Long Range Wakefields

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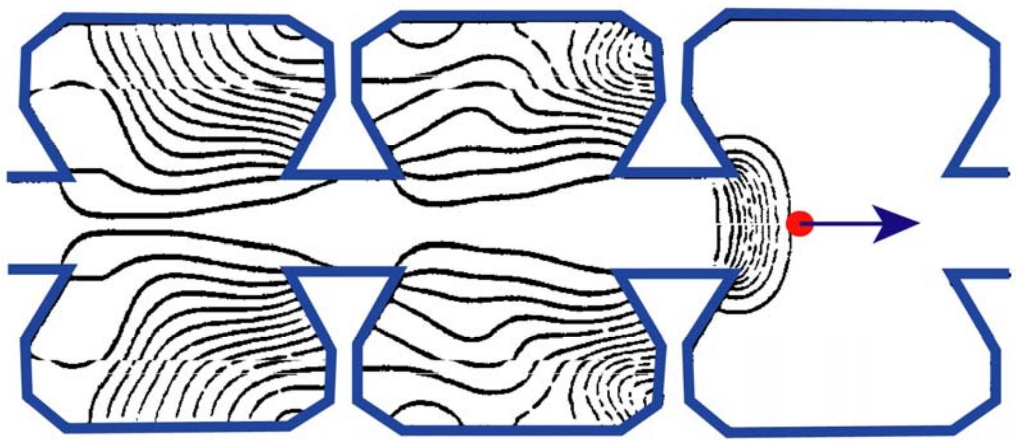
Outline of this lecture

- The concept of the long-range wakefields, causality.
- Longitudinal wakefields. Wake potential.
- Transverse wakefields. Wake potential.
- Panofsky-Wenzel theorem.
- Some examples. Beam loading. Constant impedance vs. constant gradient accelerating structure.
- Wakefield suppression: damping and detuning.
- Wakefield acceleration.

Long-range wakefields

- The electron beam traveling through the accelerating structure produces electromagnetic disturbances which can be described as a sum of the resonant modes.
- Wakefields include the higher-order modes and the beam excited accelerating mode (the effect of beam loading).



Electromagnetic fields of a point charge in free space

The charge moves with the speed of light:

$$E_r = \frac{Q\delta(z-ct)}{2\pi\varepsilon_0 r} \qquad\qquad B_\theta = \frac{\mu_0 c Q\delta(z-ct)}{2\pi r}$$

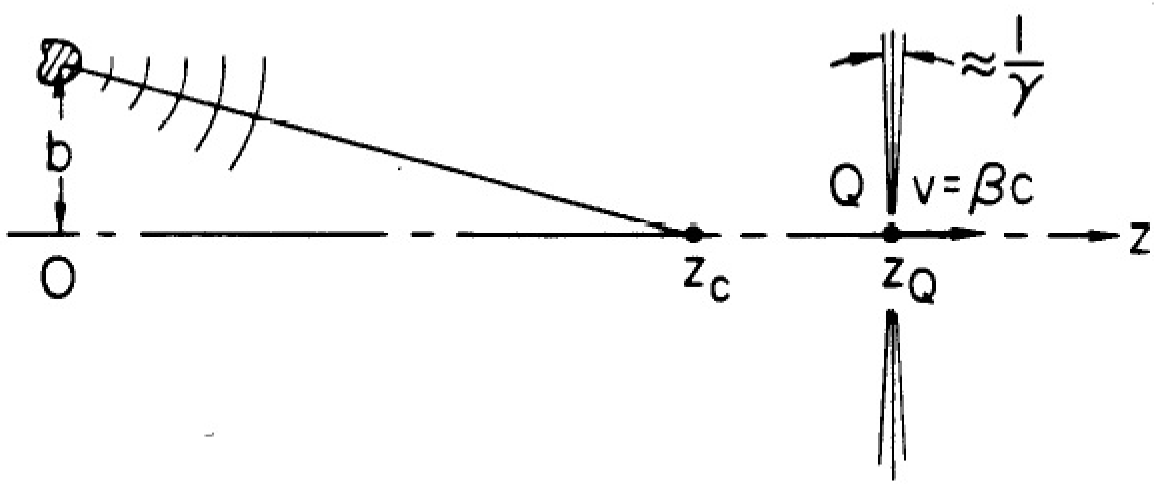
The Lorentz force acting on a test charge *Q'* that also moves with the speed of light:

$$F_r = Q'(E_r - cB_\theta) = \frac{QQ'\delta(z-ct)}{2\pi\varepsilon_0 r} - \frac{QQ'\mu_0 c^2\delta(z-ct)}{2\pi r} = 0.$$

In relativistic limit there is no electromagnetic interactions between two charge particles moving in free space.

Wakefields or no wakefields?

- The charge moves in an infinite pipe of an arbitrary cross section: no wake.
- The charge moves in a pipe with resistive wall losses: wakes.
- The charge moves by an obstacle: wakes.



Wake potential for a cavity

The longitudinal wake potential is defined as the total voltage lost by a test charge *Q* following at a distance *s* on the same path divided by the value of *Q*:

$$W_z(s) = -\frac{1}{Q}\int_0^L E_z\left(z, t = \frac{(s+z)}{c}\right) dz.$$

The transverse wake potential is defined as the momentum kick experienced by a test charge *Q* following at a distance *s* on the same path divided by the value of *Q*:

$$\vec{W}_\perp(s) = \frac{1}{Q}\int_0^L \left[\vec{E}_z + \left(\vec{v} \times \vec{B}\right)_\perp\right]_{t=(z+s)/c} dz.$$

Electromagnetic fields in the cavity

Maxwell's equations:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Express fields through potentials:

$$\vec{B} = \vec{\nabla} \times \vec{A}, \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \cdot \Phi$$

Coulomb gauge: $\vec{\nabla} \cdot \vec{A} = 0.$

$$\vec{\nabla}^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} - \frac{1}{c^2} \frac{\partial \vec{\nabla} \Phi}{\partial t} = -\mu_0 \vec{J}$$

Eigenmode expansion

$$\vec{A}(\vec{x},t) = \sum_{\lambda} q_{\lambda}(t)\vec{a}_{\lambda}(\vec{x}), \qquad \Phi(\vec{x},t) = \sum_{\lambda} r(t)\phi_{\lambda}(\vec{x}),$$

The eigenmodes $\vec{a}_{\lambda}(\vec{x}), \phi_{\lambda}(\vec{x})$ satisfy the following equation and the boundary conditions on the surface of the cavity:

$$\vec{\nabla}^2 \vec{a}_{\lambda} + \frac{\omega_{\lambda}^2}{c^2}\vec{a}_{\lambda} = 0, \qquad \vec{\nabla}^2 \phi_{\lambda} + \frac{\omega_{\lambda}^2}{c^2}\phi_{\lambda} = 0,$$

$$\vec{a}_{\lambda} \times \hat{n}|_S = 0. \qquad \phi_{\lambda}|_S = 0.$$

Coulomb gauge: $\vec{\nabla} \cdot \vec{a}_{\lambda} = 0.$

Equations for the eigenmode coefficients

$$\ddot{q}_\lambda + \omega_\lambda^2 q_\lambda = \frac{1}{2U_\lambda}\int_V \vec{j}\cdot\vec{a}_\lambda\, dV,$$

where $U_\lambda = \frac{\varepsilon_0}{2}\int_V \vec{a}_\lambda^2\, dV.$

$$r_\lambda(t) = \frac{1}{2T_\lambda}\int_V \rho\phi_\lambda\, dV,$$

where $T_\lambda = \frac{\varepsilon_0}{2}\int_V \left(\vec{\nabla}\phi_\lambda\right)^2\, dV.$

The stored energy: $\mathcal{E} = \sum_\lambda \left(\left(\dot{q}_\lambda^2 + \omega_\lambda^2 q_\lambda\right)U_\lambda + r_\lambda^2 T_\lambda\right).$

Eigenmode coefficients for a point charge

$$\rho(\vec{x},t) = Q\delta(x)\delta(y)\delta(z-ct),$$
$$\vec{j}(\vec{x},t) = \hat{z}c\rho(\vec{x},t).$$

$$\ddot{q}_\lambda + \omega_\lambda^2 q_\lambda = \frac{cQ}{2U_\lambda}\begin{cases} 0, & t<0 \\ a_{\lambda z}(0,0,ct), & 0<t<L/c \\ 0, & t>L/c \end{cases}$$

$$r_\lambda(t) = \frac{Q}{2T_\lambda}\begin{cases} 0, & t<0 \\ \phi_\lambda(0,0,ct), & 0<t<L/c \\ 0, & t>L/c \end{cases}$$

Eigenmode coefficients for a point charge (continued)

Initial conditions: no fields in the cavity before the particle enters: $q(0) = 0, \dot{q}(0) = 0.$

$$q_\lambda(t) = \begin{cases} 0, & t < 0 \\ \dfrac{cQ}{2U_\lambda \omega_\lambda} \displaystyle\int_0^t a_{\lambda z}(0,0,ct') \sin \omega_\lambda (t - t') dt', & 0 < t < L/c \\ q_\lambda(t = L/c) \cos \omega_\lambda (t - L/c) + \dfrac{\dot{q}_\lambda(t = L/c)}{\omega_\lambda} \sin \omega_\lambda (t - L/c), & t > L/c \end{cases}$$

$$\mathcal{E} = Q^2 \sum_\lambda \frac{1}{4U_\lambda} \int_0^L \int_0^L dz' dz'' a_{\lambda z}(0,0,z') a_{\lambda z}(0,0,z'') \cos \frac{\omega_\lambda (z' - z'')}{c}$$

Loss factor (or kick factor)

Introduce:

$$V_\lambda = \int_0^L exp(i\omega_\lambda z/c) a_{\lambda z}(0,0,z) dz,$$

$$\mathcal{E} = Q^2 \sum_\lambda \frac{|V_\lambda|^2}{4U_\lambda}.$$

Loss factor k_λ is the amount of energy deposited by the charge into the mode λ.

$$k_\lambda = \frac{|V_\lambda|^2}{4U_\lambda}, \qquad \mathcal{E} = Q^2 \sum_\lambda k_\lambda.$$

Wake potentials

$$W_z(s) = \frac{1}{Q}\sum_{\lambda}\int_0^L \left(\dot{q}_\lambda\left(\frac{z+s}{c}\right) a_{\lambda z}(z) + r_\lambda\left(\frac{z+s}{c}\right)\frac{\partial}{\partial z}\phi_\lambda(z)\right) dz,$$

$$W_z(s) = \sum_{\lambda} k_\lambda \cos\frac{\omega_\lambda s}{c}\begin{cases} 0, & s<0 \\ 1, & s=0 \\ 2, & s>0 \end{cases}$$

$$\vec{W}_\perp(s) = \frac{1}{Q}\sum_{\lambda}\int_0^L \left(c q_\lambda\left(\frac{z+s}{c}\right)\vec{\nabla}_\perp a_{\lambda z}(z) - r_\lambda\left(\frac{z+s}{c}\right)\vec{\nabla}_\perp\phi_\lambda(z)\right) dz,$$

$$\vec{W}_\perp(s) = \begin{cases} 0, & s<0 \\ \sum_{\lambda} c\frac{V_\lambda^* \vec{\nabla}_\perp V_\lambda}{2U_\lambda \omega_\lambda}\sin\frac{\omega_\lambda s}{c} \end{cases}$$

Generalization for the case of parallel charges

$$\rho(\vec{x},t) = Q\delta(\vec{r}-\vec{r}')\delta(z-ct),$$
$$\vec{j}(\vec{x},t) = \hat{z}c\rho(\vec{x},t).$$

$$W_z(\vec{r},\vec{r}',s) = -\frac{1}{Q}\int_0^L E_z^{\vec{r}'}\left(\vec{r},z,t=\frac{(s+z)}{c}\right)dz.$$

$$\overrightarrow{W}_\perp(\vec{r},\vec{r}',s) = \frac{1}{Q}\int_0^L \left[\vec{E}_\perp^{\vec{r}'}\left(\vec{r},z,t=\frac{(s+z)}{c}\right) + \left(\vec{v}\times\vec{B}^{\vec{r}'}\left(\vec{r},z,t=\frac{(s+z)}{c}\right)\right)_\perp\right]_{t=(z+s)/c} dz.$$

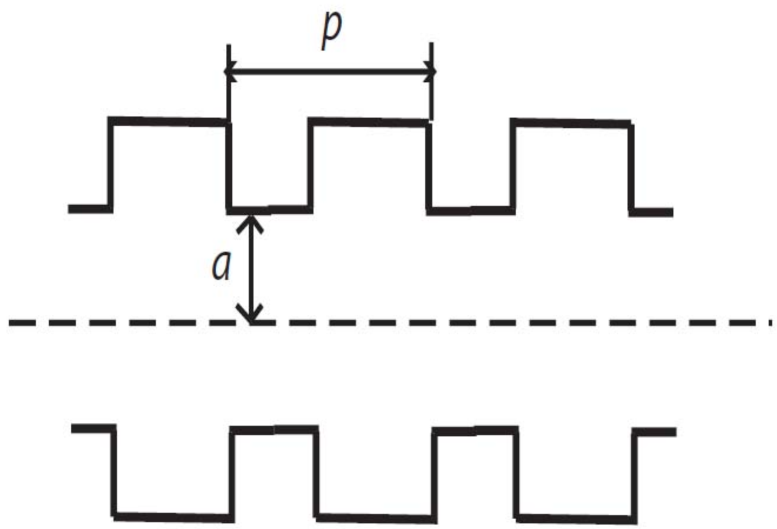
Panofsky-Wenzel Theorem

$$W_z(\vec{r},\vec{r}',s) = \begin{cases} 0, & s < 0 \\ \sum_\lambda \frac{V_\lambda^*(\vec{r}')V_\lambda(\vec{r})}{2U_\lambda}\cos\frac{\omega_\lambda s}{c} \end{cases}$$

$$\vec{W}_\perp(\vec{r},\vec{r}',s) = \begin{cases} 0, & s < 0 \\ \sum_\lambda c\frac{V_\lambda^*(\vec{r}')\vec{\nabla}_\perp V_\lambda(\vec{r})}{2U_\lambda\omega_\lambda}\sin\frac{\omega_\lambda s}{c} \end{cases}$$

$$\frac{\partial}{\partial s}\vec{W}_\perp(\vec{r},\vec{r}',s) = \vec{\nabla}_\perp W_z(\vec{r},\vec{r}',s)$$

Infinitely repeating structures



$$\vec{a}_\lambda(\vec{r}, z+p) = e^{i\beta_\lambda p}\vec{a}_\lambda(\vec{r}, z),$$
$$\phi_\lambda(\vec{r}, z+p) = e^{i\beta_\lambda p}\phi_\lambda(\vec{r}, z).$$

$\beta_\lambda p$ is the phase advance per cell.

Fourier expansion

$$\vec{a}_\lambda(\vec{r}, z) = e^{i\beta_\lambda z} \sum_{l=-\infty}^{\infty} \vec{f}_{\lambda l}(\vec{r})\, e^{2\pi i l z/p}$$

Each Fourier harmonics $\vec{a}_{\lambda n} = \vec{f}_{\lambda n}(\vec{r}) e^{\frac{2\pi i n z}{p} + i\beta_\lambda z}$

satisfied Maxwell's equations. So we can introduce

$$W_{z\lambda n}(\vec{r}, \vec{r}\,', s) = \frac{V_{\lambda n}^*(\vec{r}\,')V_{\lambda n}(\vec{r})}{2U_\lambda} \cos\frac{\omega_\lambda s}{c}$$

$$\vec{W}_{\perp\lambda n}(\vec{r}, \vec{r}\,', s) = c\,\frac{V_{\lambda n}^*(\vec{r}\,')\vec{\nabla}_\perp V_{\lambda n}(\vec{r})}{2U_\lambda \omega_\lambda} \sin\frac{\omega_\lambda s}{c}$$

Resonance condition

$$V_{\lambda n} = \lim_{N\to\infty} \frac{1}{N} \int_0^{Np} exp(i\omega_\lambda z/c) a_{\lambda z n}(0,0,z) dz$$

$$= f_{\lambda n z}(\vec{r}) \lim_{N\to\infty} \frac{1}{N} \int_0^{Np} exp(i\omega_\lambda z/c) exp\left(i\beta_\lambda z + i\frac{2\pi n z}{p}\right) dz$$

$$= \begin{cases} 0, & \frac{2\pi n}{p} + \beta_\lambda + \frac{\omega_\lambda}{c} \neq 0 \\ p f_{\lambda n z}(\vec{r}), & \frac{2\pi n}{p} + \beta_\lambda + \frac{\omega_\lambda}{c} = 0 \end{cases}.$$

Only resonant space harmonics interact with the particles.

Cylindrically symmetric waveguide

Eigenmodes of the system:

$$a_{mnz}(\vec{r}) = J_m\left(\frac{\omega_{mn}}{c} r\right)\cos(m\theta).$$

The wake potentials:

$$W_{zm}(\vec{r},\vec{r}',s) = \sum_n \frac{p^2}{2U_{mn}} J_m\left(\frac{\omega_{mn} r'}{c}\right) J_m\left(\frac{\omega_{mn} r}{c}\right)\cos m\theta \cos\left(\frac{\omega_{mn} s}{c}\right),$$

$$\vec{W}_{\perp m}(\vec{r},\vec{r}',s) = \sum_n \frac{cp^2}{2U_{mn}\omega_{mn}} J_m\left(\frac{\omega_{mn} r'}{c}\right) \vec{\nabla}_\perp \left(J_m\left(\frac{\omega_{mn} r}{c}\right)\cos m\theta\right)\sin\left(\frac{\omega_{mn} s}{c}\right).$$

Express wake potentials through the kick factor in cylindrically symmetric structures

$$k_{mn}(a) = \frac{p^2 J_m^2\left(\frac{\omega_{mn}}{c}a\right)}{4U_{mn}},$$

$$W_{zm}(\vec{r},\vec{r}',s) = \sum_n 2k_{mn}(a)\frac{J_m\left(\frac{\omega_{mn}r'}{c}\right)J_m\left(\frac{\omega_{mn}r}{c}\right)}{J_m^2\left(\frac{\omega_{mn}a}{c}\right)}\cos m\theta\cos\left(\frac{\omega_{mn}s}{c}\right).$$

$$\vec{W}_{\perp m}(\vec{r},\vec{r}',s) = \sum_n \frac{2ck_{mn}(a)}{\omega_{mn}}\frac{J_m\left(\frac{\omega_{mn}r'}{c}\right)\vec{\nabla}_\perp\left(J_m\left(\frac{\omega_{mn}r}{c}\right)\cos m\theta\right)}{J_m^2\left(\frac{\omega_{mn}a}{c}\right)}\sin\left(\frac{\omega_{mn}s}{c}\right).$$

For the electron beam that is close to axis

$$W_{zm}(\vec{r},\vec{r}',s) = \left(\frac{r'}{a}\right)^m \left(\frac{r}{a}\right)^m \sum_n 2k_{mn}(a)\cos m\theta \cos\left(\frac{\omega_{mn}s}{c}\right),$$

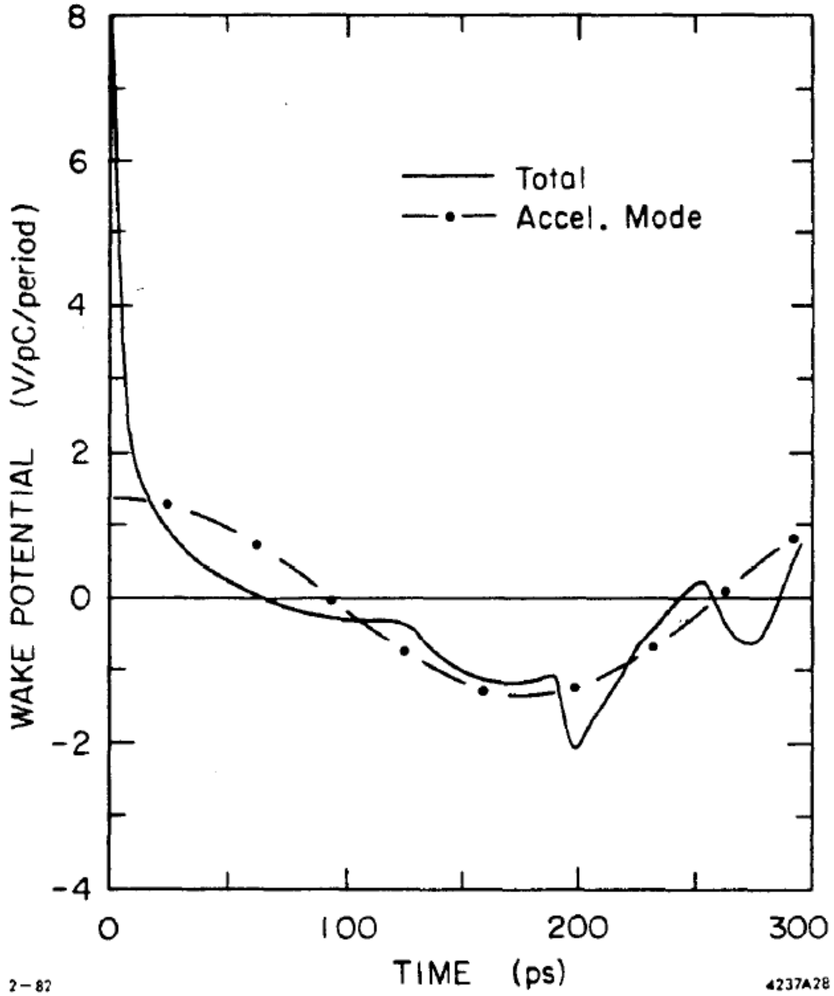
$$\vec{W}_{\perp m}(\vec{r},\vec{r}',s) = m\left(\frac{r'}{a}\right)^m \left(\frac{r}{a}\right)^{m-1} \left(\hat{r}\cos m\theta - \hat{\theta}\sin m\theta\right) \sum_n \frac{2ck_{mn}(a)}{\omega_{mn}a}\sin\left(\frac{\omega_{mn}s}{c}\right).$$

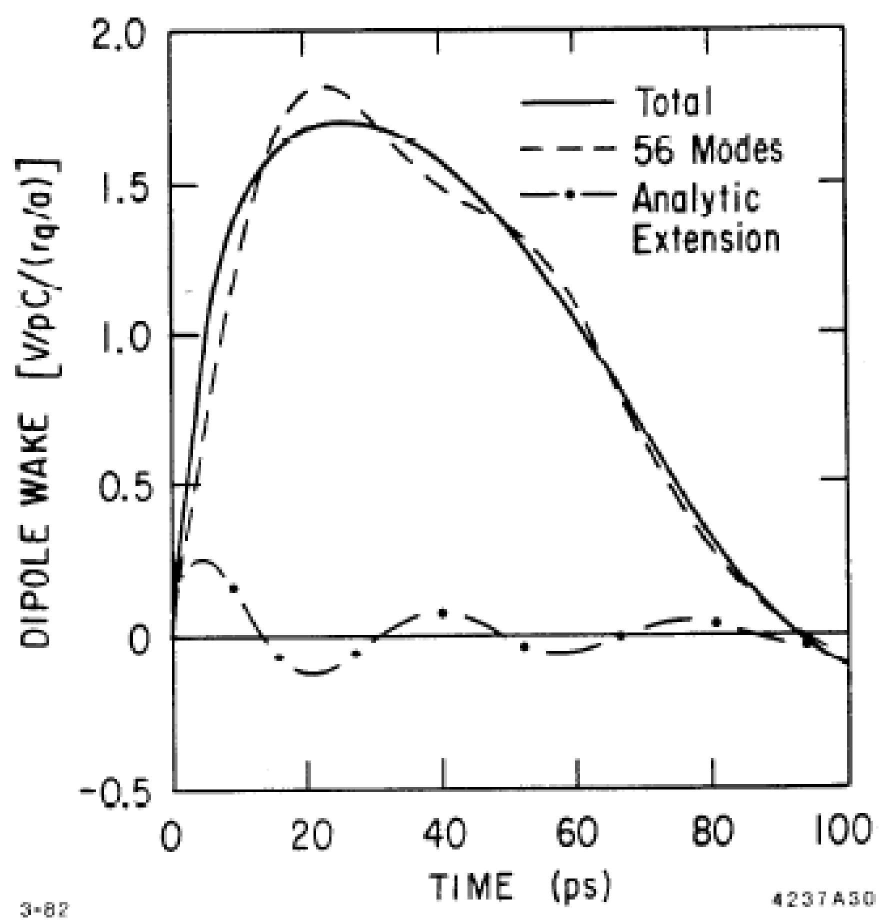
The longitudinal wake is dominated by the fundamental mode and its harmonics (m=0), while the transverse wake is dominated by the dipole modes (m=1):

$$W_{zm}(\vec{r},\vec{r}',s) \approx \sum_n 2k_{0n}(a)\cos\left(\frac{\omega_{0n}s}{c}\right),$$

$$\vec{W}_{\perp m}(\vec{r},\vec{r}',s) = \frac{r'}{a}\hat{x}\sum_n \frac{2ck_{1n}(a)}{\omega_{1n}a}\sin\left(\frac{\omega_{1n}s}{c}\right).$$

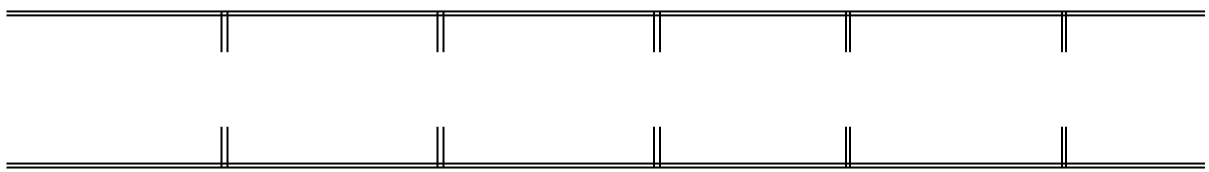
The wake fields for the SLC accelerator





Constant impedance structure

The structure with identical accelerator cells – constant impedance structure:



The power and the accelerating gradient go down along the structure due to Ohmic losses and beam loading.

$$\frac{dE_a}{dz} = -\alpha E_a, \quad \frac{dP}{dz} = -2\alpha P.$$

Constant gradient structure

$$E_a = \text{const}$$

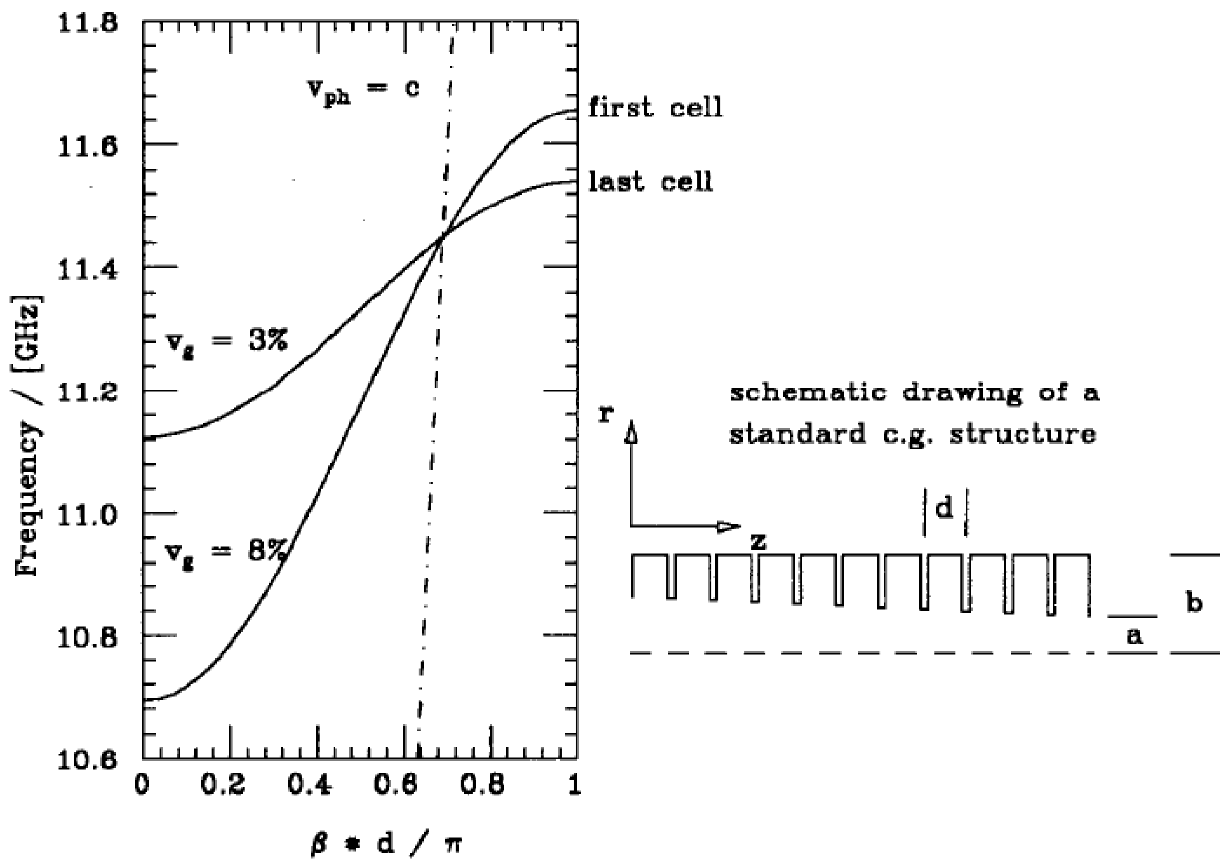
By definition

$$E_a = r_s \left|\frac{dP}{dz}\right|, \quad \left|\frac{dP}{dz}\right| = const, \quad P = P_0 - \Delta P \frac{z}{L}.$$

The group velocity goes down linearly with P:

$$v_g = -\frac{\omega P}{Q \left.\frac{dP}{dz}\right._s}$$

Cell's dispersion in a constant gradient structure



The negative effects of long-range wakefields

- Time-varying transverse deflections in the trailing bunches.
- Beam-breakup (BBU) instability.
- The wakefields extract a small fraction of the beam energy (which can be replaced by additional RF power).
- The power extracted from the beam ultimately represents additional ohmic losses in the structure walls or power delivered to the ohmic load (especially important for the superconducting structures).

Wakefield suppression

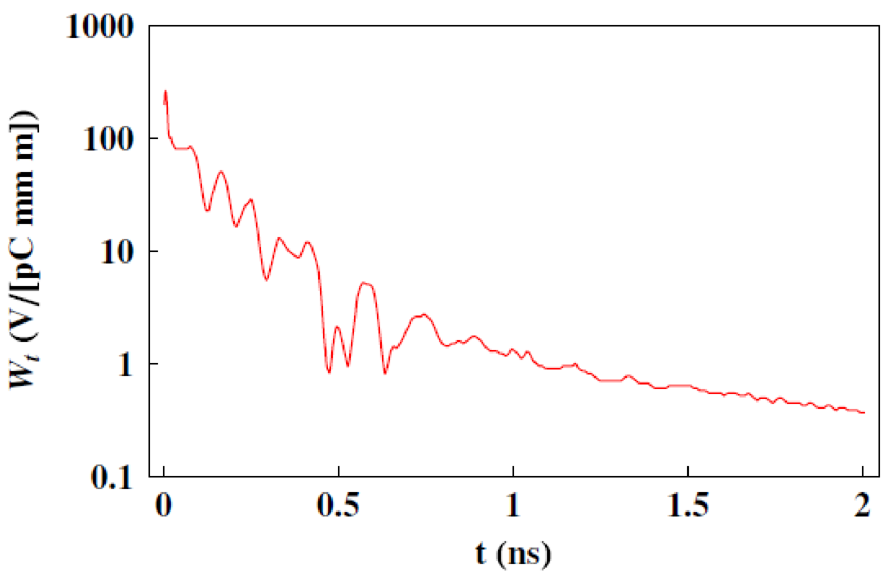
Two main methods for wakefield suppression:

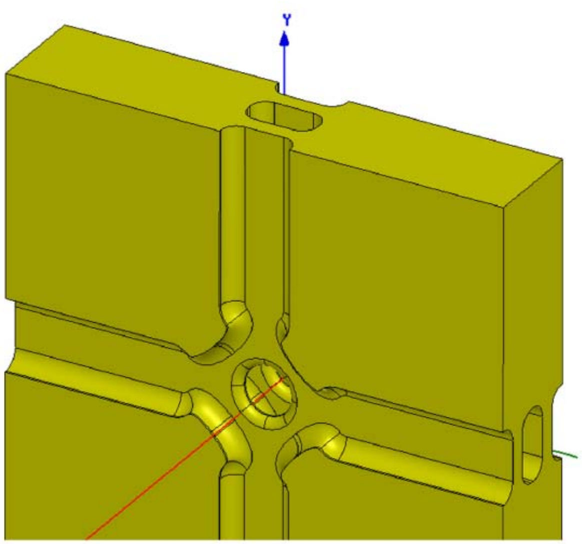
- **Detuning:** the frequencies of the higher order modes vary from cell to cell.
- **Damping:** damping waveguides are subcritical for the fundamental modes but propagate higher order modes.
- Often the method of choice is the moderate damping (Q~300 to 1000) combined with detuning.

CLIC's heavy damping scheme (Q~20)

Drawbacks:

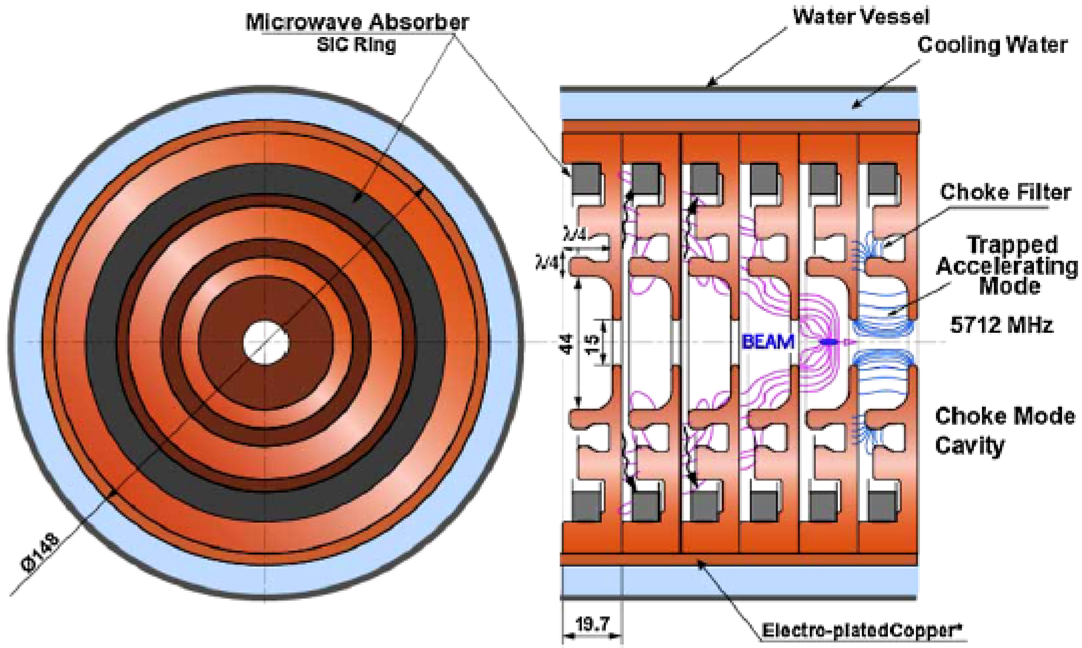
- Reduced Q for the fundamental mode.
- Enhanced surface fields.
- Complicated fabrication.





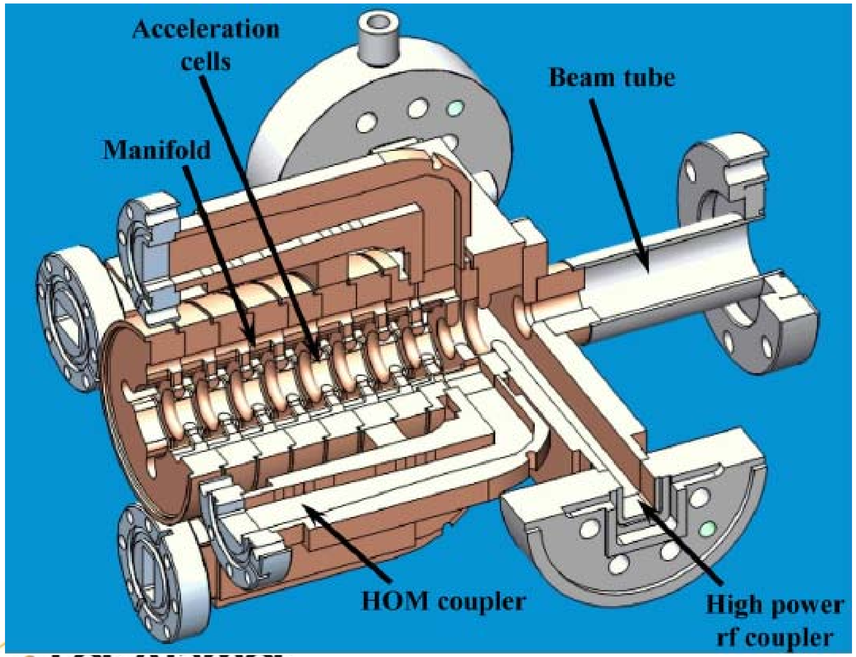
Choke-mode damping at Spring-8

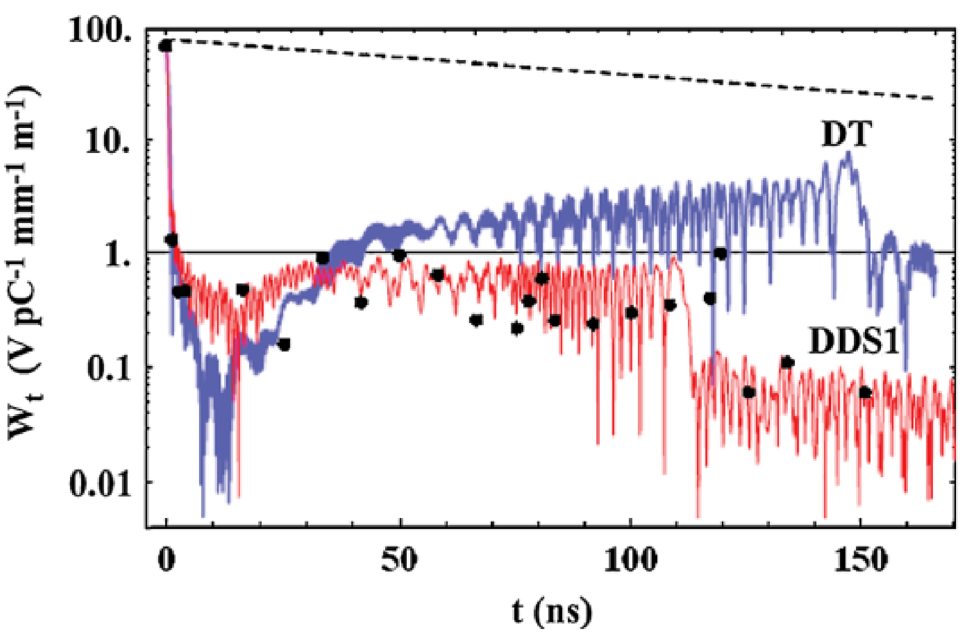
- Better confines the fundamental mode.
- Strongly damps the dipole mode.
- Less effective than CLIC's scheme with regards to other HOMs.



NLC's moderate damping and detuning

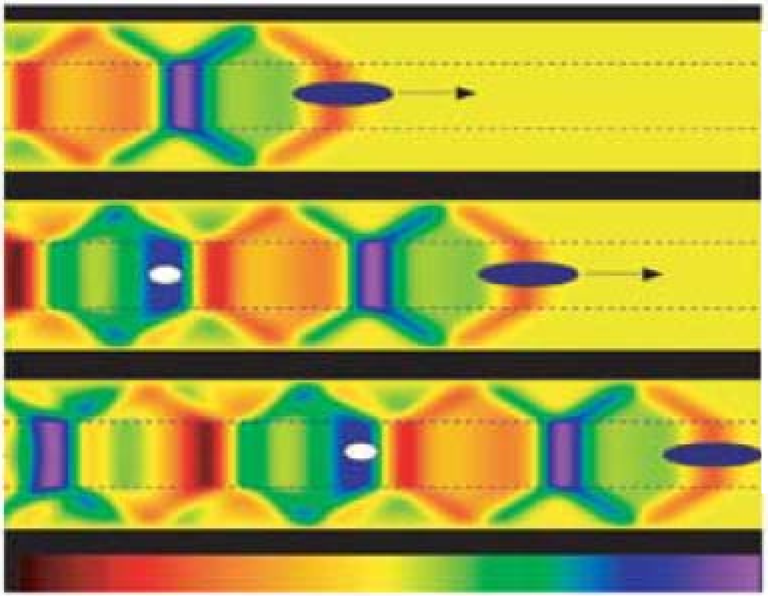
- Smaller pulse heating at the coupling irises than in heavily damped structures.
- Damping material located away from the structure.

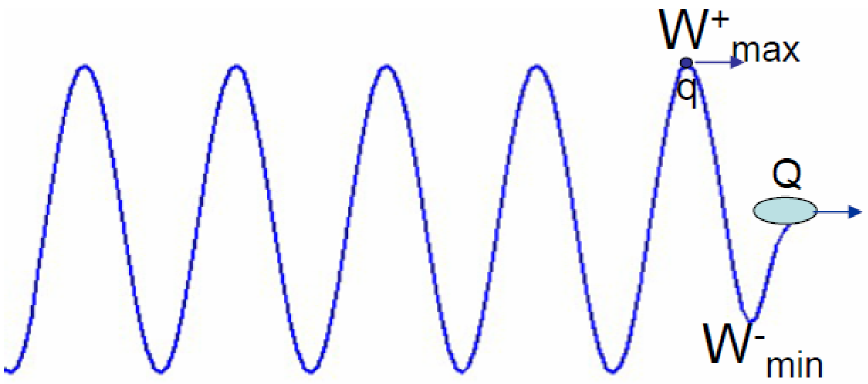




Wakefield acceleration

Transformer ratio





$$TR = \frac{Maximum\ energy\ gain\ of\ the\ witness\ bunch}{Maximum\ energy\ loss\ of\ the\ drive\ bunch}$$

Enhanced transformer ratio

Symmetric beam can never produce transformer ratios above 2. Triangular and double triangular beams are proposed to enhance the transformer ratios.