Lecture 9:

Nonlinear Resonances

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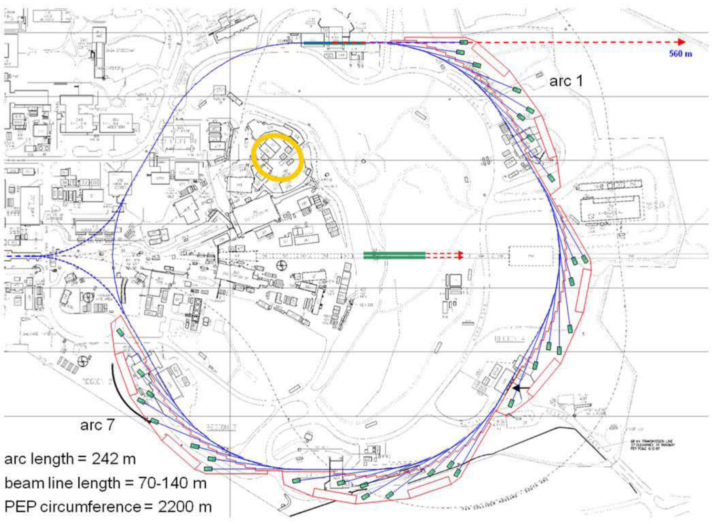
PEP-X Layout & Parameters

An ultimate storage ring

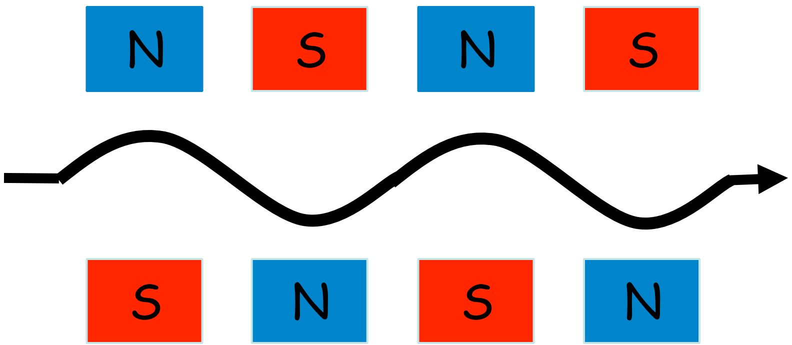


Energy, GeV	**4.5**
Circumference, m	**2199.32**
Natural emittance, pm	**11**
Beam current, mA	**200**
Emittance at 200 mA, x/y, pm	**12 / 12**
Tunes, x/y/s	**113.23 /65.14/0.007**
Bunch length, mm	**3.1**
Energy spread	**1.25×10^{-3}**
Energy loss per turn, MeV	**2.95**
RF voltage, MV	**8.3**
RF harmonic number	**3492**
Length of ID straight, m	**5.0**
Wiggler length, m	**90.0**
Beta at ID center, x/y, m	**4.92 / 0.80**
Touschek lifetime, hour	**10**
Dynamic aperture , mm	**10**

To be Built with 4^{th}-order geometrical achromats in the PEP tunnel.

Synchrotron Radiation

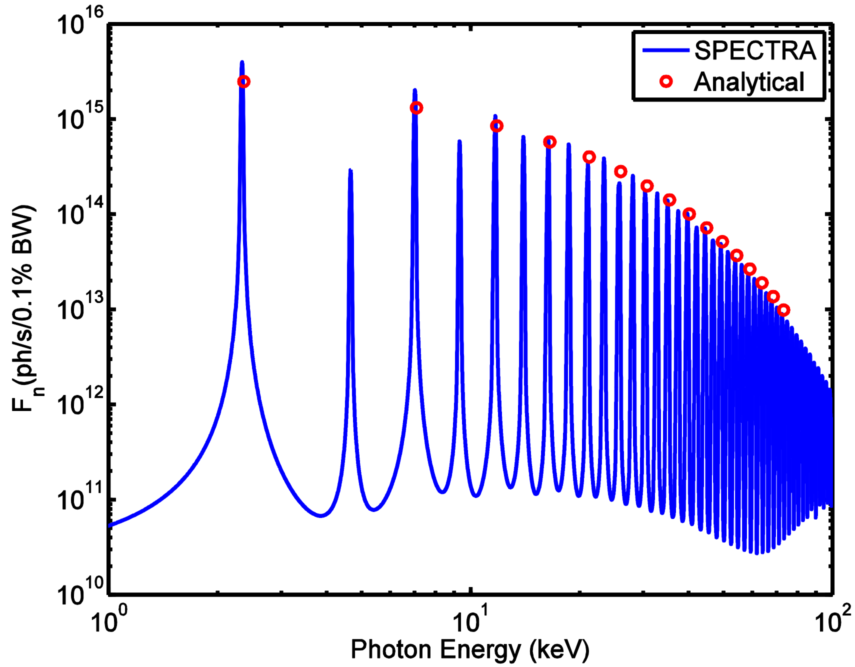
Electron beam in undulator



n^{th} harmonic wavelength:

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}\left(1+\frac{K^2}{2}\right)$$

Photon spectral flux in 0.1% BW



$$F_n = \frac{\pi}{2}\alpha N_u Q_n \left(\frac{nK^2}{4+2K^2}\right)\frac{\Delta\omega}{\omega}\frac{I}{e}$$

Spectral Brightness

Brightness of electron beam radiating at n^th (odd) harmonics in a undulator is given by

$$B_n = F_n / (4\pi^2 \Sigma_x \Sigma'_x \Sigma_y \Sigma'_y)$$

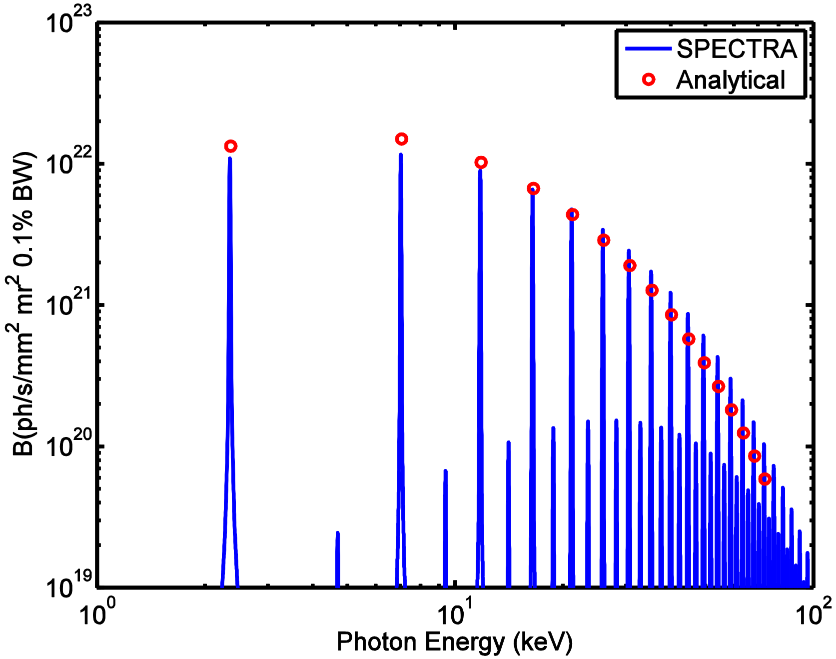
If the electron beam phase space is matched to those of photon's, the brightness becomes optimized

$$B_n = \frac{F_n}{4\pi^2 (\varepsilon_x + \lambda_n / 4\pi)(\varepsilon_y + \lambda_n / 4\pi)}$$

Finally, even for zero emittances, there is an ultimate limit for the brightness

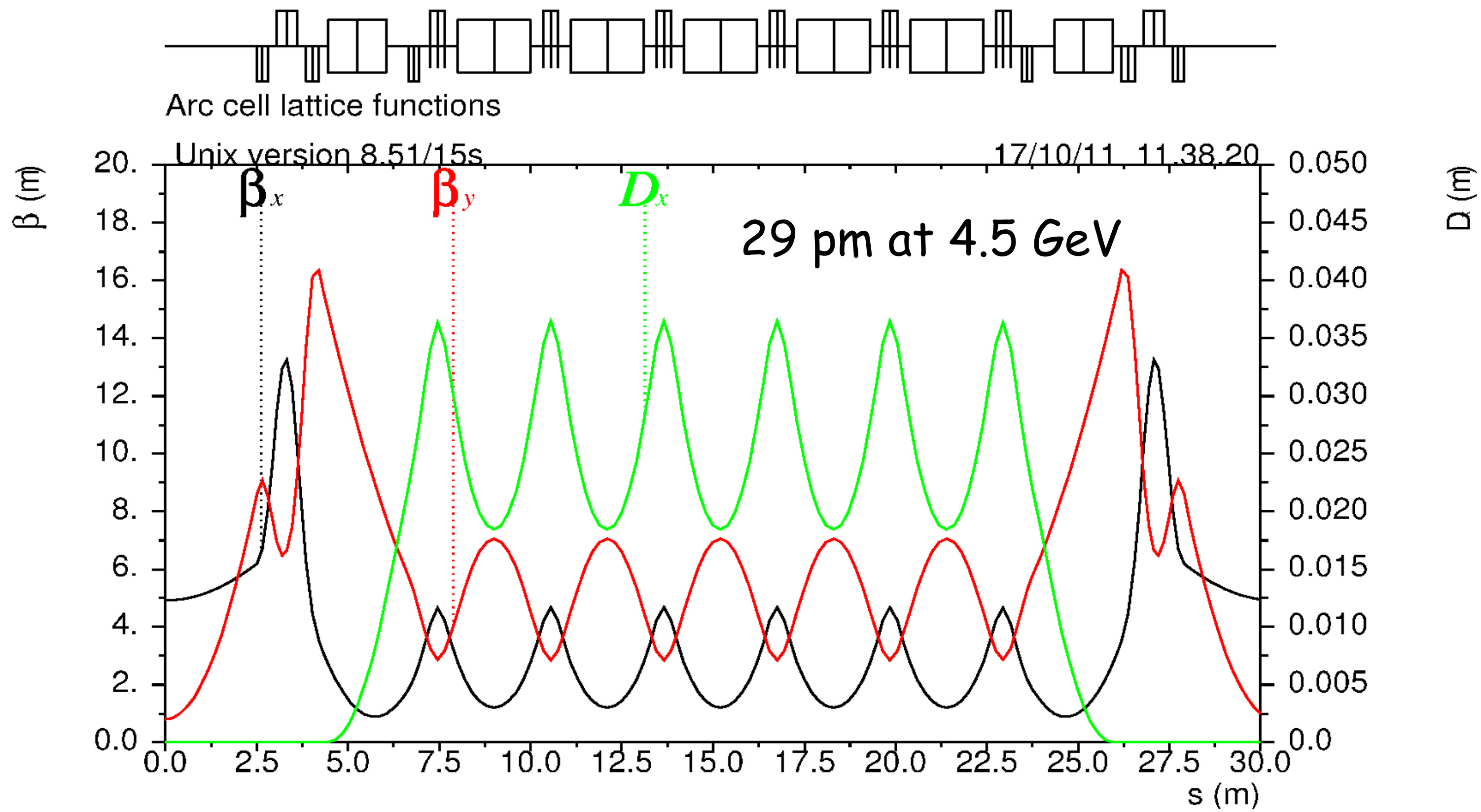
$$B_n = \frac{4F_n}{\lambda_n^2}$$

Spectral brightness of PEP-X



A diffraction limited ring at 1 angstrom or 10 pm-rad emittance

PEP-X 7 Bend Achromat



Cell phase advances: $\mu_x=(2+1/8) \times 360^0$, $\mu_y=(1+1/8) \times 360^0$.

Presentations for Magnetic Elements

Lie factors $\mathcal{M}^{1} e^{:f_3:} e^{:f_4:} \cdots$

- engine in MARYLIE (A. Dragt)
- violates symplecticity when evaluates

Dragt-Finn ↑

Taylor map $\mathcal{M}^{n}(z)$

- engine in TRANSPORT, MAD, COSY (K. Brown and M. Berz), simple R-matrix
- but high-order one violates

TPSA ↑

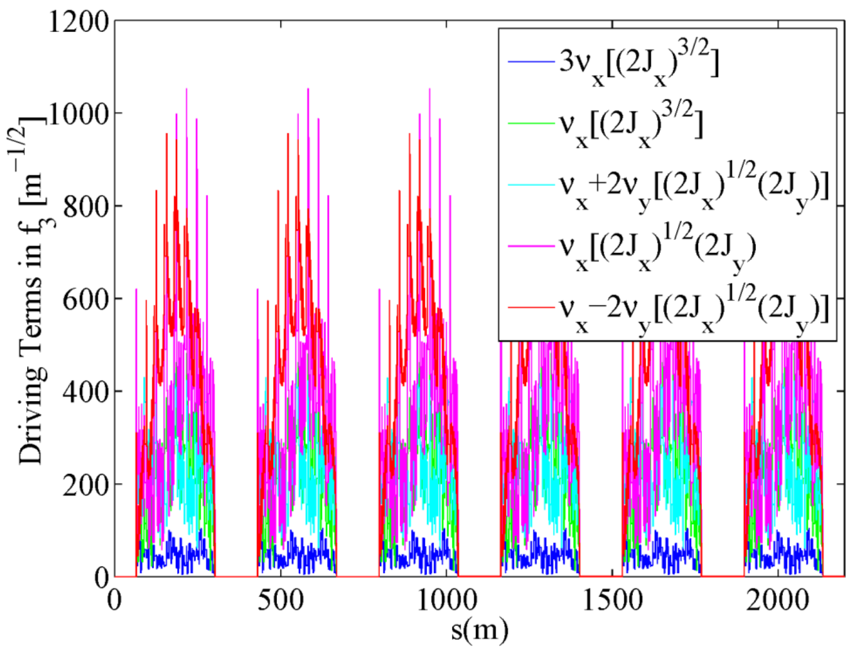
Symplectic Integrator $\prod_{i=1}^{n} e^{-\frac{:H_0:}{2}\Delta s} e^{-:H_1:\Delta s} e^{-\frac{:H_0:}{2}\Delta s}$

- engine in TEAPOT, SAD, TRACY, **LEGO**, PTC (E. Forest, R. Ruth, and K. Hirata)
- preserves symplecticity
- simple and based on several known solutions
- emphasis on numerical process

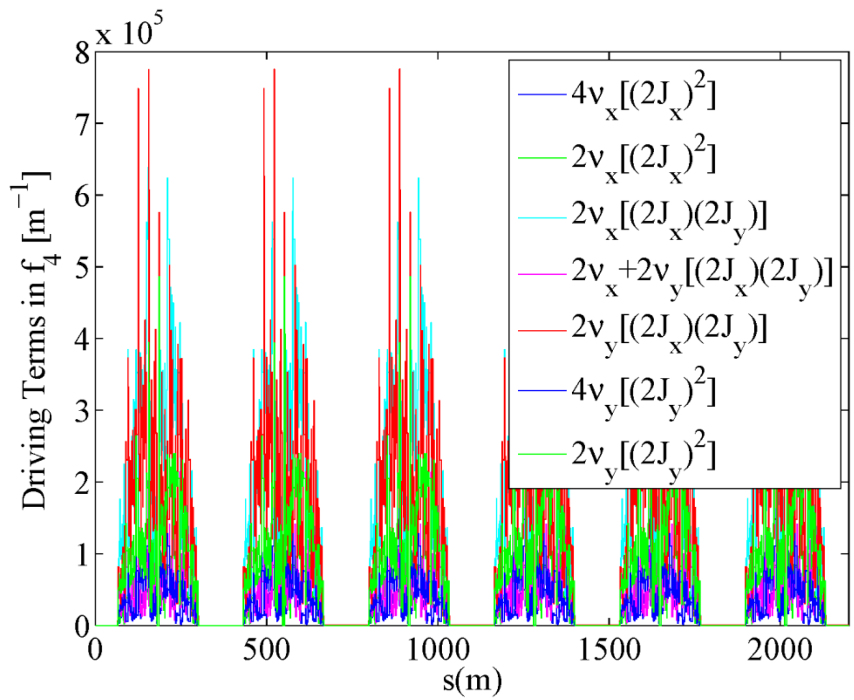
Cancellation of All Geometric 3rd and 4th Resonances Driven by Strong Sextupoles

except $2\nu_x$-$2\nu_y$

Third Order



Fourth Order



K.L. Brown & R.V. Servranckx

Nucl. Inst. Meth., A258:480–502, 1987

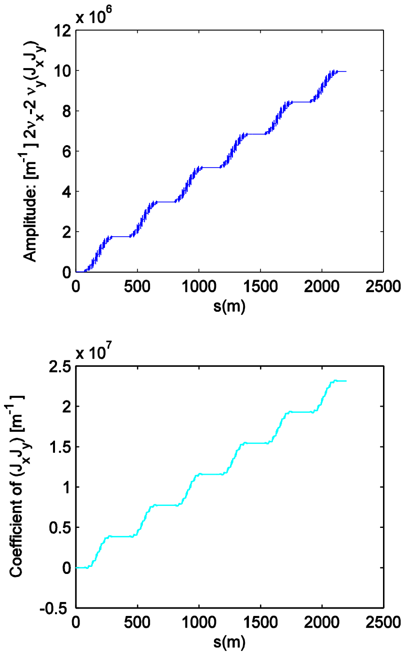
Yunhai Cai

Nucl. Inst. Meth., A645:168–174, 2011.

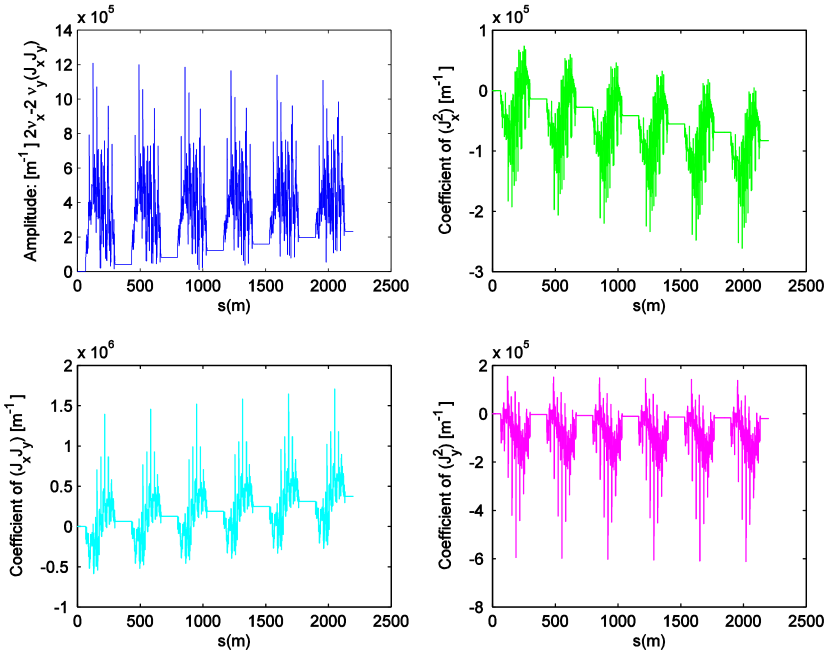
Harmonic Sextupoles

For Tune Shifts and $2\nu_x$-$2\nu_y$ Resonance

Without harmonic sextupoles



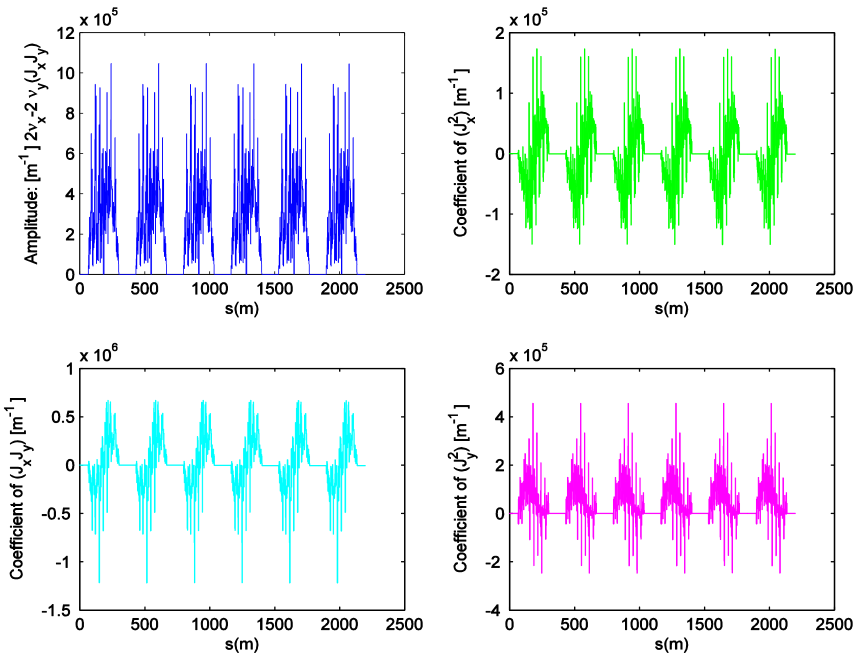
With harmonic sextupoles



OPA is used for optimizing the setting of 10 families of sextupoles. Due to the cancellation of many resonances, the optimization becomes much simpler and easier. OPA is an Accelerator Design Program from SLS PSI developed by A. Streun.

4^{th} Order Geometric Achromat

4^{th} order geometric achromat



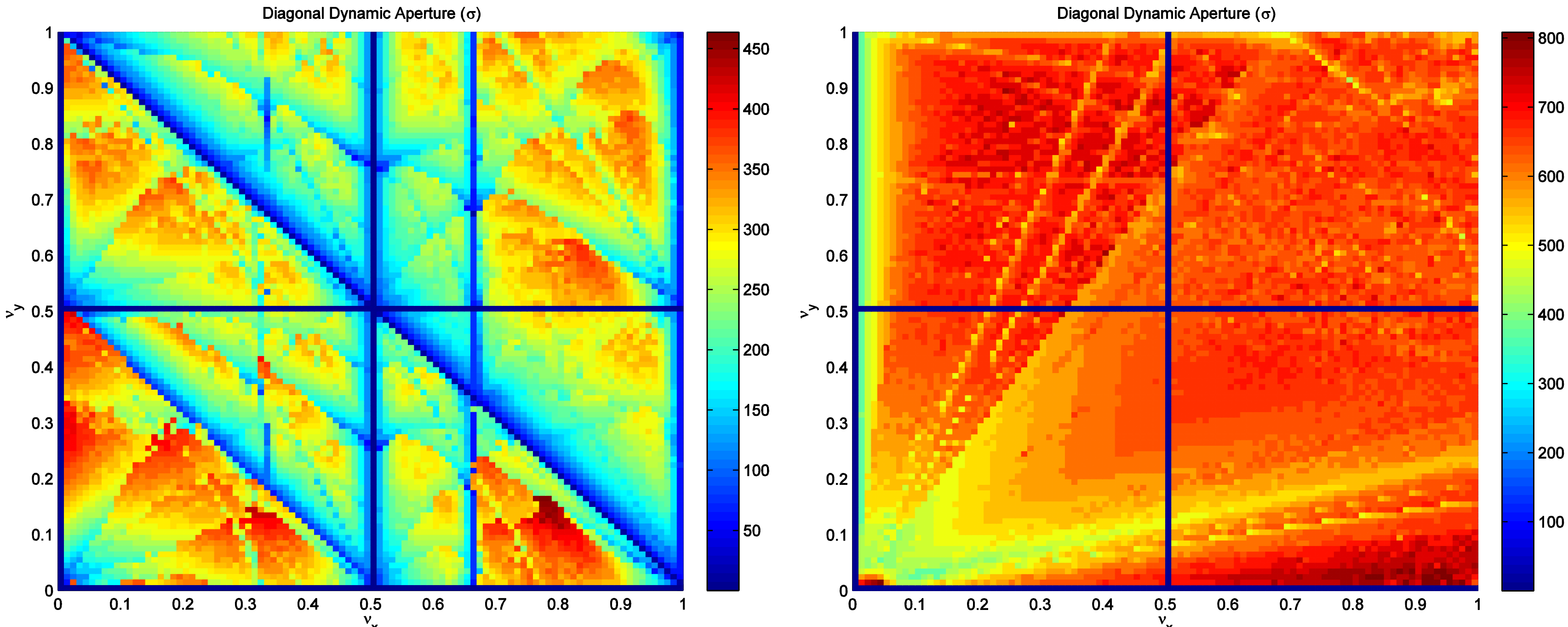
Chromatic effects

$\partial \nu_{x,y} / \partial \delta$	0,0
$\partial^2 \nu_{x,y} / \partial \delta^2$	-57,-89
$\partial^3 \nu_{x,y} / \partial \delta^3$	1332,-150
$\eta_{x,y}$	0,0
$\partial \eta_{x,y} / \partial \delta$	0,0

There are 4 families of chromatic sextupoles and 6 families of harmonic ones. The 4^{th} order geometric achromat (f_3=f_4=0) was obtained with the analytical Lie method. It was published on Yunhai Cai ey al. PRSTAB **15**, 054002 (2012).

Tune Scan of Dynamic Aperture

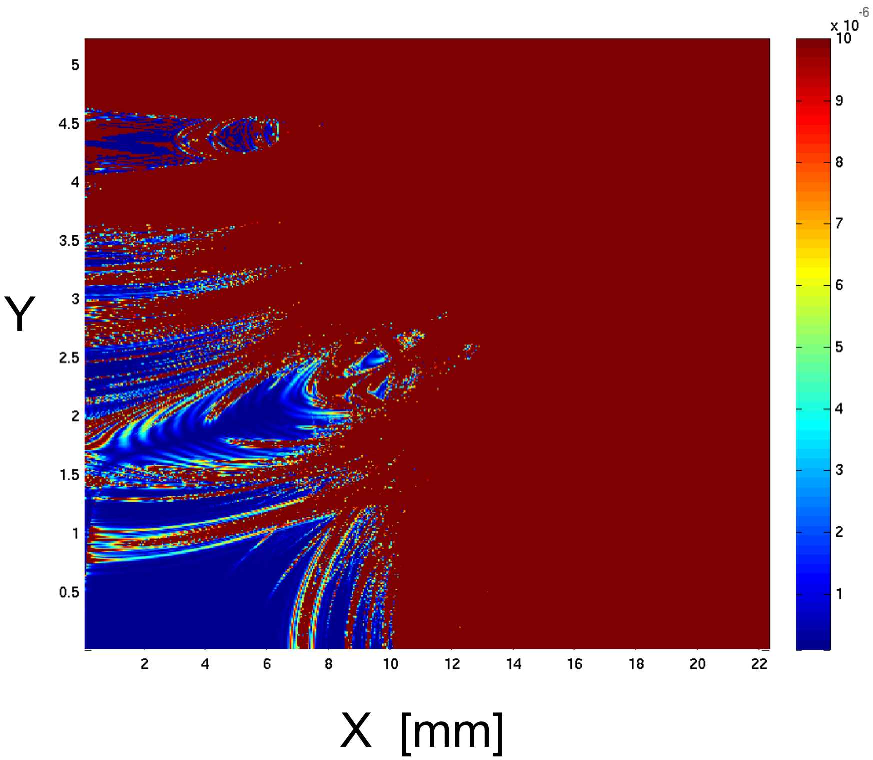
PEP-X: Baseline (2008) **PEP-X: USR (2011)**



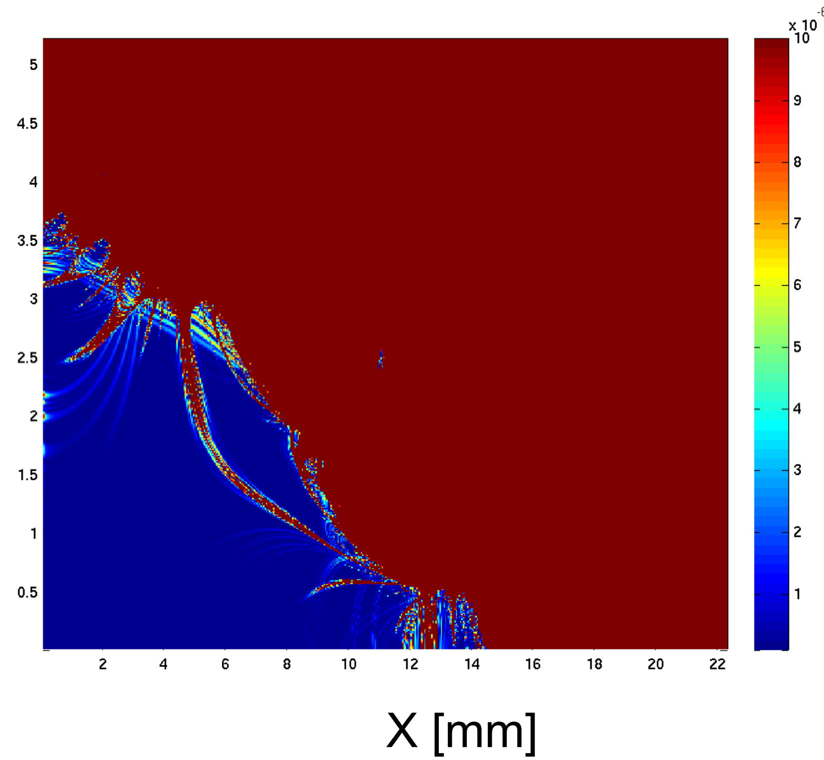
The dynamic aperture is in unit of sigma of the equilibrium beam size. The USR design is built with 4th-order geometric achromats and therefore no 3rd and 4th order resonances driven by the sextupoles seen in the scan.

Frequency Map Analysis

PEP-X: Baseline (2008)



PEP-X: USR (2011)

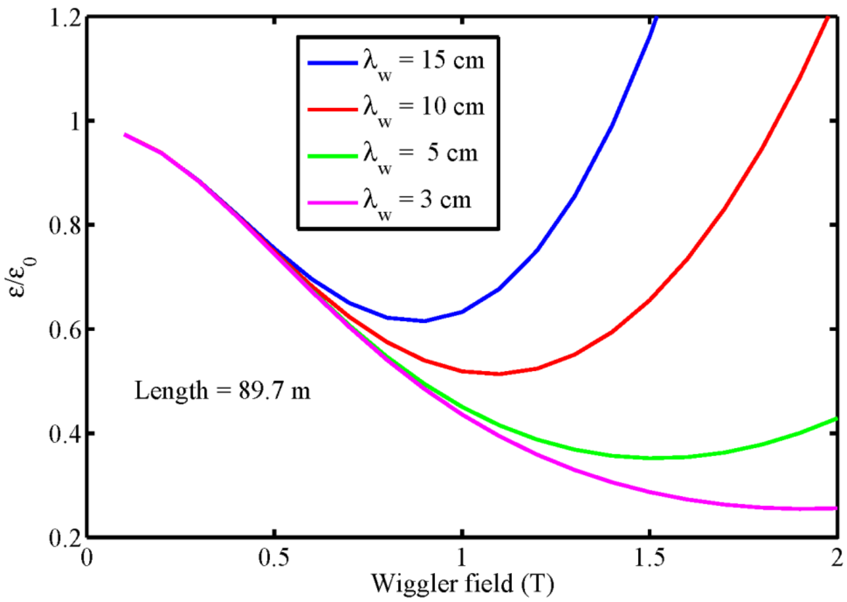


The dynamic aperture is in unit of mm at the injection. The baseline design has a factor of ten larger emittance than the one in the USR design.

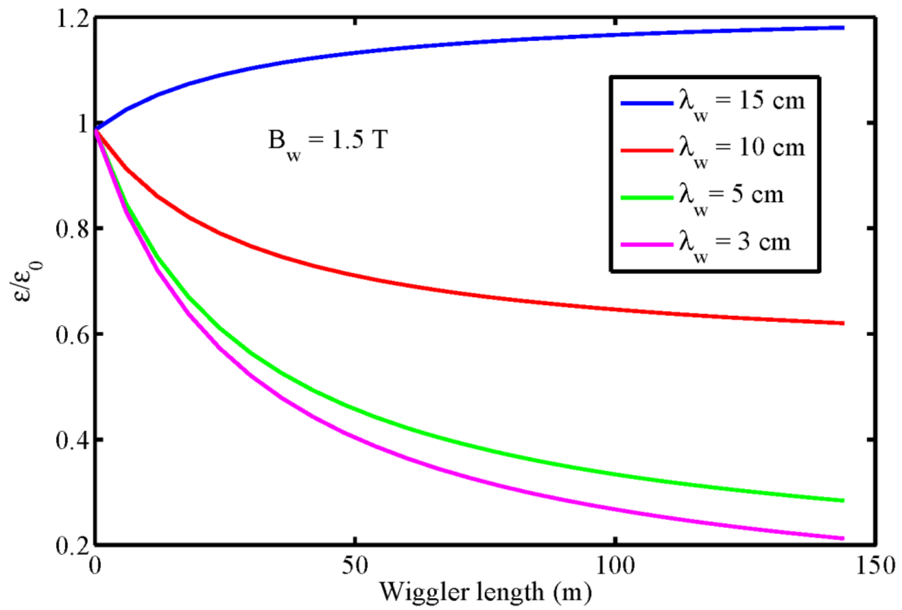
Reduce Emittance with Damping Wigglers

Emittance = 11 pm-rad at 4.5 GeV with parameters λ_w=5 cm, B_w=1.5 T

Wiggler Field Optimization



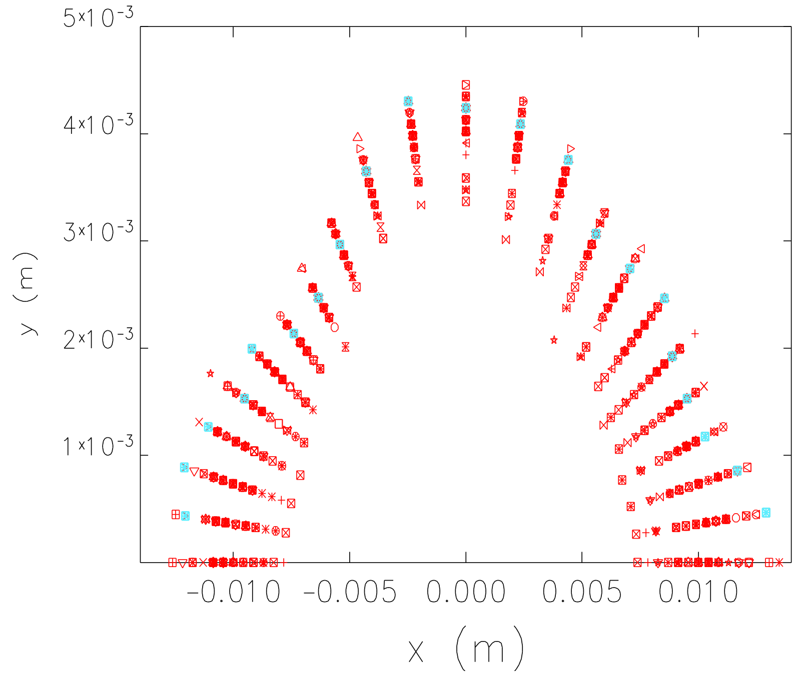
Wiggler Length Optimization



Average beta function at the wiggler section is 12.4 meter.

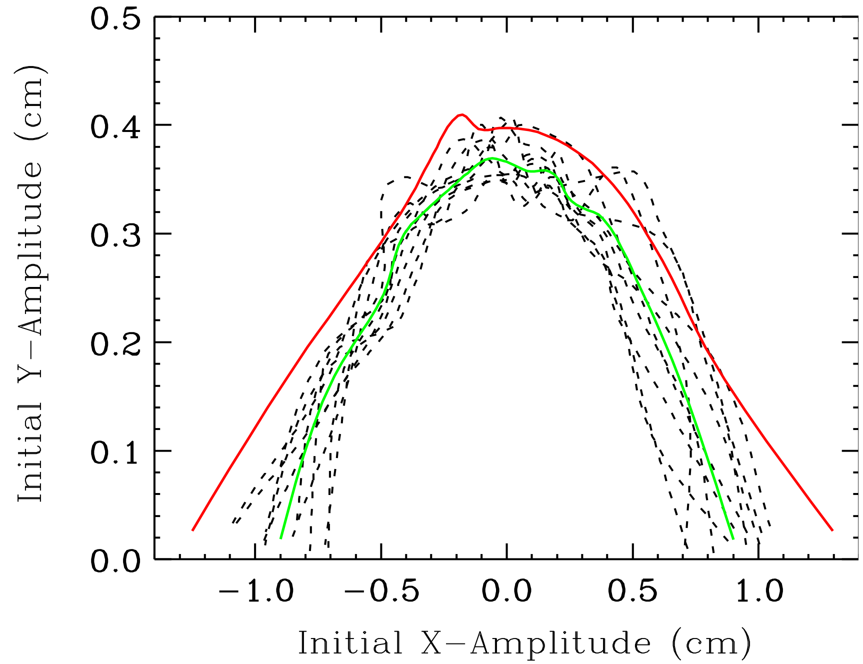
Dynamic Aperture with Machine Errors

ELEGANT Tracking



1% coupling & 1% beta beating

LEGO Tracking



Misalignments 20 microns in x.

Intra-Beam Scattering

The growth rate in the relative energy spread s_d is given by

$$\frac{1}{T_p} = \frac{r_e^2 c N_b (log)}{16\gamma^3 \varepsilon_x \varepsilon_y \sigma_z \sigma_\delta^3} < \sigma_H g(\alpha)(\sigma_x \sigma_y)^{-1/2} >,$$

where N_b is the bunch population and (log) the Coulomb log factor and the other factors are defined by

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_\delta^2} + \frac{\mathrm{H}_x}{\varepsilon_x}, \alpha = \sqrt{\frac{\varepsilon_y \beta_x}{\varepsilon_x \beta_y}},$$

$$g(\alpha) = \alpha^{(0.021 - .0044 \ln \alpha)}.$$

and the horizontal growth rate is given by

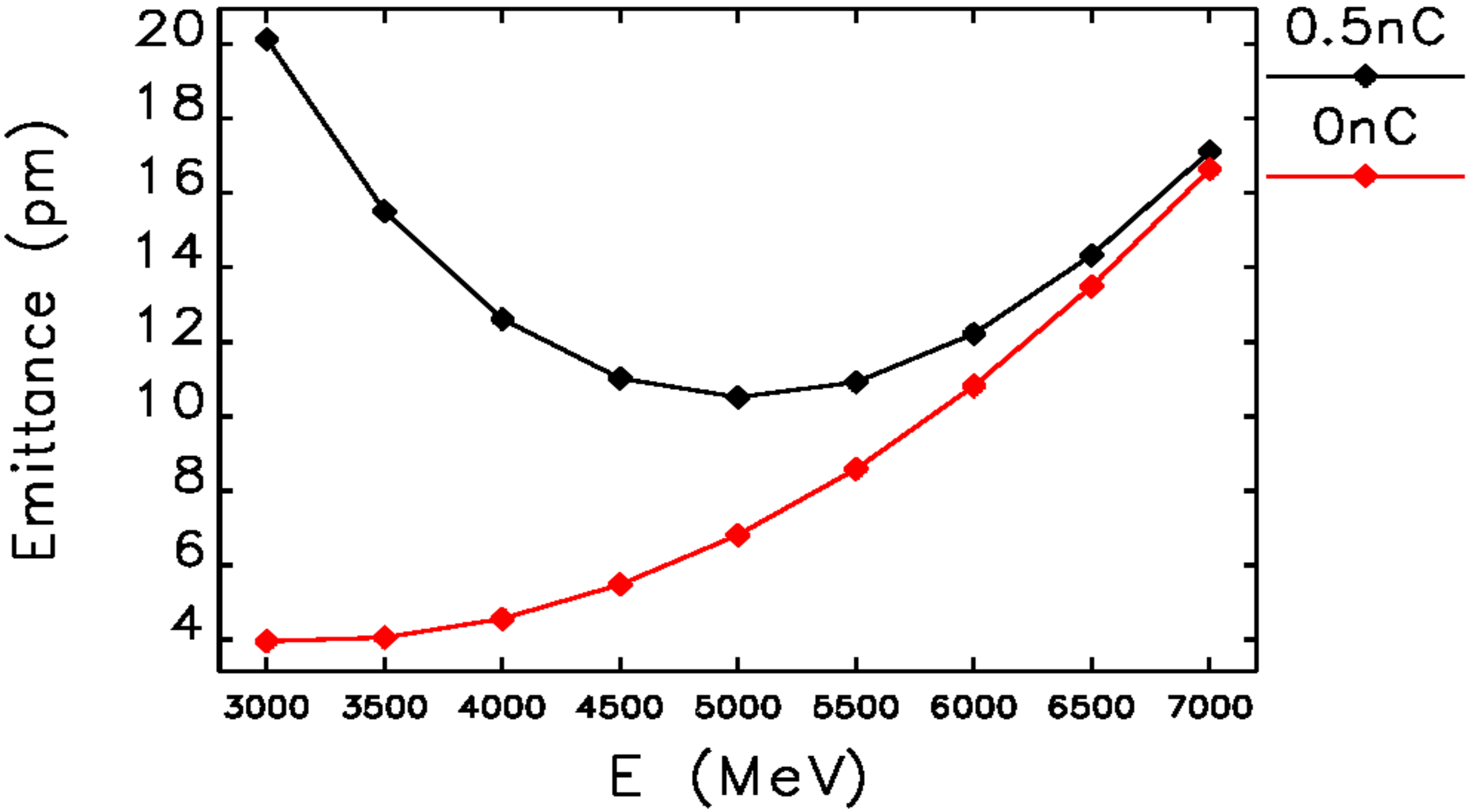
$$\frac{1}{T_x} = \frac{\sigma_\delta^2}{\varepsilon_x} < \mathrm{H}_x \Delta(\frac{1}{T_p}) > .$$

Combined with synchrotron radiation

$$\varepsilon_x = \frac{\varepsilon_{x0}}{1 - \tau_x / T_x}, \sigma_\delta^2 = \frac{\sigma_{\delta 0}^2}{1 - \tau_s / T_p},$$

$$\varepsilon_y = \kappa \varepsilon_x$$

Optimization of Energy



Touschek Lifetime

When a pair of electrons go through a hard scattering, their momentum changes are so large that they are outside the RF bucket or the momentum aperture. This process results in a finite lifetime of a bunched beam. The lifetime is given by

$$\frac{1}{\mathrm{T}} = \frac{r_e^2 c N_b}{8\sqrt{\pi}\gamma^4 \varepsilon_x \varepsilon_y \sigma_z \sigma_\delta} < \sigma_H F(\delta_m) > ,$$

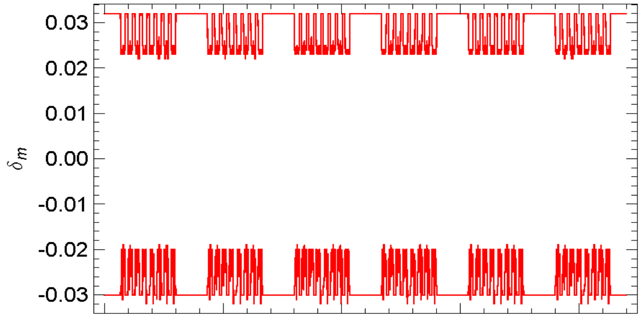
with

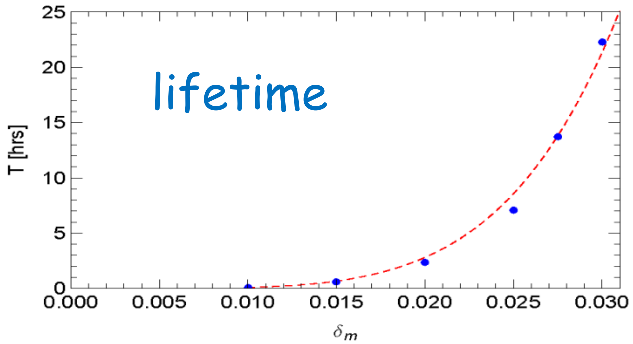
$$F(\delta_m) = \int_{\delta_m^2}^{\infty} \frac{d\tau}{\tau^{3/2}} e^{-\tau B_+} I_0(\tau B_-)\left[\frac{\tau}{\delta_m^2} - 1 - \frac{1}{2}\ln\left(\frac{\tau}{\delta_m^2}\right)\right],$$

$$B_\pm = \frac{1}{2\gamma^2}\left|\frac{\beta_x(\beta_x\varepsilon_x + \eta_x^2\sigma_\delta^2)}{\varepsilon_x(\beta_x\varepsilon_x + \beta_x \mathcal{H}_x \sigma_\delta^2)} \pm \frac{\beta_y}{\varepsilon_y}\right|,$$

where δ_m is the momentum acceptance.

momentum aperture



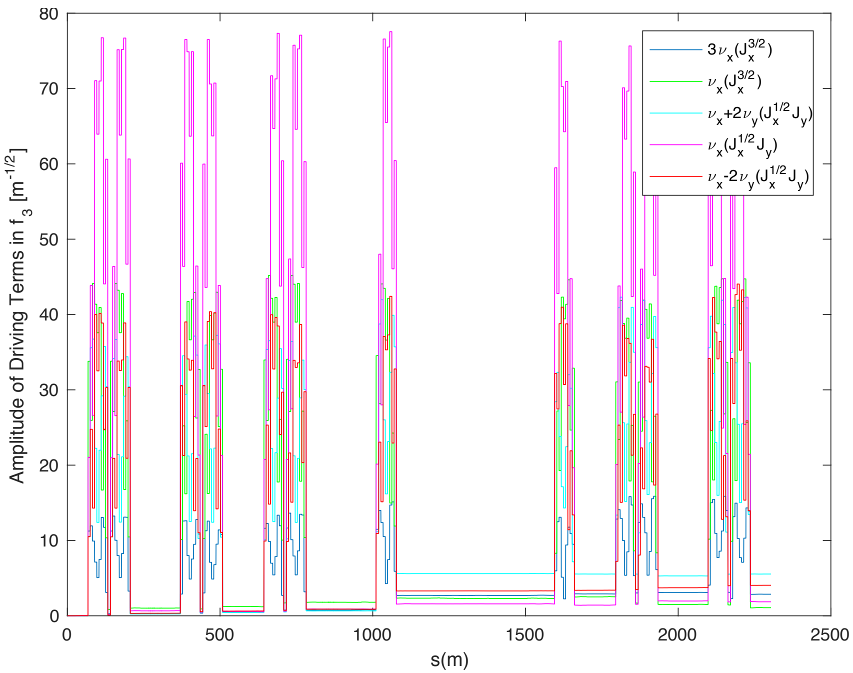


Achievements

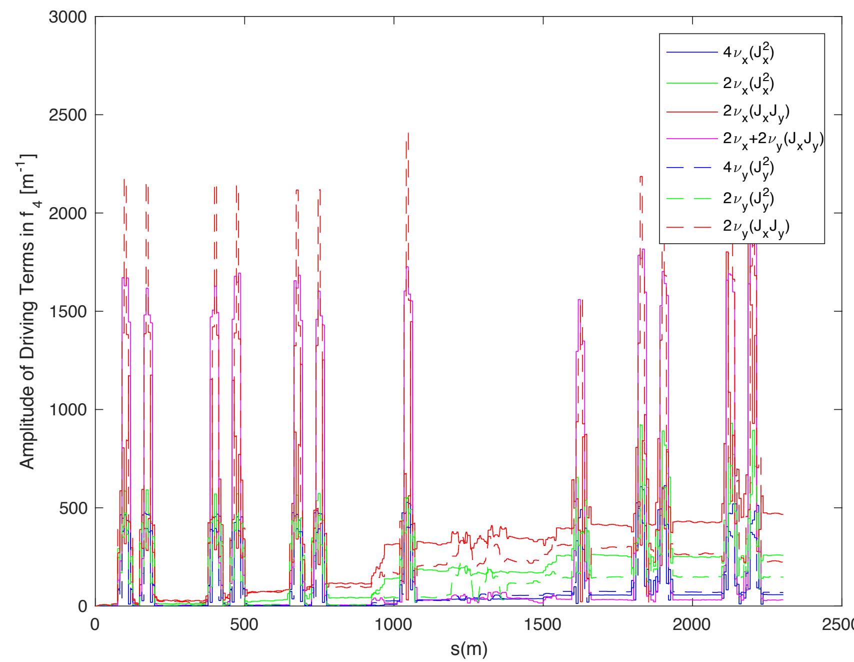
- We have developed an excellent design of an ultimate storage ring
  - Diffraction limit at 1 angstrom
  - Reasonable beam current 200 mA
  - Good beam lifetime 3 hours
  - Good injection with 10 mm acceptance
  - Achievable machine tolerances 20 microns

Resonances in PETRA-III

Third Order

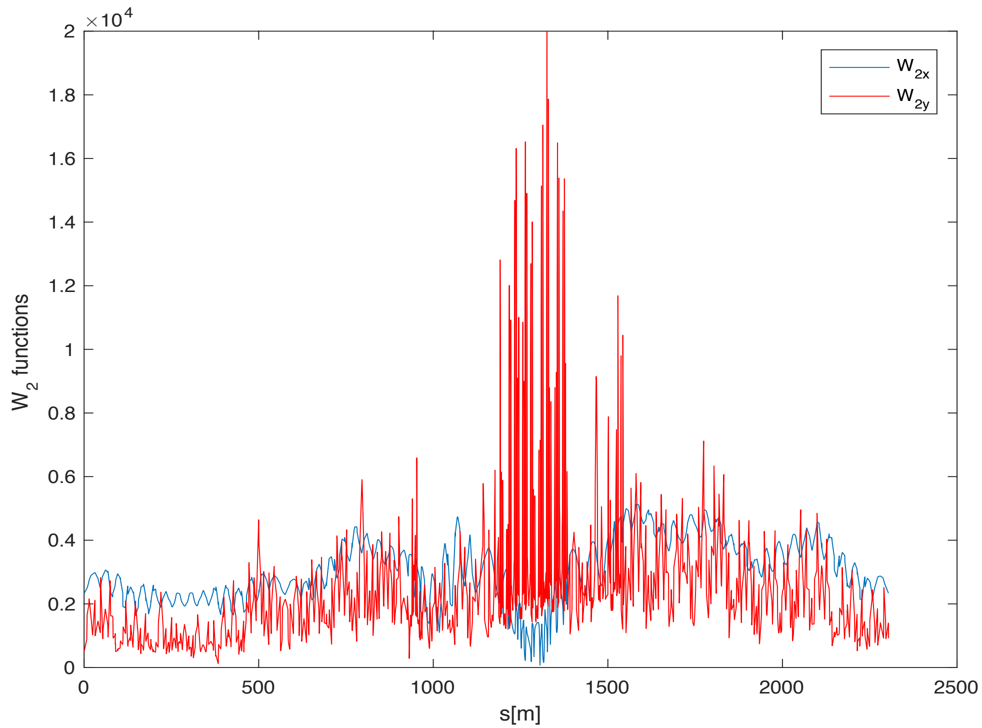


Fourth Order



What are missing?

Nonlinear Chromatic Effects in PETRA-III



Summary

- Structure resonances driven by sextupoles are determined by cell phase advances. In many cases, it can be reduced to a single 4th-order resonance in an integer units of betatron oscillation. It is very powerful and efficient way to mitigate the effects of resonances in storage rings. because it does not depend on
  - Where are the sextupoles
  - How many families of them
  - Length of sextupoles
- Known examples are $60^0/60^0$, $90^0/60^0$, $135^0/45^0$, $765^0/405^0$, $855^0/315^0$.

References

1) Yunhai Cai, "Single-particle dynamics in electron storage rings with extremely low emittance," Nucl. Instr. Meth. A 645, 168 (2011).
2) Yunhai Cai, Karl Bane, Robert Hettel, Yuri Nosochkov, Min-Huey Wang, Michael Borland, "Ultimate storage ring based on fourth-order geometric achromats," PRSTAB **15**, 054002 (2012).