Accelerator Injection and Extraction

Course given at the US Particle Accelerator School

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This collection of slides represents the material we prepared for the USPAS Course on Injection and Extraction of Beams held at the Lisle Sheraton Hotel under the Auspices of Northern Illinois University on June 19-23, 2017. A substantial fraction of the material is not our original research but rather represents the state of the art in the field. We have tried to acknowledge our sources as much as possible by either explicit mention or in the references.

To all colleagues who have willingly let us use material go our thanks and gratitude.

Revision 1, Lisle, June 24, 1017

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Accelerator and Storage Rings Basics

$$
\begin{bmatrix}
\cos\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right) & \frac{\sin\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right)\rho}{\sqrt{k\rho^2+1}} & \frac{\cos\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right)\rho}{k\rho^2+1}-\frac{\rho}{k\rho^2+1} \\
-\frac{\sqrt{k\rho^2+1}\sin\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right)}{\rho} & \cos\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right) & -\frac{\sin\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right)}{\sqrt{k\rho^2+1}} \\
0 & 0 & 1
\end{bmatrix}
$$

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Circular Machine Basics

- Lorentz Force:

$$\vec{F} = q\left(\vec{E} + \vec{\beta} \times \vec{B}\right)$$

- Momentum:

$$\vec{p} = \frac{m_0 \gamma \vec{\beta}}{c}$$

- Equation of motion:

$$\frac{d\vec{p}}{dt} = \vec{F}$$

Note: cp: momentum [eV], m_0 rest energy [eV], q charge [e_0]

Equation of Motion

- Typically, E is 0 (except in accelerating cavities)
  B is B_3 vertical guide field (except in focusing elements)
- Then the eq of motion becomes

$$q\left(\beta_2(s)B_3\hat{i}-\beta_1(s)B_3\hat{j}\right)=\frac{m_0\gamma\left(\left(\frac{d}{ds}\beta_1(s)\right)\hat{i}+\left(\frac{d}{ds}\beta_2(s)\right)\hat{j}+\left(\frac{d}{ds}\beta_3(s)\right)\hat{k}\right)}{c}$$

- Integrate this twice and get:

$$\vec{x}=-\frac{\beta_{2,0}m_0\gamma\left(\cos\left(\frac{B_3qcs}{\gamma m0}\right)\hat{i}-\sin\left(\frac{B_3qcs}{\gamma m0}\right)\hat{j}\right)}{B_3qc}$$

-

- This describes a circle with radius

$$\rho=\frac{\beta_{2,0}m_0\gamma}{B_3qc}=\frac{pc}{B_3qc}$$

- The “B-rho” value is then a property of the beam:

$$B\rho=\frac{pc}{qc}=3.33564\,pc,\;\;pc\;[\mathrm{GeV}]$$

- The circle thus defined is used as *reference orbit*. All beam dynamics can be expressed relative to this orbit.
  - this allows series expansion w/o messing up the basic geometry.

Frenet-Serret Coordinates

- To do this we transform into a beam-following coordinate system called Frenet-Serret or TNB (tangent-normal-binormal) coordinates.

tangent (longitudinal)
normal (horizontal)
binormal (vertical)

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{bmatrix} \sin\left(\frac{s}{\rho}\right) & \cos\left(\frac{s}{\rho}\right) & 0 \\ \cos\left(\frac{s}{\rho}\right) & -\sin\left(\frac{s}{\rho}\right) & 0 \\ 0 & 0 & -1 \end{bmatrix} \circ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

- and the Lorentz equation becomes

$$\left[\frac{d^2}{ds^2} X_1(s) = \frac{\frac{d}{ds} X_2(s)}{\rho}, \frac{d^2}{ds^2} X_2(s) = -\frac{\frac{d}{ds} X_1(s)}{\rho}, \frac{d^2}{ds^2} X_3(s) = 0 \right]$$

Hill's Equation

- Modern accelerators are built from discrete bending and focusing magnets. Therefore, ρ and k are functions of s.

$$\frac{d^2}{ds^2} X_2(s) = -\frac{X_2(s)}{\rho(s)^2} - k(s) X_2(s) \text{ and } \frac{d^2}{ds^2} X_3(s) = k(s) X_3(s)$$

- Mr. Hill found that solutions have the form

$$\xi_1(s) = a \cdot w(s) \cdot \cos(\psi(s))$$

$$\xi_2(s) = a \cdot w(s) \cdot \sin(\psi(s))$$

- with w(s) being given by the *envelope equation*

$$-\frac{1}{w(s)^3} - w(s)k(s) + \frac{d^2}{ds^2} w(s) = 0$$ amplitude

- and

$$\frac{d}{ds} \psi(s) = \frac{1}{w(s)^2}$$ phase

Matrix Optics

- Solutions like Mr. Hill's can be expressed by a matrix algorithm:

$$\begin{bmatrix} x(L) \\ \frac{\mathrm{d}}{\mathrm{d}L}x(L) \end{bmatrix} = R \circ \begin{bmatrix} x(0) \\ \left.\left(\frac{\mathrm{d}}{\mathrm{d}s}x(s)\right)\right|_{s=0} \end{bmatrix}, \quad R = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

- Change notation to that commonly used in accelerator work:

$$w(s) = \sqrt{\beta(s)}, \quad \frac{\mathrm{d}}{\mathrm{d}s}w(s) = -\frac{\alpha(s)}{w(s)}, \quad \gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

- and it can be shown that

$$R_p = \begin{bmatrix} \alpha(0)\sin(\mu(L)) + \cos(\mu(L)) & \sin(\mu(L))\beta(0) \\ \left(-\frac{\alpha(0)^2}{\beta(0)} - \frac{1}{\beta(0)}\right)\sin(\mu(L)) & -\alpha(0)\sin(\mu(L)) + \cos(\mu(L)) \end{bmatrix}$$

Matrix from 0 to s

- Without derivation we give the R matrix between two points of unequal β(s) and α(s):

$$\begin{bmatrix} \frac{\sqrt{\beta(s)}\left(\sin(\mu(s))\alpha(0) + \cos(\mu(s))\right)}{\sqrt{\beta(0)}} & \sqrt{\beta(s)}\sin(\mu(s))\sqrt{\beta(0)} \\ \frac{(-\alpha(0)\alpha(s) - 1)\sin(\mu(s)) + (\alpha(0) - \alpha(s))\cos(\mu(s))}{\sqrt{\beta(0)}\sqrt{\beta(s)}} & \frac{\left(-\sin(\mu(s))\alpha(s) + \cos(\mu(s))\right)\sqrt{\beta(0)}}{\sqrt{\beta(s)}} \end{bmatrix}$$

- The connection between $k(s)$ and β(s) and α(s) is:

$$k(s) = \frac{\alpha(s)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}s}\alpha(s)\right)\beta(s) + 1}{\beta(s)^2}$$

Floquet Coordinates

- The one-turn matrix R_p looks a bit like a rotation matrix. Lets make this more explicit:
  - define a matrix

$$F = \begin{bmatrix} \frac{1}{\sqrt{\beta(s)}} & 0 \\ \frac{\alpha(s)}{\sqrt{\beta(s)}} & \sqrt{\beta(s)} \end{bmatrix}$$

  - transform an arbitrary phase-space vector:

$$Q = \begin{bmatrix} q \\ p \end{bmatrix} = F \circ \begin{bmatrix} x \\ xp \end{bmatrix} = \begin{bmatrix} \frac{x}{\sqrt{\beta(s)}} \\ \frac{\alpha(s)x}{\sqrt{\beta(s)}} + \sqrt{\beta(s)}xp \end{bmatrix}$$

- Transform R_p:

$$R_n = F \circ R_p \circ F^{-1} = \begin{bmatrix} \cos(\mu(L)) & \sin(\mu(L)) \\ -\sin(\mu(L)) & \cos(\mu(L)) \end{bmatrix}$$

- Apply R_n on Q:

$$R_n \circ Q = \begin{bmatrix} \frac{\cos(\mu(L))x}{\sqrt{\beta(s)}} + \sin(\mu(L))\left(\frac{\alpha(s)x}{\sqrt{\beta(s)}} + \sqrt{\beta(s)}xp\right) \\ -\frac{\sin(\mu(L))x}{\sqrt{\beta(s)}} + \cos(\mu(L))\left(\frac{\alpha(s)x}{\sqrt{\beta(s)}} + \sqrt{\beta(s)}xp\right) \end{bmatrix}$$

- The length of the result is

$$a^2 = \frac{\beta(s)^2 xp^2 + 2\alpha(s)\beta(s)x \cdot xp + \beta(s)^2 x^2 + x^2}{\beta(s)}$$

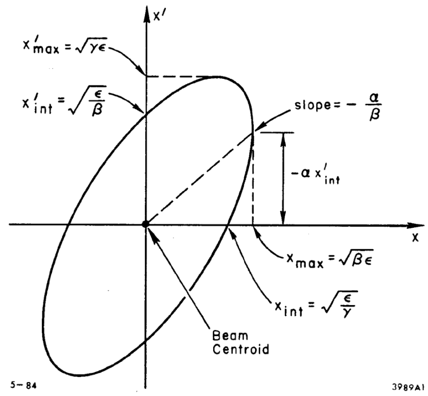
Machine Ellipse

- or, using the Twiss $\gamma(s)$:

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta}$$

$$a^2 = xp^2\beta(s) + 2x \cdot xp \cdot \alpha(s) + x^2\gamma(s)$$

- This is known as the *Courant-Snyder Invariant*. It describes an ellipse in x-xp (phase-) space.
  - a^2 is the area of the ellipse.
  - $\varepsilon = a^2/\pi$ is called the *emittance*

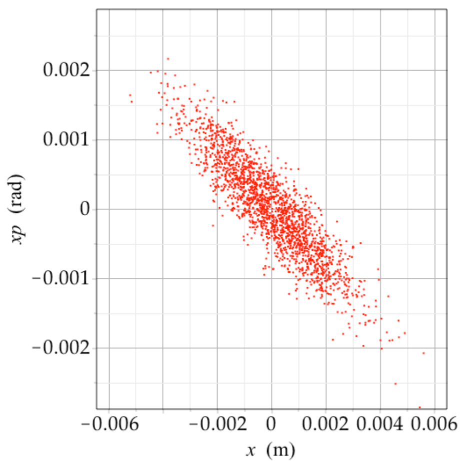


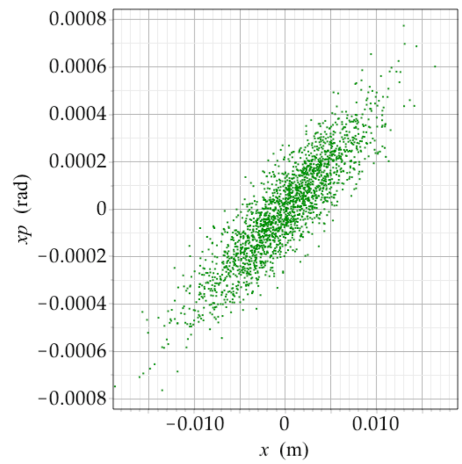
Liouville's Theorem

- A conservative system (like a beam line) does not change phase-space volume (emittance).
  - in practise, phase-space volume *can* grow due to nonlinearity & filamentation
- Once emittance has grown, there is *no way* to make it small again.
  - unless cooling techniques are used or radiation damping applies.
- Beam transfer is a significant source of emittance growth
  - (not a theorem by Liouville!)
- You cannot "merge" phase space using (static) magnets.

Machine *vs* Beam Ellipse

- The ellipse thus defined is a property of a closed ring (except for the area).
- Each particle given by (x,xp) is moving on such an ellipse.
- a *dynamic* equilibrium



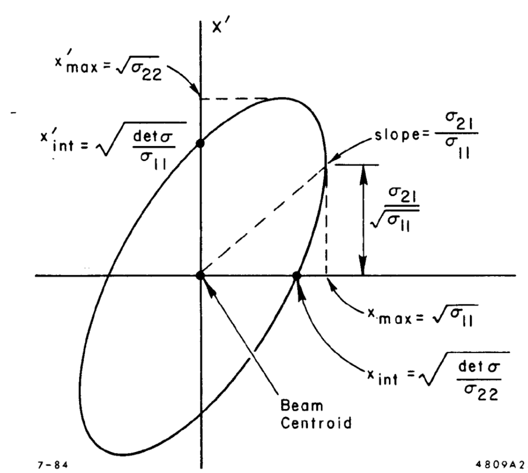


- We need to describe a *beam ellipse* as well: Σ matrix

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}, \qquad \sigma_{21} = \sigma_{12}$$

$$\sigma_{11} = \overline{x^2}, \qquad \sigma_{12} = \sigma_{21} = \overline{x \cdot xp},$$

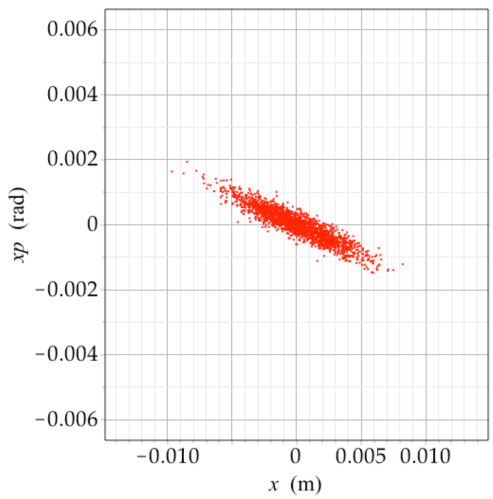
$$\sigma_{22} = \overline{xp^2}$$

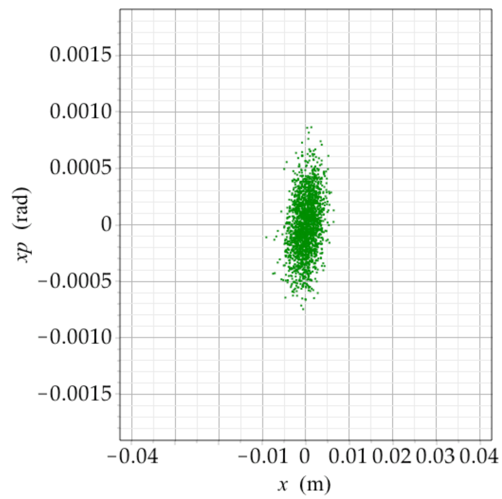


- Compare to the previous figure

$$\begin{bmatrix} \sigma 11 & \sigma 12 \\ \sigma 21 & \sigma 22 \end{bmatrix} = \begin{bmatrix} \varepsilon\beta(s) & -\varepsilon\alpha(s) \\ -\varepsilon\alpha(s) & \varepsilon\gamma(s) \end{bmatrix}$$

- What about a beam injected *off-axis* or one that has a different aspect ratio??
- A *mismatched* beam, no equilibrium





Element-Wise Description

- Drift section

$$\frac{d^2}{ds^2}X_2(s)=0 \Rightarrow R_D=\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$

- Quadrupole (watch out: cosh etc. for k<0 i.e. defocusing!)

$$\frac{d^2}{ds^2}X_2(s)=-kX_2(s)\Rightarrow R_Q=\begin{bmatrix} \cos\left(\sqrt{k}s\right) & \frac{\sin\left(\sqrt{k}s\right)}{\sqrt{k}} \\ -\sqrt{k}\sin\left(\sqrt{k}s\right) & \cos\left(\sqrt{k}s\right) \end{bmatrix}$$

- Dipole (wedge bending magnet, δ=δp/p)

$$\frac{d^2}{ds^2}X_2(s)=-\frac{X_2(s)}{\rho^2}-kX_2(s)-\frac{\delta}{\rho}\Rightarrow ?$$

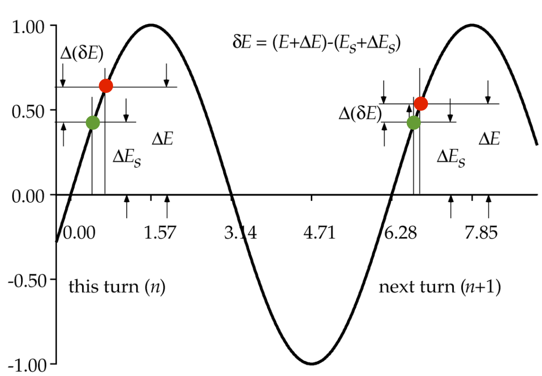
- Turns out we need a third coordinate: δ=δp/p

$$R_B = \begin{bmatrix} \cos\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right) & \frac{\sin\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right)\rho}{\sqrt{k\rho^2+1}} & \frac{\cos\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right)\rho}{k\rho^2+1} - \frac{\rho}{k\rho^2+1} \\ -\frac{\sqrt{k\rho^2+1}\sin\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right)}{\rho} & \cos\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right) & -\frac{\sin\left(\frac{\sqrt{k\rho^2+1}\cdot s}{\rho}\right)}{\sqrt{k\rho^2+1}} \\ 0 & 0 & 1 \end{bmatrix}$$

Note: the quantity $-k\rho^2$ is also known as field index n

Synchrotron Motion

- Acceleration in a synchrotron requires an rf system.
- The rf frequency is synchronous with the revolution time in the synchrotron.
- Beam particles oscillate in time and energy about the reference phase and energy
  - phase stability (Vecksler & MacMillan)



- oscillating particle
- reference particle

Equation of Motion

- The equations of motion can be written as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi(t)=\frac{\omega_{rf}^2\eta W(t)}{\beta^2 E_s} \qquad W(t)=-\delta E(t)/\omega_{rf}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}W(t)=\frac{1}{2}\frac{qV\left(\sin(\Phi_s)-\sin(\Phi(t))\right)}{h\pi} \qquad \eta=\left(\frac{1}{\gamma^2}-\frac{1}{\gamma_t^2}\right),\ \text{the } \textit{slip factor}$$

- This can be solved for $\Phi(t) = \Phi_s + \phi(t)$ and $\phi(t)$ small. If we use initial conditions $\Phi(0) = 0$ and $W(0) = W_0$, we get

$$W(t)=W_0\cos\left(\frac{1}{2}\frac{\sqrt{2}\omega_{rf}\sqrt{\eta}\sqrt{q}\sqrt{V}\sqrt{\cos(\Phi_s)}t}{\beta\sqrt{\pi}\sqrt{h}\sqrt{E_s}}\right),\quad \phi(t)=\frac{\sqrt{2}\omega_{rf}\sqrt{\eta}\sin\left(\frac{1}{2}\frac{\sqrt{2}\omega_{rf}\sqrt{\eta}\sqrt{q}\sqrt{V}\sqrt{\cos(\Phi_s)}t}{\beta\sqrt{\pi}\sqrt{h}\sqrt{E_s}}\right)W_0\sqrt{h}\sqrt{\pi}}{\beta\sqrt{E_s}\sqrt{q}\sqrt{V}\sqrt{\cos(\Phi_s)}}$$

- This describes harmonic motion
  - and an ellipse in phase space.

Rf Bucket

- Small-amplitude synchrotron oscillations have a frequency

$$\Omega_s=\sqrt{\frac{h\omega_s^2\eta qV\cos(\Phi_s)}{2\beta^2\pi E_s}}$$

η: slip factor
h: harmonic of rf
ω_s: rf frequency
V: peak rf voltage
β: relativistic velocity
E_s: beam energy
Φ_s: synchronous phase

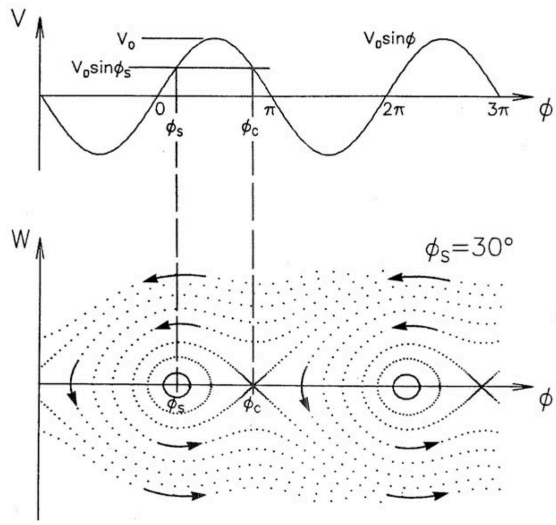
- The amplitude is limited: "bucket height":

$$\frac{\widehat{\delta(E)}}{E_s}=\frac{\sqrt{-\pi\eta hVqE_s\left(\sin(\Phi_s)\pi-2\sin(\Phi_s)\Phi_s-2\cos(\Phi_s)\right)}\beta}{E_s\pi\eta h}$$

- and a max. phase width not subject to simple analytic expression

- There are h "rf buckets" in a synchrotron.
- Energy (W in the figure below) and phase ϕ are the direct longitudinal equivalents to xp and x

M. Craddock



"Thin" Elements

- Thin quads are a useful approximation to make algebra simple

$$R_Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -kf & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & kf & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

  – a series of these converges to a regular quadrupole
- "Thin dipoles" can be defined in an ad-hoc fashion (Brown & Servranckx)

$$R_D := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sin(\theta)}{\rho} & 1 & 0 & 0 & 0 & \sin(\theta) \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

References

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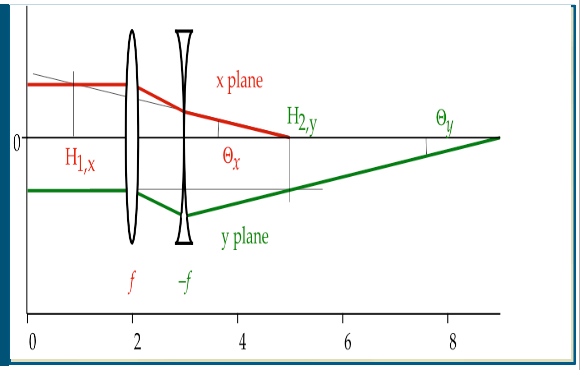
Matching Sections



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- Match injecting beam-properties to ring Twiss functions
  - $\beta_x, \alpha_x, \beta_y, \alpha_y$ match => at least 4 quadrupoles needed
  - if dispersion is involved, need at least one dipole & more quads
  - if rotation (coupling) is involved, need skew quads.
  - a workable solution is not guaranteed for any sequence of elements.
- Optical building blocks make this easier:
  - Doublet: parallel to point
  - Quarter-wave transformer: match FODOs with different parameters
  - Telescope, to magnify or demagnify a beam
- Analytic evaluation using thin-lens optics can guide the initial layout.

Insertions

- Often, the machine design can accomodate injection with an insertion
  - Dispersion suppressors
  - high-β sections
  - symmetry points with $\alpha = 0$
  - 180° sections to facilitate closed kicker bumps with 2 kickers.
- Such sections are inserted using two techniques
  - $R = I$ sections; these are transparent (often $R = -I\bullet-I$)
  - $R \neq I$ sections; but $\beta_x, \alpha_x, \beta_y, \alpha_y$ matched (changes machine tune)

Some Building Blocks

- Doublet
- Transformers
- Dispersion suppressor
- Propagation of Twiss functions:

$$T_2 = R_{12} \cdot T_1 \cdot R_{12}{}^t, \quad T_1 = \begin{bmatrix} \beta(0) & -\alpha(0) \\ -\alpha(0) & \gamma(0) \end{bmatrix}$$

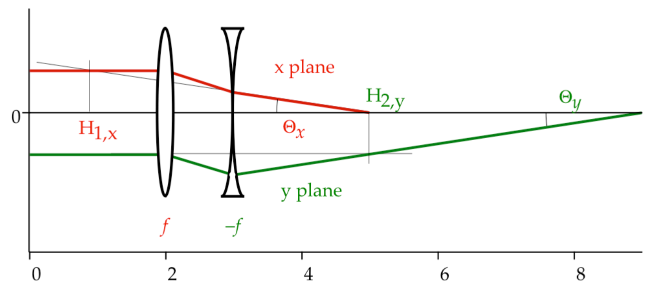
- explicit:

$$\begin{bmatrix} \beta(s) \\ \alpha(s) \\ \gamma(s) \end{bmatrix} = \begin{bmatrix} r_{11}^2 & -r_{11}r_{12} - r_{11}r_{21} & r_{12}^2 \\ -r_{21}r_{11} & r_{11}r_{22} + r_{12}r_{12} & -r_{22}r_{12} \\ r_{21}^2 & -2r_{21}r_{22} & +r_{22}^2 \end{bmatrix} \circ \begin{bmatrix} \beta(0) \\ \alpha(0) \\ \gamma(0) \end{bmatrix}$$

Doublet Lens

- A doublet lens focuses in both planes, but with different properties:

$$R=\begin{bmatrix} -L_dk_f+1 & L_d & 0 & 0 & 0 & 0\\ L_dk_dk_f-k_d-k_f & -L_dk_d+1 & 0 & 0 & 0 & 0\\ 0 & 0 & L_dk_f+1 & L_d & 0 & 0\\ 0 & 0 & L_dk_dk_f+k_d+k_f & L_dk_d+1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



- Two doublets spaced by more than their focal length make a beta transformer, with a transformation ratio roughly

$$\frac{\beta_2}{\beta_1}\approx\frac{L_2^2}{L_D^2}$$

L_D = spacing between the doublets
L_2 = space to downstream waist, β_2
β_1 = incoming β

  - (if the β in x and y are similar one may need triplets)

- The focal length of each doublet is then

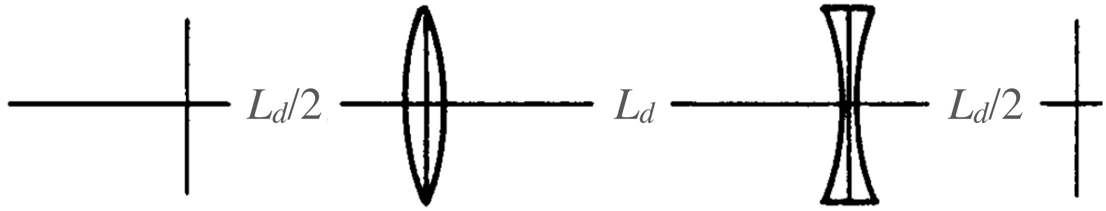
$$f_u\approx\frac{L_D^2}{L_D+L_2},\qquad f_d\approx\frac{L_DL_2}{L_D+L_2}$$

subscript u is upstream, d is downstream

- These are starting points for numerical fitting (e.g. Mad-X)
- For small β at the injection point need to move the matching quads closer else the whole array gets too long.

Quarter-Wave Transformer

- A q-w-t is a FODO cell arranged like this:



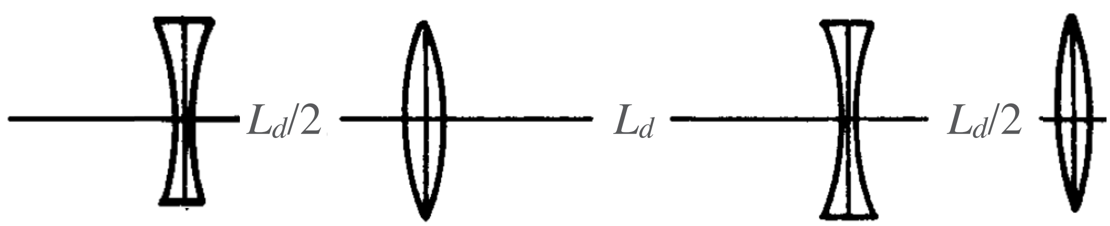
f $\qquad\qquad$ $-f$

- for $f = 1/k = \sqrt{2}/L_d$,
  its R-Matrix looks like this:

$$\begin{bmatrix} -\sqrt{2} & \frac{3Ld}{2} & 0 & 0 & 0 & 0 \\ -\frac{2}{Ld} & \sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} & \frac{3Ld}{2} & 0 & 0 \\ 0 & 0 & -\frac{2}{Ld} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- its phase advance is $\pi/2$

- To make a matching section, we add 2 quads:



$Q1$ $\qquad$ f $\qquad\qquad$ $-f$ $\qquad$ $Q2$

$$\begin{bmatrix} -\sqrt{2}-\frac{3LdkQ1}{2} & \frac{3Ld}{2} & 0 & 0 & 0 & 0 \\ \frac{2Ld(KQ2-kQ1)\sqrt{2}+3KQ2Ld^2kQ1-4}{2Ld} & -\frac{3KQ2Ld}{2}+\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2}+\frac{3LdkQ1}{2} & \frac{3Ld}{2} & 0 & 0 \\ 0 & 0 & \frac{2Ld(KQ2-kQ1)\sqrt{2}+3KQ2Ld^2kQ1-4}{2Ld} & \frac{3KQ2Ld}{2}-\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- This section propagates the beta functions as follows:

$$\beta_{x2} = \left(-\sqrt{2} - \frac{3L_d kQ1}{2}\right)^2 \beta_{x1} - 3\left(-\sqrt{2} - \frac{3L_d kQ1}{2}\right) Ld\alpha_{x1} + \frac{9L_d^2}{4}\gamma_{x1}$$

$$\beta_{y2} = \left(+\sqrt{2} + \frac{3L_d kQ1}{2}\right)^2 \beta_{y1} + 3\left(-\sqrt{2} - \frac{3L_d kQ1}{2}\right) Ld\alpha_{y1} + \frac{9L_d^2}{4}\gamma_{y1}$$

kQ1 & L_d set β_2

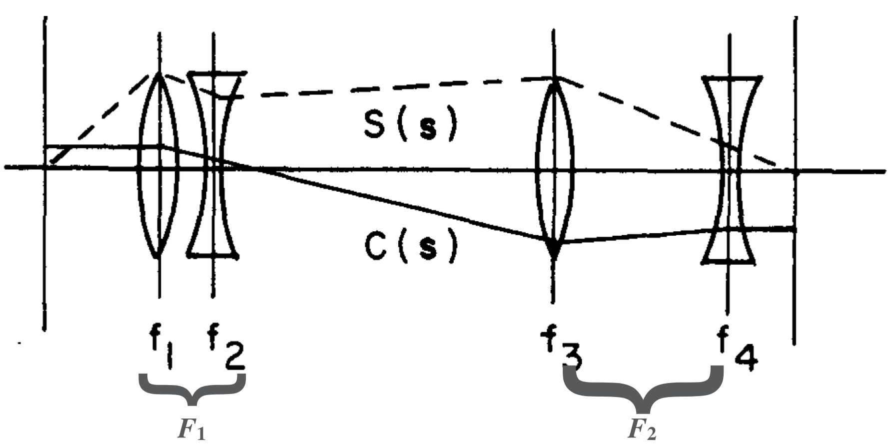
- Matching procedure:
  - set L_d, $Q1$ to achieve β_{x2}, β_{y2} as desired
  - set $Q2$ to achieve desired α_{x2}, α_{y2}.
  - for $\beta_{x1} = \beta_{y1}$ and $\alpha_{x1} = -\alpha_{y1}$ we get $\beta_{x2} = \beta_{y2}$ and $\alpha_{x2} = -\alpha_{y2}$
  - Putting this at the symmetry point of a FODO matches one FODO to another one with different parameters
- Analytic expressions for L_d, $Q1$ as $f(\beta_{x2}, \beta_{y2})$ can be found but are not insightful.

Half-Wave Transformer

- A Half-wave transformer is characterized by a Matrix

$$R = \begin{bmatrix} -R_{11} & 0 \\ 0 & -1/R_{11} \end{bmatrix}$$

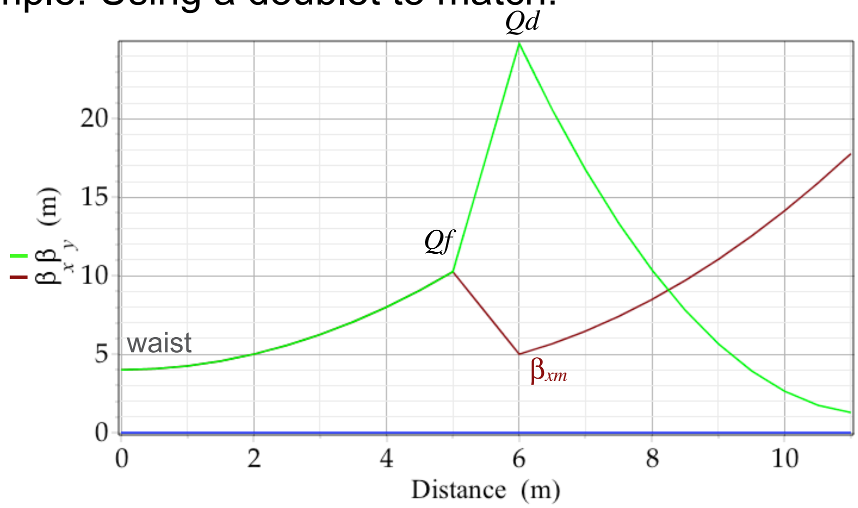
R_{11} is the magnification $F_2/F_1 = \sqrt{(\beta_2/\beta_1)}$
F_1, F_2 is the effective focal length of each doublet
Phase advance is $\mu = \pi$
R_{33} may differ from R_{11}



- The distance to the waist is about the focal length of the 2nd doublet.
- The distance between the doublets is the sum of the focal lengths of each doublet, and the magnification, the ratio of the two.
- Such transformers work well between points with $\alpha_x = \alpha_y = 0$.
- As before, these considerations help getting starting values for the numerical fitting.
-

Match of a FODO to a Waist

- Consider a ring where an insertion has been provided with a double-waist, which we want to match to. The incoming beam has FODO-like parameters.
- Example: Using a doublet to match:



- R matrix for the matching section:

$$\begin{bmatrix} -Ld2kQf+1 & -Ld1Ld2kQf+Ld1+Ld2 & 0 & 0 & 0 & 0 \\ Ld2kQdkQf-kQd-kQf & (Ld2kQdkQf-kQd-kQf)Ld1-Ld2kQd+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & Ld2kQf+1 & Ld1Ld2kQf+Ld1+Ld2 & 0 & 0 \\ 0 & 0 & Ld2kQdkQf+kQd+kQf & (Ld2kQdkQf+kQd+kQf)Ld1+Ld2kQd+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- β_x at the 2nd quadrupole (Q_d) is

$$\beta_{xm} = \left(-L_{d2}k_{Qf}+1\right)^2 \beta_{xw} + \frac{\left(L_{d1}+L_{d2}\left(-L_{d1}k_{Qf}+1\right)\right)^2}{\beta_{xw}}$$

- we can find the value for the 1st matching quad:

$$k_{Qf} = \frac{L_{d1}^2 + L_{d1}L_{d2} + \beta_{xw}^2 - \sqrt{L_{d1}^2\beta_{xm}^2\beta_{xw}^2 - L_{d2}^2\beta_{xw}^2 + \beta_{xm}^2\beta_{xw}^3}}{\left(L_{d1}^2 + \beta_{xw}^2\right)L_{d2}}$$

- not very instructive in itself, but we use this result to look at α_{xm} after Qd:

$$\alpha_{xm} = \frac{\sqrt{L_{d1}^2\beta_{xm}\beta_{xw} - L_{d2}^2\beta_{xw}^2 + \beta_{xm}\beta_{xw}^3} + \beta_{xm}\beta_{xw}\left(L_{d2}k_{Qd}-1\right)}{L_{d2}\beta_{xw}}$$

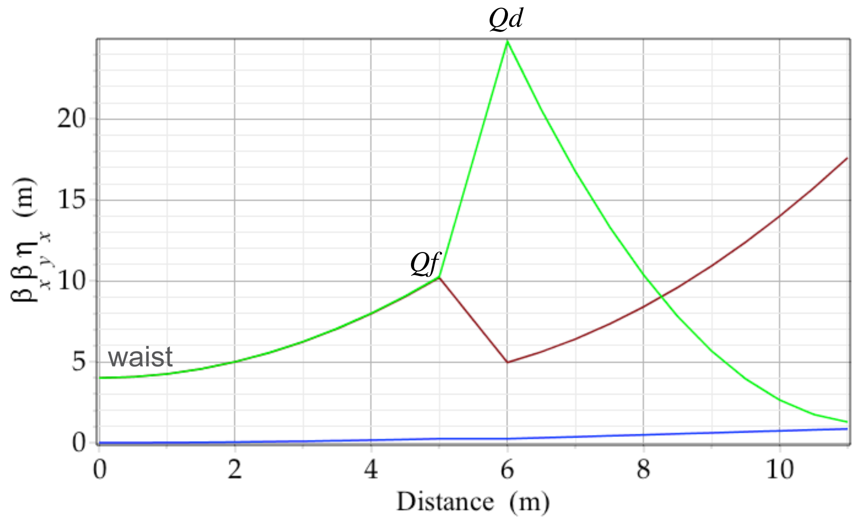
- We (usually) want α_x to be ≤0 after Qd, so we can solve:

$$k_{Qd} < \frac{\beta_{xm}\beta_{xw} - \sqrt{L_{d1}^2\beta_{xm}\beta_{xw} - L_{d2}^2\beta_{xw}^2 + \beta_{xm}\beta_{xw}^3}}{\beta_{xm}\beta_{xw}L_{d2}}$$

- At which point we have expressions for the two quadrupoles & need to put in numbers.
- If we use $L_{d1} = 5$ m, $L_{d2} = 1$ m, $\beta_{xm} = \beta_{ym} = 4$ m, we get $k_{Qf} = 0.43/\text{m}$ and $k_{Qd} < -0.42/\text{m}$. The previous figure was calculated using $k_{Qd} = -0.54/\text{m}$.
- The following cells will be the FODO array we match into, with the first cell likely needing slight adjustments.
- This exercise shows that even simple matching problems have complex algebra unless we restrict the parameter space

Dispersion Matching

- Injection regions may have 0 or finite dispersion that we need to match to. The situation is made more complicated by septa that create dispersion of their own.



Dispersion Suppressors

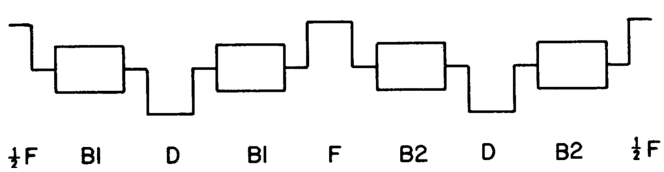
- We demonstrate dispersion matching by introducing dispersion suppressors. Techniques to match to finite dispersion are similar.
- A FODO cell has dispersion given by

$$\eta_{Qf} = \frac{L\theta}{4}\frac{1+\frac{1}{2}\sin\left(\frac{\mu}{2}\right)}{\sin\left(\frac{\mu}{2}\right)^2}$$

θ = bending angle of cell

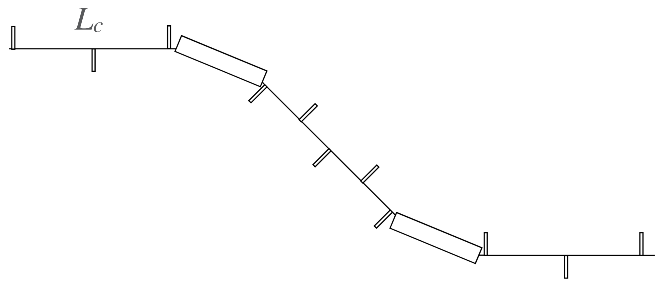
- It can be shown that such a cell transforms 0 dispersion to twice its matched value.
  - a cell with half bending angle can match dispersion to 0 (!)
- In more detail:

$$\theta_1 = \theta_D\left(1-\frac{1}{4\sin\left(\frac{\mu}{2}\right)^2}\right),\ \theta_2 = \theta_D\left(\frac{1}{4\sin\left(\frac{\mu}{2}\right)^2}\right)$$



A Dispersion-Matched DogLeg

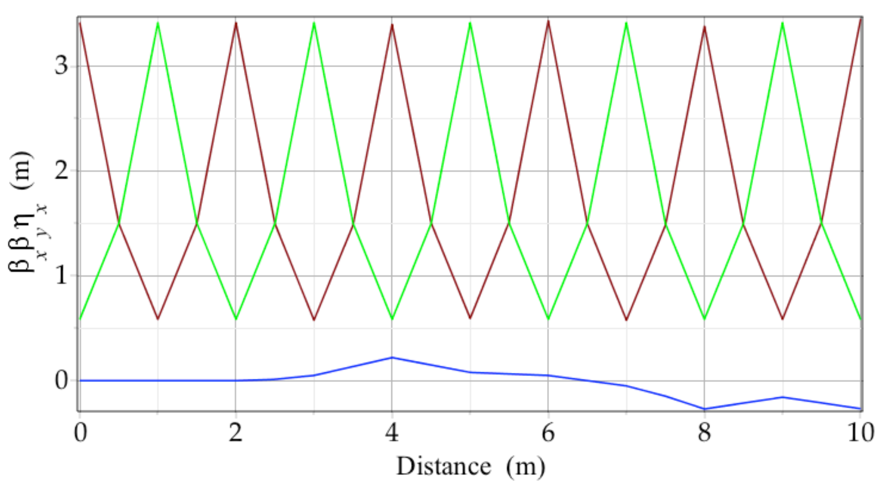
- Consider a (horizontal) offset in the geometry of a beam line
- without the optics: large dispersion at end.
  - roughly $\theta*2.5*L_c$
  - How can optics make this 0 to 1st order ??



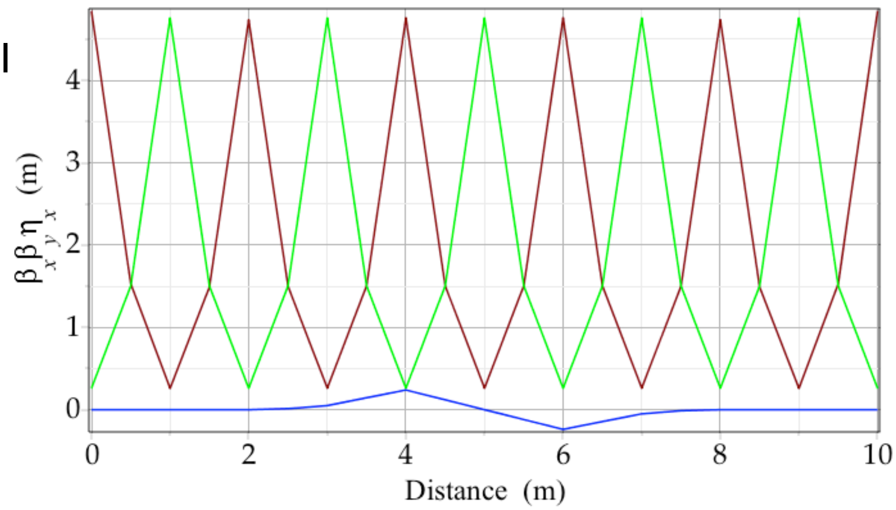
- A 180° section in between the dipoles will flip dispersion in sign
  - the 2nd dipole then makes it 0.
  - by symmetry dispersion should be 0 at the center but *not* the slope of dispersion, which is <0!

Thin-Quad Model of DogLeg

- One approach is to make the FODO cells 90° & see how far we get with this:
  - for a symmetric (QD = –QF) cell, $kQ = \frac{2\sqrt{2}}{L_c}$
- Result (for θ_{bend} = 0.1 mrad and L_c = 2 m):

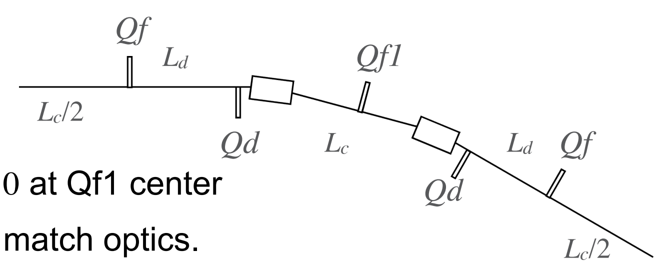


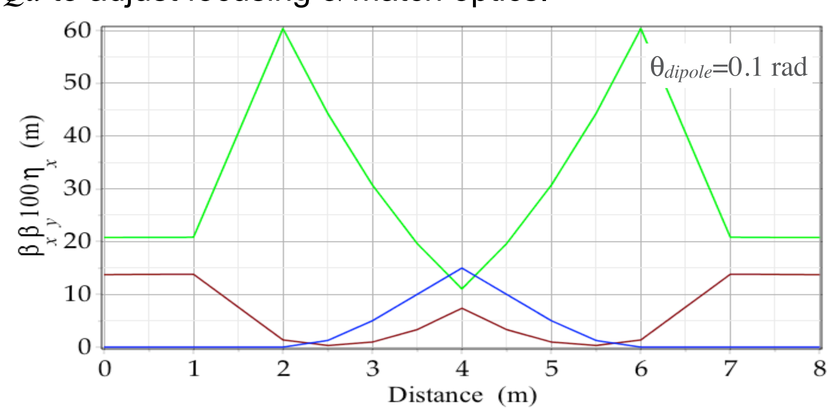
- So this did not work (because η'≠0 at the end of the dipole)
- In this case, it is easy to find an analytic solution to make dispersion 0 at the symmetry point: $kQ = \frac{3.582}{L_c}$
- the phase advance/cell is just over 127°.



Bending Section

- For a bending section, achromat cells are a good starting point:
- Example: DBA cell:
  - use Ld=1 m, Lc= 4 m:
  - $k_{Qf1} = 1.33\ \mathrm{m}^{-1}$ to make $\eta' = 0$ at Qf1 center
  - Qf, Qd to adjust focusing & match optics.

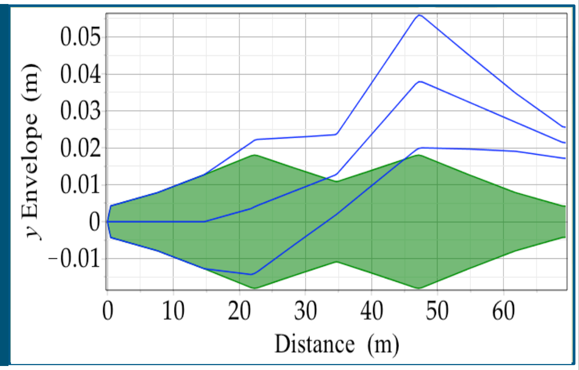




References

- K.L. Brown and R.V. Servranckx, SLAC-PUB-3381, 1984.

Single-turn Injection



U. WIENANDS
ANL

E. MARIN-LACOMA
CERN

19-Jun-2017
USPAS, Lisle Il.

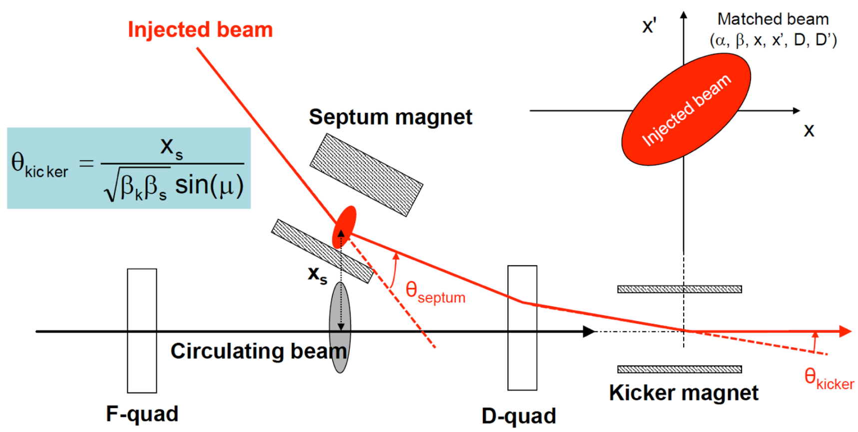
- We need a time-varying field
- On what time scale?
  - A: short enough to get the injecting beam onto the axis.

- A different way to look at this:
  - A time-varying field briefly puts the orbit of the ring onto the axis of the incoming beam
  - The beam is guided into the ring
  - At the next turn this field has vanished & the injected beam continues circulating

Simplest Injection

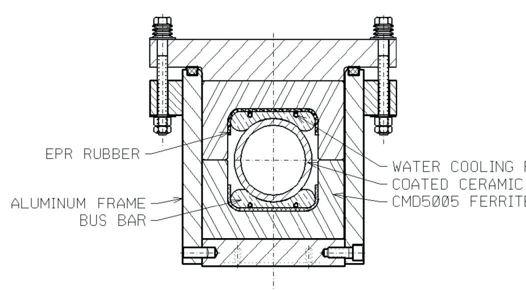
- The Kicker guides the incoming bunch into the ring
- after a turn, the kicker field has collapsed & the circulating beam passes straight through.
  - ~~note that kicker bending is opposite to that of ring magnets!~~
- Vertical injection is also possible and done.

C. Bracco, CERN



"Real-World" Limitations

vertical kicker, H<–>h



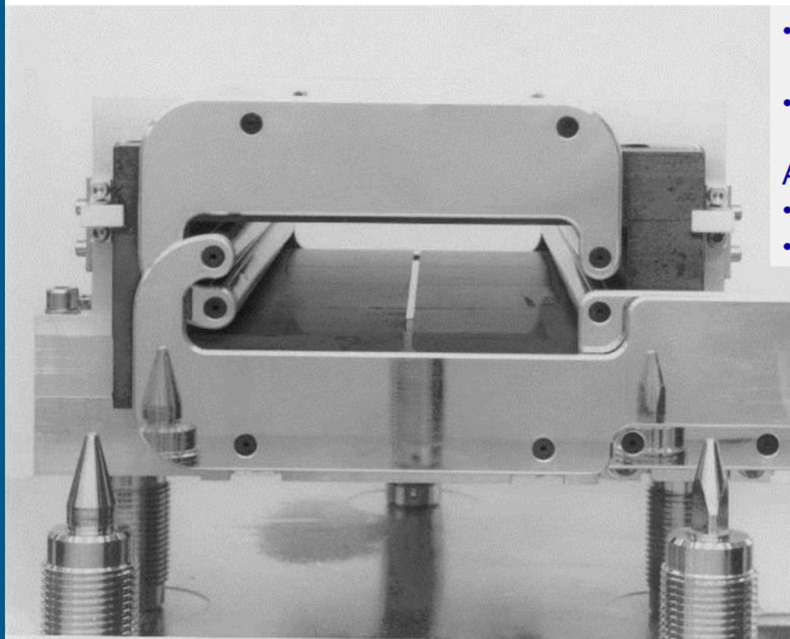
- Kicker Parameters
- Field: $$B \approx \mu_0 \frac{N \cdot I}{h}$$
- Inductance $$L \approx \mu_0 \frac{N^2 H}{h} l$$

h: full gap height
H: full gap width
l: length

- It turns out that for any reasonably fast rise/fall time the voltage requirement is prohibitive if N>1
- This limits the B field, lest I becomes prohibitive.

Lumped Inductance Kicker

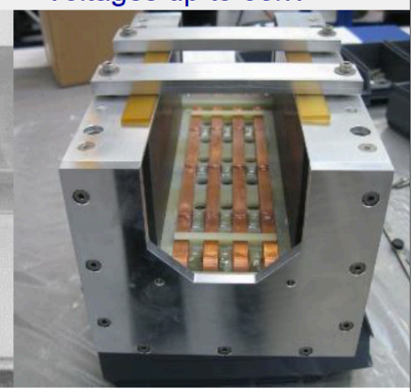
M. Barnes, CERN



- Used for “slower” systems (typically > ~1μs rise/fall).
- “Simple” and “robust”.

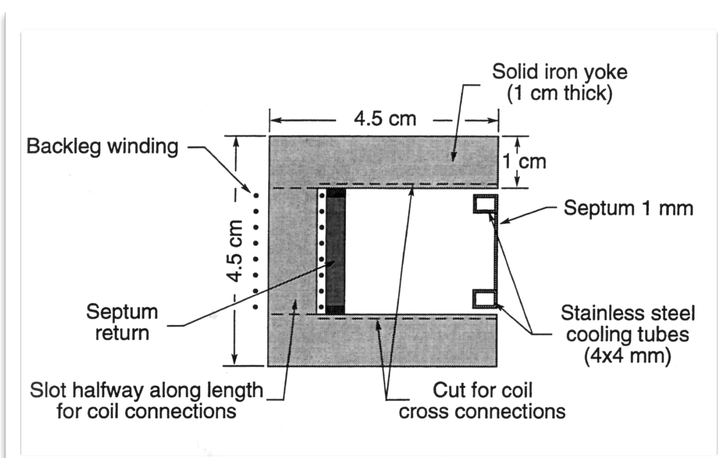
At CERN:

- Currents up to 18.5 kA
- Voltages up to 30kV



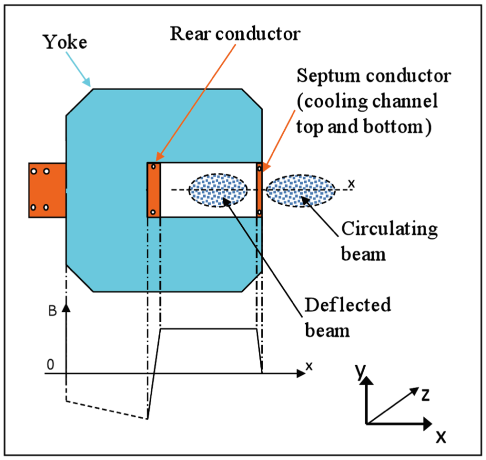
Septum Magnets

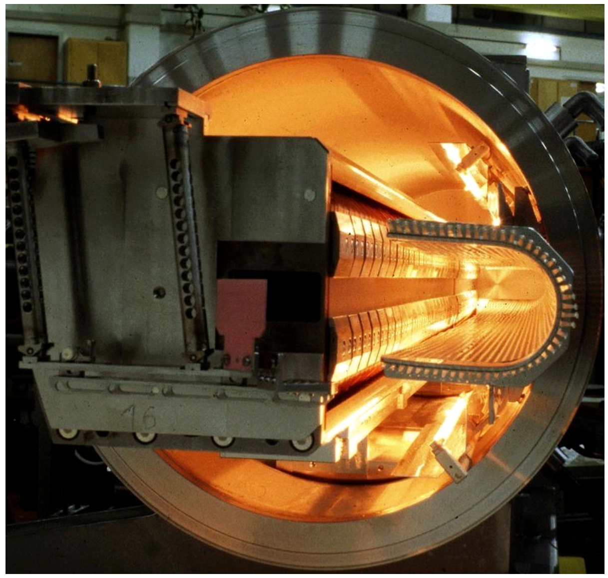
- With the limited kicker angle we only have a small gap between the injecting and circulating beam.
- => use (one or more) septum magnet(s) to line up the incoming beam.



Pulsed Septum Magnet

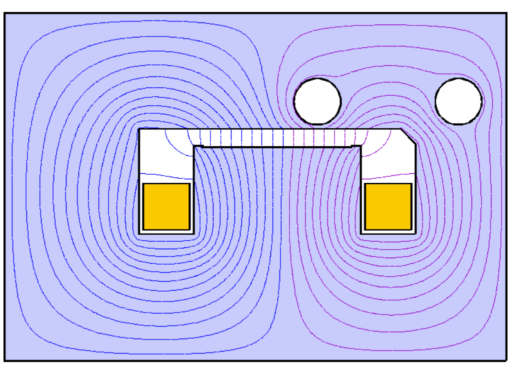
M. Barnes, CERN

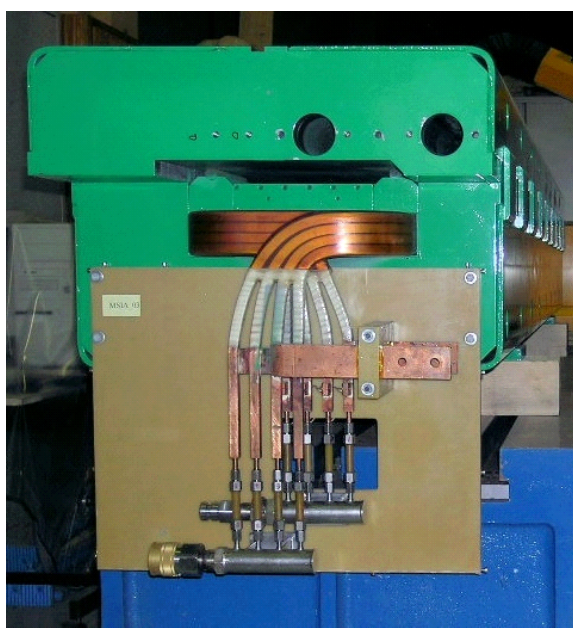




Lambertson Septum (LHC)

M. Barnes, CERN





Optimizing a Machine for Injection

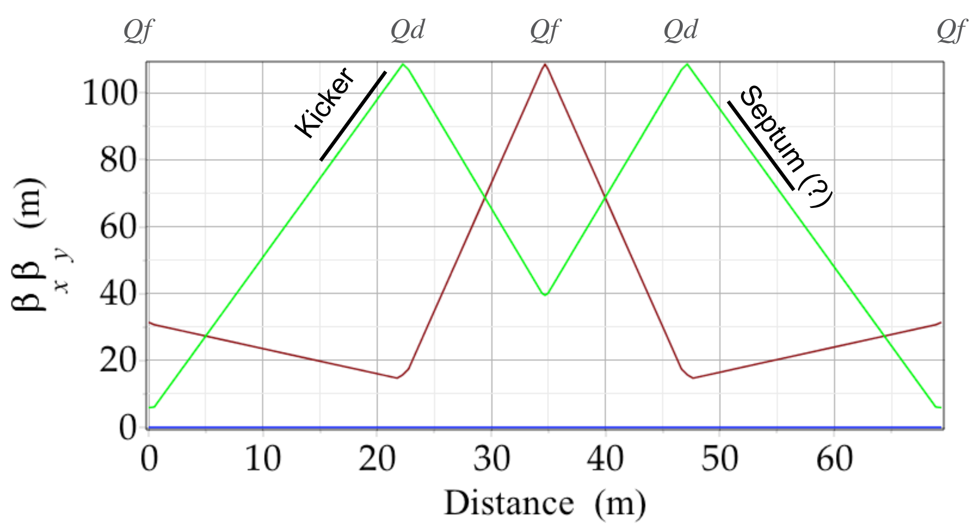
- Want maximum effect from the limited kicker angle
  - kicker @ high β, most parallel beam
- Want maximum clearance for the septum
- Partial transformation through a kicker followed piece of ring:

$$\begin{pmatrix} x \\ xp \end{pmatrix}_2 = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}\left(\sin(\mu_{12})\alpha_1 + \cos(\mu_{12})\right) & \sqrt{\beta_1\beta_2}\sin(\mu) \\ \frac{(-\alpha_1\alpha_2 - 1)\sin(\mu_{12}) + (\alpha_1 - \alpha_2)\cos(\mu_{12})}{\sqrt{\beta_1\beta_2}} & \sqrt{\frac{\beta_1}{\beta_2}}\left(\cos(\mu_{12}) - \alpha_2\sin(\mu_{12})\right) \end{pmatrix} \bullet \begin{pmatrix} x \\ xp + \delta xp \end{pmatrix}_1$$

- => high β at kicker & septum; 90° phase advance
- Insertions help, if the lattice allows it

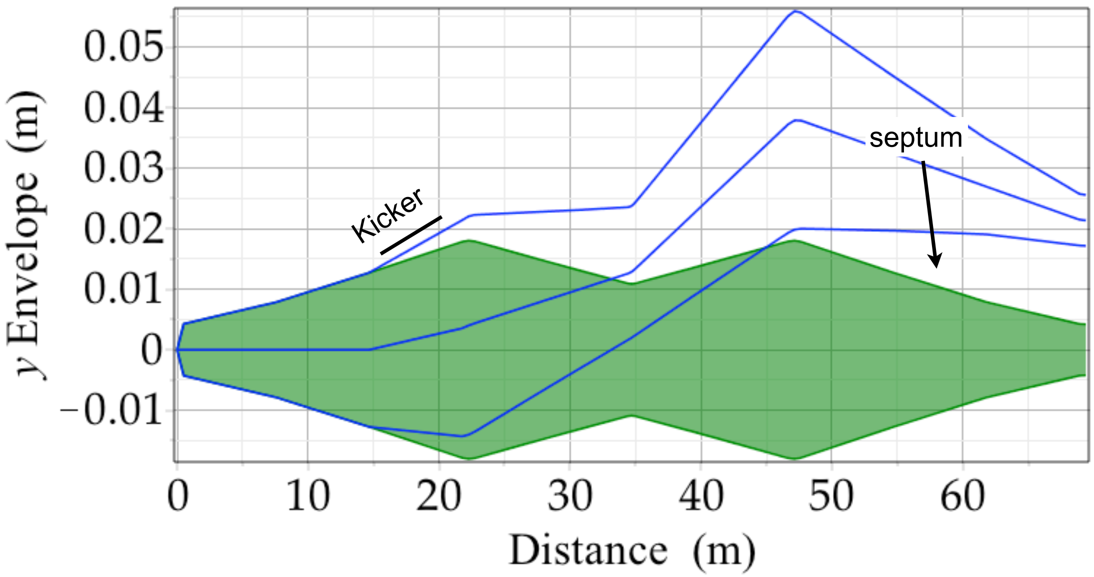
Simple Insertion

- A matched insertion transforming β from its ring value to a higher value
  - typically have η=0 in straight.

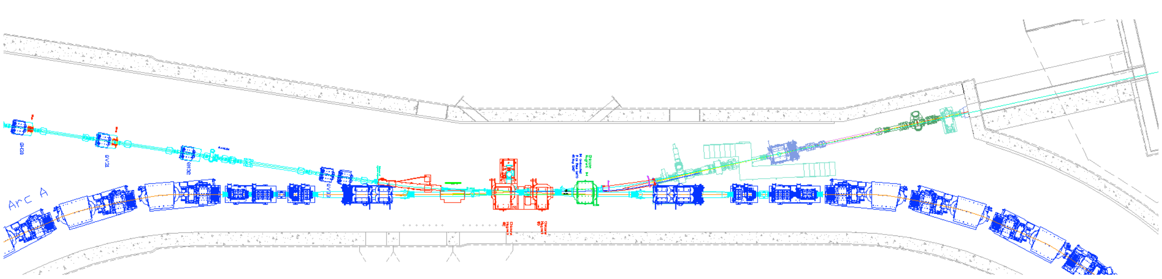


Envelope plot

- It is often better to sacrifice β for phase angle
  - beam envelope shrinks with √β

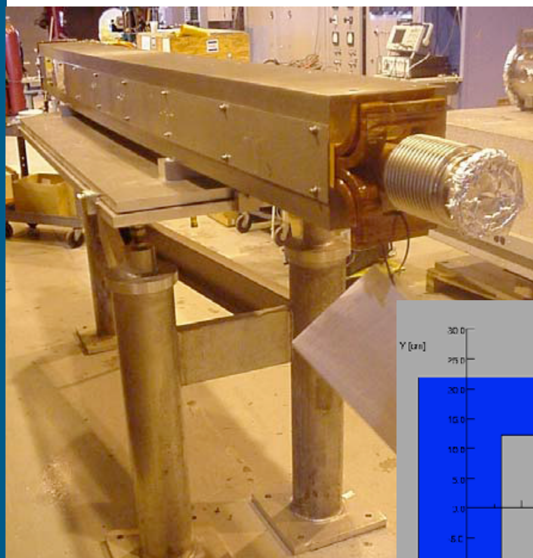


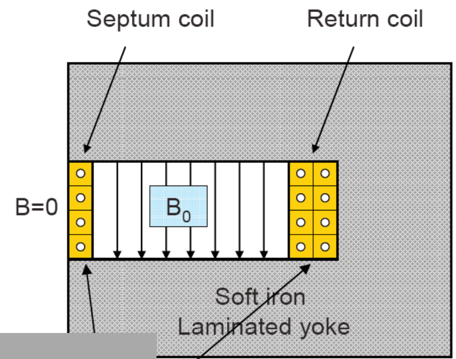
Example: SNS Ring Injection System

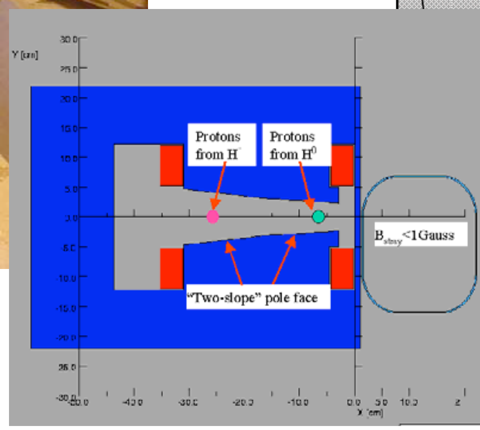


Septum Magnets (SNS)

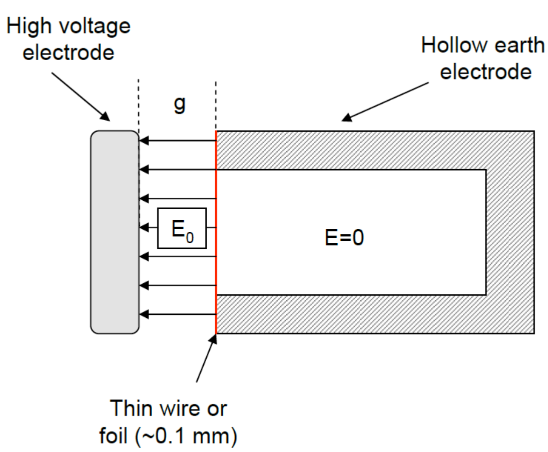
M. Plum, ORNL



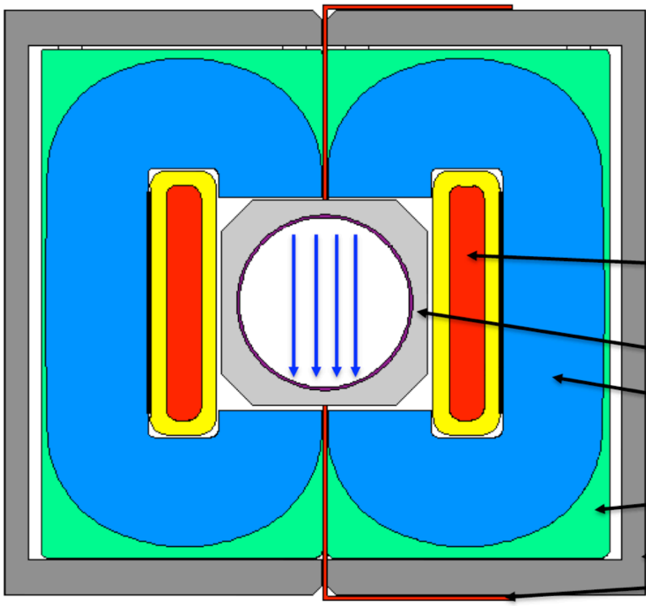




Electrostatic Wire Septum



Kicker Magnet (LHC MKD)



Magnets are built around a metallized ceramic vacuum chamber

Variations on the Theme

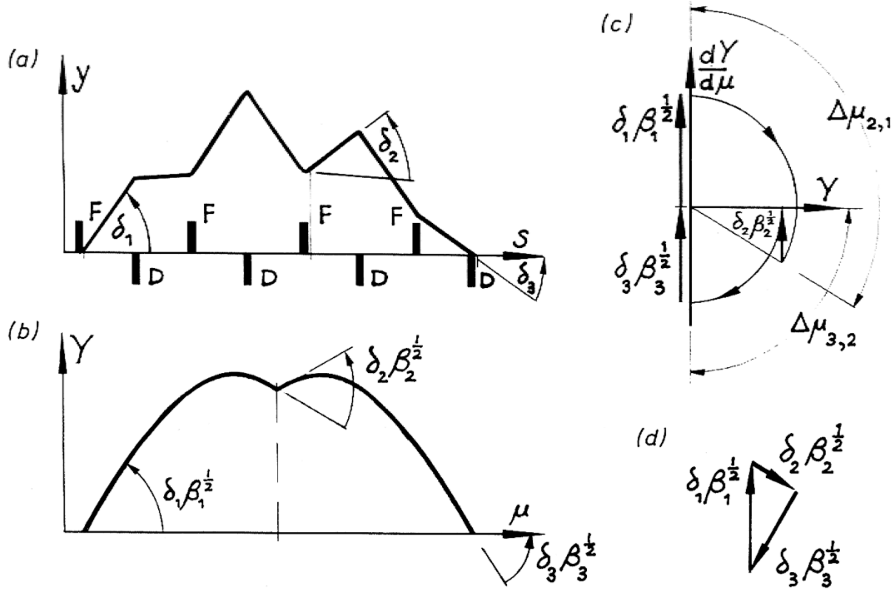
- In many cases the kicker angle is limiting
  - Use a slower but stronger closed bump to assist.
- How to make a "closed bump"?
- Use Matrix optics:

$$\begin{bmatrix} x \\ xp \end{bmatrix}_2 + \begin{bmatrix} 0 \\ \delta xp_2 \end{bmatrix} = \begin{bmatrix} \sin(\mu)\alpha(0)+\cos(\mu) & \beta(0)\sin(\mu) \\ \left(-\frac{\alpha(0)^2}{\beta(0)} - \frac{1}{\beta(0)}\right)\sin(\mu) & -\sin(\mu)\alpha(0)+\cos(\mu) \end{bmatrix} \circ \left(\begin{bmatrix} 0 \\ \delta xp_1 \end{bmatrix} + \begin{bmatrix} x \\ xp \end{bmatrix} \right)$$

- need $[x,xp]_2$ to be $[0,0]$ to close the bump
  - > $\mu=\pi$, $\delta xp_2=\delta xp_1$

Three-Bump

- 3-Bump allows freedom in phase advance.



Specific Injection Issues

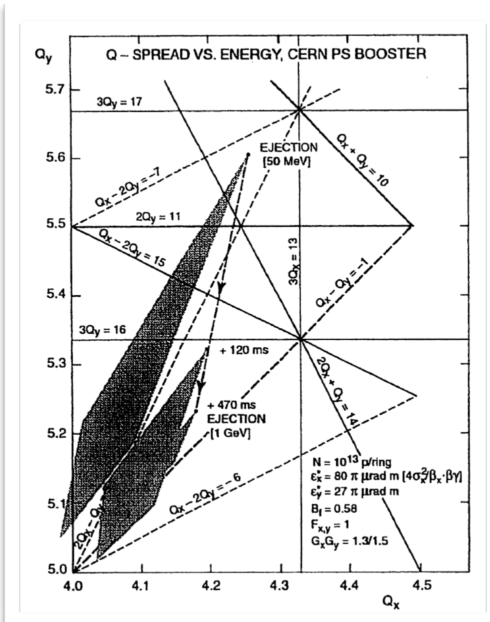
- Beams are larger in size
  - geometric emittance is $\propto 1/\gamma$
- Space-charge forces are stronger
  - biggest effect is tune spread covering larger part of working area.
  - tune spread can lead to distorted distributions: mismatch
  - sign is usually reduced injection efficiency
  - effect is difficult to assess => tracking needed
- Beam loss at beginning of acceleration
  - longitudinal acceptance shrinks, sometimes dramatically.
- Transient beam loading causes longitudinal mismatch
  - Rf voltage changes upon a slug of beam entering machine.

Space Charge

- Non-relativistic charges repel each other
  - a defocusing force, reduces betatron tune

$$\delta Q_{sc} = -\frac{R^2 n_0 r_0}{2Q\beta^2\gamma^3\sigma^2 l_b}$$

  - amplitude-dependent
    => becomes a tune spread
  - also can modulate the Twiss functions:
    => amplitude-dependent mismatch
- Mitigation:
  - better correction may help
  - increase injection energy

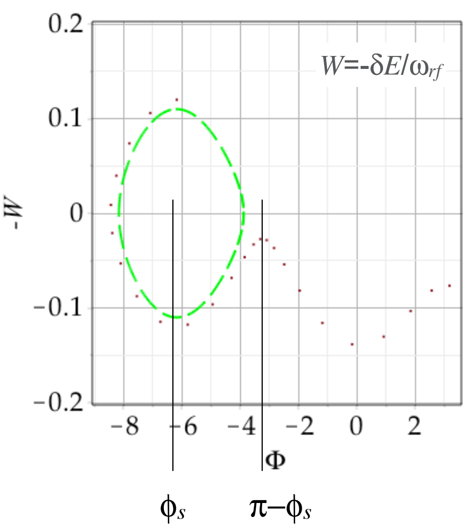


Rf Acceptance

- longitudinal match similar to transverse match
  - bucket height (max. $\delta E/E$):

$$\frac{\delta E}{E} < \frac{\beta\sqrt{V\cdot q}}{\sqrt{\pi h E_s \eta}}\sqrt{-(\pi-2\phi_s)\sin(\phi_s)+2\cos(\phi_s)}$$

  - bucket length: no closed soln; fixed points are ϕ_s and $\pi-\phi_s$, "left side" found numerically.

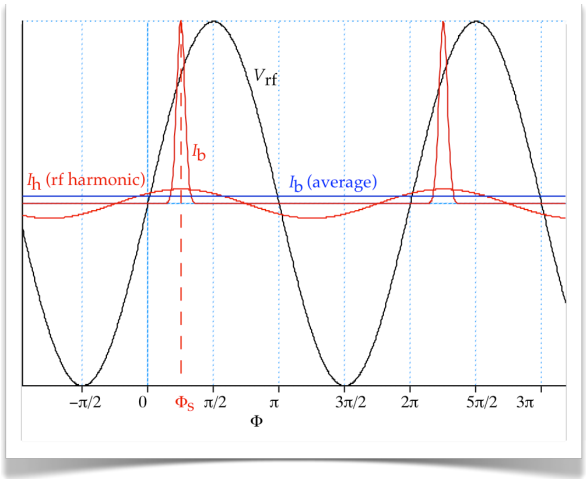


Transient Beam Loading

- Beam current induces voltage in cavity
  - > not in phase with rf voltage

$$V_b = \frac{2 i_b R_s \cos(\Psi) e^{\frac{I}{2}(\pi - 2\Phi_s + 2\Psi)}}{1+\beta}$$

  - ψ: detuning angle
    β: coupling factor
    ϕ_s: synchronous angle
    R_s: shunt resistance
    i_b: beam current
- The sum voltage is different in magnitude and phase
- compensate by feed-forward



Steering Error (Offset)

(see V. Kain, CERN lectures)

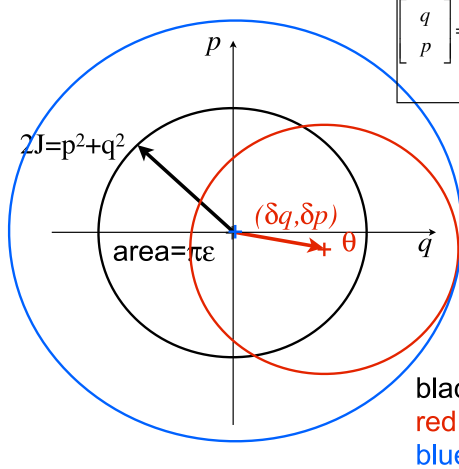
- Work in normalized coordinates

$$q_{new} := q_0 + \delta\cos(\theta)$$
$$p_{new} := p_0 + \delta\sin(\theta)$$
$$J := \frac{1}{2}\bar{p}^2 + \frac{1}{2}\bar{q}^2$$
$$J = \frac{1}{2}\delta^2 + \left(\cancel{p_0\sin(\theta) + q_0\cos(\theta)}\right)\delta$$

=0 on average

$$+\frac{1}{2}p_0^2 + \frac{1}{2}q_0^2$$
$$= \varepsilon_0 + \frac{1}{2}\delta^2$$

$$\begin{bmatrix} q \\ p \end{bmatrix} = \begin{bmatrix} \frac{x}{\sqrt{\beta(s)}} \\ \frac{\alpha(s)x}{\sqrt{\beta(s)}} + \sqrt{\beta(s)}xp \end{bmatrix}$$



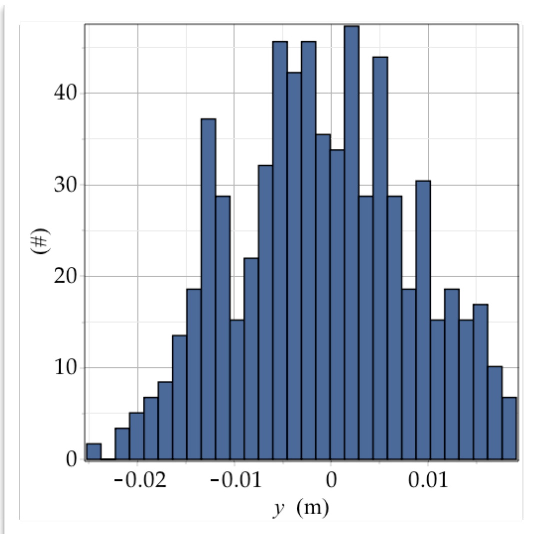
black: ring
red: injected
blue: ring, enlarged

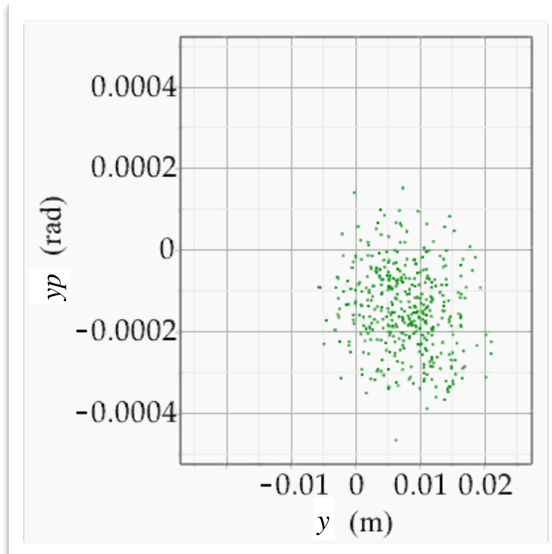
$$\delta^2 = \frac{1}{2}\left(\frac{\alpha\delta_x}{\sqrt{\beta}} + \sqrt{\beta}\delta_{xp}\right)^2 + \frac{1}{2}\frac{\delta_x^2}{\beta}$$

$$\frac{\varepsilon_{new}}{\varepsilon_0} := 1 + \frac{1}{2}\frac{\delta_x^2 + (\alpha\delta_x + \beta\delta_{xp})^2}{\beta\varepsilon_0}$$

small β: sensitive to x
large β: sensitive to xp

Injection Offset



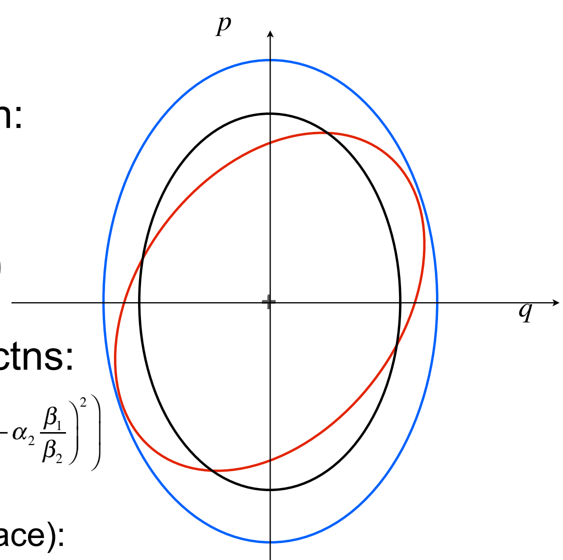


Phase-Space Mismatch

- Ring: $\alpha_1, \beta_1, \gamma_1$
- injected: $\alpha_2, \beta_2, \gamma_2$
- start from betatron oscillation:

$$x_2 = \sqrt{2\beta_2 J_2}\cos(\phi)$$

$$xp_2 = -\sqrt{\frac{2J_2}{b_2}}\left(\sin(\phi) + \alpha_2\cos(\phi)\right)$$



- normalize using *ring* Twiss fctns:

$$J_x = \bar{q}_2^2\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2\right) + \bar{p}_2^2\frac{\beta_2}{\beta_1} - 2\bar{q}_2\bar{p}_2\left(\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2\right)$$

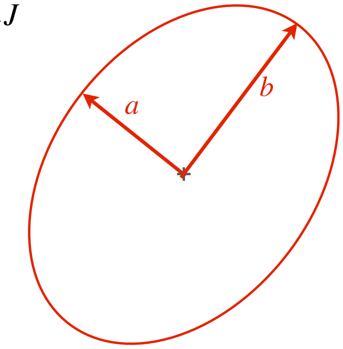
  - new "Twiss functions" (in *q*-*p* space):

$$\alpha_{new} = -\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right), \quad \beta_{new} = \frac{\beta_2}{\beta_1}, \quad \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2$$

- Define H such that

$$a=\frac{A}{\sqrt{2}}\left(\sqrt{H+1}+\sqrt{H-1}\right),\ b=\frac{A}{\sqrt{2}}\left(\sqrt{H+1}-\sqrt{H-1}\right),\ A=\sqrt{2J}$$

$$H=\frac{1}{2}\left(\gamma_{new}+\beta_{new}\right)=\frac{1}{2}\left(\frac{\beta_1}{\beta_2}+\frac{\beta_2}{\beta_1}+\frac{\beta_2}{\beta_1}\left(\alpha_1-\alpha_2\frac{\beta_1}{\beta_2}\right)^2\right)$$



- Then define λ

$$\lambda=\frac{1}{\sqrt{2}}\left(\sqrt{H+1}+\sqrt{H-1}\right),\ \frac{1}{\lambda}=\frac{1}{\sqrt{2}}\left(\sqrt{H+1}-\sqrt{H-1}\right)$$

- and get

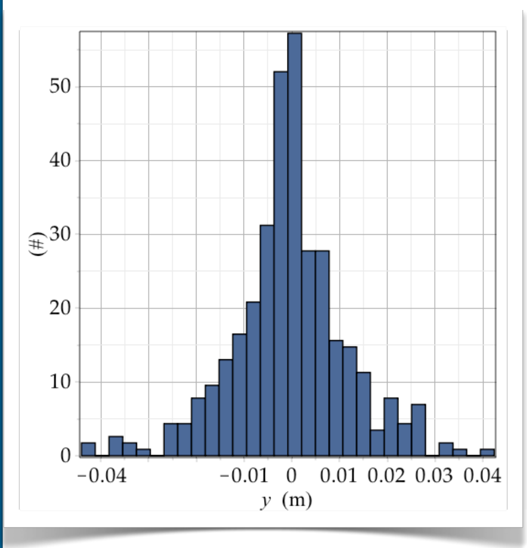
$$x_{new}=\lambda\cdot A\sin\left(\phi+\phi_1\right),\ xp_{new}=\frac{1}{\lambda}\cdot A\cos\left(\phi+\phi_1\right)$$

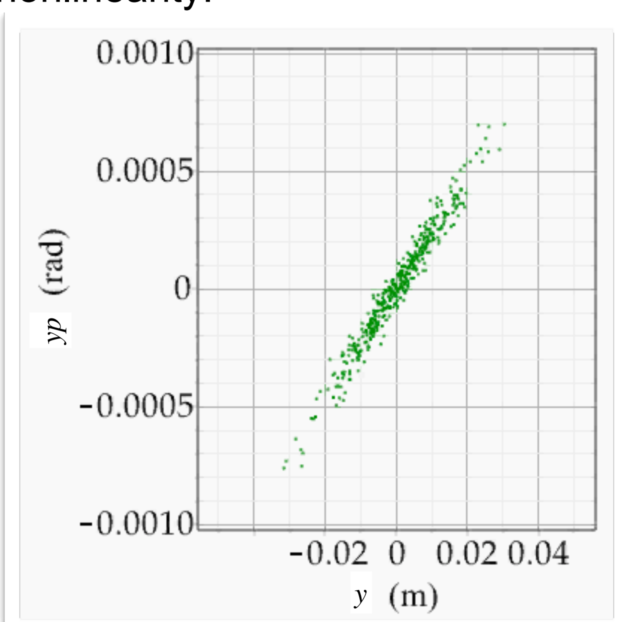
- and finally

$$\boxed{\varepsilon_{new}=\frac{1}{2}\varepsilon_0\left(\lambda^2+\frac{1}{\lambda^2}\right)=\frac{1}{2}\varepsilon_0\left(\frac{\beta_1}{\beta_2}+\frac{\beta_2}{\beta_1}+\frac{\beta_2}{\beta_1}\left(\alpha_1-\alpha_2\frac{\beta_1}{\beta_2}\right)^2\right)}$$

Matching issues

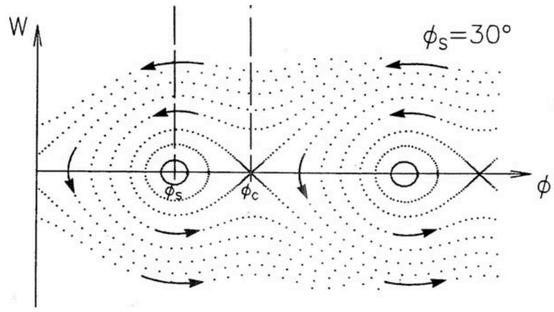
- Here is what happens when we inject a mismatched beam into a machine with some nonlinearity:





Longitudinal Plane

- The same matching issues exist in the longitudinal plane:
  - position –> phase (=time)
  - angle –> energy (=dϕ/dt)



- If rf frequencies are the same or a multiple of each other; phase the systems wrt. each other.
- Bunch aspect ratio should match bucket aspect ratio
  - this can be tricky for injection from a linac

Longitudinal Matching

- Usually, the injectee ring is larger than the injector ring.
  - It is also not uncommon that f_{rf}(injectee) ≠ f_{rf}(injector)
- Match the aspect ratio of bunch & bucket to prevent emittance growth.
- Since the bunch usually only fills the linear part of the bucket, his can be done analytically:
  - from the solution to the small-amplitude motion we define the aspect ratio as the ratio of the extreme energy and phase deviations:

$$A = \frac{\widehat{W}}{\hat{\phi}} = \frac{1}{2}\frac{\sqrt{2}\sqrt{\omega_{rev}}\,\beta\sqrt{E_s}\sqrt{q}\sqrt{V}\sqrt{\cos(\Phi_s)}}{\omega_{rf}^{(3/2)}\sqrt{\eta}\sqrt{\pi}}$$

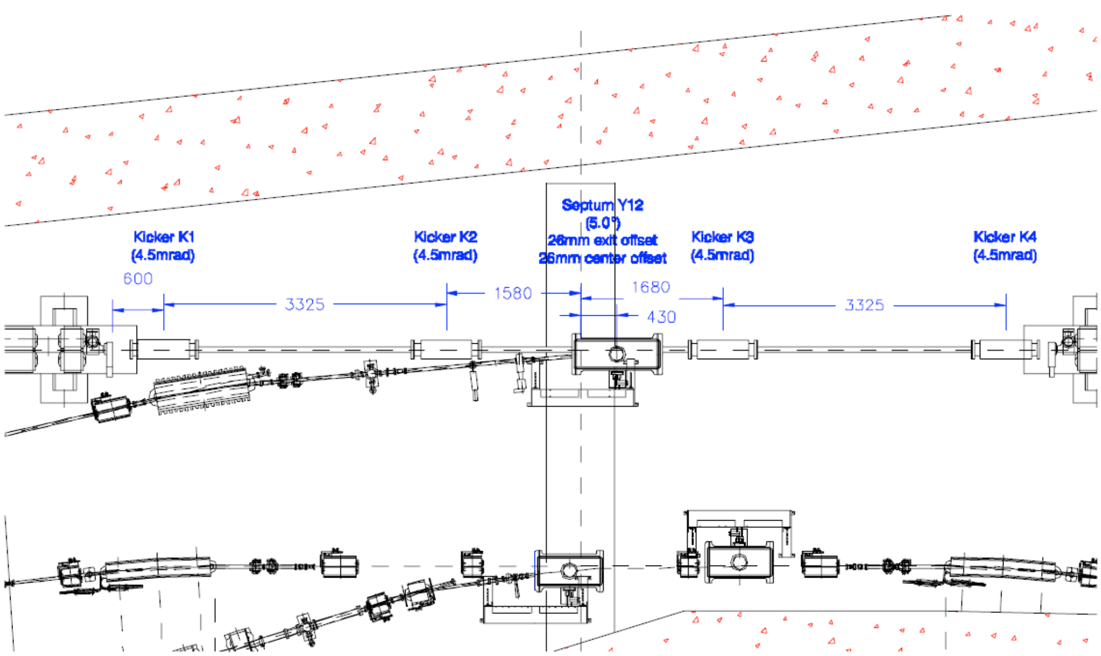
- We can now find the ratio for two different rings (1 and 2) of the aspect ratios, for the same rf frequency in both rings:

$$\frac{A_2}{A_1} = \frac{\sqrt{\omega_{rev2}}\sqrt{V_2}\sqrt{\eta_1}}{\sqrt{\eta_2}\sqrt{\omega_{rev1}}\sqrt{V_1}}$$

ω_{rev}: revolution frequency
η: slip factor
V; rf voltage

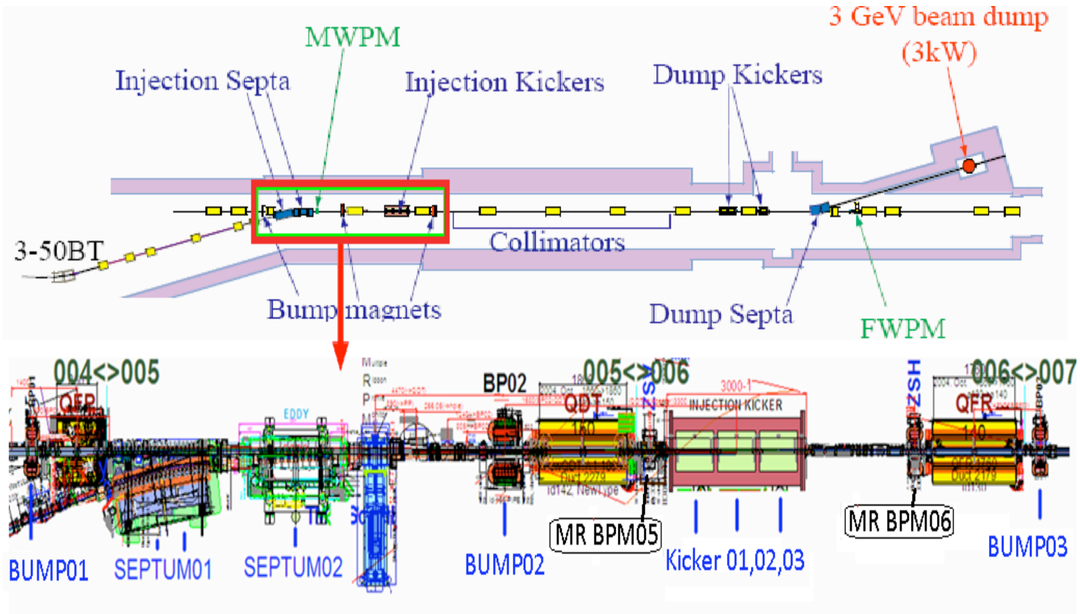
- unless one or both rings are close to transition, or one or both rings have lattice that manipulate the transition energy, this ratio is near unity for equal rf voltages.
  - since then $\eta \approx 1/\gamma_t^2 = \alpha_p \approx 1/\nu_x^2 \approx 1/R$
- If the frequencies differ, the frequency ratio becomes another parameter in the equation.

SLS Injection Section

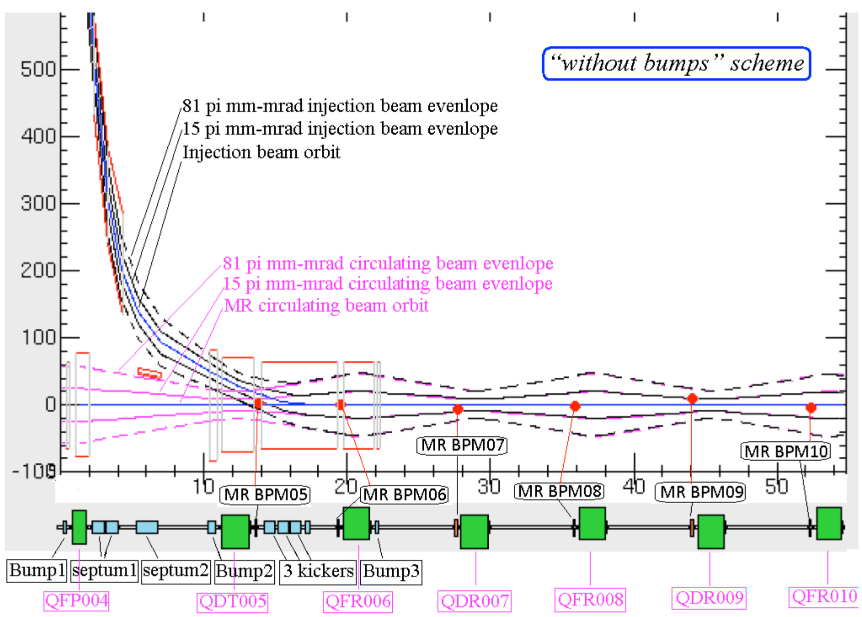


Some *real-life* Injection Systems

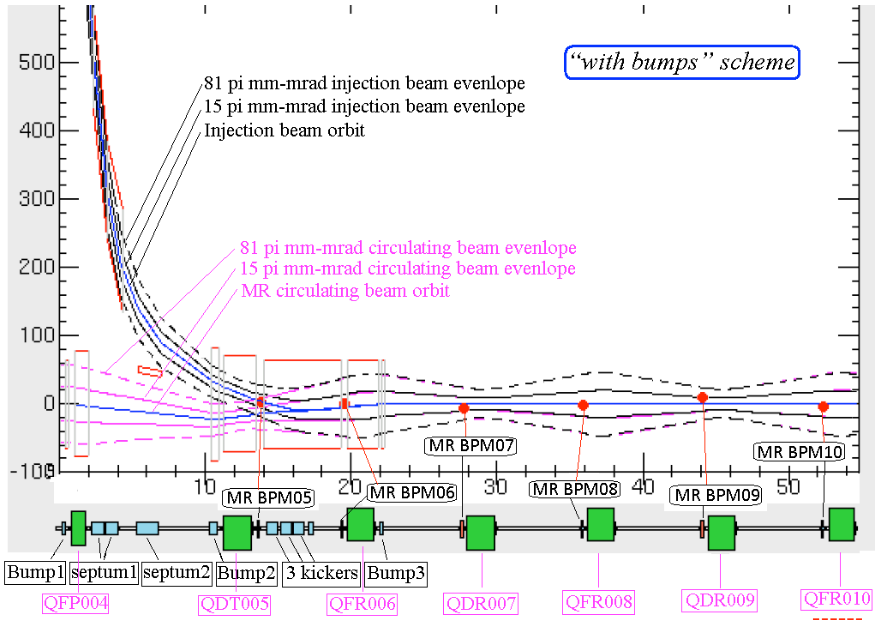
- JPARC Main ring (3 GeV protons -> 50 GeV)



- Only fast kicker, no slow bumps used



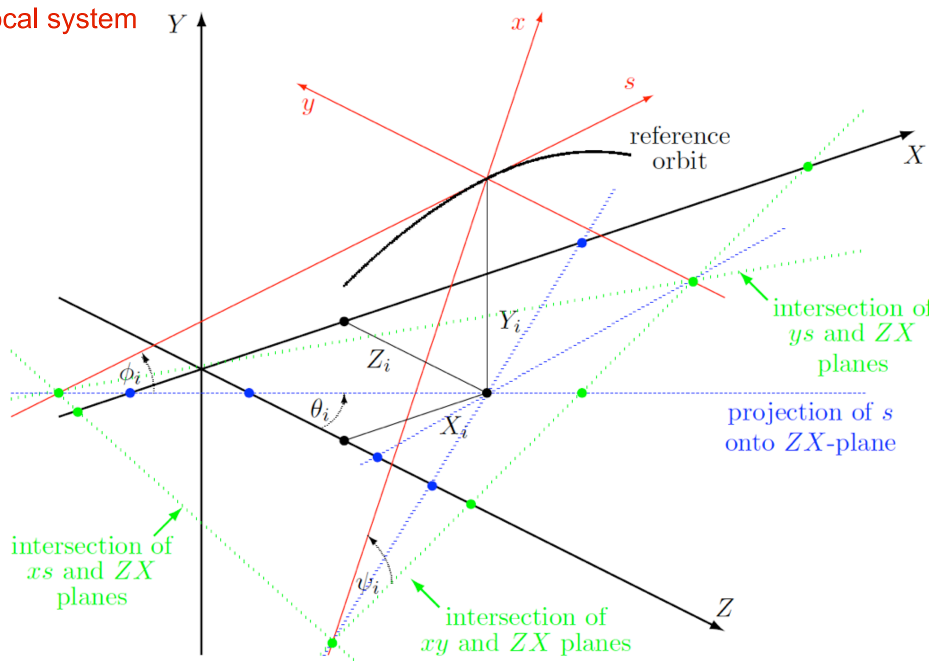
- Slow bumps make room for low-energy circulating beam
  - during acceleration the beam shrinks -> bump is collapsed)



Local & Global Coordinate System

X,Y,Z: global system

x,y,s: local system



Local-Global Transformations

- At each point, the displacement of the ref. orbit is given by a vector V and a matrix W:

$$V = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \qquad W = \Theta \quad \Phi \quad \Psi$$

$$\Theta = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \quad \Phi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix}, \quad \Psi = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- θ, ϕ and ψ are often called “pitch, yaw and roll”
- Roll will lead to coupling that needs to be compensated for a complete match
  - operationally difficult: best to avoid in final matching section
  - Mad-X SROTATION handles beam matrix properly.

Some Practical Considerations

- “Treaty Point”: hand-off from the beam-line designer to the machine designer.
  - often a symmetry point in the ring, or the downstream end of the injection septum.
- Coordinate matching:
  - Programs like Mad allow arbitrary starting point.
  - Difficulty: if injection line and ring are not in the same plane.

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- B. Goddard, “Overview of Injection & Extraction Techniques” in CERN Accelerator School on Beam Injection, Extraction and Transfer, Erice, IT, Mar-2017, https://indico.cern.ch/event/451905/timetable/
- C. Bracco, “Injection: Hadron Beams”, ibid.
- M. Barnes, “Kicker Magnets”, ibid.
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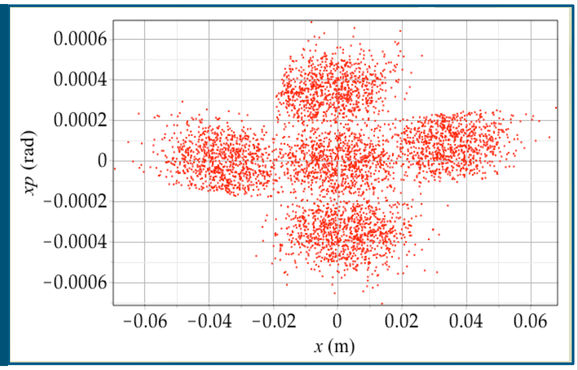
- P.J. Bryant and K. Johnsen, “The Principles of Circular Accelerators and Storage Rings”, Cambridge University Press, U.K., 1993.
- M. Tomizawa et al., “Injection and Extraction Orbit of the J-PARC Main Ring”, Proc. EPAC2006 Edinburgh, GB, 1987.
- The MAD-X Program User’s Reference Manual, CERN May-2017.

Multi-turn Injection



U. WIENANDS
ANL

E. MARIN-LACOMA
CERN

19-Jun-2017
USPAS, Lisle Il.

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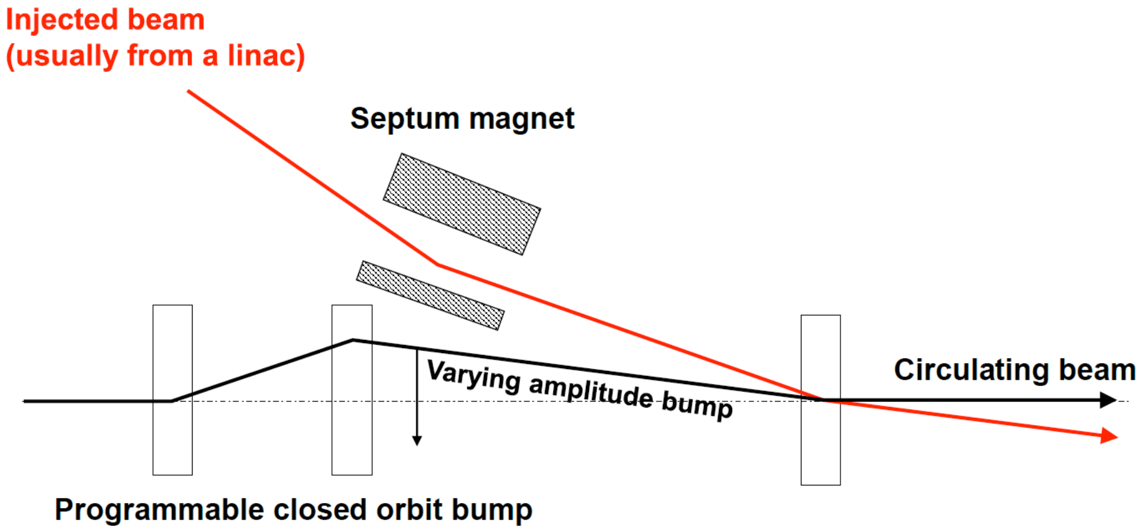
Why and when Multi-turn Injection?

- Injector is short
  - Inject subsequent bunches, box-car fashion
  - mostly an issue of kicker rise/fall times.
- Injector does not have enough intensity
  - accumulate more particles
  - How to do that?
    - Liouville limits what can be done, no “merging” of phase space!
    - new beam has to occupy different region in phase space, longitudinal or transverse (transverse stacking, slip-stacking)
    - Charge-exchange injection is one way around this (common for protons)
    - Damping makes this easy for electrons

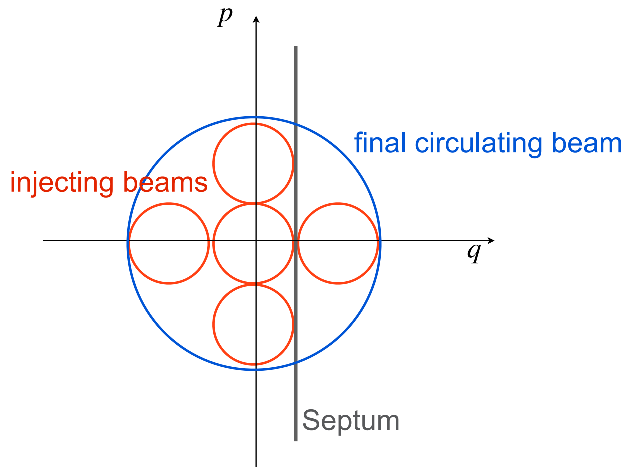
Transverse Multi-turn Injection

- Simplest implementation (CERN PS Booster)



Basic Scheme

- Inject off axis, let betatron oscillation pull the injected beam off the septum
- The simplest case: Q = 0.25, inject centered beam and 4 turns around it.
- For simplicity assume $\beta_1 = \beta_2$ and $\alpha = 0$, angle offset $= 0$
  - usually the case.



Analysis of 5-turn injection

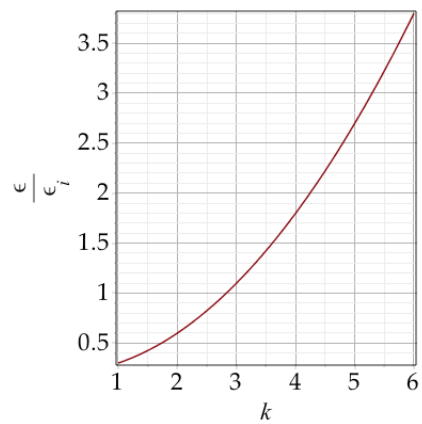
- Assume Gaussian beam:

$$I(x) = \frac{I_0}{2}\frac{\sqrt{2}e^{-\frac{1}{2}\frac{(x)^2}{\sigma^2}}}{\sigma\sqrt{\pi}}$$

this is the spatial distribution

- cutting off at $k\sigma/2$ due to the septum:

$$\frac{I}{I_0} = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{1}{4}k\sqrt{2}\right)$$



- We can write the final emittance using the formula from the previous lecture:

$$\frac{\varepsilon}{\varepsilon_i} = 1 + \frac{1}{2}\frac{\delta_x^{\ 2}}{\beta\varepsilon_i} = 1 + \frac{1}{2}k^2$$

$\alpha = 0$, $\delta xp = 0$ and $\delta x = k\sigma_x$

Injection Efficiency

- We lose a fraction x each time the beam passes the septum
  - but not if it is "on the other side"!

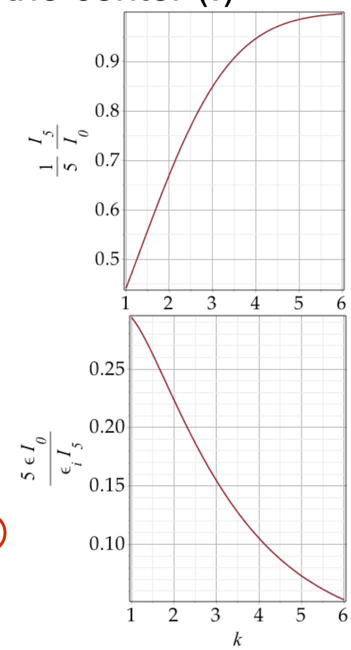
turn	C	1	2	3	4
0	1				
1	(1-x)	(1-x)			
2	(1-x)	(1-x)	(1-x)		
3	(1-x)	1	(1-x)	(1-x)	
4	(1-x)	(1-x)	1	(1-x)	(1-x)
Total	$(1-x)^4$	$(1-x)^3$	$(1-x)^2$	$(1-x)^2$	(1-x)

- The total beam loss involves 4 time scraping the injecting beam & 4 times the circulating beam at the center (!)

$$\frac{I_5}{5 \cdot I_0} = \left(1 - \frac{I}{I_0}\right)^{4*2}$$

- Often beam, brightness (int/emittance) is what counts

- beam loss has a knee near $k = 3.5$; brightness favors $k \approx 1$ but >50% loss ☹



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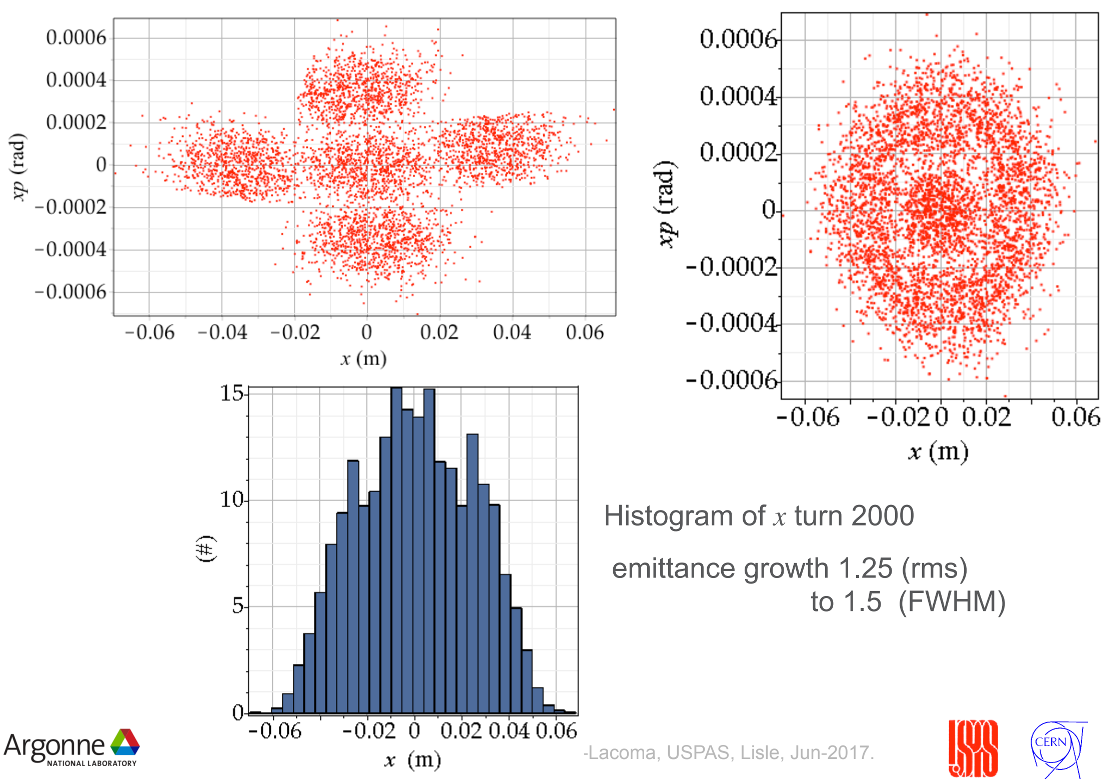
Result of a tracking run

Turn 5

Turn 2000



Histogram of x turn 2000

emittance growth 1.25 (rms)
to 1.5 (FWHM)

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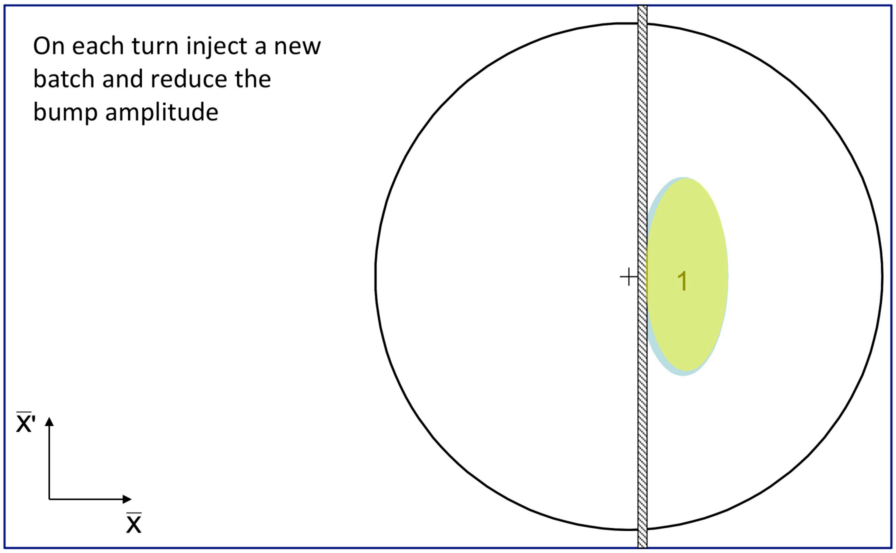
Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$

Beam rotates π/2 per turn in phase space

C. Bracco

Turn 1



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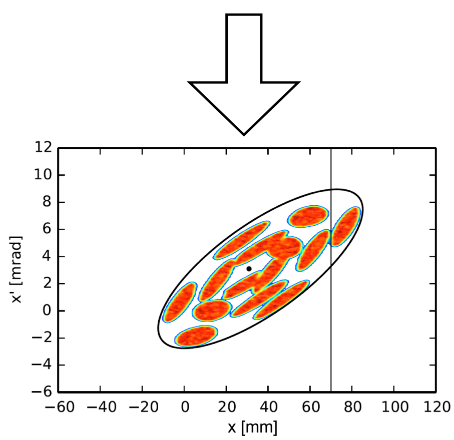
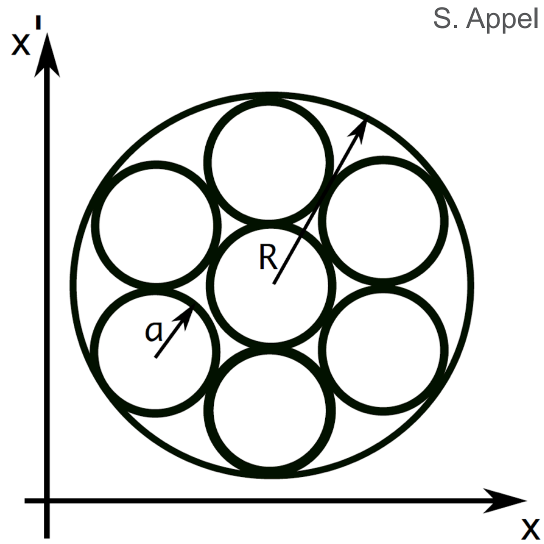
SIS 18 Injection (GSI)

S. Appel

- Hexagonal dense packing of 7 beam-pulses
  - tune = 1/6; match as on-axis
- Optimization using g.a.

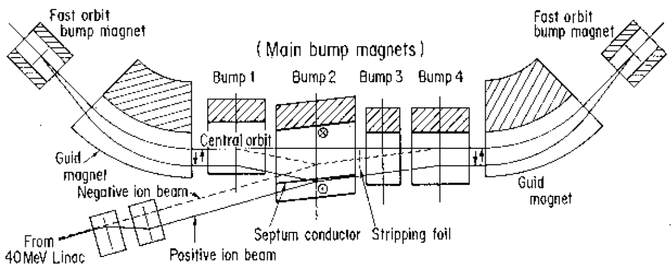




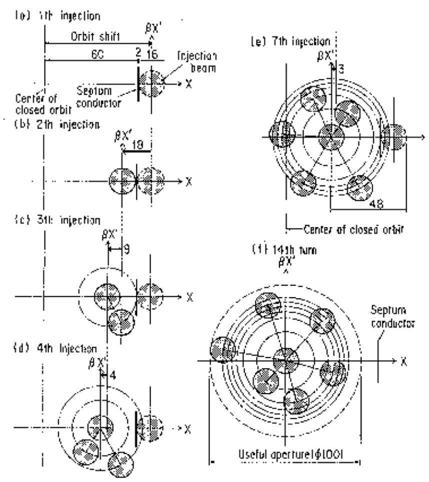
Requires relatively large gaps to make work

KEK PS Injection

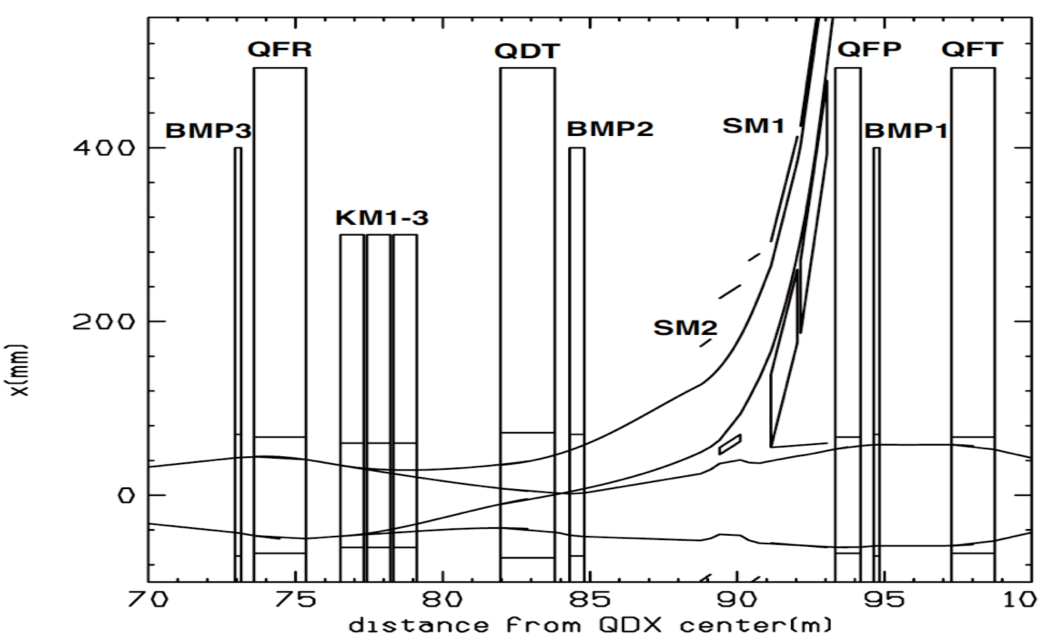
- Double-bump system (fast-slow)



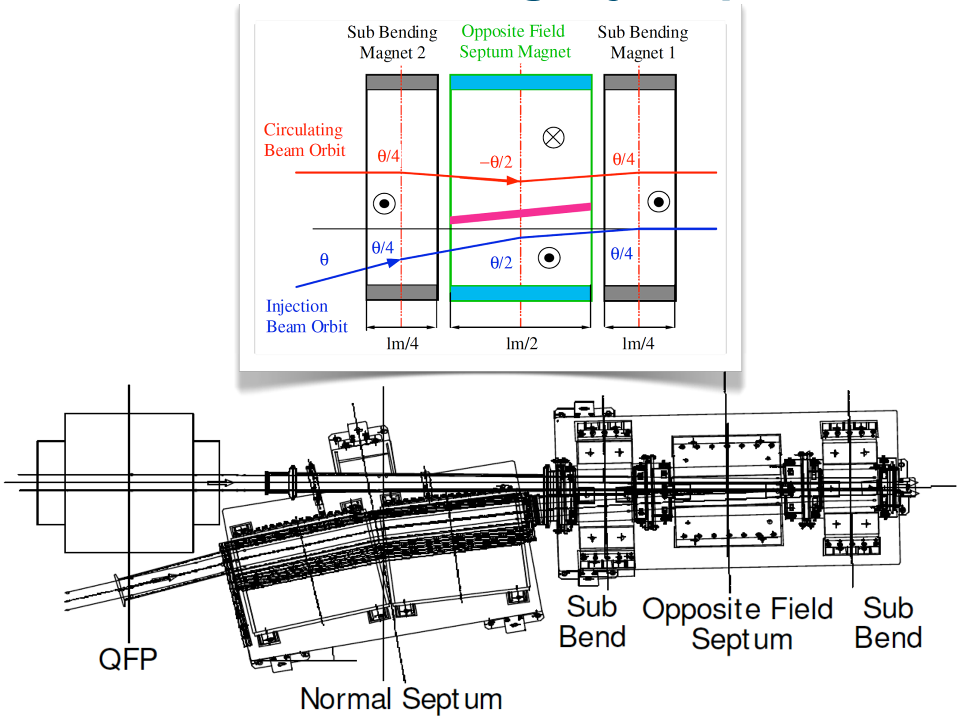
14-turn injection

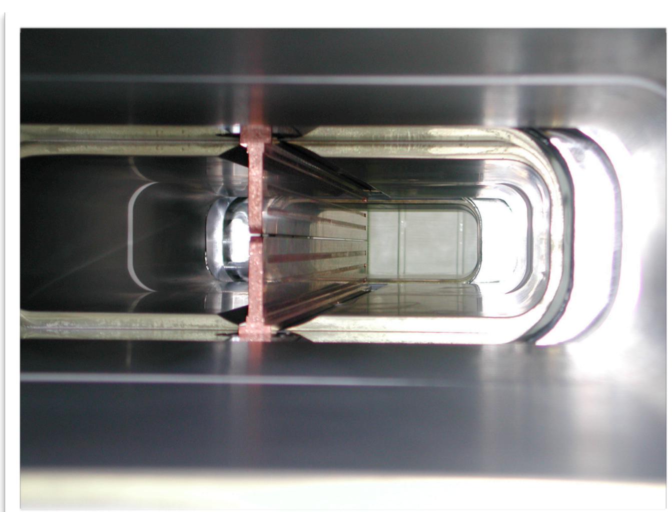


JPARC Main Ring Injection



JPARC Main-Ring Inj. Septum





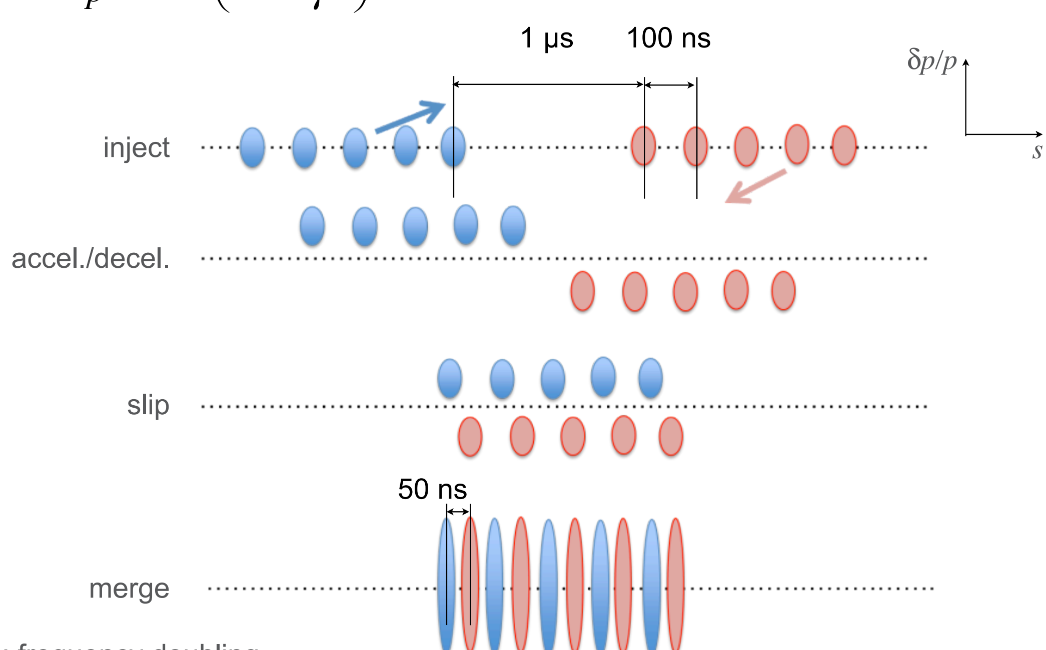
Slip Stacking

- Inject batches in box-car fashion
- "slip" the second batch in azimuth to overlap with the first one using a 2nd rf frequency
- "merge" the two batches using the rf.
- Requires the slip factor h to be large enough and momentum acceptance.
- May involve debunching of the stack

Example: CERN SPS (LHC ions, proposed)

$$\frac{\Delta t}{t} = \eta \frac{\Delta p}{p}, \quad \eta = \left(\alpha_p - \frac{1}{\gamma^2} \right)$$

T. Argyropoulos, CERN

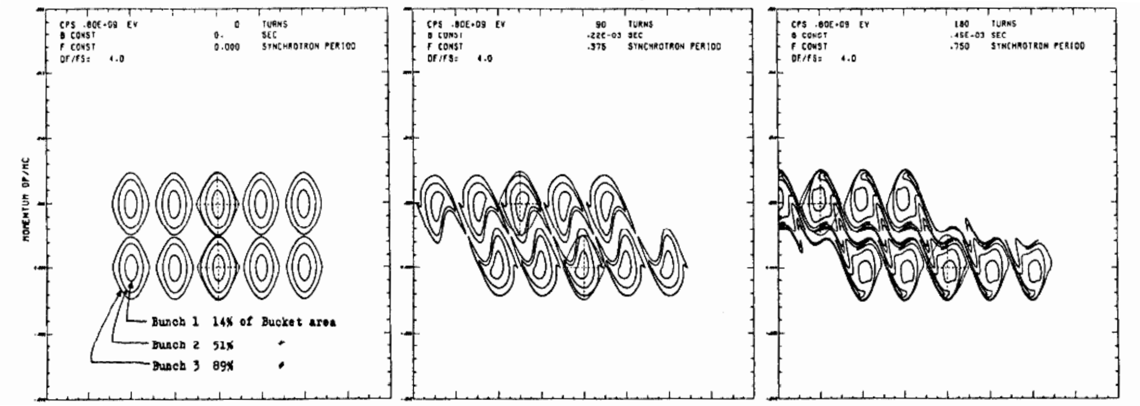


here: frequency doubling

Longitudinal phase space

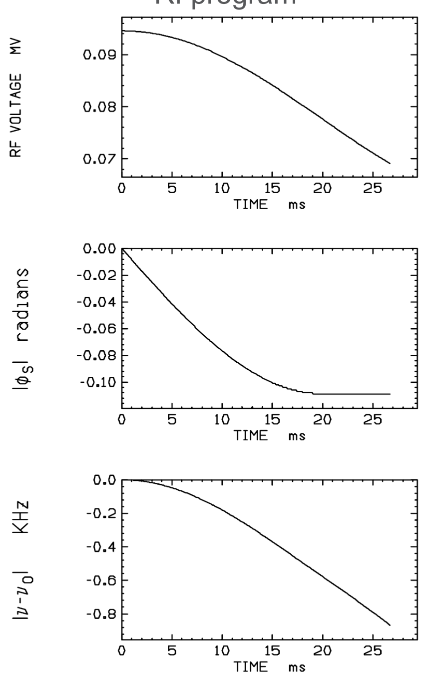
- modelling for CERN SPS (for p-bar production)
  - $\alpha = \Delta f / f_s > 4.$
- note existence of two series of rf buckets, offset in $\delta p/p$

turn 0 turn 90 turn 180

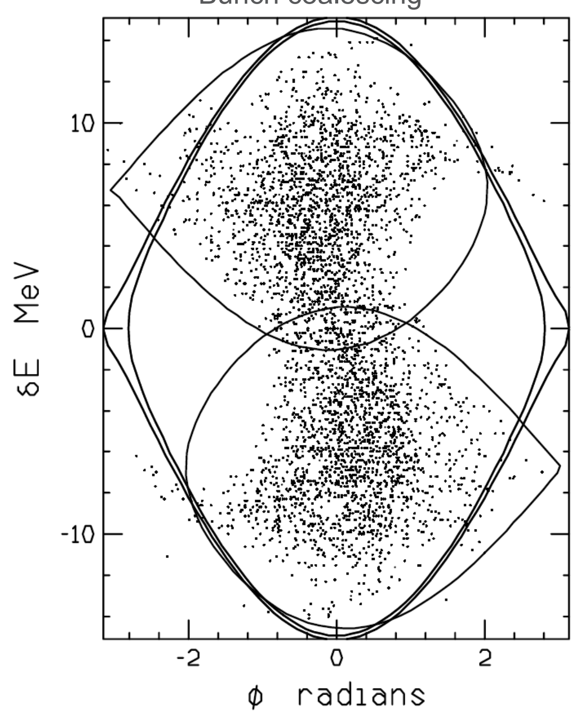


FNAL Main Injector

Rf program



Bunch coalescing



Multi-turn Parameters

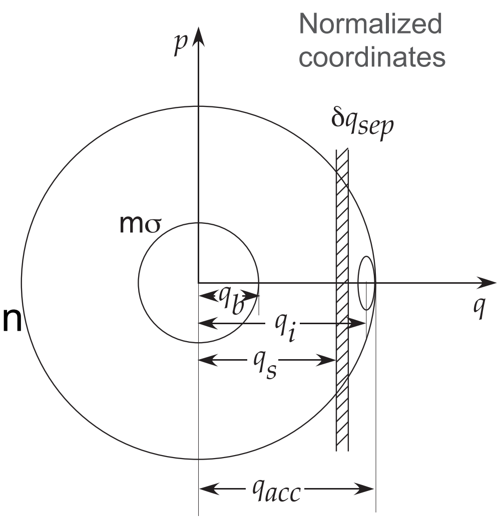
- Tune of the ring
  - each turn, the injected beam is displaced by 2Pi/n in phase space.
  - needs to be enough to move beam off the septum.

- Betatron match of offset beam:

$$\frac{\beta_i}{\beta_r} = \left(\frac{\varepsilon_i}{\varepsilon_r}\right)^{\frac{1}{3}} \qquad \frac{\alpha_i}{\alpha_r} = \frac{\beta_i}{\beta_r}$$

- The minimum phase rotation is then

$$\mu_{\min} = \frac{\pi}{\arccos\left(\frac{q_s}{q_{acc}}\right)}$$



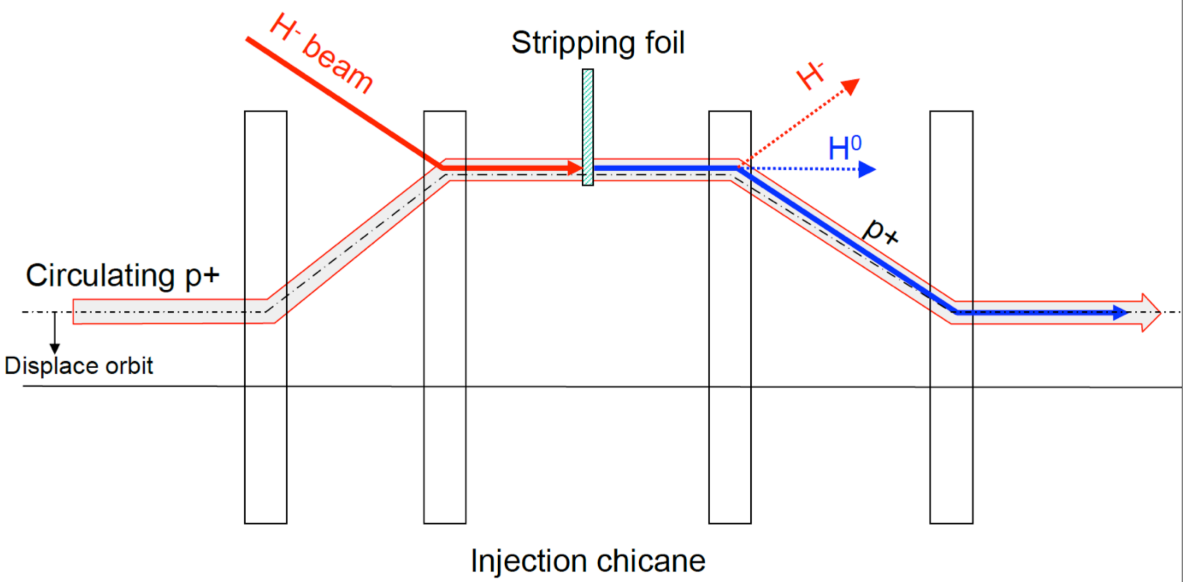
Charge-Exchange Injection

- Accelerate H⁻ ions up to a moderately high energy
  - 10s…1000 MeV, typically
  - upper limit set by Lorentz stripping
  - lower limit set by stripper efficiency

- Send them through a stripper foil
  - both weakly-bound electrons will get striped off, $H^- \rightarrow H^+$
  - stripper foil is thin => protons can pass through with minimal scattering
    - 50 μg/cm^2 @ 50 MeV to 200 μg/cm^2 @ 800 MeV.
  - ability to merge phase space; charge exchange is non-Liouvillian.

- This works with heavier ions as well
  - often use a multi-stage approach to fully strip ions for efficiency

H– injection Schematic



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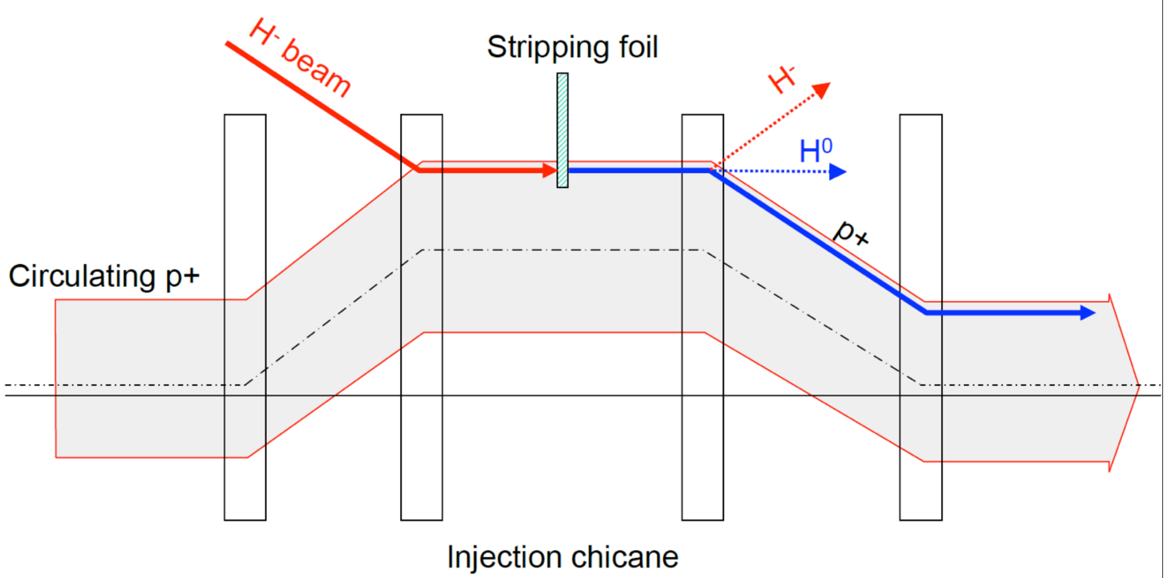

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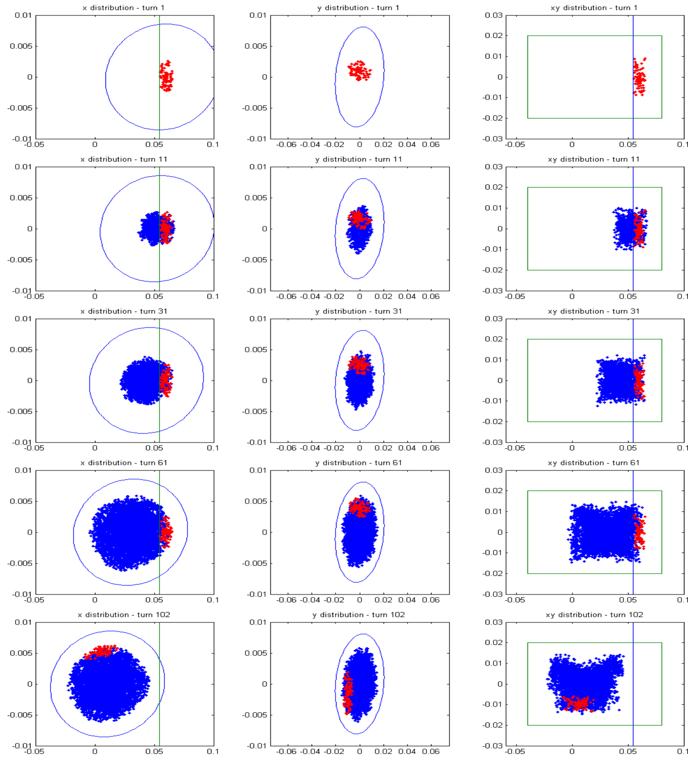
Charge exchange H- injection painting

Time



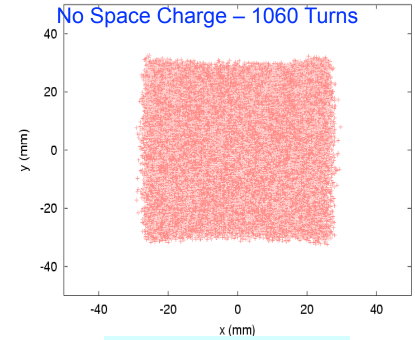
M. Plum, ORNL

SNS Painting with Space-Charge

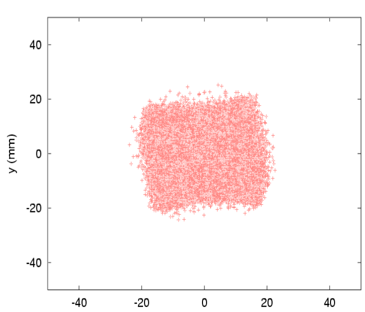
M. Plum, ORNL

- Injection painting scheme optimized to minimize space charge and beam loss: Paint with hole in the center to help create uniform density.
- Also try to keep circulating beam foil intercepts to a minimum (~7-10 foil hits per proton).
- Footprint suits stringent target requirements.

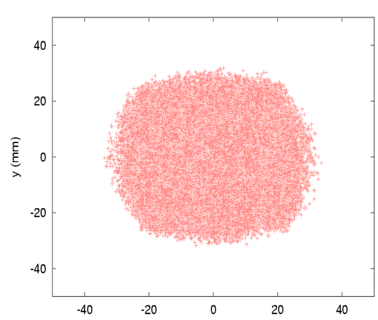
No Space Charge – 1060 Turns



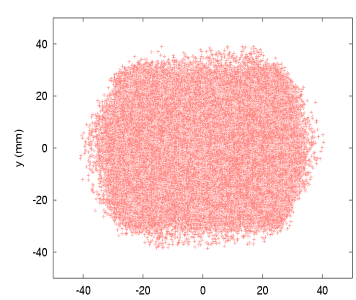
200 Turns



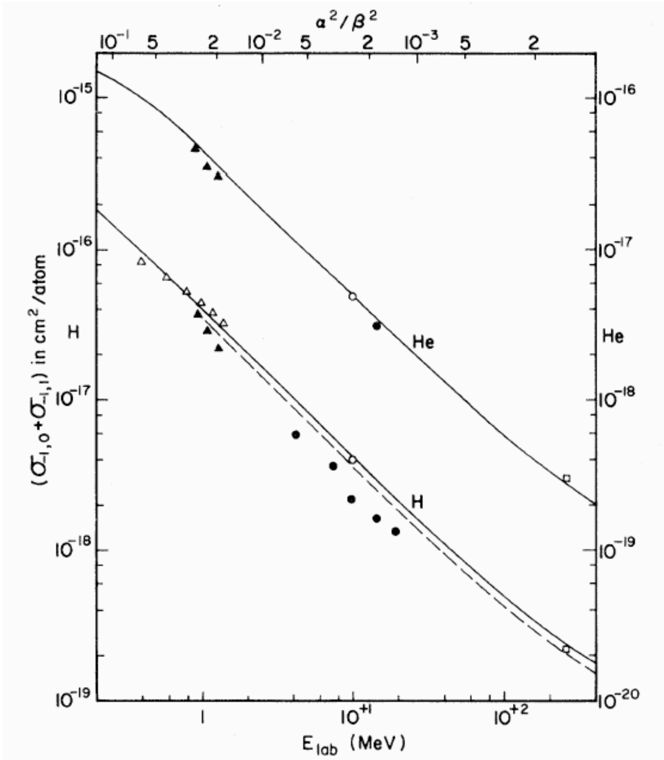
600 Turns



1060 Turns



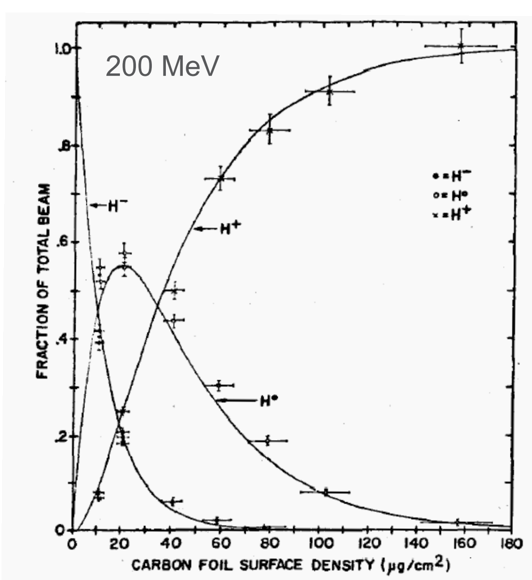
Stripping Cross Section *vs* Energy

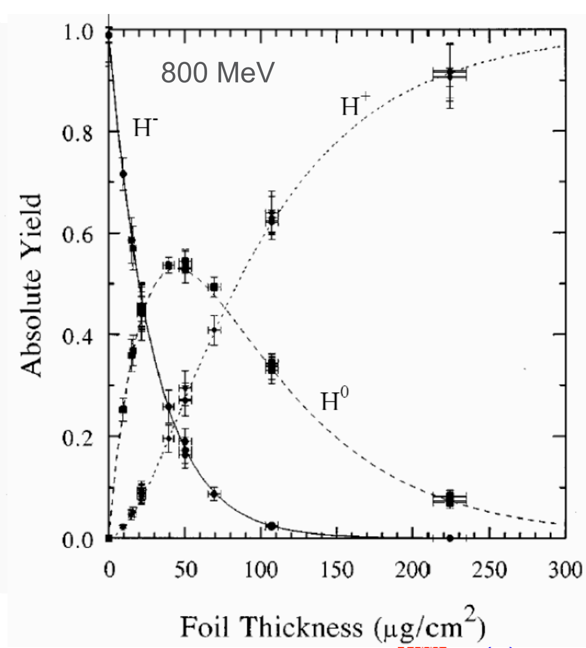


Chou et al., NIMA, 2008

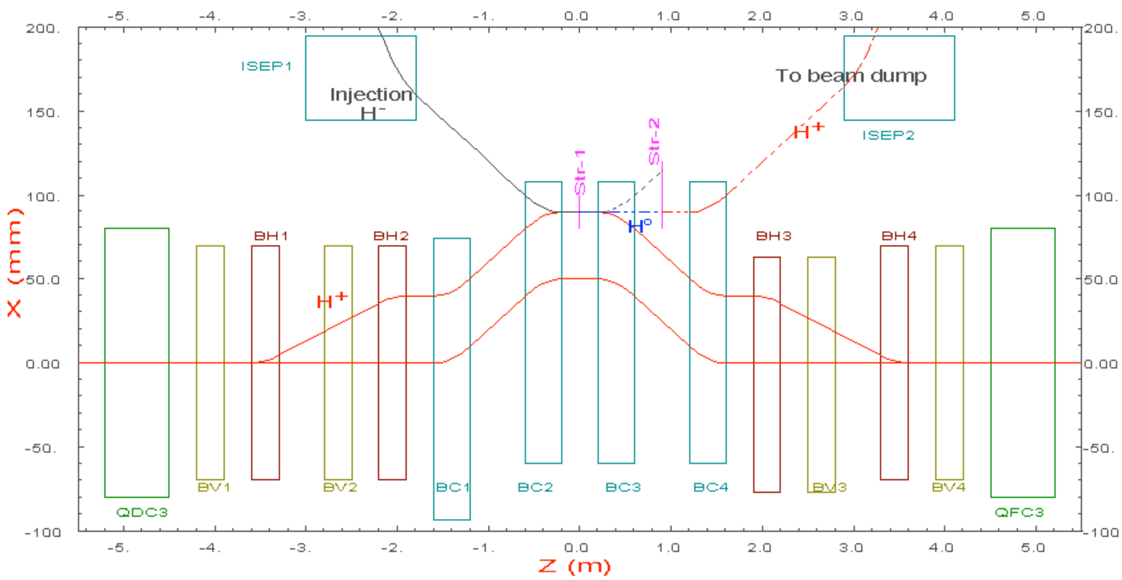
Stripping Efficiency @ 200 MeV and 800 MeV

Chou et al.

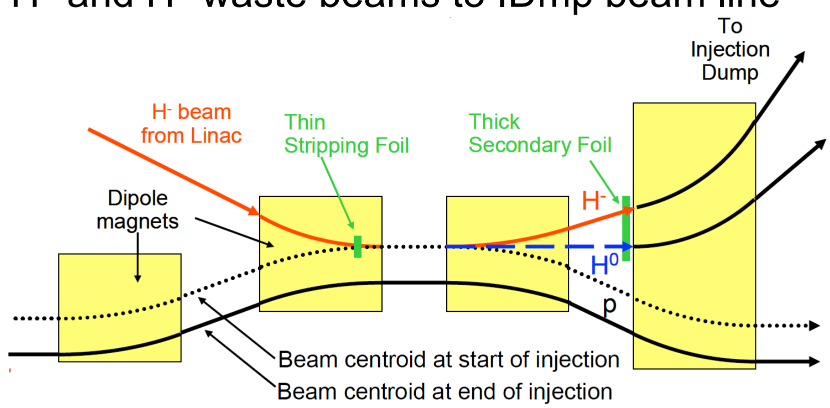




H– Injection Layout (SNS)

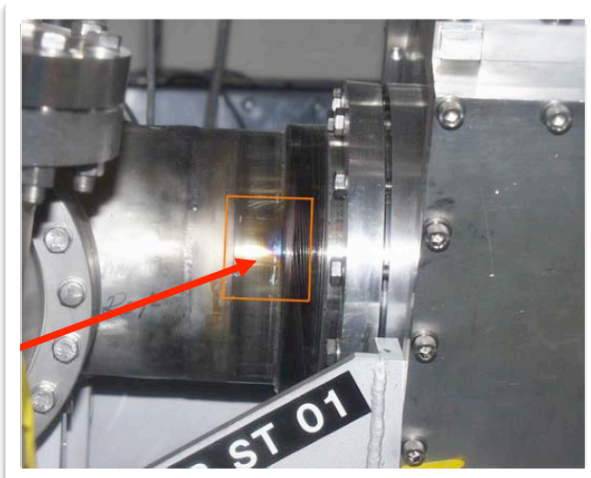


- Closed orbit bump of about 100 mm
- Merge H^- and circulating beams with zero relative angle
- Place foil in 2.5 kG field and keep chicane #3 peak field <2.4 kG for H^0 excited states
- Field tilt [arctan(By/Bz)] >65 mrad to keep electrons off foil
- Funnel stripped electrons down to electron catcher
- Direct H^- and H^0 waste beams to IDmp beam line



Where do the Stripped e⁻ go??

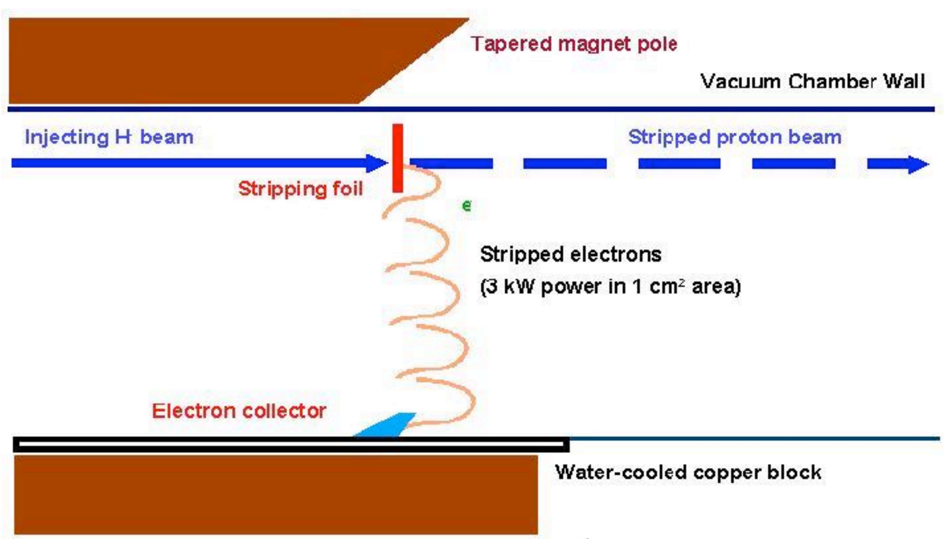
- 2 e^- per stripped proton @ incident beam energy
  - 1/938 time the total energy of the proton beam: ≈ 1.5 MeV*100 mA
  - At SNS: P_{e-} up to 1.5 kW… not negligible!



Burn mark from stripped electrons in LANL PSR

Control of Electrons

- The SNS primary stripper foil is in a tapered magnetic field, which directs the electrons down to a watercooled catcher.



Effect of Foil on the Beam

Particle Data Group

- Any matter in the beam path will scatter:

$$\theta_{rms} = \frac{0.0136\sqrt{\frac{X_0}{x}}\left(1+0.038\ln\left(\frac{X_0}{x}\right)\right)}{\beta cp}$$

cp=momentum in GeV
X_0=radiation length
x = thickness
$\beta = v/c$

  - Note: the above is optimistic for thin foils as large-angle scatters are underestimated => “plural” scattering

- This increases the beam emittance:

$$\varepsilon = \sigma_{xp}^2 \beta_{Twiss} = \varepsilon_0 + \beta_{Twiss}\theta_{ms}^2 n_{foil}$$

- Also, particles lose energy in the foil

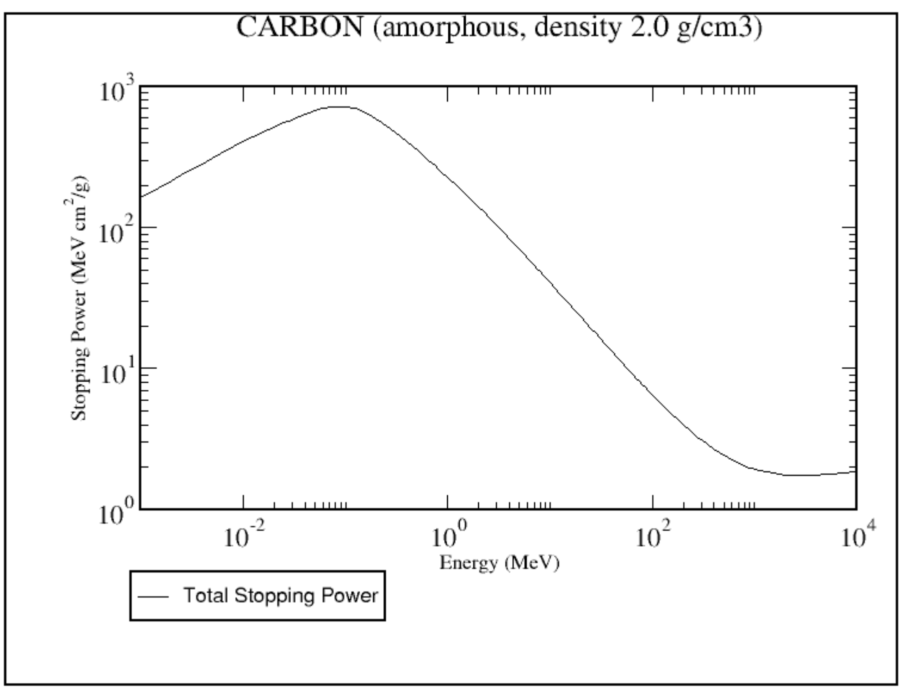
$$\left\langle -\frac{dE}{dx}\right\rangle = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right] . \quad \text{[MeV/(g/cm}^2\text{)]}$$

$K = 0.307$ [MeV/(mol/cm^2)] $\quad W_{\max} = \dfrac{2m_ec^2\beta^2\gamma^2}{1+2\gamma m_e/M + (m_e/M)^2}$. for a particle of mass M

  - This generates significant power that has to be dissipated (radiation)

dE/dx for ^{12}C

NIST PSTAR



0^{th}-Order Design of H⁻ injection

- Decide on the number of turns needed (intensity, emittance)
- Decide on stripper foil thickness needed
  - mostly depends on minimum efficiency desired
- Decide on the final emittance
  - space-charge consideration
- Evaluate the scattering for the # of turns needed
  - in general, scattering should not dominate the final emittance
- Evaluate foil heating
- iterate and hopefully converge
- Modelling (ACCSIM or other codes)
  - many labs write their own taylored to their specific needs

Foil Heating

C.J. Liaw et al.

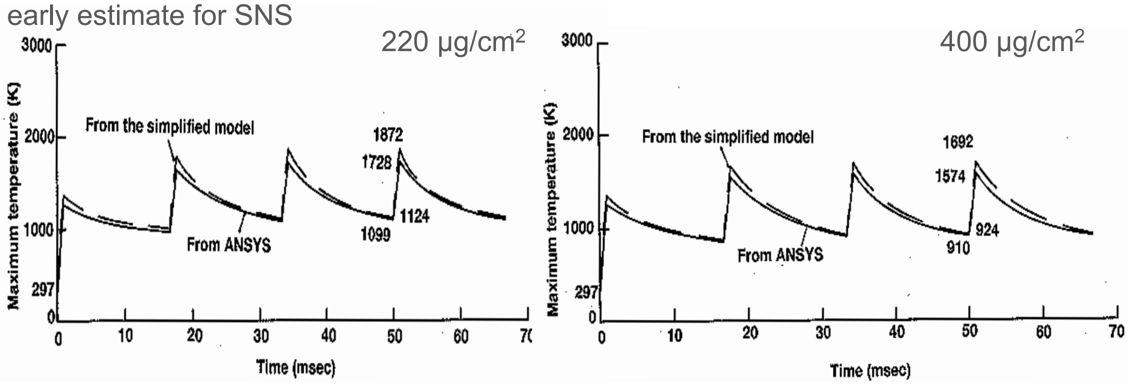
P: Power, *T*: abs temp
A: area, *V*: volume
ε: emissivity, ρ: density
σ: Stefan-Boltzmann
c: spec. heat capacity
$_c$: carbon foil, $_0$: ambient

- Foil heating due to a pulsed beam

$$\rho_c V_c c_c \, dT_c/d\tau = -2\sigma \, f\varepsilon_c A_c (T_c^4 - T_0^4) + PA_c$$

  - *T*-rise due to specific heat of foil, *T*-fall due to radiative cooling

early estimate for SNS



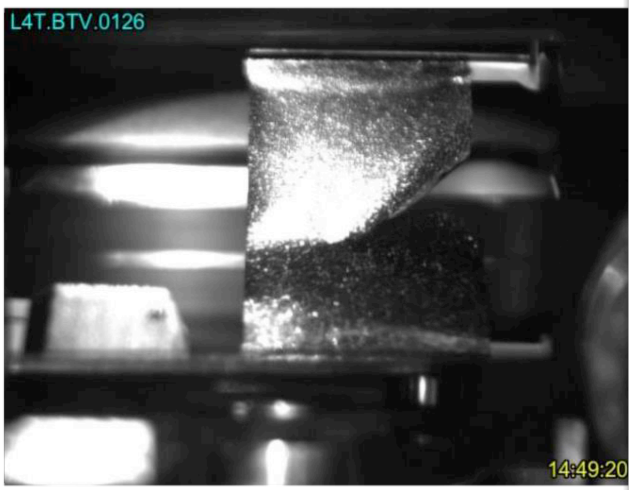
Liaw et al., proc. PAC 1999, New York

Stripper Foils

- Stripper foil damage

B. Goddard

CERN PS Booster



SNS

- is there a way to strip without a foil??

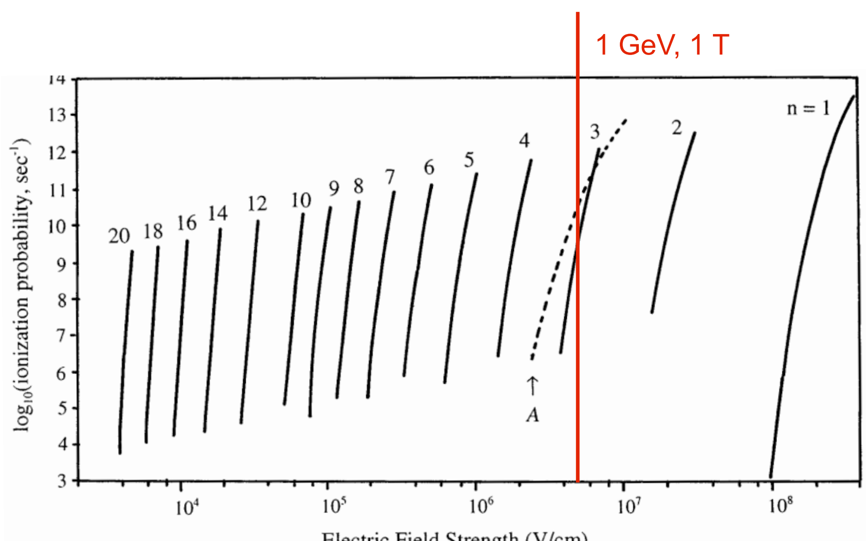
Argonne NATIONAL LABORATORY

Alternatives to Stripper Foils

- Lorentz stripping in a strong magnetic field

B. Goddard

  - works at high energy, but only $H^- \to H^0$
  - H^0 not amenable to Lorentz stripping as is; however, excited H^0 atoms are



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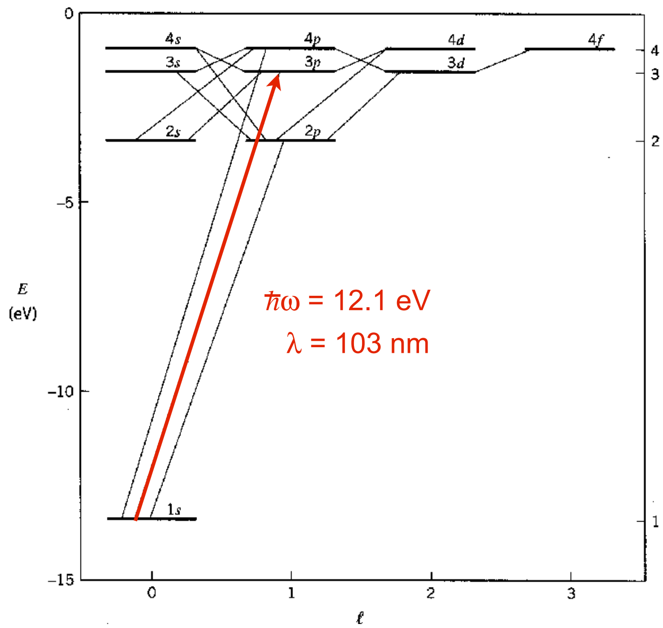
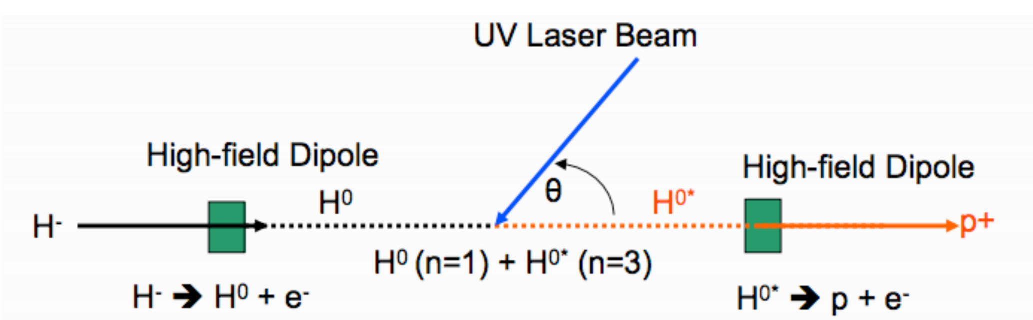
Laser Stripping

- H^0 can be excited by a laser of suitable wavelength
- lifetime $10^{-9} \ldots 10^{-10}$s
  - long enough to travel a foot or so
- then strip in a 2nd strong dipole



Laser Stripping

- Strip H^- to H^0 in a strong B-field
- Excite H^0 with a laser of the right (Lorentz-shifted!) wavelength
- Strip excited H^0 to H^+ in another strong B-field



- Doppler shift shortens the laser wavelength
- use angle θ to “tune” the laser on resonance

$$h\omega \rightarrow h\omega\left(1+\beta\cos(\theta)\right)\gamma$$

Details

- Fundamentally, the resonance is narrow,
  - the laser line width is narrow as well
  - particles have different γ and angle so low probability of excitation
  - > would need enormous laser power to make this work efficiently

- The key to success is to taylor the laser beam divergence and to dispersion-match the angle θ.

- SNS has shown this can actually work, 90% efficiency, 7 ns
  - working on 10 μs system
  - will need an optical cavity to get to cw.

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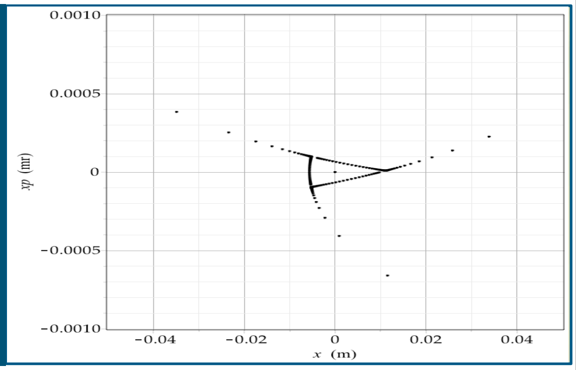
- W. Chou et al.,”Stripping Efficiency and Lifetime of Carbon Foils“, arXiv:physics/0611157
- C.J. Liaw et al., “Calculation of the Maximum Temperature on the Carbon Stripping Foil of the Spallation Neutron Source”, Proc. PAC99, New York, NY 3300(1999).

Slow Extraction



U. WIENANDS
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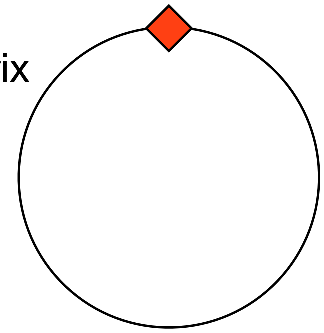
19-Jun-2017
USPAS, Lisle Il.

Slow Extraction

- Single turn extraction from a ring => very small duty factor
  - t_{rev}/t_{cycle}: 1E-5 or similar
- This can be an issue for coincidence experiments
  - random coincidences increase with peak rate, actual coincidences with the average rate.
- Need a method to "peel off" the beam slowly, ms to seconds.
- General idea: run beam onto a resonance & peel off the unstable particles.
  - on an isolated resonance, the phase space topology is easily understood and controlled.

1/3-Integer Extraction

- 1/3-integer and 1/2-integer resonances are being used for extraction. We will discuss the 1/3 integer extraction in some detail.
- Consider a ring with a single (thin) sextupole:
- The ring is described by its 1st-order matrix M, the sextupole by its transfer map:

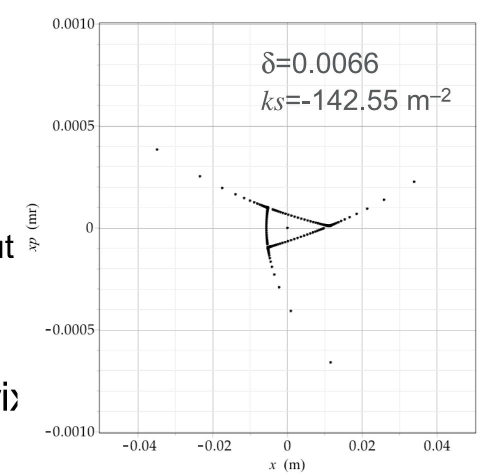


$$\begin{bmatrix} x \\ xp \end{bmatrix} = \begin{bmatrix} x \\ -ksx^2 + xp \end{bmatrix}$$

- The quadratic term distorts the phase-space topology and separates stable & unstable particles

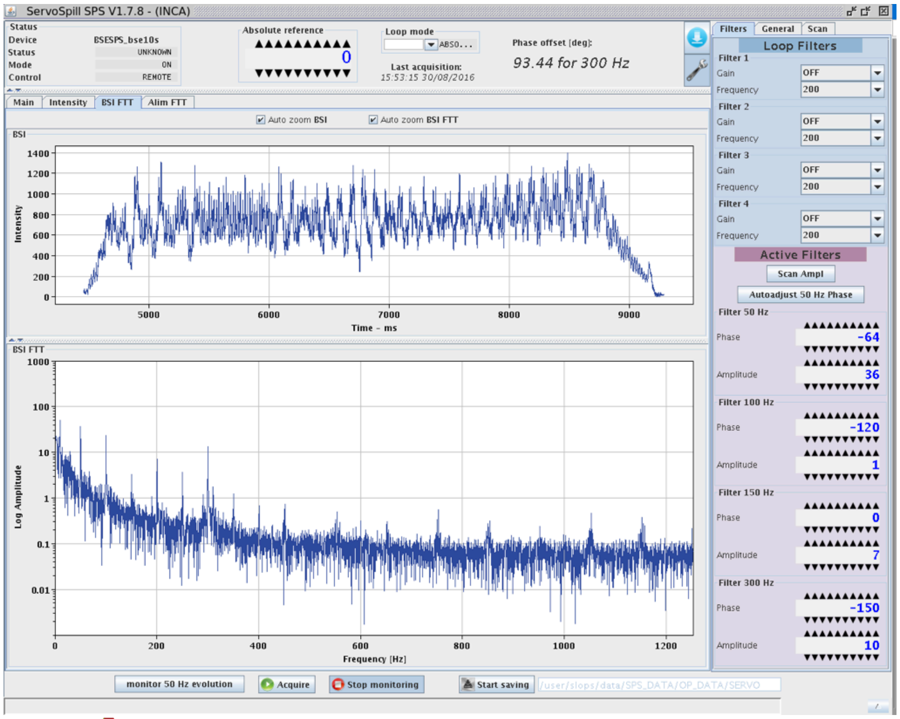
Fixed Points

- We can look at the phase space for this system:
  - [0,0] is stationary
  - small (x,xp) is nearly ellipsoidal
  - there are three fixed points that repeat every 3 turns.
    - Qx = 1/3 exactly
  - at larger amplitude, particles stream out
  - separation lines from bounded to unbounded motion: “separatrices”
- A septum to intercept the separatri*x* extracts the beam
- change the tune to shrink stable area => slow extraction



SPS Slow Extraction Spill

M. Fraser



Note the oscillations in the intensity.

The FFT reveals power lines (50 Hz*n) but also others

Third-Integer Resonance Analysis

- Either use Hamiltonian mechanics or analyse a simple model like ring+1 sextupole
  - either way will give starting values, then use numeric tracking to get actual solution.
- Consider a ring with a tune (1/3+δ), a sextupole with integrated strength ks, beta = β_x. Its map is

$$\begin{bmatrix} x \\ xp \end{bmatrix} = \begin{bmatrix} \left(-5.44x - \pi xp\beta_x\right)\delta - 0.5x + 0.866xp\beta_x \\ \dfrac{-0.75ks\beta_x^{\ 3}xp^2 + 0.866ks\beta_x^{\ 2}xxp + \left(\left(-0.5-5.44\delta\right)xp - 0.25ksx^2\right)\beta_x + \left(-0.866+\pi\delta\right)x}{\beta_x} \end{bmatrix}$$

- We find the fixed points by applying this map 3 times to (x,xp)
- The result is too messy to use directly, but we can taylor-expand and keep only up to 2nd order

- The truncated three-turn map is then

$$\begin{bmatrix} x \\ xp \end{bmatrix} = \begin{bmatrix} x - 1.5\beta_x^{\,2} ksxpx + 18.850 xp\delta\beta_x \\ \dfrac{\beta_x xp - 0.75 ks\beta_x x^2 - 18.850 x\delta + 0.75 ksxp^2 \beta_x^{\,3}}{\beta_x} \end{bmatrix}$$

- and we can solve for the 1st fixed point & its conjugate:

$$x_{fp} = \frac{12.566\delta}{\beta_x ks}, \quad xp = \pm \frac{21.766\delta}{ks\beta_x^{\,2}}$$

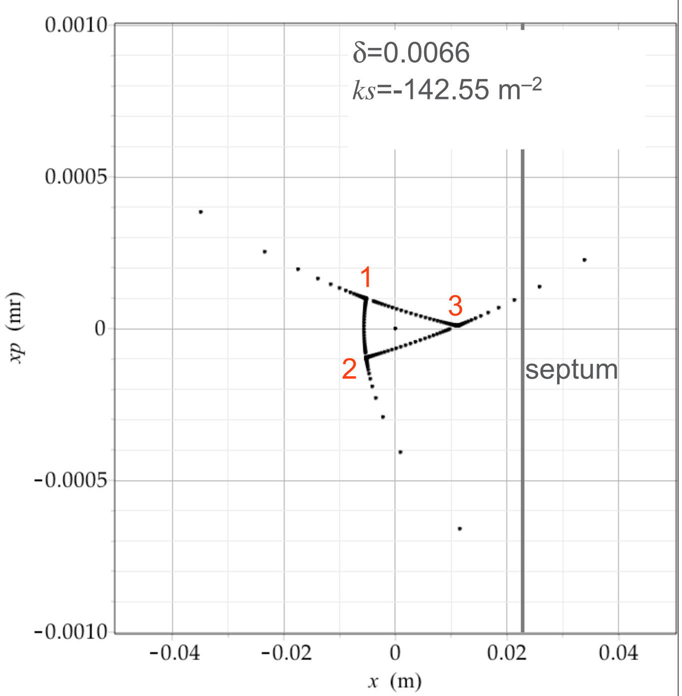
- and the third one:

$$x_{fp} = -\frac{25.133\delta}{ks\beta_x}$$

Resonance Triangle

- Putting the numbers in we indeed get the three fixed points.
- The area of the triangle is:

$$\pi\varepsilon = \frac{820.544\delta^2}{ks^2\beta_x^{\,3}}$$



Stepsize

- Extraction efficiency is directly calculated from the stepsize:

$$\varepsilon = 1 - \frac{w}{\Delta x} \qquad w\text{: septum thickness}$$

- With the stepsize

$$\Delta x = 11.1\delta\left(x_s - x_{ufp}\right) - 0.866\left(x_s - x_{ufp}\right)^2 ks \cdot \beta_x$$

- These formulae give us starting values for the design

Emittance

- Liouville tells us that the minimum extracted emittance is

$$\frac{\varepsilon_r}{n}$$

- but to get this we need a programmed bump in x and xp.
  - follow the movement of the UFP, i.e.

$$\delta x(\delta) = -\frac{25.133(\delta - \delta_0)}{ks\beta_x}$$

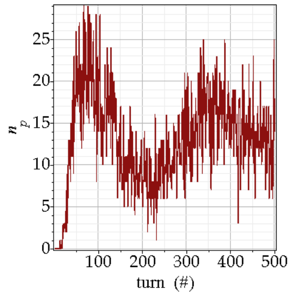
- the rate of change in δ in turn controls the intensity of the extracted beam
  - for maximum duty factor, δ is a function of the beam distribution.

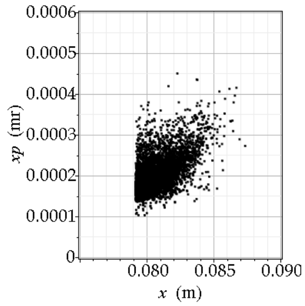
Modelling of a Slow-Extraction Cycle

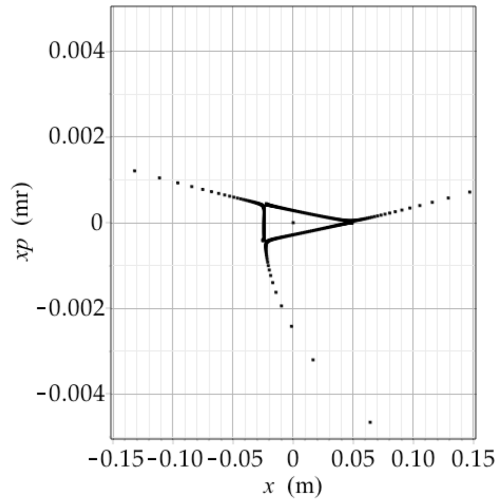
- Assume β_x=100 m, ε=10^{-5}. Chose a stepsize of 2 mm and put the septum 3 cm away from the fixed point.
- Solve for the stepsize and the triangle area to find

$$\delta = 0.002735, ks = -0.01398$$

- The UFP is then at $x_{fp} = 0.049$
- A first spill plot:

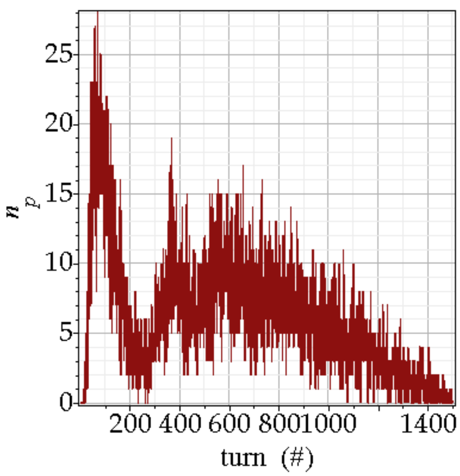


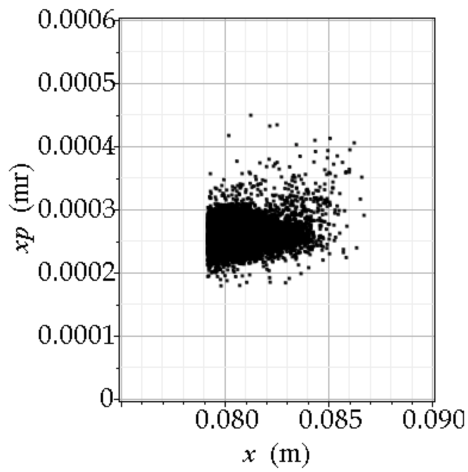




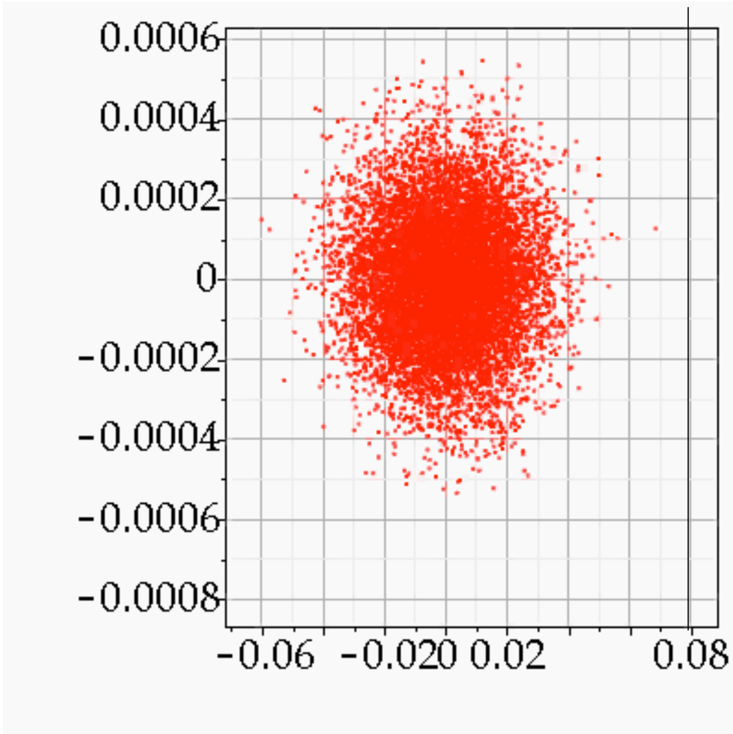
Another demo run

- "Overrun the resonant tune to reduce residual beam"





Movie



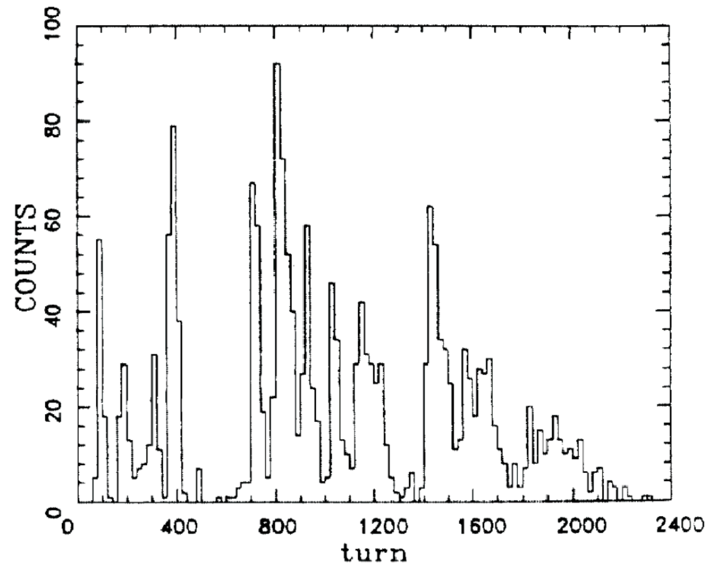
Chromatic Slow Extraction

- If the chromaticity of the machine is not 0, δ and the momentum of the particles are correlated.
  - Beam-lets get extracted according to their momentum.
- If the chromaticity and the dispersion fulfill the Hardt condition, the longitudinal emittance of the extracted beam can be reduced in addition to the transverse.

$$\xi = \frac{ks}{4\pi\nu}\left(\eta_s^{'}\cos(\phi_s) - \eta_s\sin(\phi_s)\right)$$

Noise sensitivity

- High sensitivity to tune makes system sensitive.
- Ex: Simulation with δ=0.011, 2x10–4 noise & ac ripple on quadrupoles:

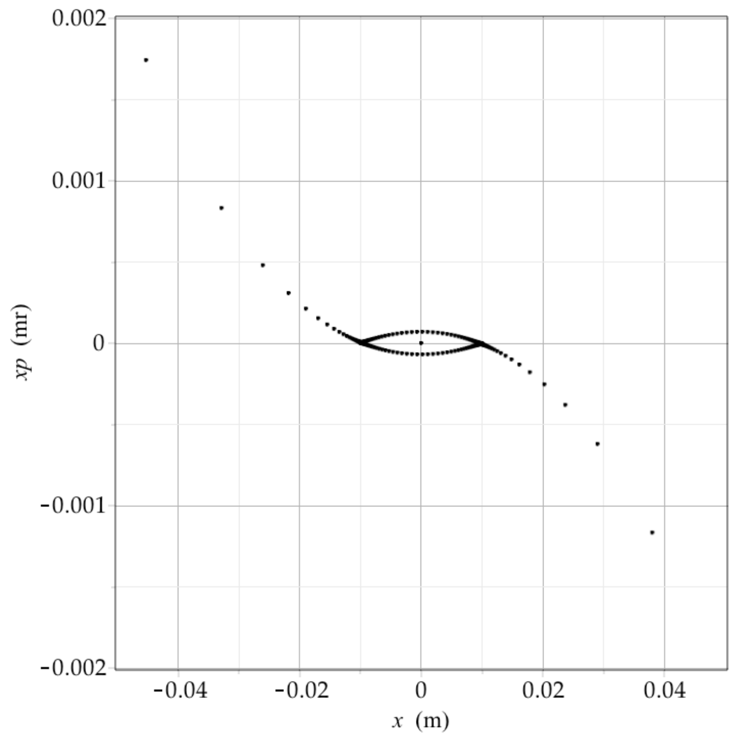


1/2-Integer Extraction

- It is also possible to extract on a 1/2 integer resonance
  - stronger resonance => easier to avoid residual beam left in ring.
- But it is a linear resonance => no separatrices
- This is overcome by using an octupole to drive the resonance & provide nonlinearity.
- 1/2-integer is a stop band: easier to completely empty the ring

Half Integer Resonance

- Driven by an octupole



Schemes for Very High Extraction Efficiency (low beam loss)

(Masahito Tomizawa)

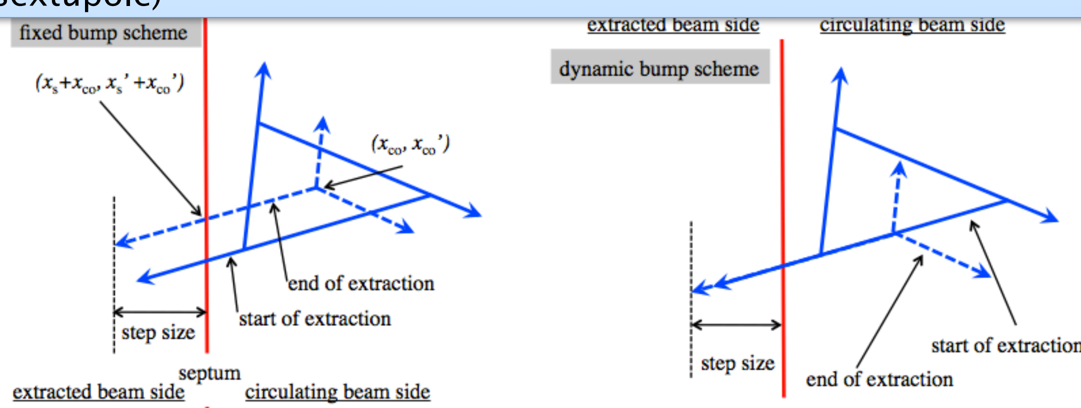
◎Electrostatic Septum (ESS) QF-QF high β (small α) 40m

-> large step size (20mm)

◎dispersion free at ESS + low horizontal chromaticity

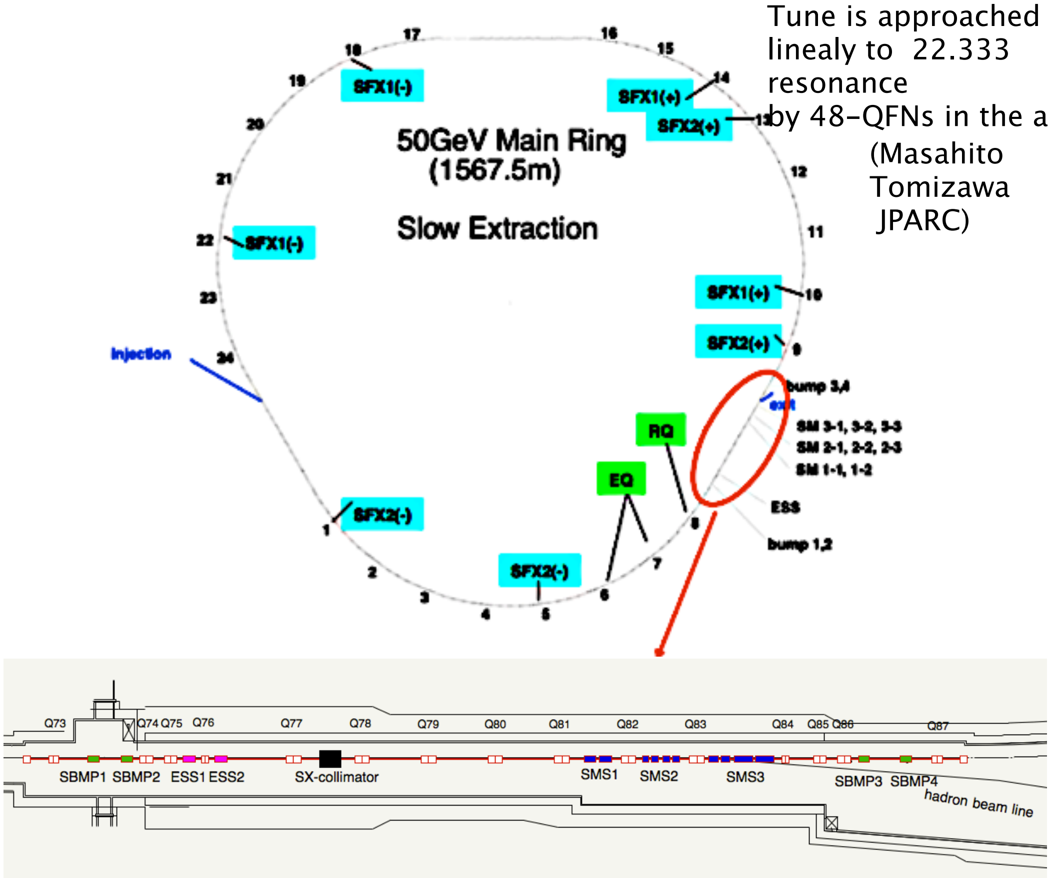
-> Separatrix is independent of Δp/p

1/3 resonant extraction onstant resonant sextupole)



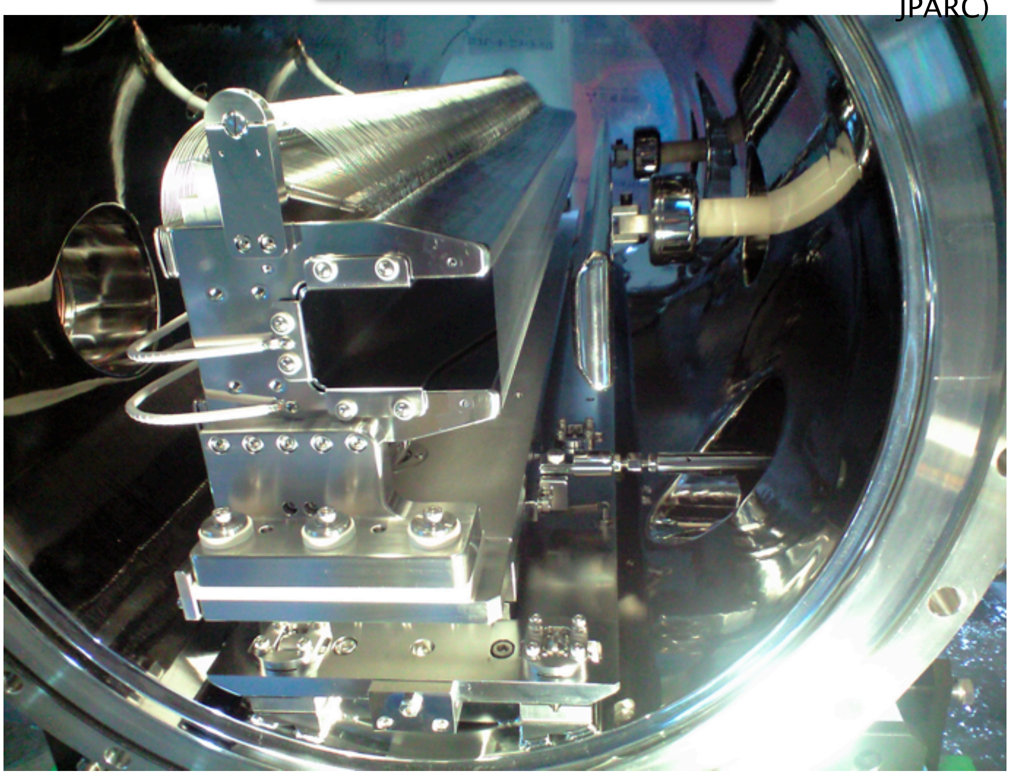
Tune is approached linealy to 22.333 resonance by 48-QFNs in the arcs

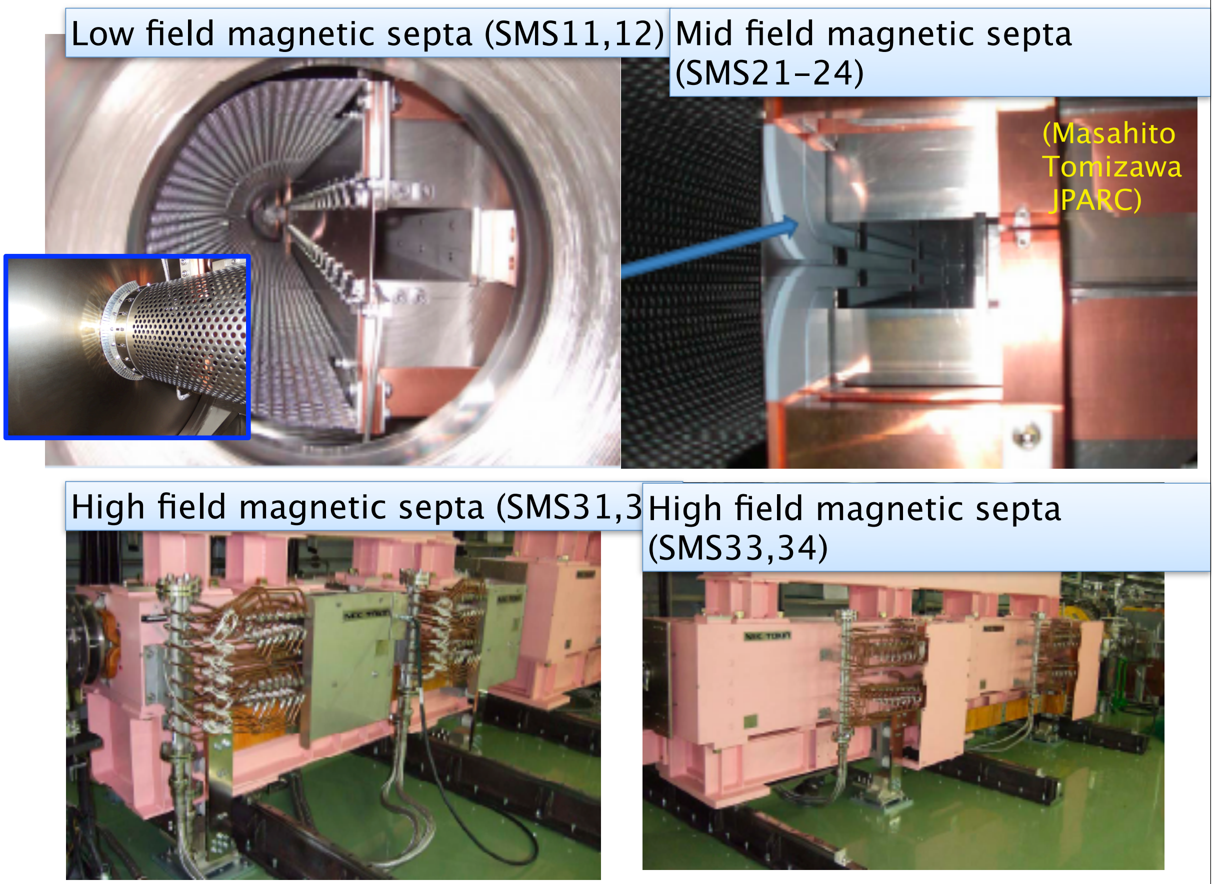
(Masahito Tomizawa JPARC)



Electro Static Septa (ESS1,2)

(Masahito Tomizawa JPARC)

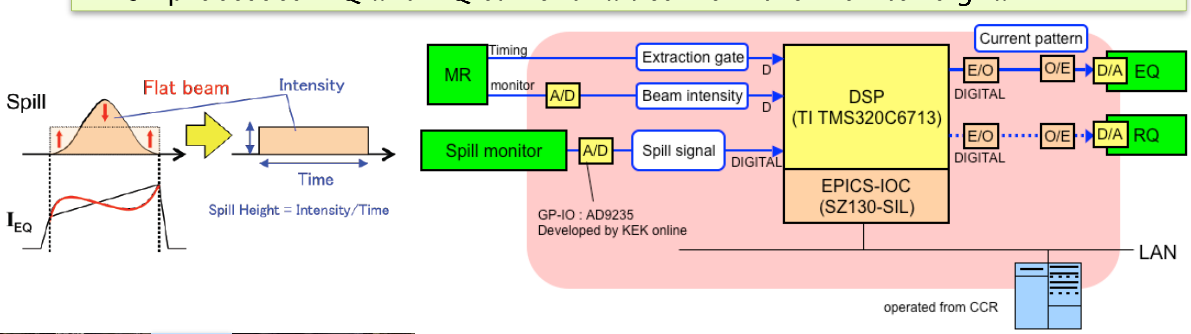


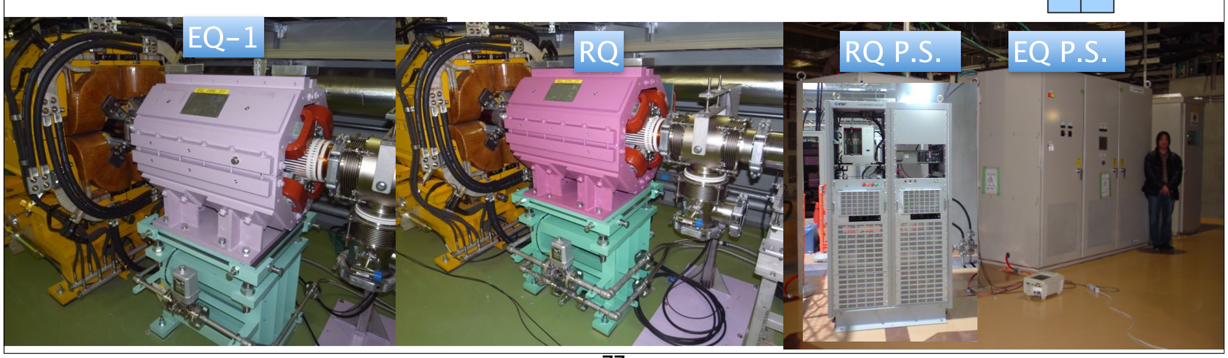


Beam Spill Feedback System

(Masahito Tomizawa JPARC)

A beam intensity monitor is placed in external beam line.
Uniform beam spill shape is obtained from tune modulation by quadrupoles EQ
Tune ripples are compensated by quadrupole RQ (and EQ)
A DSP processes EQ and RQ current values from the monitor signal





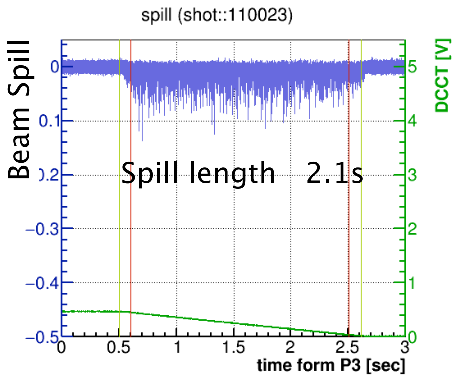
41.6kW User Operation Performances

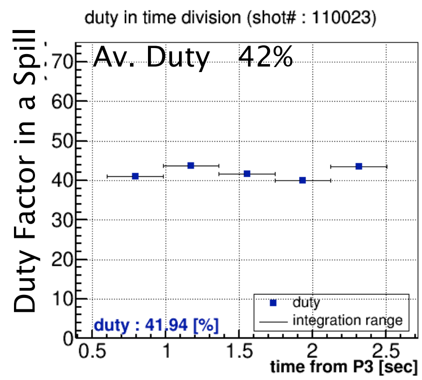
(Masahito Tomizawa JPARC)

I(t): PM signal sampled at 100KHz through 10KHz LPF

$$Spill\,Duty\,Factor = \left[\int_0^T I(t)dt\right]^2 \Big/ \left[\int_0^T dt \cdot \int_0^T I^2(t)dt\right]$$

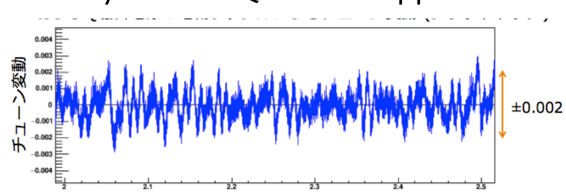
ideal spill -> 100%





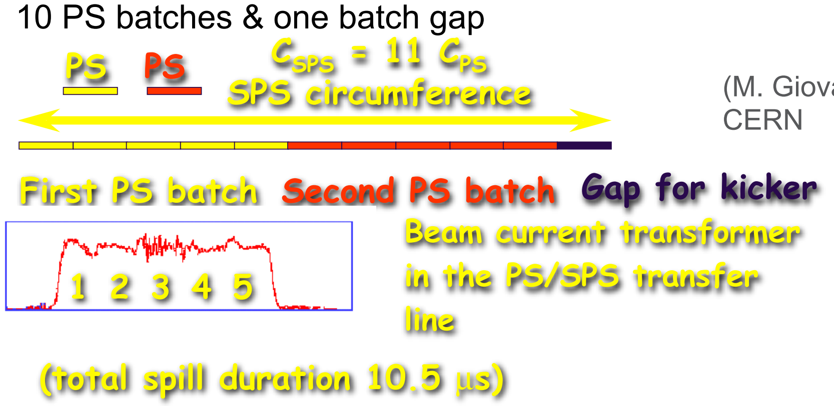
New Result 5/29 Study Duty 58%

Large tune ripple produced by BM and Q current ripple



Multi-Turn Extraction (CERN PS)

- CERN SPS is 11 times as long as PS.
  - 10 PS batches & one batch gap



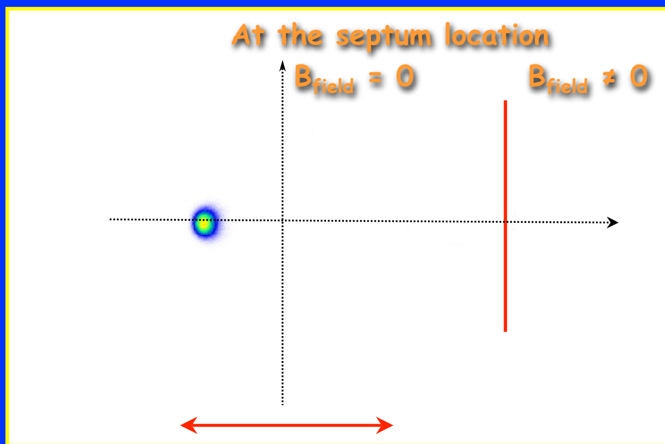
(M. Giovannozzi)
CERN

- How to extract over 5 turns?
  - slice the bunches, or
  - split the beam into 4+1 beamlets using 4th order resonance.
- Splitting in principle allows loss-less extraction of 5 turns
- Proven to work @ the SPS

Novel CERN multi-turn extraction

CERN PS MULTI TURN EXTRACTION

Final stage after 20000 turns (about 42 ms for CERN PS)

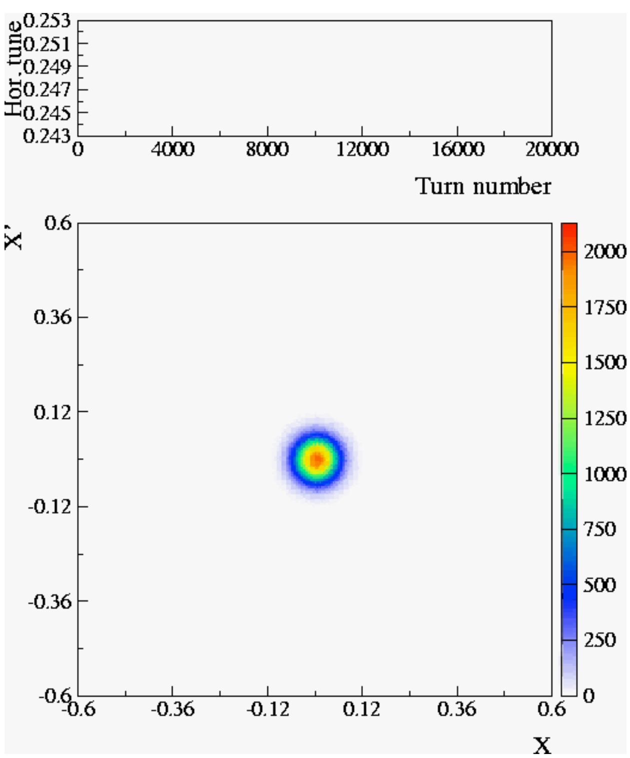


About 6 cm in physical space

Slow (few thousand turns) bump first (closed distortion of the periodic orbit)

Fast (less than one turn) bump afterwards (closed distortion of periodic orbit)

MTE Demo



M. Giovanozzi et al., CERN

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- M. Fraser & B. Goddard, “SPS slow-extraction: Challenges and possibilities for improvement”, Physics Beyond Colliders Kick-off Workhop, CERN, Sep-2016.
- M. Tomizawa, “J-PARC Slow Extraction”, Slow-Extraction Workshop, Darmstadt, DE, Jun-2016.
- M. Giovannozzi, “Resonant extraction: review of principles and experimental results”, ibid.

e^- Machine Injection

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Wednesday, June 21st
Summer 2017 USPAS
Course: Injection and Extraction of Beams

†Acknowledgments: Brennan GODDARD, Masamitsu AIBA, Wolfgang BARTMANN

Outline

INTRODUCTION

Motivation

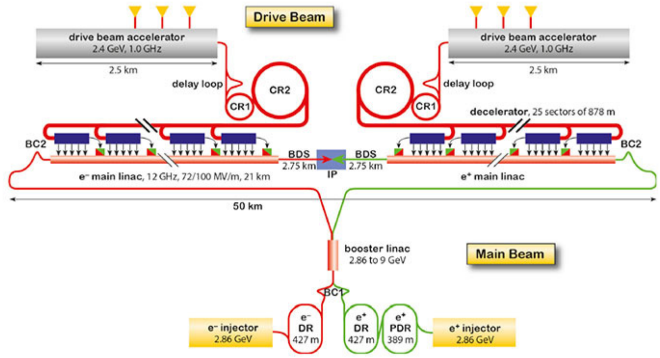
- Machines are usually designed to operate over a certain regime
  - Energy (Linear accelerators)
  - Emittance (Damping Rings)
  - Charge (Storage Rings)
  - ...
- Chain of accelerators are required to satisfy the requirements from the final users
  - Energy
  - Intensity
  - Luminosity (Colliders)
  - Brilliance (Light sources)
  - ...

A good example are lepton linear colliders and synchrotron light sources

Colliders

- Compact Linear Collider[a]
  - e+e- Collider at 3 TeV
  - 2-beam acceleration
  - CDR published in 2012

$$L \approx \frac{N_b n^2 f_{rep}}{4\pi\sigma_x\sigma_y} \quad (1)$$



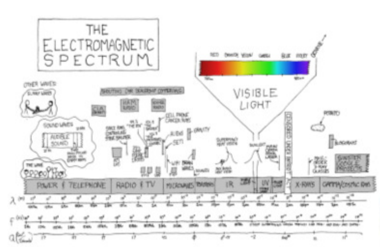
[a] http://clic-study.web.cern.ch/

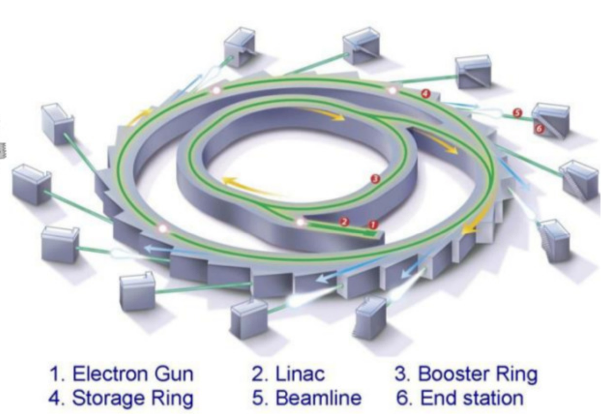
Parameter	Unit	Source	Damping Ring	Main Linac	Beam Delivery System
Energy	[GeV]	2.86	2.86	1500	1500
σ_x	[nm]	$3 \cdot 10^5$	$3 \cdot 10^4$	$4 \cdot 10^3$	40
σ_y	[nm]	$4 \cdot 10^5$	$3 \cdot 10^3$	$4 \cdot 10^2$	1
Luminosity	$[\frac{10^{34}}{s^1 cm^2}]$	10^{-9}	10^{-6}	10^{-4}	5.9

INTRODUCTION BASIC CONCEPTS INJECTION REFERENCES

Synchrotron Light Sources

Linac ⇒ Booster ⇒ Storage ⇒ Dump
TL1 TL2 TL3



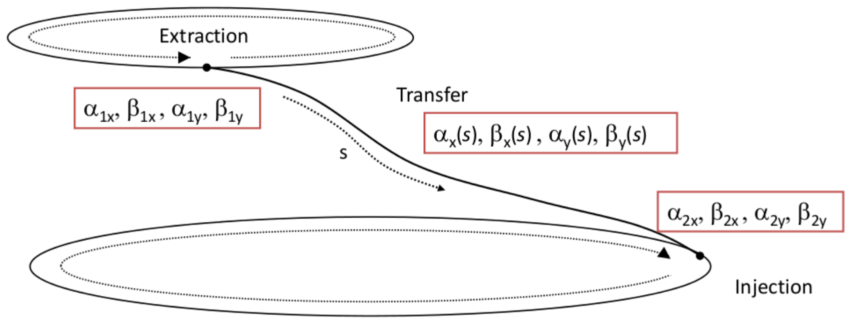


BASIC CONCEPTS

TRANSFER LINE

A transfer line (TL) transports the beam from extraction of one machine to injection of the next one

- Trajectories must be matched ($\beta_{x,y}$, $\alpha_{x,y}$, $\eta_{x,y}$ and $\eta'_{x,y}$)
- While satisfying additional constraints as minimum bend radius, maximum quadrupole gradient, magnet aperture, cost, geology...



We are going to focus on the last section of the TL, **Injection**

TL transports the beam from s_1 to s_2 through n elements
Each element can be expressed as a matrix, thus the TL can be represented by the product of n matrices

$$\begin{bmatrix} x_2 \\ x'_2 \end{bmatrix} = \bar{M} \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} = \prod_{i=1}^{n} M_i \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} \tag{2}$$

$\bar{M}$ can be parametrized by the Twiss functions as;

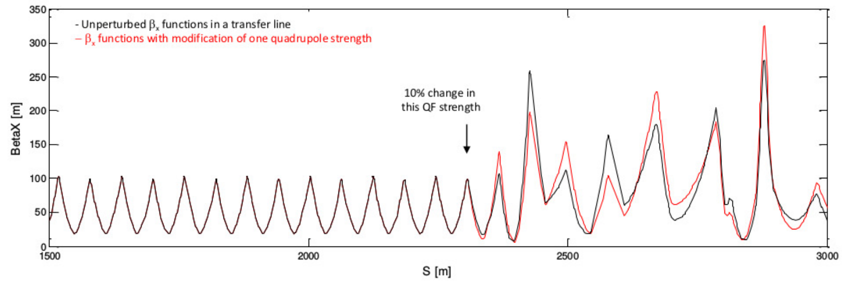
$$\bar{M} = \begin{bmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(cos(\Delta\mu) + \alpha_1 sin(\Delta\mu)) & \sqrt{\beta_2\beta_1} sin(\Delta\mu)) \\ \sqrt{\frac{1}{\beta_2\beta_1}}((\alpha_1 - \alpha_2)cos(\Delta\mu) - (1 + \alpha_1\alpha_2)sin(\Delta\mu)) & \sqrt{\frac{\beta_1}{\beta_2}}(cos(\Delta\mu) - \alpha_2 sin(\Delta\mu)) \end{bmatrix} \tag{3}$$

Twiss and Dispersion Propagation

Transfer lines are

- Single pass machines ⇒ no periodic solution exists
- Twiss parameters $\beta_{x,y}$, $\alpha_{x,y}$, $\eta_{x,y}$ and $\eta'_{x,y}$ are propagated through $\bar{M}$
- Twiss dispersion values at any point depend on
  - Machine elements
  - Initial coordinates

Unlike circular machines, a change of an element only affects the downstream Twiss and dispersion values

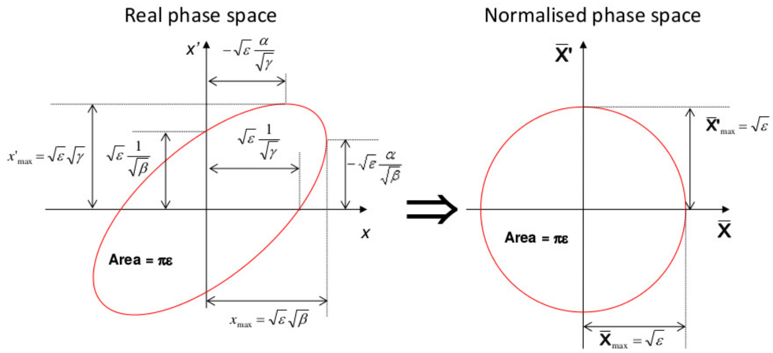


PHASE SPACE

Normalized Coordinates

Normalized coordinates are frequently used to analyse injection and extraction schemes

$$x, x' \xrightarrow{N} X, X' \begin{pmatrix} X \\ X' \end{pmatrix} = N \begin{pmatrix} x \\ x' \end{pmatrix} = \frac{1}{\sqrt{\beta_s}} \begin{pmatrix} 1 & 0 \\ \alpha_s & \beta_s \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$



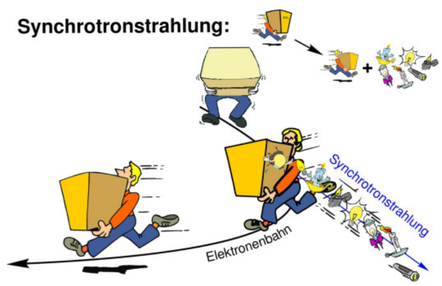
$$\varepsilon = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

$$\varepsilon = \bar{X}^2 + \bar{X}'^2$$

SYNCHROTRON RADIATION

Leptons vs Hadrons

- Important difference: Lepton motion is damped while Hadrons' is not
- Space charge effects less severe in e^- as they become relativistic at lower E



Damping radiation allows for:

- Different injection schemes and techniques
- Relax tolerances on injection precision and matching

Radiation Power

A point-like particle travelling under acceleration radiates a total power as;

$$P_\gamma = \frac{2 r_c m_0 \gamma^6 \left(\dot{\vec{\beta}}^2 - \left(\vec{\beta} \times \dot{\vec{\beta}} \right)^2 \right)}{3c} \quad (4)$$

first derived by Lienhard in 1898
Transverse and longitudinal radiated power can be expressed as;

$$P_\gamma = \frac{2\ r_c\ c\ \gamma^2 \dot{p_\perp}^2}{3\ m_0} \quad (5)$$

$$P_\gamma = \frac{2\ r_c\ \dot{p_\parallel}^2}{3\ m_0\ c} \quad (6)$$

Being the transverse power a factor γ^2 more severe than the longitudinal

Power Emitted

The variation of $p_\perp$ is related to the bending radius (ρ) as,

$$\frac{\partial}{\partial t} p_\perp = \frac{m\,\gamma\,\beta^2}{\rho} \tag{7}$$

Substituting Eq. (7) into Eq. (5) and assuming $\beta \approx 1$ leads to:

$$P_\gamma = \frac{E^4\,C_\gamma\,c}{2\,\pi\,\rho^2} \tag{8}$$

being $C_\gamma = \frac{4\,\pi\,r_c}{3\,m_0^3}$. *What is the ratio between $C_\gamma(e^-)$ and $C_\gamma(p^+)$?* Just look at the following table....

Machine	Particle	Circum. [km]	Energy [GeV]	Synch.Rad Critical Energy [eV]	Total Power emitted SR [kW]
LEP	e^+e^-	26.7	100	$7\cdot10^5$	$1.7\cdot10^4$
LHC	p	26.7	7000	44	7.5

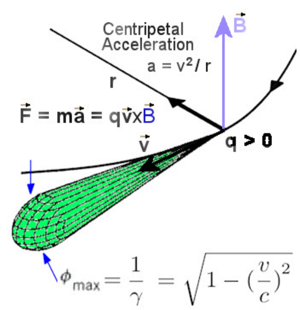
Energy Loss

The energy loss due to radiation over 1 turn is obtained by integrating Eq. (8) over 2π

$$U_\gamma = \frac{E^4\,C_\gamma\,c}{\rho} \tag{9}$$

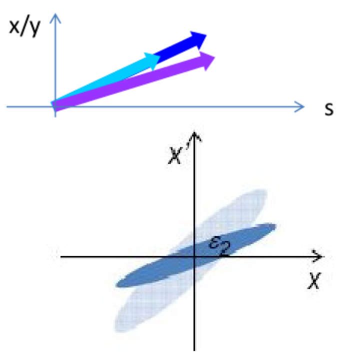
The light emitted by particles on a bend trajectory is within a forward cone of angle θ_{SR}

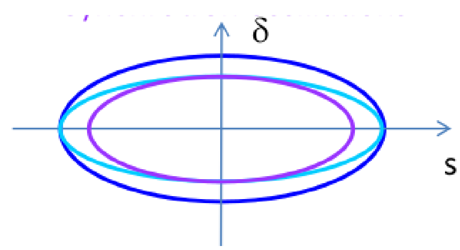
$$\theta_{\mathrm{SR}} = \frac{1}{\gamma}$$



Radiation Damping

This effect takes place on circular machines at energies where synchrotron radiation is emitted (e.g. synchrotron light source)
The beam energy is kept *constant* thanks to the accelerating cavities, which provide the exact energy lost by SR per turn, see Eq. (9)
The angle of a particle against the reference orbit is the ratio of transverse over longitudinal momentum $yp_0 = \frac{p_\perp}{p}$





Radiation Damping

However when the particle changes its momentum by Δp

$$yp = yp_0 \frac{p_\perp}{p + \Delta p} \approx yp_0 \left(\frac{p_\perp}{p} - \frac{p_\perp}{p^2} \Delta p \right) = \left(1 + \frac{\Delta p}{p} \right) yp_0 \quad (10)$$

The position (y) and angle (yp) at a given position can be expressed in terms of $A = \sqrt{\epsilon}$, β and ϕ as;

$$y = A\sqrt{\beta} cos(\phi) \quad (11)$$

$$yp = -\frac{A(sin(\phi) + cos(\phi))}{\sqrt{\beta}} \quad (12)$$

(if one neglects the contribution equal or higher than $O(\Delta p^3)$)

Emittance Reduction

The Courant-Snyder invariant reads as;

$$A^2 = \beta\ yp^2 + 2\alpha\ y\ yp + \gamma\ y^2 \qquad (13)$$

When crossing the cavity, the invariant is modified by $(A + \Delta(A))^2 - A^2$ which is equal to taking the total derivative of Eq. (13), this leads to

$$2A\Delta(A) = 2\alpha\ y\Delta(yp)yp^2 + 2\beta yp + \Delta(yp) \qquad (14)$$

It has been assumed $\Delta y = 0$ at the cavity, in fact

$$\Delta yp = -\frac{U_\gamma}{E_s} yp \qquad (15)$$

Plug Eqs. (15, 11, 12) into Eq.(14) and integrating over all phases ($\phi = 0..2\pi$) leads to

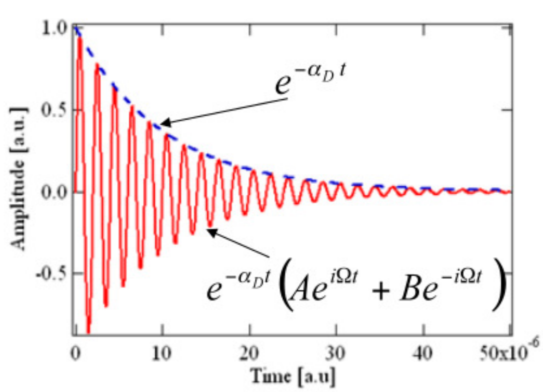
$$2\Delta(A) = -\frac{A\ U_\gamma}{E_s} \xrightarrow{\text{Diff.Eq.}} 2\frac{d}{dt}A(t) = -\frac{A(t)U_\gamma}{\tau_s\ E_s} \qquad (16)$$

where τ_s is the revolution time of the synchronous particle

Damping Time

Solving Eq. (16) and assuming $A(t = 0) = A_0$

$$A(t) = A_0 \cdot e^{t \cdot D_y} \qquad (17)$$



We define the vertical damping rate (D_y) as,

$$D_y = -\frac{U_\gamma}{2\tau_s\ E_s} = \frac{J_y}{2\tau_s} \qquad (18)$$

The resulting betatron motion is damped in time

Damping Time

Motion in the horizontal and longitudinal planes are also damped
However the derivation is more complex, as dispersion links both planes (see Ref. [1], Ch.8)

$$D_x = \frac{(1-D)U_\gamma}{2\tau_s\, E_s} = \frac{J_x}{2\tau_s} \tag{19}$$

$$D_z = \frac{(2+D)U_\gamma}{2\tau_s\, E_s} = \frac{J_z}{2\tau_s} \tag{20}$$

where D depends on the dispersion ($\eta(s)$), bending radius ($\rho(s)$) and the focusing elements ($k(s)$) of the ring as,

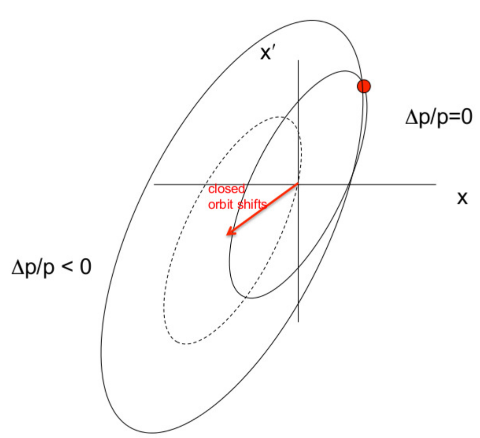
$$D = \frac{\int \frac{\eta(s)(1+2\rho(s)^2)k(s)}{\rho(s)^3} ds}{\int \frac{1}{\rho(s)^2} ds} \tag{21}$$

D_x, D_y and D_z are related by Robinson′s damping criterion:

$$J_x + J_y + J_z = 4 \tag{22}$$

Quantum Excitation

Eq. (17) tells us that emittance$\Rightarrow$ 0 for sufficient time
In reality there is a competing process between radiation damping and quantum excitation that determines the equilibrium ϵ_x, ϵ_y and ϵ_z
When an e^- emitts a photon with energy (u_γ) on a dispersive region there are 2 effects



- x changes as $\frac{\eta(s)u_\gamma}{E_s}$
- xp changes and so $\frac{\eta'(s)u_\gamma}{E_s}$ due to $\frac{1}{\gamma}$

Quantum Excitation

Following the same strategy as used to solve Eq. (13) (but now for the horizontal plane), we arrive at

$$\Delta(A^2) = \frac{(\beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2)u_\gamma^2}{E_s^2} \tag{23}$$

The final emittance depends on the Twiss and dispersion functions For convenience we define

$$\mathcal{H}(s) = \beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2 \tag{24}$$

Integrating Eq.(23) and weighting over the number of emitted photons ($N_\gamma(u_\gamma(s))$) we arrive at the following equation

$$\frac{\Delta(A^2)}{\tau_s} = \frac{\int \frac{\mathcal{H}u_\gamma(s)^2 N_\gamma(u_\gamma(s))}{E_s^2} ds}{c\tau_s} \tag{25}$$

Equilibrium Emittance

After converting Eq. (25) into a differential equation and adding the damping contribution Eq. (16) we arrive at,

$$2\frac{d}{dt}A(t)\ A(t) = -\frac{A(t)^2 U_\gamma}{\tau_s\ E_s} + \frac{\int \frac{\mathcal{H}u_\gamma(s)^2 N_\gamma(u_\gamma(s))}{E_s^2} ds}{c\tau_s} \tag{26}$$

After solving this differential equation,

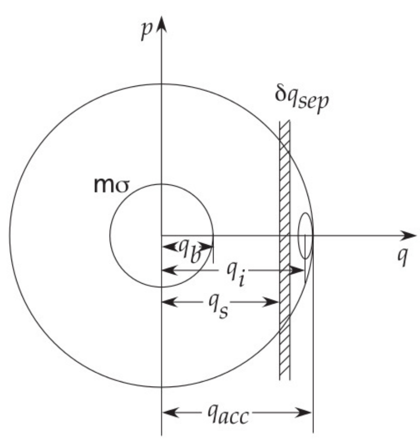
$$A(t)^2 = A_0\ e^{-\frac{U_\gamma\ t}{\tau_s\ E_s}} + \frac{\int \mathcal{H}u_\gamma(s)^2 N_\gamma(u_\gamma(s))ds}{c\tau_s\ E_s^2} \tag{27}$$

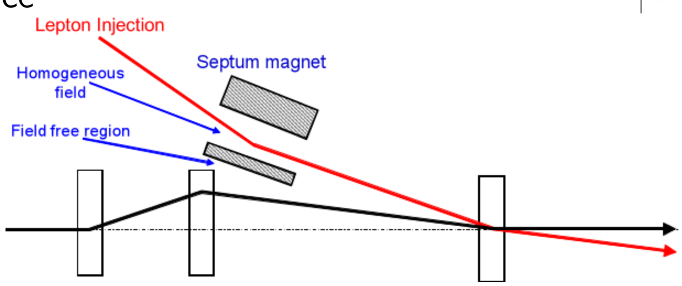
It is now clear that $A^2(t) = \epsilon(t) \neq 0$ when $t \to \infty$

INJECTION

Injection Scheme

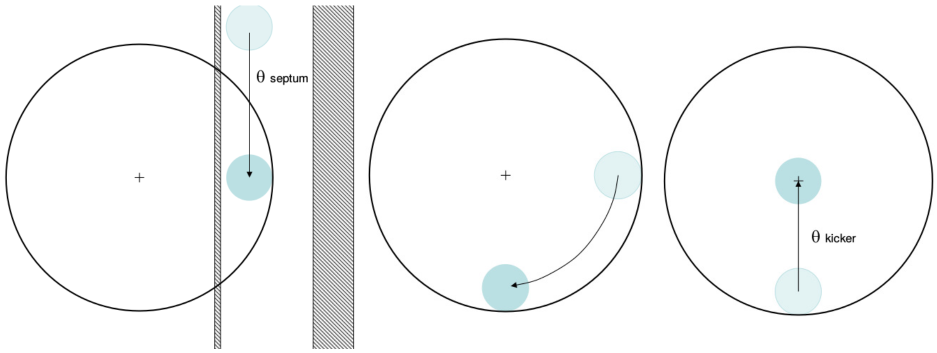
- Bring injected beam as close as possible to reference orbit, by
  - Septum (strong DC (or low) B field)
  - Kicker (low field fast rise/fall times)
- Bumped circulating beam to relax septum/kicker
- Inject beam into machine acceptance





Injection Steps

Step 1	Step 2	Step 3
Septum	$\phi = \frac{\pi}{2}$	Kicker



Beam Losses

- Injection process should minimize beam losses for both injected or circulating beams to avoid irradiation, activation or even direct damage of machine components
  - A thin septum is desirable to align the incoming beam to the current beam onto the orbit bump
  - Orbit bump is usually constructed by 3 (or 4) correctors to bring stored beam close to septum (and as parallel as possible)
  - Injected beam should fit into the acceptance of the machine (e.g. storage rings >10 σ of stored damped beam)
  - Acceptance of injection system should at least stay above a few σ except for very brief moments to minimize beam losses
    - by sustaining low values of quantum life time (τ_q)

Quantum Life Time

The equilibrium emittance obtained in Eq. (27) determines the distribution of the electrons which will be Gaussian (Central Limit Theorem)

There is a constant exchange of particles in the core of the beam and in the tail

e^- stored beams are inevitably Gaussian beams. If beam's tail is collimated, it will be replenished at expenses of intensity

The Quantum Life Time (τ_q) is found to be [2],

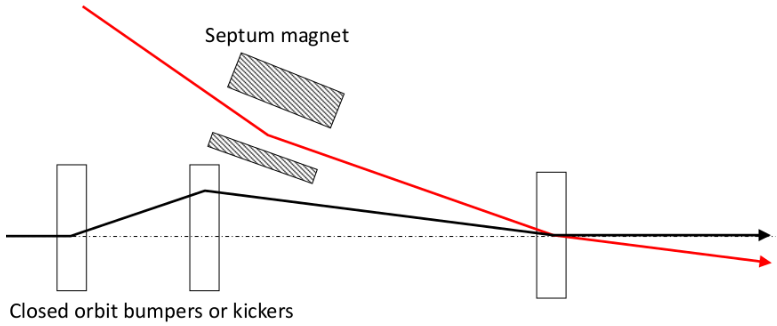
$$\frac{1}{\tau_q} = \frac{A_0^2}{D_x \sigma_x^2} e^{-\frac{A_0^2}{2\sigma_x^2}} \quad (28)$$

in absence of resonance, being D_x the horizontal damping time, Eq. (19) and A_0 the physical aperture of the machine

A_0/σ_x	5	5.5	6	6.5	7
τ_q	1.8 min	20.4 min	5.1 h	98.3 h	103 days

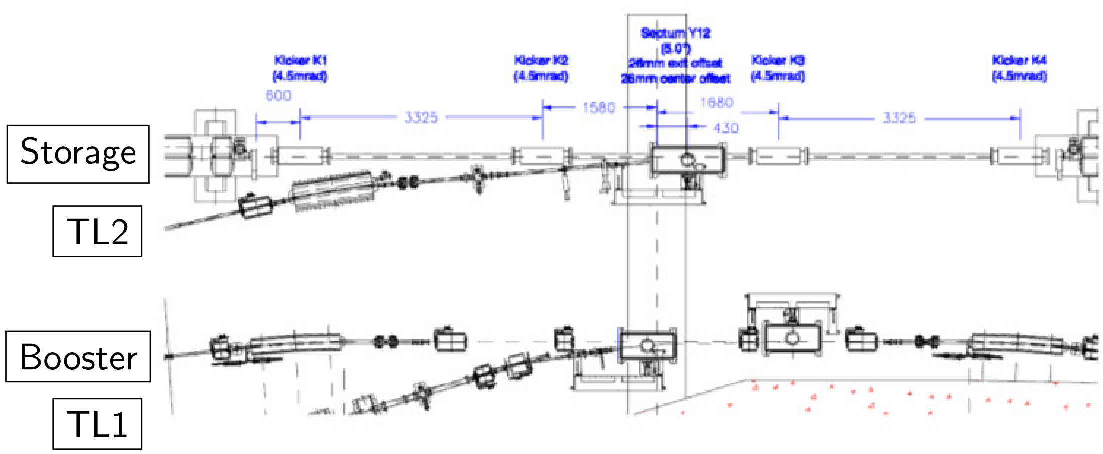
BETATRON INJECTION

Scheme



- Injected beam is offset at the septum with its own Twiss, Dispersion and emittance
- Injected beam is injected with an angle with respect to the closed orbit
- Injected beam performs damped betatron oscillations about the closed orbit

Example: Swiss Light Source 2.4 GeV



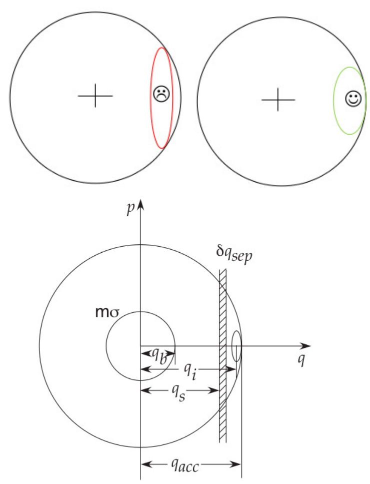
- 10 m long straight section
- 4-kicker orbit bump
- 5° Septum

Optimum Injection

There exist an optimum injection where the miss-matched at the septum is minimised

Optimum conditions:

- Circle curvature (circulating)=Ellipse curvature (injected)
- Upright ellipse



circulating: $\epsilon_{acc} = q_{acc}^2 + p_{acc}^2$

injected: $\epsilon_i = b_i p_i^2 + \frac{q_i^2}{b_i}$

where b_i represents the beta function into norm. phase space $b_i = \frac{\beta_i}{\beta_r}$

Optimum condition is expressed as:

$$\frac{d^2 q_{acc}}{dp_{acc}^2}\Big|_{p=0} = \frac{d^2 q_i}{dp_i^2}\Big|_{p_i=0} \qquad (29)$$

INTRODUCTION BASIC CONCEPTS INJECTION REFERENCES
Betatron Injection

Optimum Injection

$$\frac{d^2 q_{acc}}{dp_{acc}^2}\Big|_{p_{acc}=0} = -\frac{1}{\sqrt{\epsilon_{acc}}} \qquad (30)$$

$$\frac{d^2 q_i}{dp_i^2}\Big|_{p_i=0} = -\frac{b_i^{3/2}}{\sqrt{\epsilon_i}} \qquad (31)$$

Which leads to

$$\frac{\beta_i}{\beta_{acc}} = \left(\frac{\epsilon_i}{\epsilon_{acc}}\right)^{1/3} \qquad (32)$$

if injection happens at a point where $\alpha_r \neq 0$:

$$a_i = \alpha_{acc} - \alpha_i \frac{\beta_i}{\beta_{acc}} \qquad (33)$$

The optimum is when ellipse is not tilted ($a_i = 0$), therefore

$$\frac{\alpha_i}{\alpha_{acc}} = \frac{\beta_i}{\beta_{acc}} \qquad (34)$$

Eqs. (32) and (34) *solve* the matching problem for off-axis injection

General rules are:

- Injection (as extraction) are located on straight sections
- Septum is usually placed at a high beta point to reduce the phase space taken by the width of the septum

Injection Parameters

- Machine acceptance ($\sqrt{\epsilon_{acc}}$) should exceed the injection septum (q_s) in order to inject the beam into the closed orbit
- This condition is assured by shifting the closed orbit towards the septum by means of 180°-bump (upstream and downstream kickers are located at phase advanced ±90°) w.r.t. septum
- At the septum we need a displacement of

$$\delta q_s = q_s - q_b \tag{35}$$

- The angle required by the upstream kicker is

$$\delta x_k' = \frac{\delta p_k}{\sqrt{\beta_k}} = \frac{\delta q_s}{\sqrt{\beta_k}} = \frac{q_s - q_b}{\sqrt{\beta_k}} \tag{36}$$

as they are 90° apart. Taking into account that $q_s = \frac{x_s}{\beta_r}$ and $q_b = n\sqrt{\epsilon_b}$, we arrive at

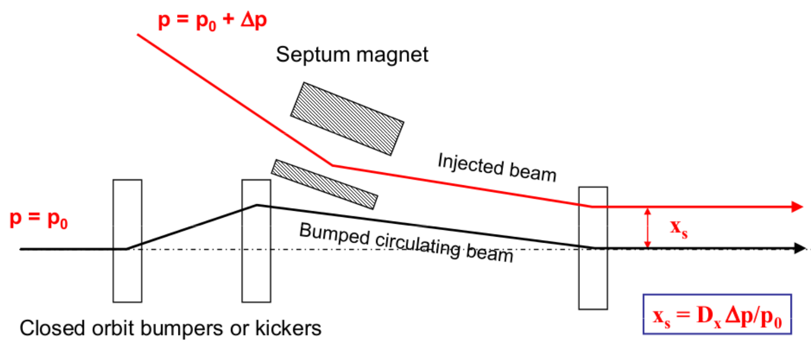
$$\delta x_k' = \frac{x_s}{\sqrt{\beta_k \beta_r}} - \frac{n\sqrt{\epsilon_b}}{\sqrt{\beta_k}} \tag{37}$$

SYNCHROTRON INJECTION

Scheme

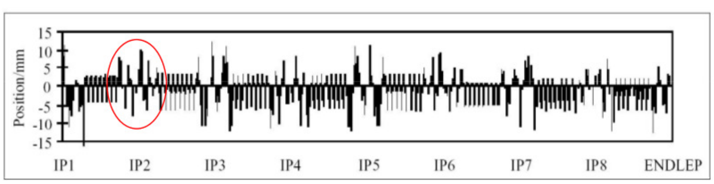
An alternative injection scheme that avoids off-axis injection in the transverse plane is the synchrotron or longitudinal injection. In this case the beam is centered in x/y but off-axis in the z-plane

- Beam injected parallel to circulating beam
- Synchrotron oscillations at Q_s
- Beam does not perform betatron oscillations
- Energy loss due to SR is proportional to $(1+\delta)^3$
- Dispersion at injection is not 0

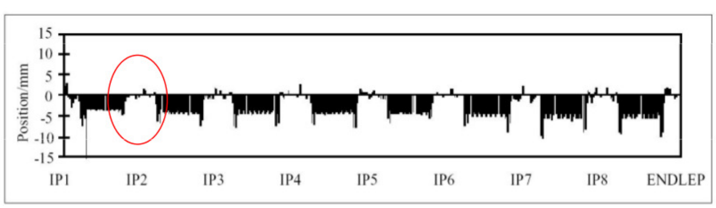


Example: LEP

Both schemes were actually implemented in LEP [3], [4] at 20 GeV
Betatron Injection : 6000 turns (0.6 s)



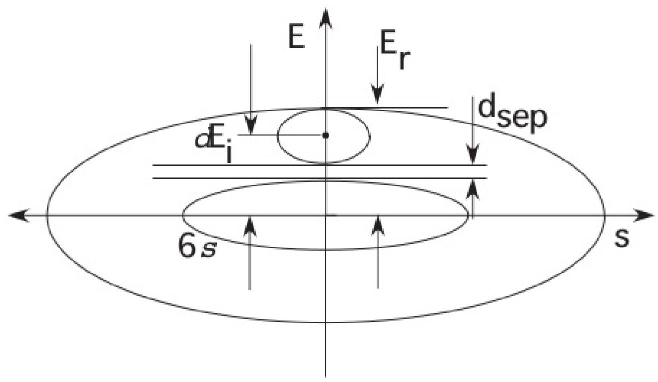
Synchrotron Injection: 3000 turns (0.3 s)



Synchrotron Injection in LEP gave improved background for experiments due to small orbit offsets in zero dispersion straight sections

Phase Space

In this scenario the phase space (z, E) look like



the septum appears as a horizontal line of a thickness given by

$$d_s = \frac{th_s}{\eta} \qquad (38)$$

being th_s the physical thickness of the septum and η the dispersion at the septum location

Energy Offset

The horizontal offset required is

$$\delta x = \sqrt{(m\sigma_E\eta)^2 + m^2\epsilon_x\beta_x} + n\sigma_{E_{inj}}\eta \qquad (39)$$

In terms of energy offset

$$\delta_E = m\sqrt{\sigma_E^2 + \frac{\epsilon_x}{\mathcal{H}}} + n\frac{\sigma_E}{\mathcal{H}} \qquad (40)$$

being

- m the number of minimum σ acceptance during injection
- n the number of σ accepted of the injecting beam

Eq. (40) shows the importance of $\mathcal{H}$. Since $\mathcal{H} = \frac{\eta^2}{\beta}$ is the energy resolution, where β acts as a scaling factor ($\pm$ adjustable by optics) Colliders are suitable for synchrotron injection (as $\eta(IP) = 0$) whereas circular light sources not that much since the value of $\mathcal{H}$ is dictated by the low emittance requirements

QUADRUPOLE KICKER

INTRODUCTION BASIC CONCEPTS INJECTION REFERENCES
Multipole Kicker

Motivation

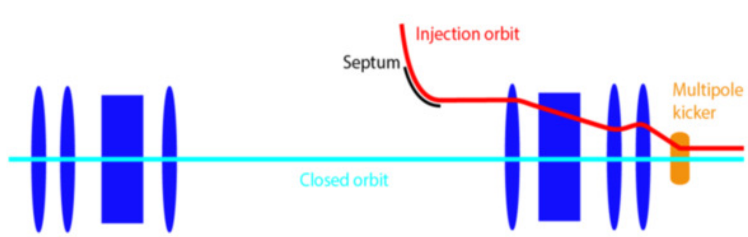
The hardware implemented is a septum plus a pulsed quadrupole

Pros:

- Stored beam is unperturbed, since the multipole magnet has 0 field on axis
- Betatron [5] or synchrotron [6] injection schemes could be implemented
- Reduced space

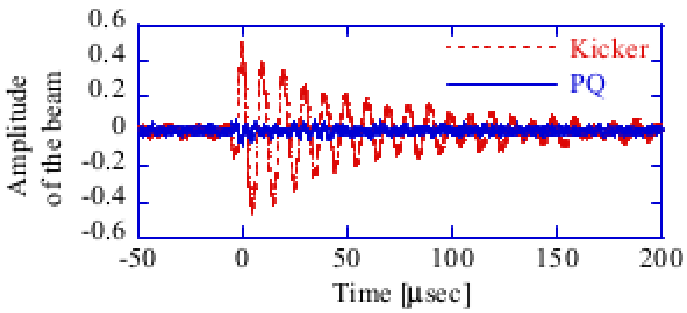
Cons:

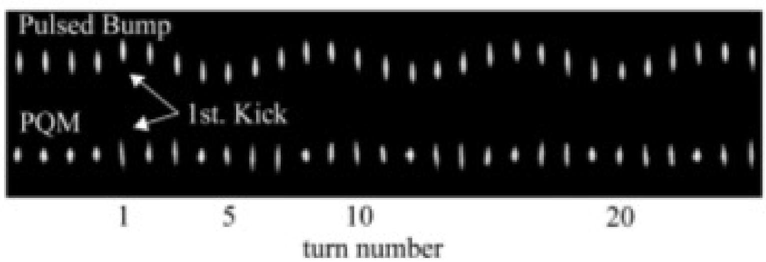
- Alignment of the pulsed magnet (distortion of stored beam)
- Beam profile modulation [7]
- Transient emittance growth



Example: Photon Factory Advanced Ring (PF-AR) 2007

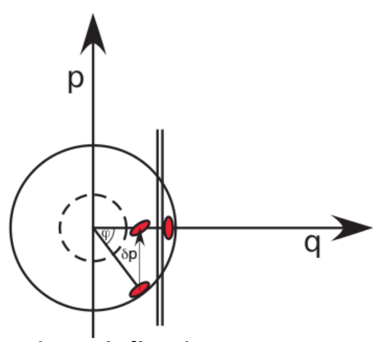
This scheme was experimentally tested at PF-AR in KEK, Japan [8]
Beam injection at 3 GeV





Phase Space

Injected beam usually enters at $q = q_i, p = 0$
After rotating ϕ the quadrupole kicks the beam closer to closed orbit
It also focuses/defocuses the injected beam $\Rightarrow$ changing its matching condition



$$q = q_i \qquad p = p_i \qquad (41)$$

$$q_{i,1} = q_i cos(\phi) + p_i sin\phi \qquad (42)$$
$$p_{i,1} = p_i cos(\phi) + q_i sin\phi \qquad (43)$$

$$q_{i,2} = q_{i,1} \qquad (44)$$
$$p_{i,2} = p_{i,1} + k_q q_{i,1} \qquad (45)$$

Initial and final emittances

$$\epsilon_0 = q^2 + p^2 \qquad (46)$$
$$\epsilon_2 = q_{i,2}^2 + p_{i,2}^2 = (1 + k_q^2)q^2 + 2k_q pq + p^2 \qquad (47)$$

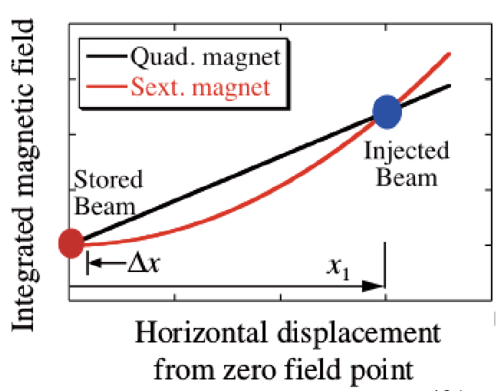
Although the beam is miss-matched it will be damped!

SEXTUPOLE KICKER

Motivation

The hardware implemented is a septum plus a pulsed sextupole
Pros:

- Stored beam is unperturbed, since the multipole magnet has 0 field on axis
- Betatron [5] or synchrotron [6] injection schemes could be implemented
- Extended field-free region on-axis (less distortion of storem beam)



Comparison field gradient (k) and the field strength (kl) on the stored beam

$$k_2 l = 0.05 k_1 l$$
$$k_2 = 0.1 k_1$$

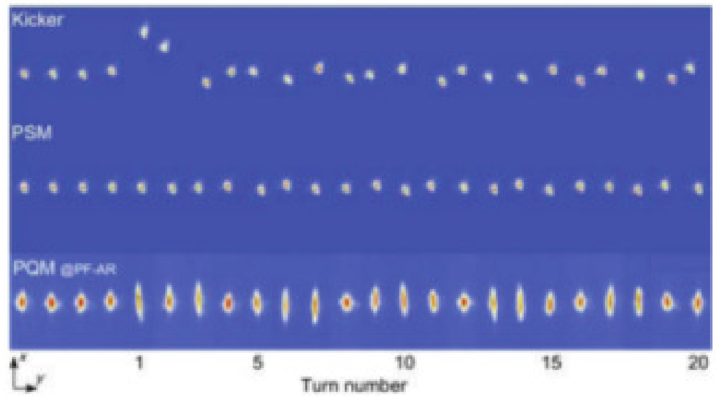
Example: Photon Factory Advanced Ring (PF-AR)

Installation of pulse sextupole magnet at the Photon Factory in 2008 [7]

4-kicker

PSM

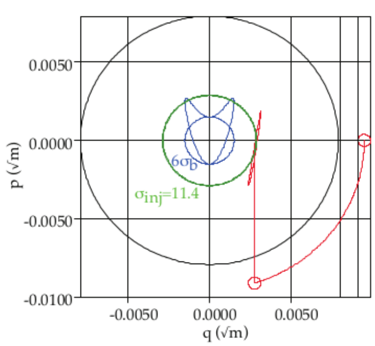
PQM



- coherent dipole oscillations of the stored beam in both planes are much smaller
- Top-up injection 0.02% in peak to peak during two hours
- Amplitude of the stored beam oscillation in the injection was much reduced

Phase Space

Analysis is very similar to the PQM scheme



$$q = q_i \qquad p = p_i \tag{48}$$

$$q_{i,1} = q_i cos(\phi) + p_i sin\phi \tag{49}$$
$$p_{i,1} = p_i cos(\phi) + q_i sin\phi \tag{50}$$

$$q_{i,2} = q_{i,1} \tag{51}$$
$$p_{2,1} = p_{i,1} + k_s q_{i,1}^2 \tag{52}$$

Initial and final emittances

$$\epsilon_0 = q^2 + p^2 \tag{53}$$

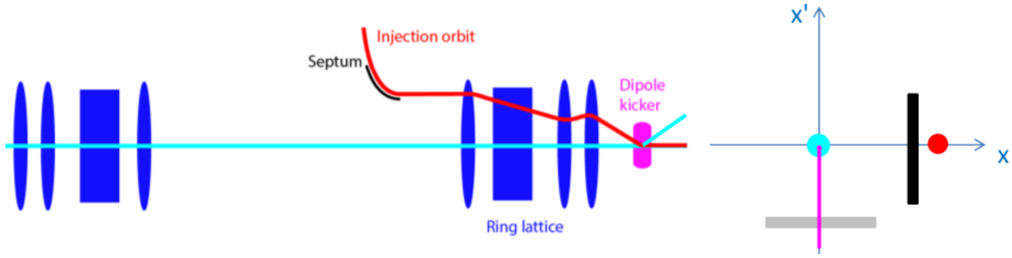
$$\epsilon_2 = q_{i,2}^2 + p_{i,2}^2 = (1 + k_s^2 q_b^2) q_b^2 \tag{54}$$

SWAP-OUT

Motivation

Swap-out [9] injection technique enables to inject bunches into very small aperture rings where acceptance is very limited

- Injection in the transverse plane
- Septum plus dipole kicker
- Spent circulating beam (low charge) is replaced by a fully-charged beam
- No disturbance on stored beam



- No examples yet but it is planned for ALS and APS upgrades

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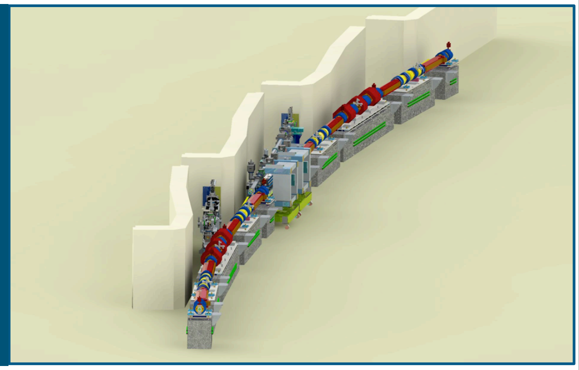
Top-up Injection



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Fill & Coast Cycles

- The rate of beam loss for a ring with current I and beam lifetime τ_b is:

$$\frac{dI}{dt} = \frac{I}{\tau_b}$$

- Each injection pulse increases the circulating current:

$$\Delta i_{inj} = \frac{Q_{inj}}{\tau_{rev}}$$

Q_{inj}: charge per injector pulse
Δi_{inj}: change in stored-beam current

- The total fill time is then

$$t_f = \frac{I}{\Delta i_{inj} f_{inj}} = \frac{I}{Q_{inj}} \frac{\tau_{rev}}{f_{inj}}$$

- Fill-and-coast average intensity

$$\int_0^T I dt = \frac{T}{t_c + t_f} \int_0^{t_c} I_0 \exp\left(-\frac{t}{\tau_b}\right) dt$$

t_c: coast time
t_f: fill time
T: averaging time

- t_c is optimal when average over peak intensity is maximized

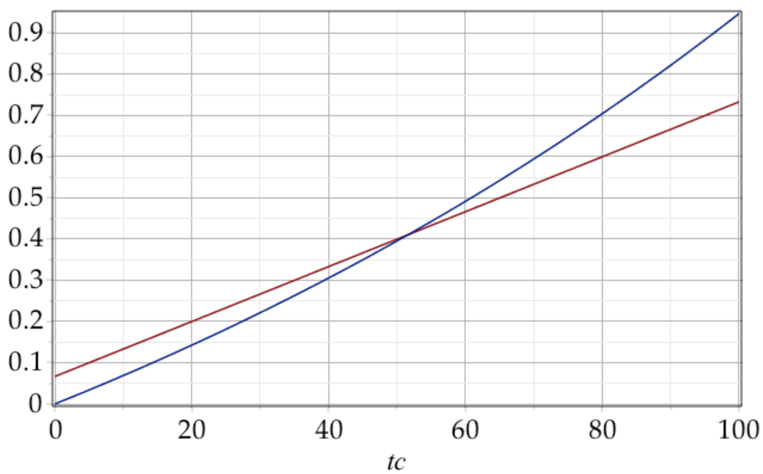
$$\frac{1}{I_0 T} \int_0^T I dt = \frac{\tau_b}{t_f + t_c} \left(1 - \exp\left(-\frac{t_c}{\tau_b}\right)\right)$$

- This is the case when

$$\frac{t_f + t_c}{\tau_b} = \exp\left(\frac{t_c}{\tau_b}\right) - 1$$

Optimum Condition

- Ex: τ_b=150 [min], t_f=10 [min]



Top-up injection

- The injector is running (almost) all the time. Intensity of a bunch varies exponentially:

$$Q_b = Q_0 \exp(-t_i / \tau_b)$$

- For a given injector charge, each bunch needs the average injection rate:

$$Q_{inj} = Q_0 \left(1 - \exp\left(-\frac{t_c}{\tau_b}\right)\right) \Rightarrow t_c = \frac{1}{f_{i,b}} = -\ln\left(\frac{Q_{inj}}{Q_0}\right)\tau_b$$

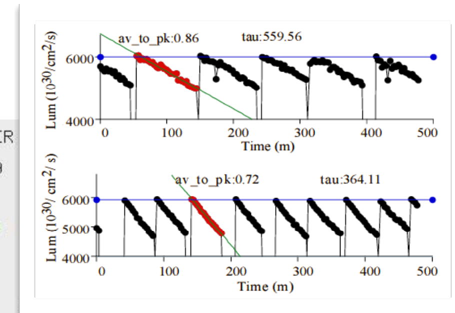
- therefore the average injection rate needed for n_b bunches is

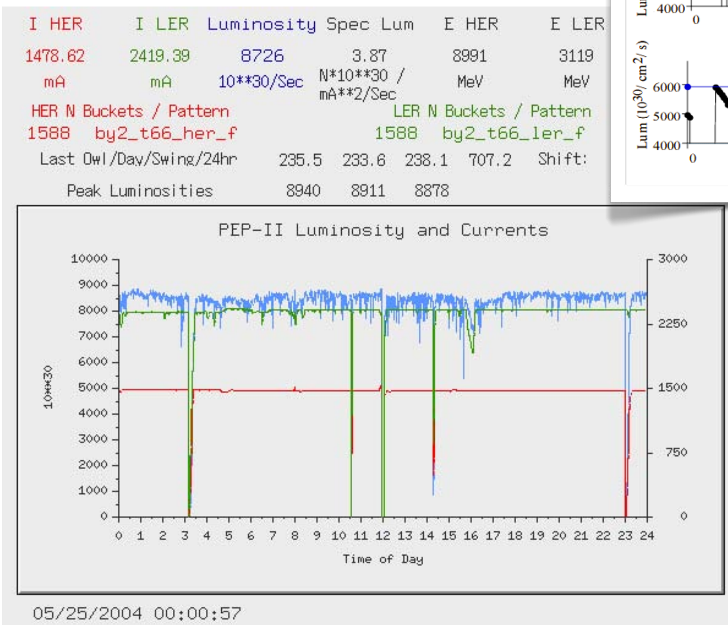
$$f_{inj} = \Sigma f_{i,b} = \sum_b \frac{1}{-\ln\left(\frac{Q_{inj}}{Q_0}\right)\tau_b}$$

Injector and Control Requirements

- The injector has to be programmable to inject into any rf bucket.
- A bunch-current monitor is needed to monitor charge in every bunch to select the next candidate for refill.
- Light sources have special safety requirements:
  - block top-up if magnet currents are out of spec.
  - block top-up if there is no beam in the ring
  - clearing magnets in photon beam lines, if possible.
  - avoidance of possibility to get injecting beam into the expt. hutches
- In colliders, a state machine allows top-up only when it is safe to do so.

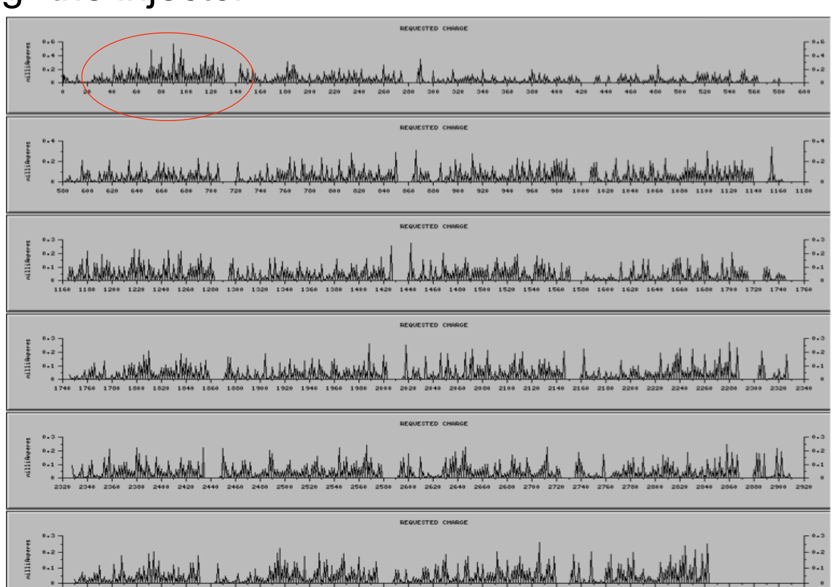
PEP-II "Trickle Charge"(Tm)





Top-up rate as a Diagnostic

- Top-up rate indicates bunch lifetime. Can show when bunches "hog" the injector



e^- Machine Extraction

Edu Marin[1] Uli Wienands[2] †

[1]CERN, (Switzerland)
[2]ANL, (USA)

Wednesday, June 21st
Summer 2017 USPAS
Course: Injection and Extraction of Beams

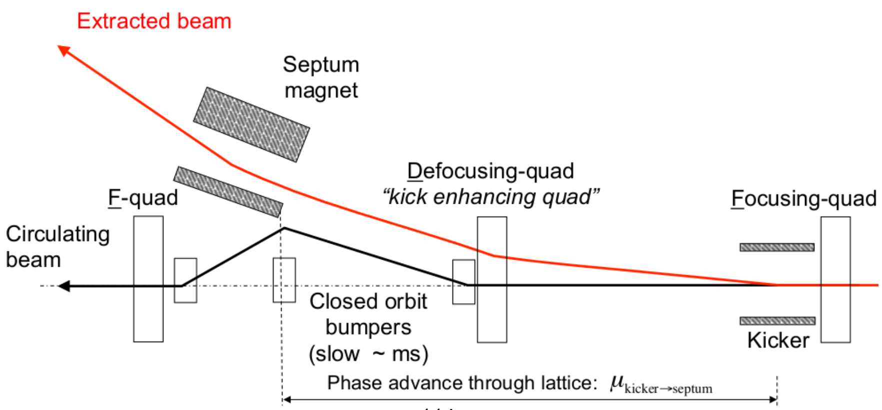
†Acknowledgments: Brennan GODDARD, Matthew Fraser, Wolfgang BARTMANN

Outline

INTRODUCTION

INTRODUCTION KICKER REQUIREMENTS REFERENCES

- In essence extraction is the reverse process of injection, although:
  - No need to close the bump
- Usually it takes place at higher energies
  - Stronger elements are required
    - Orbit bump might be needed
  - Less space charge effect (usually not a concern for e^-)
- Power density issues due to small emittances
- Single-turn (Fast extraction) is typically used for e^- machines



KICKER REQUIREMENTS

Kick Optimisation

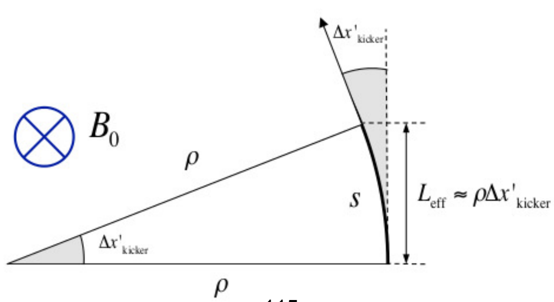
To minimise the kicker deflection required:

$$\Delta x'_{kicker} = \frac{x_{extr} - x_{bump}}{\sqrt{\beta_{kicker}\beta_{septum}} sin\mu_{kicker,septum}} \quad (1)$$

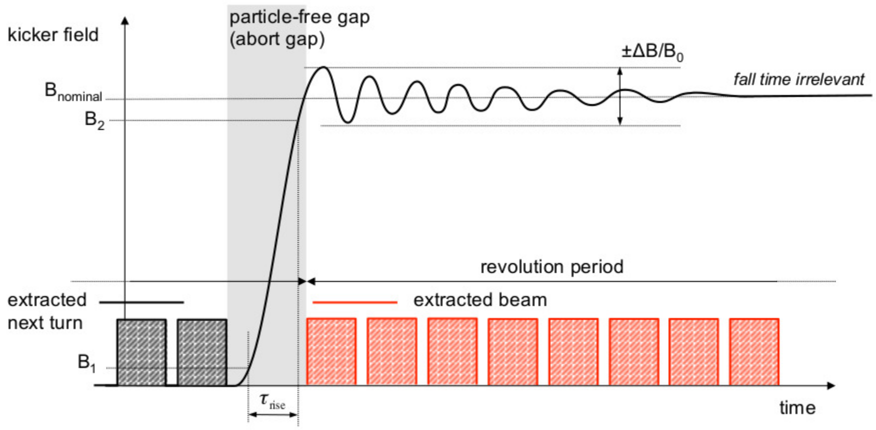
- Optimum phase advanced between kicker and septum ($\approx \pi/2$)
- Defocusing quad in between to contribute to extraction
- Large β at the kicker (small divergence) and septum

The kicker integrated strength is (small angles approximation)

$$\Delta x'_{kicker} = \frac{s}{\rho} \approx \frac{B_0 \int_0^s dl}{B_0 \rho} = \frac{q}{p} \int_0^s B \cdot dl = \frac{q}{p} B_0 L_{eff} \quad (2)$$



Kick Pulse Shape



- Rise-time, τ_{rise} usually defined between given limits [%] of B nominal
- Ripple definitions depends on the tolerable emittance growth
  - Very challenging for damping rings provide since they provide extremely small emittances

Jitter Tolerances DR

Example at Next Linear Collider Project (NLC) [1]

- In order to preserve small emittances coming out of DR
  - Kicker jitter $\leq 10\%$ $(1 \cdot \sigma)$

$$\frac{\delta x'}{x'} \leq \frac{1}{10}\frac{\sigma}{\delta x} = \frac{1}{10}\frac{\sqrt{\epsilon_{ext}\beta}}{d_s + m\sqrt{\epsilon_{inj}\beta}} \qquad (3)$$

being m the number of σ that the extracted beam has to clear from the injected beam

Damping Rings of linear collider work at a regime where

$$\frac{\epsilon_{ext}}{\epsilon_{inj}} \approx 10^{-3}$$

If we apply the design NLC DR values [2];

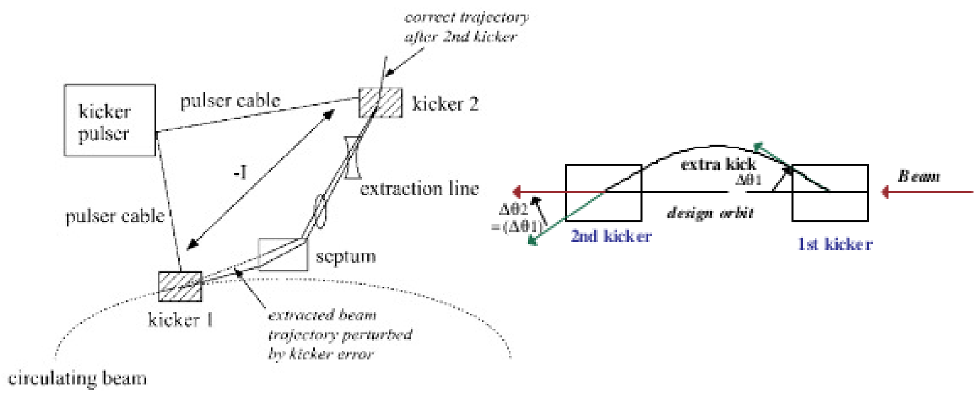
- β=3 m
- ϵ_{inj}=3 mm
- ϵ_{ext}=3 μm
- m=7

$$\boxed{\frac{\delta x'}{x'} = 3 \cdot 10^{-4}}$$

Which has not been achieved operationally yet

Double Kicker System

A compensating kicker system (double kicker system) in the extraction line could relax the required tolerance



A second kicker located at 180° phase advanced from the first kicker could compensate for angle variations (angle jitter) induced by the 1st kicker

Both kickers should be fed by the same modulator

INTRODUCTION KICKER REQUIREMENTS REFERENCES

Double Kicker System

The matrix that transport the beam between the kickers is

$$M_{k_1 \to k_2} = \begin{bmatrix} -\sqrt{\frac{\beta_2}{\beta_1}} & 0 \\ -\frac{\alpha_2 - \alpha_1}{\sqrt{\beta_2 \beta_1}} & -\sqrt{\frac{\beta_1}{\beta_2}} \end{bmatrix} \tag{4}$$

Position and angle after the second kicker is:

$$\begin{bmatrix} \delta x \\ \delta x' \end{bmatrix} = \begin{bmatrix} -\sqrt{\frac{\beta_2}{\beta_1}} & 0 \\ -\frac{\alpha_2 - \alpha_1}{\sqrt{\beta_2 \beta_1}} & -\sqrt{\frac{\beta_1}{\beta_2}} \end{bmatrix} \begin{bmatrix} 0 \\ \delta x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \delta x_2 \end{bmatrix} \tag{5}$$

Leading to

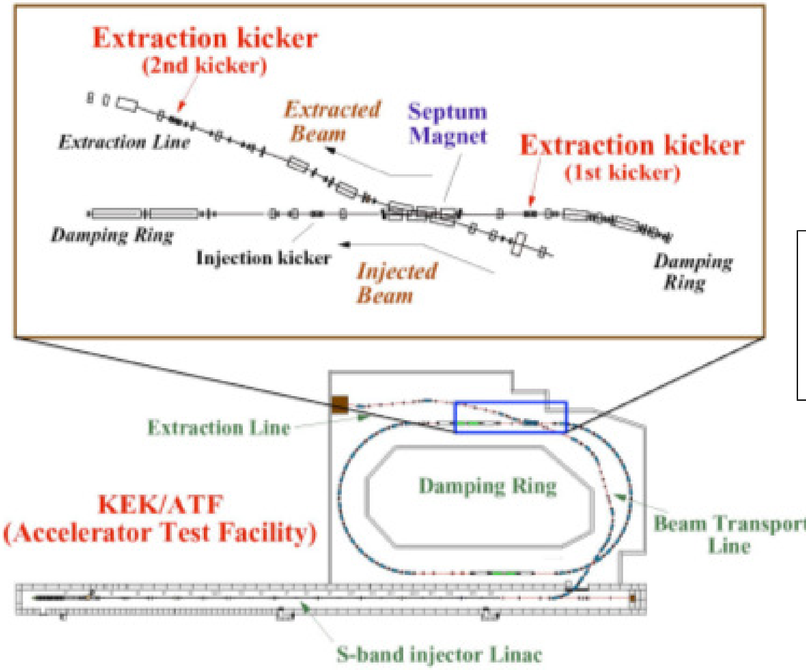
$$\delta x = 0 \qquad \delta x' = -\sqrt{\frac{\beta_1}{\beta_2}} \delta x_1' + \delta x_2'$$

Since it is desired that $\delta x' = 0$ then the

$$\delta x_2' = \sqrt{\frac{\beta_1}{\beta_2}} \delta x_1'$$

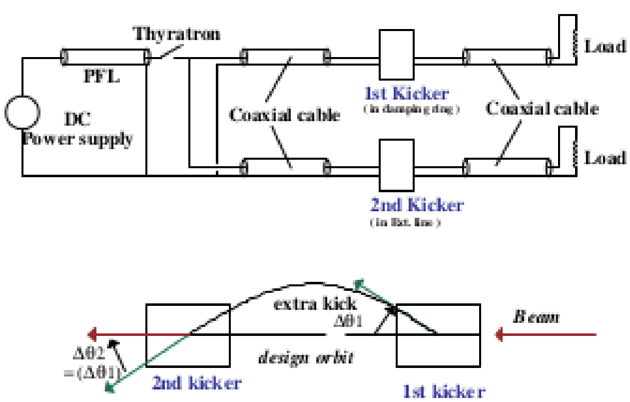
Kicker System @ ATF

This compensating scheme has been experimentally tested at the Accelerator Test Facility (ATF) in Japan [3]



Tolerance [4]
$\frac{\delta x'}{x'} = 5 \cdot 10^{-4}$
$(\beta = 10\ m)$

ATF Double Kicker Experience



Orbit jitter was measured at a BPM downstream 2$^{\text{nd}}$ kicker
$\Delta\theta_1$, $\Delta\theta_2$ and $\frac{\Delta p}{p}$ were fitted from the BPM readings

$$\Delta x_{bpm} = R_{12}(1, bpm)\Delta\theta_1 + R_{12}(2, bpm)\Delta\theta_2 + \frac{\Delta p}{p}\eta_{bpm} \quad (6)$$

	$\Delta\theta < 0.007 mrad$		$\Delta\theta \geq 0.007 mrad$	
Mode	# of meas.	σ_{kicker} [μm]	# of meas.	σ_{kicker} [μm]
Double	115	37	181	124
Single	60	78	248	34

ATF Double Kicker Discussion

It is crucial to keep the ratio between betas at optimum
$\beta_{2-kicker}$ is scanned to reduce the orbit jitter variation at BPM

$\beta_{2-kicker}$ $[m]$	σ_{kicker} $[\mu m]$
5	11.4
6	11
8	10.9
10	11.4
12	12.9

Assuming model optics and $k = \frac{\Delta\theta_1}{\Delta\theta_2}$
Measured $\beta_{1-kicker}$=4.95 m
Resolution of incoming position/angle jitter and monitor $= 10.7\mu m$

It is obtained

$$\frac{\Delta\theta_1}{\Delta\theta_2} = k = 0.83 \tag{7}$$

which is not explained by cable length difference
It was suspected that difference of ceramic coating between the two kickers caused the much difference of the field strength

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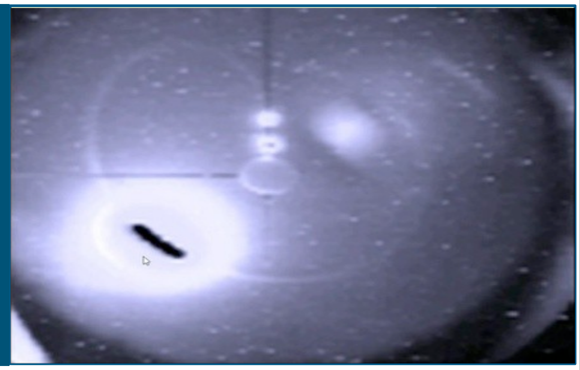
[4] H.Nakayama, KEK Proceedings 92-6,1992,p326-p334

Beam-Abort Systems



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- Many machines need a safe way to get rid of the beam quickly
  - a critical component overheats
  - background/radiation in the detector becomes too high
  - the rf system trips (esp. in e^-/e^+ machines)
  - the beam orbit leaves a safe region (lightsources)
  - …
- In modern machines the power density to be absorbed in the dump is significant.
  - high charge, small emittance

Energy Density

Machine	Stored energy	Beam size (avg.) (m^2)	Energy density (J/m^2)
LHC	360 MJ	10^{-7}	3.6×10^{15}
SKEKB	200 kJ	2.7×10^{-9}	7.5×10^{13}
PEP-II	180 kJ	2×10^{-7}	9×10^{9}
APS-U	4.4 kJ	10^{-10}	4.5×10^{13}
ILC	4.5 MJ, 25 MW	6.5×10^{-7}	7×10^{12}
CLIC	0.3 MJ, 14 MW	4.3×10^{-6}	5×10^{10}

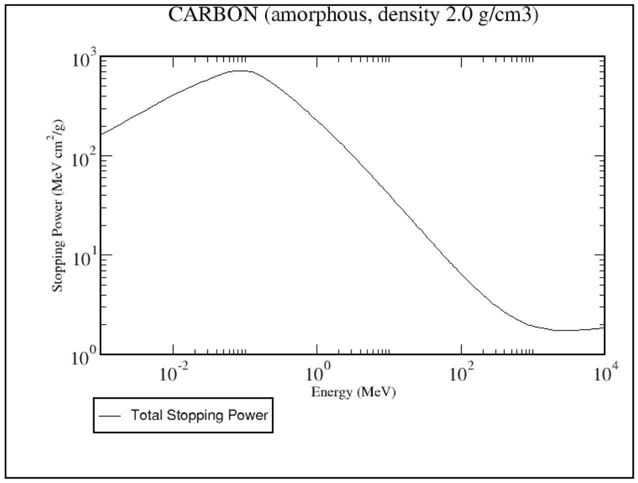
- (The beam sizes are average numbers for the circulating beam)
- Even PEP-II needed special effort to allow beam dumps

- Low Z dump material
  - Al, C, Be: good; Cu, Fe, W: bad
  - keep the power density as low as possible

- Energy density is reduced by
  - defocusing the beam
  - "painting" the beam across the surface of the dump
  - running beam through a spoiler before the dump

Energy Loss Mechanisms

- For thin materials (e.g. windows or the surface layer of a dump):
  - dE/dx energy-loss described by Bethe-Bloch.
- Impulsive heating of material due to power density



- In thicker materials, showers develop.
  - highest power density at a few radiation lengths

- Shower from 30 GeV beam

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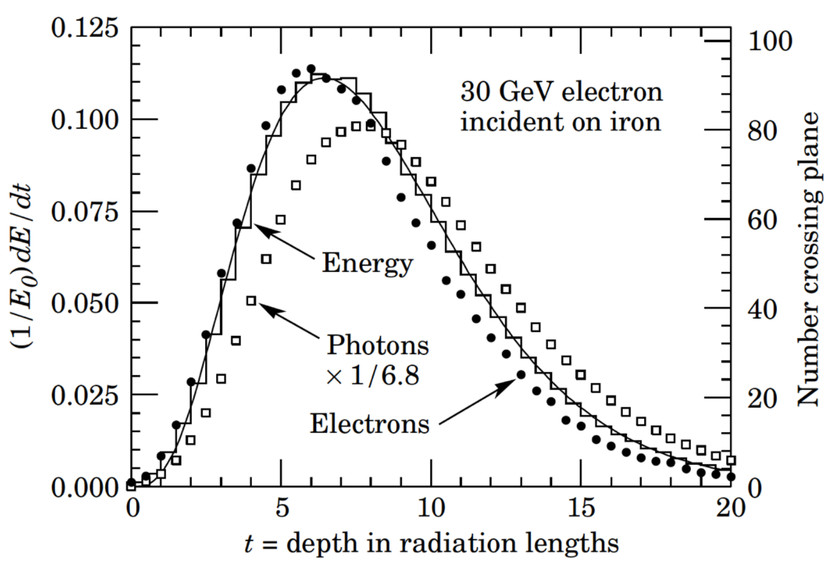
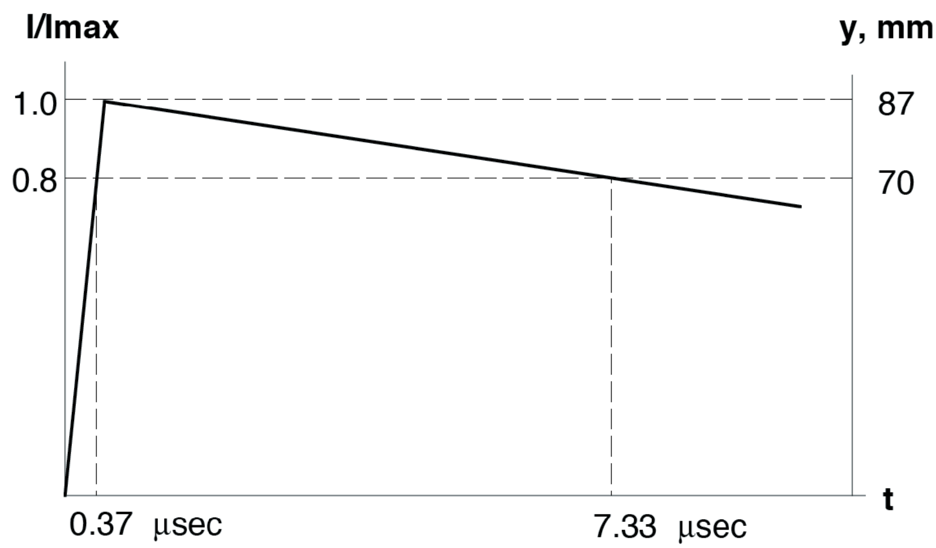
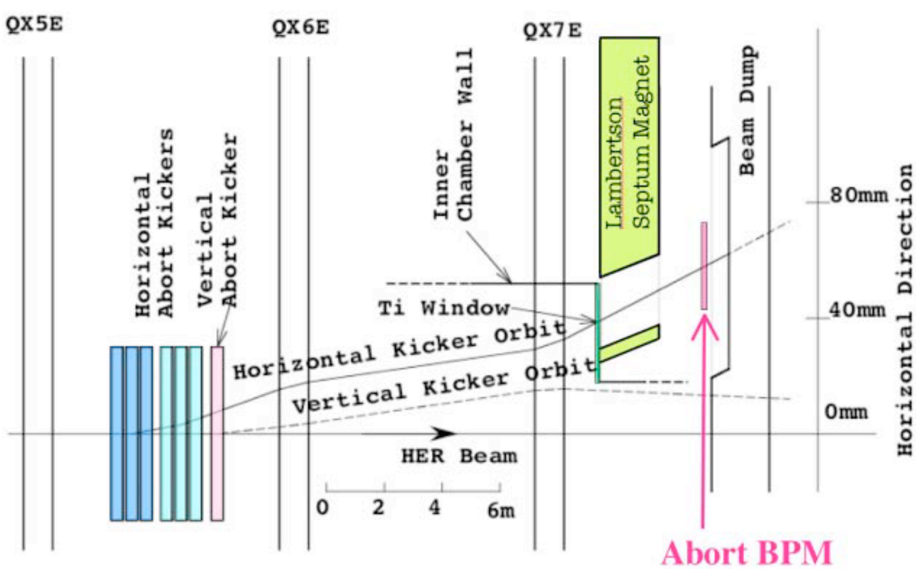


- synchronized with a gap in the beam



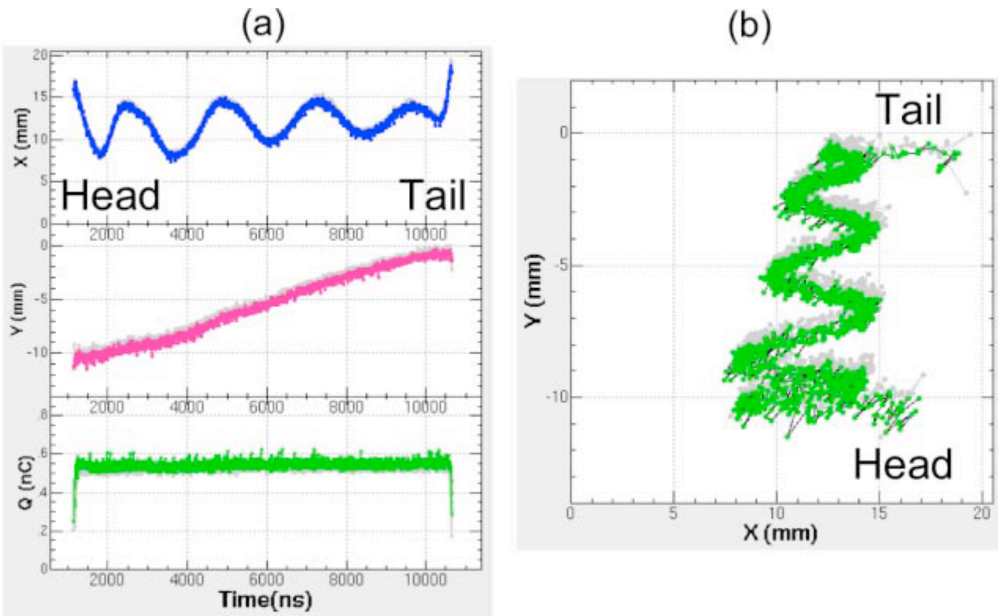
KEKB Abort Layout

Iida et al.



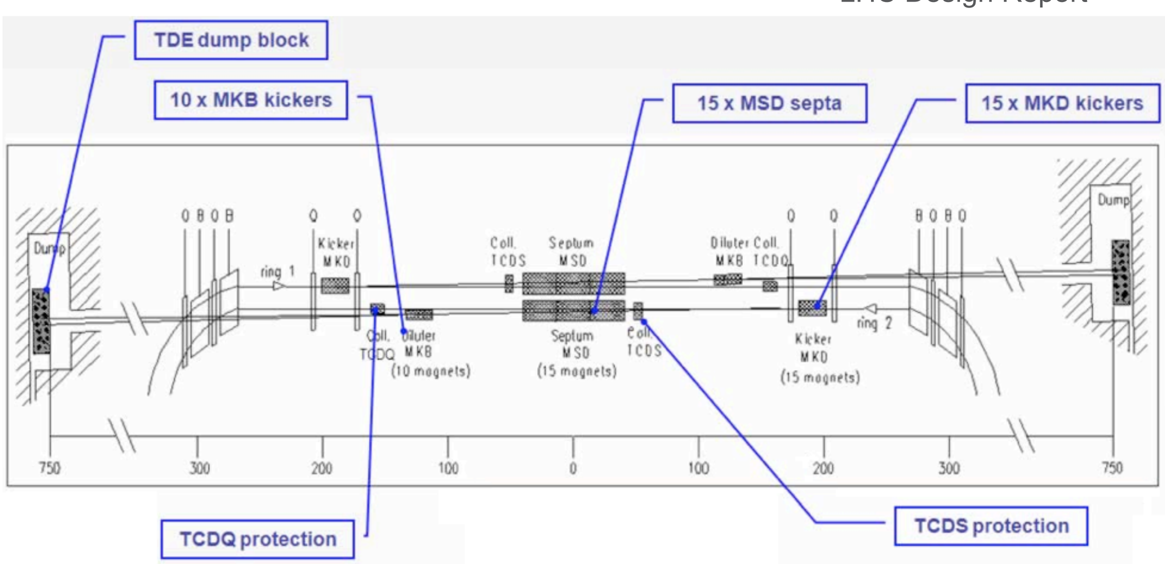
KEKB Abort System

Iida et al.

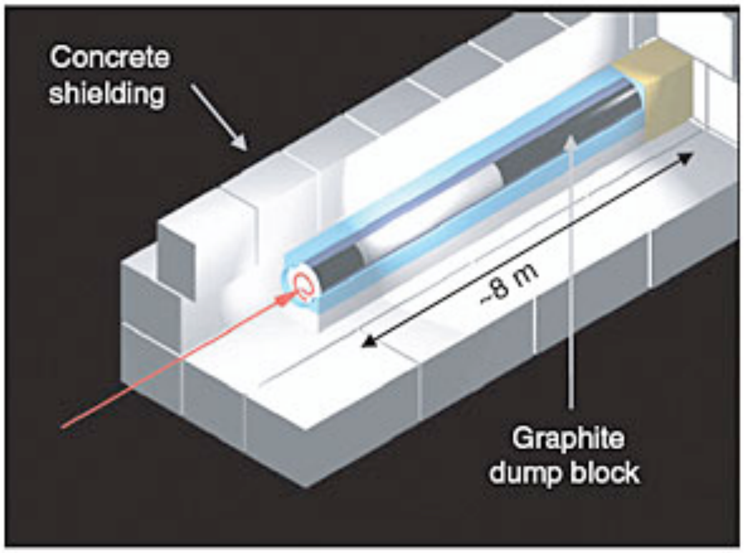


LHC Beam Abort Layout

LHC Design Report

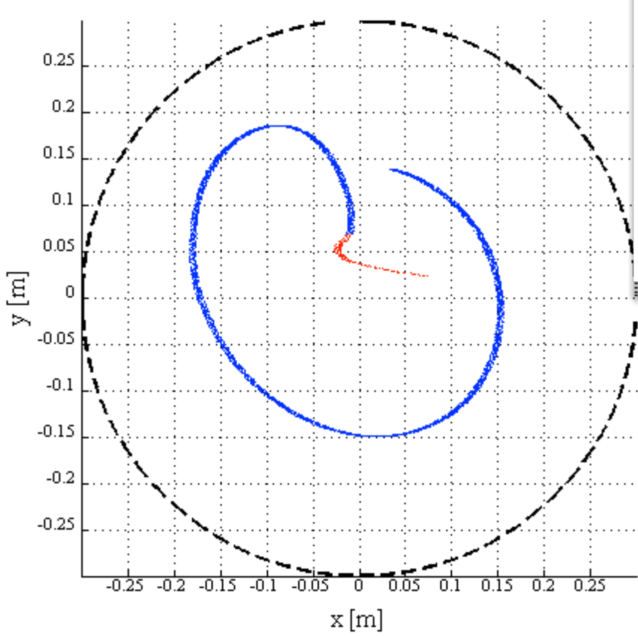


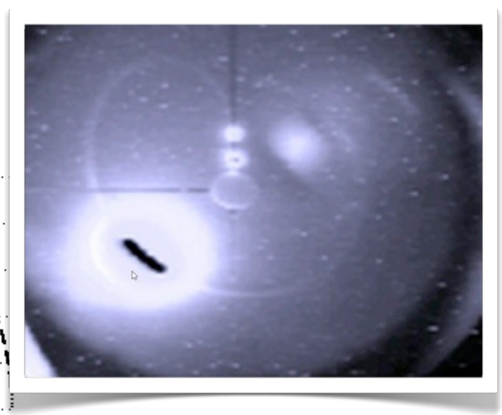
LHC Dump



LHC Beam Abort Spiral

- Beam enlarged to 1.6x1.4 mm^2

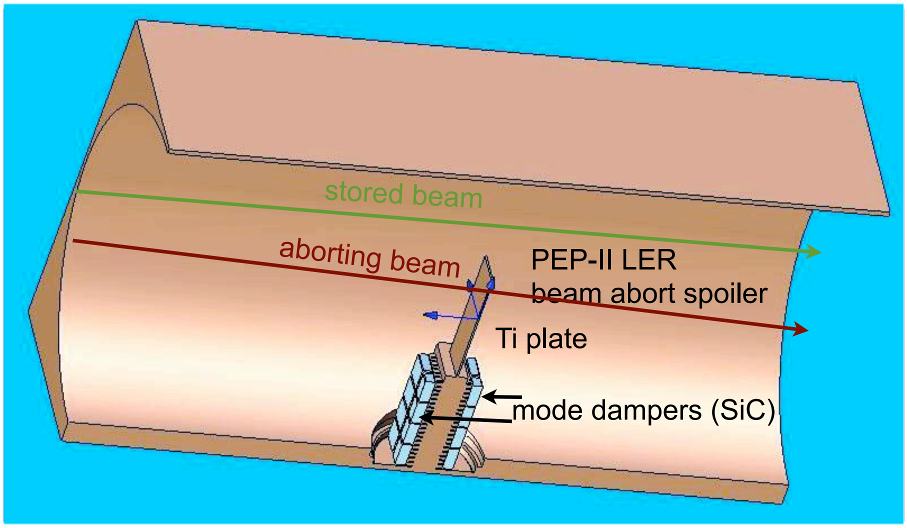




Spoiler

- A plate of suitable material can act as scatterer & dilute the phase-space density of the beam
- protect the exit window in this case (PEP-II LER)

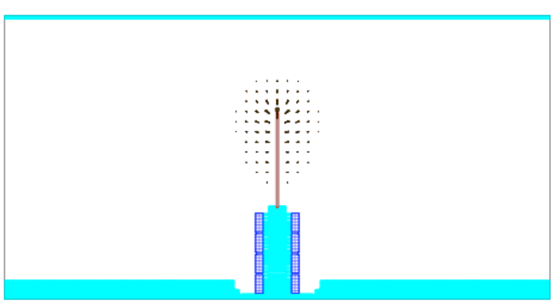
A. Novokhatski

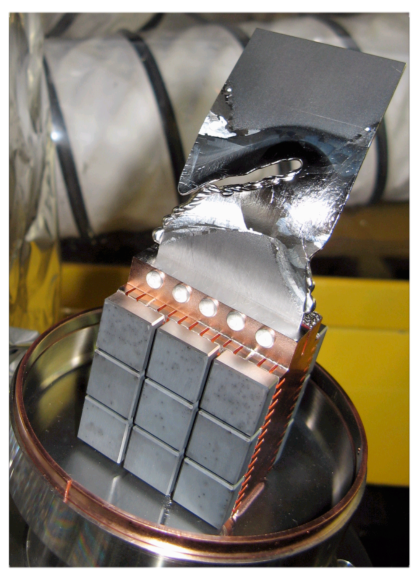


But there are Pitfalls…

A. Novokhatski

- A specific bunch pattern excited a strong e-m mode
  - the damage is *not* from direct beam hit
  - spoiler made nice $\lambda/4$ antenna
  - estimated power into absorber: 400 W

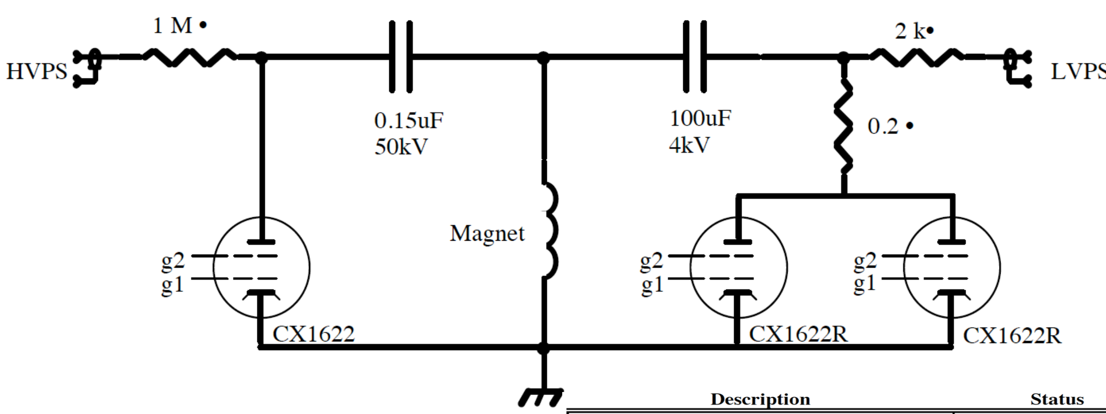


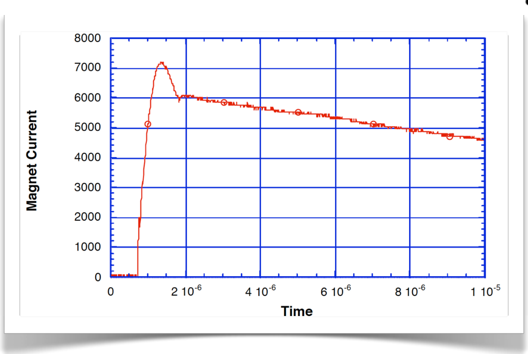


Reliability

- Beam abort systems often need to be relied on to prevent damage to machine components.
- Redundant triggers
- Sufficient stored energy to be able to fire in case of charging supply failure
- self triggering of secondary switches (e.g. to provide the ramp to paint across the dump)
- Redundant systems often firing with a one-turn delay.
- Monitor every parameter important for the system to work. Fire the abort if any parameter goes out of spec.

PEP-II Abort Pulser



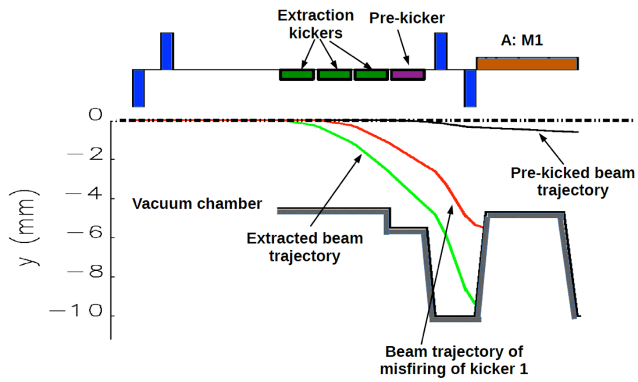


Description	Status
HV Fail High (V > 102%)	Not Ready
HV OK	Trigger Ready
HV Falling (0 > Slope • -1%/ms)	Failing
HV Fail Low (V < 98%)	Not Ready
LV Fail High (V > 102%)	Not Ready
LV OK	Trigger Ready
LV Falling (0 > Slope • -1	Failing
LV Fail Low (V < 98%)	Not Ready
HV Keep Alive Current Hi	Failing
HV Keep Alive Current OK	Trigger Ready
HV Keep Alive Current Low	Failing
LV1 Keep Alive Current High	Failing
LV1 Keep Alive Current OK	Trigger Ready
LV1 Keep Alive Current Low	Failing
LV2 Keep Alive Current High	Failing
LV2 Keep Alive Current OK	Trigger Ready
LV2 Keep Alive Current Low	Failing

Fire while still able to!

APS-U Bunch Swap-Out

- Because of limited machine acceptance, APS-U will inject on-axis
  - implies the bunches get replaced rather than topped-up
- Power density for even one extracted bunch is too high
  - even ^{12}C would reach > 5700 K for one bunch
  - this also prevents spoiler schemes from working
- Solution: pre-kick the beam and let it decohere & grow in size



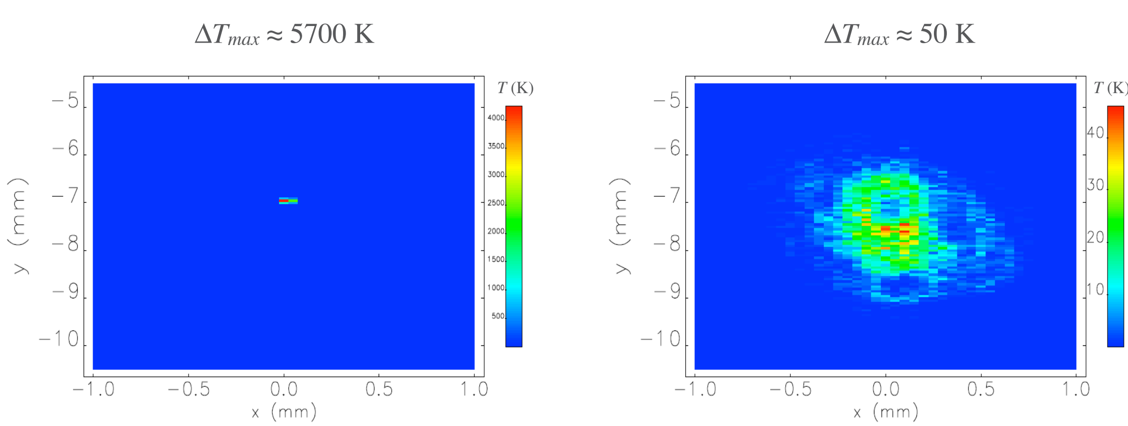
A. Xiao

side view

APS-U Swap-Out Beam Dump

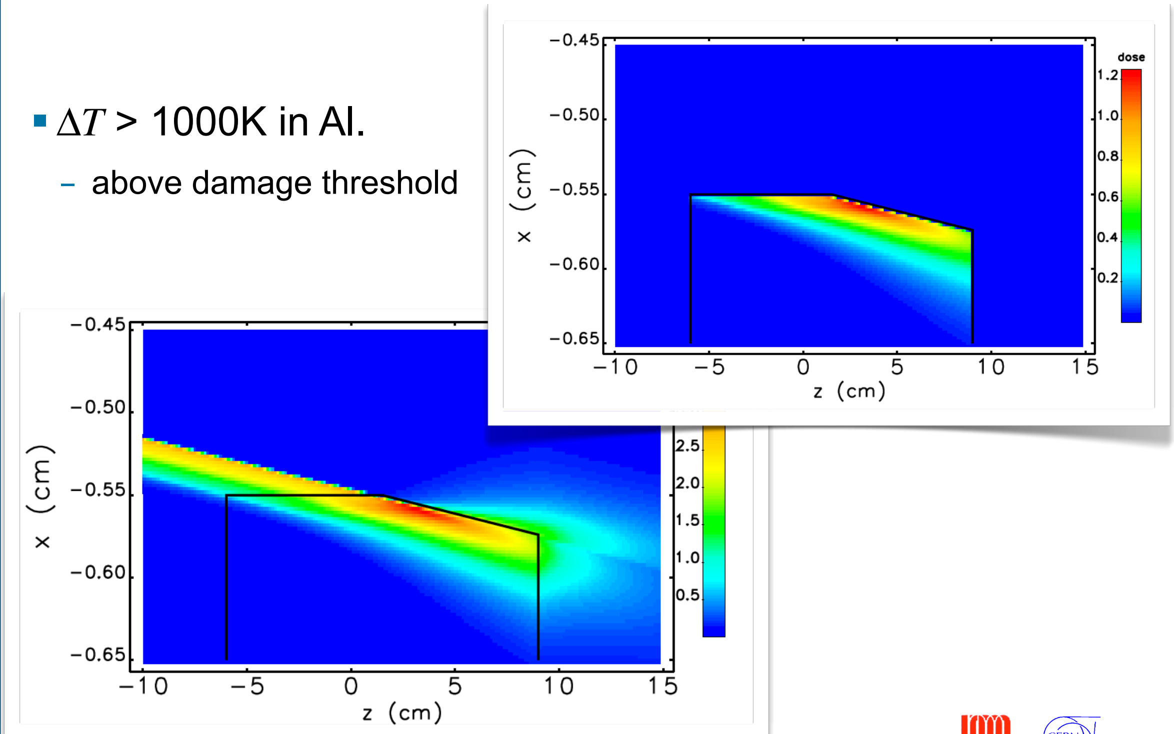
Lack of space forces internal dump => no optics to enlarge beam

J. Dooling



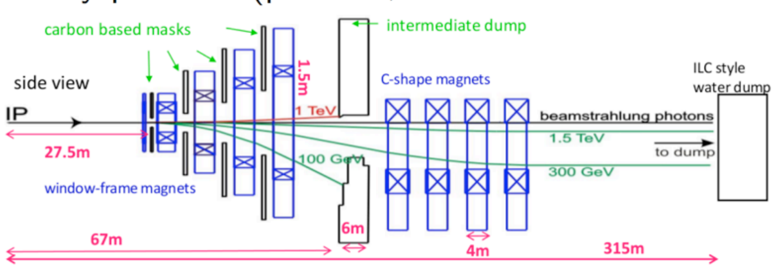
APS-U Abort Dump (Concept)

- ΔT > 1000K in Al.
  - above damage threshold



Linear Collider Beam Disposal

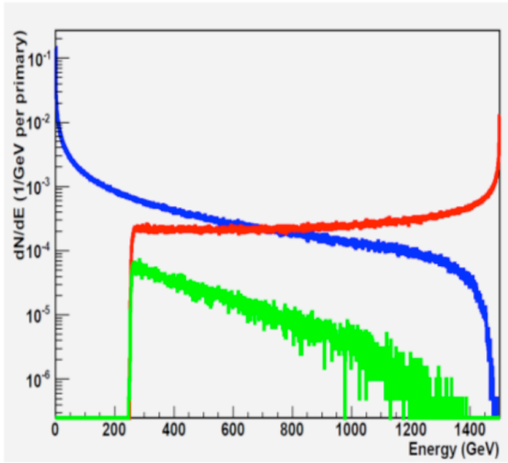
- Although there is not stored energy, there is a large beam power to be extracted/transported/dumped from the IP
- Beams collide at an angle of few tens of *mrad* for safety extraction ($\mathcal{L}_{\rm loss}$ recovered by crab cavities)
- Stay clear of incoming beam
- Need to dump 2 type of beams
  - Collided (large spread in energy and angle) transport challenging
  - Un-collided (same energy but in small spot sizes) dump challenging
- Secondary particles (photons, coherent and in-coherent pairs)



Particle Species

Several beams should be dump, disrupted, photons and pairs

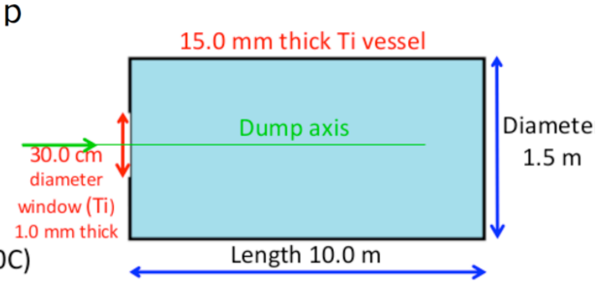
- e^+e^- collision creates **disrupted beam**
  - Huge energy spread, large x,y div in outgoing beam
    → total power of ~10MW
- High power divergent **beamstrahlung photons**
  - 2.2 photons/incoming e+e-
    → 2.5 E12 photons/bunch train
    → total power of ~4MW
- **Coherent** e+e- pairs
  - 5E8 e+e- pairs/bunchX
  → 170kW opposite charge
- Incoherent e+e- pairs
  - 4.4E5 e+e- pairs/bunchX
    → 78 W



Main Dump

Example of the 18 MW main dump

- Cylindrical vessel
- Volume: $18m^3$, Length: 10m
- Diameter of 1.8m
- Water pressure at 10bar (boils at 180C)
- Ti-window, 1mm thick, 30cm diameter

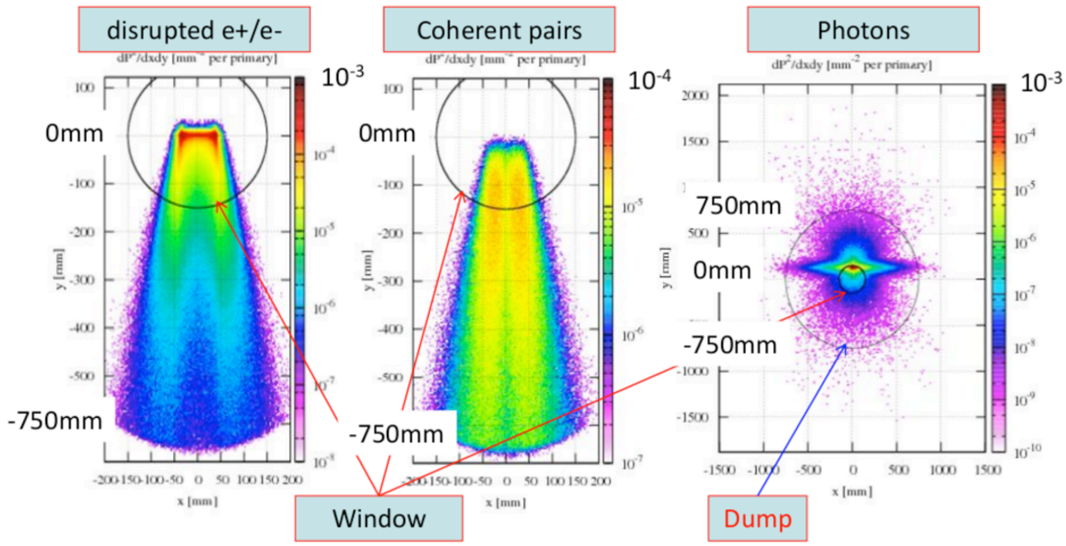


	CLIC	ILC
Beam energy	1500 GeV	250 GeV
# particles per bunch	3.7×10^9	2×10^{10}
# bunches per train	312	2820
Duration of bunch train	156 ns	950 μs
Uncollided beam size at dump σ_x, σ_y	1.56 mm, 2.73 mm	2.42 mm, 0.27 mm
# bunch trains per second	50	5
Beam power	14 MW	18 MW

Dump Window

Mechanical design very challenging:

- water pressure (10 bar)
- pressure wave from instantaneous heat deposition
- beams (disrupted, photons, pairs) power (ILC: 25 W, 21 J/cm^3)



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Diagnostics

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Thursday, June 22^{nd}
Summer 2017 USPAS
Course: Injection and Extraction of Beams

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Outline

Diagnostics

Motivation

User needs...

- Final users of the beam always pushing machine performance
- High-quality, long term stability and flexibility

So the Accelerator Physicist requires...

- Instrumentation to diagnose the beam
  - It depends on particle beam
  - Energy
  - Single or multi-pass
- Fast and non-destructive (beam and instrument) methods are preferred

Most common measurements are:

- Current
- Beam loss
- Profile
- **Transverse emittance**
- Pick-ups
- Longitudinal parameters

Emittance

Diagnostics Emittance Emittance Measurement ATF2 Example

Emittance

Recap from Monday lecture...

Emittance (ϵ) is related to the area (a) occupied by the beam in phase space as:

$$\epsilon = a^2\pi$$

Slide-14: Σ matrix was defined as,

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \epsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix} \Rightarrow det|\Sigma| = \epsilon \qquad (1)$$

it is a function of s

It can be identified,

$$\sigma_{11} = \overline{x^2} \qquad \sigma_{12} = \sigma_{21} = \overline{x \cdot x'} \qquad \sigma_{22} = \overline{x'^2} \qquad (2)$$

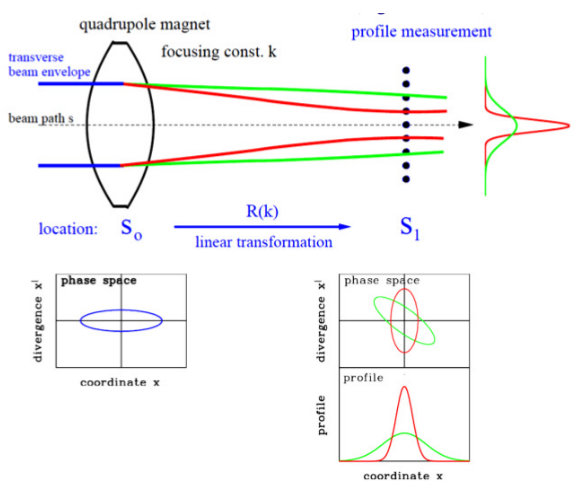
Emittance Measurement

- Quadrupole Scan (single monitor)
- Multi-position measurement

Quadrupole Scan Emittance Measurement

Quadrupole Scan

From a profile determination, the emittance can be calculated via linear transformation, if a well known and constant distribution is assumed



Quadrupole is scanned for different values of k

Beam width σ is changed according to the focusing strength of the quad

R is the transport matrix from s_0 to s_1 $\Rightarrow R(k)$

ϵ_0 is obtained from the different σ^i measurements

Emittance

The beam width (x_{rms}) is measured at s_1, thus $\sigma_{11} = x_{rms}^2$
Emittance (ϵ) is related to the area (a) occupied by the beam in phase space as:

$$\epsilon = a^2\pi$$

Slide-14: Σ matrix was defined as,

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \epsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix} \Rightarrow det(\Sigma) = \epsilon \quad (3)$$

it is a function of s

It can be identified,

$$\sigma_{11} = \overline{x^2} \qquad \sigma_{12} = \sigma_{21} = \overline{x \cdot x'} \qquad \sigma_{22} = \overline{x'^2} \quad (4)$$

Emittance Measurement

The beam width (x_{rms}) is measured at s_1, thus $\sigma_{11} = x_{rms}^2$
Different values of quadrupole strength are sampled
$k_1,\ k_2,\ k_3,\ k_4$... so the transfer matrix from s_0 to s_1 is,

$$R(k_i) = R_{\mathrm{drift}} \cdot R_{\mathrm{quad}}(k) \tag{5}$$

The Σ matrix transforms as,

$$\Sigma_{s_1} = R(k_1) \cdot \Sigma_{s_0} \cdot R^T(k_1) \tag{6}$$

We can construct a system of equations for all k_n values as

$$\sigma_{11}^{s_1}(k_1) = R_{11}^2(k_1) \cdot \sigma_{11}^{s_0} + 2R_{11}(k_1)R_{12}(k_1) \cdot \sigma_{12}^{s_0} + R_{12}^2(k_1) \cdot \sigma_{22}^{s_0} \tag{7}$$

$$\vdots$$

$$\sigma_{11}^{s_1}(k_n) = R_{11}^2(k_n) \cdot \sigma_{11}^{s_0} + 2R_{11}(k_n)R_{12}(k_n) \cdot \sigma_{12}^{s_0} + R_{12}^2(k_n) \cdot \sigma_{22}^{s_0} \tag{8}$$

Diagnostics Emittance Emittance Measurement ATF2 Example

Emittance Measurement

More than 3 values of k_n are needed if we want to estimate the error of our calculation
$R(k_n)$ can be obtained using thin-lens approximation,

$$R_{\mathrm{quad}}(k_n) = \begin{bmatrix} 1 & 0 \\ k_n & 0 \end{bmatrix} \tag{9}$$

$$\Downarrow$$

$$R(k_n) = R_{\mathrm{drift}} \cdot R_{\mathrm{quad}}(k_n) = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ k_n & 1 \end{bmatrix} = \begin{bmatrix} 1 + k_n L & L \\ k_n & 1 \end{bmatrix} \tag{10}$$

The Σ matrix transforms as Eq. (6) thus

$$\sigma_{11}^{s_1} = R_{11}(k_n)(\sigma_{11}^{s_0} R_{11}(k_n) + \sigma_{12}^{s_0} R_{12}(k_n)) + R_{12}(k_n)(\sigma_{21}^{s_0} R_{11}(k_n) + \sigma_{22}^{s_0} R_{12}(k_n)) \tag{11}$$

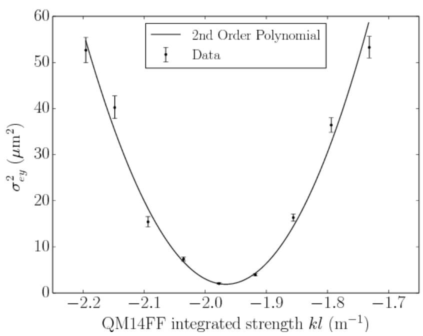
Emittance Measurement

Substituting $R_{ij}(k_n)$ components of Eq. (10) into Eq. (11) and collecting terms in power of k,

$$\sigma_{11}^{s_1}(k_n) = \sigma_{11}^{s_0} L^2 \cdot k_n^2 + (2L\sigma_{11}^{s_0} + 2L^2 2L\sigma_{12}^{s_0}) \cdot k_n + L^2\sigma_{22}^{s_0} + 2L\sigma_{12}^{s_0} + \sigma_{11}^{s_0} \quad (12)$$

Our fitting function:
(parabola opening upwards, symmetry axis x=b and vertex (b,c)),

$$\sigma_{11}^{s_1}(k_n) = a(k_n - b)^2 + c = ak_n^2 - 2abk_n + (c + ab^2) \quad (13)$$



The Σ at s_0 is,

$$\sigma_{11}^{s_0} = \frac{a}{L^2} \quad (14)$$

$$\sigma_{12}^{s_0} = -\frac{a}{L^2}\left(\frac{1}{L} + b\right) \quad (15)$$

$$\sigma_{22}^{s_0} = \frac{1}{L^2}\left(ab^2 + c + \frac{2ab}{L} + \frac{a}{L^2}\right) \quad (16)$$

Diagnostics Emittance Emittance Measurement ATF2 Example

Emittance Measurement

ϵ is finally obtained by substituting Eq (16) into Eq. (3),

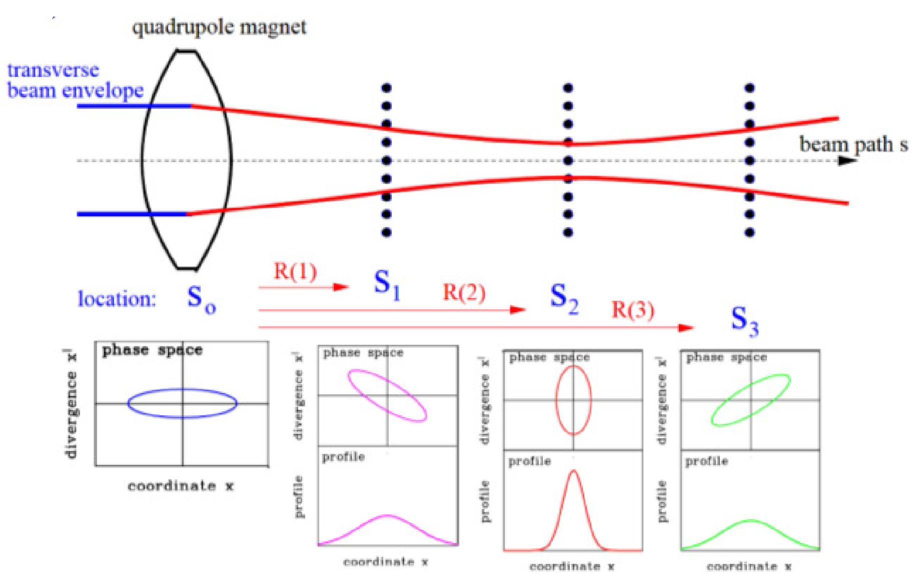
$$\epsilon^{s_0} = \sqrt{\sigma_{11}^{s_0}\sigma_{22}^{s_0} - \sigma_{12}^{s_0}\sigma_{12}^{s_0}} = \sqrt{\frac{ac}{L}} \quad (17)$$

Exercise:
Express the values of β^{s_0}, α^{s_0} and γ^{s_0} in terms of a, b and c?
How to distinguish between emittance growth and Twiss miss-match?

Multipole Locations
Emittance Measurement

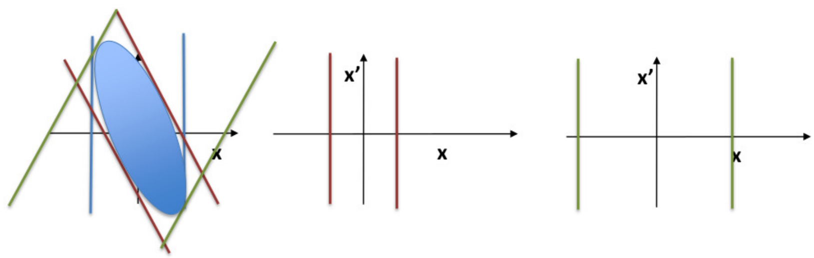
Multiple locations

An alternative method is to use multiple locations (s_1, s_2 and s_3)
Minimum number of monitors 3, for an estimation of the measurement error more monitors are required



Multiple locations

- Usually 3 profile measurements around a waist
- Spot width corresponds to vertical lines in phase-space at each location
- Vertical lines become tangents to the beam ellipse at initial location s_0



Strategy is the same as Quadrupole Scan, difference is that $Rs_0 \rightarrow s_i$ are assumed to be known

Emittance

The beam width (x_{rms}) is measured at $s_1,\ldots,\, s_n$, thus $\sigma_{11}^{s_1} = (x_{rms}(s_i))^2$
We can construct a system of equations for all locations as

$$\sigma_{11}^{s_1}(s_i) = R_{11}^2(s_i) \cdot \sigma_{11}^{s_0} + 2R_{11}(s_1)R_{12}(s_i) \cdot \sigma_{12}^{s_0} + R_{12}^2(s_i) \cdot \sigma_{22}^{s_0} \quad (18)$$

If we define

$$M_x = \begin{bmatrix} R_{11}^2(s_1) & 2R_{11}(s_1)R_{12}(s_1) & R_{12}^2(s_1) \\ \ldots & \ldots & \ldots \\ R_{11}^2(s_n) & 2R_{11}(s_n)R_{12}(s_n) & R_{12}^2(s_n) \end{bmatrix} \quad (19)$$

Then

$$\begin{bmatrix} \sigma_{11}^{s_1} \\ \ldots \\ \sigma_{11}^{s_n} \end{bmatrix} = M_x \begin{bmatrix} \sigma_{11}^{s_0} \\ \sigma_{12}^{s_0} \\ \sigma_{22}^{s_0} \end{bmatrix} \quad (20)$$

Emittance Measurement

The problem then reduces to a set of 3 uncoupled systems of equations for x, y and $x-y$ planes,

$$\begin{bmatrix}\sigma_{33}^{s_1}\\ \dots\\ \sigma_{33}^{s_n}\end{bmatrix} = M_y \begin{bmatrix}\sigma_{33}^{s_0}\\ \sigma_{34}^{s_0}\\ \sigma_{44}^{s_0}\end{bmatrix} \quad (21) \qquad \begin{bmatrix}\sigma_{13}^{s_1}\\ \dots\\ \sigma_{13}^{s_n}\end{bmatrix} = M_{xy} \begin{bmatrix}\sigma_{13}^{s_0}\\ \sigma_{14}^{s_0}\\ \sigma_{23}^{s_0}\\ \sigma_{24}^{s_0}\end{bmatrix} \quad (22)$$

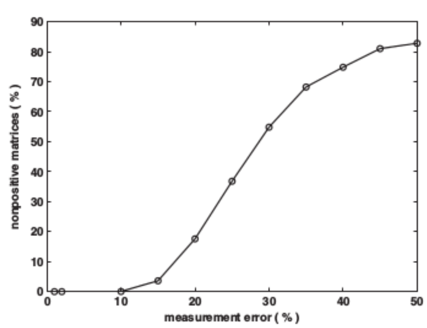
$$M_y = \begin{bmatrix} R_{33}^2(s_1) & 2R_{33}(s_1)R_{34}(s_1) & R_{34}^2(s_1)\\ \dots & \dots & \dots\\ R_{33}^2(s_n) & 2R_{33}(s_n)R_{34}(s_n) & R_{34}^2(s_n)\end{bmatrix} \quad (23)$$

$$M_{xy} = \begin{bmatrix} R_{11}(s_1)R_{33}(s_1) & R_{11}(s_1)R_{34}(s_1) & R_{12}(s_1)R_{33}(s_1) & R_{12}(s_1)R_{34}(s_1)\\ \dots & \dots & \dots & \dots\\ R_{11}(s_n)R_{33}(s_n) & R_{11}(s_n)R_{34}(s_n) & R_{12}(s_n)R_{33}(s_n) & R_{12}(s_n)R_{34}(s_n)\end{bmatrix} \quad (24)$$

Eqs (20), (21) and (22) are solved separately by a least-squares fit

Optimum Diagnostics

The solution found by the least-squares fit may be non-physical (negative beam size at s_0) due to the noisy measurements



Example of dependency of non positive solutions on the relative measurement error

The diagnostics section can be designed to minimize the number of non-physical solutions

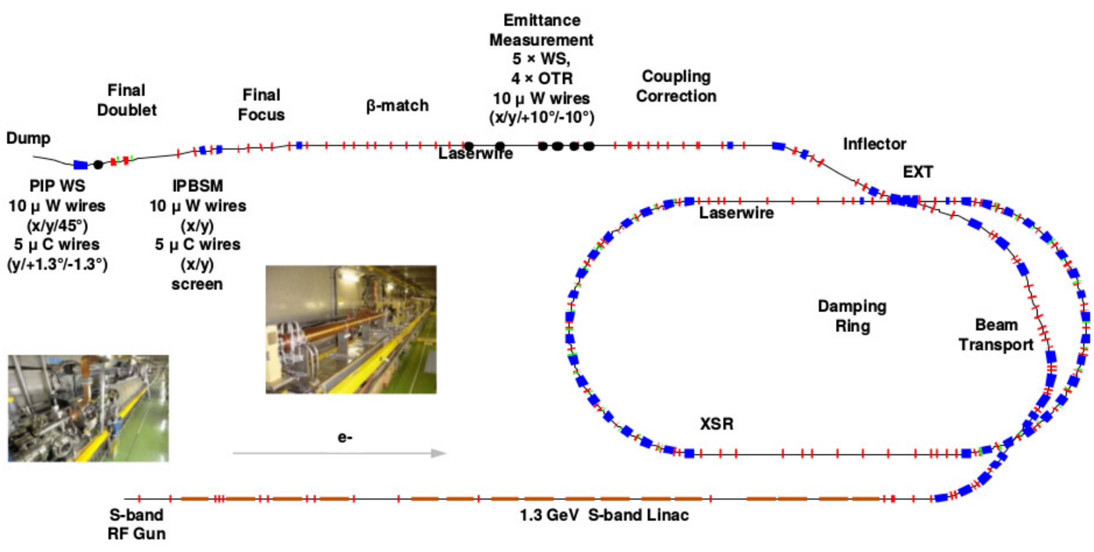
In [1] is found that the optimum phase advanced between measurement locations is

$$\Delta\mu = \frac{180^\circ}{n} \quad (25)$$

where n is the number of locations

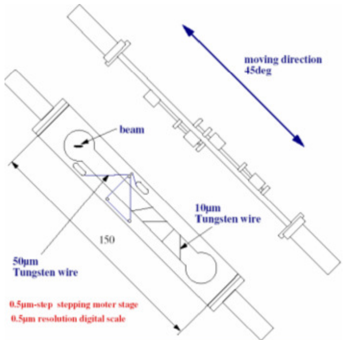
ATF2 Example

Beam Profile Measurement

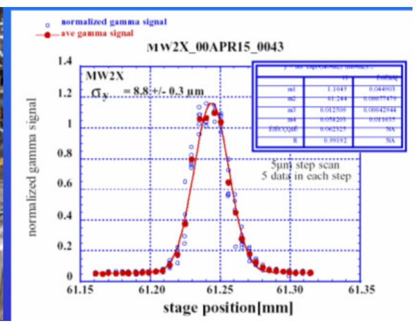


Wires Scan

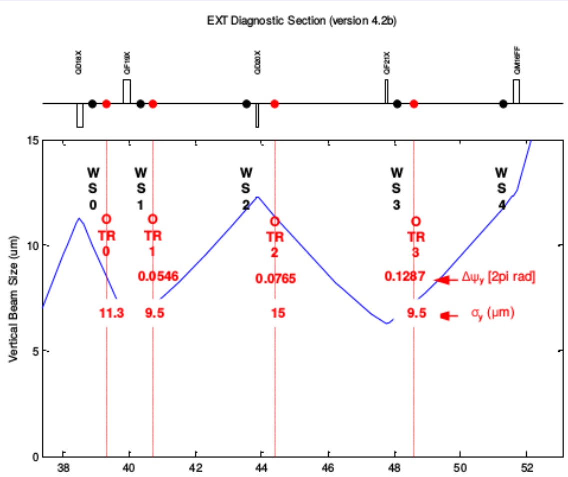
Change in voltage on wire induced by secondary emission of γ detected by Cerenkov



thin W wires and 5 μm precision stepper-motors (courtesy H. Hayano, 2003)

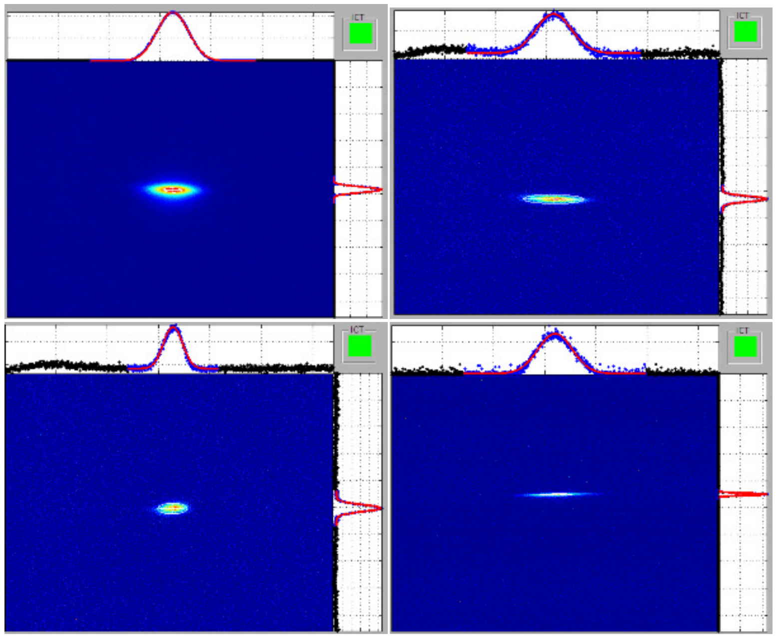


Optical Transition Radiation



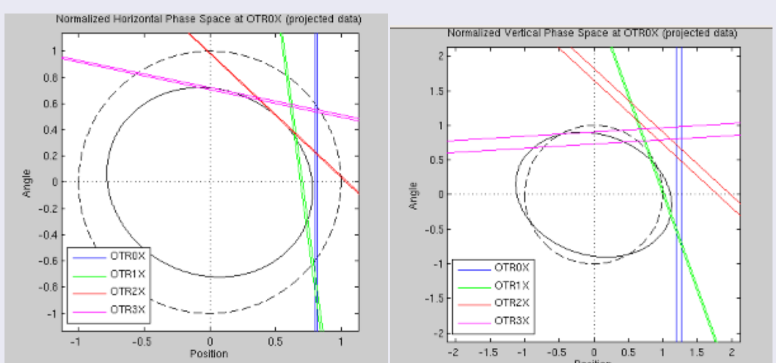
Targets are made with aluminium (2um) or aluminized kapton (3-5um with 1200 Amstrongs Al coating)

OTRs Measurements



Example of Emittance Measurement @ ATF2

After applying the tuning procedure (camera tilt, dispersion, coupling):

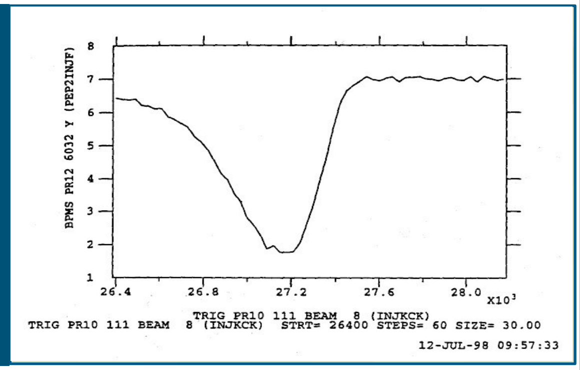


ϵ_x=**1.1nm (Bmag=1.00)** ϵ_y=**12pm (Bmag=1.0)**

REFERENCES

[1] I. AGAPOV, G. A. BLAIR, AND M. WOODLEY, "Beam emittance measurement with laser wire scanners in the International Linear Collider beam delivery system", Phys. Rev. ST Accel. Beams 10, 112801 (2007)

Diagnostics



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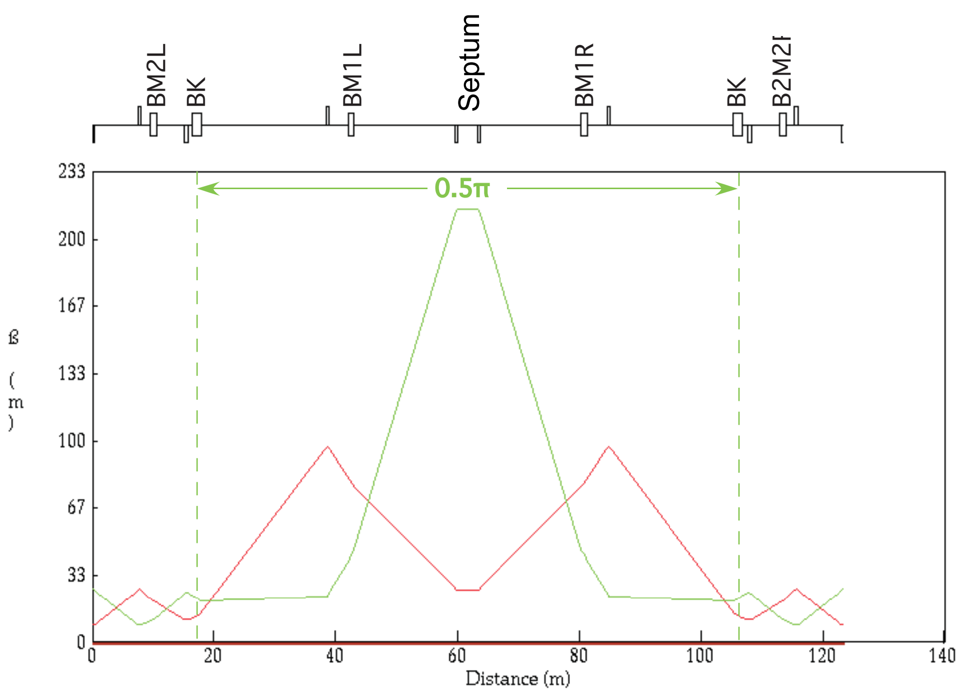
Diagnostics of Injection Performance

- Kicker timing & bump closure
- First-turn steering & matching
- Energy and bunch-bucket timing match
- Setup of off-axis injection
- Quadrupole matching

Injection Tuning

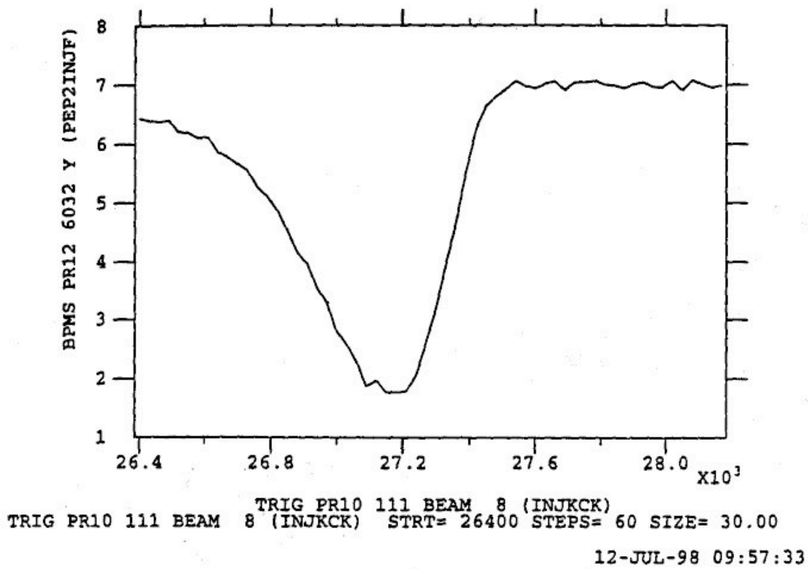
- Typically, injection setup follows a straight-forward strategy
  - Put the incoming beam onto its design trajectory.
  - Make sure the kicker(s) are timed correctly wrt. the injecting beam
  - Put the injecting beam on-axis using kicker(s) and bumps
  - If needed, use orbit correctors just upstream of the injection to make the turn-2 orbit like the turn-1 orbit. The injected bunch should now store.
  - With rf on, analyse the motion of the injecting beam for synchrotron oscillations.
    - these indicate either a phase or an energy offset.
    - reduce by adjusting either incoming beam or the ring parameters (rf phase, energy).
  - For off-axis injection, collapse the orbit bump until the desired injection orbit (1st turn) is reached; or until injection efficiency is optimized.

PEP-II Injection Straight



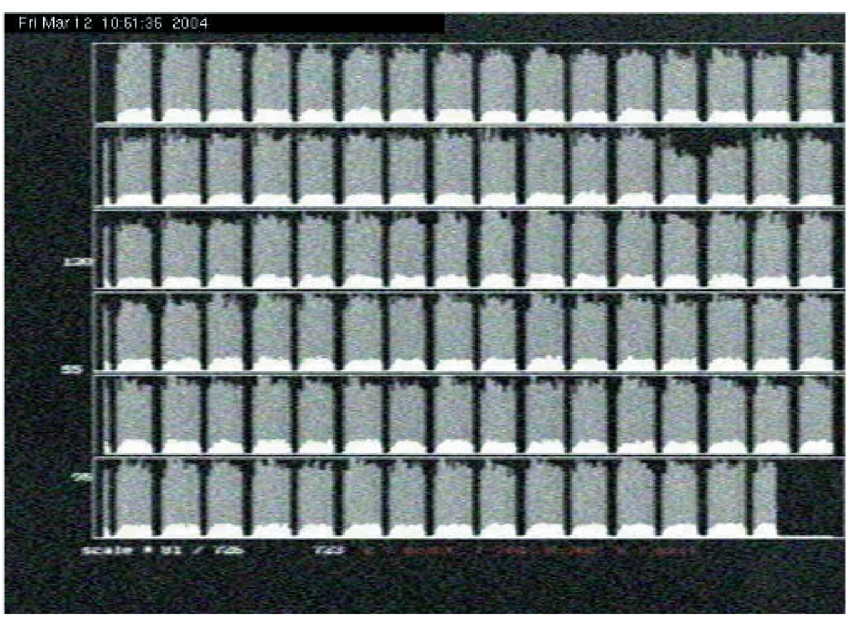
Tuning Aids

- Kicker Timing Curve using triggered BPM (1 bunch)



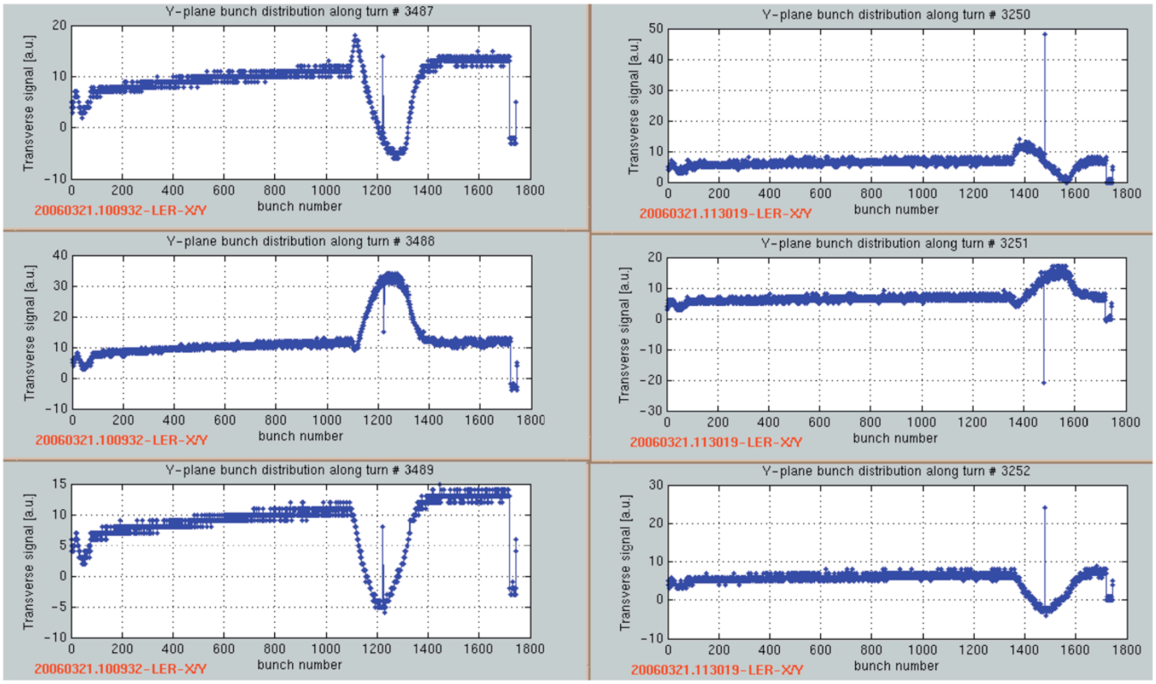
Mismatched Kickers

- Dips in luminosity caused by kicker mismatch



Kicker Matching (Bump Closure)

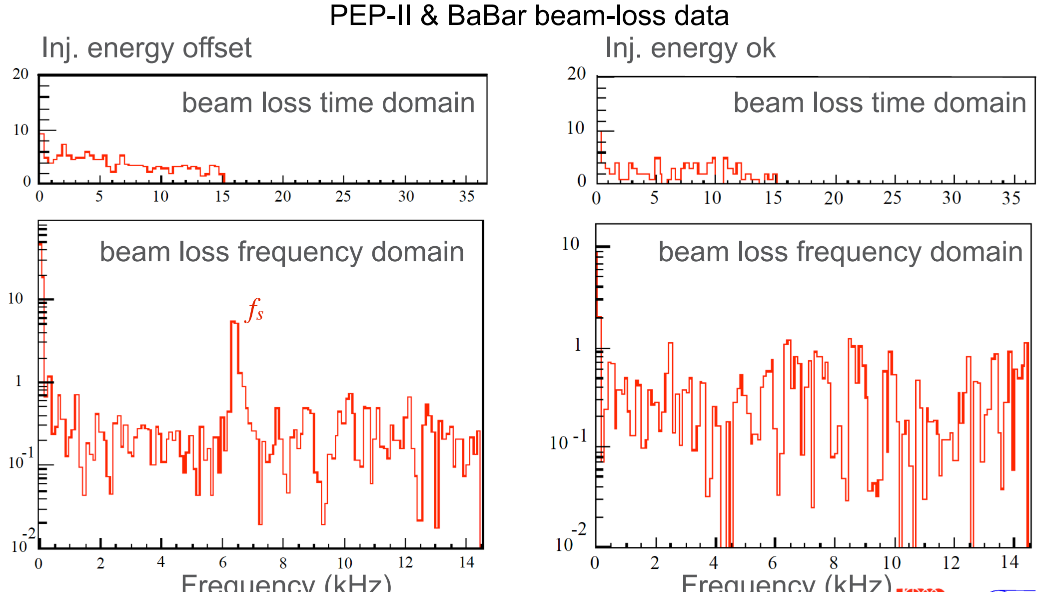
- Bunch-by-bunch BPM, triggerable



Fourier Transform

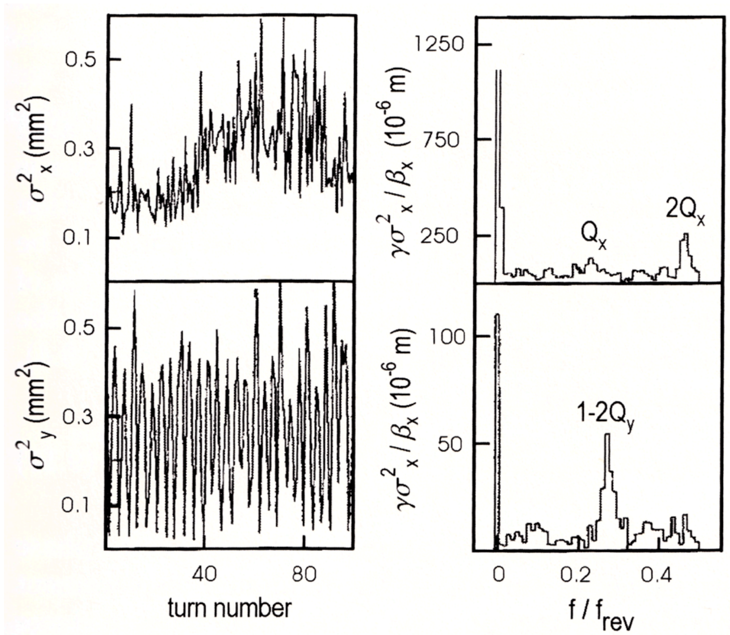
- A powerful way of diagnosing injection trouble is to use FFT of either BPM signals or of beam-loss signals.

PEP-II & BaBar beam-loss data



Transverse Mismatch Diagnostics

- SLC Damping Ring Gated Camera data (Minty et al.)



References

- F. Zimmermann, M. Minty, "Measurement and Control of Charged Particle Beams", Springer, 2003.

Exercise:
Design Extraction Line from ATF Damping Ring

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(Dated: June 16, 2017)

Abstract

The goal of this exercise is to design an extraction line (EXT) for the 1.3 GeV electron damping ring (DR). Taking into consideration the machine constraints and hardware limitations. The extraction line should preserve the emittance delivered by the DR. To this end an emittance measurement section should be present in the extraction line.

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I. INTRODUCTION

The accelerator and particle physics communities are considering a lepton linear collider as the most appropriate machine to carry out high precision particle physics research in the high energy regime. There exist two proposals for the next generation of e^+-e^- linear collider (LC), the International Linear Collider (ILC) [1], [2] and [3] and the Compact Linear Collider (CLIC) [4], [5] and [6]. In order to reach the required luminosity ($\mathcal{L}$) for the experiments, the vertical spot size at the IP (σ_y^*) is of the order of a few nanometers. The final focus systems (FFS) of both LC projects reduce the $\beta_{x,y}$ functions $\leq 100\mu m$. Although this strong focusing is quite challenging, it is equally important to inject a beam with extremely small vertical emittance $\epsilon_y \approx pm$. To this end, the Accelerator Test Facility (ATF), was designed to experimentally verify the generation of such a small emittance.

II. DESCRIPTION OF ATF

The ATF damping ring receives a 1.3 GeV electron beam from the ATF linac. There is a common point in the ring for injection and extraction of the beam. The extraction beam line extends over about 52 m and delivers the beam to the final focus system (ATF2

TABLE I. Comparison between relevant parameters of different final focus systems.

Project	Status	Beam Energy [GeV]	$\gamma\epsilon_y$ [nm]	σ_y^* [nm]	β_y^* [mm]	L* [m]	ξ_y []
FFTB	Designed	46.6	2000	52	0.1	0.4	4000
FFTB	Measured	46.6	2000	70	-	0.4	-
ATF2 Nominal	Designed	1.3	30	37	0.1	1.0	10000
ATF2 Nominal	Measured	1.3	30	65[a]	0.1	1.0	10000
ATF2 Ultra-low β^*	Proposed	1.3	30	23	0.025	1.0	40000
CLIC L*= 3.5 m	Designed	1500	20	1	0.069	3.5	50000
ILC	Designed	250	35	5.9	0.48	3.5	7500

[a] This value is considered as an upper limit of the actual beam size due to relative phase jitter between the laser fringe pattern and the e^- beam, see more details in [**?**].

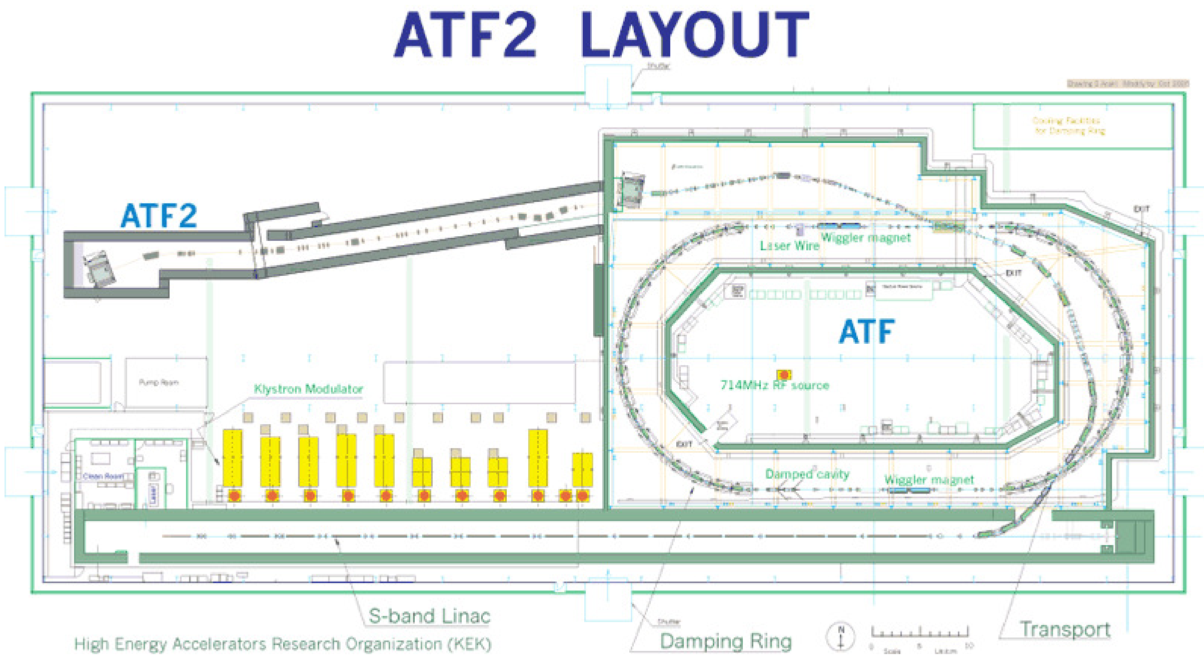


FIG. 1. Scheme of the ATF facility.

beamline). Figure 1 shows a layout of the ATF and ATF2 facility.

Figure 2 shows a layout of the ATF2 (Extraction and Final Focus System) beam line.

The ATF2 beam line is divided into two sections, the extraction beam line (EXT) and the final focus system. The EXT extends over 52 m, it comprises an extraction and diagnostic sections. The diagnostic section is used for measuring the emittance and the Twiss parameters and for correcting the dispersion and transverse coupling of the electron beam. The ATF2 FFS beam line extends over 40 m, it is responsible for transporting and vertically focusing the beam at the IP to tens of nanometres.

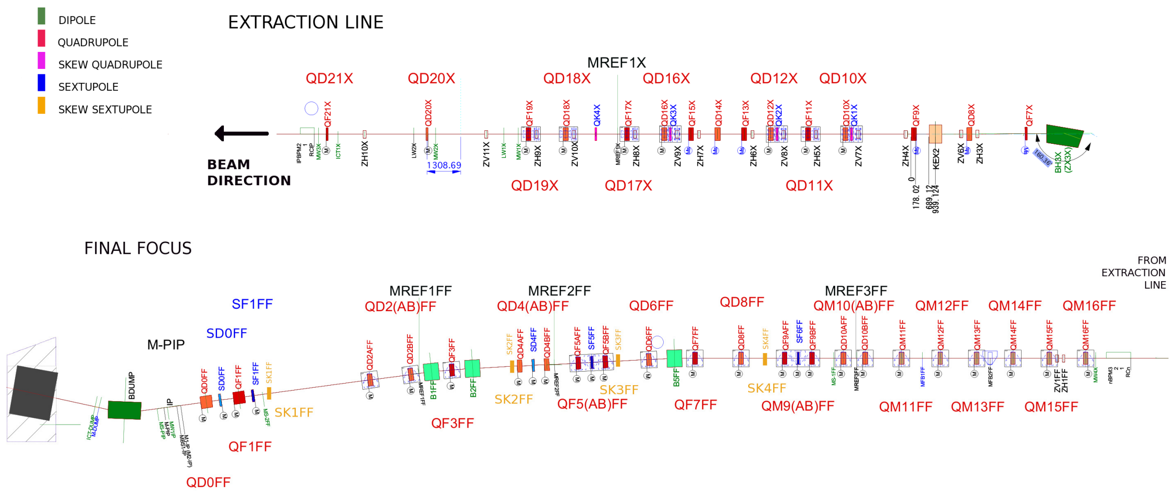


FIG. 2. Scheme of the ATF2. The beam line on the left represents the extraction beam line (EXT). The beam line on the right represents the FFS as the continuation of the EXT line (Figure courtesy of S. Araki).

III. ATF PARAMETERS

Damping Ring

Figure 3 shows a close-up at the extraction point (label as **kicker 1**) of the ATF ring.

Septum is divided in 3 sector bend magnets BS1X, BS2X and BS3X to complete the extraction. The distances between BS1X and BS2X and between BS2X and BS3X are the same and equal to 0.2 m.

IV. EXERCISE OBJECTIVES

This exercise is composed on the following 6 steps:

- First thing is to decide which extraction scheme is best suited for the ATF damping ring. To this end we need to know the available space at the extraction location, and physical constraints for our equipment.
- Decide kicker requirements according to the necessary horizontal offset at the septum for safety extraction.

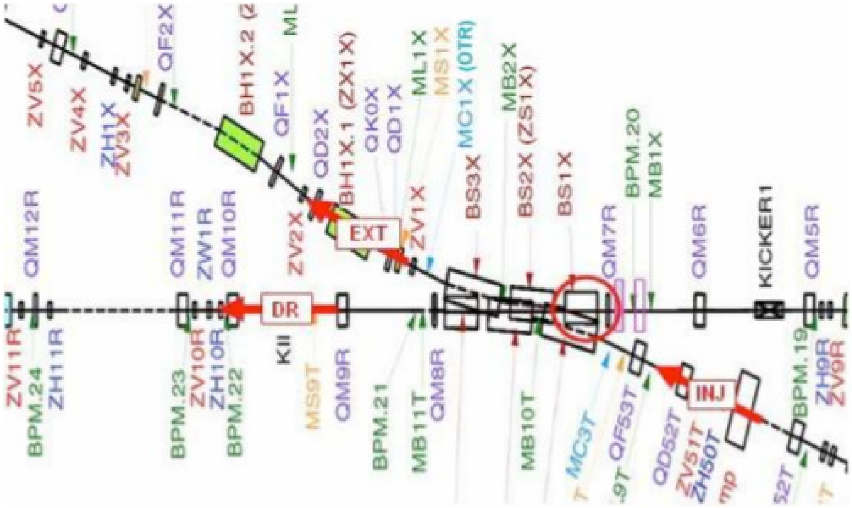


FIG. 3. Extraction configuration for ATF.

- Since we need to transport the beam to the FFS beamline located parallel to the side of the DR where the extraction takes place, decide which insertion object is best to do the job.

TABLE II. Damping Ring Parameters at extraction point **kicker 1**.

Energy	[GeV]	1.3
$\delta p/p$	[%]	0.08
$\gamma\epsilon_x$	[μm]	5.06
$\gamma\epsilon_y$	[nm]	30
β_x	[m]	6.54
β_y	[m]	3.23
α_x	[]	0.99
α_y	[]	-1.92
η_x	[m]	10^{-6}
η'_x	[]	10^{-6}
η_y	[m]	10^{-6}
η'_y	[]	10^{-6}

Once you have the sketch of the extraction line, match the Twiss at the extraction point to the Twiss at the exit of the entrance of the FFS.

- Designed an emittance measurement section that consists of 4 beam size measurements, preserving the match conditions from previous step. Decide on cell structure, phase advanced in both planes.
- Obtain plots for the survey coordinates, Twiss functions ($\beta_{x,y}$) and dispersion ($\eta_{x,y}$). Plot the aperture and beam size ($\sigma_{x,y}$) along the extraction line.
- Determine what is the jitter tolerance to preserve the emittance delivered by the DR. How would you relax this tolerance?

V. SOLUTION

A. Step 1

kicker + plus offset quadrupoles due to space constraints from the DR

B. Step 2

kicker length 0.5 m. Angle equals to 0.005. Beam offset at QM6R and QM7R is 0.65 cm and 2.25 cm respectively.

C. Step 3

Dogleg section.

TABLE III. Damping Ring Parameters at extraction point **kicker 1**.

Magnet	Length [m]	Angle [rad]
BS1X	0.6	0.028
BS2X	0.8	0.072
BS3X	1.0	0.234

D. Step 4

Match command MAD-X

E. Step 5

Figure 4 shows the survey coordinates along the ATF2.

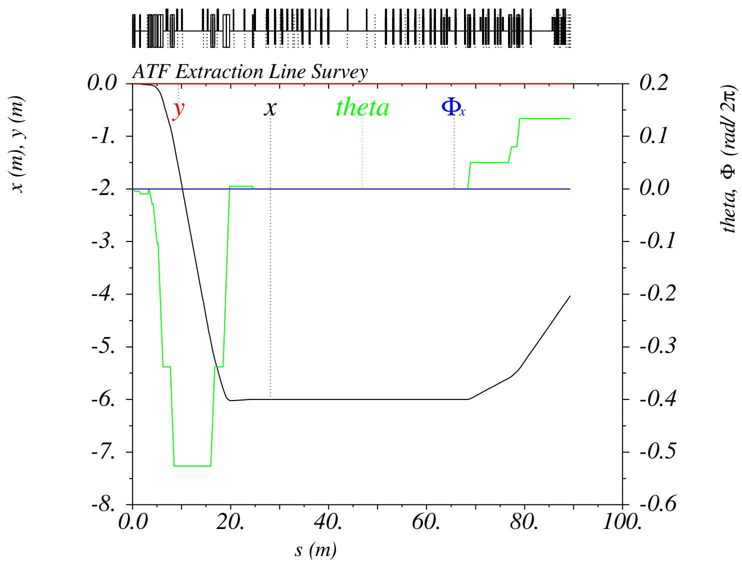


FIG. 4. Survey coordinates of the ATF2 beam line.

Figure 5 shows the $\beta_{x,y}$ and η_x-functions along the extraction and final focus system beam line.

Figure 6 shows the aperture and $10{\cdot}\sigma_{x,y}$ along the ATF2 beam line.

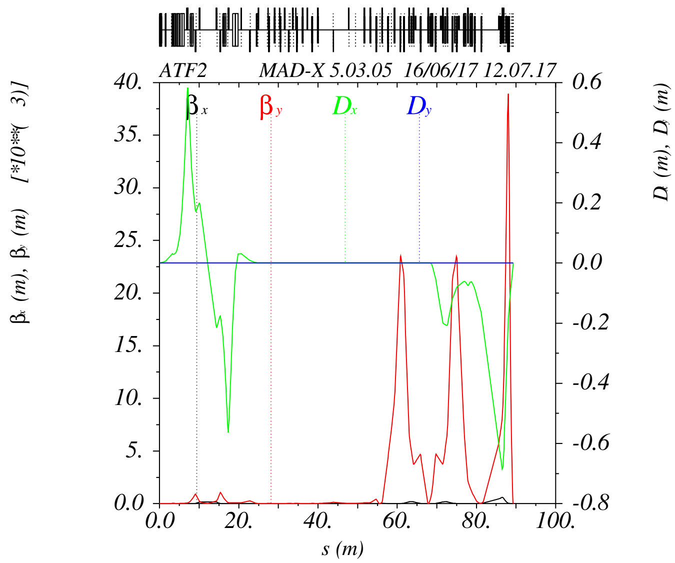


FIG. 5. The $\beta_{x,y}$-functions and the η_x-function for the ATF2 nominal lattice throughout the ATF2 beam line.

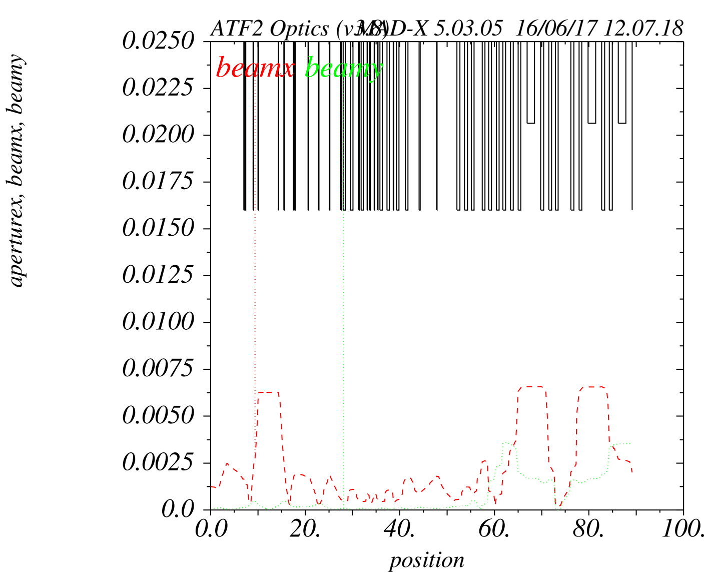


FIG. 6. Machine aperture and beam size throughout the ATF2 beam line.

F. Step 6

MAD-X script

[1] Phinney, Nan and Toge, Nobukasu and Walker, Nicholas, "ILC Reference Design Report Volume 3 - Accelerator", physics.acc-ph, arXiv:0712.2361 (2007).

[2] B. Barish for the Global Design Effort and S. Yamada for the Research Directorate, "The International Linear Collider Technical Design Report", CERN-ATS-2013-037, (2013).

[3] "ILC Home page", `http://www.linearcollider.org/ILC/What-is-the-ILC/The-project.`"

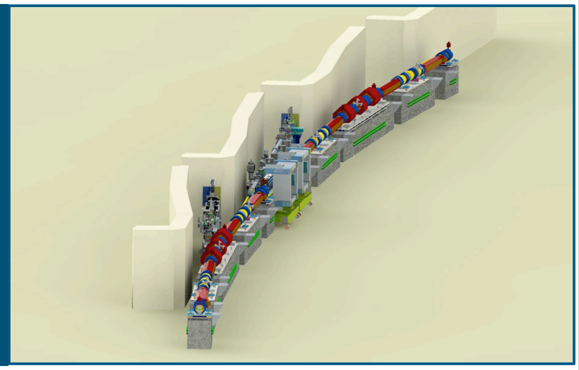
[4] P. Lebrun, L. Linssen, A. Lucaci-Timoce, D. Schulte, F. Simon, S. Stapnes, N. Toge, H. Weerts, J. Wells, "The CLIC Programme: towards a staged e+ e- Linear Collider exploring the Terascale, CLIC Conceptual Design Report", CERN-2012-005, (2012).

[5] "R. Tomás, "Overview of the Compact Linear Collider, Phys. Rev. ST Accel. Beams, volume 13 - 1 (2010).

[6] "CLIC Home page, `http://clic-study.org.`"

[7] Raimondi, Pantaleo and Seryi, Andrei, "Novel Final Focus Design for Future Linear Colliders", Phys. Rev. Lett., volume 86- 17, (2001)

The APS-U Injection & Swap-Out System



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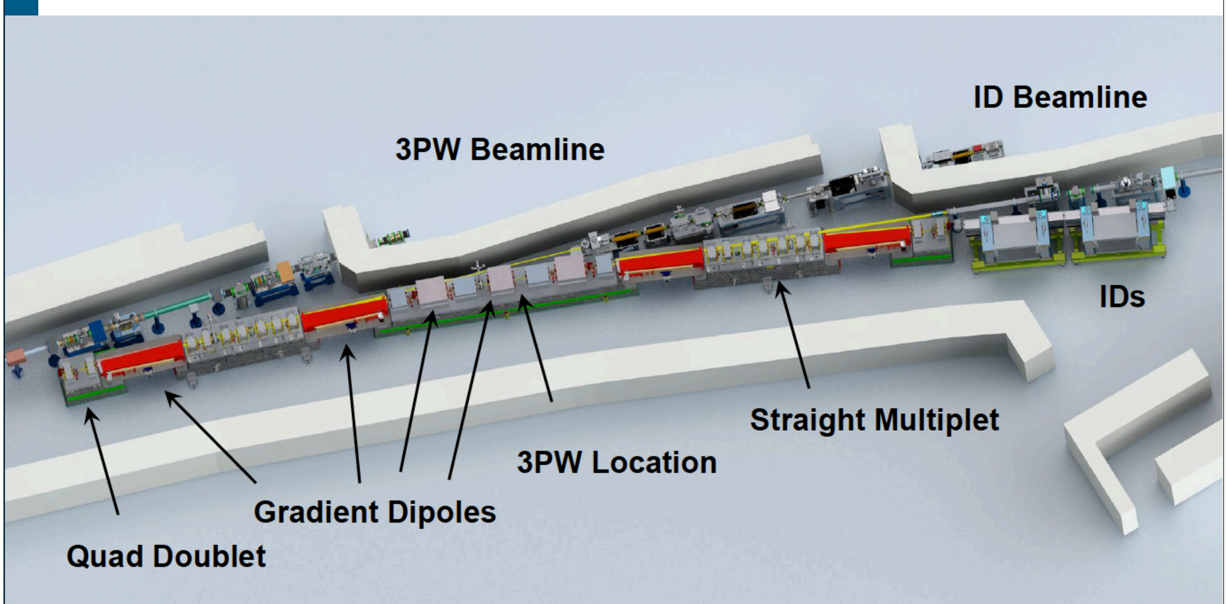
Acknowledgments: M. Borland, Y. Sun, A. Xiao, J. Dooling, X. Sun, Z. Conway, M. Abliz, M. Jaski

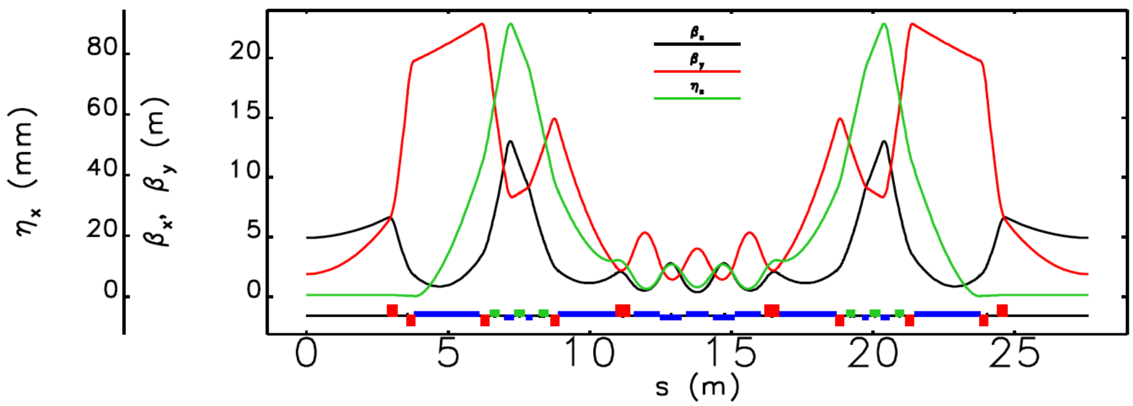
Argonne NATIONAL LABORATORY

APS-U One Sector (of 40)



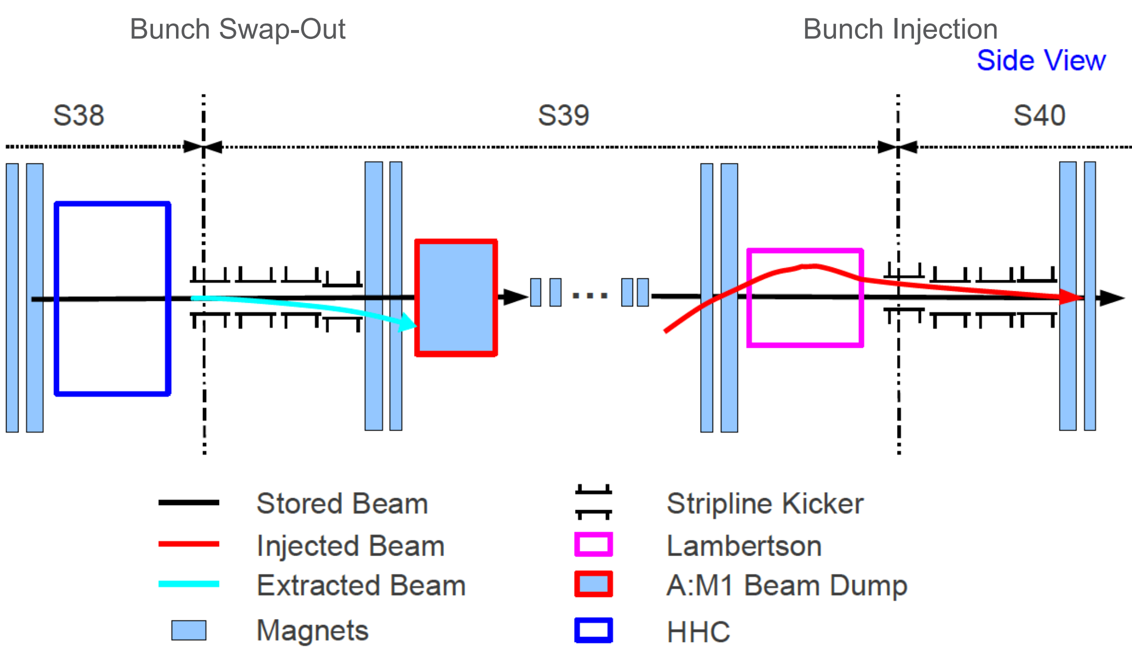
APS-U Magnet Lattice

- 7-Bend Achromat Lattice, 6 GeV, 41 pm-rad emittance
- 6 GeV injector, 60 nm-rad emittance
  - swap-out injection, extraction in one straight

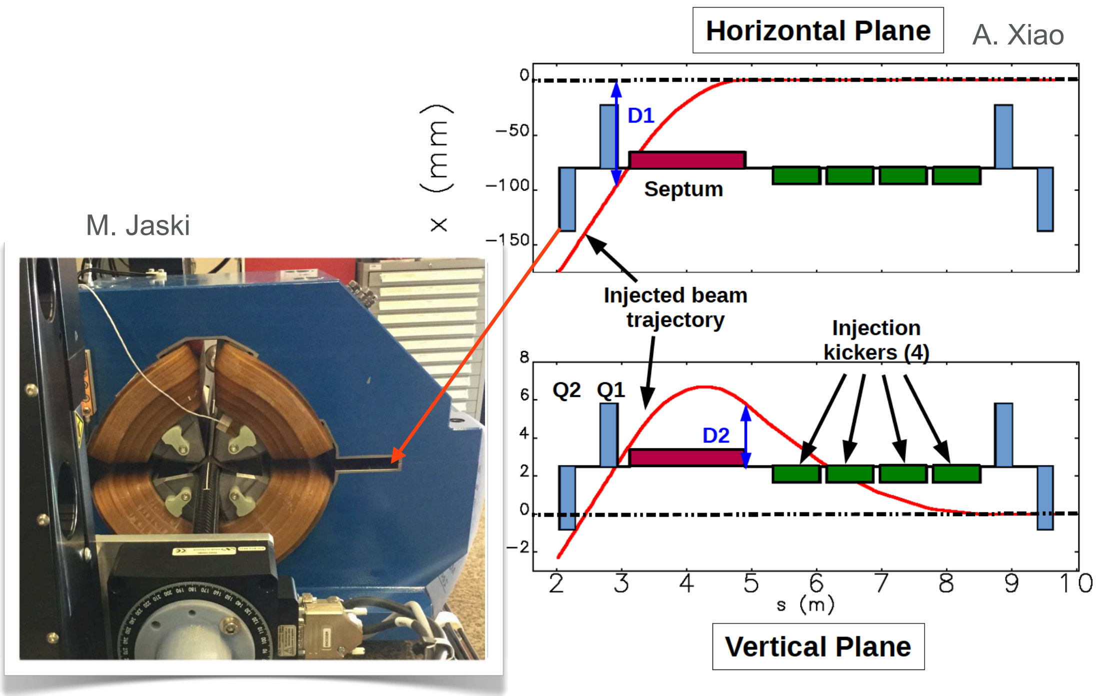


Beam-Transfer Sections

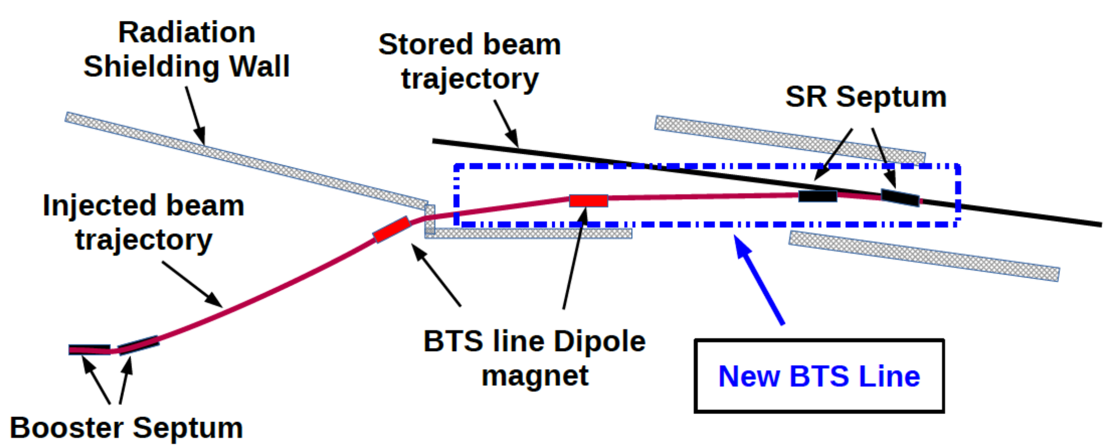
A. Xiao



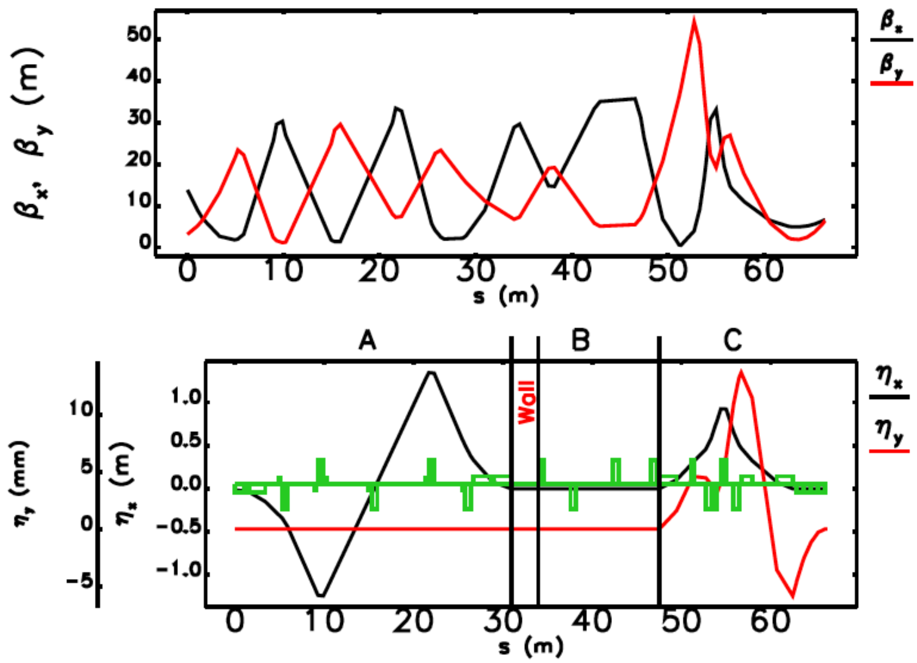
Injection Details



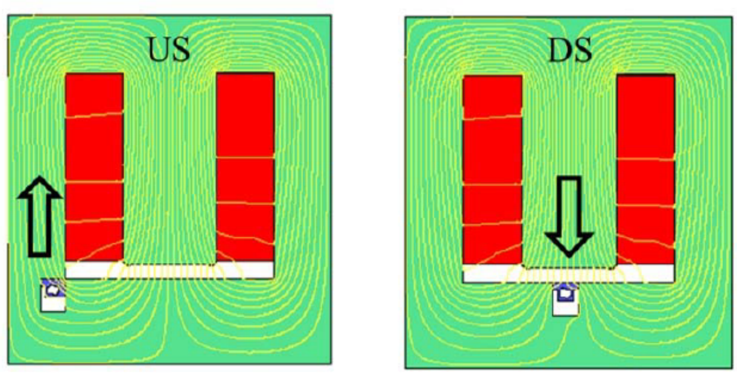
Transfer Line (BTS) Layout



Transfer Line (BTS) Lattice

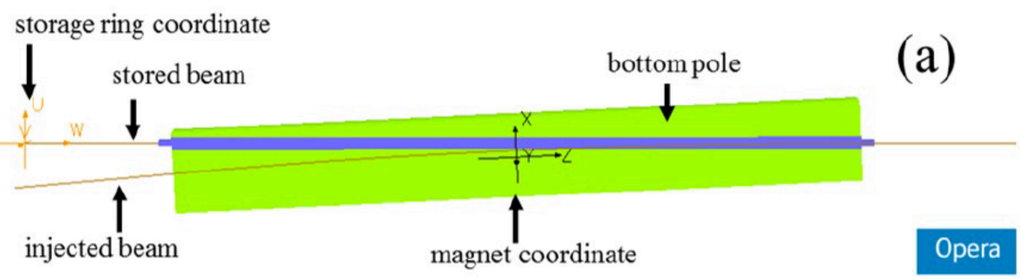


Lambertson Septum



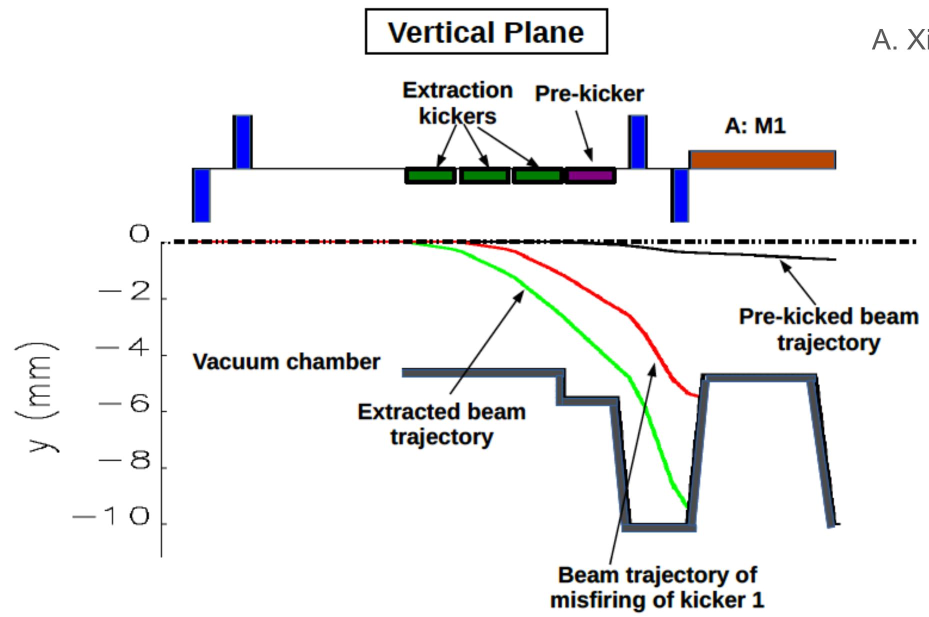
M. Abliz

89 mrad bending
93 mrad roll



Opera

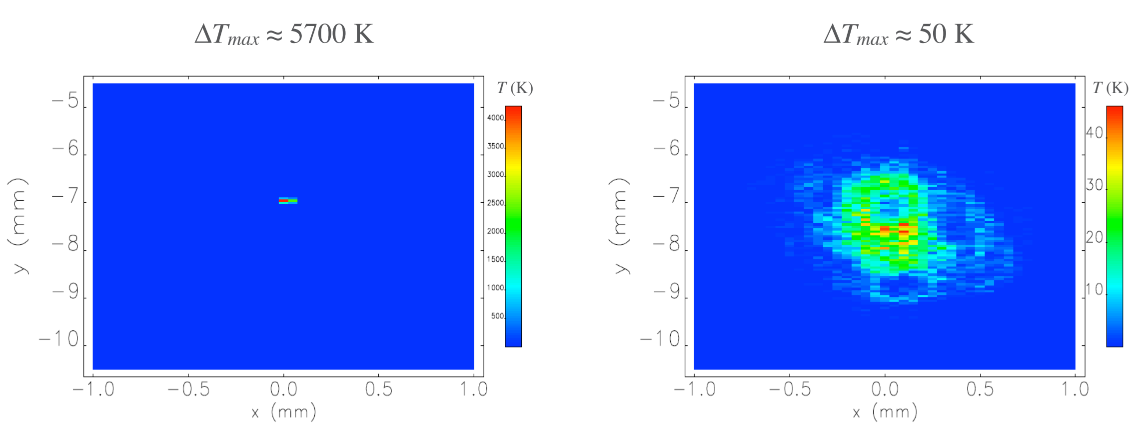
A. Xiao



APS-U Swap-Out Beam Dump

Lack of space forces internal dump => no optics to enlarge beam

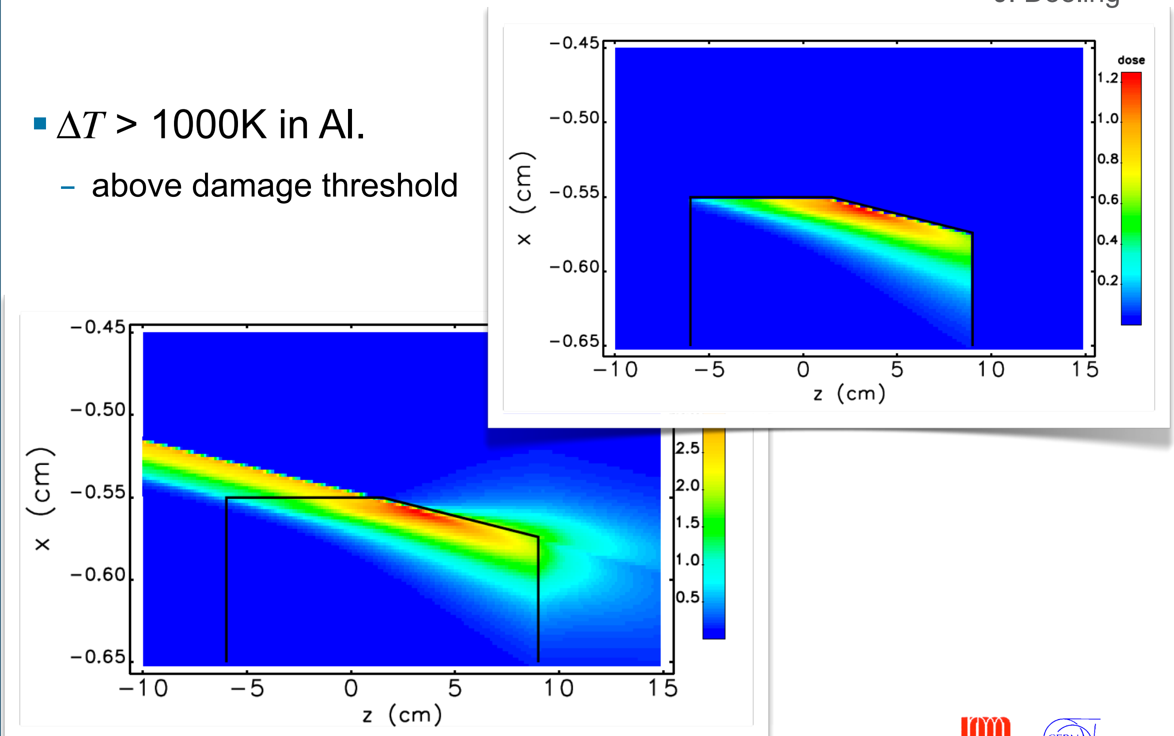
J. Dooling



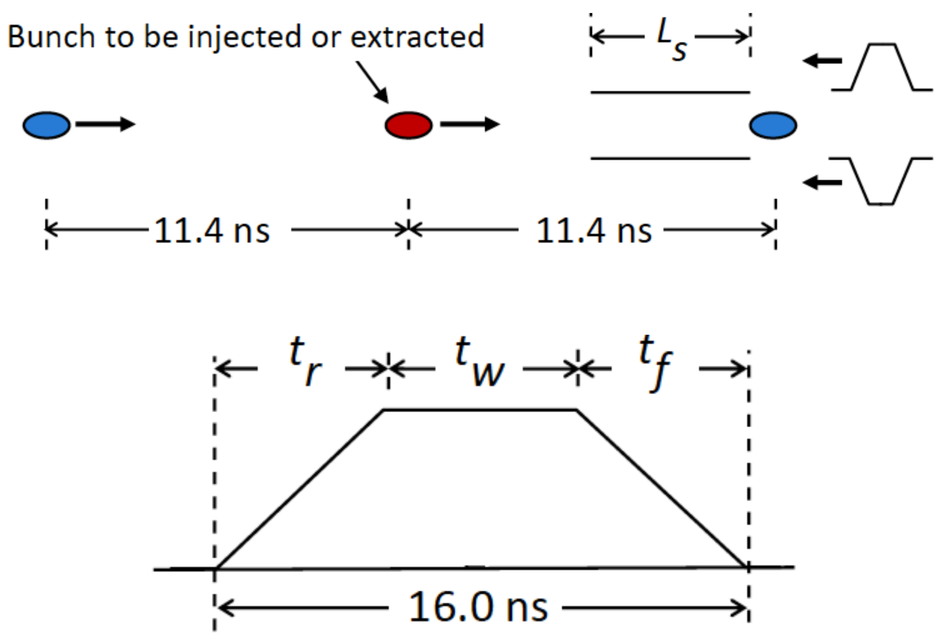
APS-U Abort Dump (Concept)

J. Dooling

- ΔT > 1000K in Al.
  - above damage threshold

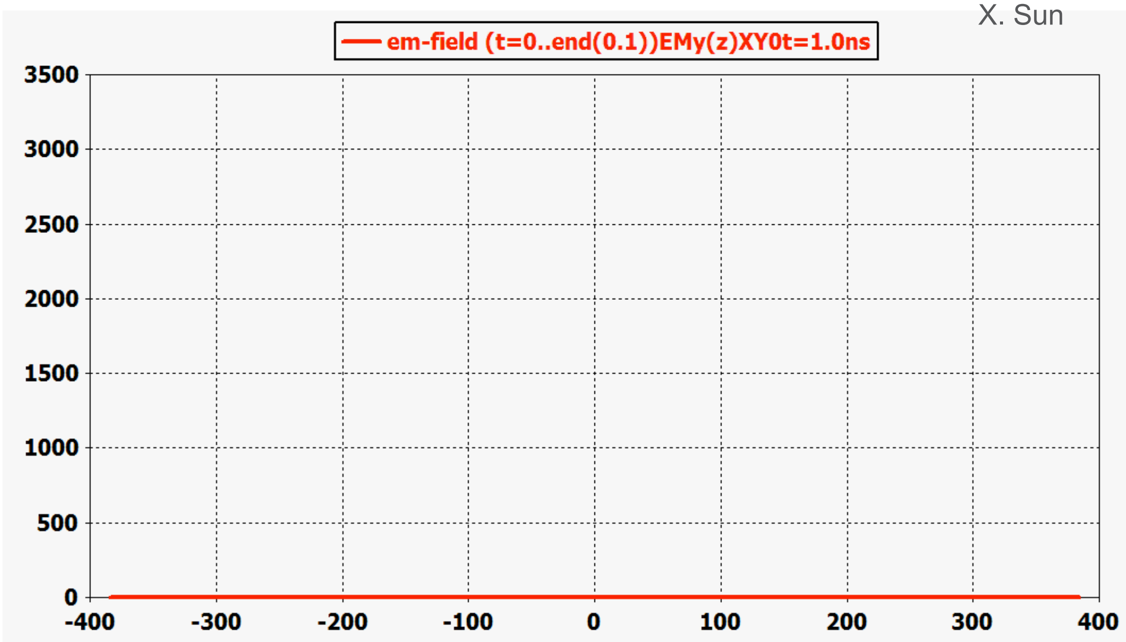


Kicker Waveforms



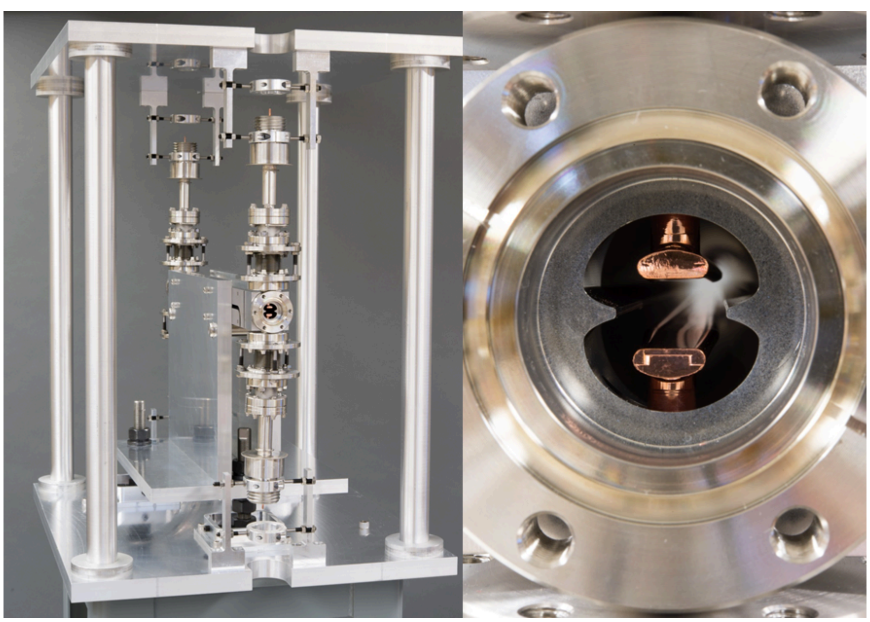
Kicker Waveform

X. Sun

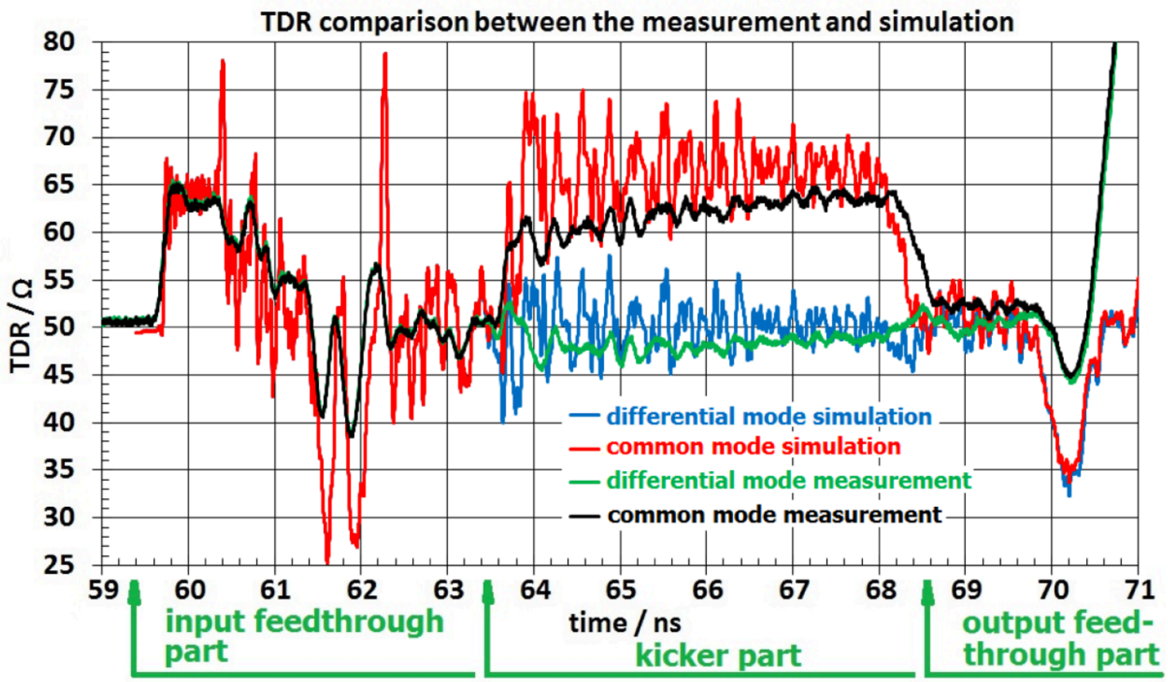


APS-U Stripline Kicker

Z. Conway



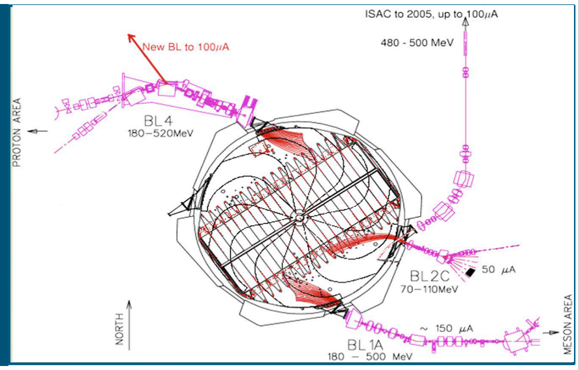
X. Sun



U. Wienands & E. Marin-Lacoma, USPAS, Lisle, Jun-2017.

15

Cyclotron Injection & Extraction



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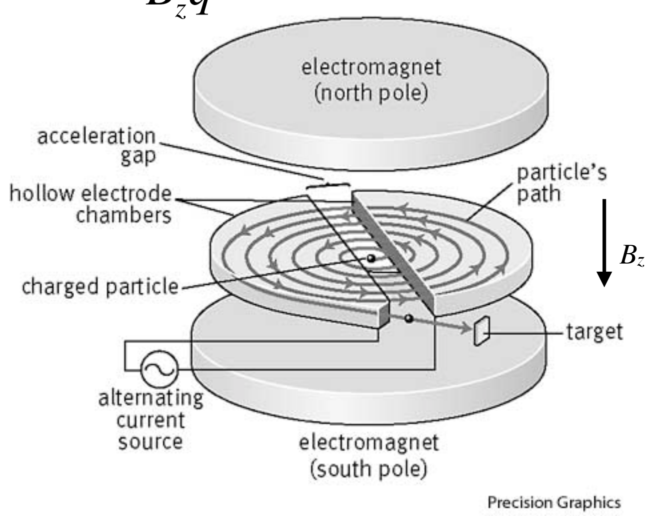
19-Jun-2017
USPAS, Lisle Il.

Cyclotron

- Circular accelerator with a spiral beam trajectory
  - this keeps the revolution frequency constant, until relativity kicks in:

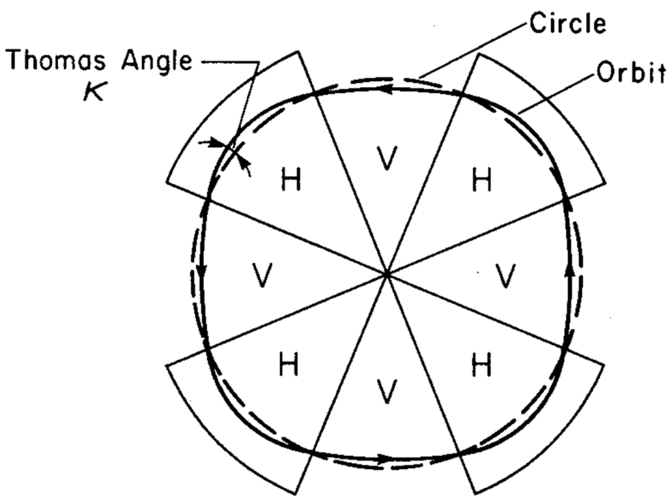
$$\gamma m_0 \omega = qB_z \qquad R = \frac{mv}{B_z q}$$



Precision Graphics

Isochronous Cyclotron

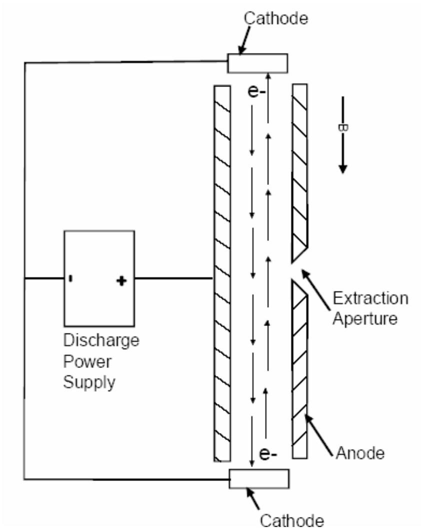
- Alternating field provides for stronger focusing
  - allows for higher beam energy as spiral pitch gets tighter



- For even higher energy need spiral-shaped fields
  - even higher focusing
  - turn separation at large radius is lost

Cyclotron Injection

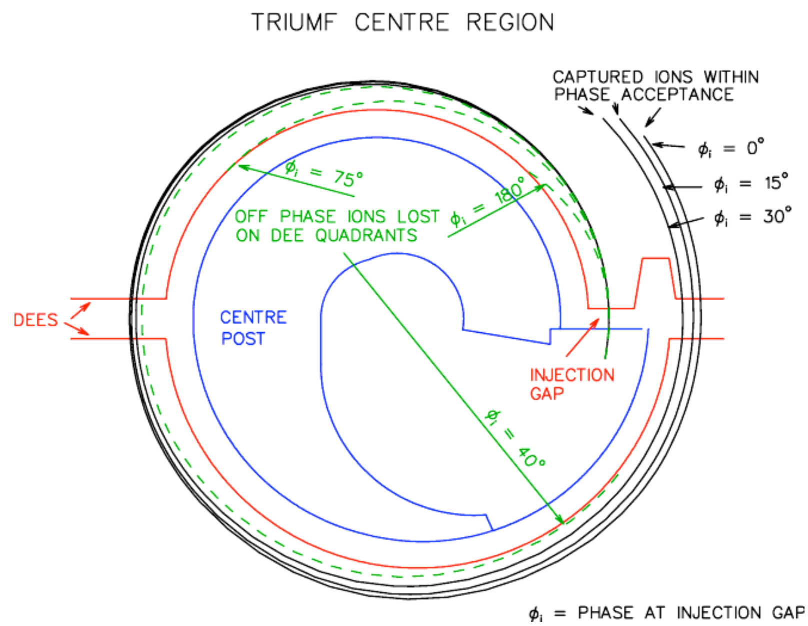
- internal source: simple arrangement; typically protons or light ions



Penning Ion Source (PIG)

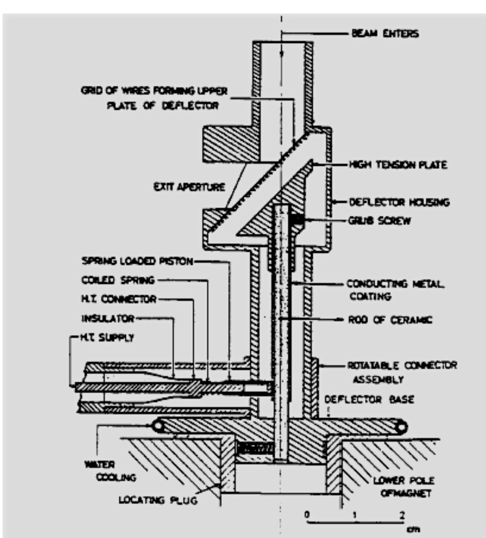
- Center region of an industrial cyclotron (IBA C18/9)

Center Region



Electric Mirror Inflector

- Simple, high fields (≈ beam energy), delicate extraction grid



not really used anymore

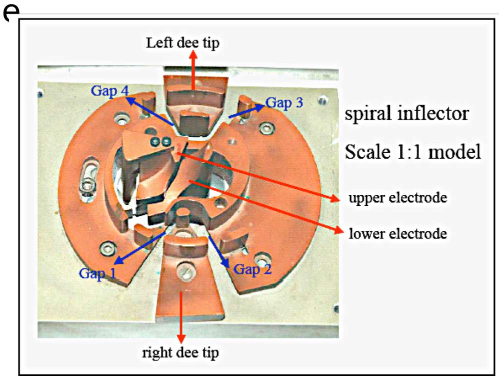
Spiral Inflector

- A twisted capacitor, following the particle path in the e-m field
  - E-field always normal to particle path
  - Voltage required:

$$V = 2\frac{E}{q}\frac{d}{A}$$

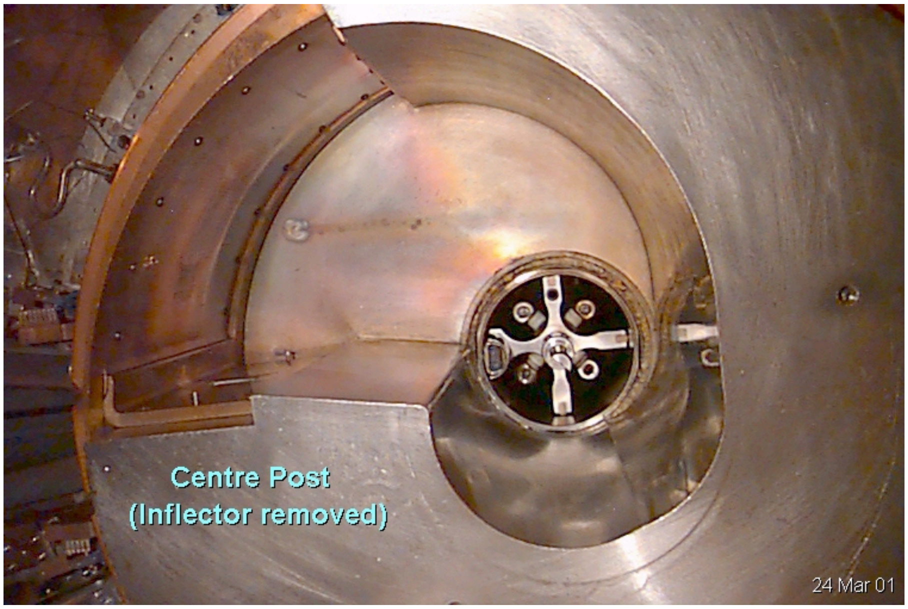
E: particle energy
q: particle charge
d: gap of inflector
A: radius of inflector

  - The field can also tilt around the particle orbit in the inflector to fine-tune the direction (tilt parameter k')
  - A and k' are design parameters.



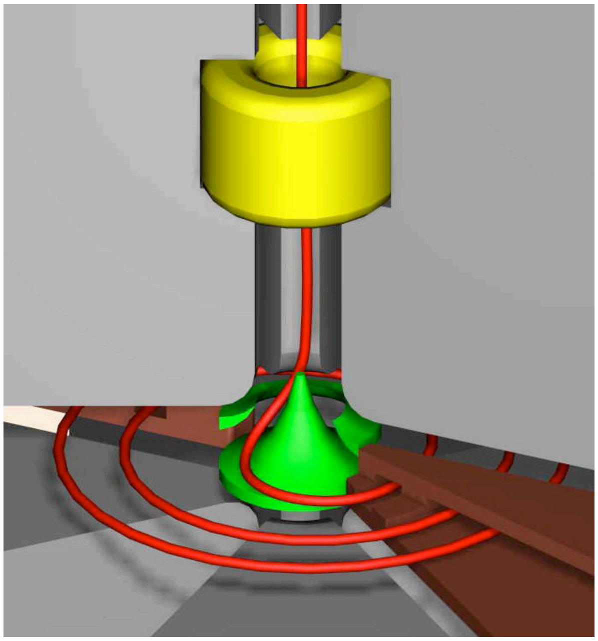
Spiral Inflector

Center Region



Magnetostatic Inflector

W. Kleeven, IBA



- Easier at higher energy
- No high voltage; field by main coils
- under study

Transport of a Beam through the Inflector

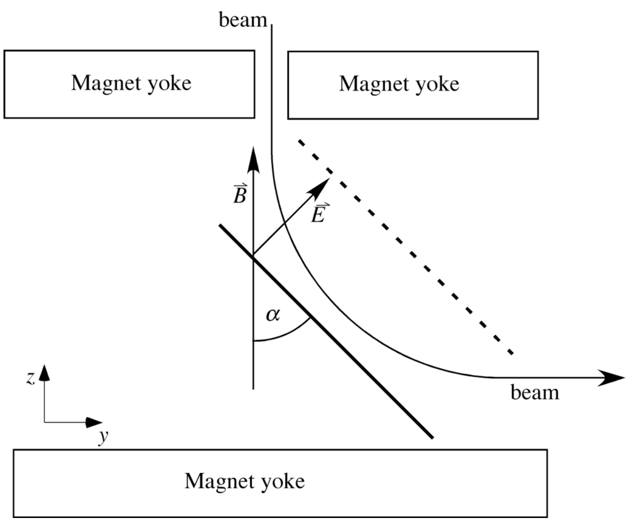
- No R-matrix for a beam through a (spiral) inflector exists
  - > an infinitesimal F matrix is used.

$$F(s) = \frac{R(s+ds,s)-1}{ds}$$

$$\Sigma(s+ds) = R\Sigma(s)R^T \quad \Rightarrow \quad \frac{d\Sigma}{ds} = F(s)\Sigma(s) + \Sigma(s)F(s)$$

  - we can then integrate to transport Σ along the inflector

Mirror Inflector



$$x = E_y / B\omega(\omega t - \sin \omega t)$$

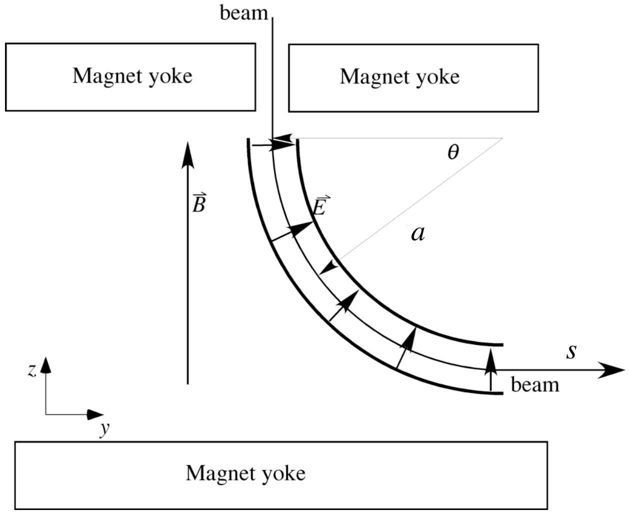
$$y = E_y / B\omega(1 - \cos \omega t)$$

$$z = (\omega / 2B)E_z t^2 - v_0 t + z_0$$

with $\omega = qB/m$

Spiral Inflector

- Motion is circular about B and follows E => spiral



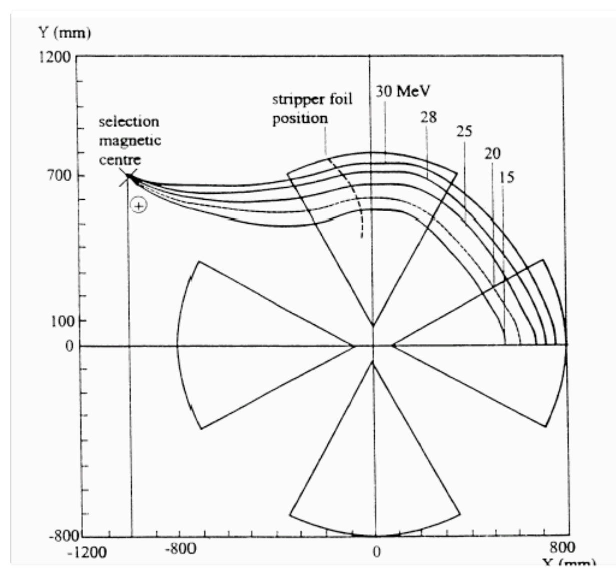
$$F = \begin{bmatrix} 0 & 1 & -Ck & 0 & 0 & 0 \\ -S^2k^2 & 0 & -Sk/a & 0 & 0 & Sk \\ Ck & 0 & 0 & 1 & 0 & 0 \\ -Sk/a & 0 & 0 & 0 & 0 & 2/a \\ -Sk & 0 & -1/a & 0 & 0 & 1 \\ -Ck/a & 0 & 0 & -1/a & 0 & 0 \end{bmatrix}$$

$$C = \cos\frac{s}{a},\; S = \sin\frac{s}{a},\; k = \frac{1}{\rho}$$

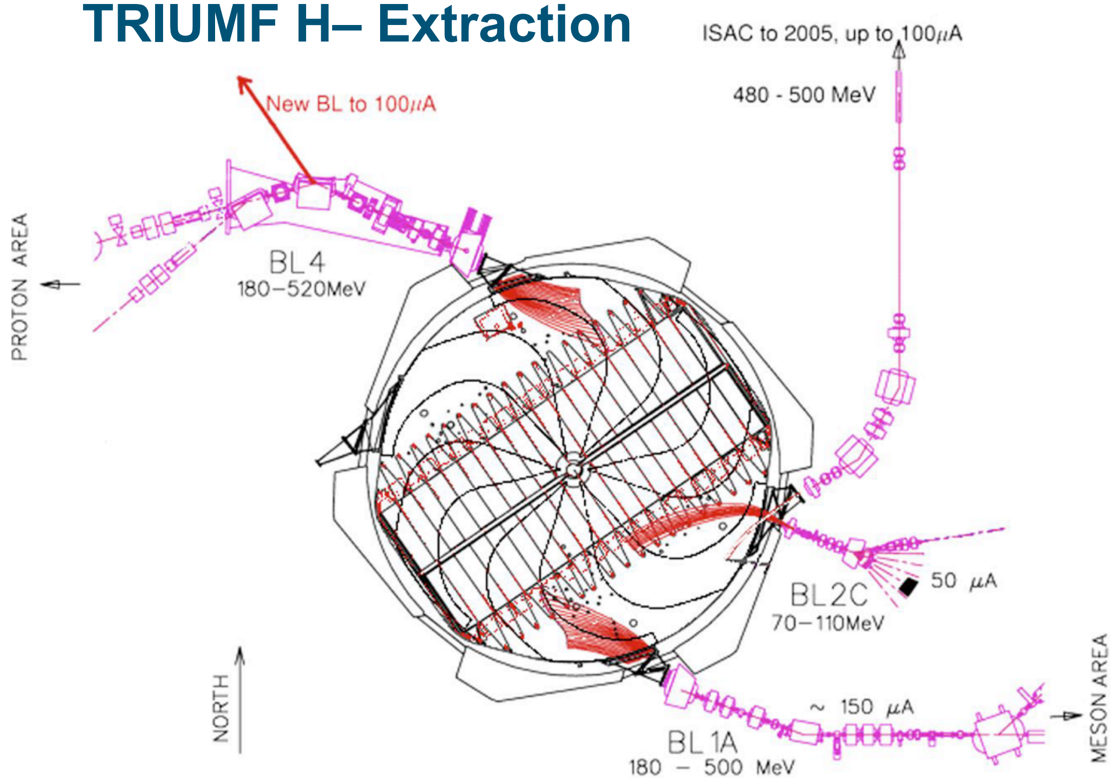
a: electric radius of curvature
ρ: magnetic radius of curvature
s: path length

H– Extraction

- Simplest extraction is by stripping H– ions:
- quite efficient
- no turn separation needed
- can extract several beams
- can select intensity by partial interception of beam
- varian: accelerate H_2^+ and strip to H^+



TRIUMF H– Extraction



Cyclotron Extraction

- For the deflector, *turn separation* is needed to avoid the deflector electrode being hit by beam.

$$\Delta r(\theta_n) = \Delta r_0(\theta_n) + \Delta x \sin(2\pi n(\nu_r - 1) + \theta_0) + 2\pi(\nu_r - 1)x\cos(2\pi n(\nu_r - 1) + \theta_0)$$

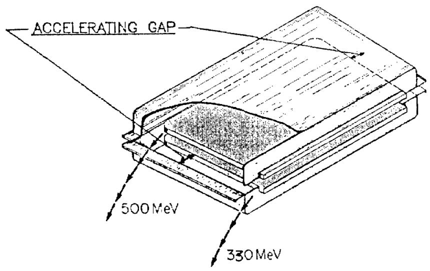
- The acceleration part is given by

$$\Delta r_0 \approx \frac{r}{2}\frac{\Delta E_{turn}}{E}$$

  - in an isochronous cyclotron, r grows slower than E so the turns bunch up towards the top end.
  - The higher V_{rf}, the higher is ΔE and thus turn separation.

TRIUMF AAC

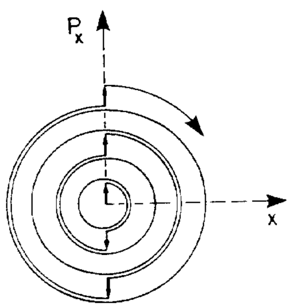
- Increase DE/turn from 320 keV to 620 keV using a 4th harmonic cavity

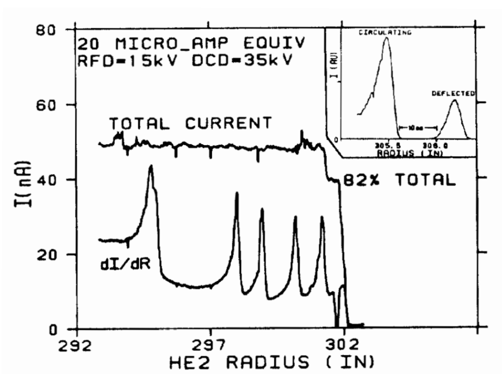


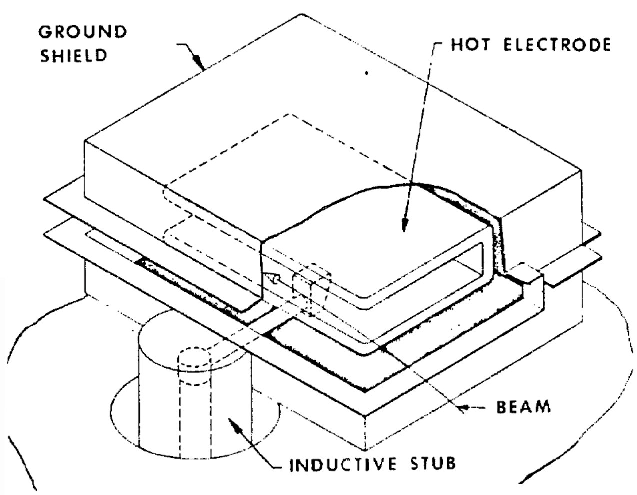
- double turn separation; lower Lorentz stripping.

Excitation of the 3/2 radial Resonance

- 11.5 MHz, 25 kV, Δr from 1.5 mm to 5 mm (at 440 MeV)







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- M.K. Craddock, "High Intensity Circular Proton Accelerators", TRI-87-2, TRIUMF, Vancouver, BC, Canada, 1987.
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- P. Mandrillon, "Injection into Cyclotrons".
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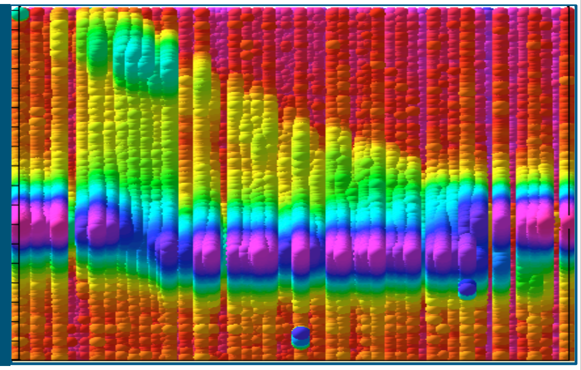
- R. Baartman, W. Kleeven, "A Canonical Treatment of the Spiral Inflector for Cyclotrons", Particle Accelerators, 41(1993).

Crystal Extraction



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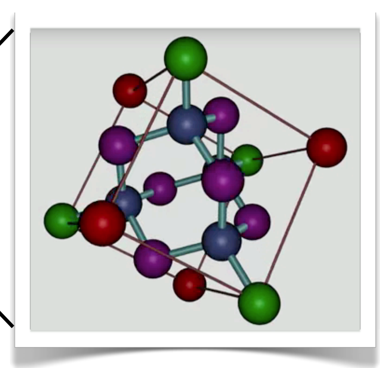
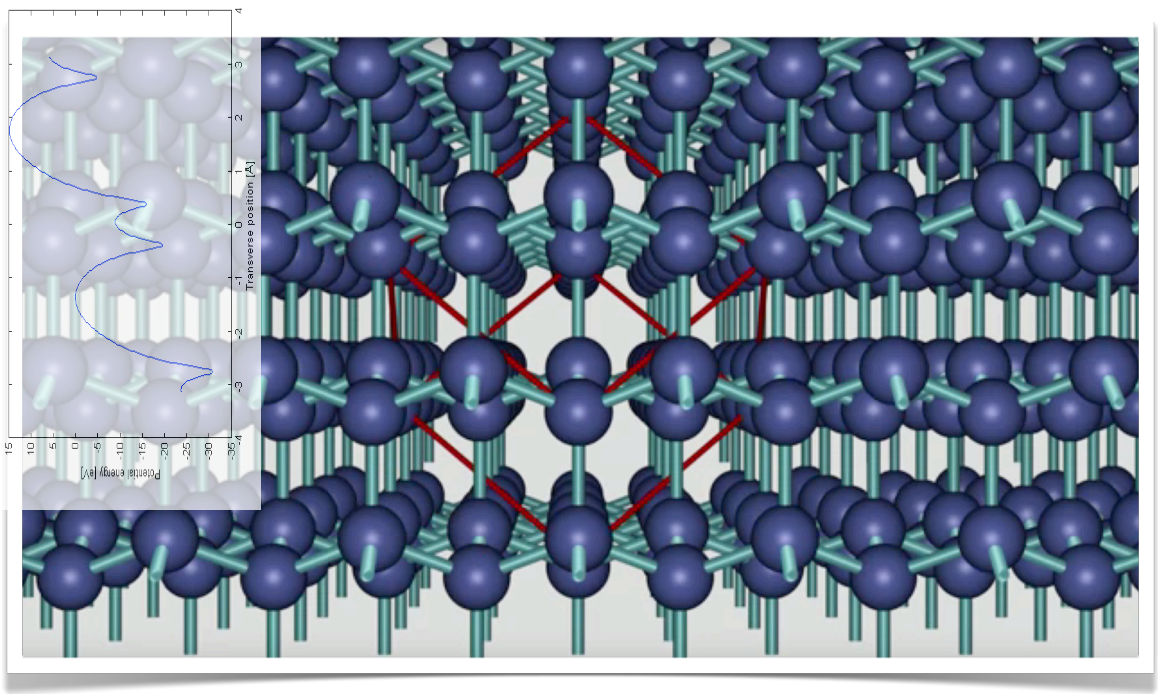
Crystalline Potentials

Si crystals

Si unit cell



Si (111) Planes



- The electric field near a nucleus is

$$E(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{r^2}$$

e.g. 2*10[12] V/cm at 0.1Å

and for a crystalline plane can be approximated by a continuum potential:

$$U(\vec{r}) = \frac{1}{d}\int V(\vec{r},z)dz$$

which is about 20..25 eV for a Si(110) crystal

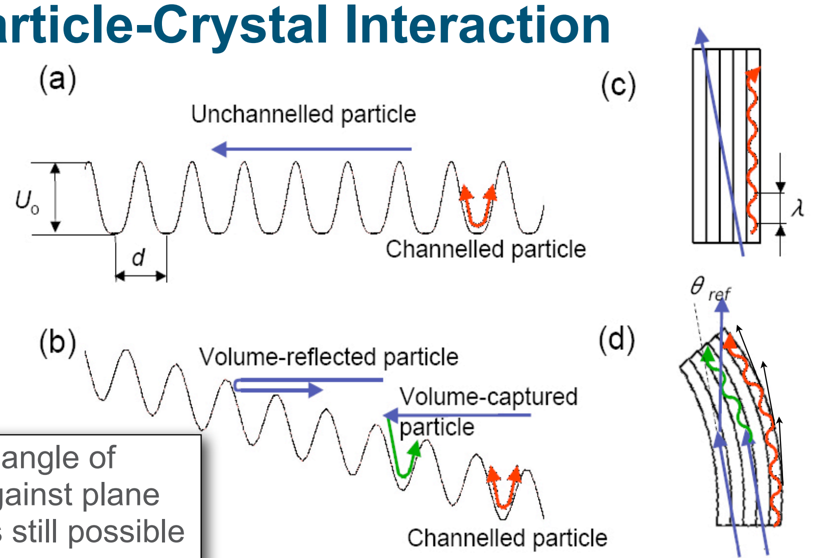
- The transverse energy is then

$$E_\perp = \frac{p_\perp}{2\gamma M} + U\left(\vec{r}_\perp\right) = \frac{1}{2}pv\Theta^2 + U\left(r_\perp^2\right)$$

Particle-Crystal Interaction

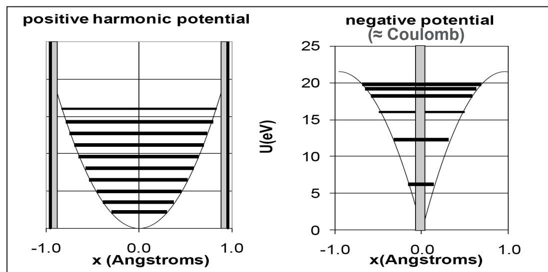
Possible processes:

- multiple scattering
- channeling
- volume capture
- de-channeling
- volume reflection



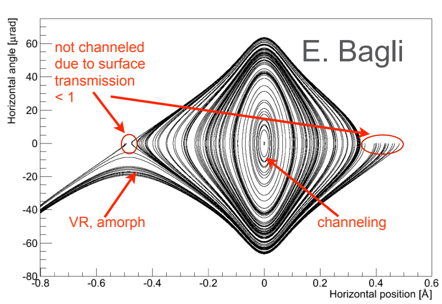
Critical angle: max. angle of incoming particle against plane where channeling is still possible
$\theta_{crit} = \sqrt{2U_0/E}$

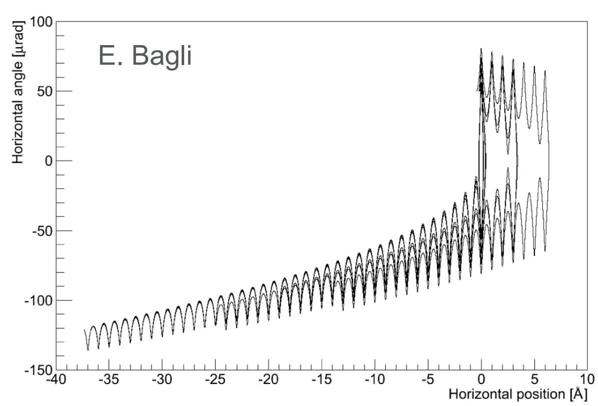
Potential shape differs depending on polarity

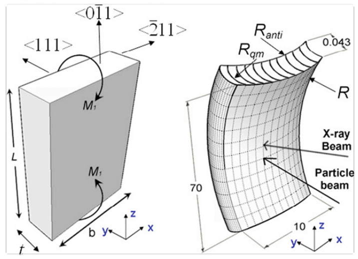


Phase Space (bent crystal)

- Same topology as a (moving) rf bucket



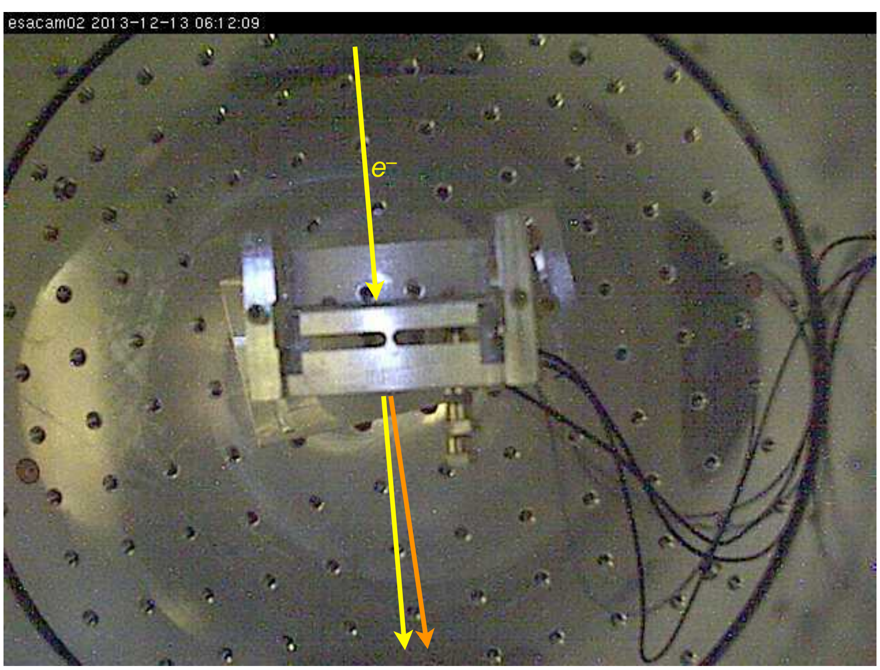




Main crystal features

- **Crystal thickness 60±1 μm**
  Once the crystal will be back in Ferrara we will measure crystal thickness with accuracy of a few nm.
- **(111) bent planes (the best planes for channeling of negative particles).**
- **Bending angle 402±9 μrad (x-ray measured). If needed I can provide a value with lower uncertainty.**

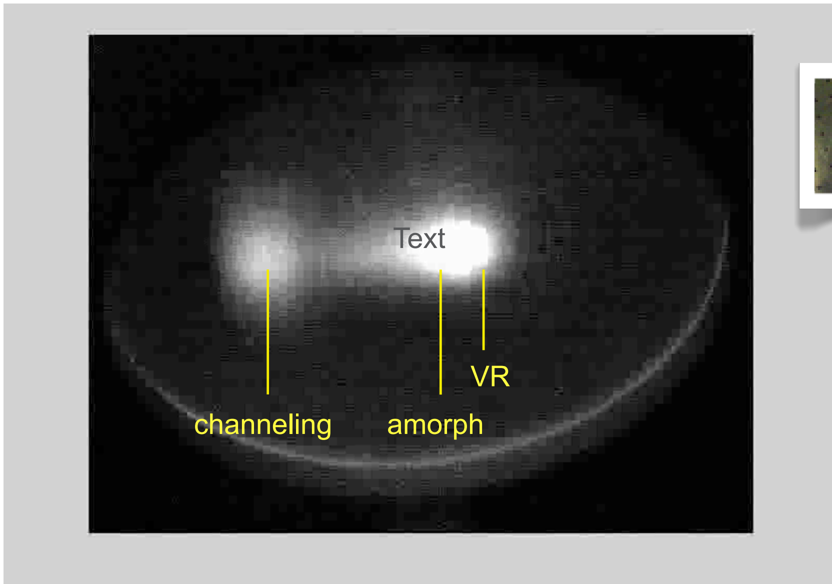
T513 Expt. @ SLAC ESA

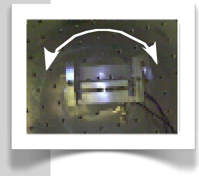


Electron Deflection @ 4.2 GeV

https://www.sciencedaily.com/releases/2015/02/150225132110.htm

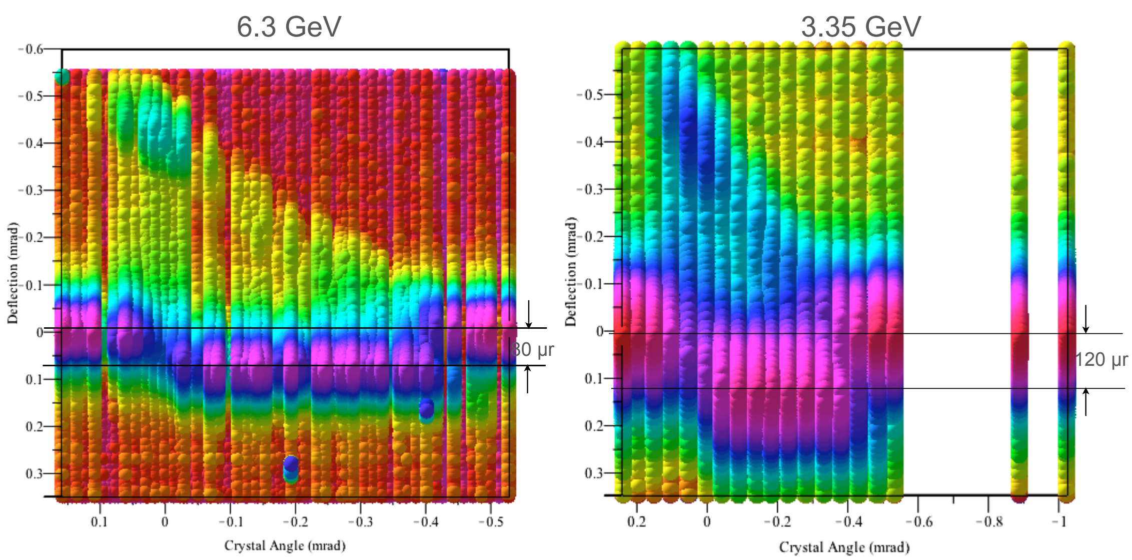
(Movie credit: T. Wistisen)



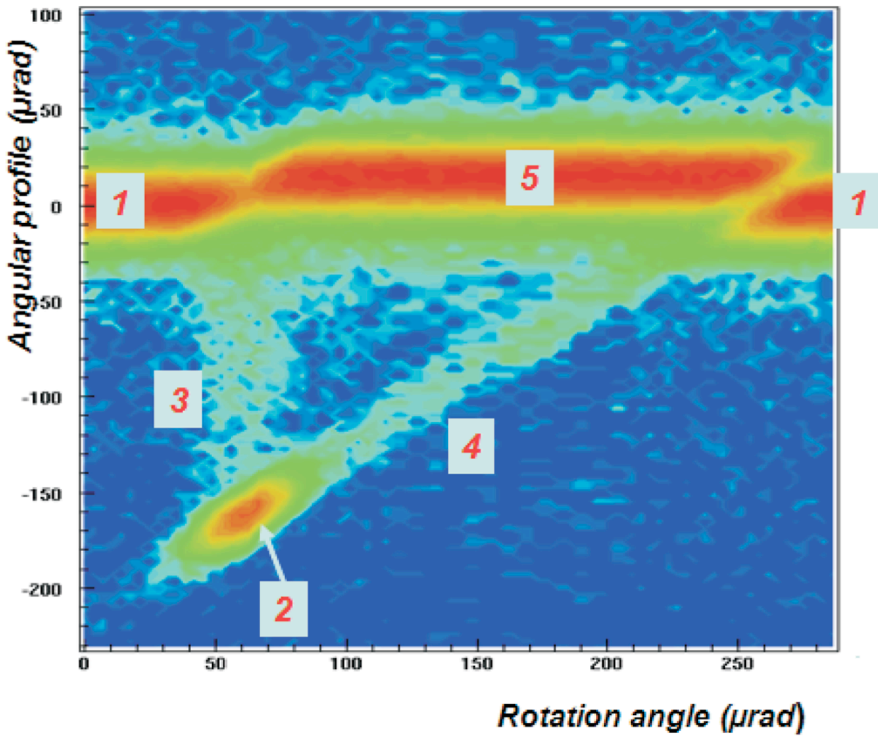


Triangle Plots

Colors rep. log(intensity).
Crystal angles from fit to laser spot (est'd uncertainty 2…5 μrad)



Deflection Triangle (Protons)

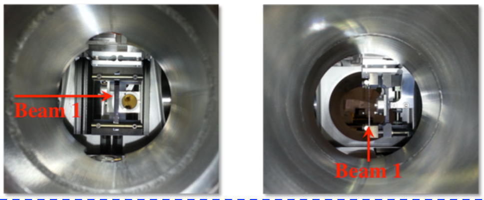


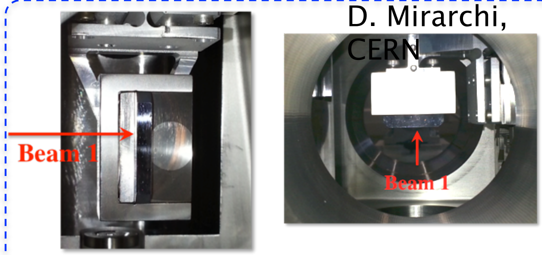
- Protons channel reasonably well channeling has been used to extract from h.e. synchrotrons
  - U70 in Protvino
  - proposed for LHC halo extraction (expt. in place)
    - critical angle 2.4 μrad
    - Tsyganov's radius ≈ 15 m
    - > 10s of μr bending achievable.
- Electron channeling efficiency only ≈ 25%, not enough
  - but volume reflection about 95% albeit at maybe 1/4 the angle
  - extraction using a VR array may be possible
  - this is interesting for beam collimation

Hardware installed in the LHC

Two crystals installed in the **IR7 (Beam1)** during **April 2014: (developed in the UA9 framework)**

Silicon Strip crystal in the horizontal plane

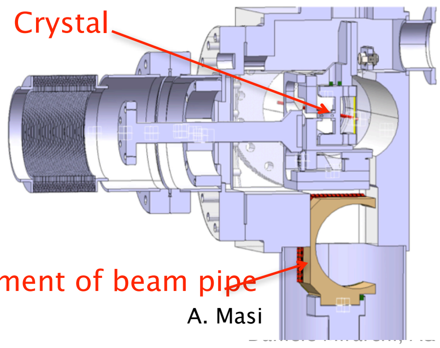


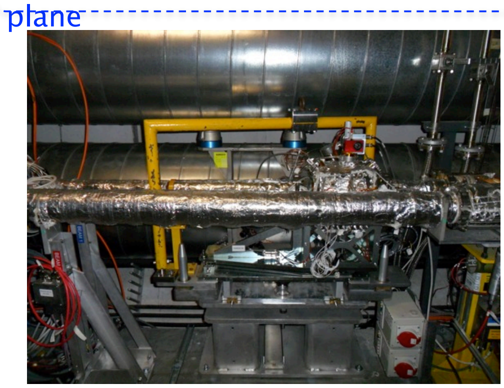


Quasi-mosaic crystal in the vertical plane

And relative goniometers: (UA9 framework)

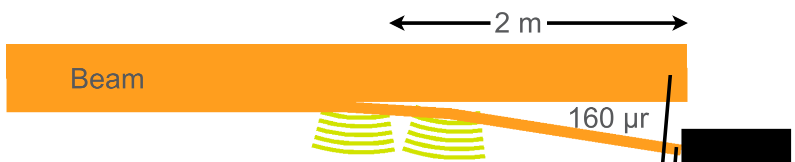
- ✓ Piezo actuator in closed loop (angular stage)
- ✓ Transparent during normal operation



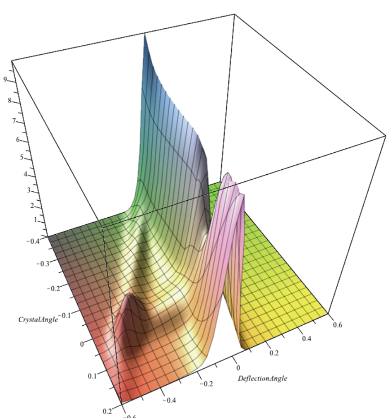


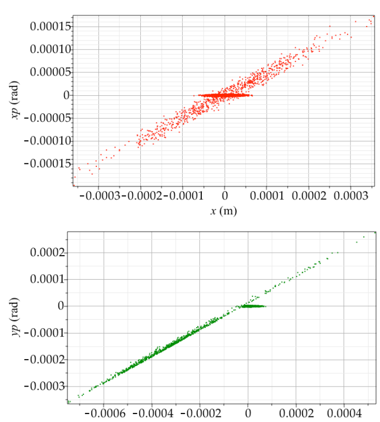
VR Collimator Concept

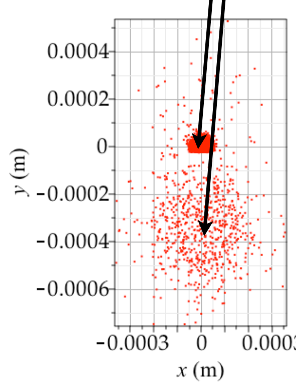
- The T513 data can help designing beam collimation for e^-:



pdf to generate deflections







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- W. Scandale et al., Deflection of 400 GeV/c proton beam with bent silicon crystals at the CERN Super Proton Synchrotron, Phys. Rev. ST Accel. Beams 11, 063501 (2008).
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