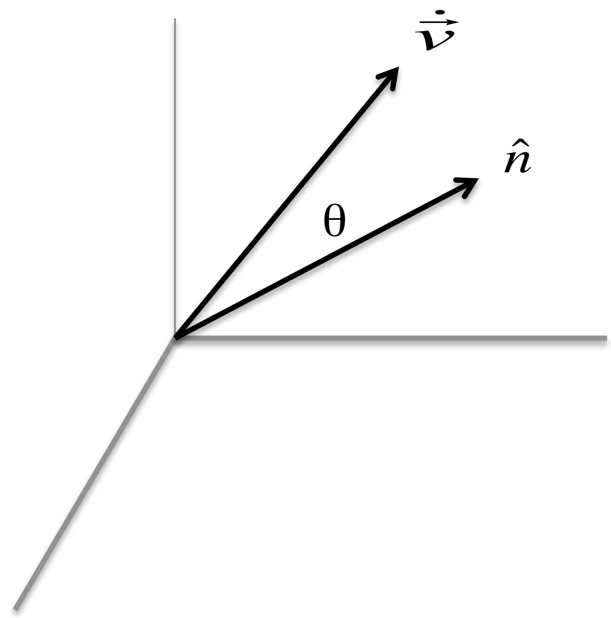


Synchrotron Radiation Power

Using the Lienard-Wiechert formula of the radiated field at a low velocity

$$\vec{E} = \left(\frac{e}{c}\right)\left[\frac{\vec{n}\times(\vec{n}\times\dot{\vec{\beta}})}{R}\right]_{ret}$$

$$\vec{B} = \vec{n}\times\vec{E}$$



One can derived the Lamor's formula,

$$P_\gamma = \frac{2}{3}\frac{e^2}{c^3}\left|\dot{\vec{v}}\right|^2$$

(Lienard 1898)

Its relativistic counterpart,

$$P_\gamma = \frac{2}{3}\frac{e^2}{m^2c^3}\gamma^2\left|\frac{d\vec{p}}{dt}\right|^2 \xrightarrow[\text{motion}]{\text{circular}} P_\gamma = \frac{2}{3}\frac{e^2c}{\rho^2}\beta^4\gamma^4$$

Tracking with Classical Radiation

From the relativistic Lamor formula, we have

$$\frac{d\delta}{d\ell} = -C_K (1+\delta)^2 \left| \frac{\vec{B}_\perp}{B\rho} \right|^2$$

where $C_K=2r_e\gamma_0^3/3$. The magnetic field is known inside a tracking procedure of the element. Further more, using the Hamilton equation for the sixth coordinate, we can rewrite it as

$$\frac{d\delta}{ds} = C_K (1+\delta)^2 \left| \frac{\vec{B}_\perp}{B\rho} \right|^2 \frac{\partial H}{\partial \delta}$$

It can be used for step of the integration. The radiation damping with the proper partition is a result of this change of the momentum. Note that there Is no dependence on the Planck constant.

Radiation Damping

Instantaneous synchrotron radiated power is given by

$$P_\gamma = \frac{2}{3} r_e mc^2 \frac{\gamma^4}{\rho^2},$$

Energy loss per turn is

$$U_0 = \frac{2\pi\rho}{c} P_\lambda = \frac{4\pi}{3} \frac{r_e mc^2}{\rho} \gamma^4.$$

or

$$\frac{U_0}{E} = \frac{4\pi}{3} \frac{r_e}{\rho} \gamma^3.$$

which is at order of the damping increments. Therefore the damping time $t \sim T_0 E/U_0$.The damping of the emittance is

$$\varepsilon_{ext} = \varepsilon_{inj} e^{-2t/\tau} + \varepsilon_{equ}(1 - e^{-2t/\tau})$$

Longitudinal Radiation Damping

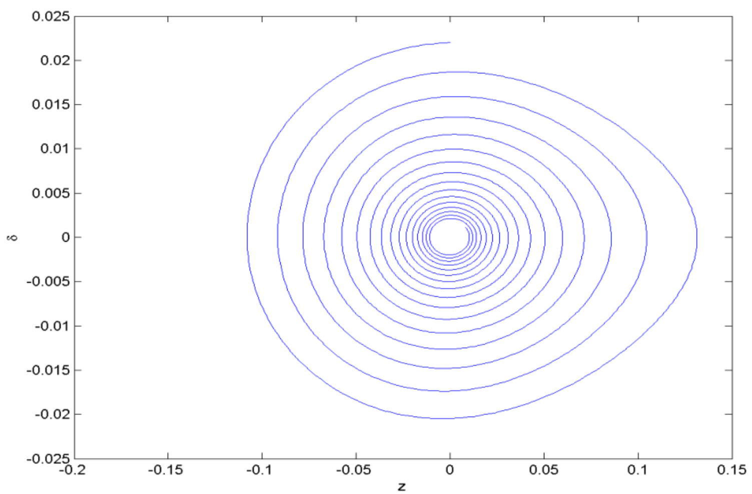
For a single RF in a ring, every turn we have

$$\begin{cases} \delta_{n+1} = \delta_n + \dfrac{eV_{RF}}{E_0}\sin(k_{RF}z_n + \varphi_s) - \dfrac{U_0}{E_0} - D_s\delta_n \\ z_{n+1} = z_n - \alpha_p C\delta_{n+1} \end{cases}$$

D_s is due to the fact that the energy loss depends in the deviation of the energy from the synchronous particle.

$$\Rightarrow \begin{cases} \dot{\delta} = \dfrac{eV_{RF}k_{RF}}{T_0E_0}\cos\varphi_s z - D_s\delta \\ \dot{z} = -\dfrac{\alpha_p C}{T_0}\delta \end{cases}$$

RF Bucket



Synchrotron tune is given by

$$\upsilon_s = \sqrt{\frac{h\alpha_p}{2\pi}\frac{eV_{RF}}{E_0}\cos\varphi_s},$$

where $\omega_s = \nu_s\omega_0$.

Radiation Damping

As we have illustrated that energy loss as a function of the energy deviation results in the radiation damping, it easily see that the damping increments are given by

$$\alpha_x = -\frac{1}{2}\frac{<P_\gamma>}{E_0}(1-\vartheta) = -\frac{1}{2}\frac{<P_\gamma>}{E_0}J_x,$$

$$\alpha_y = -\frac{1}{2}\frac{<P_\gamma>}{E_0} = -\frac{1}{2}\frac{<P_\gamma>}{E_0}J_y,$$

$$\alpha_s = -\frac{1}{2}\frac{<P_\gamma>}{E_0}(2+\vartheta) = -\frac{1}{2}\frac{<P_\gamma>}{E_0}J_s,$$

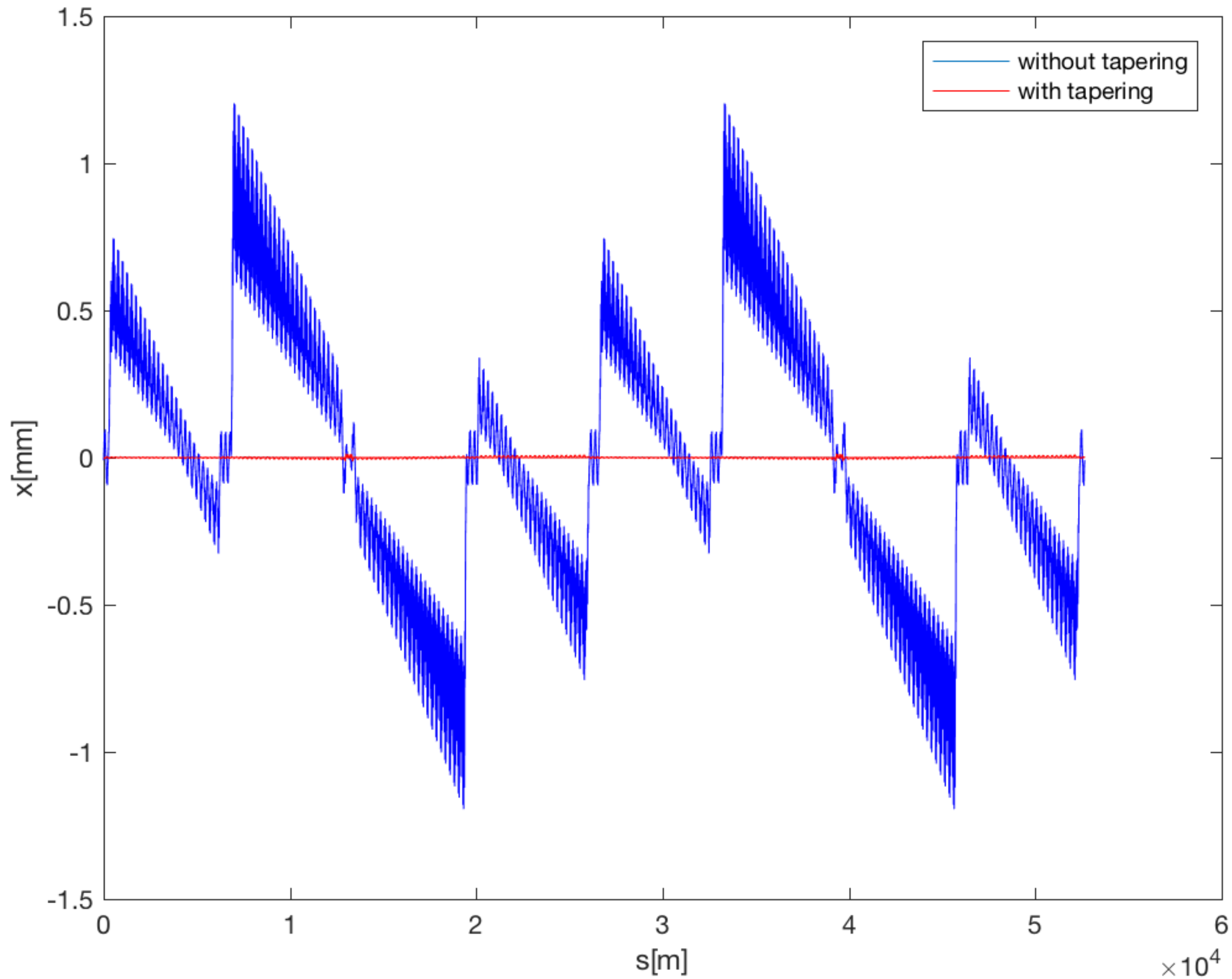
where

$$\vartheta = \frac{<\frac{\eta_x}{\rho^3}(1+2\rho^2 K_1)>_s}{<\frac{1}{\rho^2}>_s}$$

← Only important one combined function magnets are used. Note that $K_1 < 0$ reduces the horizontal emittance.

J_x, J_y, and J_s are called the damping partitions and $J_x+J_y+J_s=4$. The damping time is given by $\tau=|1/\alpha|$.

Sawtooth and Tapering (120 GeV)



Quantum Effects of Synchrotron Radiation

Instantaneous radiated power is given by

$$P_\gamma = \frac{2}{3} r_e mc^2 \frac{c\gamma^4}{\rho^2},$$

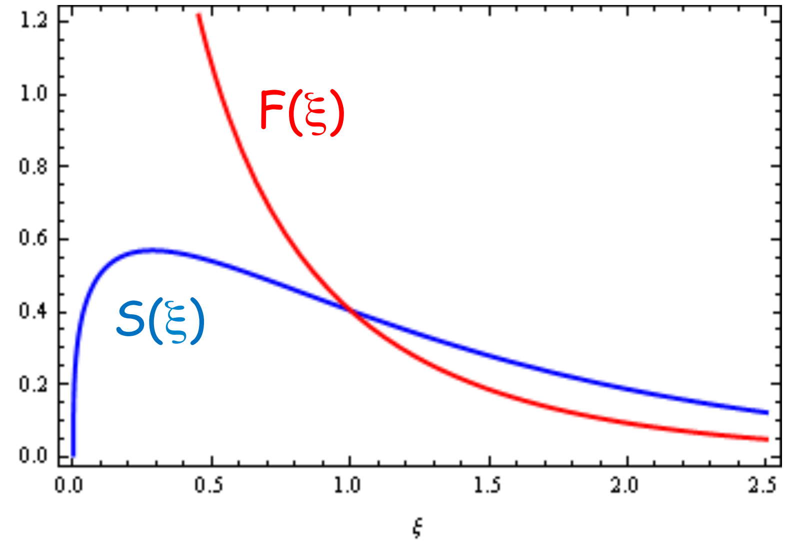
and spectrum,

$$\frac{dP_\gamma}{d\omega} = \frac{P_\gamma}{\omega_c} S\left(\frac{\omega}{\omega_c}\right),$$

where $\omega_c = 3c\gamma^3/2\rho$ and S is defined as,

$$S(\xi) = \frac{9\sqrt{3}}{8\pi} \xi \int_\xi^\infty K_{5/3}(\bar{\xi}) d\bar{\xi}.$$

Normalized power spectrum S and photon number spectrum F



$K_{5/3}$ is the modified Bessel function. Then the quantum distribution function is

$$n(u) = \frac{P_\gamma}{u_c^2} F\left(\frac{u}{u_c}\right), F(\xi) = S(\xi)/\xi, u_c = \omega_c \hbar.$$

key →

$$\dot{N}_{ph} < u^2 > = \frac{55}{24\sqrt{3}} u_c P_\gamma$$

Envelope Formulation

Propagation of the sigma matrix or the second moment of a Gaussian distribution is given by,

$$\Sigma_{s_2} = M_{s_1 \to s_2} \Sigma_{s_1} \tilde{M}_{s_1 \to s_2} + d\Sigma_q$$

where M is the transfer matrix between the position s_1 and s_2 and $d\Sigma_q$ is the contribution due to the quantum diffusion. For each integration step, we have,

$$d\Sigma_{q55} = -C_D (1+\delta)^4 \left| \frac{B_\perp}{B\rho} \right|^3 \frac{\partial H}{\partial \delta} ds$$

where

$$C_D = \frac{55 r_e \lambda\!\!\!^{-}_e \gamma_0^5}{24\sqrt{3}}$$

An equilibrium can be reached after a few damping times.

Energy Spread and Emittance

Balance between the quantum excitation and radiation damping results in an equilibrium Gaussian distribution with relative energy spread σ_δ and horizontal emittance ε_x:

$$\sigma_\delta^2 = \frac{\tau_s}{2E_0^2} < \dot{N}_{ph} < u^2 >>_s = C_q \frac{\gamma^2}{J_s} \frac{\left\langle 1/\rho^3 \right\rangle_s}{\left\langle 1/\rho^2 \right\rangle_s},$$

$$\varepsilon_x = \frac{\tau_x}{4E_0^2} < \dot{N}_{ph} < u^2 > \mathcal{H}_x >_s = C_q \frac{\gamma^2}{J_x} \frac{\left\langle \mathcal{H}_x / \rho^3 \right\rangle_s}{\left\langle 1/\rho^2 \right\rangle_s},$$

where and

$$C_q = \frac{55 \lambda\!\!\!^{-}_e}{32\sqrt{3}}, \qquad \mathcal{H}_x = \beta_x \eta_{px}^2 + 2\alpha_x \eta_x \eta_{px} + \gamma_x \eta_x^2$$

- The quantum constant $C_q = 3.8319 \times 10^{-13}$ m for electron
- γ is the Lorentz factor (energy)

Minimization of Emittance

For an electron ring without damping wigglers, the horizontal emittnace is given by

$$\varepsilon_0 = F \frac{C_q \gamma^2}{J_x} \theta^3$$

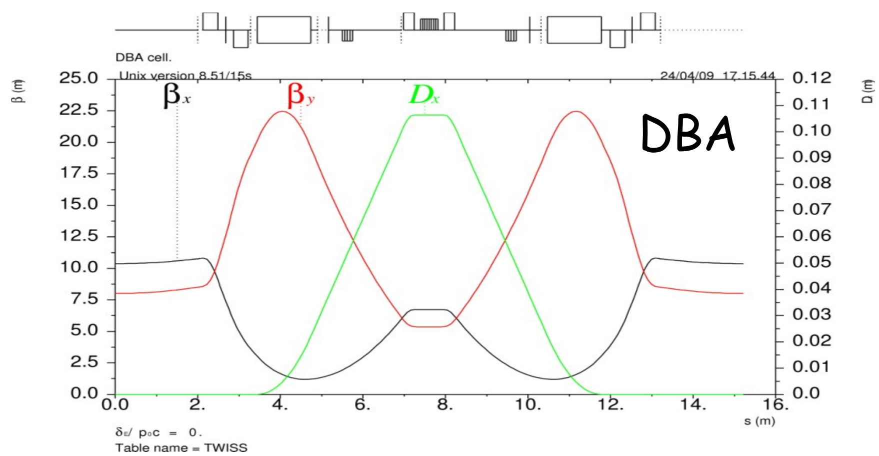
where F is a form factor determined by choice of cell and θ is bending angle of dipole magnet in cell. In general, stronger focusing makes F smaller. Often there is a minimum achievable value of F for any a given type of cell. For example, we have

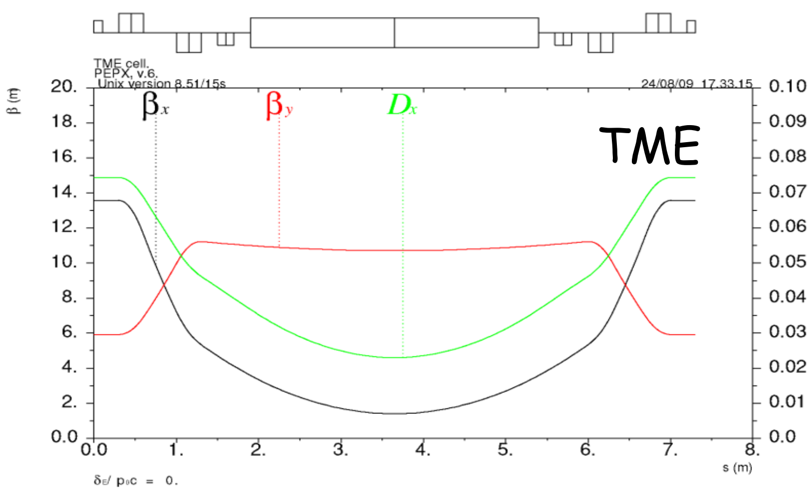
$$F_{min}^{DBA} = \frac{1}{4\sqrt{15}}$$

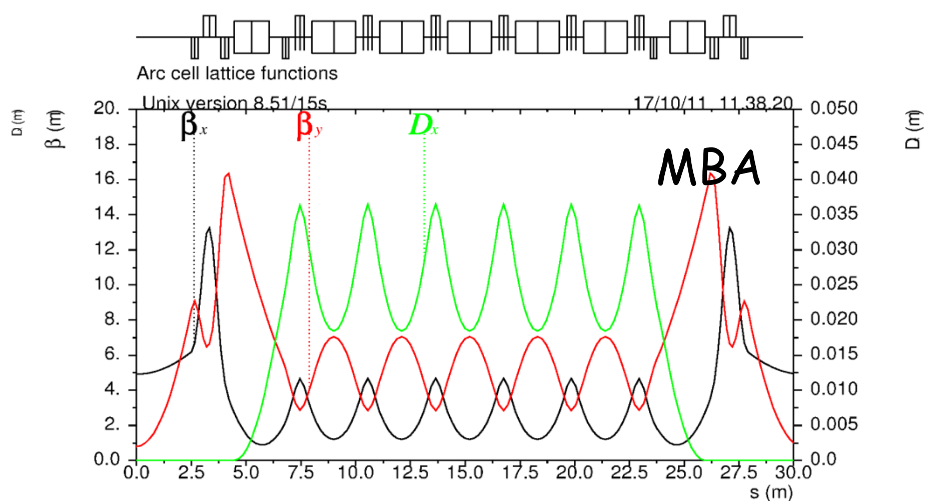
$$F_{min}^{TME} = \frac{1}{12\sqrt{15}}$$

There is a factor of three between the minimum values of DBA and TME cells. That's the price paid for an achromat, namely fixing the dispersion and its slop at one end of dipole.

Types of Periodic Cell







Emittance Reduction Using Damping Wiggler

$$\frac{\varepsilon}{\varepsilon_0} = \frac{1 + \dfrac{4C_q}{15\pi J_x} N_p \dfrac{\beta_x}{\varepsilon_0 \rho_w} \gamma^2 \dfrac{\rho_0}{\rho_w} \theta_w^3}{1 + \dfrac{1}{2} N_p \dfrac{\rho_0}{\rho_w} \theta_w}$$

N_p, is the total number of wiggler poles

β_x, is the average horizontal beta function in the wiggler

ρ_w, is wiggler bending radius at the peak field

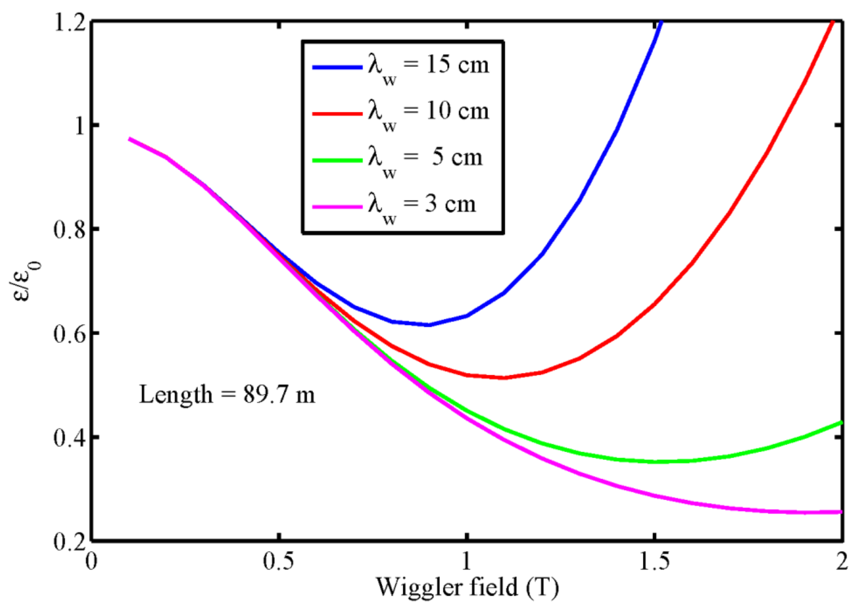
$\theta_w = \dfrac{\lambda_w}{2\pi\rho_w}$, is the peak trajectory angle in the wiggler

λ_w, is the wiggler period length

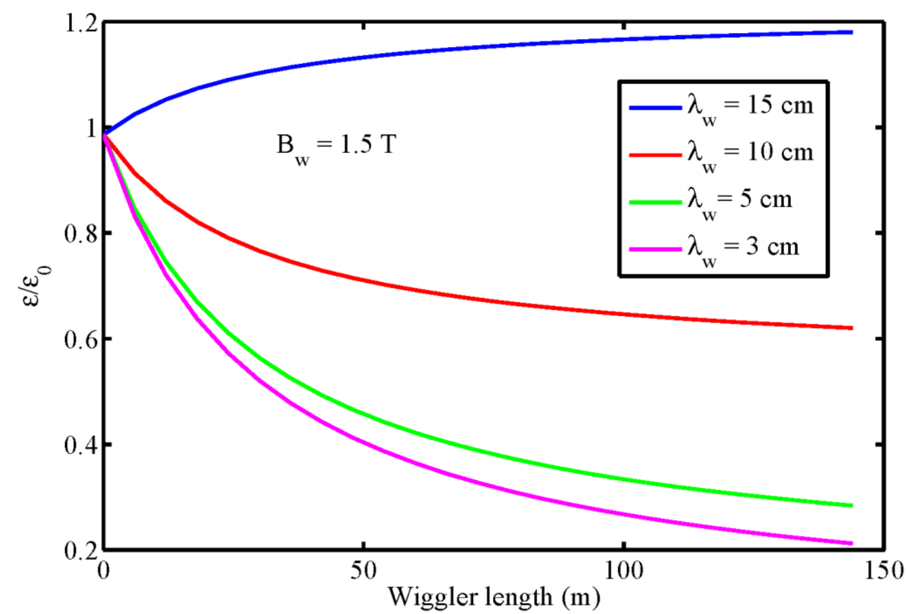
Optimization of Wigglers Parameters

Emittance = 11 pm-rad at 4.5 GeV with
parameters λ_w=5 cm, B_w=1.5 T

Wiggler Field Optimization



Wiggler Length Optimization



Average beta function at the wiggler section is 12.4 meter.

Positron Damping Ring
for FACET-II

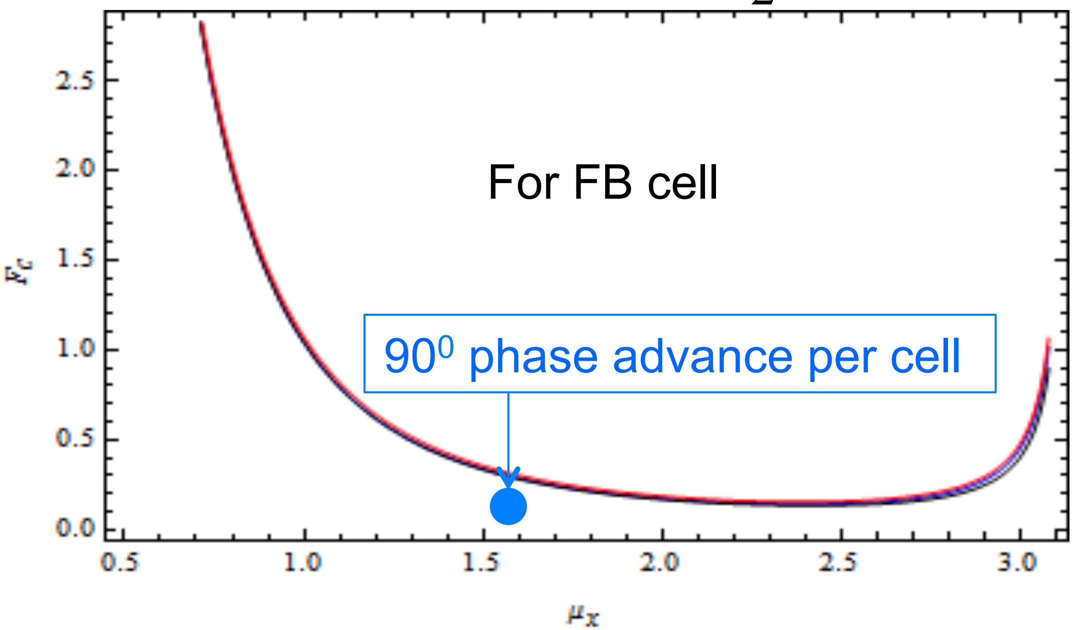
Energy, E [MeV] 335.0
Circumference, C [m] 21.4137
Tune, ν_x ν_y, 4.58, 2.58
Emittance, $\gamma\epsilon_{x,y}$[μm-rad] 5.5
Bunch length, σ_z [mm] 3.9
Energy spread, σ_δ [%] 0.062
Damping partition, J_x, J_y, J_z 2.15, 1.0, 0.85
Damping time, τ_x, τ_y, τ_z [ms] 16.9, 36.4, 43.0
Natural Chromaticity, ξ_{x0}, ξ_{y0} -6.5, -4.4
Energy loss per turn, U_0 [keV] 1.362
RF voltage, V_{RF} [MV] 1.1
RF frequency, f_{RF} [MHz] 714.0
Synchrotron Tune 0.037

Diameter: 3 meter

Emittance Scaling

Natural emittance: $$\varepsilon_0 = F_c \frac{C_q \gamma^2}{J_x} \theta^3$$

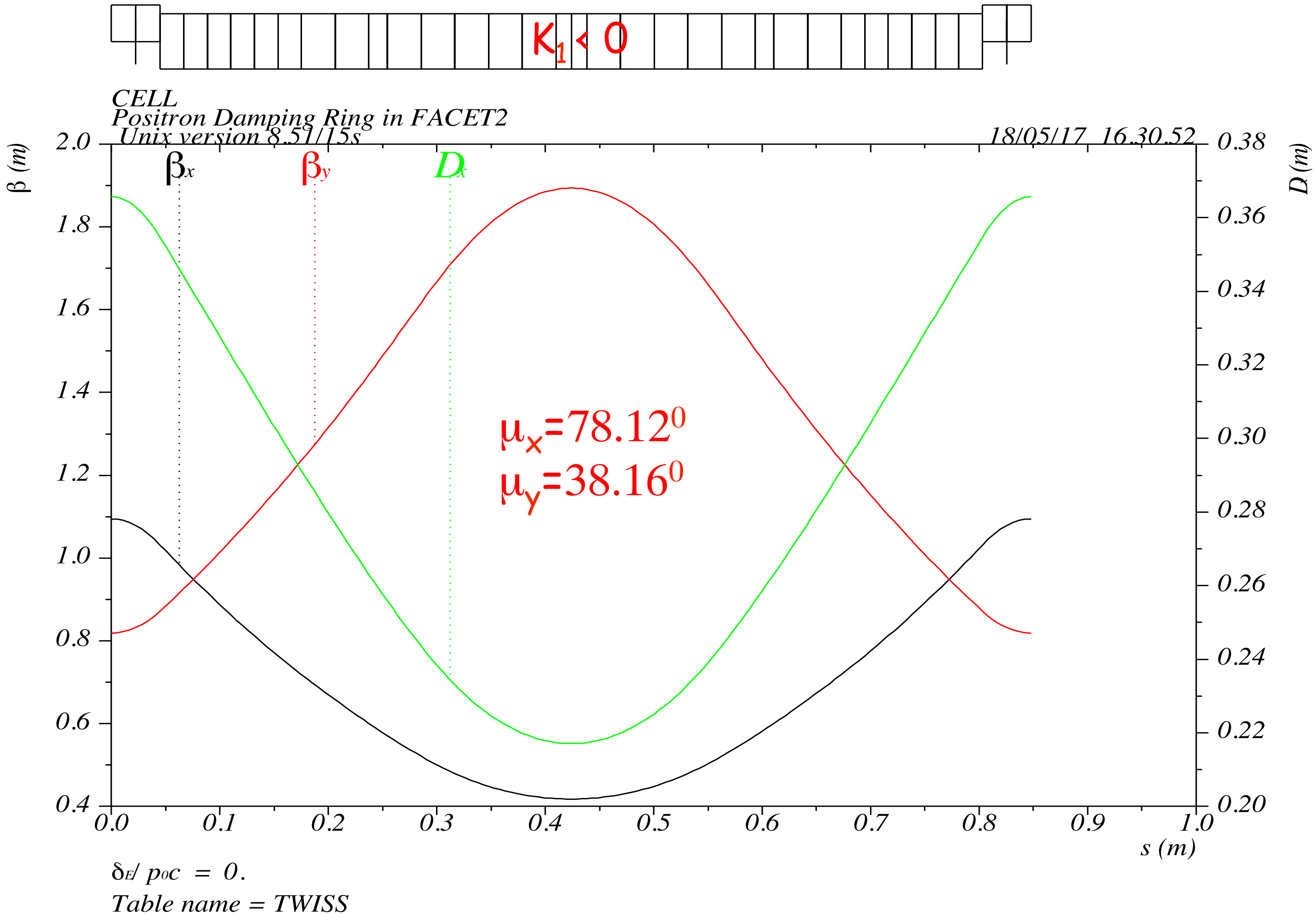
For FODO cell: $$F_c^{FODO} = \frac{1 - \frac{3}{4}\sin^2\frac{\mu}{2} + \frac{1}{60}\sin^4\frac{\mu}{2}}{4\sin^2\frac{\mu}{2}\sin\mu}$$



--- $\mu_y=\mu_x/2$

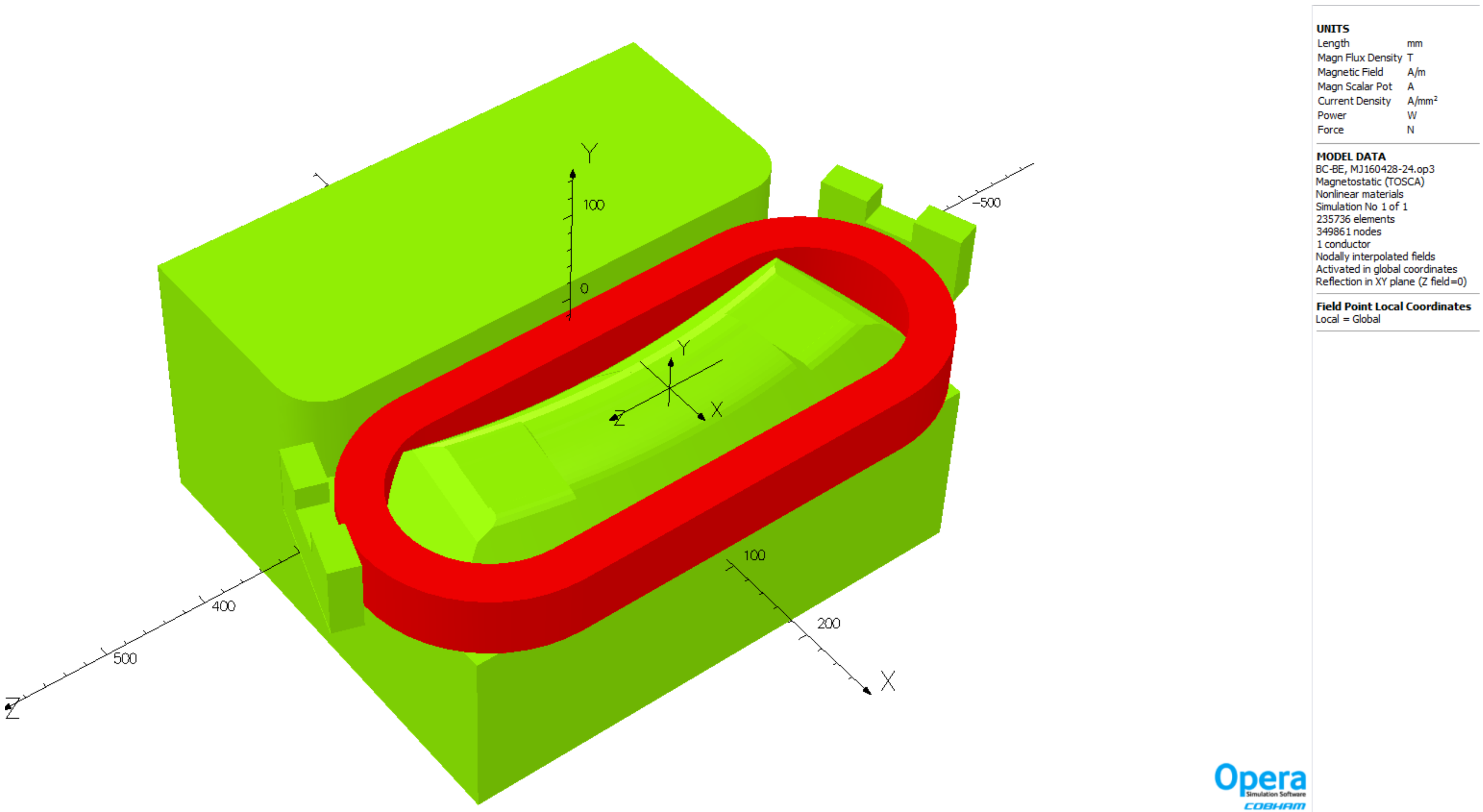
--- $\mu_y=\mu_x/3$

FB Cell

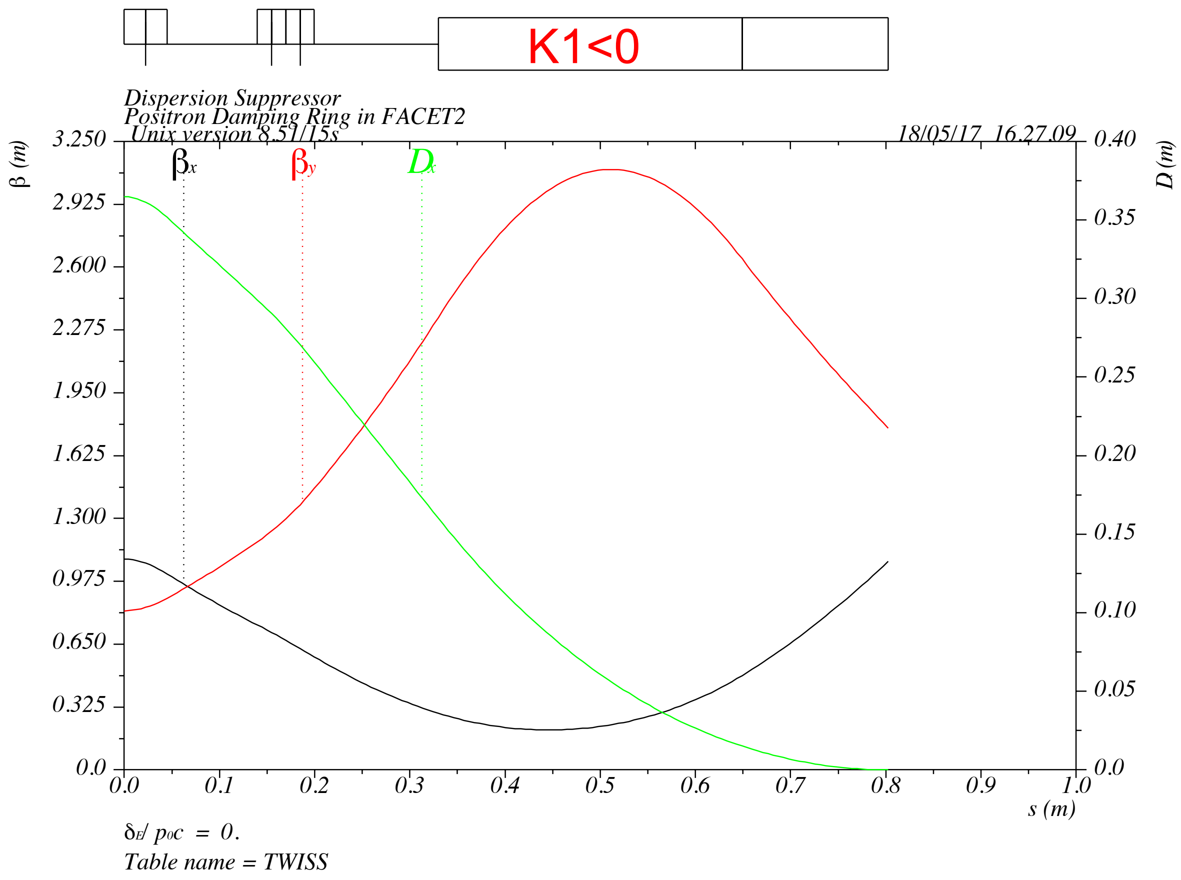


K1 < 0
CELL
Positron Damping Ring in FACET2
Unix version 8.51/15s
18/05/17 16.30.52
β (m)
D (m)
βx
βy
Dx
μx=78.12⁰
μy=38.16⁰
s (m)
δE/ p0c = 0.
Table name = TWISS

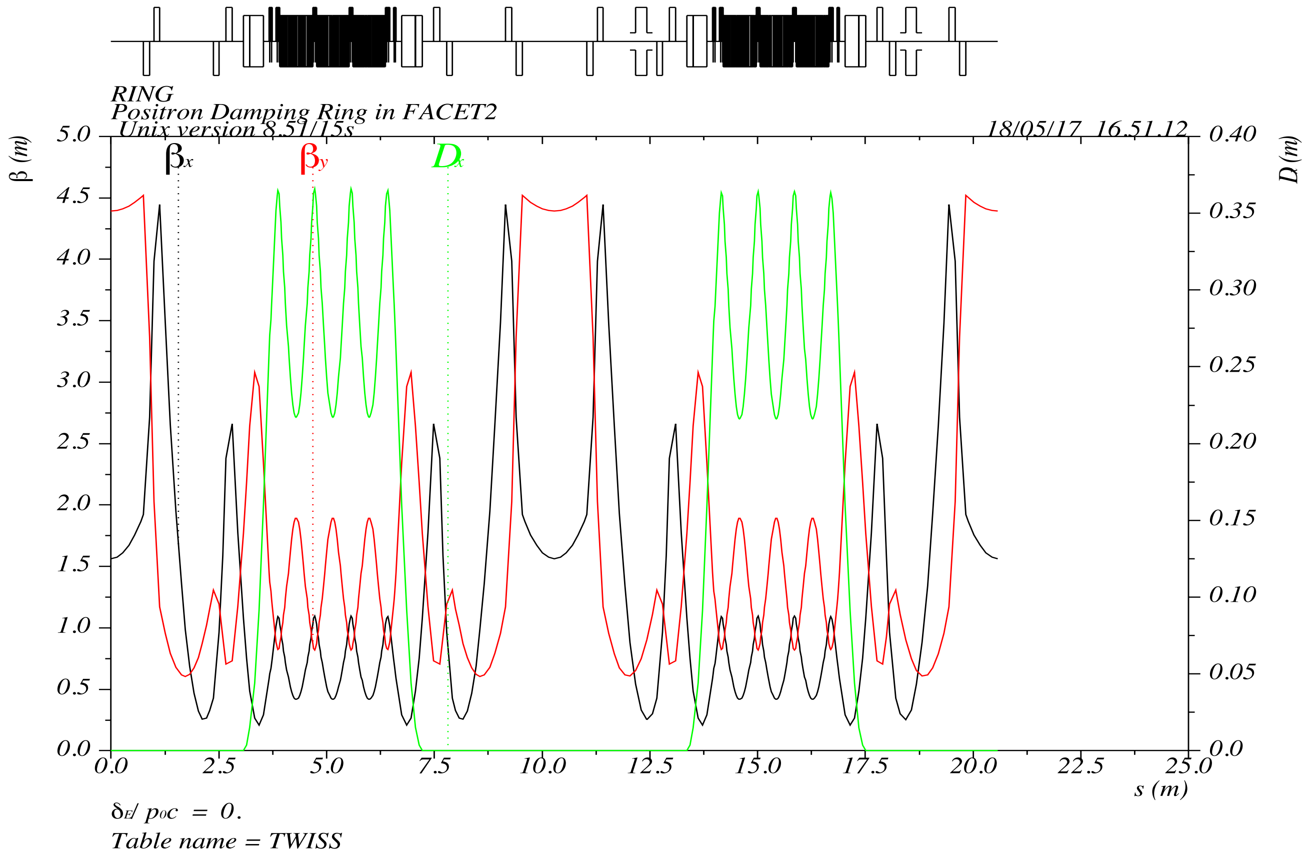
Combine Function Magnet



Dispersion Suppressor



FACET-II Positron Damping Ring



- Ring consists of two “5-bend” achromats
- Injection and extraction in vertical plane

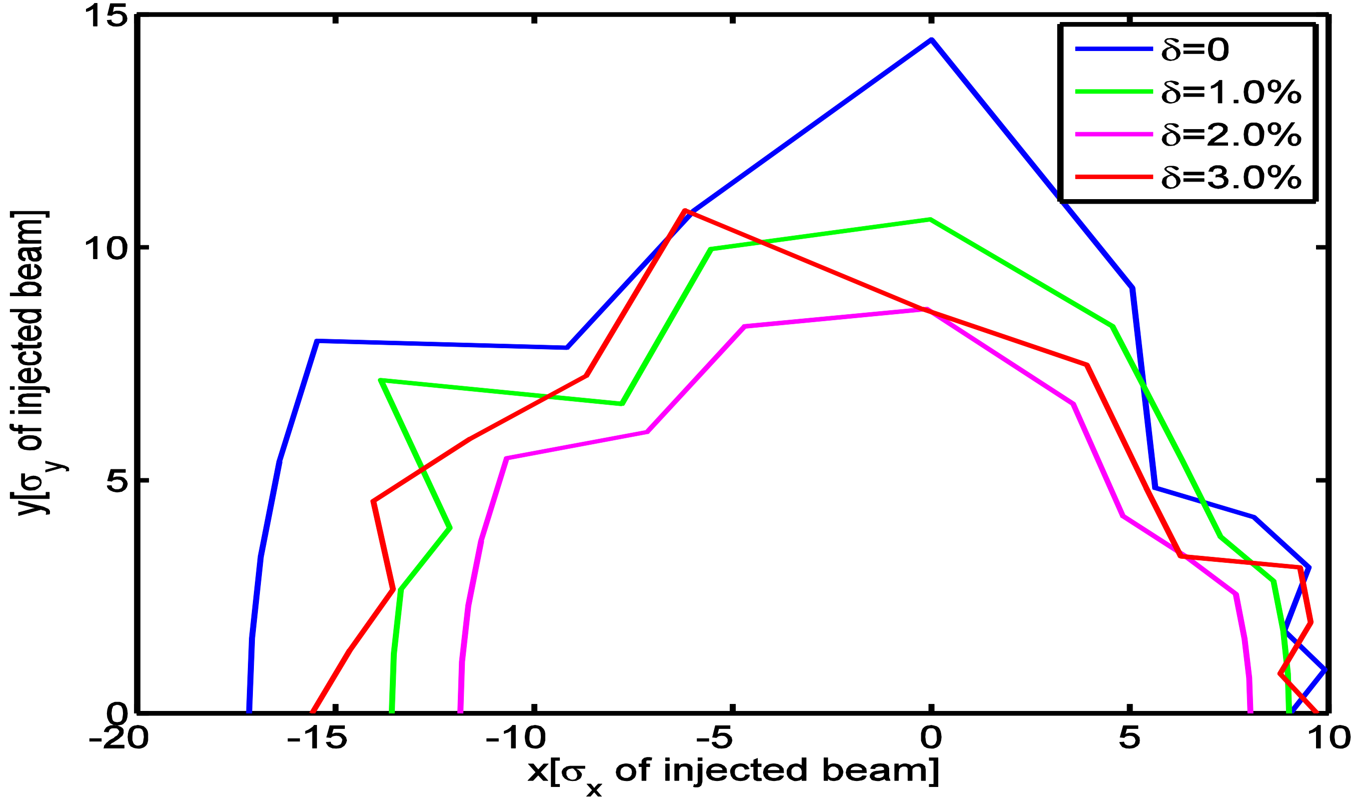
Nonlinear Analysis

Nonlinear chromaticities and tune shifts due to betatron amplitudes:

Derivatives of tunes	Values
$\partial \nu_{x,y} / \partial \delta$	+0.5,+0.5
$\partial^2 \nu_{x,y} / \partial \delta^2$	+16.3, +25.6
$\partial^3 \nu_{x,y} / \partial \delta^3$	-279.0, -1734.6
$\partial \nu_x / \partial J_x$ [m^{-1}]	+49.2
$\partial \nu_{x,y} / \partial J_{y,x}$ [m^{-1}]	-78.6
$\partial \nu_y / \partial J_y$ [m^{-1}]	+213.1

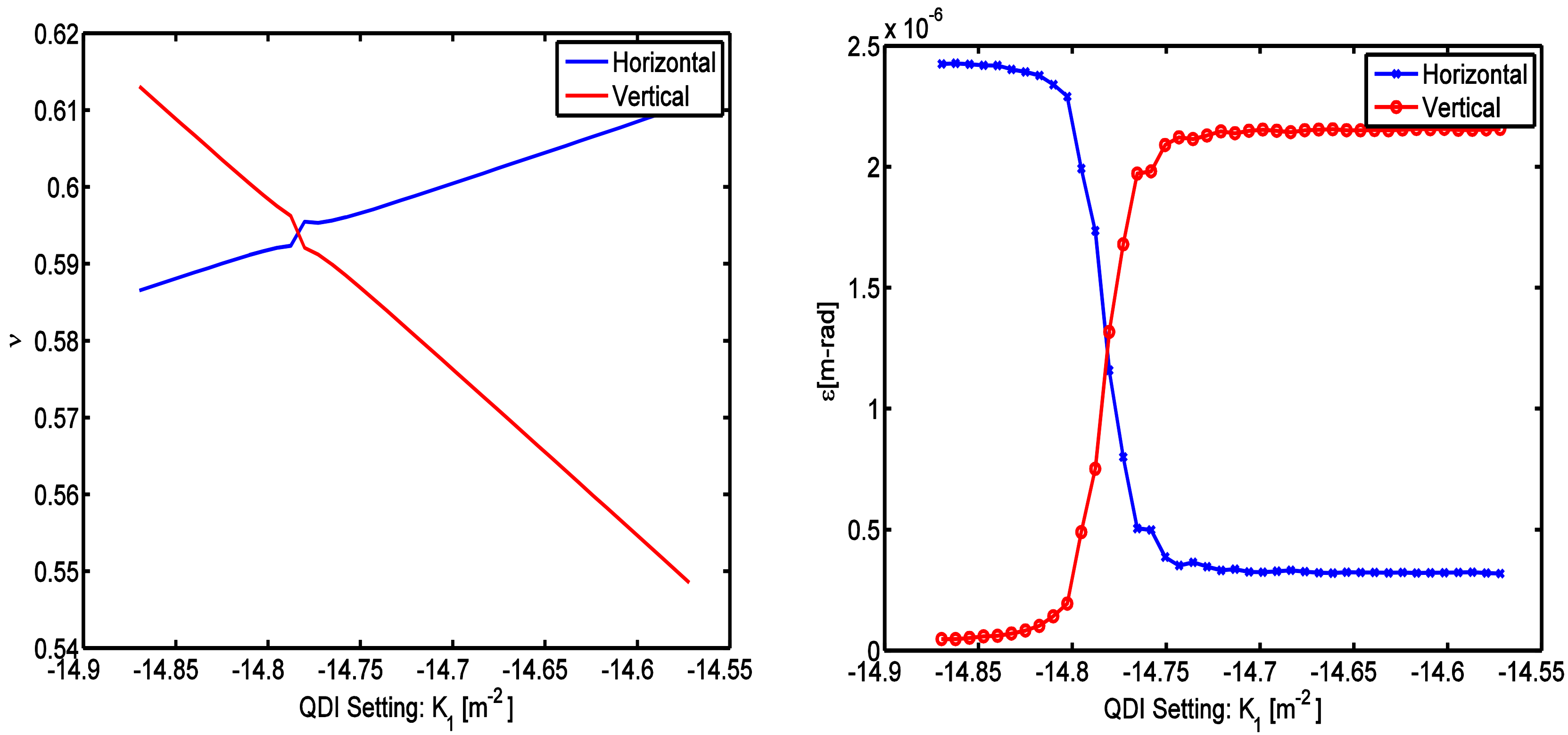
- Two families of sextupoles are used for chromatic compensation
- These nonlinear terms are very modest

Dynamic Aperture



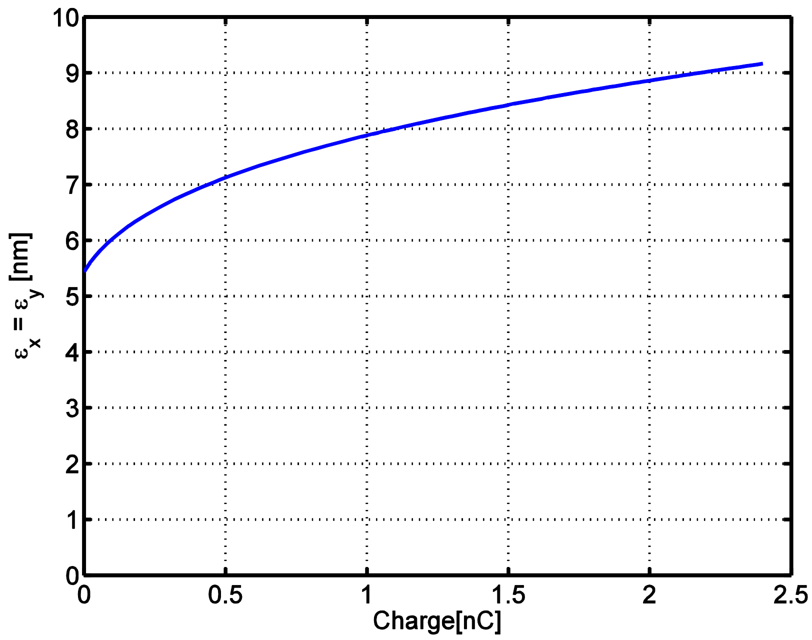
- Synchrotron oscillation is included in simulation
- Normalized emittances: 2.5/2.2 mm-rad

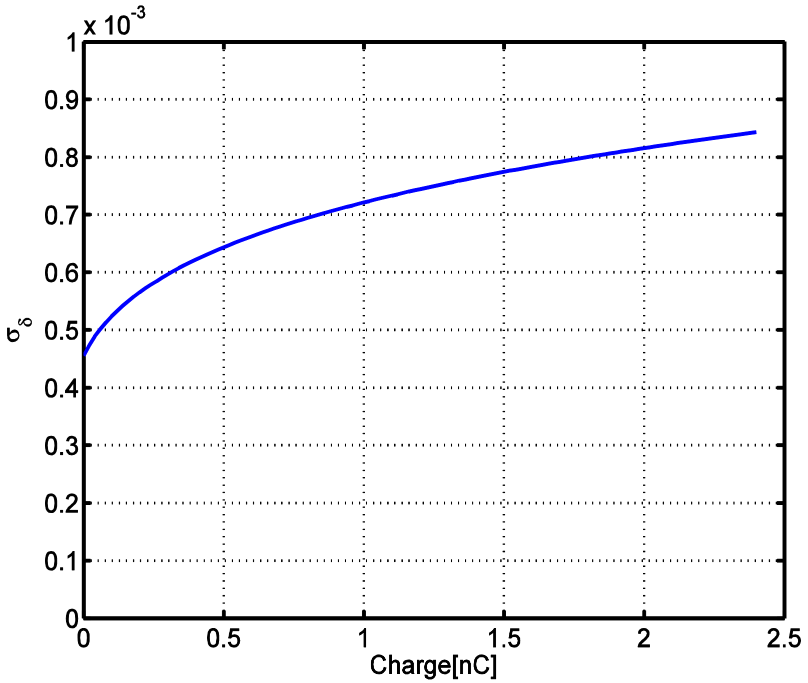
Round Beam



- About 1% coupling initially
- A family of quadrupoles is adjusted
- 1000 particles used in simulation
- No loss seen

Intra-Beam Scattering





- Growth rates are calculated using the Bjorken-Mtingwa formulas and the Nagaitsev algorithm for efficient computation

Summary

- Synchrotron radiation modifies the harmonic oscillators to the damped ones. Their damping rates are determined by the relativistic Lamor formula, which does not depend on the Planck constant: h or quantum physics.
- The quantization of the emitted photons generate heating in the electron motion. Balancing with the radiation damping, the beam reaches its equilibrium with finite energy spread and emittance. The relevant physical constant is the reduced Compton length.
- The art is how to reduce the emittance while retaining a large region of stability.

References

1) Matthew Sands, "The physics of electron storage rings," SLAC-121, November 1970.
2) Alexander W. Chao, "Evaluation of beam distribution parameters in an electron storage ring," J. Appl. Phys. 50, 595 (1979).
3) K. Ohmi, K. Hirata, and K. Oide, "From the beam-envelope matrix to synchrotron-radiation integral," Phys. Rev. E **49**, 751 (1994).
4) E. Forest, M.F. Reusch, D. Bruhwiler, A. Amiry, "The correct local description for tracking in rings," (1994).

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