January 28,2017

To the participants of the January 2017 USPAS class on LLRF Systems, Technology and Applications to Particle Accelerators:

Greetings, and I thought that some of you would want to see the solutions to the class problems. I hope this set of handwritten solutions is helpful. In general I think that most of you who turned in the problems had the general idea of each problem, but here's my brief thinking.

We also have a short individual note for you on your final paper topic and presentation.

We want to thank everybody who participated, shared your experience and made this such an interesting class for us.

Notes from Assignment #1

Problem #1, spectrum of a bunched beam - generally the issue is that the revolution of the beam creates a sequence of lines 1.52 MHz apart - the finite bunch length has the line spectrum falling off at roughly 1/bunch length , for 50 ps about 20 GHz. For the fully populated ring, the line spacing is the RF frequency (500 MHz). The bunch length is the same so the fall-off is at the same 20 GHz frequency.

For part (c) - the easy way to do this is graphically - the square wave modulation is a product in the time domain, so convolve the spectrum from (b) with the square wave components at the revolution frequency, 3X, 5X, 7X, etc. So sidebands at each RF harmonic are created falling off with the harmonics. Jon Daniels also did this as a numeric problem in matlab (attached) - but here you need a lot of data points because of the big ratio in time intervals between the revolution period, the bunch spacing, and the short bunches. So sometimes doing things analytically with a pencil isn't such a bad thing.

for part d - while in principle you could deconvolve the beam current structure from the observed spectrum, and exact knowledge of the fill pattern, it would be hard to figure out a bucket number dependence (the N dependence) if the fill had rotational symmatery. Even if it had a defined gap, and you could say bucket 0 is alinged with the gap, it is a messy inverse problem. using the time domain instrument (like a scope) really helps. Note you don't need to have the full frequency response in the scope to cover all the beam spectrum. If you can resolve the individual bunches, the relative amplitudes of the band-limited signals still tells you the relative current population of each bunch.

Problem #2

The situation with modulations on a cavity, or both cavities, is best thought of by drawing a vector diagram of the two cavity fields. I made my own crude drawings for you.

for part a) - the two vectors just add up in a line, so the accelerating voltage is just twice the voltage in each cavity. The beam has to find the voltage just equal to the energy loss/turn, so if Vcav(t)= V_RFsin(omega_RF * t) you can find the beam phase with respect to the RF with the inverse sin. set the voltage equal to the energy loss/turn. For the ALS, above transition, this has to be the falling slope of the waveform. Don't forget the 2*pi in converting angular frequncy to Hz.

part b) If the voltage in 1 cavity is amplitude modulated, this raises and lowers the RF votage at the modulation frquency. As sketched, the beam still needs to ride at the exact voltage to make up the energy loss. This means the synchronous phase is modulated at the modulation frequency, the beam just moves forward and back relative to the nominal phase position. As long as the modulation frequency is well below the synchrotron frequency, the beam just follws the modulation. But as the modulation gets close to the synchrotron frequency, you wiull excite very large amplitude motion as you get near the resonant frequency.

Noise on a klystron HV power supply can often excite this synchrotron motion at some multiple of the line frequency, if it is a three phase supply at 360 Hz, the ripple on the supply is at 720 Hz.

Looking at the beam signal off a BPM, you would see the phase modulation spectrum of the beam, so sidebands at +/- n*omega_synchrotron. For small modulations, practically hard to see many orders of n.

If one cavity had a phase modulation on it, the beam would also be moving to keep at the synchronous voltage, so the BPM spectrum would also have the phase modulation on it. Again, the effective RF voltage (the vector sum) would change in amplitude as the two cavities moved in phase against each other. So the synchronous phase of the beam moves, too.

Part c) if you move one cavity in phase with respect to the other as a DC offset, the vector sum is as sketched, so the total cavity voltage is now reduced. The beam still needs to make up exactly the same voltage/ energy as case a). So it finds the position with respect to the vector sum, and this gives a phase in each cavity with is unique relative to the RF waveform phase. This means the two slopes in the beam crossings are different, so the synchrotron frequency will be lower when the net vector sum cavity voltage is lower as is in this case c). So by measuring the beam synchrotron frequency as you move a cavity phase relative to another, maximizing the synchrotron frequency will bring the RF cavities into phase alignment.

If you look at the cavity power delivered to the beam, by looking at the cavity regulated voltage and the forward power, you could also align the cavity phases by finding the relative phase where the two cavities had the same RF voltage magnitude and had the same forward power (assuming they were detuned the same amount).

Problem #3, PI attenuator
Pozar did the "T", (Example 4.4), so I thought we should do a "PI". To have the same input and output impedance the PI section has symmetry. In general you can make either section, though the values may be easier to implement in one or the other, or parasitics may be an issue for very big resistor values, etc.

Problem #4, TDR responses
These are good to think through - the notes from Agilent are also attached to emphasize the time constants for the capacitors in shunt. Note that the problem with an RC parallel termination, and the C shunting another transmission line, are the same sort of problem. That's because a properly terminated transmission line looks resistive.

Problem #5 Pozar 6.16
This is illustrating a different sort of planar resonator. The essential issue is that when the resonator circumference is some multiple of a wavelength, it is resonant. As we discussed in class, you can couple to this inductively with a loop, or capacitively from an adjacent transmission line. Picking the coupling strength is significant because it defines the external Q.

The propagation velocity in this line depends on the effective dieletric constant, so once you know the substrate thickness, and dielectric constant, you can calculate using the transmission line dimensions to get the "effective" dielectric constant for the transmission line. It is a little different than just the board, as some field leaks out into the free space above the board. This is in Pozar if you want to look at the equations for estimating this. Once you have the effective dielectric contrant, you have the velocity and the wavelength, and knowing the circumference of a circle you have the resonant frequencies.

If you couple with a structure like a directional coupler, you could choose to excite modes in either clockwise or counterclockwise propagation.. They would have identical properties, same frequency, etc. If you had excitation of both modes, in some superposition, I think you would create a standing wave pattern on the transmission line of the resonator, just like for a 1/4 resonator with the forward and backward waves in it.

Problem #6) Pozar 6.1 (with second edition values)
Again, a straightforward illustration of the section on lumped element resonators. This is really almost the same problem as Claudio assigned, his is specific to an accelerating cavity

with a beam - this simpler example helps to see how the internal losses in the resonator (the internal Q) and the external losses from the resonator (the external Q) together set the system Q.

If anyone wants to discuss the class, offer ideas, etc. please do contact me by email (jdfox@stanford.edu) or phone (650-926-2789). Thanks again for your participation and humor. I also thank Ozhan Turgut for his help throughout the class, and Claudio Rivetta and Themis Mastoridis for their lectures about stability, beams and various RF control system approaches.

Sincerely,

John D. Fox
SLAC/Stanford Applied Physics

Figues and Notes for Problem #2 HW #1

J. Fox

USPAS PROBLEM SET #2

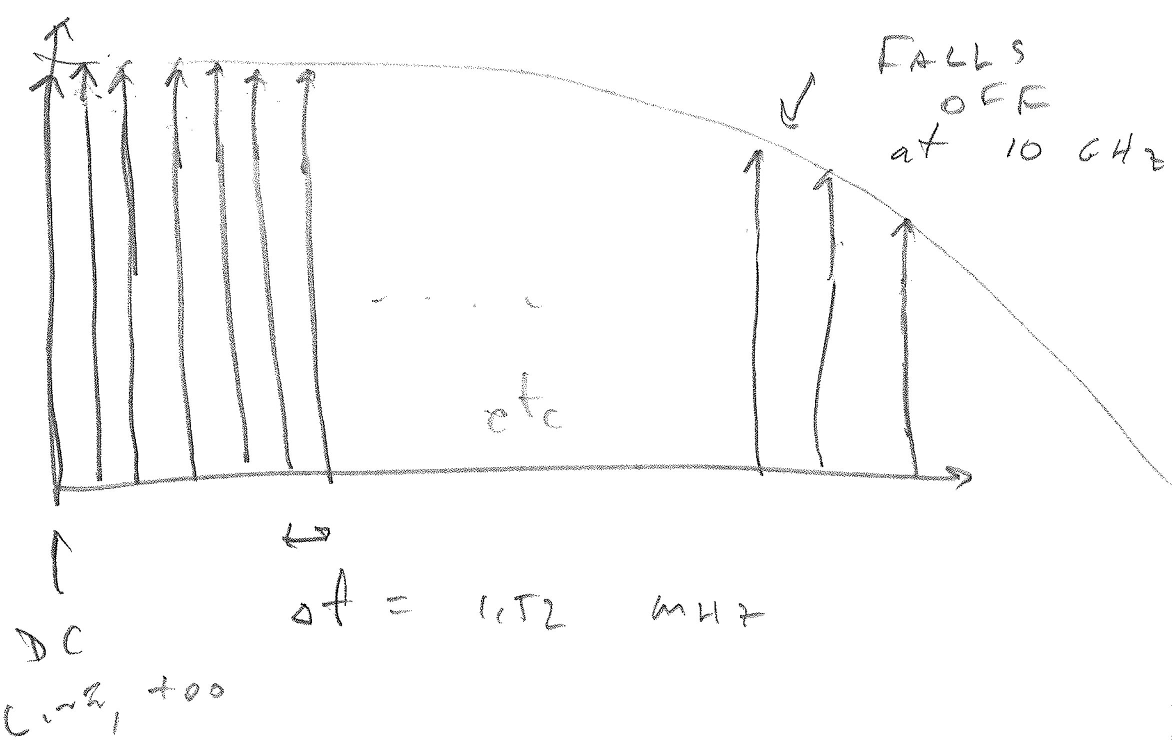
1) BEAM SPECTRUM FROM ALS

RF FREQUENCY 500 MHz
HARMONIC NUMBER 328

SO REVOLUTION FREQUENCY = $\frac{500 \cdot 10^6}{328}$ = 1.52 MHz

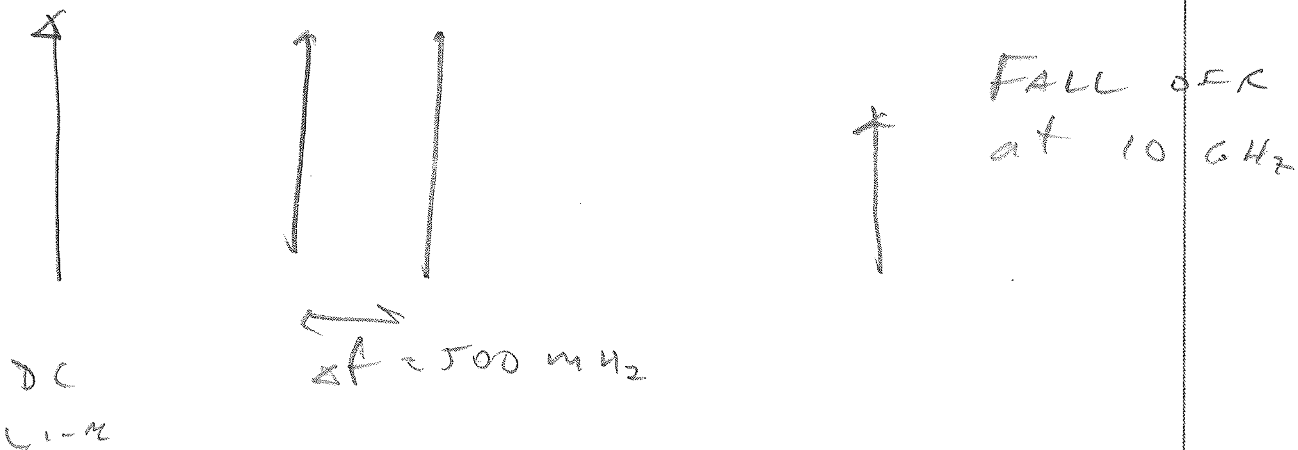
IF THE BUNCH IS ~ 100 ps

THE SPECTRUM IS A SERIES OF LINES EVERY 1.52 MHz, WITH A GAUSSIAN ENVELOPE - THE FALL OFF IS AT ROUGHLY 10 GHz $\left(\frac{1}{100 \cdot 10^{-12}}\right)$

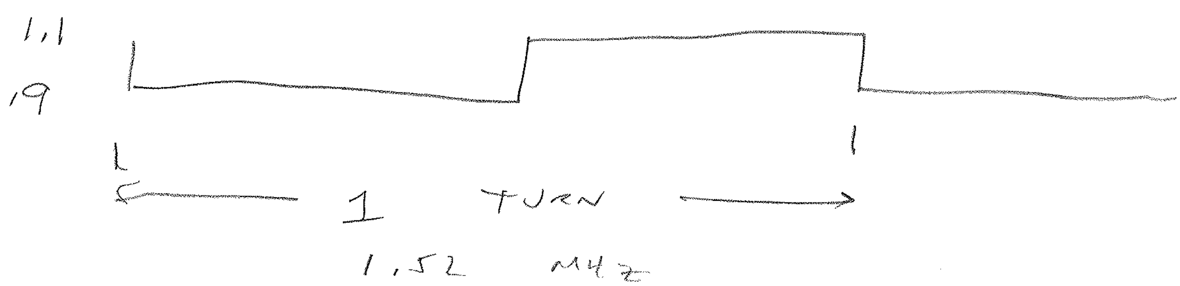


b) IF UNIFORMILLY FILLED (328 BUCKETS) SPECTRUM HAS SAME ENVELOPE, BUT LINES SPACED $\Delta f = 500$ MHz

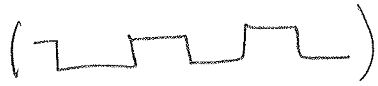
(RING LOOKS $\frac{1}{328}$ AS BIG)



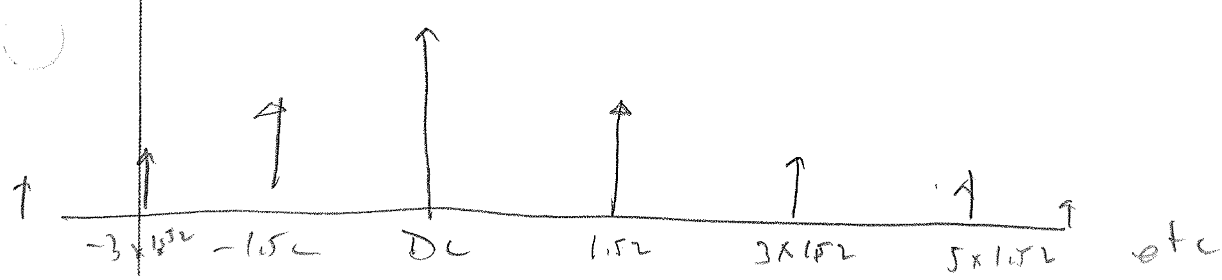
c) IF FILL IS SQUARE WAVE MODULATED



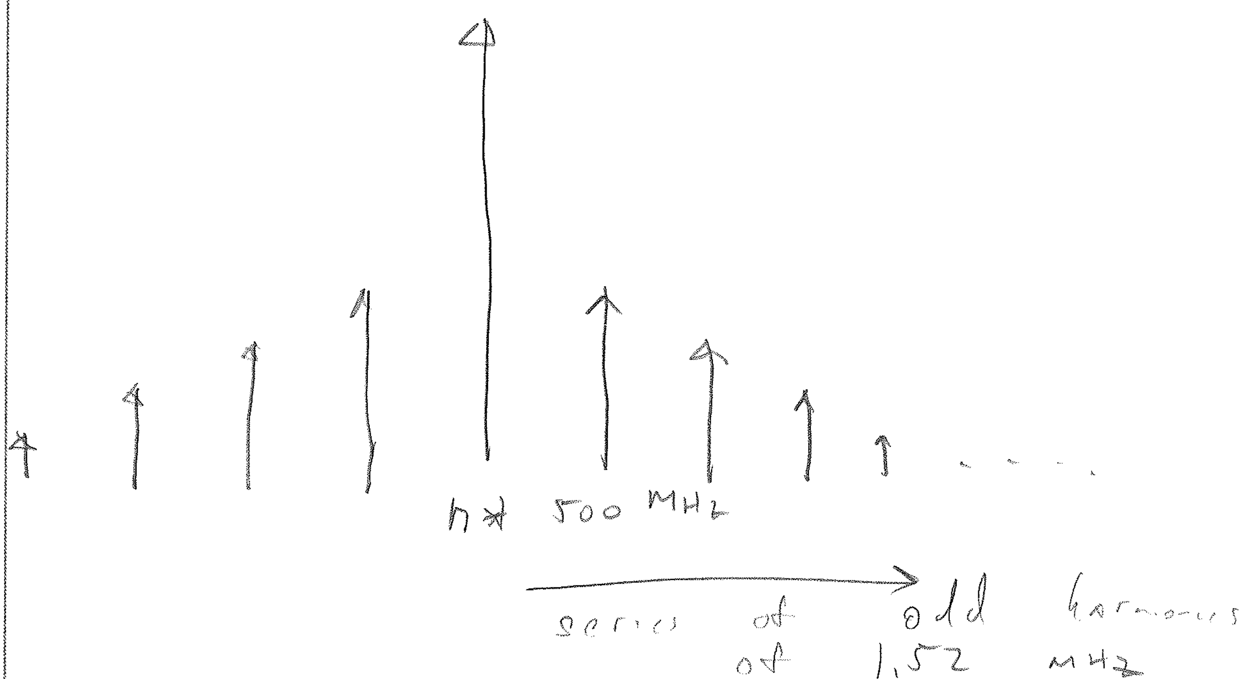
RESPONSE IS CONVOLUTION OF

CASE (b) ** F.T. ()
1.52 MHz

THE SQUARE WAVE HAS COMPONENTS at
1.52 MHz, 3* 1.52 MHz, 5* 1.52 MHz



So RESULT is like case "b",
where around each line is a set of sidebands



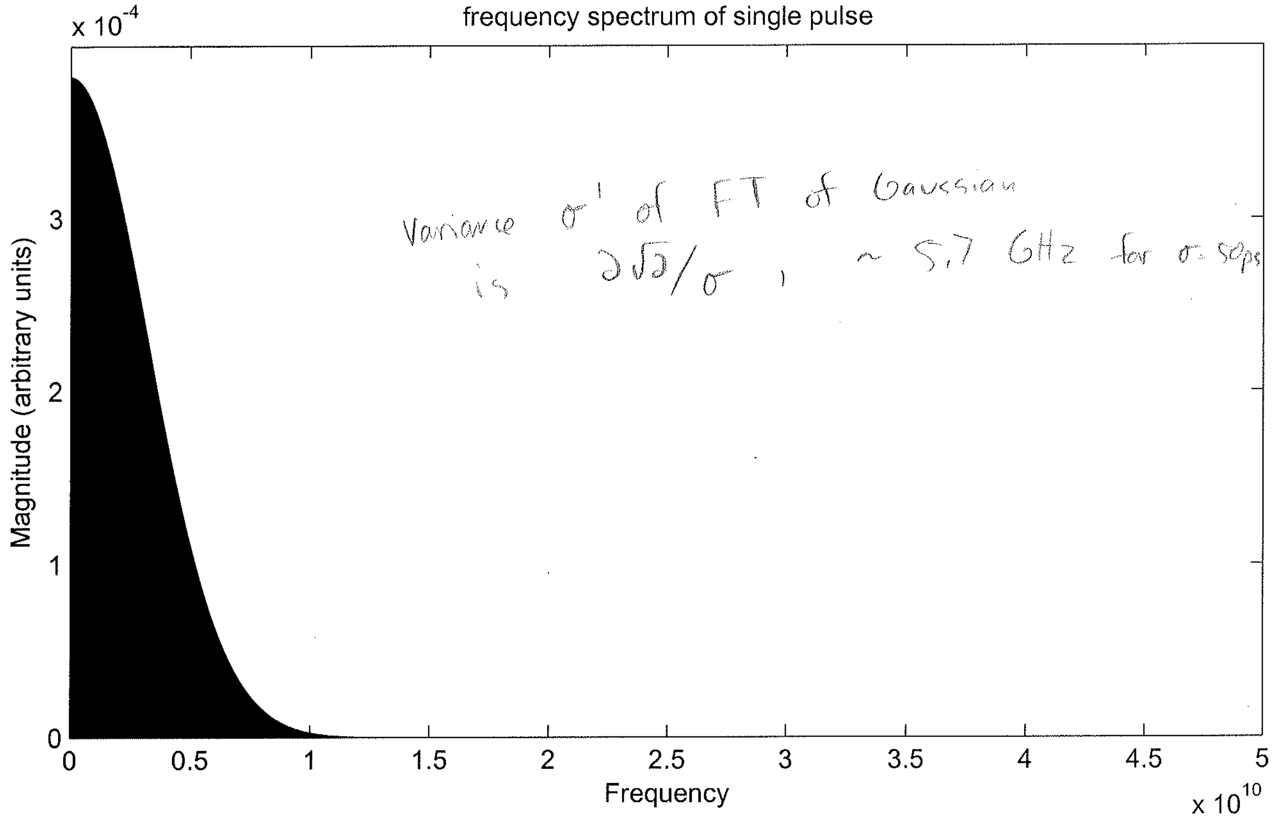
Overall envelope still falls off with the 10 GHz from the bunch length,

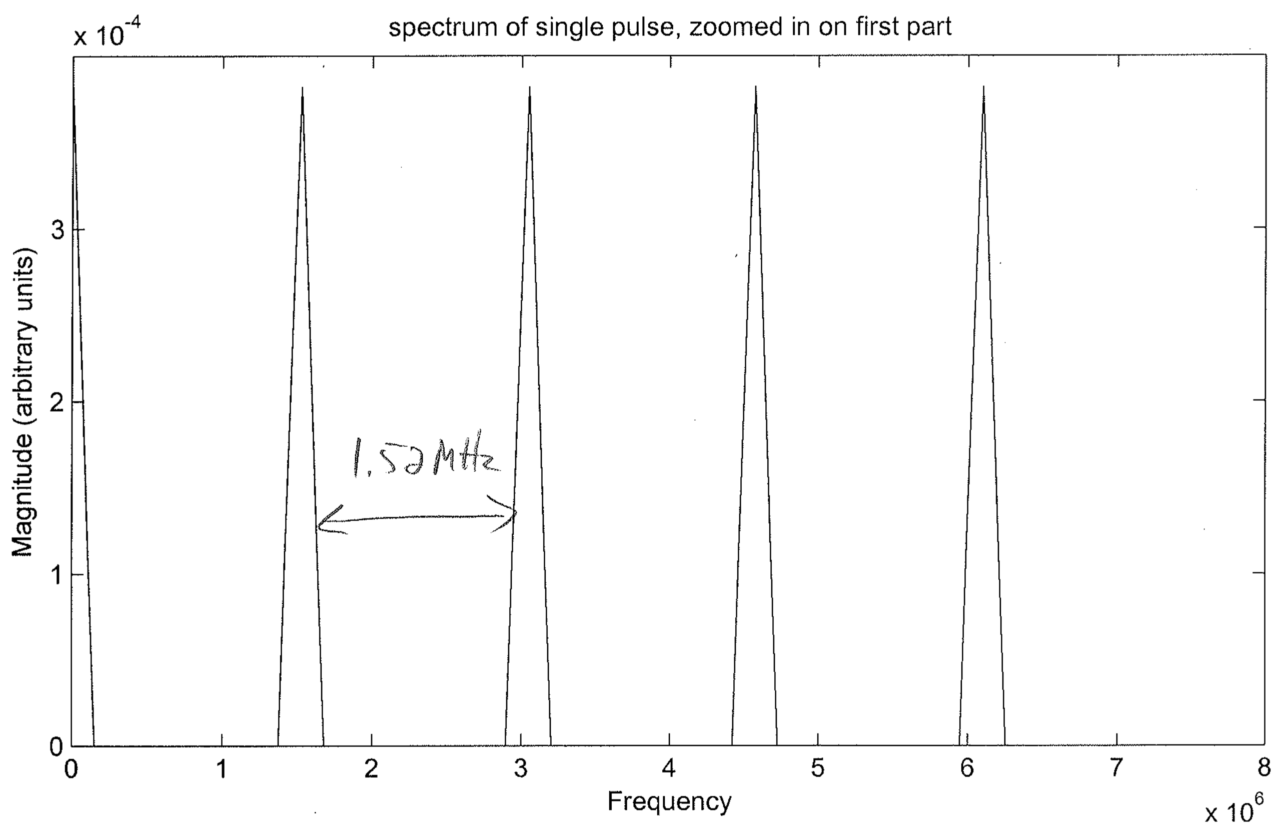
```
% Jon Daniels (jon.daniels@stanford.edu)
% solves the 1st HW problem on HW #2 for USPAS June 2005
%    John Fox's class on RF engineering and signal processing

close all;
clear all;

% basic problem: have 500 MHz RF ring with 328 buckets

% part a %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% want to calculate spectrum of single bucket
timestep = 10e-12;  % 10 ps timestep
freq = 500e6/328;
totaltime = 1/freq;
time = timestep:timestep:totaltime;
sigma = 50e-12; % sigma of gaussian pulse
center = 300e-12;   % arbitrary choice for center time of pusle
centerarray = [center];

% create signal with gaussian
signal = exp(-(time-center).^2/2/sigma^2);

% make 10 copies of the signal
% by making more copies we increase the frequency resolution
% with only one copy then FFT looks like Gaussian
% with 10 copies, the Gaussian envelope will be every 10th point,
%    and other points between will be zero, and so forth
signal2 = repmat(signal, 1, 10);

% compute the FFT based on my helpful function to clean things up
[freqs, f] = myfft(timestep, signal2);

% now plot results
figure(1);
subplot(2,1,1);plot(freqs, f);
title('frequency spectrum of single pulse');
xlabel('Frequency');
ylabel('Magnitude (arbitrary units)');
subplot(2,1,2);plot(freqs(1:50), f(1:50));
title('spectrum of single pulse, zoomed in on first part');
xlabel('Frequency');
ylabel('Magnitude (arbitrary units)');



% part b %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% want to calculate spectrum of all buckets filled
% basically just like part a except frequency of period changes
timestep = 10e-12;  % 10 ps timestep
freq = 500e6;  % note that this time don't divide by # buckets
totaltime = 1/freq;
time = timestep:timestep:totaltime;
```

```
sigma = 50e-12; % sigma of gaussian pulse
center = 300e-12;   % arbitrary choice for center time of pusle
centerarray = [center];

signal = exp(-(time-center).^2/2/sigma^2);
signal2 = repmat(signal, 1, 10);
[freqs, f] = myfft(timestep, signal2);
figure(2);
subplot(2,1,1);plot(freqs, f);
title('frequency spectrum of all buckets filled');
xlabel('Frequency');
ylabel('Magnitude (arbitrary units)');
subplot(2,1,2);plot(freqs(1:100), f(1:100));
title('spectrum of all buckets filled, zoomed in on first part');
xlabel('Frequency');
ylabel('Magnitude (arbitrary units)');


% part c %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% want to calculate spectrum of half buckets filled to 0.9,
%   second half filled to 1.1
timestep = 10e-12;  % 10 ps timestep
freq = 500e6;
totaltime = 1/freq;
time = timestep:timestep:totaltime;
sigma = 50e-12; % sigma of gaussian pulse
center = 300e-12;   % arbitrary choice for center time of pusle
centerarray = [center];

signal = exp(-(time-center).^2/2/sigma^2);

% make signal now by replicating 164 times scaled by 0.9,
%   then replicating 164 times scaled by 1.1
signal2 = [repmat(0.9*signal, 1, 164), repmat(1.1*signal, 1, 164)];
signal2 = repmat(signal2, 1, 10);
[freqs, f] = myfft(timestep, signal2);

% now plot results
figure(3);
subplot(2,1,1);plot(freqs, f);
title('frequency spectrum of all buckets filled');
xlabel('Frequency');
ylabel('Magnitude (arbitrary units)');
subplot(2,1,2);plot(freqs(1:100), f(1:100));
title('spectrum of all buckets filled, zoomed in on first part');
xlabel('Frequency');
ylabel('Magnitude (arbitrary units)');
```

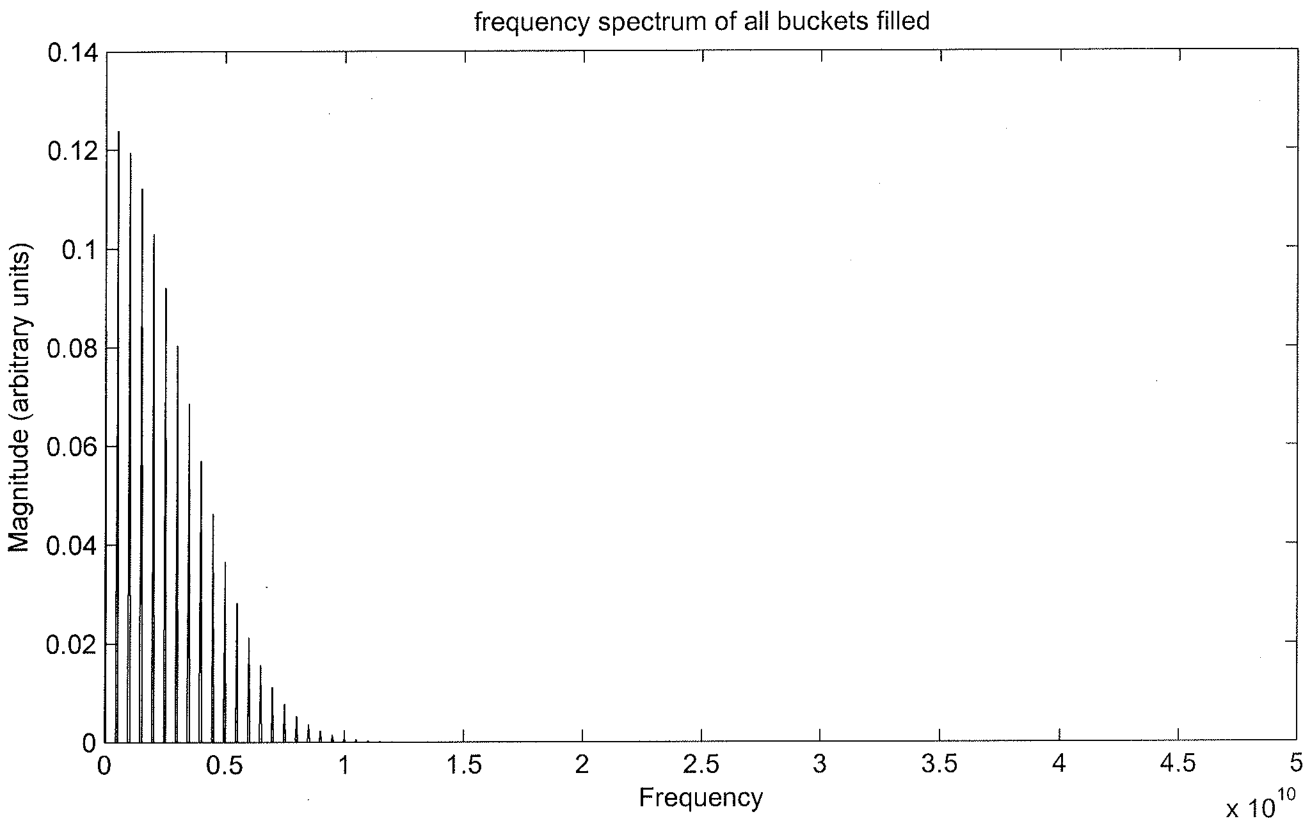
```
% Jon Daniels (jon.daniels@stanford.edu)
% returns a "nice" FFT of a real-valued input signal along with the
%    the corresponding frequency scale
% since the input data is real (assumed), the FFT is conjuate symmetric

function [freqs, result] = myfft(timestep, array)
  % compute the FFT, shifted and normalized
  a = fftshift(fft(array))/length(array);
  % now DC component is in center
  cen = floor(length(a)/2)+1;
  neg = a(cen-1:-1:1);
  pos = a(cen+1:1:end);
  % if even number of terms, pad the positive matrix
  % I'm not sure if correct to pad with 0 or conj of negative value
  if length(pos) < length(neg)
      pos = [pos, conj(neg(end))];
  end
  result = abs([a(cen)*2, pos+conj(neg)]);
  % now compute the freq vector
  spacing = 1/(timestep*length(array));
  freqs = 0:spacing:spacing*(length(result)-1);
end
```

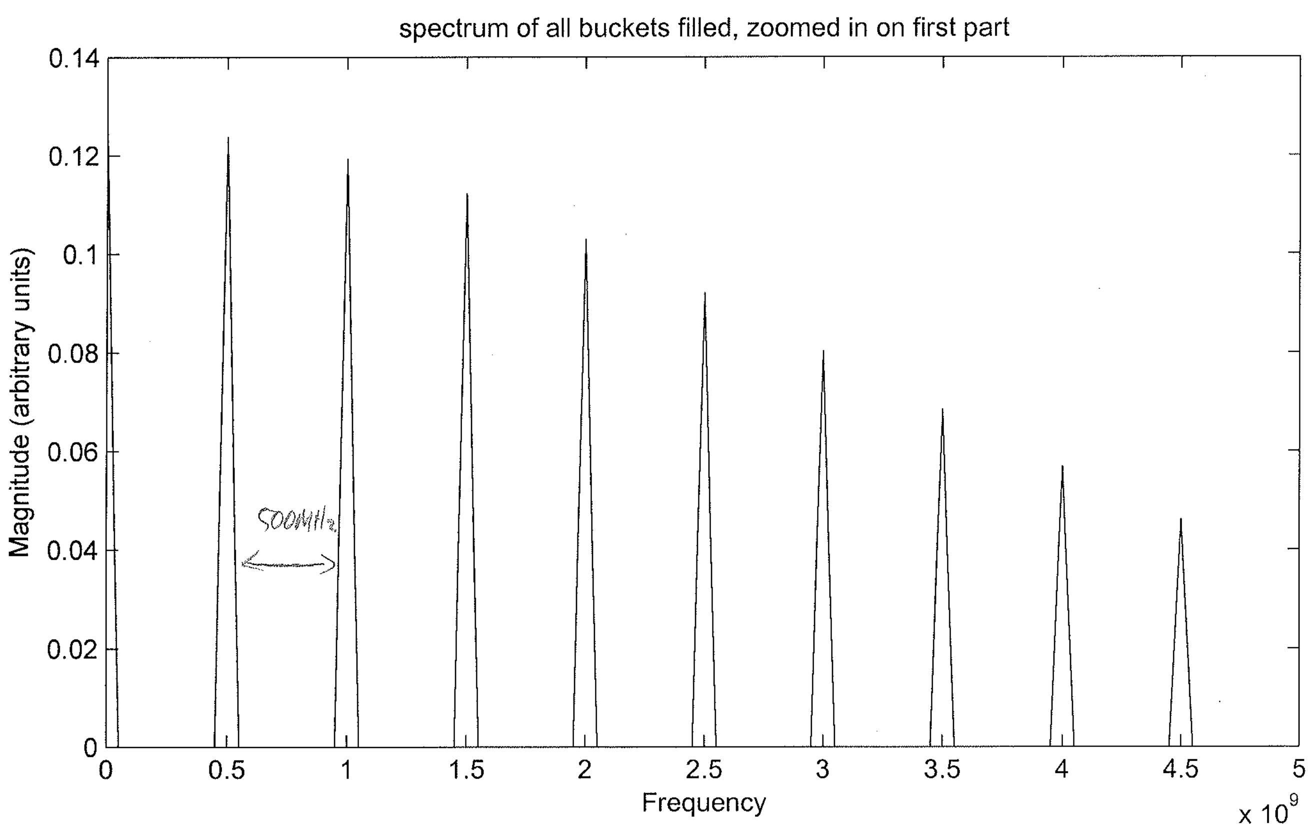
Part A


x 10^-4
frequency spectrum of single pulse
Variance σ' of FT of Gaussian
is 2√2/σ, ~ 5.7 GHz for σ=50ps
Magnitude (arbitrary units)
Frequency
x 10^10

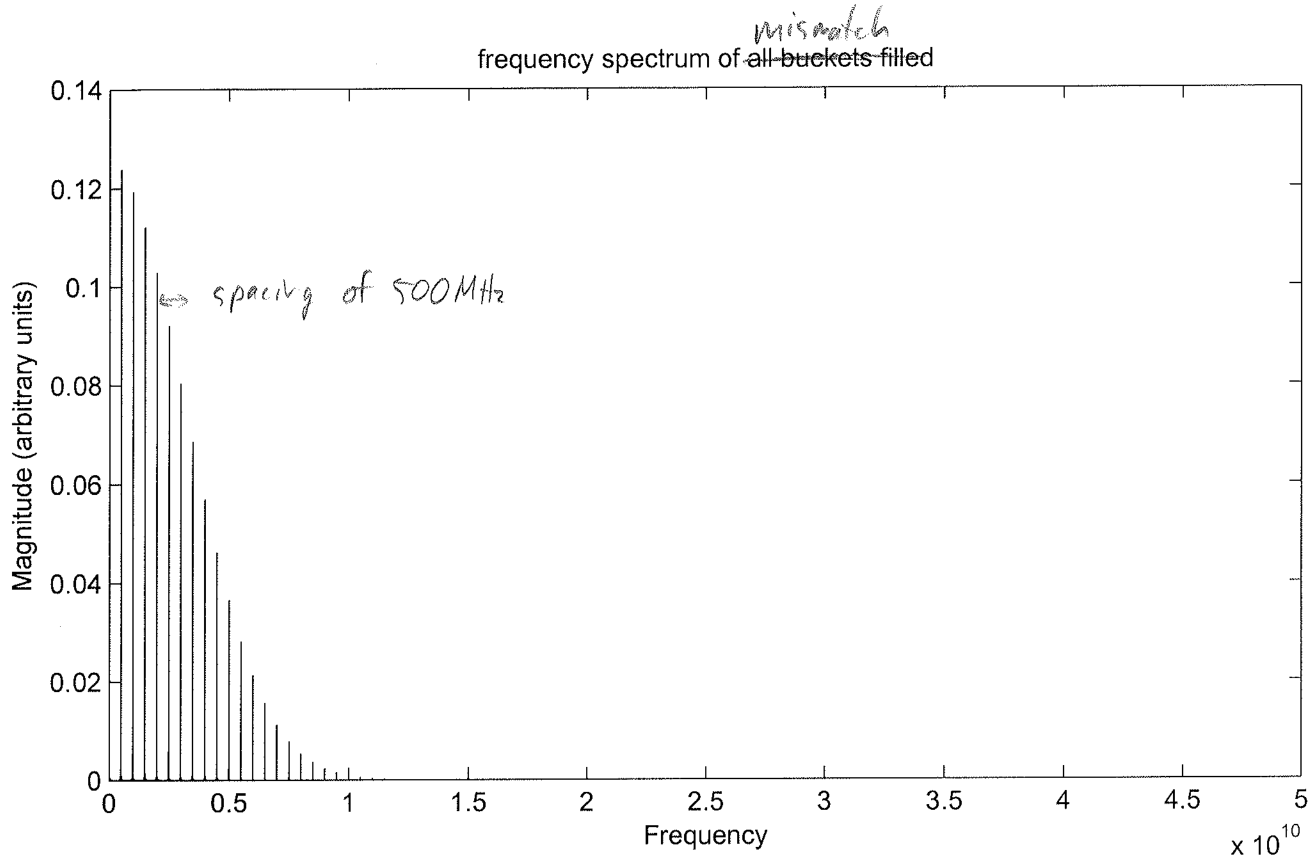

x 10^-4
spectrum of single pulse, zoomed in on first part
1.52 MHz
Magnitude (arbitrary units)
Frequency
x 10^6

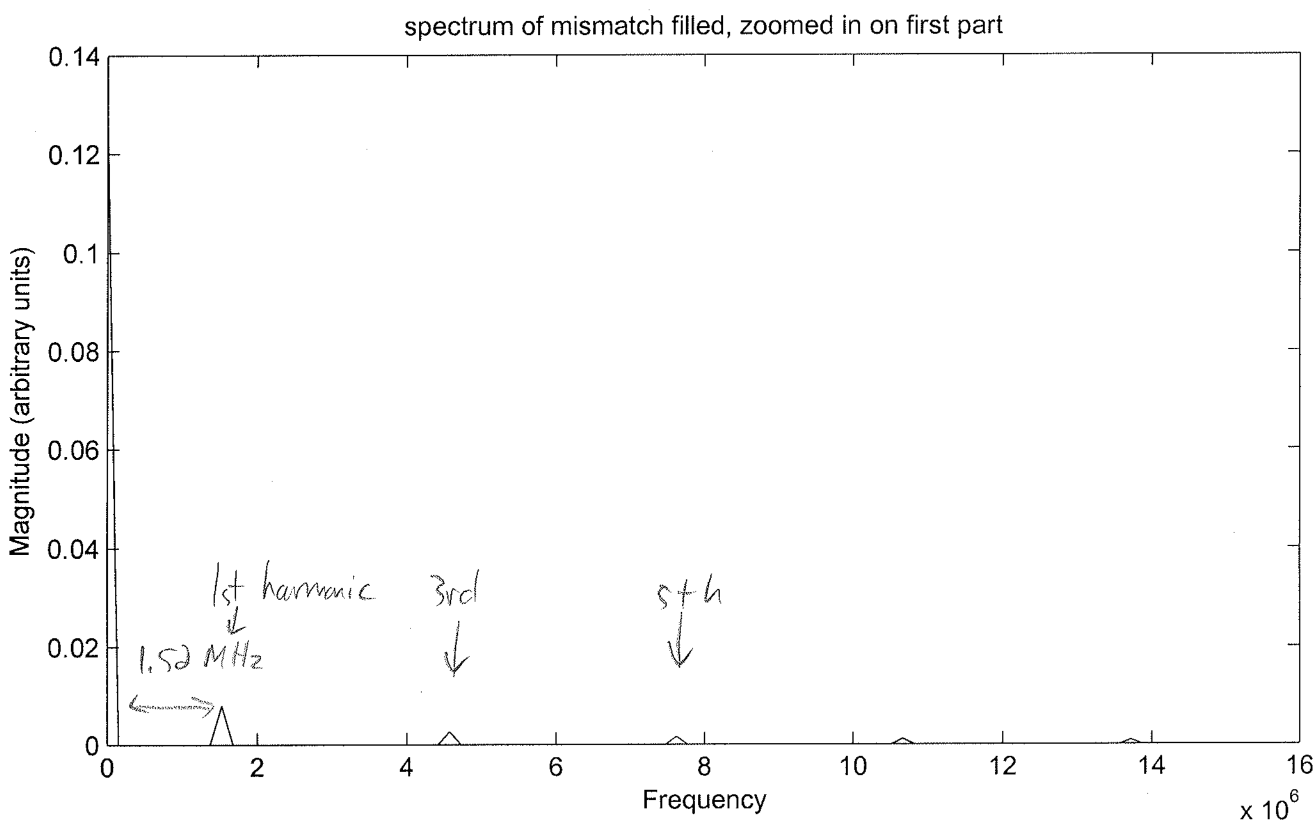
Part B


frequency spectrum of all buckets filled
Magnitude (arbitrary units)
Frequency
x 10^10


spectrum of all buckets filled, zoomed in on first part
Magnitude (arbitrary units)
500MHz
Frequency
x 10^9

Part C


frequency spectrum of ~~all buckets filled~~ mismatch
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Magnitude (arbitrary units)
spacing of 500MHz
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
Frequency
x 10^10


spectrum of mismatch filled, zoomed in on first part
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Magnitude (arbitrary units)
1st harmonic
1.52 MHz
3rd
5th
0 2 4 6 8 10 12 14 16
Frequency
x 10^6

Figures

Figues and Notes for Problem #2 HW #1

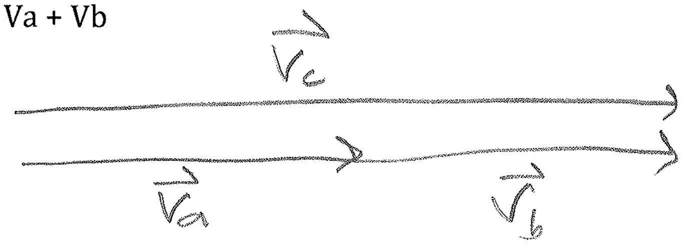
USPAS 2017

J. Fox

Case a) Two cavities in phase

Consider the effective cavity voltage

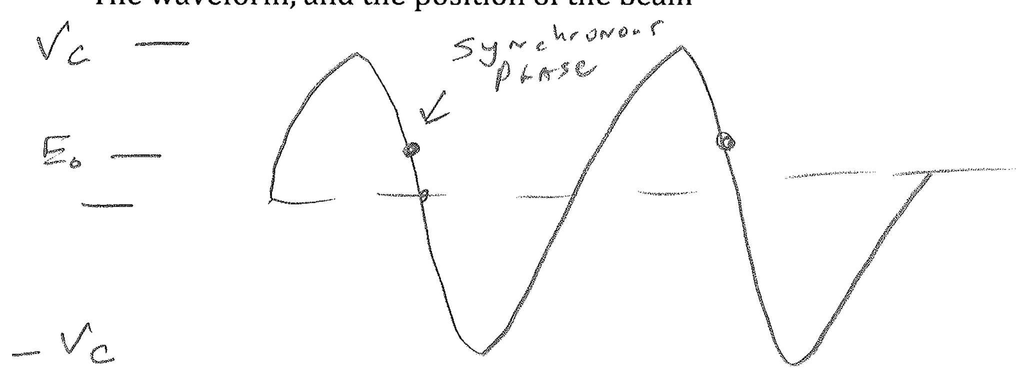
$\vec{Vc} = \vec{Va} + \vec{Vb}$



In this case

$|V_c| = |V_a| + |V_b|$

The waveform, and the position of the beam



Where E0=energy loss/turn 92keV

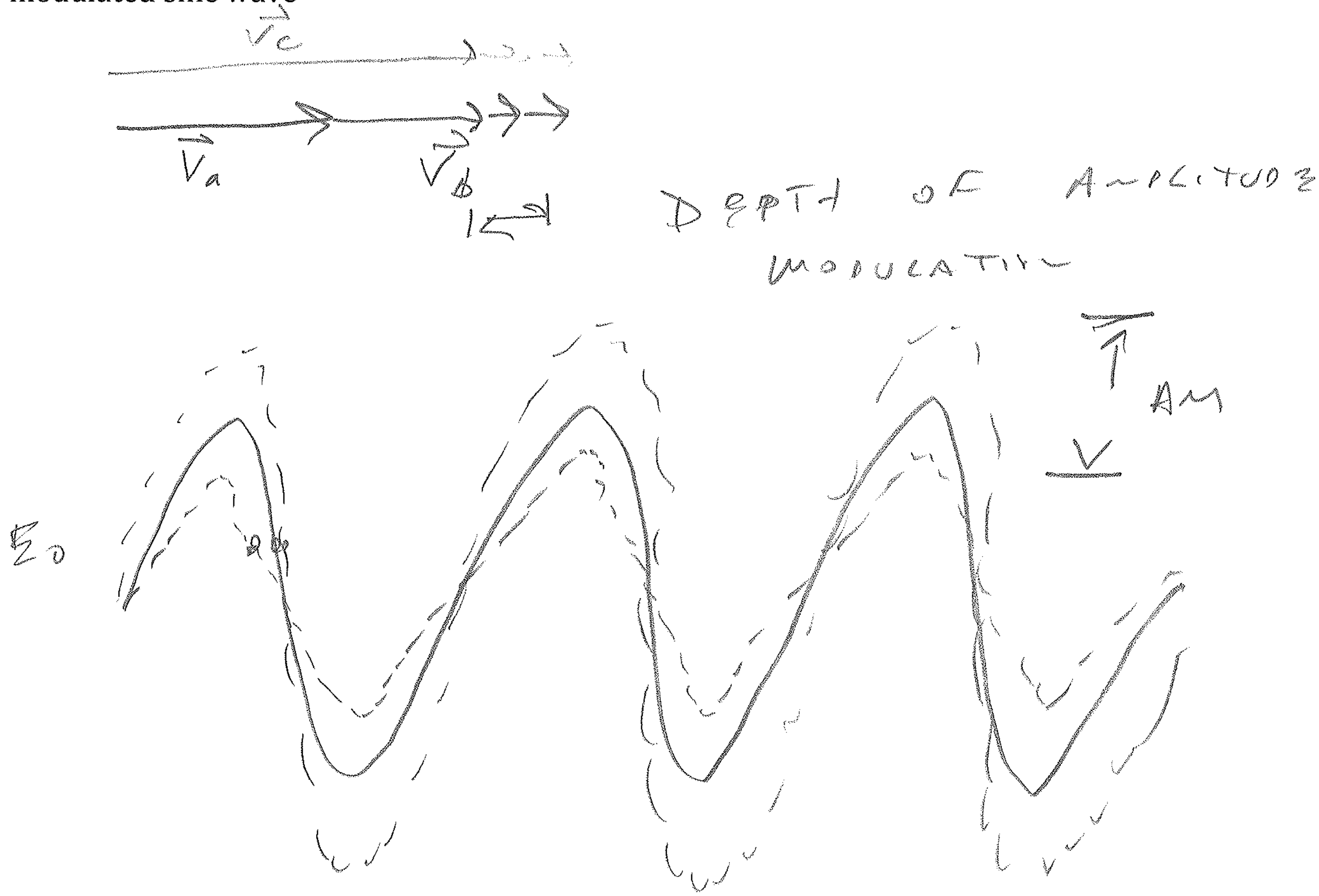
Vc= 1.1MV

The beam finds the condition where ϕ_s

$$V_c \sin(\phi_s) = E_0$$

Negative slope is stable above transition

Part b) If the RF Cavity has amplitude modulation (say 5% at 720 Hz) – one vector gets longer and shorter with time. The resulting vector sum of cavity voltage is an amplitude modulated sine wave

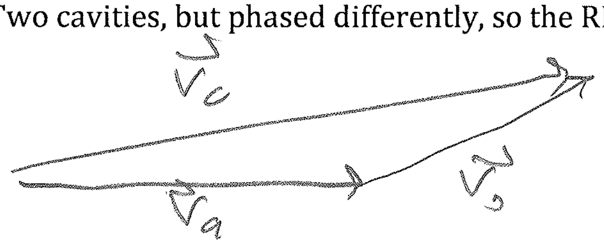


The beam still needs to be at the right synchronous voltage, 92 keV, so the beam undergoes a phase modulation at the 720 Hz modulation frequency. The bigger the amplitude modulation, the bigger the phase modulation and shift of synchronous phase at the modulation frequency.

If a cavity is Phase Modulated, think of the vector sum and note the total RF voltage is now also modulated in amplitude and phase.

Case c)

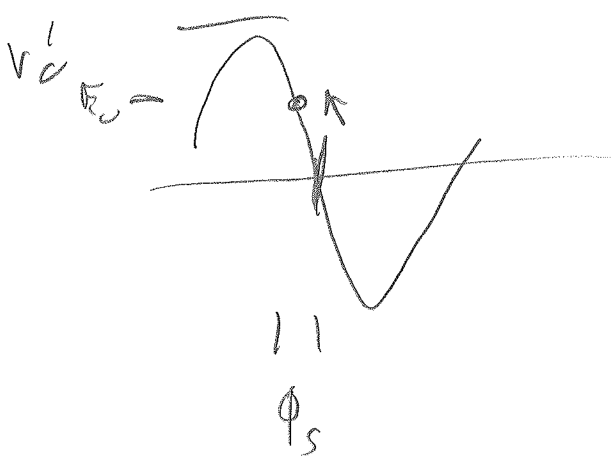
Two cavities, but phased differently, so the RF vector sum of the two cavity fields is like this:



Note that $|V_c| < |V_a| + |V_b|$

So the effective voltage in the RF system is now less than the perfectly phased cavities. The beam still needs the same E0, 92 keV, so it moves to find a new phase position higher up on the accelerating waveform (the total voltage is now lower)

$V_c' < |V_a| + |V_b|$



THE BEAM MOVES UP HIGHER TO GET TO E_0

J. Fox
1-25-04
6/28/05

-10 dB ATTENUATOR

1)

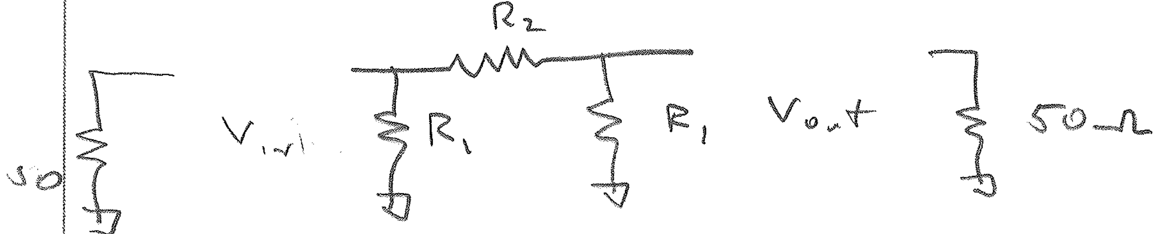
$$dB = 20 \log \left(\frac{V_{out}}{V_{IN}} \right)$$

OR $$V_{out} = \frac{1}{\sqrt{10}} V_{IN}$$

CONSIDER THE BASIC π SECTION

(THE T SECTION IS WORKED OUT IN POLAR SECTION 4.3, EXAMPLE 4.4)

BASIC FORM

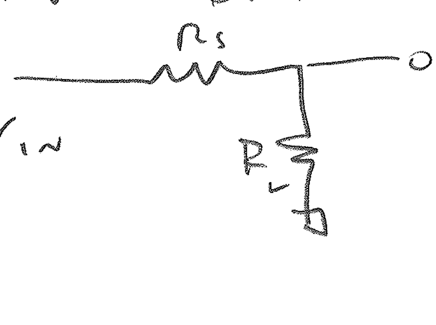


$\rightarrow$ $Z_{IN} = 50\Omega$

$\leftarrow$ $Z_{IN} = 50\Omega$

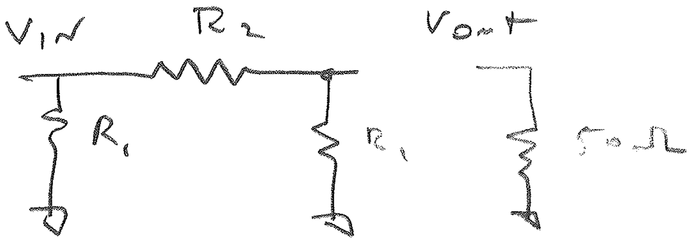
WE WANT $V_{out} = \frac{1}{\sqrt{10}} V_{IN}$

VOLTAGE DIVIDER EQUATION

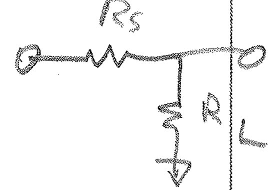


$$V_{out} = \frac{R_L}{R_S + R_L} V_{IN}$$

JF
1/25/04



THINK OF A VOLTAGE DIVIDER



$$\frac{50\Omega // R_1}{R_2 + 50\Omega // R_1} = \frac{1}{\sqrt{10}}$$

AND CONSIDER THE INPUT, OUTPUT Z

$$R_1 // (R_2 + 50\Omega // R_1) = 50\Omega$$

THIS IS TWO EQUATIONS, 2 UNKNOWNS

IN GENERAL, FOR ARBITRARY COUPLING FACTOR K, INPUT/OUTPUT IMPEDANCE RIMAGE

$$R_1 = R_{IMAGE} \frac{K+1}{K-1}$$

$$R_2 = R_{IMAGE} \frac{(K^2-1)}{2K}$$

K^2 = POWER RATIO

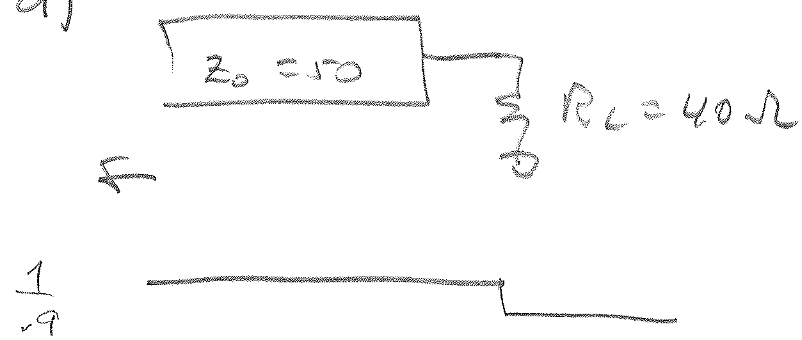
K = VOLTAGE RATIO

So $\boxed{\begin{array}{l} R_1 = 96\,\Omega \\ R_2 = 71\,\Omega \end{array}}$ IS A SOLUTION

As to "T" vs "π" - both are implementable. For some imaginable situations one may have more convenient values than the other.

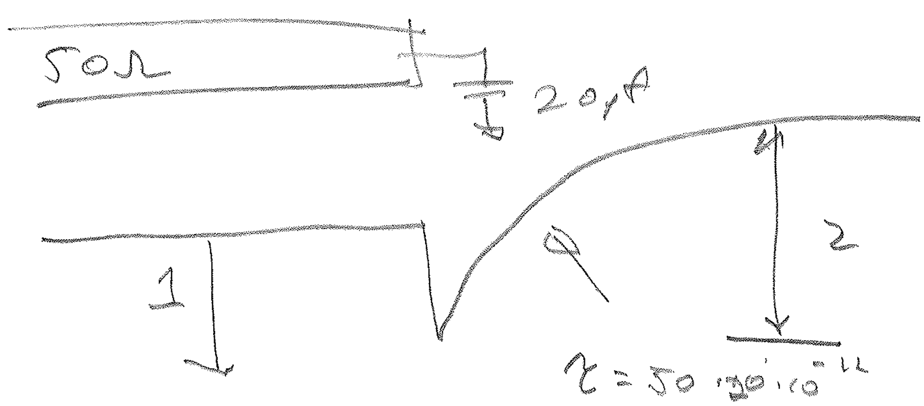
#3 TDR Problems

a)



$$\rho = \frac{40-50}{40+50} = -\frac{1}{9}$$

b)

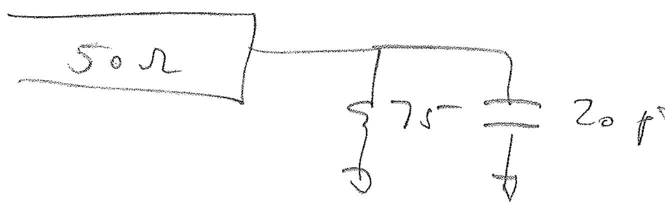


$\tau = 50 \cdot 20 \cdot 10^{-12}$

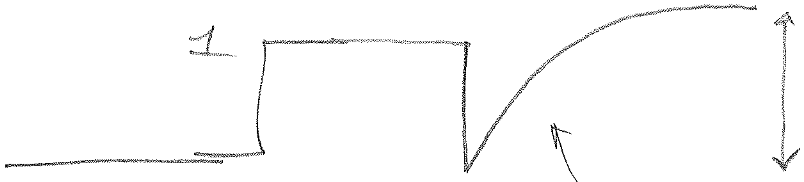
Looks like a short at first, then an open, so goes to 2 x initial amplitude

RC is $50\,\Omega$ and 20 pF

c)



Like "b" but final voltage is from $\frac{50-75}{}$ match

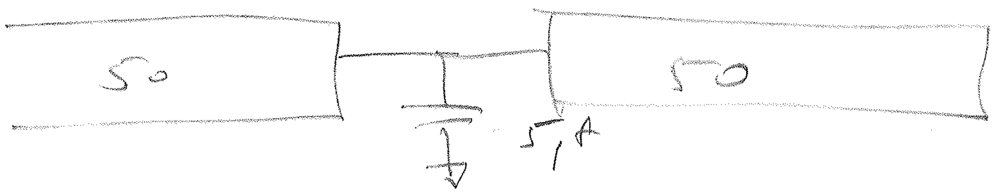


$1 + \frac{25}{125}$

$\tau = (50 \| 75)\, 20\,pF$

C looks like a short

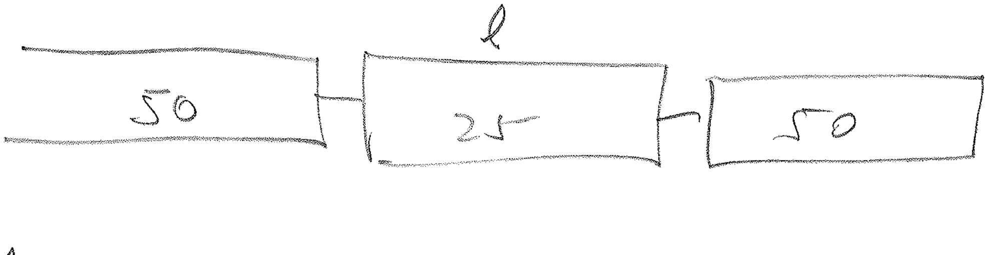
d.

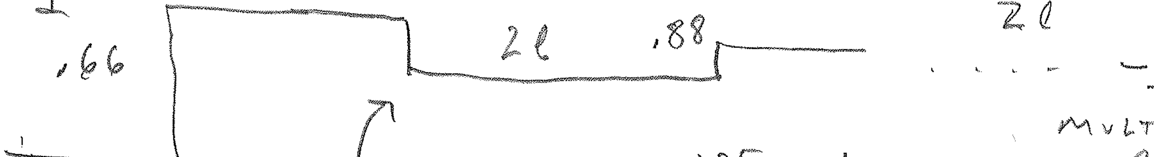


Like "c" but matched

$\tau = (50 \| 50)\ 5\,pF$

e





$-\frac{25}{75}$

$\frac{+25}{75} * .66 = .22$

Multiple reflections occur

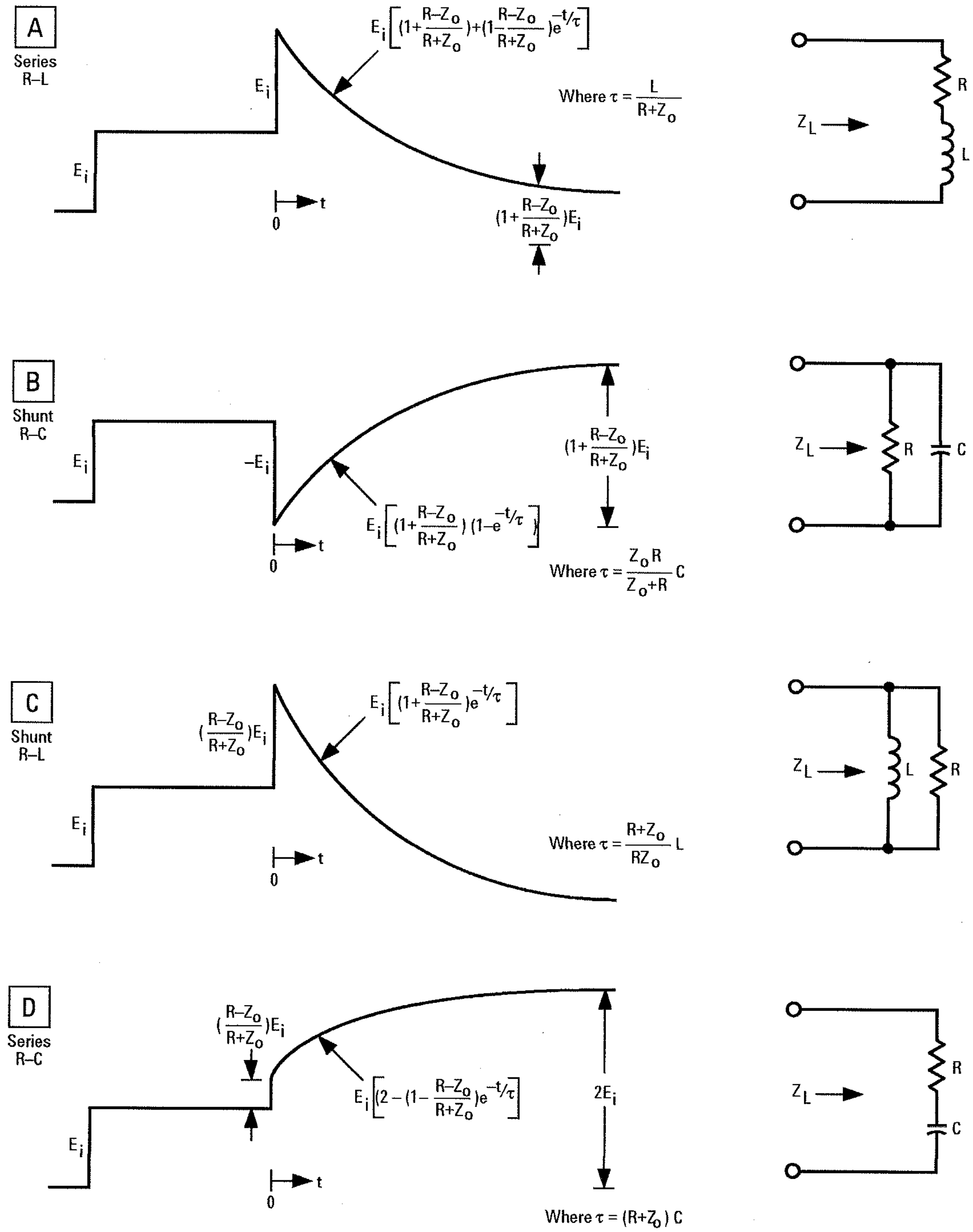


Figure 8. Oscilloscope displays for complex Z_L.

Going to an equivalent circuit (Figure 13) valid at $t = 0^+$.

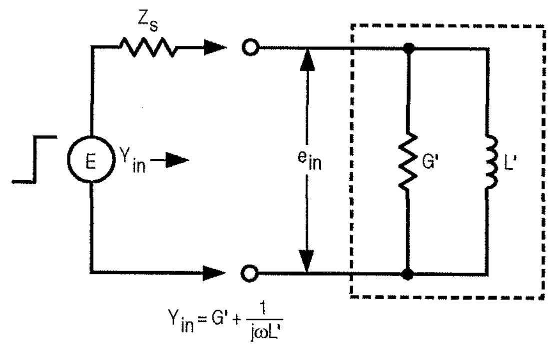


Figure 13. A simple model valid at $t = 0^+$ for a line with shunt losses

A qualitative interpretation of why $e_{in}(t)$ behaves as it does is quite simple in both these cases. For series losses, the line looks more and more like an open circuit as time goes on because the voltage wave traveling down the line accumulates more and more series resistance to force current through. In the case of shunt losses, the input eventually looks like a short circuit because the current traveling down the line sees more and more accumulated shunt conductance to develop voltage across.

Multiple Discontinuities

One of the advantages of TDR is its ability to handle cases involving more than one discontinuity. An example of this is Figure 14.

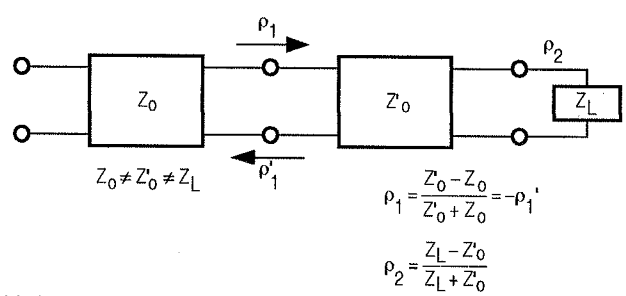


Figure 14. Cables with multiple discontinuities

The oscilloscope's display for this situation would be similar to the diagram in Figure 15 (drawn for the case where $Z_L < Z_o < Z'_o$:

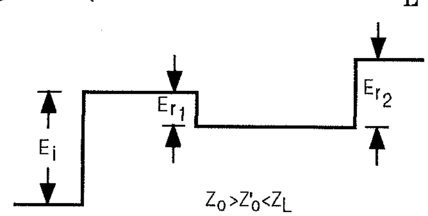


Figure 15. Accuracy decreases as you look further down a line with multiple discontinuities

It is seen that the two mismatches produce reflections that can be analyzed separately. The mismatch at the junction of the two transmission lines generates a reflected wave, E_{r^1}, where

$$E_{r^1} = \rho_1 E_i = \left(\frac{Z'_o - Z_o}{Z'_o + Z_o}\right) E_i$$

Similarly, the mismatch at the load also creates a reflection due to its reflection coefficient

$$\rho_2 = \frac{Z_L - Z'_o}{Z_L + Z'_o}$$

Two things must be considered before the apparent reflection from Z_L, as shown on the oscilloscope, is used to determine ρ2. First, the voltage step incident on Z_L is $(1 + \rho_1) E_i$, not merely E_i. Second, the reflection from the load is

$$[\ \rho_2 (1 + \rho_1) E_i\ ] = E_{r_L}$$

but this is not equal to E_{r_2} since a re-reflection occurs at the mismatched junction of the two transmission lines. The wave that returns to the monitoring point is

$$E_{r_2} = (1 + \rho_1') E_{r_L} = (1 + \rho_1') [\ \rho_2 (1 + \rho_1) E_i\ ]$$

Since $\rho_1' = -\rho_1$, E_{r_2} may be re-written as:

$$E_{r_2} \quad E_{r_2} = [\ \rho_2\ (1 - \rho_1 2)\ ]\ Ei$$

The part of E_{r_L} reflected from the junction of

$$E_{r_L} \quad Z'_o \text{ and } Z_o \ \text{(i.e., } \rho_1' E_{r_L})$$

is again reflected off the load and heads back to the monitoring point only to be partially reflected at the junction of Z_o' and Z_o. This continues indefinitely, but after some time the magnitude of the reflections approaches zero.

In conclusion, this application note has described the fundamental theory behind time domain reflectometry. Also covered were some more practical aspects of TDR, such as reflection analysis and oscilloscope displays of basic loads. This content should provide a strong foundation for the TDR neophyte, as well as a good brush-up tutorial for the more experienced TDR user.

PAC 03

J.F.

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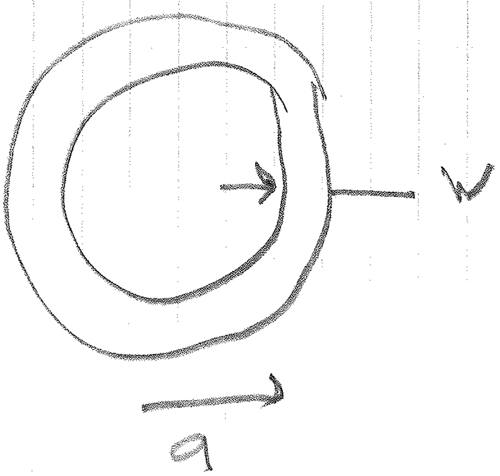

dielectric $\epsilon = \epsilon_r$
thickness d

microstrip resonator

System is resonant if
circumference is $n\lambda$
Long

FIRST RESONANCE $2\pi a = \lambda$

for microstrip

$$v_p = \frac{c}{\sqrt{\epsilon_e}}$$

$$\beta = k_0 \sqrt{\epsilon_e}$$

for resonance

$$2\pi = \beta 2\pi a = \sqrt{\epsilon_e}\, k_0\, 2\pi a$$

$$k_0 = \frac{2\pi f}{c}$$

or

$$\cancel{2\pi} = \sqrt{\epsilon_e}\, \frac{2\pi}{c} f\, \cancel{2\pi} a$$

$$f = \frac{c}{2\pi\sqrt{\epsilon_e}\; a} \quad \text{where } \epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\,\frac{1}{\sqrt{1 + 12d/w}}$$

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J. Fox

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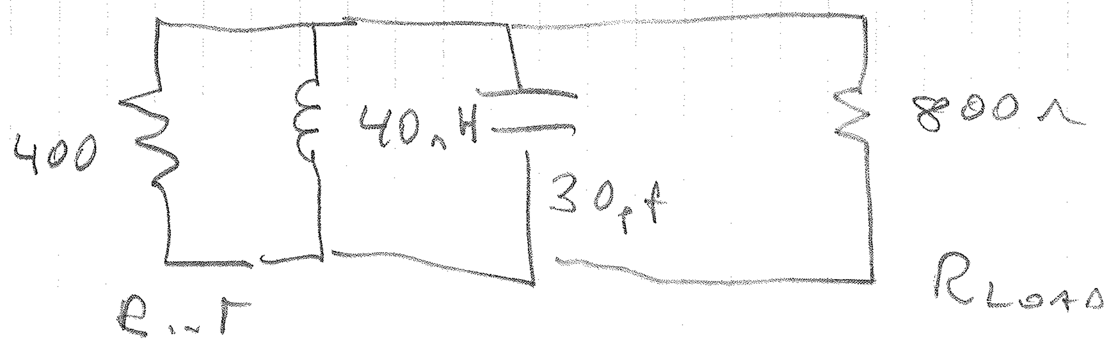

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{40\cdot 10^{-9}\cdot 30\cdot 10^{-12}}} = 9.1\cdot 10^{8} \text{ RAD/SEC}$$

$$\boxed{(145 \text{ MHz})}$$

$$Q_{int} = \omega_0 R_I C = 9.1\cdot 10^{8}\cdot 400\cdot 30\cdot 10^{-12}$$

$$\boxed{Q_{int} = 10.9}$$

$$Q_{ext} = \omega_0 R_L C = \boxed{21.8}$$

$$Q_{TOTAL} = \frac{1}{\frac{1}{Q_{int}} + \frac{1}{Q_{ext}}} = \underline{7.3}$$

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