Unit 10
Electro-magnetic forces and stresses in superconducting accelerator magnets

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With significant re-use of material from the same unit lecture by Ezio Todesco, USPAS 2017

Outline

1. Introduction
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5. Stored energy and end forces
6. Stress and strain
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8. Conclusions

References

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Introductory comments

- Superconducting accelerator magnets are characterized by high fields and high current densities.
- As a results, the coil is subjected to strong electro-magnetic forces, which tend to move the conductor and deform the winding.
- The resulting deformations can affect field quality and/or serve as energy sources in the "disturbance spectrum"
- A good knowledge of the magnitude and direction of the electro-magnetic forces, as well as of the stress of the coil, is mandatory for the mechanical design of a superconducting magnet.
- In this unit we will describe the effect of these forces on the coil through simplified approximation of practical winding, and we will introduced the concept of pre-stress, as a way to mitigate their impact on magnet performance.

Recap

- In the presence of a magnetic field B, an electric charged particle q in motion with a velocity v is acted on by a force F_L called electro-magnetic (Lorentz) force [N]:

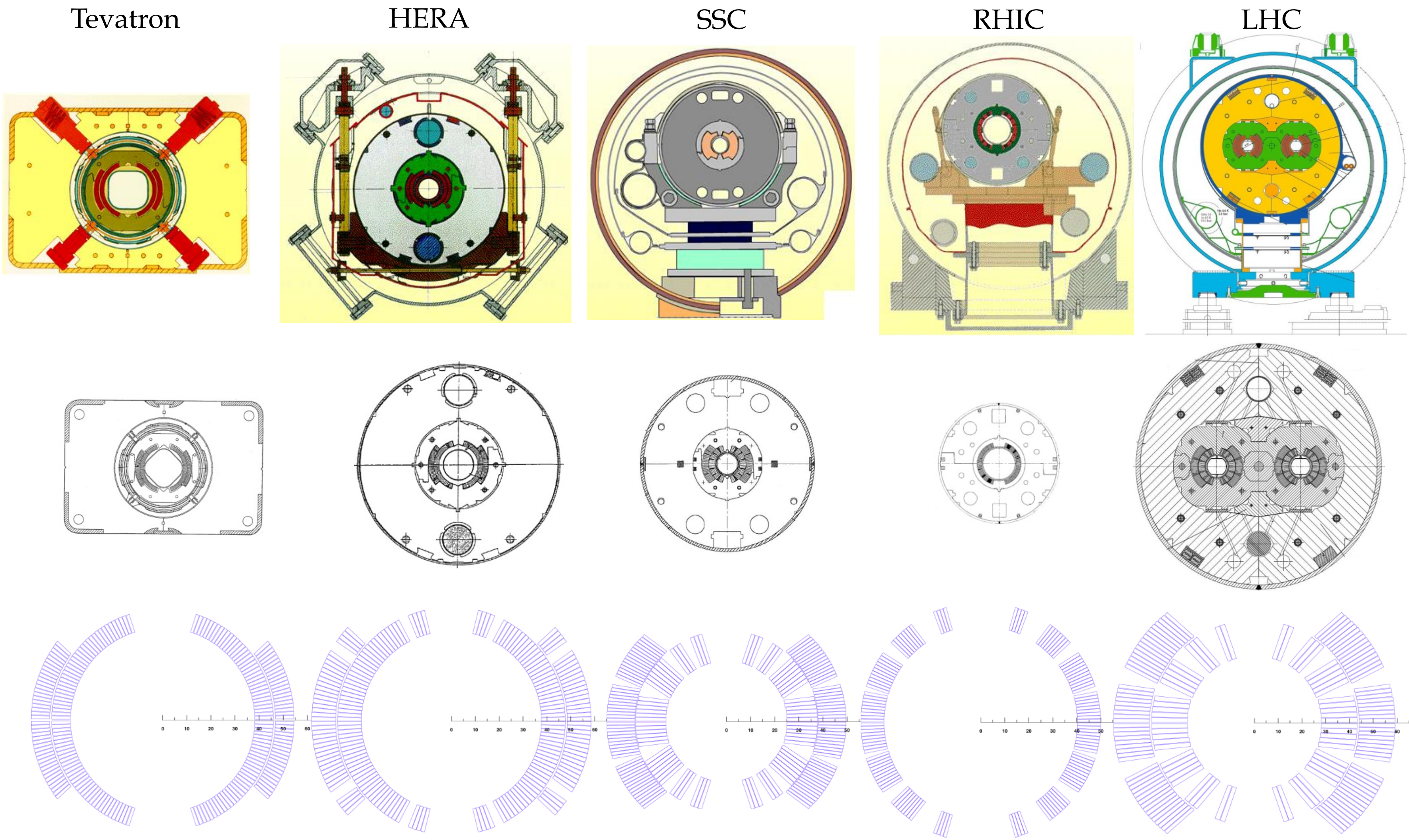
$$\vec{F}_L = q\vec{v} \times \vec{B}$$

- A conductor element carrying current density J (A/mm^2) is subjected to a force density f_L [N/m^3]

$$\vec{f}_L = \vec{J} \times \vec{B}$$

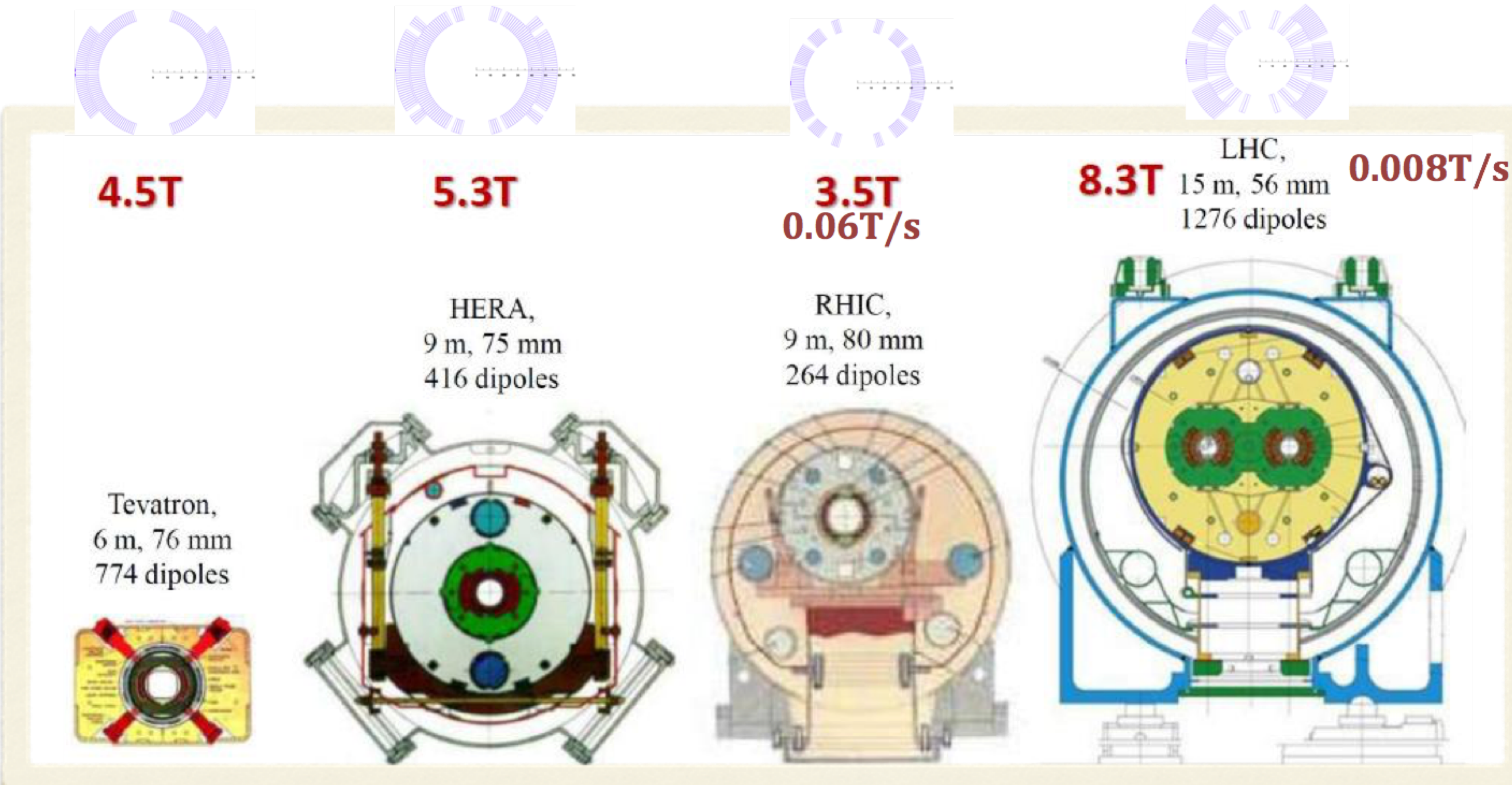
- The e.m. force acting on a coil is a body force, i.e. a force that acts on all the portions of the coil (like the gravitational force).

Overview of accelerator dipole magnets

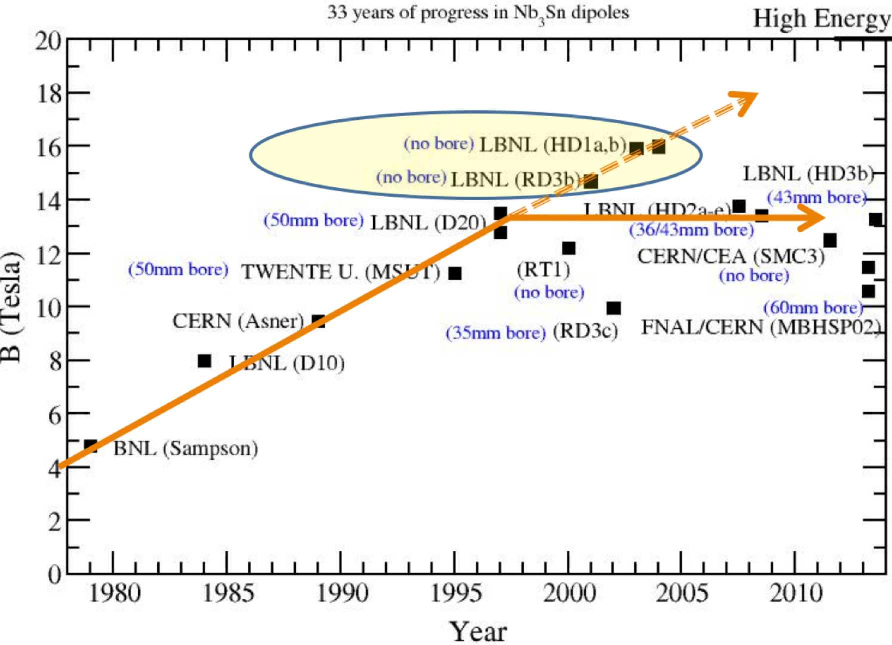


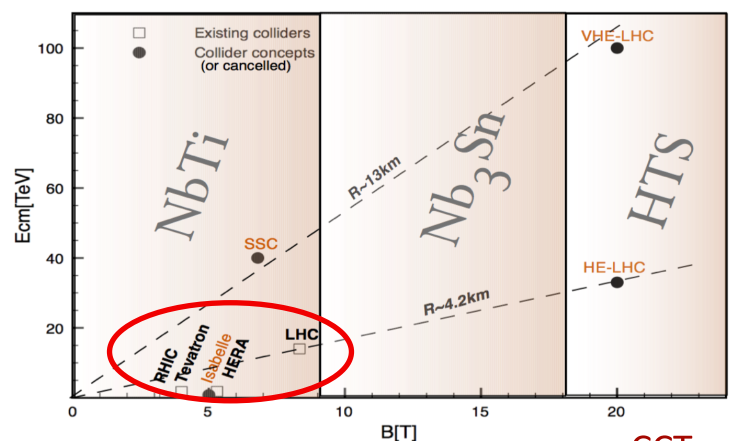
All magnets in colliders to-date are based on the cos(θ) concept

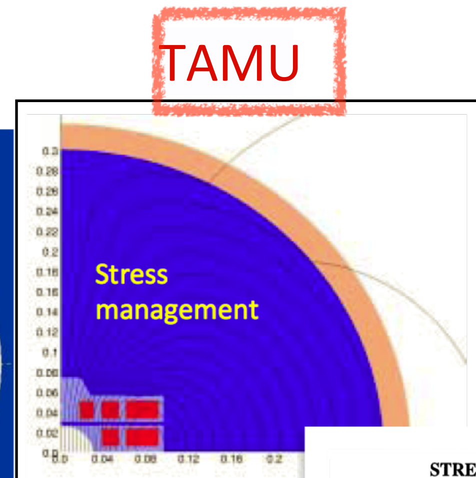
$$E_{mag} \propto r_0^2 B^2$$

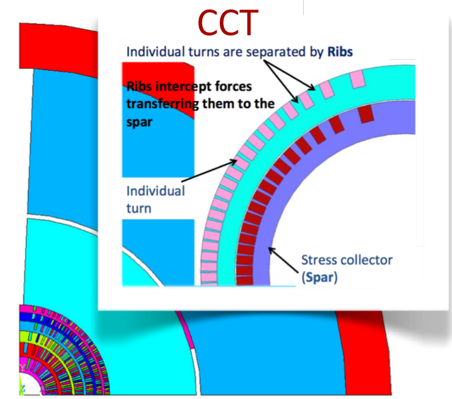


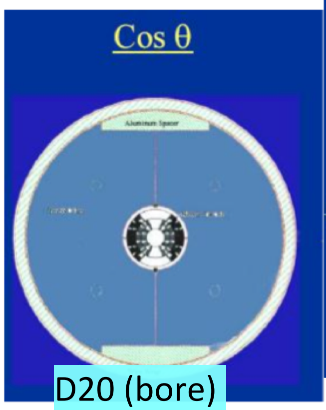
For high-field dipoles a number of concepts have been, and are being, investigated

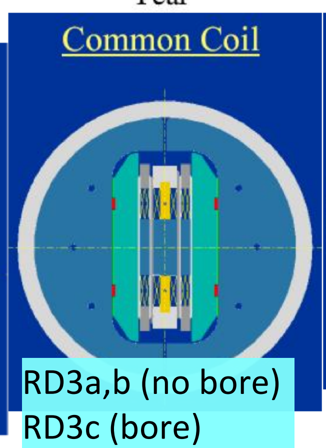


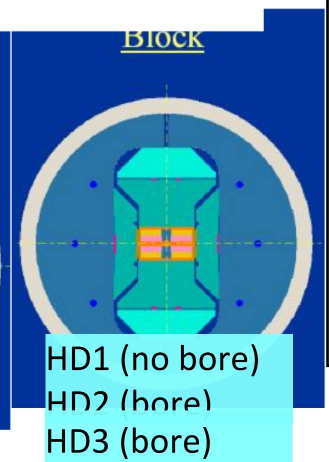


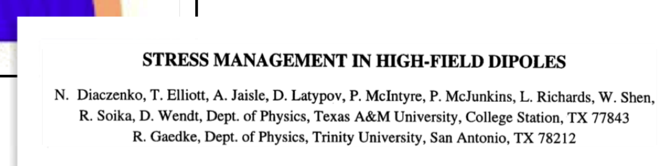











STRESS MANAGEMENT IN HIGH-FIELD DIPOLES

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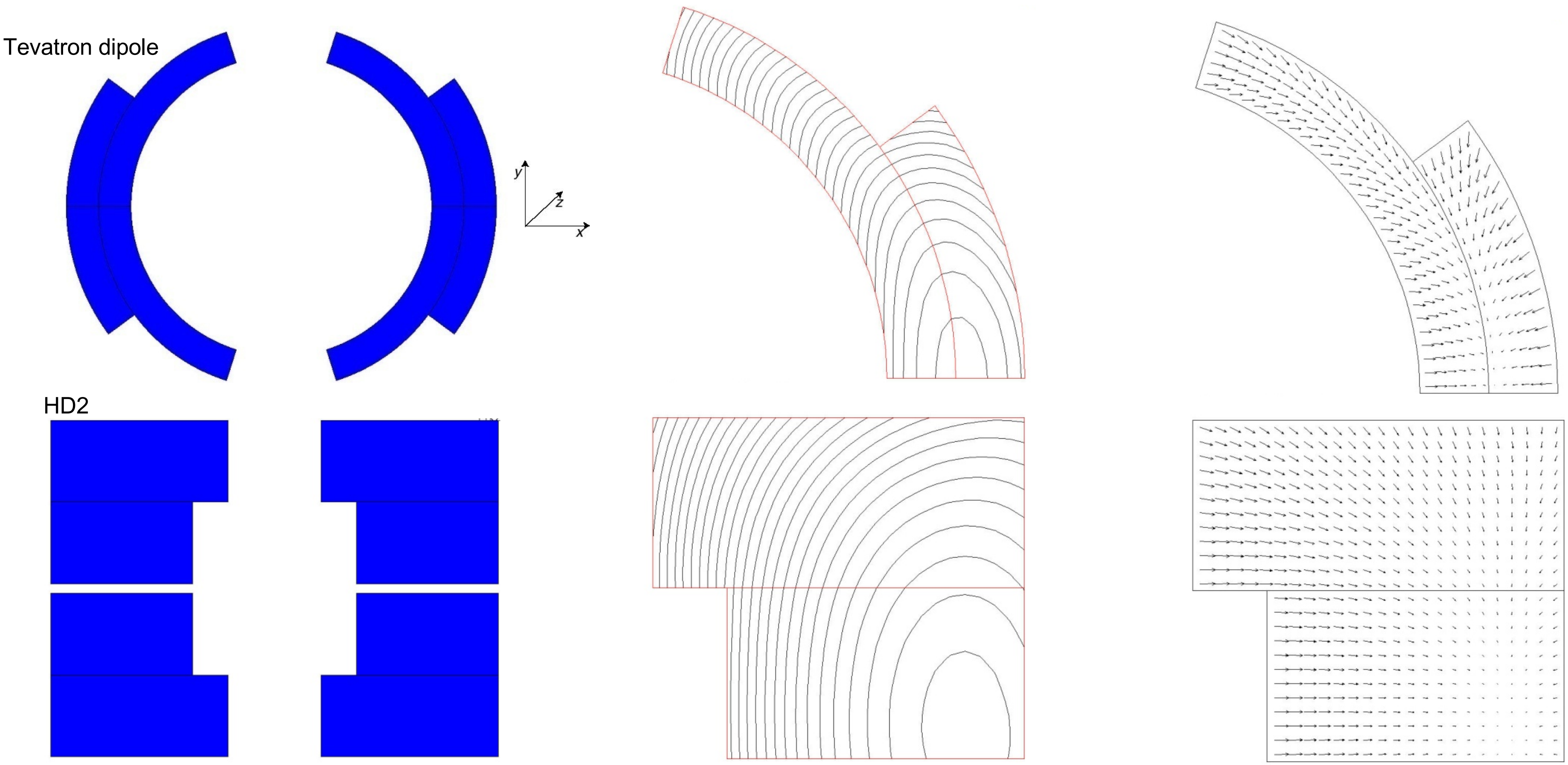
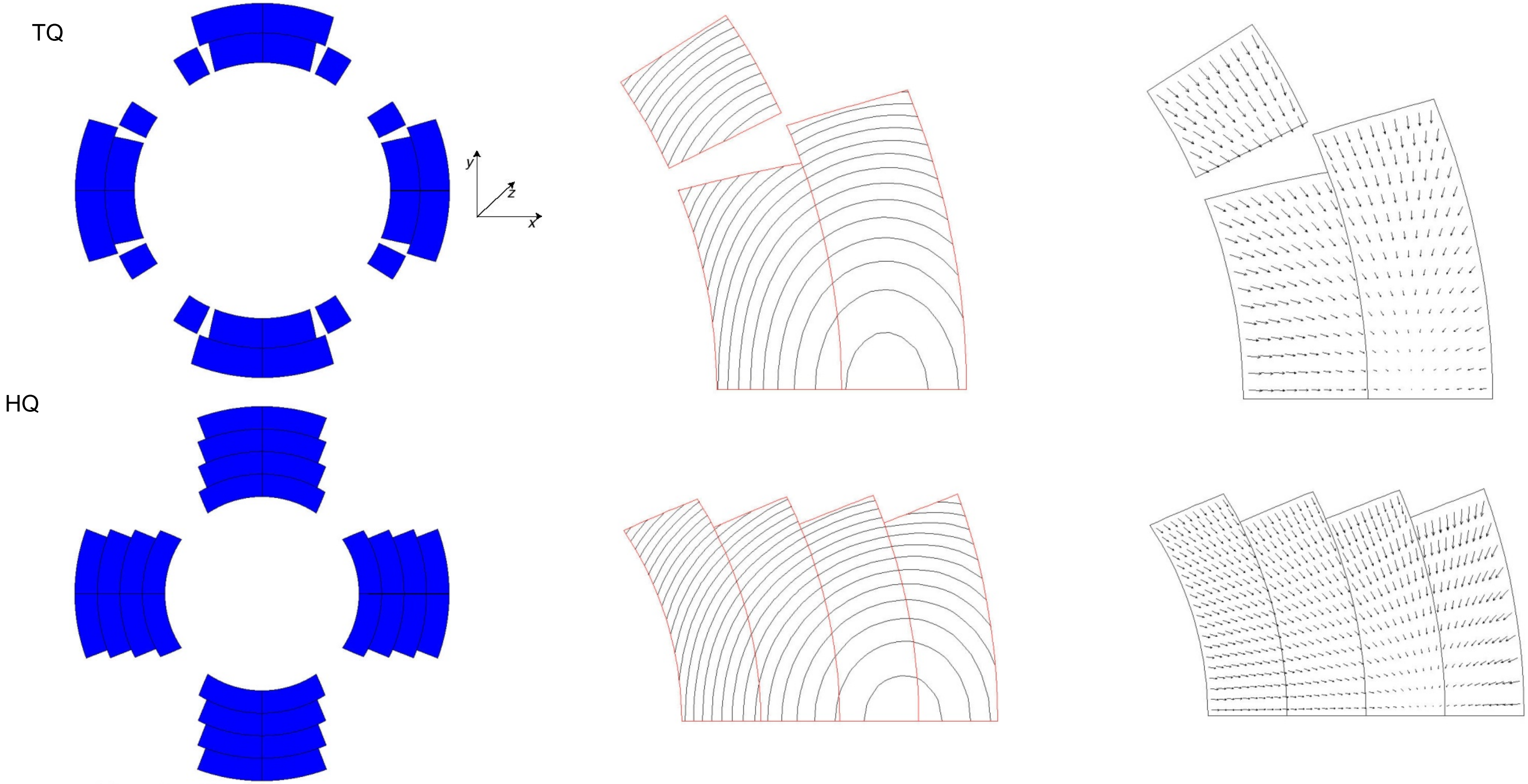
Forces acting on a dipole magnet

- The e.m. forces in a dipole magnet tend to push the coil
  - Towards the mid plane in the vertical-azimuthal direction (F_y, $F_\theta < 0$)
  - Outwards in the radial-horizontal direction (F_x, $F_r > 0$)



Forces on a quadrupole magnet

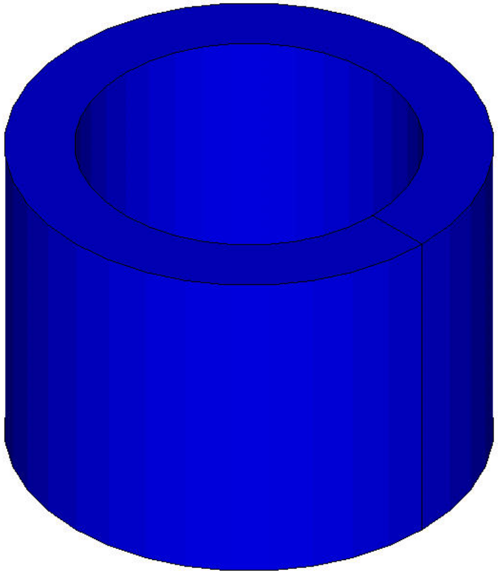
- The e.m. forces in a quadrupole magnet tend to push the coil
  - Towards the mid plane in the vertical-azimuthal direction (F_y, $F_\theta < 0$)
  - Outwards in the radial-horizontal direction (F_x, $F_r > 0$)

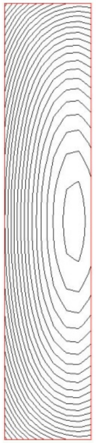


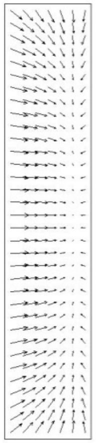
Lets start with solenoids

- The e.m. forces in a solenoid tend to push the coil
  - Outwards in the radial-direction $(F_r > 0)$
  - Towards the mid plane in the vertical direction $(F_y < 0)$

- Important difference between solenoids and dipole/quadrupole magnets
  - In a dipole/quadrupole magnet the horizontal component pushes outwardly the coil
    - The force must be transferred to a support structure
  - In a solenoid the radial force produces a circumferential hoop stress in the winding
    - The conductor, in principle, could support the forces with a reacting tension







The magnetic force in solenoids can often be viewed as a pressure

- The magnetic field is acting on the coil as a pressurized gas on its container.
- Let's consider the ideal case of a infinitely long "thin-walled" solenoid, with thickness *d*, radius *a*, and current density J_θ .
- The field outside the solenoid is zero. The field inside the solenoid B_0 is uniform, directed along the solenoid axis and given by

$$B_0 = \mu_0 J_\theta \delta$$

- The field inside the winding decreases linearly from B_0 at a to zero at $a + \delta$. The average field on the conductor element is $B_0/2$, and the resulting Lorentz force is radial and given by

$$f_L = \frac{J_0 B_o}{2}$$

We can define an effective magnetic pressure

- We can therefore define a magnetic pressure p_m acting on the winding surface:

$$f_L \delta = p_m$$

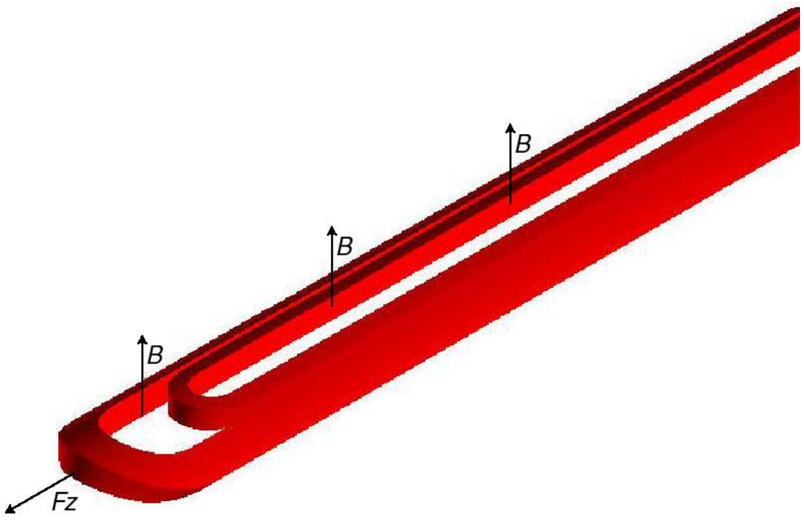
where

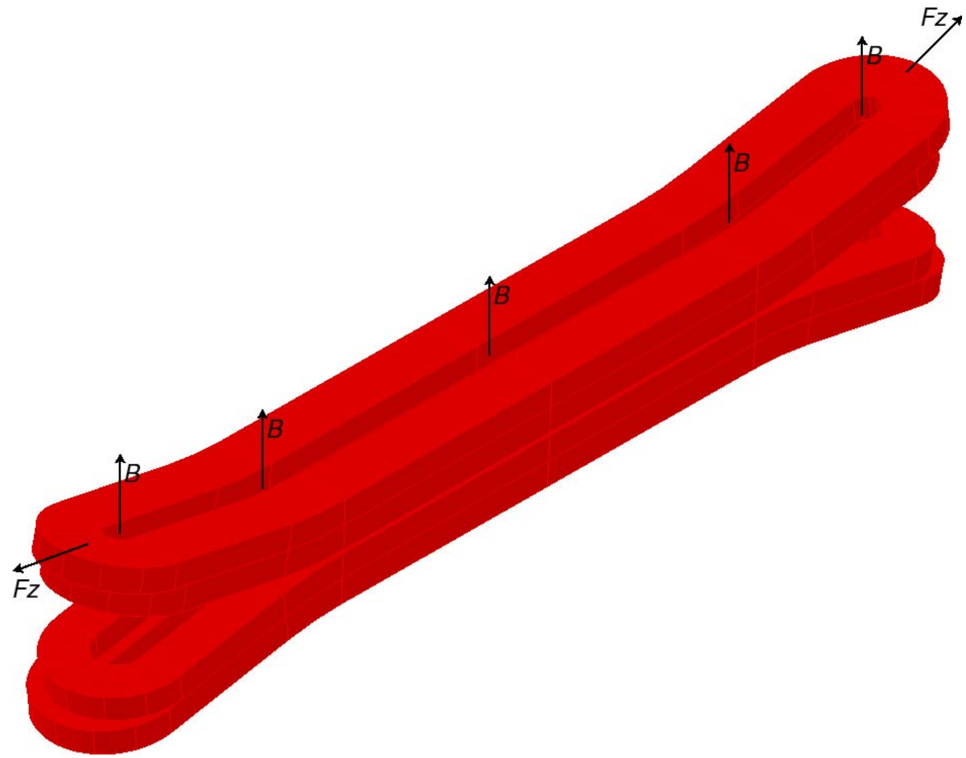
$$p_m = \frac{B_0^2}{2\mu_0}$$

- So, with a 10 T magnet, the windings undergo a pressure $p_m = (10^2)/(2 \cdot 4\,\pi \times 10^{-7}) = 4 \times 10^7$ Pa = 390 atm.
- The force pressure increase with the square of the field
- Note that this model is built on some assumptions – not valid for thick and short coils

End forces in accelerator magnets push outward

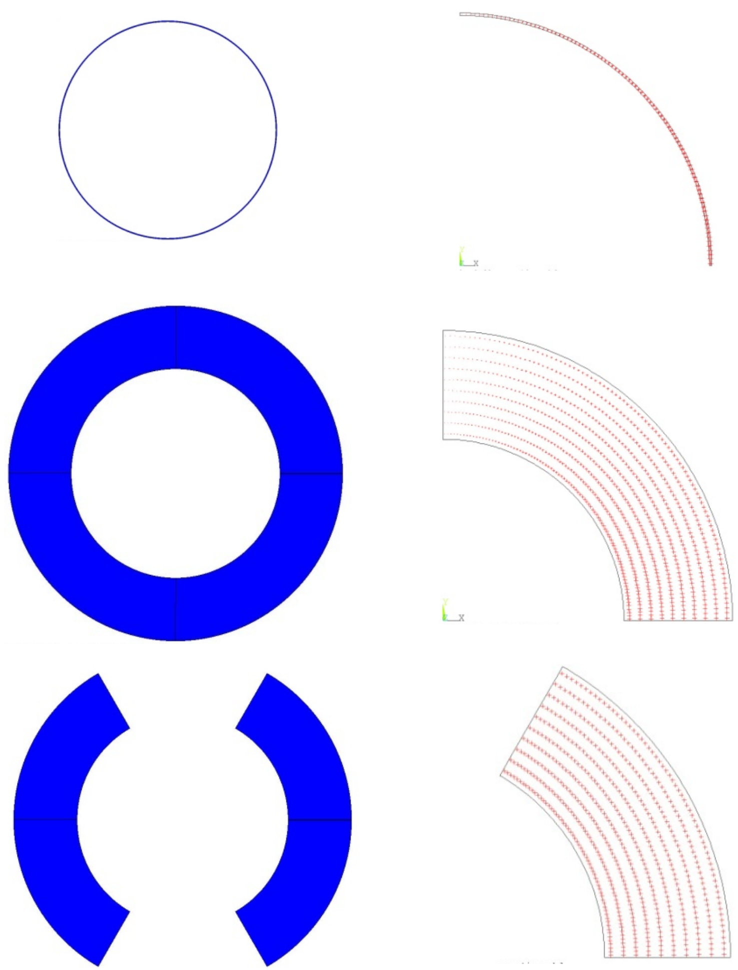
- In the coil ends the Lorentz forces tend to push the coil
  - Outwards in the longitudinal direction ($F_z > 0$)
- Similarly as for the solenoid, the axial force produces an axial tension in the coil straight section.





Lets consider various configurations

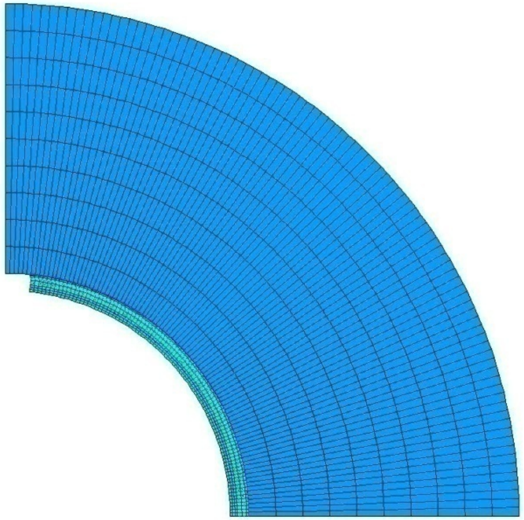
- In order to estimate the force, let's consider three different approximations for a n-order magnet
  - Thin shell (see Appendix I)
    - Current density $J = J_0 \cos(n\theta)$ (A per unit circumference) on a infinitely thin shell
      - Orders of magnitude and proportionalities
  - Thick shell (see Appendix II)
    - Current density $J = J_0 \cos n\theta$ (A per unit area) on a shell with a finite thickness
      - First order estimate of forces and stress
  - Sector (see Appendix III)
    - Current density $J = const$ (A per unit area) on a a sector with a maximum angle $\theta = 60^o/30^o$ for a dipole/quadrupole
      - First order estimate of forces and stress



In the thin-shell cos(θ) approximation we find…

- In a dipole, the field inside the coil is

$$B_y = -\frac{\mu_0 J_0}{2}$$



- The total force acting on the coil [N/m] is

$$F_x = \frac{B_y^2}{2\mu_0}\frac{4}{3}a \qquad F_y = -\frac{B_y^2}{2\mu_0}\frac{4}{3}a$$

- The Lorentz force on a dipole coil varies
  - with the square of the bore field
  - linearly with the magnetic pressure
  - linearly with the bore radius.
- In a rigid structure, the force determines an azimuthal displacement of the coil and creates a separation at the pole.
- The structure sees F_x.

And in the thin-shell cos(2θ) approximation we find…

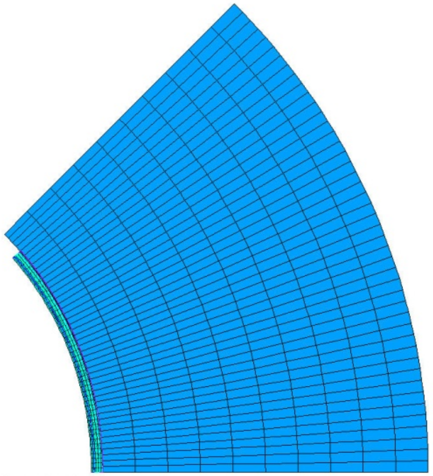
- In a quadrupole, the gradient [T/m] inside the coil is

$$G = \frac{B_c}{a} = -\frac{\mu_0 J_0}{2a}$$

- The total force acting on the coil [N/m] is

$$F_x = \frac{B_c^2}{2\mu_0} a \frac{4\sqrt{2}}{15} = \frac{G^2}{2\mu_0} a^3 \frac{4\sqrt{2}}{15}$$

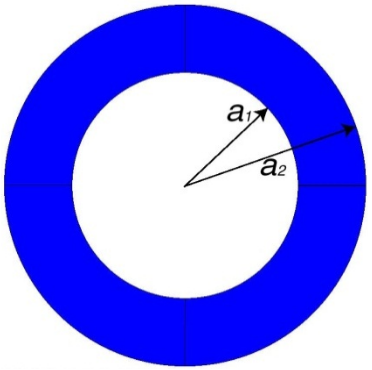
$$F_y = -\frac{B_c^2}{2\mu_0} a \frac{4\sqrt{2}+8}{15} = -\frac{G^2}{2\mu_0} a^3 \frac{4\sqrt{2}+8}{15}$$



- The Lorentz force on a quadrupole coil varies
  - with the square of the gradient or coil peak field
  - with the cube of the aperture radius (for a fixed gradient).
- Keeping the peak field constant, the force is proportional to the aperture.

For thick coils the solutions become more complex, but can still be solved

- We assume
  - $J = J_0 \cos n\theta$ where J_0 [A/m²] is ⊥ to the cross-section plane
  - Inner (outer) radius of the coils = *a1 (a2)*
  - No iron
- The field inside the aperture is



$$B_{ri} = -\frac{\mu_0 J_0}{2} r^{n-1}\left(\frac{a_2^{2-n}-a_1^{2-n}}{2-n}\right)\sin n\vartheta \qquad B_{\theta i} = -\frac{\mu_0 J_0}{2} r^{n-1}\left(\frac{a_2^{2-n}-a_1^{2-n}}{2-n}\right)\cos n\vartheta$$

- The field in the coil is

$$B_r = -\frac{\mu_0 J_0}{2}\left[r^{n-1}\left(\frac{a_2^{2-n}-r^{2-n}}{2-n}\right)+\frac{1}{2+n}\left(\frac{r^{2+n}-a_1^{2+n}}{r^{1+n}}\right)\right]\sin n\vartheta \qquad B_\theta = -\frac{\mu_0 J_0}{2}\left[r^{n-1}\left(\frac{a_2^{2-n}-r^{2-n}}{2-n}\right)-\frac{1}{2+n}\left(\frac{r^{2+n}-a_1^{2+n}}{r^{1+n}}\right)\right]\cos n\vartheta$$

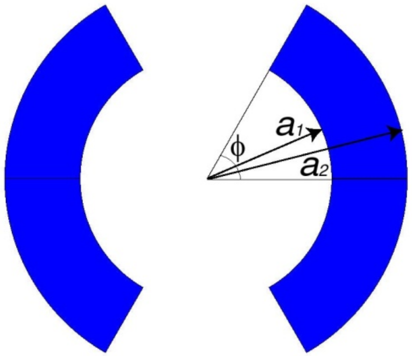
- The Lorentz force acting on the coil [N/m³] is

$$f_r = -B_\theta J = \frac{\mu_0 J_0^2}{2}\left[r^{n-1}\left(\frac{a_2^{2-n}-r^{2-n}}{2-n}\right)-\frac{1}{2+n}\left(\frac{r^{2+n}-a_1^{2+n}}{r^{1+n}}\right)\right]\cos^2 n\vartheta \qquad f_x = f_r\cos\theta - f_\theta\sin\theta$$

$$f_\theta = B_r J = -\frac{\mu_0 J_0^2}{2}\left[r^{n-1}\left(\frac{a_2^{2-n}-r^{2-n}}{2-n}\right)+\frac{1}{2+n}\left(\frac{r^{2+n}-a_1^{2+n}}{r^{1+n}}\right)\right]\sin n\theta\cos n\vartheta \qquad f_y = f_r\sin\theta + f_\theta\cos\theta$$

And similarly for the sector coils…

- We assume
  - $J=J_0$ is ⊥ the cross-section plane
  - Inner (outer) radius of the coils = *a1 (a2)*
  - Angle ϕ = 60^o (third harmonic term is null)
  - No iron



- The field inside the aperture

$$B_r = -\frac{2\mu_0 J_0}{\pi}\left[(a_2-a_1)\sin\phi\,\sin\theta + \sum_{n=1}^{\infty}\frac{r^{2n}}{(2n+1)(2n-1)}\left(\frac{1}{a_1^{n-1}}-\frac{1}{a_2^{n-1}}\right)\sin(2n+1)\phi\,\sin(2n+1)\theta\right]$$

$$B_\theta = -\frac{2\mu_0 J_0}{\pi}\left[(a_2-a_1)\sin\phi\,\cos\theta + \sum_{n=1}^{\infty}\frac{r^{2n}}{(2n+1)(2n-1)}\left(\frac{1}{a_1^{n-1}}-\frac{1}{a_2^{n-1}}\right)\sin(2n+1)\phi\,\cos(2n+1)\theta\right]$$

- The field in the coil is

$$B_r = -\frac{2\mu_0 J_0}{\pi}\left\{(a_2-r)\sin\phi\,\sin\theta + \sum_{n=1}^{\infty}\left[1-\left(\frac{a_1}{r}\right)^{2n+1}\right]\frac{r}{(2n+1)(2n-1)}\sin(2n-1)\phi\,\sin(2n-1)\theta\right\}$$

$$B_\theta = -\frac{2\mu_0 J_0}{\pi}\left\{(a_2-r)\sin\phi\,\cos\theta - \sum_{n=1}^{\infty}\left[1-\left(\frac{a_1}{r}\right)^{2n+1}\right]\frac{r}{(2n+1)(2n-1)}\sin(2n-1)\phi\,\cos(2n-1)\theta\right\}$$

Lets look at stored energy

- The magnetic field possesses an energy density [J/m³]

$$U = \frac{B_0^2}{2\mu_0}$$

- The total energy [J] is given by

$$E = \int_{all\_space} \frac{B_0^2}{2\mu_0} dV$$

- Or, considering only the coil volume, by

$$E = \frac{1}{2}\int_V \vec{A} \cdot \vec{j} dV$$

This is used extensively in numerical codes

- Knowing the inductance L, it can also be expresses as

$$E = \frac{1}{2} L I^2$$

- The total energy stored is strongly related to mechanical and protection issues (see later lectures)

We can use the stored energy expression to derive the end forces

- In general the force acting on a system with potential energy U is

$$\vec{F} = \nabla U$$

- The magnetic stored energy can therefore be used to evaluate the Lorentz forces

$$F_r = \frac{\partial E}{\partial r} \qquad F_\theta = \frac{1}{r}\frac{\partial E}{\partial \theta} \qquad F_z = \frac{\partial E}{\partial z} = \frac{\partial}{\partial z}\left(\frac{LI^2}{2}\right)_{z=1m}$$

- This means that, considering a "long" magnet, one can compute the end force F_z [N] from the stored energy per unit length W [J/m]:

$$B_r(r,\theta) = \frac{1}{r}\frac{\partial A_z}{\partial \theta}$$

$$W = \frac{1}{2}\int_{coil\_area} \vec{A}_z \cdot \vec{j}\, dA$$

$$B_\theta(r,\theta) = -\frac{\partial A_z}{\partial r}$$

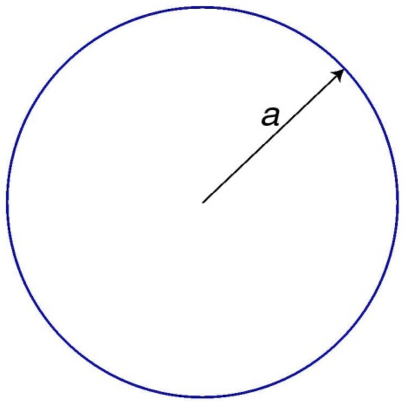
Application to the thin shell

- For a dipole, the vector potential within the thin shell is

$$A_z = +\frac{\mu_0 J_0}{2} a \cos\vartheta$$

and, therefore,

$$F_z = \frac{B_y^2}{2\mu_0} 2\pi a^2$$



  - The axial force on a dipole coil varies
    - with the square of the bore field
    - linearly with the magnetic pressure
    - with the square of the bore radius.

- For a quadrupole, the vector potential within the thin shell is

$$A_z = +\frac{\mu_0 J_0}{2} \frac{a}{2} \cos 2\vartheta$$

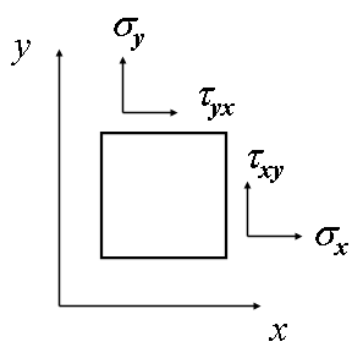
and, therefore,

$$F_z = \frac{B_c^2}{2\mu_0} \pi a^2 = \frac{G^2 a^2}{2\mu_0} \pi a^2$$

  - Being the peak field the same, a quadrupole has half the F_z of a dipole.

Some basic definitions to set the stage

- A stress σ or τ [Pa] is an internal distribution of force [N] per unit area [m^2]
  - When the forces are perpendicular to the plane the stress is called normal stress (σ) ; when the forces are parallel to the plane the stress is called shear stress (τ).
  - Stresses can be seen as way of a body to resist the action (compression, tension, sliding) of an external force.



- A strain ε ($\delta l/l_0$) is a forced change dimension δl of a body whose initial dimension is l_0.
  - A stretch or a shortening are respectively a tensile or compressive strain; an angular distortion is a shear strain.

Definitions…

- The *elastic modulus* (or Young modulus, or modulus of elasticity) *E* [Pa] is a parameter that defines the stiffness of a given material. It can be expressed as the rate of change in stress with respect to strain (Hook's law):

$$\varepsilon = \sigma / E$$

- The Poisson's ratio ν is the ratio between "axial" to "transverse" strain. When a body is compressed in one direction, it tends to elongate in the other direction. Vice versa, when a body is elongated in one direction, it tends to get thinner in the other direction.

$$\nu = -\varepsilon_{axial} / \varepsilon_{trans}$$

Connecting stress and strain

- By combining the previous definitions, for a biaxial stress state we get

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \qquad \varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E}$$

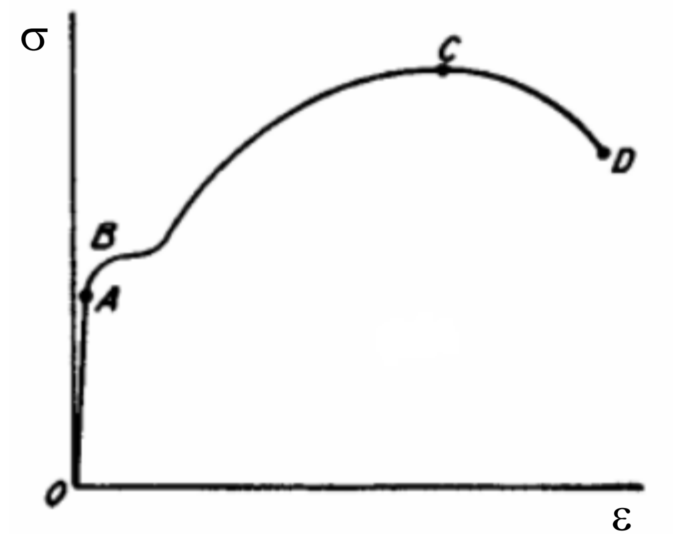
  and

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) \qquad \sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right)$$

- Compressive stress is negative, tensile stress is positive.
- The Poisson's ration couples the two directions
  - A stress/strain in x also has an effect in y.
- For a given stress, the higher the elastic modulus, the smaller the strain and displacement.

Basic characteristics of common structural materials

- The proportionality between stress and strain is more complicated than the Hook's law
  - A: limit of proportionality (Hook's law)
  - B: yield point
    - Permanent deformation of 0.2 %
  - C: ultimate strength
  - D: fracture point



- Several failure criteria are defined to estimate the failure/yield of structural components, as
  - Equivalent (Von Mises) stress $\sigma_v < \sigma_{yield}$, where

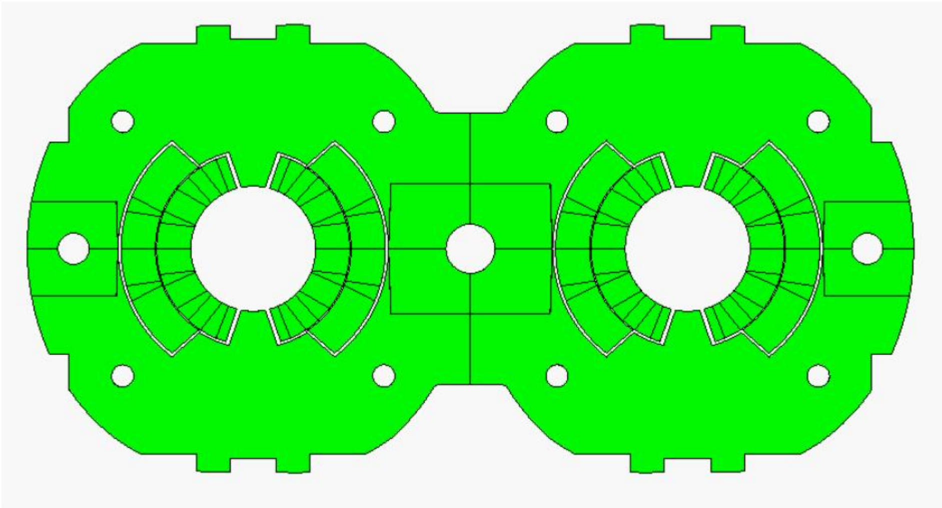
$$\sigma_v = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

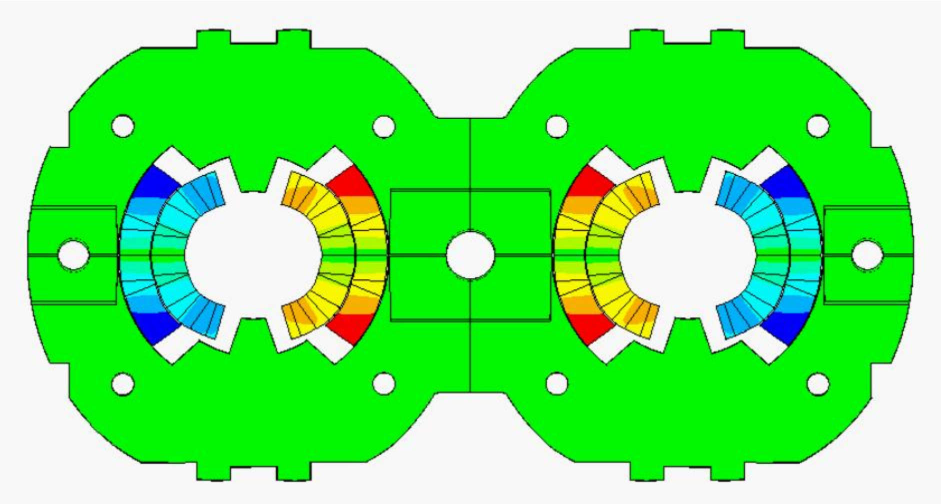
Some basic comments on accelerator magnet mechanics

- The e.m forces push the conductor towards mid-planes and supporting structure.
- The resulting strain is an indication of
  - a change in coil shape
    - effect on field quality
  - a displacement of the conductor
    - potential release of frictional energy and consequent quench of the magnet
- The resulting stress must be carefully monitored
  - In NbTi magnets, possible damage of kapton insulation at about 150-200 MPa.
  - In Nb_3Sn magnets, possible conductor degradation at about 150-200 MPa.
  - In general, al the components of the support structure must not exceed the stress limits.
- Mechanical design has to address all these issues.

Basic mechanical deformations seen in accelerator magnets

LHC dipole at 0 T | LHC dipole at 9 T





Displacement scaling = 50

- Usually, in a dipole or quadrupole magnet, the highest stresses are reached at the mid-plane, where all the azimuthal e.m. forces accumulate (over a small area).

We can calculate the midplane stress on thick cos(θ) coils under hypothesis of no shear

No shear

- For a dipole,

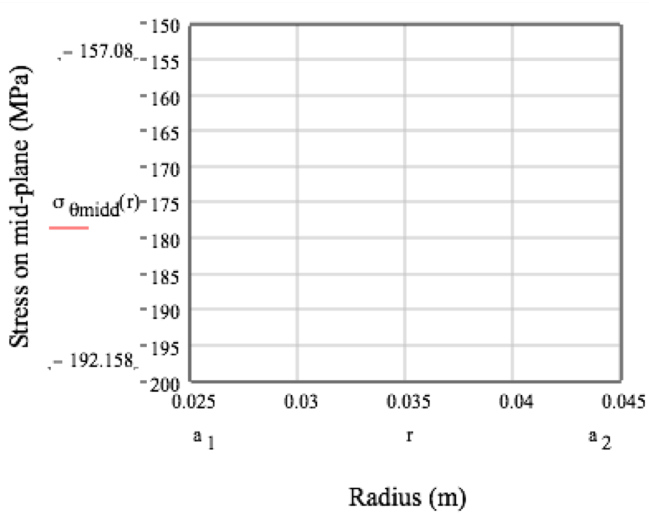
$$\sigma_{\theta\_mid-plane} = \int_0^{\pi/2} f_\theta r d\theta = -\frac{\mu_0 J_0^2}{2}\frac{r}{2}\left[(a_2 - r) - \frac{r^3 - a_1^3}{3r^2}\right]$$

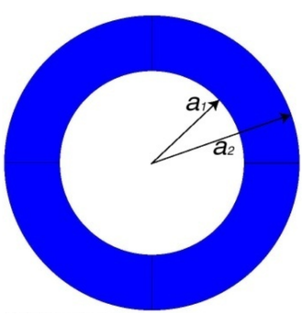
$$\sigma_{\theta\_mid-plane\_av} = -\frac{\mu_0 J_0^2}{2}\left[\frac{5}{36}a_2^3 + \frac{1}{6}\left(\ln\frac{a_1}{a_2} + \frac{2}{3}\right)a_1^3 - \frac{1}{4}a_2 a_1^2\right]\frac{1}{a_2 - a_1}$$

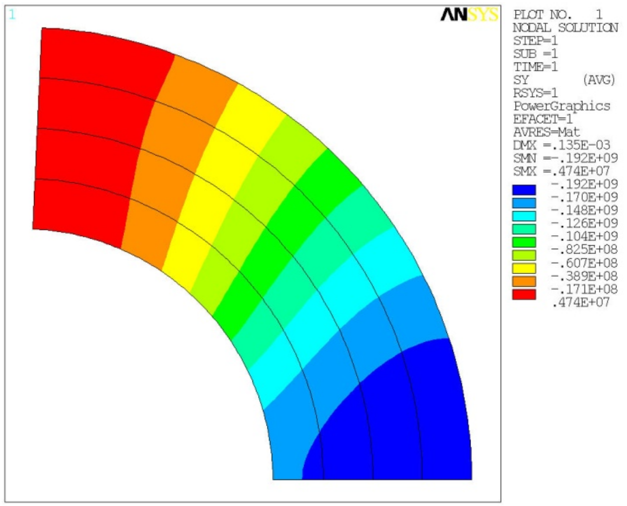
- For a quadrupole,

$$\sigma_{\theta\_mid-plane} = \int_0^{\pi/4} f_\theta r d\theta = -\frac{\mu_0 J_0^2}{2}\frac{r}{4}\left(r\ln\frac{a_2}{r} - \frac{r^4 - a_1^4}{4r^3}\right)$$

$$\sigma_{\theta\_mid-plane\_av} = -\frac{\mu_0 J_0^2}{2}\left[\frac{1}{144}\frac{7a_2^4 + 9a_1^4}{a_2} + \frac{1}{12}\left(\ln\frac{a_1}{a_2} + \frac{4}{3}\right)a_1^3\right]\frac{1}{a_2 - a_1}$$







A first perspective on pre-stress – motivation for field quality

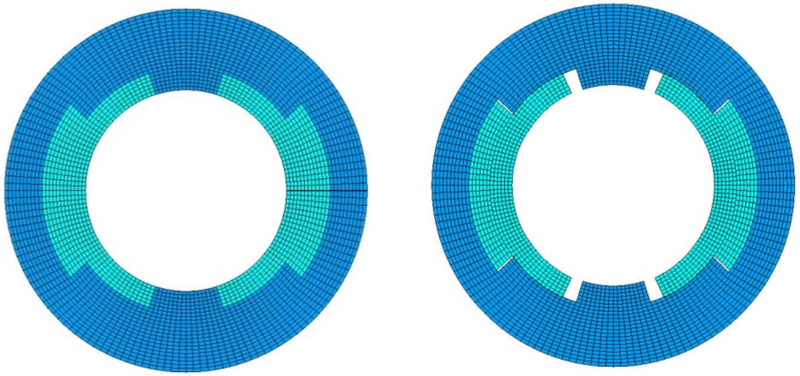
A.V. Tollestrup, [3]

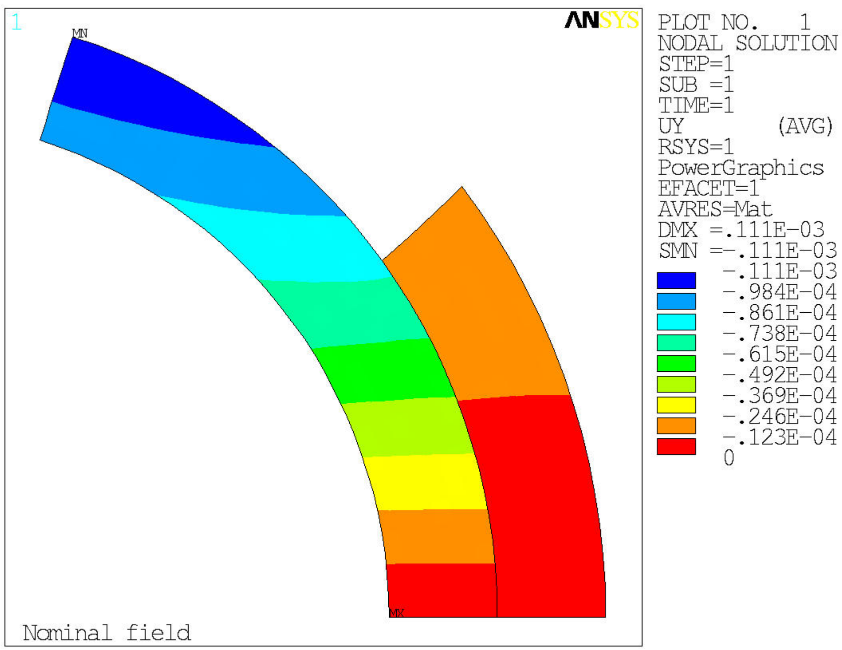
Annu. Rev. Nucl. Part. Sci. 1984.34:247-284.

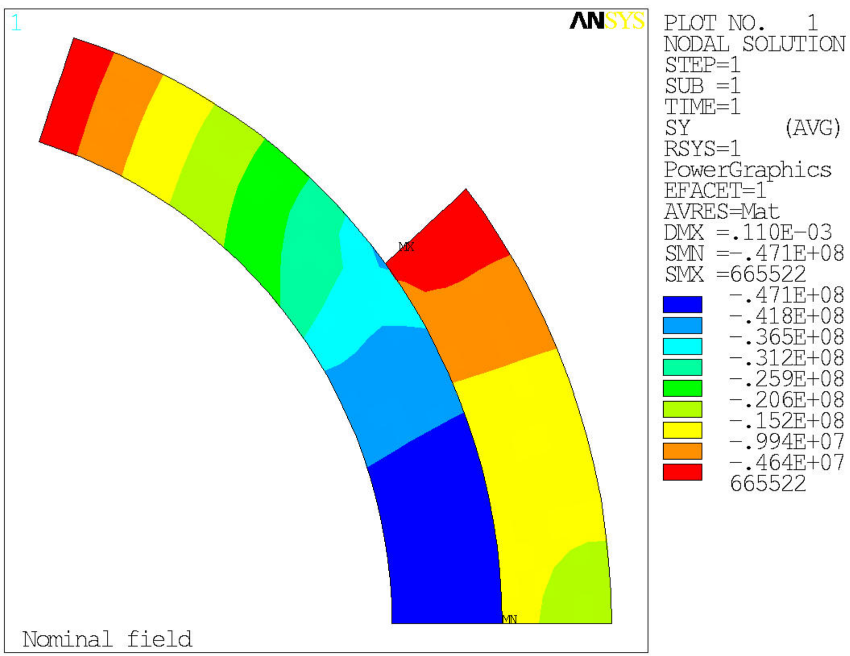
We are now in a position to discuss a difficult problem in magnet construction. If the azimuthal forces of the last section were allowed to act on a coil supported only in the radial direction, the coil would compress itself as it was excited and the angles of the shell would change. If these change symmetrically, a sextupole moment is induced and, if asymmetrically, quadrupole terms appear as well. The field is enormously sensitive to these angles—they must be maintained to an accuracy of $\sim 25\ \mu$rad for adequate field quality. The forces are so large that, with the elastic modulus available in the insulated coil packages ($E = 10^6$ psi), the compression of the coil would far exceed this limit. As a result, when the coil is constructed, it is preloaded in the azimuthal direction to the extent that the elastic forces are greater than the magnetic forces. This ensures that the boundaries of the coil package will stay in contact with the collars during excitation. The Tevatron coils (18) were assembled in a large press, and the CBA magnets were bolted together with a similar pressure. Elastic motion of the coil relative to its support can still take place but at a much reduced level. (It is similar to fixing two ends of a loaded beam, compared to fixing only one end.) Elastic motion can also be reduced by making the elastic modulus high. The group at LBL has had successes in achieving $E \approx 4 \times 10^6$ psi, which is perhaps four times larger than achieved in the Tevatron magnet; the CBA coils had 2×10^6 psi.

Tevatron main dipole

- Let's consider the case of the Tevatron main dipole (B_{nom}= 4.4 T).
- In a infinitely rigid structure without pre-stress the pole would move of about -100 μm, with a stress on the mid-plane of -45 MPa.

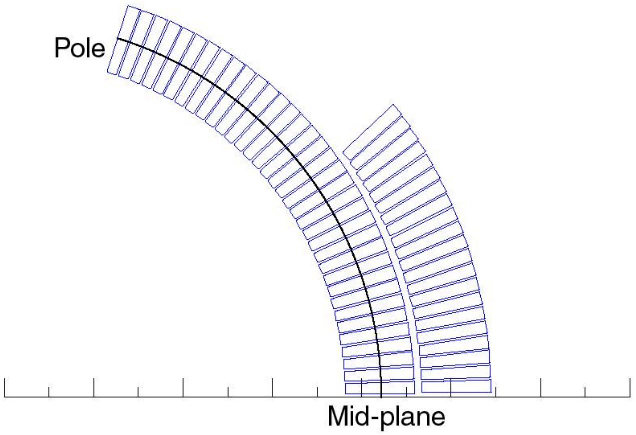


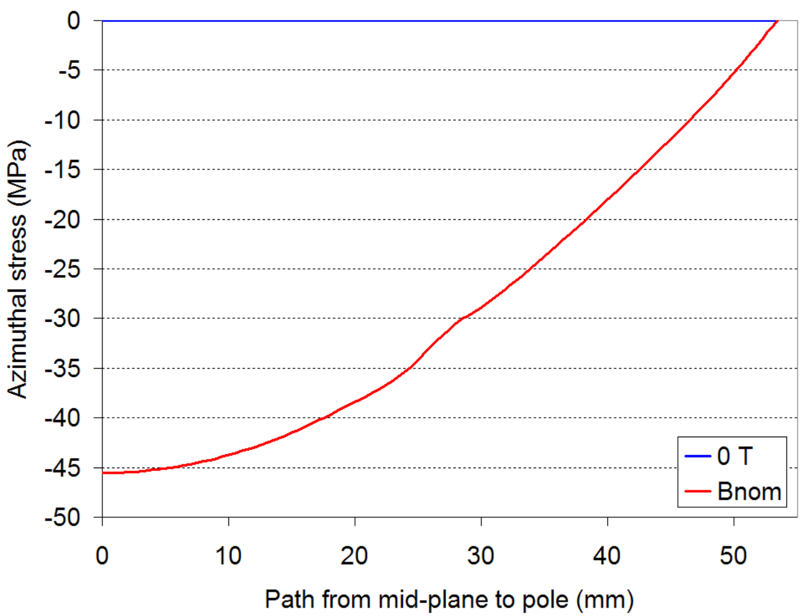


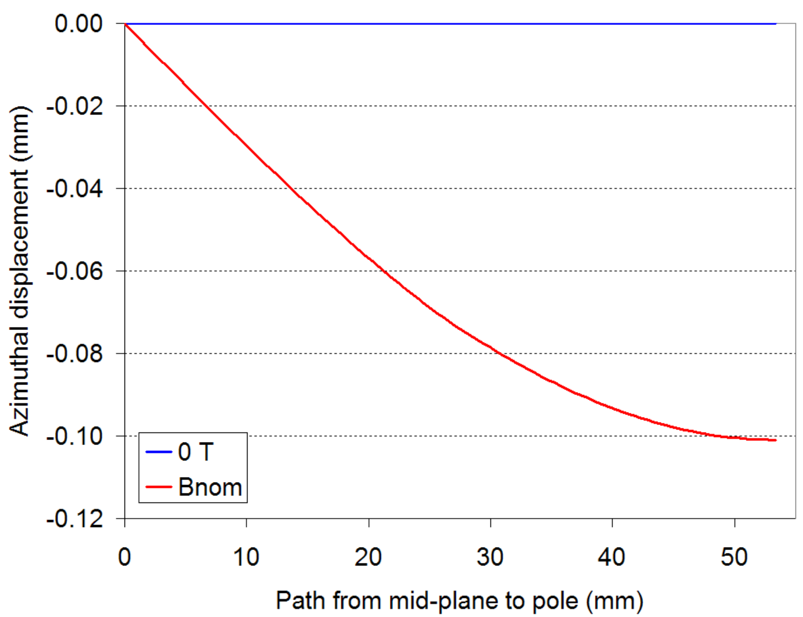


7. Pre-stress
Tevatron main dipole

- We can plot the displacement and the stress along a path moving from the mid-plane to the pole.
- In the case of no pre-stress, the displacement of the pole during excitation is about -100 μm.

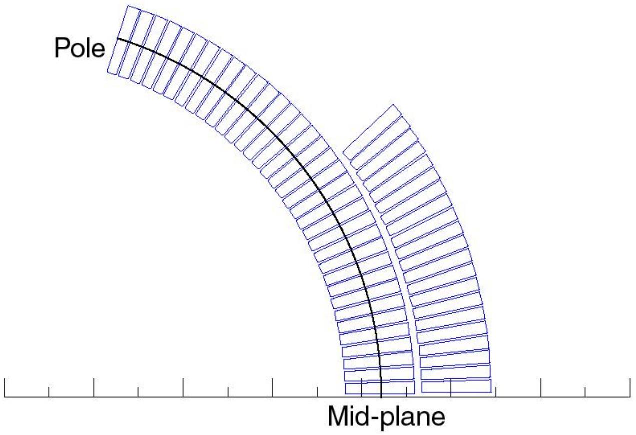


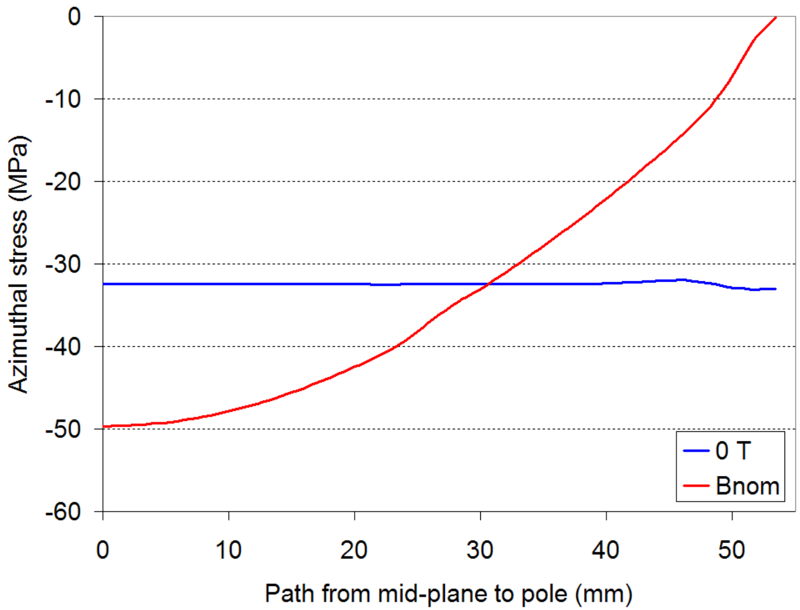


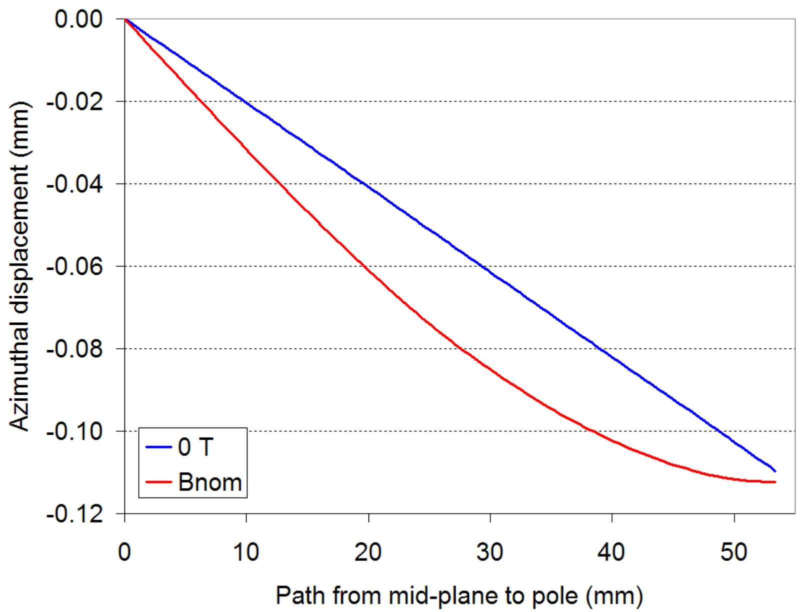


7. Pre-stress
Tevatron main dipole

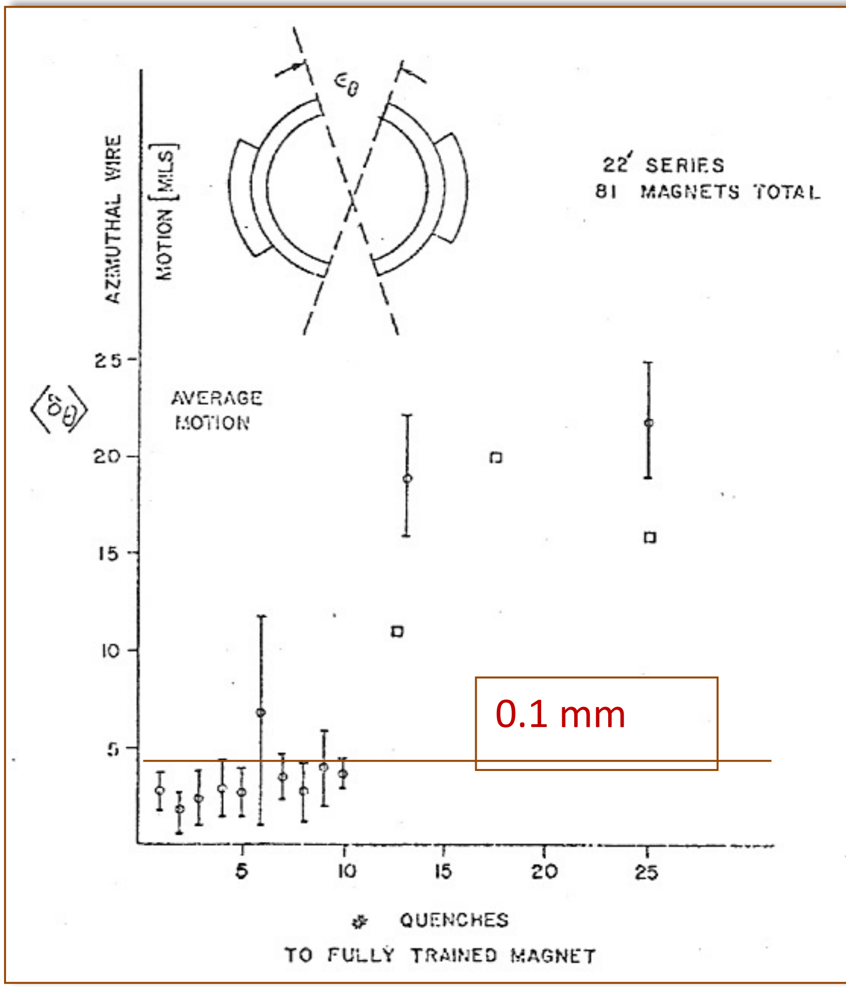
- We now apply to the coil a pre-stress of about -33 MPa, so that no separation occurs at the pole region.
- The displacement at the pole during excitation is now negligible, and, within the coil, the conductors move at most of -20 μm.







The correlation between coil separation and training as a second motivation for prestress: Experience in Tevatron



A.V. Tollestrup, [10]

- Above movements of 0.1 mm, there is a correlation between performance and movement: the larger the movement, the longer the training

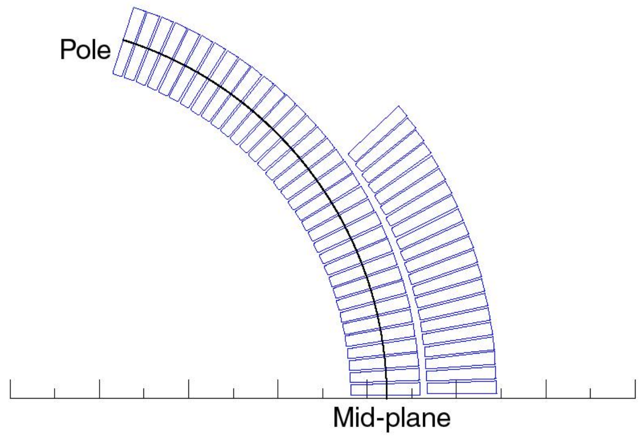
elastic. It shows very little hysteresis. However, the azimuthal motion has given more trouble. Early in the program, magnets were built such that the elastic forces when cold were less than the magnetic forces, and the conductor at the key moved. Fig. 11 shows data from 81 magnets whose training took from one quench to over 25. Some magnets in this series had preload small

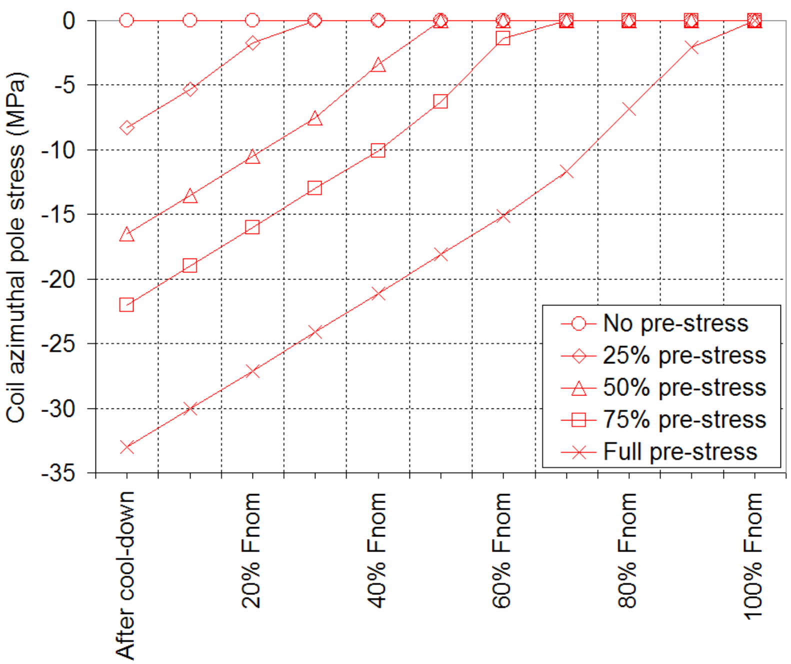
- Conclusion: one has to prestress to avoid movements that can limit performance

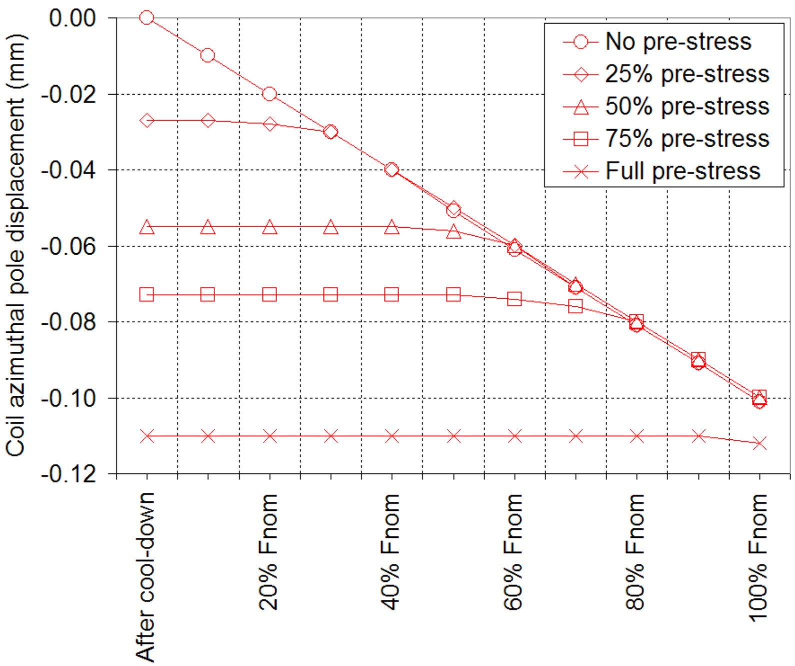
error bars are just a square root of the number of magnets at each point. It is seen that this motion does not couple into the training until it is large enough so that the conductor is completely unclamped. Why it takes some magnets one quench and others 10 quenches to train when the conductor remains clamped is a mystery.

Data from the Tevatron

- We focus now on the stress and displacement of the pole turn (high field region) in different pre-stress conditions.
- The total displacement of the pole turn is proportional to the pre-stress.
  - A full pre-stress condition minimizes the displacements.

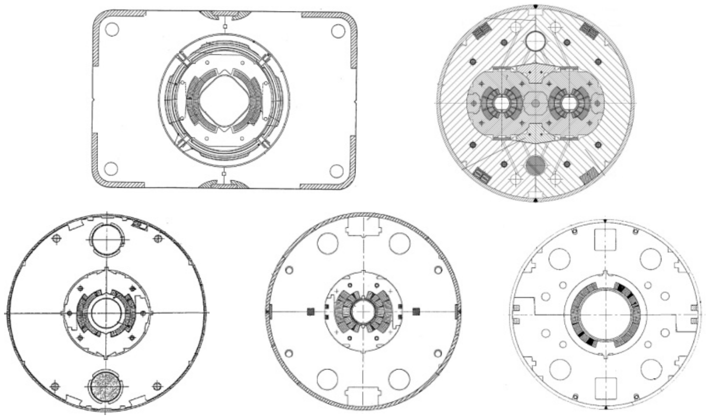


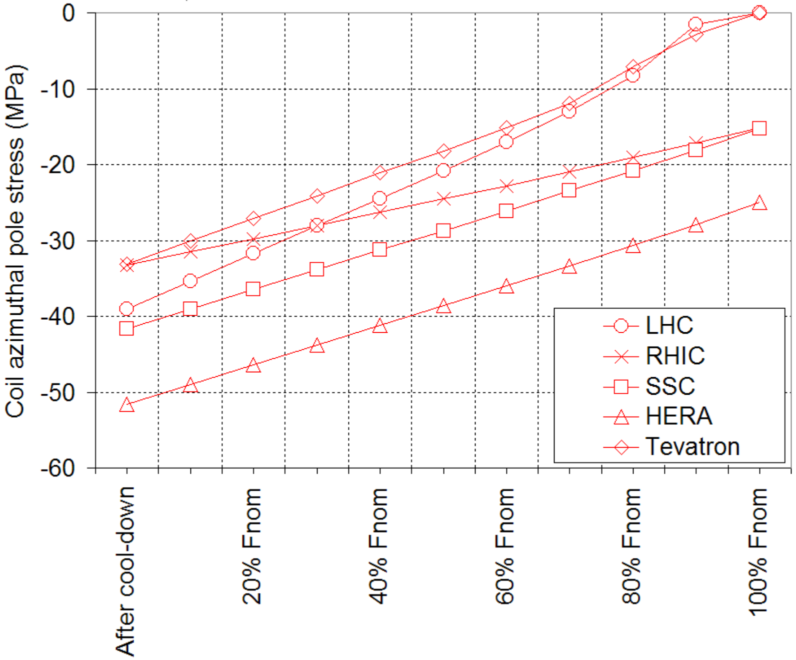




Pre-stress on various collider magnet configurations

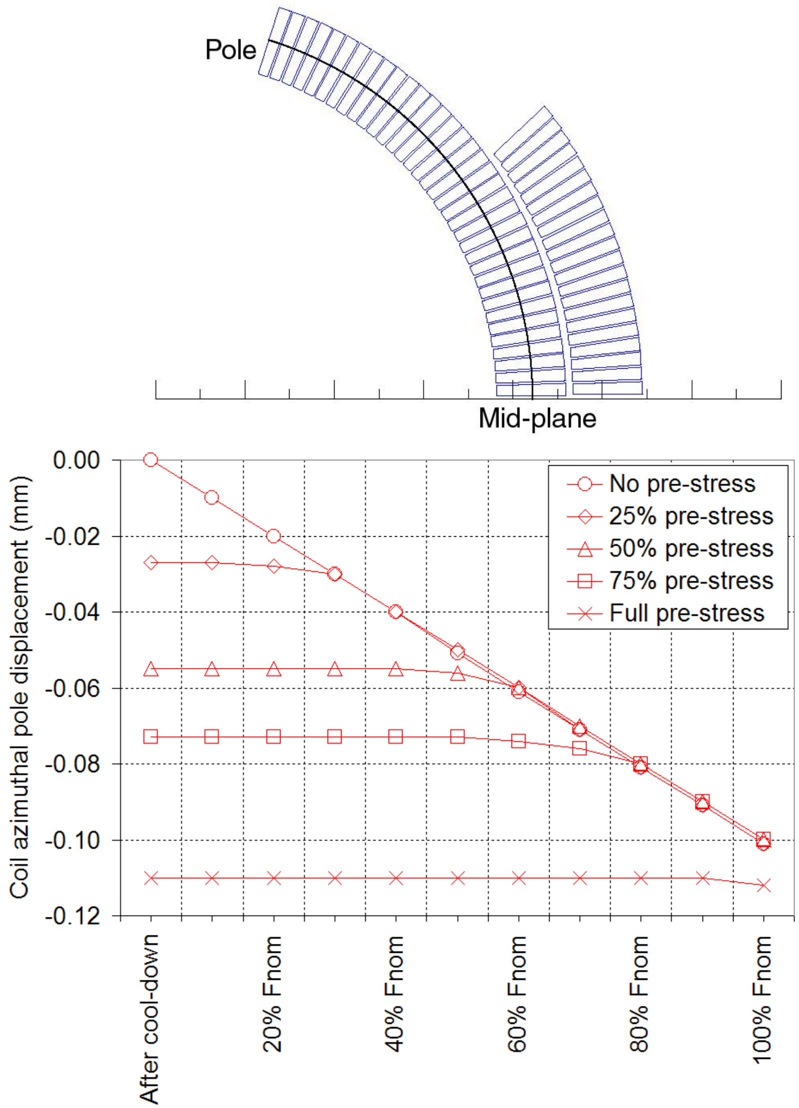
- The practice of pre-stressing the coil has been applied to all the accelerator dipole magnets
  - Tevatron [4]
  - HERA [5]
  - SSC [6]-[7]
  - RHIC [8]
  - LHC [9]
- The pre-stress is chosen in such a way that the coil remains in contact with the pole at nominal field, sometime with a "mechanical margin" of more than 20 MPa.



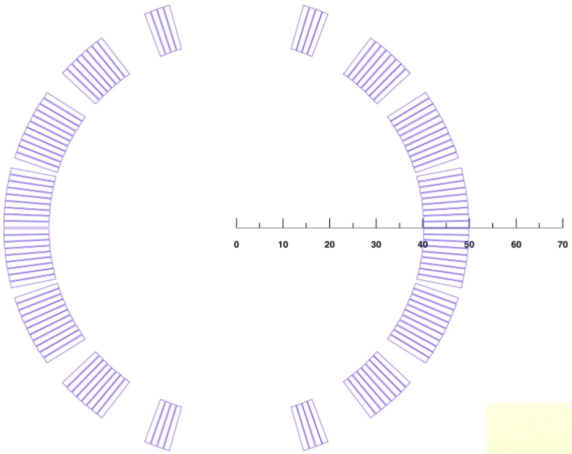


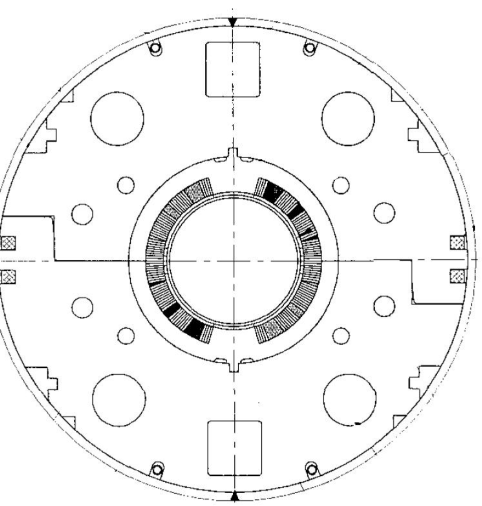
Prestress to minimize displacement

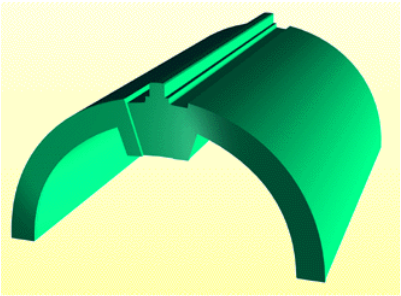
- As we pointed out, the pre-stress reduces the coil motion during excitation.
- This ensure that the coil boundaries will not change as we ramp the field, and, as a results, minor field quality perturbations will occur.
- What about the effect of pre-stress on quench performance?
  - In principle less motion means less frictional energy dissipation or resin fracture.
  - Nevertheless the impact of pre-stress on quench initiation remains controversial

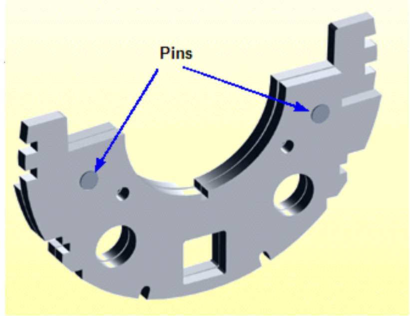


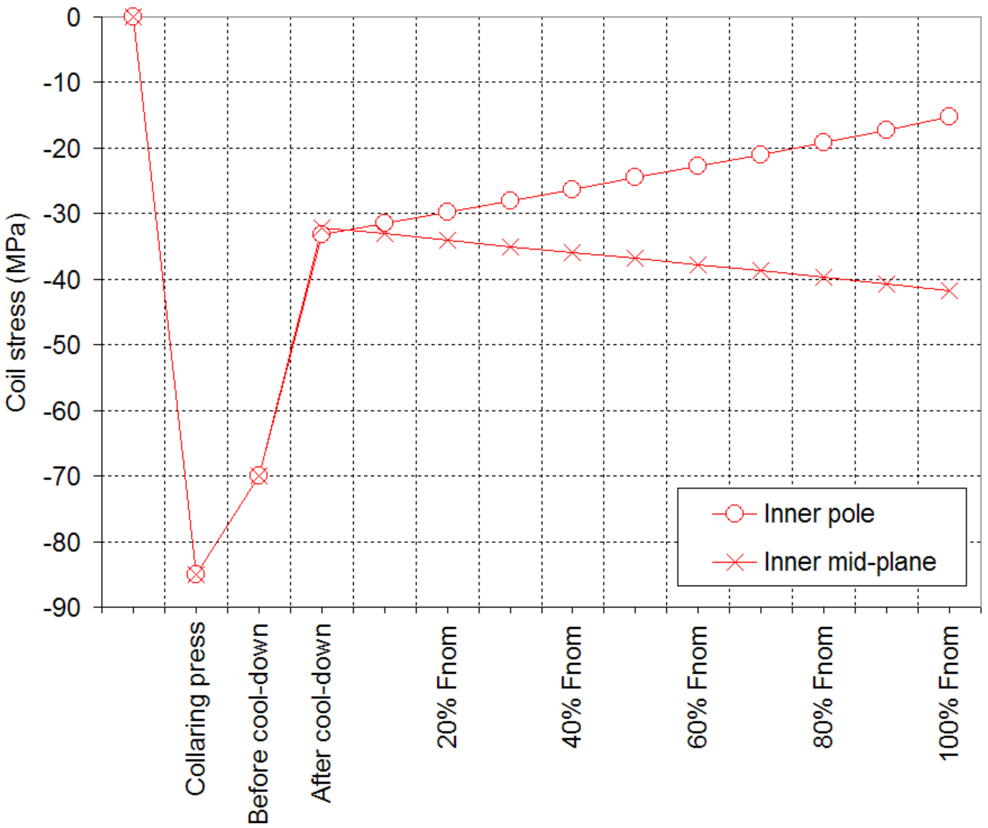
Application of prestress to eliminate separation: RHIC main dipole





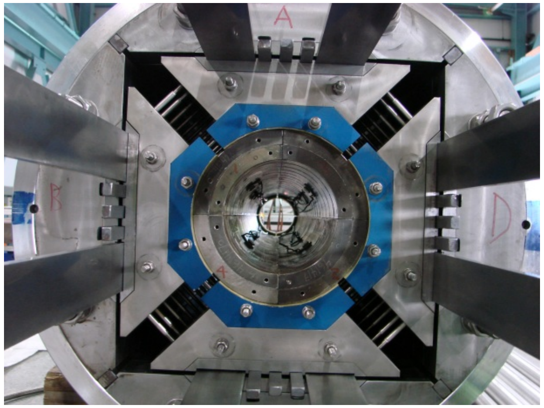


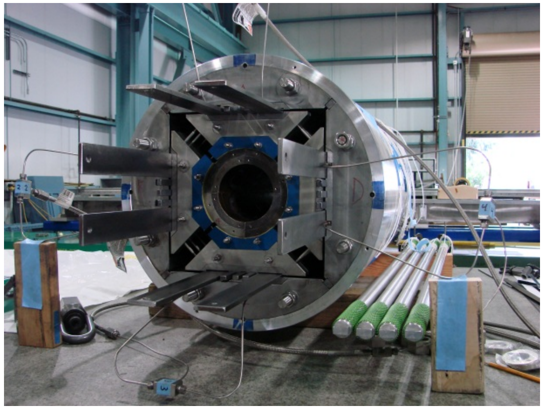


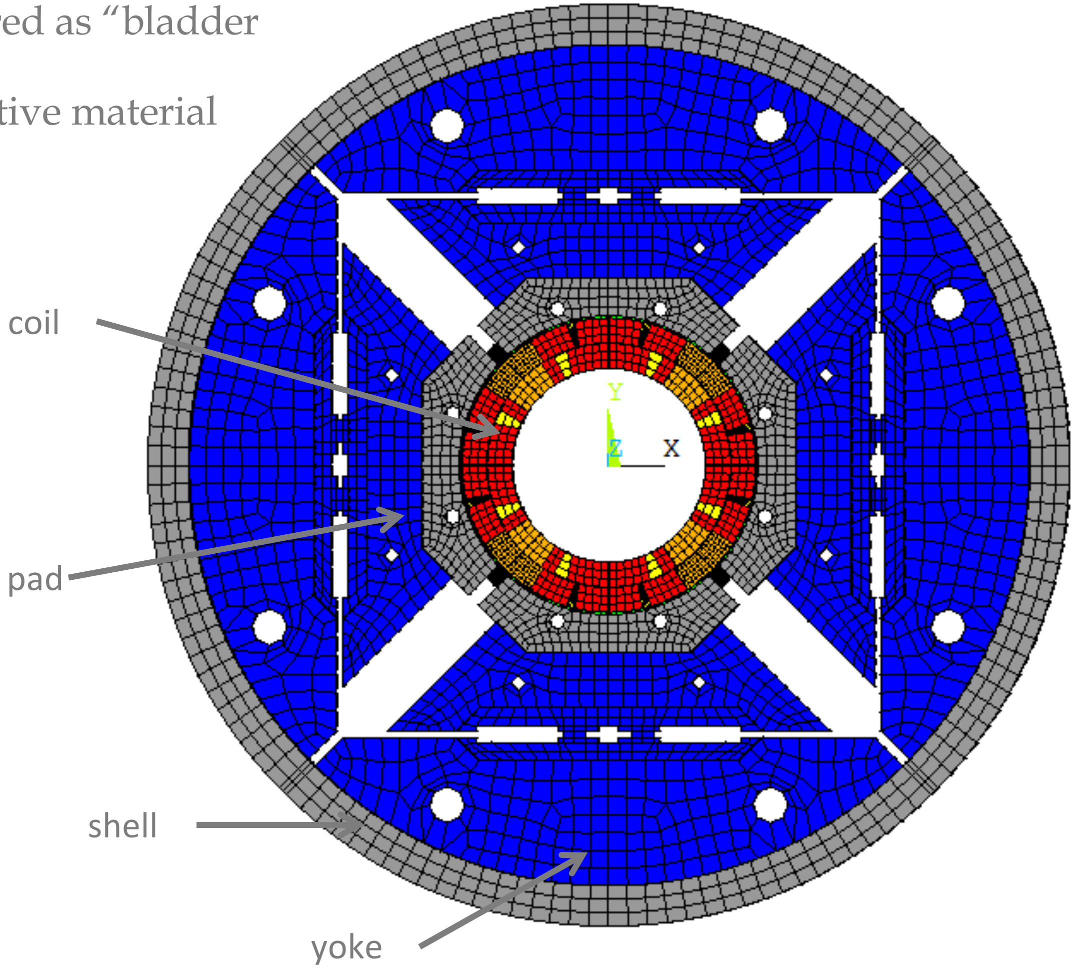


Shell –based, bladder-and-key support structure concept designed for strain-sensitive material

- Shell-based support structure often referred as "bladder and keys" structure
  - developed at LBNL for strain sensitive material

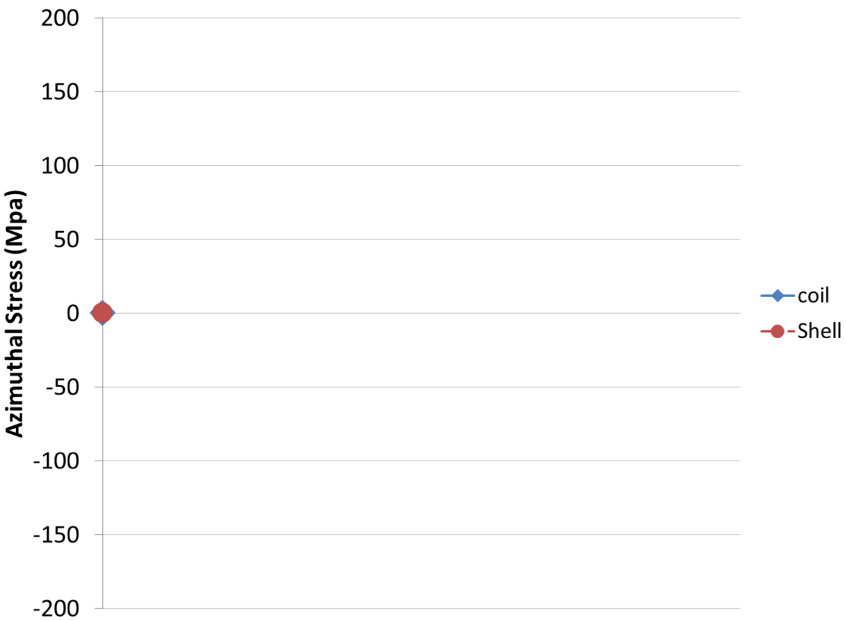


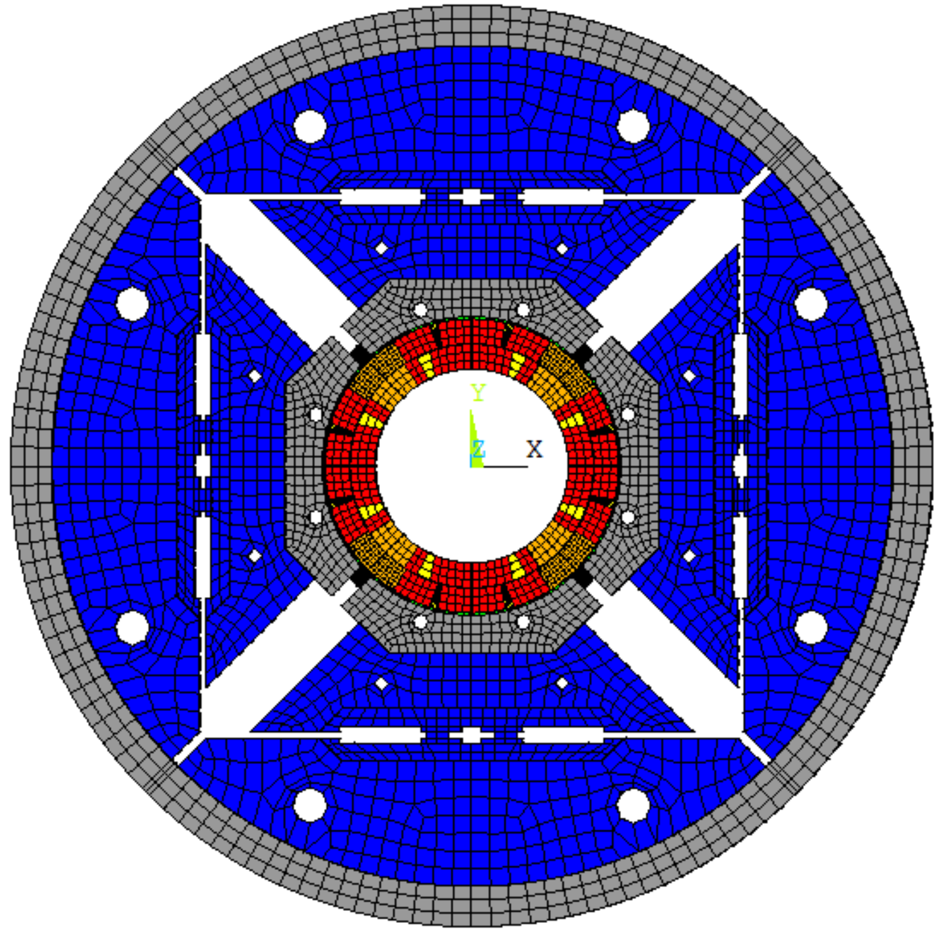




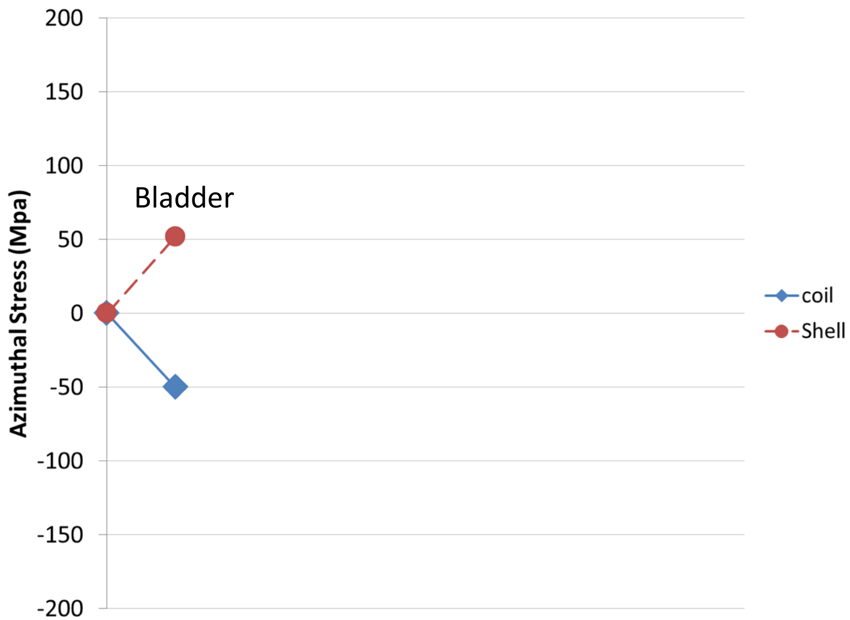
Example: LARP HQ

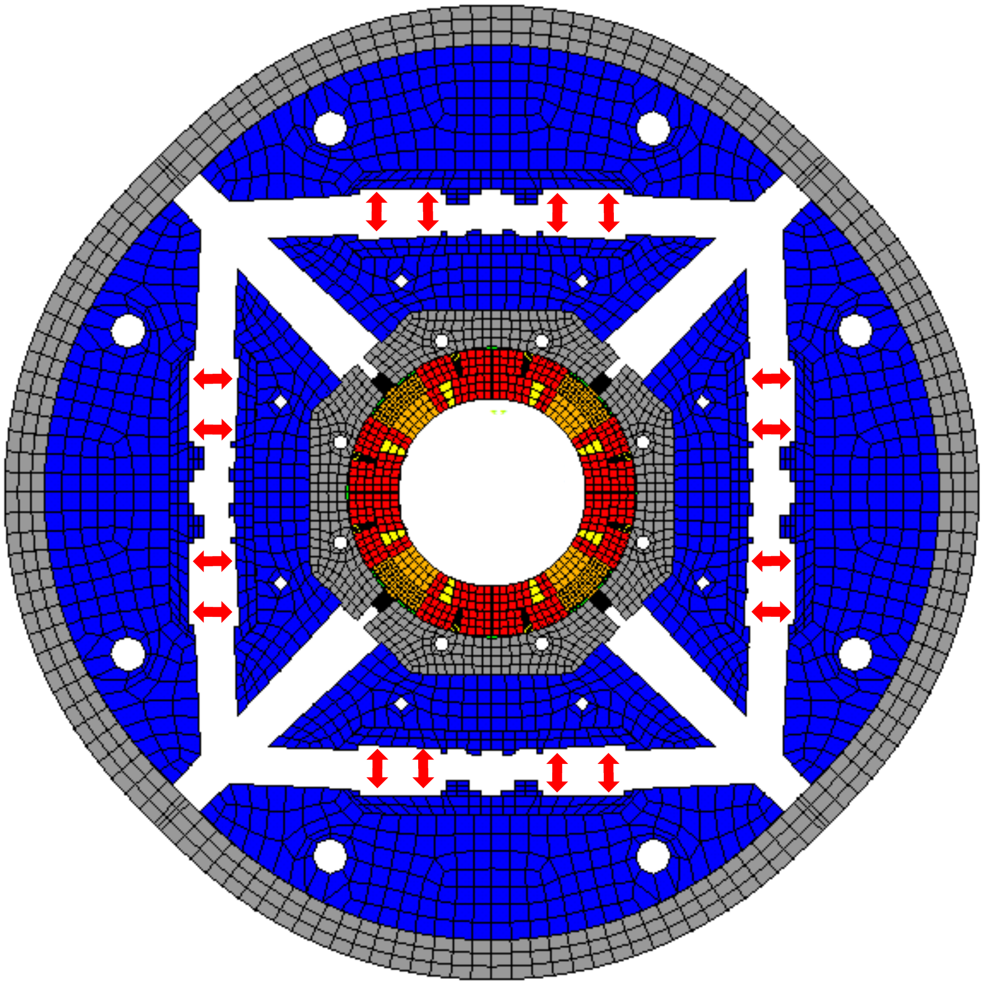
Shell-based support structure Concept





Shell-based support structure Concept

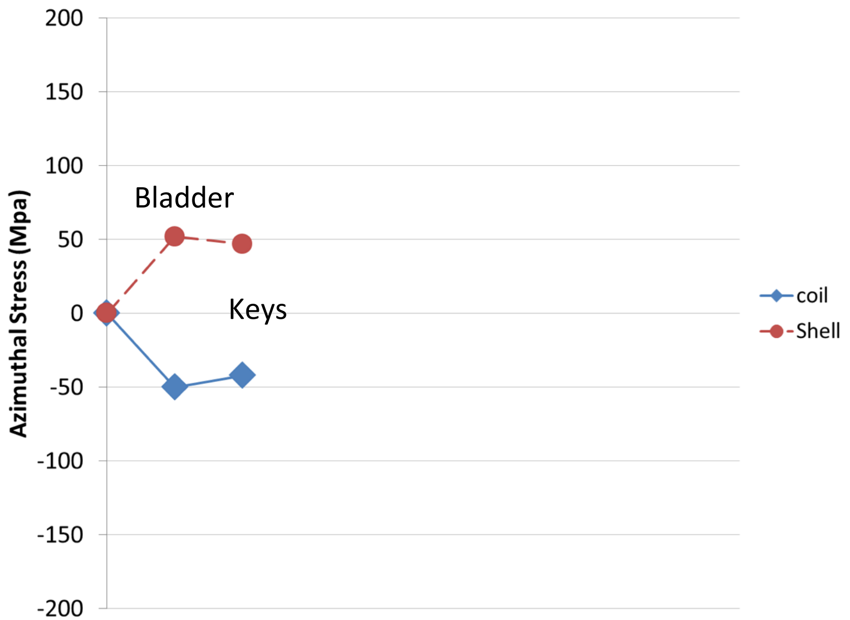


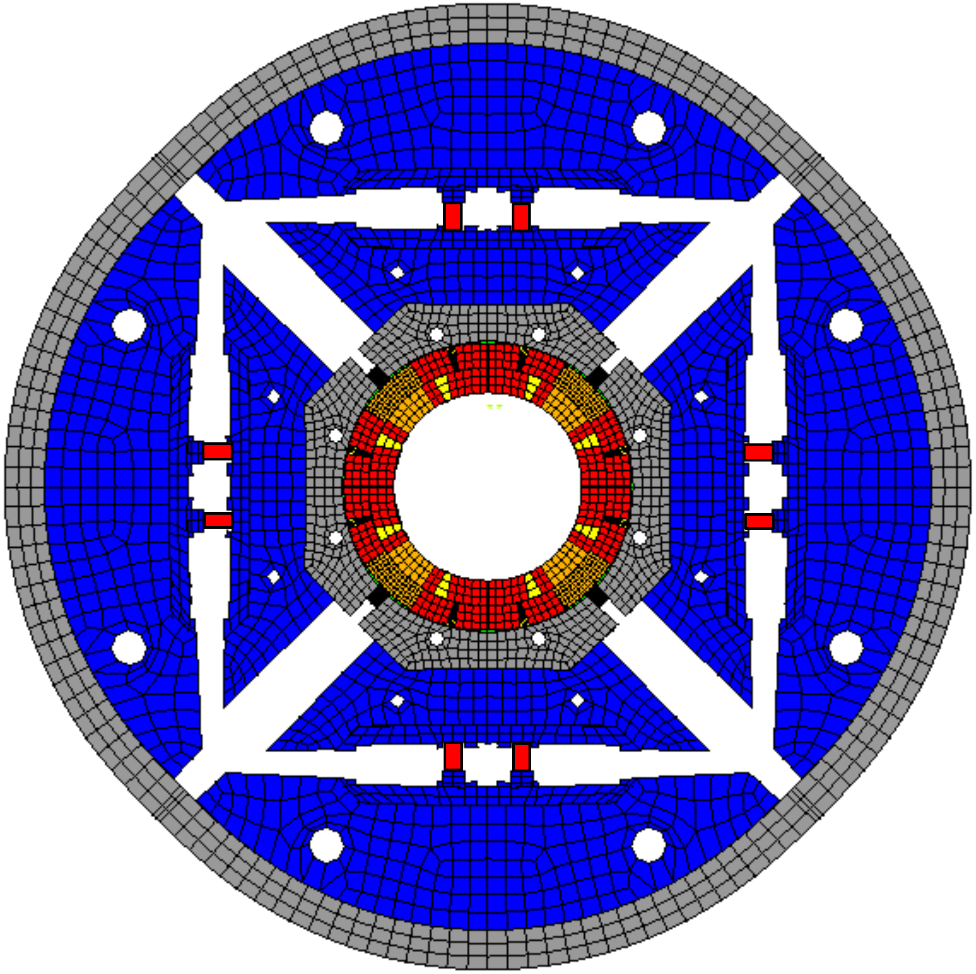


Displacement scaling 30

Shell-based support structure Concept

Shimming of the load leys

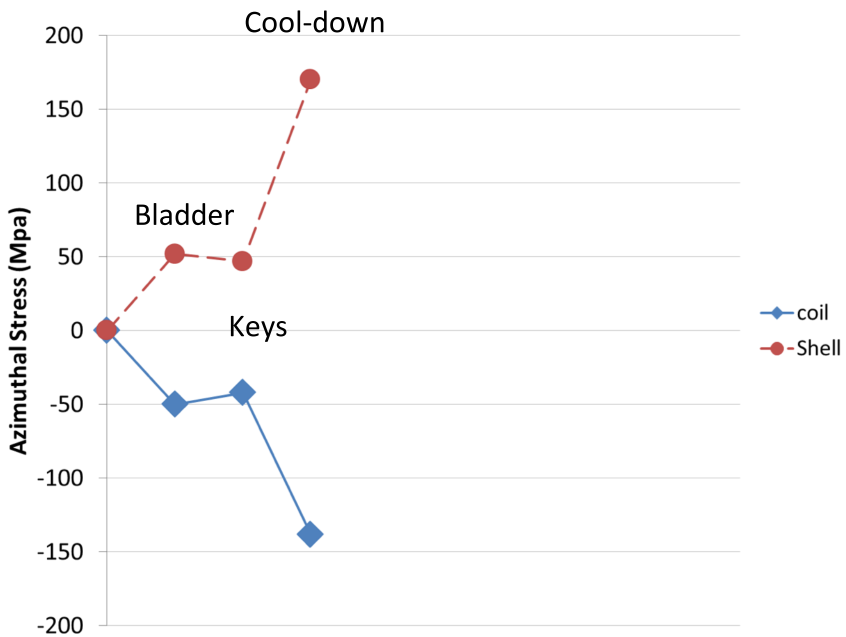


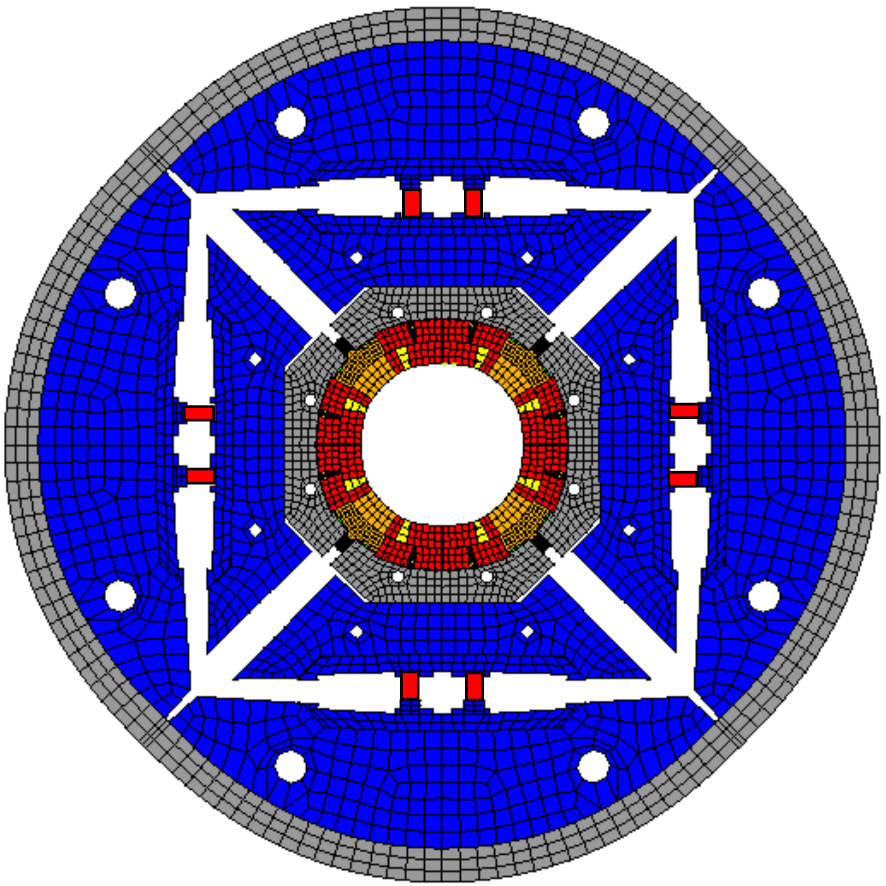


Displacement scaling 30

Shell-based support structure Concept

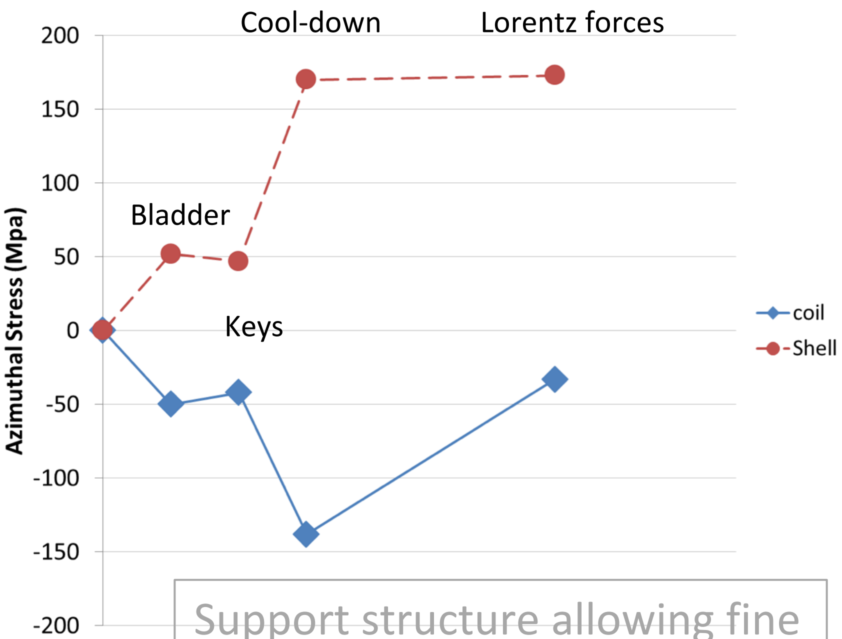
Cool-down

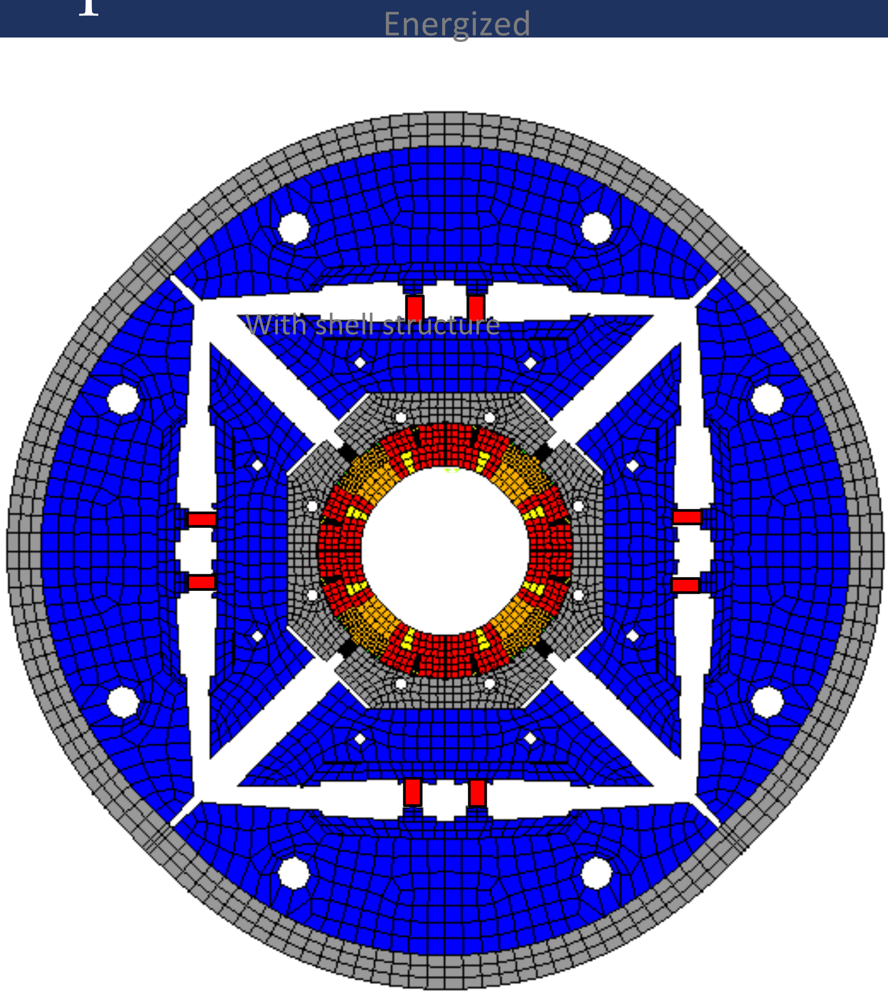




Displacement scaling 30

Shell-based support structure Concept



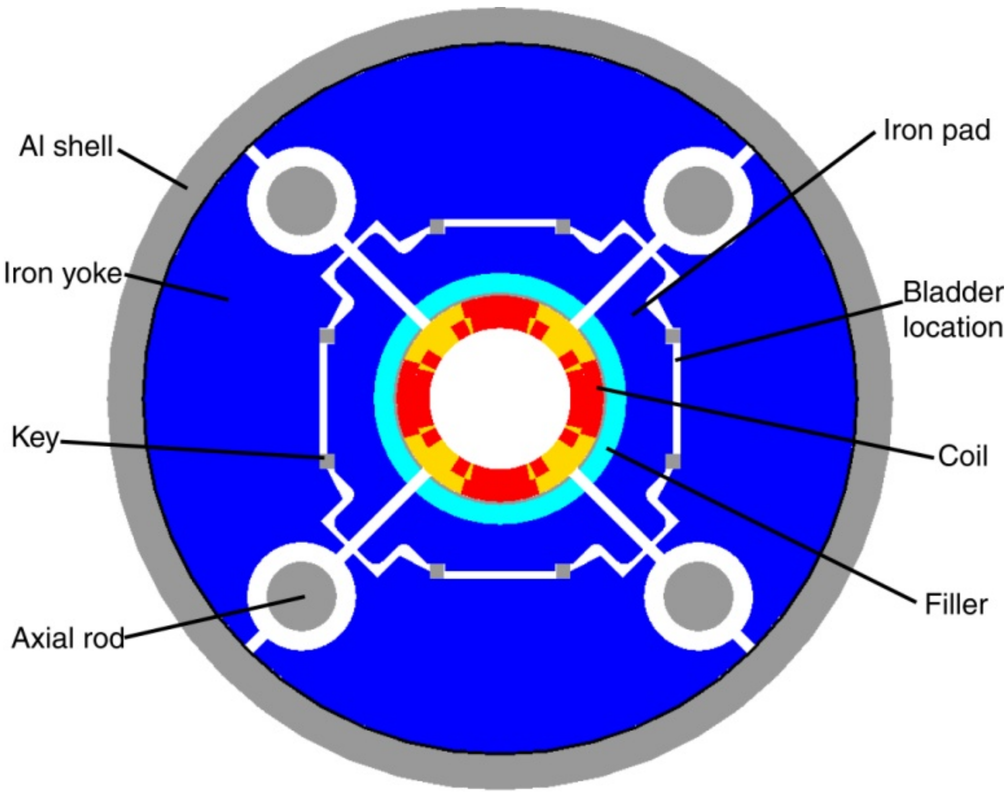


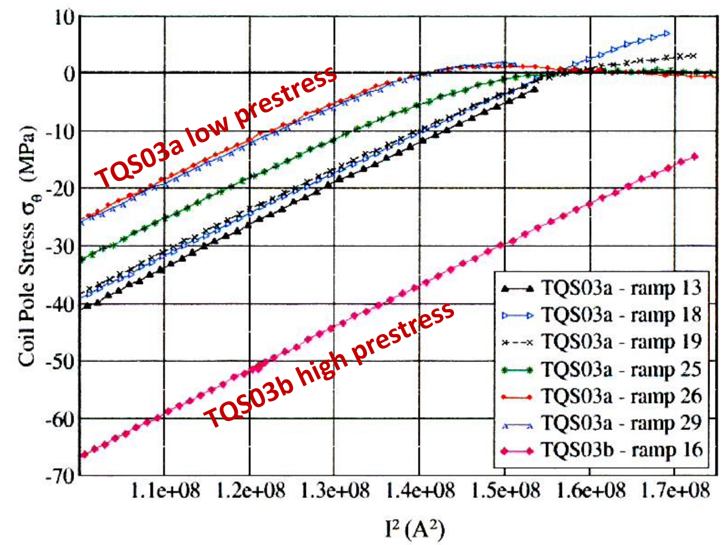
Support structure allowing fine tuning of the pre-stress

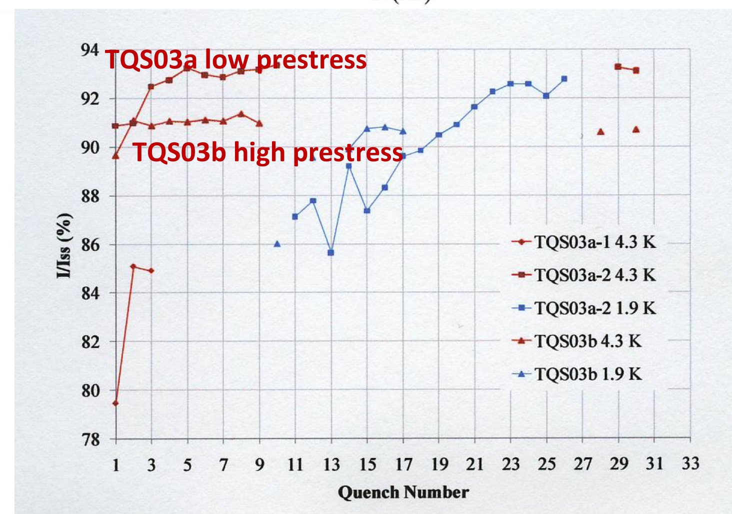
Displacement scaling 30

Application of prestress to eliminate separation: LARP TQ quadrupole

- With low pre-stress, unloading but still good quench performance
- With high pre-stress, stable plateau but small degradation

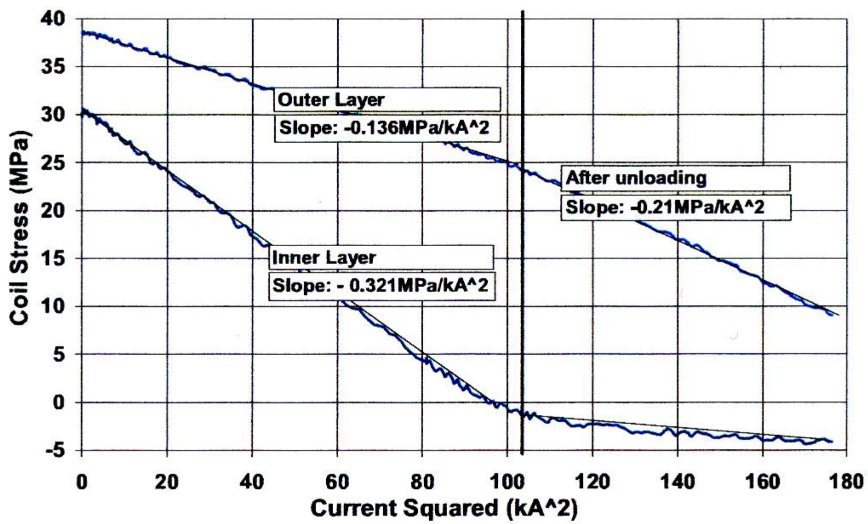






Experience in the LHC short dipoles

- Study of variation of pre-stress in several models – evidence of unloading at 75% of nominal, but no quench



N. Andreev, [11]

- "It is worth pointing out that, in spite of the complete unloading of the inner layer at low currents, both low pre-stress magnets showed correct performance and quenched only at much higher fields"

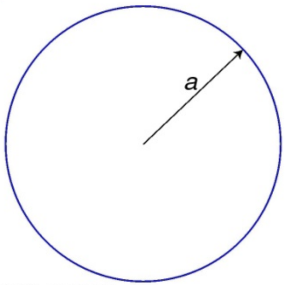
Conclusions

- We presented the force profiles in superconducting magnets
  - A solenoid case can be well represented as a pressure vessel
  - In dipole and quadrupole magnets, the forces are directed towards the mid-plane and outwardly.
    - They tend to separate the coil from the pole and compress the mid-plane region
    - Axially they tend to stretch the windings
- We provided a series of analytical formulas to determine in first approximation forces and stresses.
- The importance of the coil pre-stress has been pointed out, as a technique to minimize the conductor motion during excitation.

Appendix I
Thin shell approximation

Appendix I: thin shell approximation
Field and forces: general case

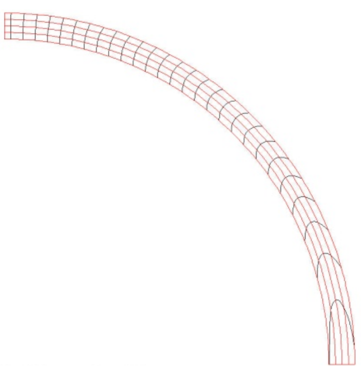
- We assume
  - $J = J_0 \cos n\theta$ where J_0 [A/m] is ^ to the cross-section plane
  - Radius of the shell/coil = a
  - No iron
- The field inside the aperture is



$$B_{ri} = -\frac{\mu_0 J_0}{2}\left(\frac{r}{a}\right)^{n-1} \sin n\vartheta \qquad B_{\theta i} = -\frac{\mu_0 J_0}{2}\left(\frac{r}{a}\right)^{n-1} \cos n\vartheta$$

- The average field in the coil is

$$B_r = -\frac{\mu_0 J_0}{2} \sin n\vartheta \qquad B_\theta = 0$$

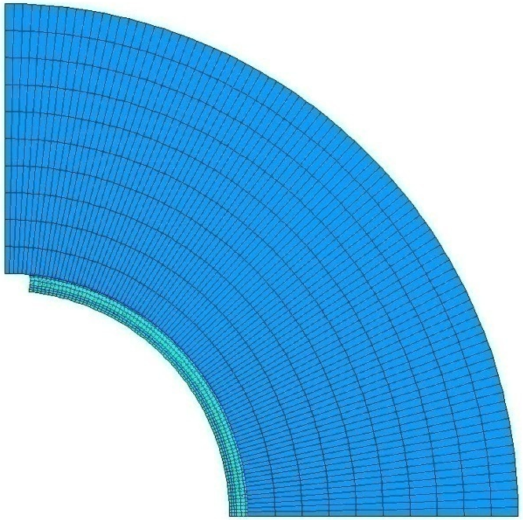


- The Lorentz force acting on the coil [N/m^2] is

$$f_r = -B_\theta J = 0 \qquad f_\theta = B_r J = -\frac{\mu_0 J_0}{2} \sin n\vartheta \cos n\theta$$

$$f_x = f_r \cos\theta - f_\theta \sin\theta \qquad f_y = f_r \sin\theta + f_\theta \cos\theta$$

Appendix I: thin shell approximation
Field and forces: dipole



- In a dipole, the field inside the coil is

$$B_y = -\frac{\mu_0 J_0}{2}$$

- The total force acting on the coil [N/m] is

$$F_x = \frac{B_y^2}{2\mu_0}\frac{4}{3}a \qquad F_y = -\frac{B_y^2}{2\mu_0}\frac{4}{3}a$$

- The Lorentz force on a dipole coil varies
  - with the square of the bore field
  - linearly with the magnetic pressure
  - linearly with the bore radius.
- In a rigid structure, the force determines an azimuthal displacement of the coil and creates a separation at the pole.
- The structure sees F_x.

Appendix I: thin shell approximation
Field and forces: quadrupole

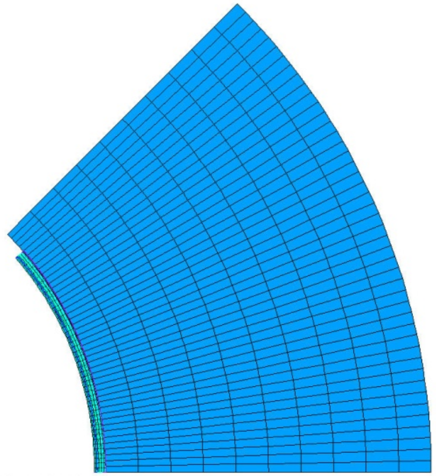
- In a quadrupole, the gradient [T/m] inside the coil is

$$G = \frac{B_c}{a} = -\frac{\mu_0 J_0}{2a}$$

- The total force acting on the coil [N/m] is

$$F_x = \frac{B_c^2}{2\mu_0} a \frac{4\sqrt{2}}{15} = \frac{G^2}{2\mu_0} a^3 \frac{4\sqrt{2}}{15}$$

$$F_y = -\frac{B_c^2}{2\mu_0} a \frac{4\sqrt{2}+8}{15} = -\frac{G^2}{2\mu_0} a^3 \frac{4\sqrt{2}+8}{15}$$



- The Lorentz force on a quadrupole coil varies
  - with the square of the gradient or coil peak field
  - with the cube of the aperture radius (for a fixed gradient).
- Keeping the peak field constant, the force is proportional to the aperture.

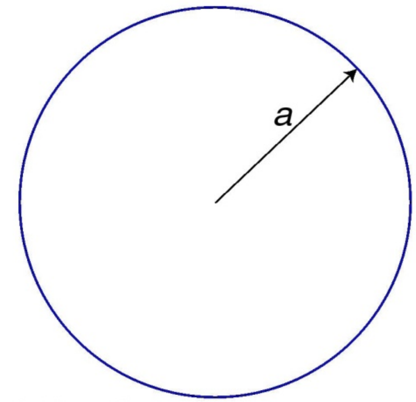
Stored energy and end forces: dipole and quadrupole

- For a dipole, the vector potential within the thin shell is

$$A_z = +\frac{\mu_0 J_0}{2} a \cos\vartheta$$

and, therefore,

$$F_z = \frac{B_y^2}{2\mu_0} 2\pi a^2$$



  - The axial force on a dipole coil varies
    - with the square of the bore field
    - linearly with the magnetic pressure
    - with the square of the bore radius.
- For a quadrupole, the vector potential within the thin shell is

$$A_z = +\frac{\mu_0 J_0}{2} \frac{a}{2} \cos 2\vartheta$$

and, therefore,

$$F_z = \frac{B_c^2}{2\mu_0} \pi a^2 = \frac{G^2 a^2}{2\mu_0} \pi a^2$$

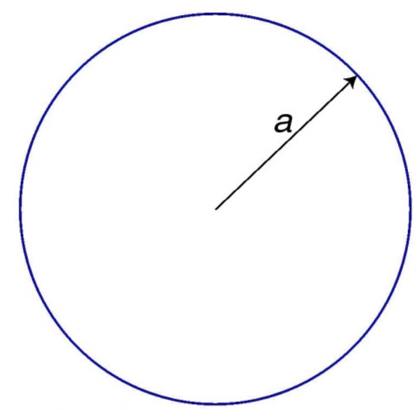
  - Being the peak field the same, a quadrupole has half the F_z of a dipole.

Appendix I: thin shell approximation
Total force on the mid-plane: dipole and quadrupole

- If we assume a "roman arch" condition, where all the f_θ accumulates on the mid-plane, we can compute a total force transmitted on the mid-plane F_θ [N/m]

- For a dipole,

$$F_\theta = \int_0^{\frac{\pi}{2}} f_\theta a d\theta = -\frac{B_y^2}{2\mu_0} 2a$$



- For a quadrupole,

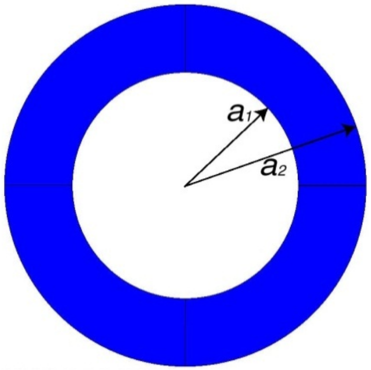
$$F_\theta = \int_0^{\frac{\pi}{4}} f_\theta a d\theta = -\frac{B_c^2}{2\mu_0} a = -\frac{G^2}{2\mu_0} a^3$$

  - Being the peak field the same, a quadrupole has half the F_θ of a dipole.
  - Keeping the peak field constant, the force is proportional to the aperture.

Appendix II
Thick shell approximation

Appendix II: thick shell approximation Field and forces: general case

- We assume
  - $J = J_0 \cos n\theta$ where J_0 [A/m²] is ^ to the cross-section plane
  - Inner (outer) radius of the coils = *a1 (a2)*
  - No iron
- The field inside the aperture is



$$B_{ri} = -\frac{\mu_0 J_0}{2} r^{n-1}\left(\frac{a_2^{2-n} - a_1^{2-n}}{2-n}\right)\sin n\vartheta \qquad B_{\theta i} = -\frac{\mu_0 J_0}{2} r^{n-1}\left(\frac{a_2^{2-n} - a_1^{2-n}}{2-n}\right)\cos n\vartheta$$

- The field in the coil is

$$B_r = -\frac{\mu_0 J_0}{2}\left[r^{n-1}\left(\frac{a_2^{2-n} - r^{2-n}}{2-n}\right) + \frac{1}{2+n}\left(\frac{r^{2+n} - a_1^{2+n}}{r^{1+n}}\right)\right]\sin n\vartheta \qquad B_\theta = -\frac{\mu_0 J_0}{2}\left[r^{n-1}\left(\frac{a_2^{2-n} - r^{2-n}}{2-n}\right) - \frac{1}{2+n}\left(\frac{r^{2+n} - a_1^{2+n}}{r^{1+n}}\right)\right]\cos n\vartheta$$

- The Lorentz force acting on the coil [N/m³] is

$$f_r = -B_\theta J = \frac{\mu_0 J_0^2}{2}\left[r^{n-1}\left(\frac{a_2^{2-n} - r^{2-n}}{2-n}\right) - \frac{1}{2+n}\left(\frac{r^{2+n} - a_1^{2+n}}{r^{1+n}}\right)\right]\cos^2 n\vartheta \qquad f_x = f_r \cos\theta - f_\theta \sin\theta$$

$$f_\theta = B_r J = -\frac{\mu_0 J_0^2}{2}\left[r^{n-1}\left(\frac{a_2^{2-n} - r^{2-n}}{2-n}\right) + \frac{1}{2+n}\left(\frac{r^{2+n} - a_1^{2+n}}{r^{1+n}}\right)\right]\sin n\theta \cos n\vartheta \qquad f_y = f_r \sin\theta + f_\theta \cos\theta$$

Appendix II: thick shell approximation
Field and forces: dipole

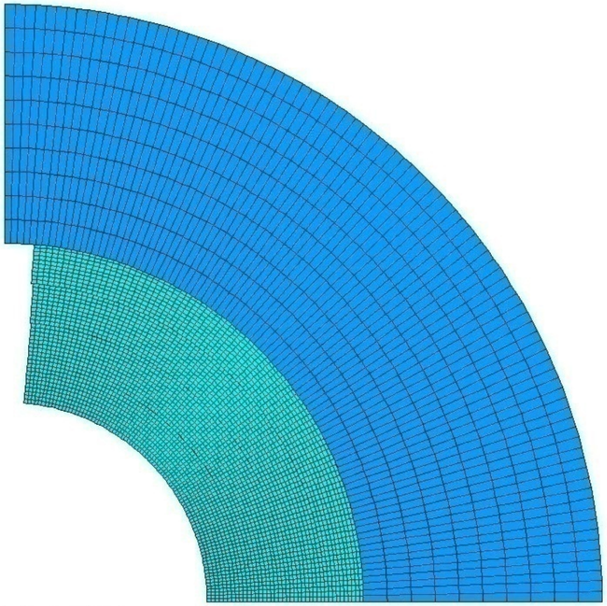
- In a dipole, the field inside the coil is

$$B_y = -\frac{\mu_0 J_0}{2}(a_2 - a_1)$$

- The total force acting on the coil [N/m] is

$$F_x = \frac{\mu_0 J_0^2}{2}\left[\frac{7}{54}a_2^3 + \frac{1}{9}\left(\ln\frac{a_2}{a_1} + \frac{10}{3}\right)a_1^3 - \frac{1}{2}a_2 a_1^2\right]$$

$$F_y = -\frac{\mu_0 J_0^2}{2}\left[\frac{2}{27}a_2^3 + \frac{2}{9}\left(\ln\frac{a_1}{a_2} + \frac{1}{3}\right)a_1^3\right]$$

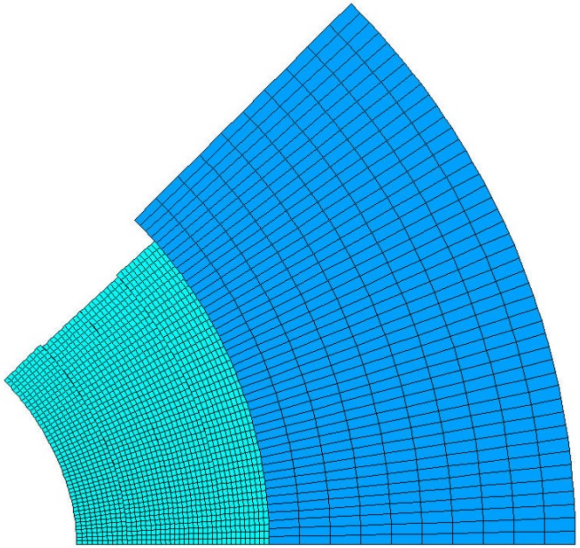


- In a quadrupole, the field inside the coil is

$$G = \frac{B_y}{r} = -\frac{\mu_0 J_0}{2} \ln\frac{a_2}{a_1} \qquad \text{being} \qquad \frac{a_2^{2-n} - a_1^{2-n}}{2-n} = \ln\frac{a_2}{a_1}$$

- The total force acting on the coil [N/m] is

$$F_x = \frac{\mu_0 J_0^2}{2}\left[\frac{\sqrt{2}}{540}\frac{11a_2^4 - 27a_1^4}{a_2} + \frac{5\sqrt{2}}{45}\left(\ln\frac{a_1}{a_2} + \frac{4}{15}\right)a_1^3\right]$$



$$F_y = -\frac{\mu_0 J_0^2}{2}\left\{\frac{1}{540}\frac{\left(11\sqrt{2}+7\right)a_2^4 + \left(81 - 27\sqrt{2}\right)a_1^4}{a_2} + \frac{1}{45}\left[\left(5\sqrt{2} - 5\right)\ln\frac{a_1}{a_2} + \frac{4\sqrt{2}-22}{3}\right]a_1^3\right\}$$

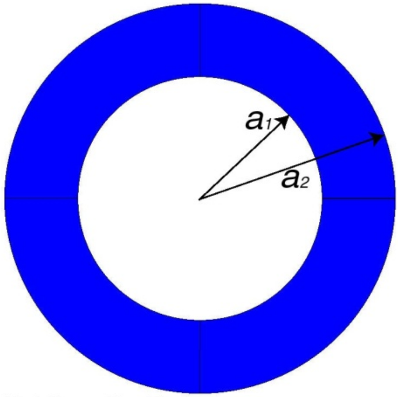
Stored energy, end forces: dipole and quadrupole

- For a dipole,

$$A_z = +\frac{\mu_0 J_0}{2} r\left[(a_2 - r) - \frac{r^3 - a_1^3}{3r^2}\right]\cos\theta$$

and, therefore,

$$F_z = +\frac{\mu_0 J_0^2}{2}\frac{\pi}{2}\left(\frac{a_2^4}{6} - \frac{2}{3}a_1^3 a_2 + \frac{a_1^4}{2}\right)$$



- For a quadrupole,

$$A_z = +\frac{\mu_0 J_0}{2}\frac{r}{2}\left(r\ln\frac{a_2}{r} - \frac{r^4 - a_1^4}{4r^3}\right)\cos 2\theta$$

and, therefore,

$$F_z = +\frac{\mu_0 J_0^2}{2}\frac{\pi}{8}\left[\frac{a_2^4}{8} + \left(\ln\frac{a_1}{a_2} - \frac{1}{4}\right)a_1^4\right]$$

- For a dipole,

$$\sigma_{\theta\_mid-plane} = \int_0^{\pi/2} f_\theta r d\theta = -\frac{\mu_0 J_0^2}{2}\frac{r}{2}\left[(a_2 - r) + \frac{r^3 - a_1^3}{3r^2}\right]$$

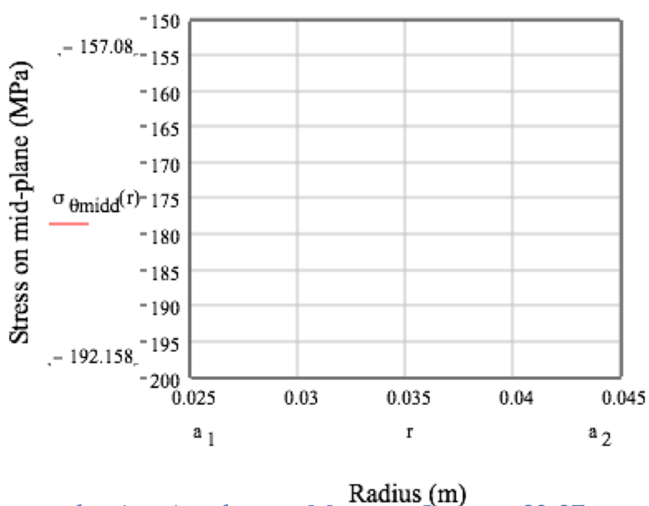
No shear

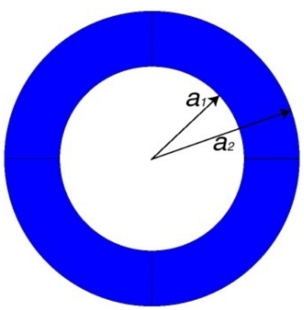
$$\sigma_{\theta\_mid-plane\_av} = -\frac{\mu_0 J_0^2}{2}\left[\frac{5}{36}a_2^3 + \frac{1}{6}\left(\ln\frac{a_1}{a_2} + \frac{2}{3}\right)a_1^3 - \frac{1}{4}a_2 a_1^2\right]\frac{1}{a_2 - a_1}$$

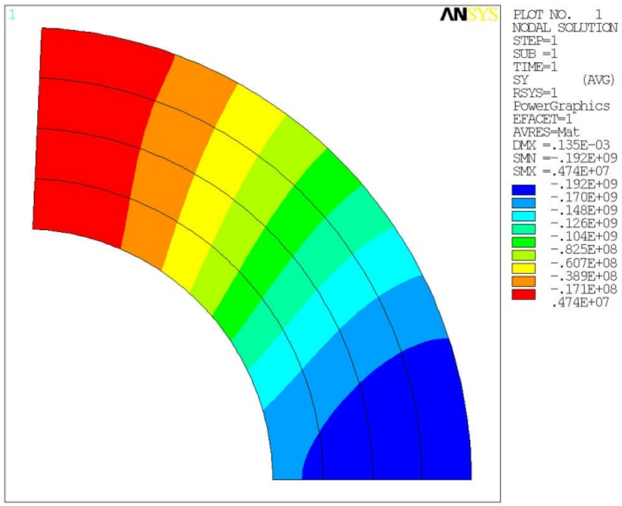
- For a quadrupole,

$$\sigma_{\theta\_mid-plane} = \int_0^{\pi/4} f_\theta r d\theta = -\frac{\mu_0 J_0^2}{2}\frac{r}{4}\left(r\ln\frac{a_2}{r} - \frac{r^4 - a_1^4}{4r^3}\right)$$

$$\sigma_{\theta\_mid-plane\_av} = -\frac{\mu_0 J_0^2}{2}\left[\frac{1}{144}\frac{7a_2^4 + 9a_1^4}{a_2} + \frac{1}{12}\left(\ln\frac{a_1}{a_2} + \frac{4}{3}\right)a_1^3\right]\frac{1}{a_2 - a_1}$$



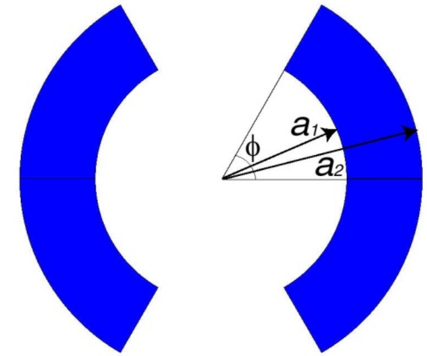




Appendix III
Sector approximation

Appendix III: sector approximation Field and forces: dipole

- We assume
  - $J=J_0$ is ^ the cross-section plane
  - Inner (outer) radius of the coils = *a1 (a2)*
  - Angle $\phi = 60^o$ (third harmonic term is null)
  - No iron



- The field inside the aperture

$$B_r = -\frac{2\mu_0 J_0}{\pi}\left[(a_2-a_1)\sin\phi\sin\theta + \sum_{n=1}^{\infty}\frac{r^{2n}}{(2n+1)(2n-1)}\left(\frac{1}{a_1^{n-1}}-\frac{1}{a_2^{n-1}}\right)\sin(2n+1)\phi\,\sin(2n+1)\theta\right]$$

$$B_\theta = -\frac{2\mu_0 J_0}{\pi}\left[(a_2-a_1)\sin\phi\cos\theta + \sum_{n=1}^{\infty}\frac{r^{2n}}{(2n+1)(2n-1)}\left(\frac{1}{a_1^{n-1}}-\frac{1}{a_2^{n-1}}\right)\sin(2n+1)\phi\,\cos(2n+1)\theta\right]$$

- The field in the coil is

$$B_r = -\frac{2\mu_0 J_0}{\pi}\left\{(a_2-r)\sin\phi\sin\theta + \sum_{n=1}^{\infty}\left[1-\left(\frac{a_1}{r}\right)^{2n+1}\right]\frac{r}{(2n+1)(2n-1)}\sin(2n-1)\phi\,\sin(2n-1)\theta\right\}$$

$$B_\theta = -\frac{2\mu_0 J_0}{\pi}\left\{(a_2-r)\sin\phi\cos\theta - \sum_{n=1}^{\infty}\left[1-\left(\frac{a_1}{r}\right)^{2n+1}\right]\frac{r}{(2n+1)(2n-1)}\sin(2n-1)\phi\,\cos(2n-1)\theta\right\}$$

Appendix III: sector approximation
Field and forces: dipole

- The Lorentz force acting on the coil [N/m³], considering the basic term, is

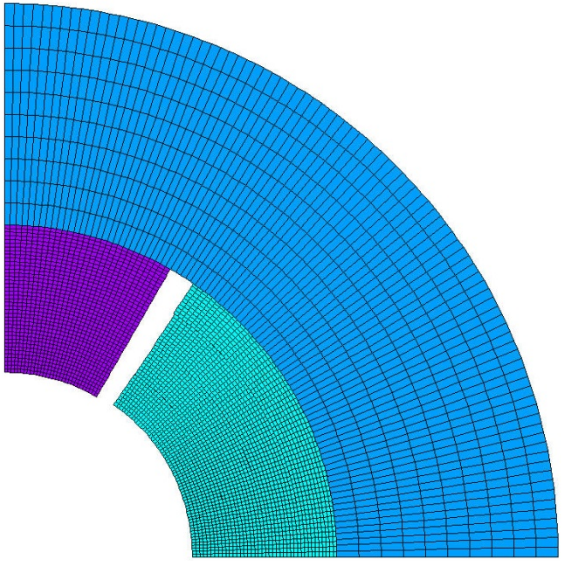
$$f_r = -B_\theta J = +\frac{2\mu_0 J_0^2}{\pi}\sin\phi\left[(a_2 - r) - \frac{r^3 - a_1^3}{3r^2}\right]\cos\theta \qquad f_x = f_r\cos\theta - f_\theta\sin\theta$$

$$f_\theta = B_r J = -\frac{2\mu_0 J_0^2}{\pi}\sin\phi\left[(a_2 - r) + \frac{r^3 - a_1^3}{3r^2}\right]\sin\theta \qquad f_y = f_r\sin\theta + f_\theta\cos\theta$$

- The total force acting on the coil [N/m] is

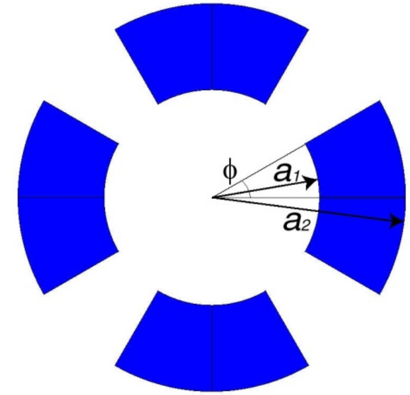
$$F_x = +\frac{2\mu_0 J_0^2}{\pi}\frac{\sqrt{3}}{2}\left[\frac{2\pi - \sqrt{3}}{36}a_2^3 + \frac{\sqrt{3}}{12}\ln\frac{a_2}{a_1}a_1^3 + \frac{4\pi + \sqrt{3}}{36}a_1^3 - \frac{\pi}{6}a_2 a_1^2\right]$$

$$F_y = -\frac{2\mu_0 J_0^2}{\pi}\frac{\sqrt{3}}{2}\left[\frac{1}{12}a_2^3 + \frac{1}{4}n\frac{a_1}{a_2}a_1^3 - \frac{1}{12}a_1^3\right]$$



Appendix III: sector approximation Field and forces: quadrupole

- We assume
  - $J=J_0$ is ^ the cross-section plane
  - Inner (outer) radius of the coils = *a1 (a2)*
  - Angle $\phi = 30^{\circ}$ (third harmonic term is null)
  - No iron



- The field inside the aperture

$$B_r = -\frac{2\mu_0 J_0}{\pi}\left\{ r \ln\frac{a_2}{a_1}\sin 2\phi \sin 2\theta + \sum_{n=1}^{\infty}\frac{r}{2n(4n+2)}\left[\left(\frac{r}{a_1}\right)^{4n} - \left(\frac{r}{a_2}\right)^{4n}\right]\sin(4n+2)\phi \sin(4n+2)\theta \right\}$$

$$B_\theta = -\frac{2\mu_0 J_0}{\pi}\left\{ r \ln\frac{a_2}{a_1}\sin 2\phi \cos 2\theta + \sum_{n=1}^{\infty}\frac{r}{2n(4n+2)}\left[\left(\frac{r}{a_1}\right)^{4n} - \left(\frac{r}{a_2}\right)^{4n}\right]\sin(4n+2)\phi \cos(4n+2)\theta \right\}$$

- The field in the coil is

$$B_r = -\frac{2\mu_0 J_0}{\pi}\left\{ r \ln\frac{a_2}{r}\sin 2\phi \sin 2\theta + \sum_{n=1}^{\infty}\frac{r}{2n(4n-2)}\left[1-\left(\frac{a_1}{r}\right)^{4}\right]\sin(4n-2)\phi \sin(4n-2)\theta \right\}$$

$$B_\theta = -\frac{2\mu_0 J_0}{\pi}\left\{ r \ln\frac{a_2}{r}\sin 2\phi \cos 2\theta - \sum_{n=1}^{\infty}\frac{r}{2n(4n-2)}\left[1-\left(\frac{a_1}{r}\right)^{4}\right]\sin(4n-2)\phi \cos(4n-2)\theta \right\}$$

Appendix III: sector approximation
Field and forces: quadrupole

- The Lorentz force acting on the coil [N/m³], considering the basic term, is

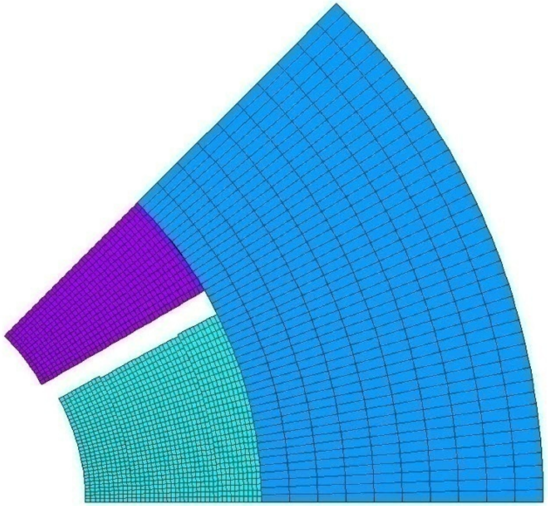
$$f_r = -B_\theta J = +\frac{2\mu_0 J_0^2}{\pi}\sin 2\phi\left(r\ln\frac{a_2}{r}-\frac{r^4-a_1^4}{4r^3}\right)\cos 2\theta \qquad f_x = f_r\cos\theta - f_\theta\sin\theta$$

$$f_\theta = B_r J = -\frac{2\mu_0 J_0^2}{\pi}\sin 2\phi\left(r\ln\frac{a_2}{r}+\frac{r^4-a_1^4}{4r^3}\right)\sin 2\theta \qquad f_y = f_r\sin\theta + f_\theta\cos\theta$$

- The total force acting on the coil [N/m] is

$$F_x = +\frac{2\mu_0 J_0^2}{\pi}\frac{\sqrt{3}}{12}\left[\frac{1}{72}\frac{12a_2^4-36a_1^4}{a_2}+\left(\ln\frac{a_1}{a_2}+\frac{1}{3}\right)a_1^3\right]$$

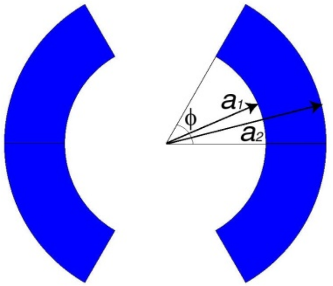
$$F_y = -\frac{2\mu_0 J_0^2}{\pi}\frac{\sqrt{3}}{2}\left[\frac{5-2\sqrt{3}}{36}a_2^3+\frac{1}{12}\frac{a_1^4}{a_2}+\frac{2-\sqrt{3}}{6}\ln\frac{a_1}{a_2}a_1^3+\frac{1}{9}\left(\frac{\sqrt{3}}{2}-2\right)a_1^3\right]$$



Appendix III: sector approximation
Stored energy, end forces: dipole and quadrupole

- For a dipole,

$$A_z = +\frac{2\mu_0 J_0}{\pi} r \sin\phi \left[(a_2 - r) - \frac{r^3 - a_1^3}{3r^2} \right] \cos\theta$$



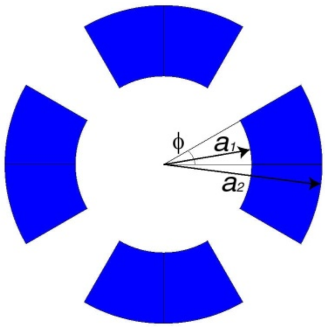
and, therefore,

$$F_z = +\frac{2\mu_0 J_0^2}{\pi} \frac{3}{2} \left(\frac{a_2^4}{6} - \frac{2}{3} a_1^2 a_2 + \frac{a_1^4}{2} \right)$$

- For a quadrupole,

$$A_z = +\frac{2\mu_0 J_0}{\pi} \frac{r}{2} \sin 2\phi \left(r \ln\frac{a_2}{r} - \frac{r^4 - a_1^4}{4r^3} \right) \cos 2\theta$$

and, therefore,



$$F_z = +\frac{2\mu_0 J_0^2}{\pi} \frac{3}{8} \left[\frac{a_2^4}{8} + \left(\ln\frac{a_1}{a_2} - \frac{1}{4} \right) a_1^4 \right]$$

Appendix III: sector approximation
Stress on the mid-plane: dipole and quadrupole

- For a dipole,

No shear

$$\sigma_{\theta\_mid-plane} = \int_0^{\pi/3} f_\theta r d\theta = -\frac{2\mu_0 J_0^2}{\pi}\frac{\sqrt{3}}{4} r\left[(a_2 - r) - \frac{r^3 - a_1^3}{3r^2}\right]$$

$$\sigma_{\theta\_mid-plane\_av} = -\frac{2\mu_0 J_0^2}{\pi}\frac{3}{4}\left[\frac{5}{36}a_2^3 + \frac{1}{6}\left(\ln\frac{a_1}{a_2} + \frac{2}{3}\right)a_1^3 - \frac{1}{4}a_2 a_1^2\right]\frac{1}{a_2 - a_1}$$

- For a quadrupole,

$$\sigma_{\theta\_mid-plane} = \int_0^{\pi/6} f_\theta r d\theta = -\frac{2\mu_0 J_0^2}{\pi}\frac{\sqrt{3}}{8} r\left(r\ln\frac{a_2}{r} - \frac{r^4 - a_1^4}{4r^3}\right)$$

$$\sigma_{\theta\_mid-plane\_av} = -\frac{2\mu_0 J_0^2}{\pi}\frac{\sqrt{3}}{2}\left[\frac{1}{36}\frac{7a_2^4 + 9a_1^4}{a_2} + \frac{1}{3}\left(\ln\frac{a_1}{a_2} + \frac{4}{3}\right)a_1^3\right]\frac{1}{a_2 - a_1}$$