Photonic Band Gap Structures

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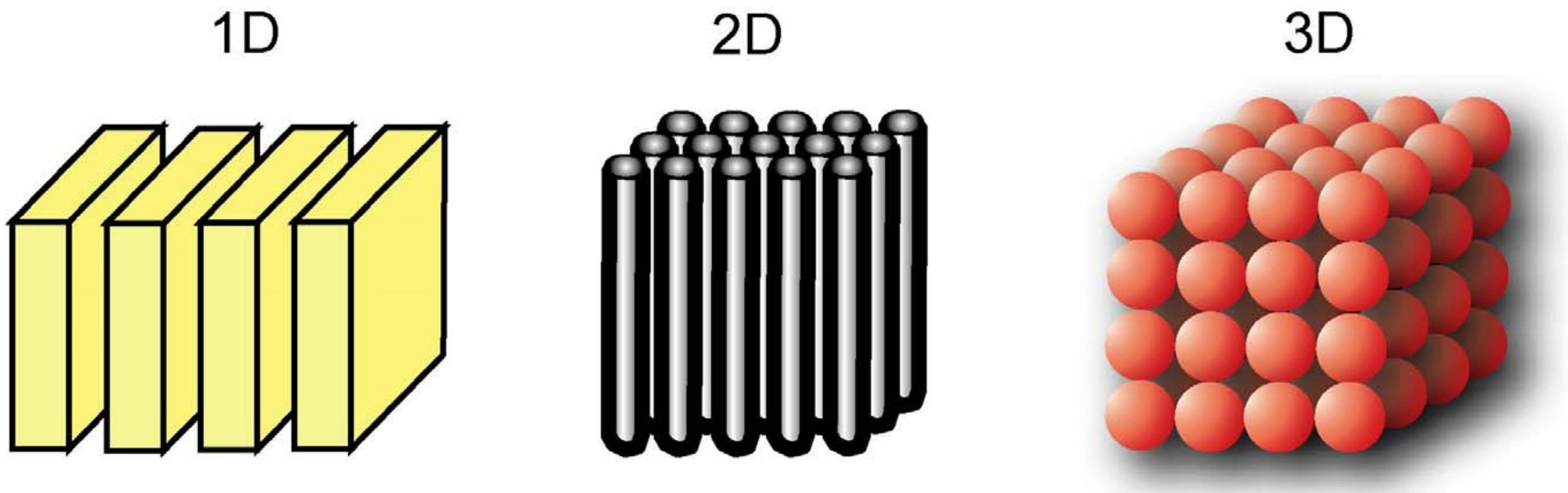
Outline of this lecture

- **Photonic band gap structures: definition and examples.**
- **Basic theory of 1D and 2D photonic band gap structures.**
- **Band gaps and global band gaps.**

Photonic Band Gap Structures

A photonic bandgap (PBG) structure is a one-, two- or three-dimensional periodic metallic and/or dielectric system (for example, of plates, rods or balls).

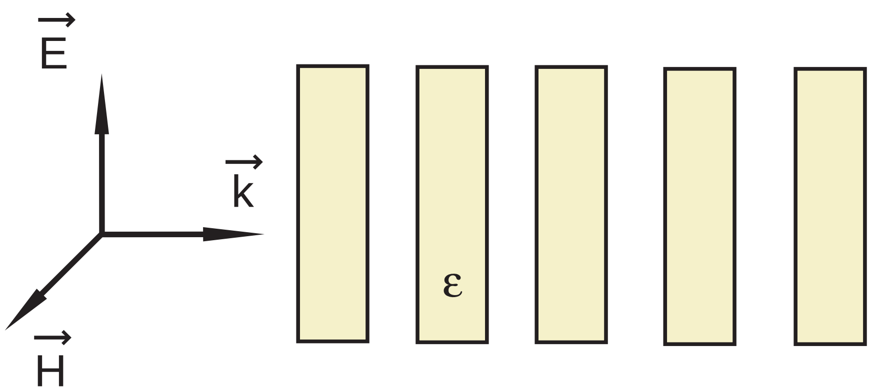


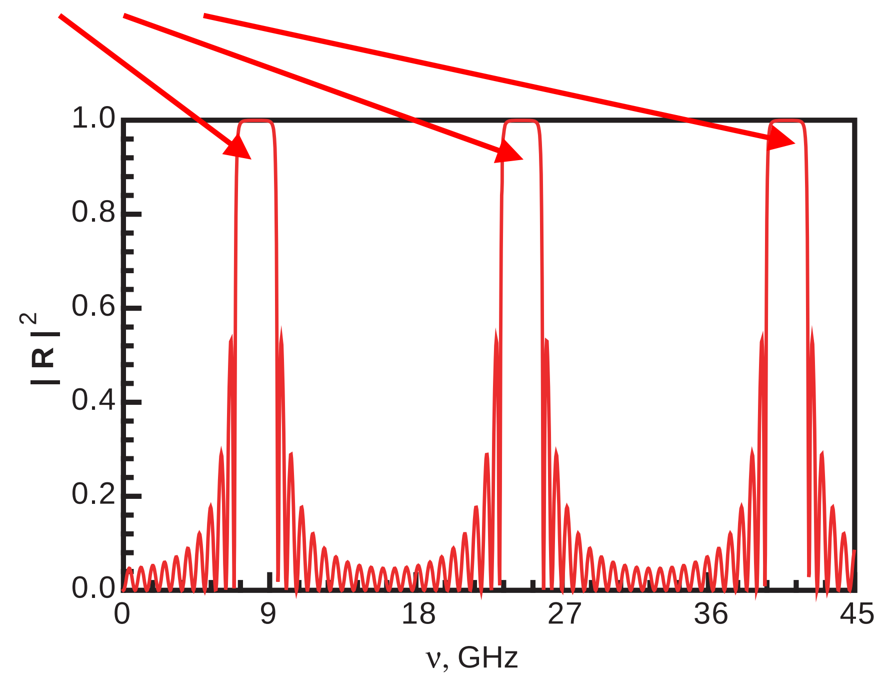
J.D.Joannopoulos, R.D.Meade, and J.N.Winn, *Photonic Crystals: Molding the Flow of Light* (Princeton Univ. Press, Princeton, 1995).

Band Gaps

PBG structure arrays reflect waves of certain frequencies while allowing waves of other frequencies to pass through.

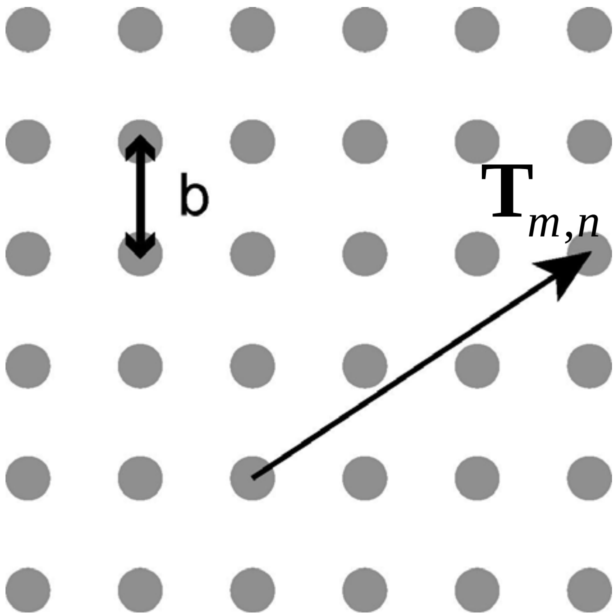
Band Gaps





1D example: Bragg reflector

Maxwell equations in PBG structures



Fields in PBG structures satisfy Maxwell's equations:

$$\begin{cases} \nabla \times \mathbf{E} = -i\mu_0 \omega \mathbf{H} \\ \nabla \times \mathbf{H} = i\varepsilon \omega \mathbf{E} \\ \nabla \cdot (\varepsilon \mathbf{E}) = 0 \\ \nabla \cdot \mathbf{H} = 0 \end{cases}$$

2D square lattice:

$$\mathbf{T}_{m,n} = \hat{e}_x bm + \hat{e}_y bn,$$

m,n - integers

$\psi = E_i,\ H_i$ must satisfy the Floquet theorem

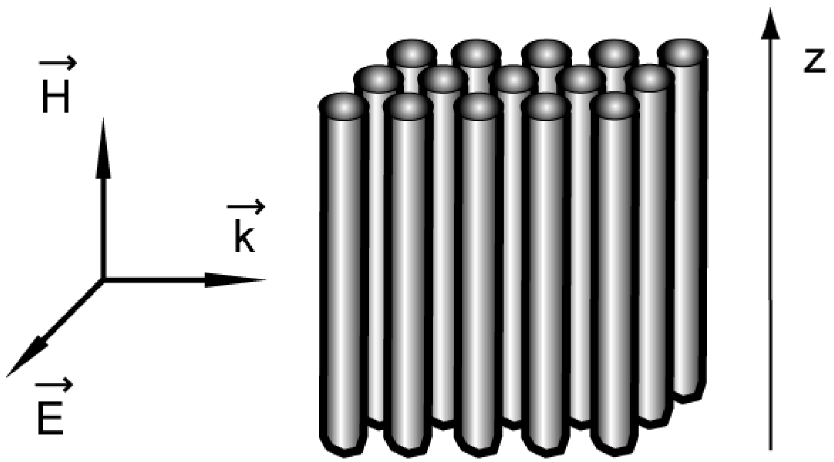
$$\psi(\mathbf{x}_\perp + \mathbf{T}_{m,n}) = \psi(\mathbf{x}_\perp) e^{i\mathbf{k}\cdot\mathbf{T}_{m,n}}$$

Maxwell equations solved for $\omega(\mathbf{k})$

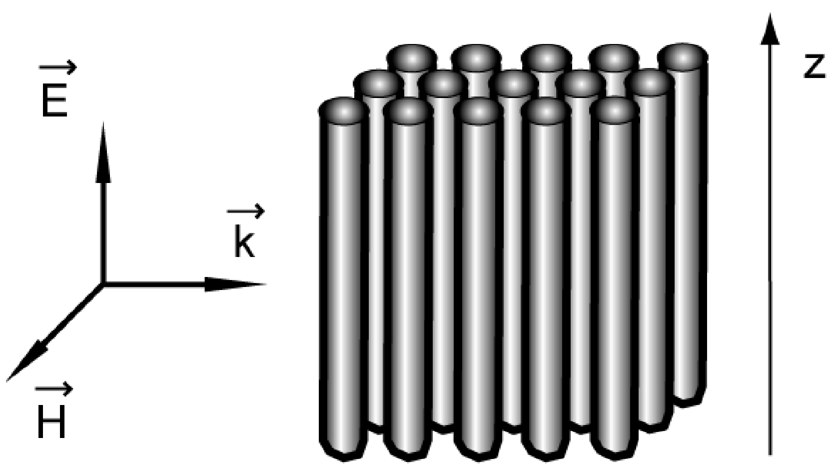
2D metal PBG structures

Lattice of metal rods:

The TE wave



The TM wave



2D metal PBG structures

Maxwell's equation: $$\nabla_\perp^2 \psi(\mathbf{x}_\perp) + \frac{\omega^2}{c^2}\psi(\mathbf{x}_\perp) = 0$$

Boundary conditions:

The TE wave $\quad \psi = H_z\,, \quad \left.\frac{\partial \psi}{\partial n}\right|_S = 0$

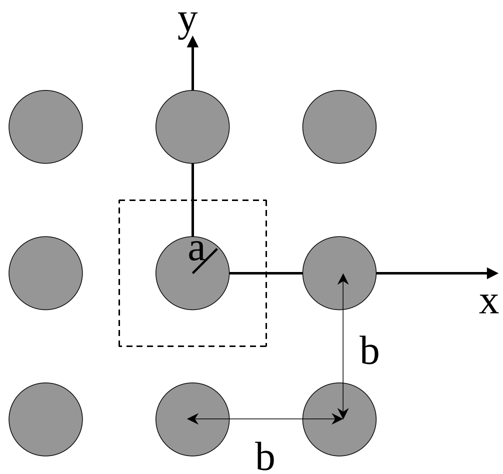
The TM wave $\quad \psi = E_z\,, \quad \left.\psi\right|_S = 0$

Periodic boundary conditions:

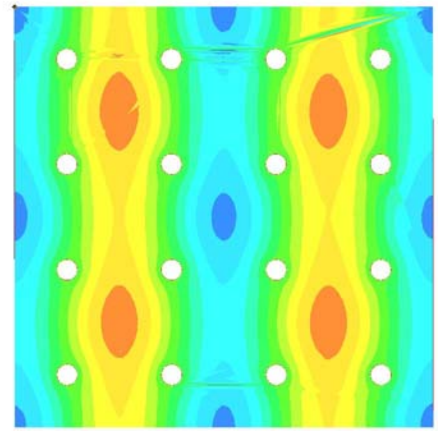
$$\psi(\mathbf{x}_\perp + \mathbf{T}) = \psi(\mathbf{x}_\perp) e^{i\mathbf{k}\cdot\mathbf{T}}$$

Reciprocal lattice

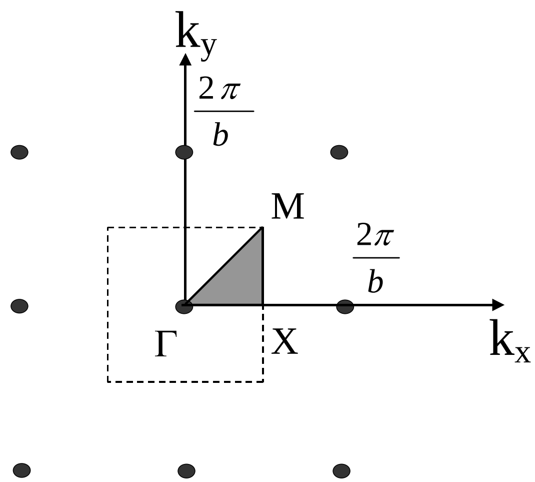
Crystal structure



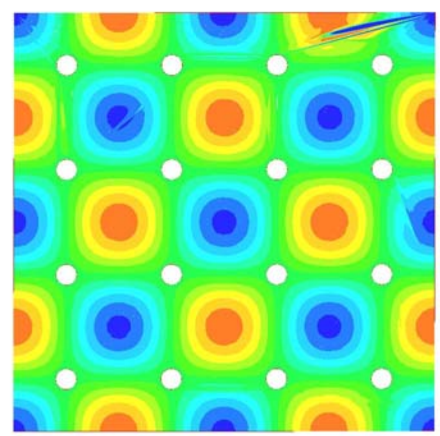
X-point



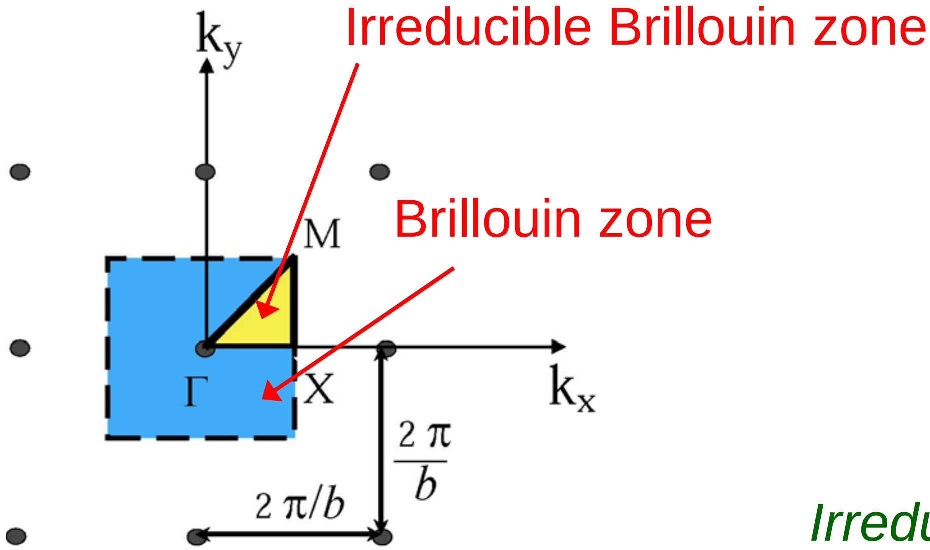
Reciprocal lattice



M-point



Brillouin diagrams



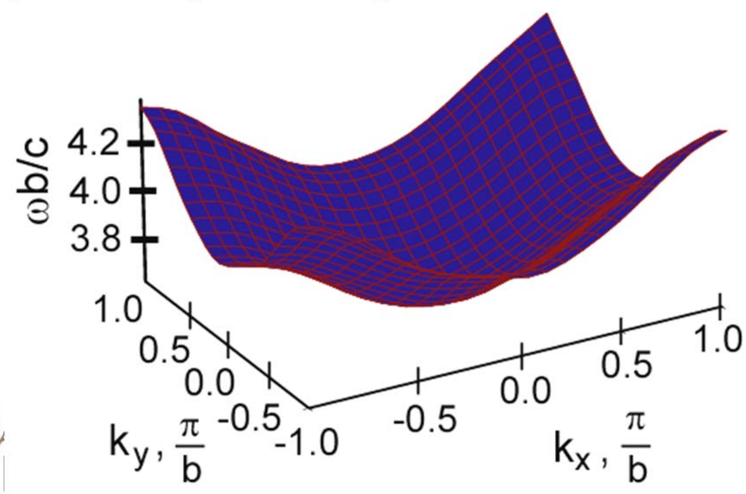
$$\psi\left(\mathbf{x}_\perp+\mathbf{T}\right)=\psi\left(\mathbf{x}_\perp\right)e^{i\mathbf{k}\cdot\mathbf{T}_{m,n}}$$

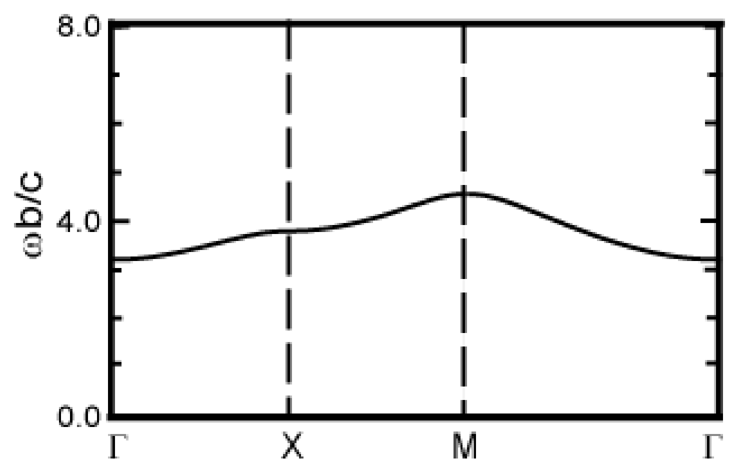
$e^{i\mathbf{k}\cdot\mathbf{T}_{m,n}}$ is periodic, only

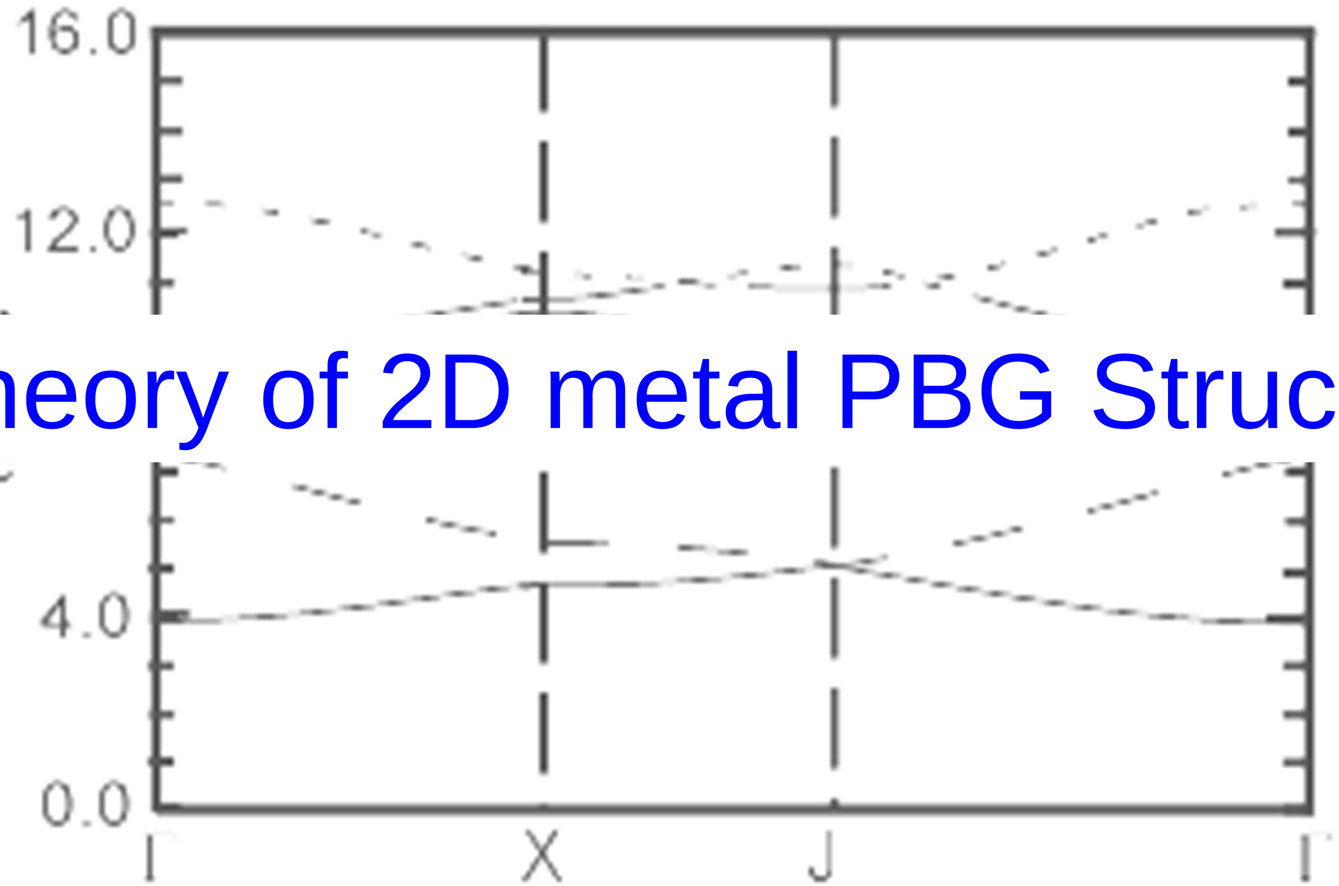
$\mathbf{k}$ inside the *Brillouin zone* matter.

Brillouin diagram:

$\omega(\mathbf{k})$ is plotted along the *Irreducible Brillouin zone's* boundary







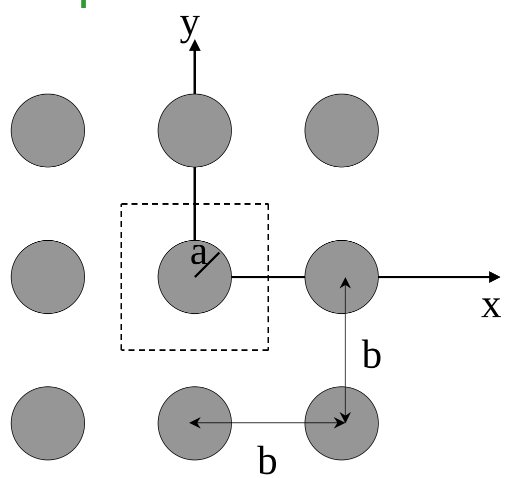
Theory of 2D metal PBG Structures

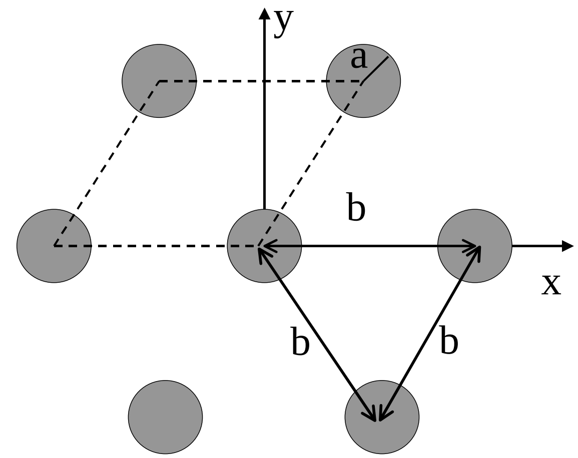
Square and triangular lattices

Square lattice

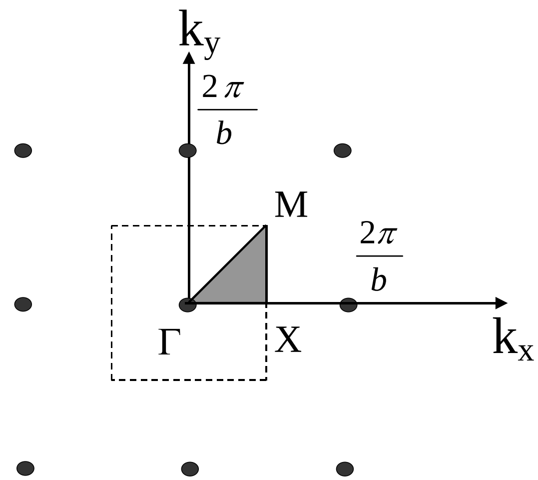
Triangular lattice

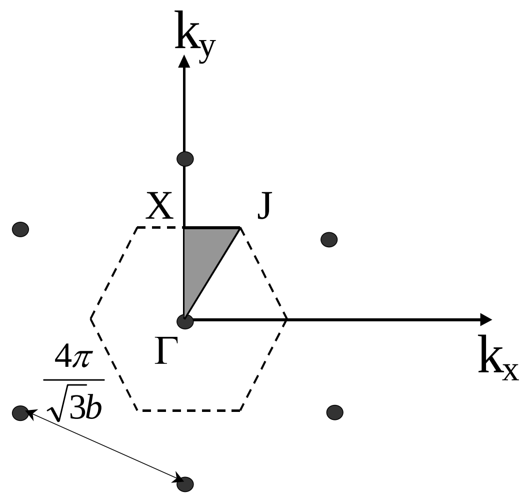
Crystal lattice





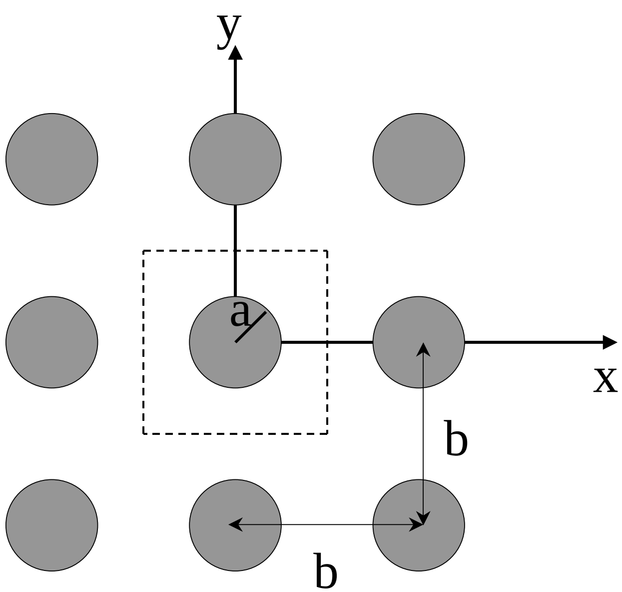
Reciprocal lattice

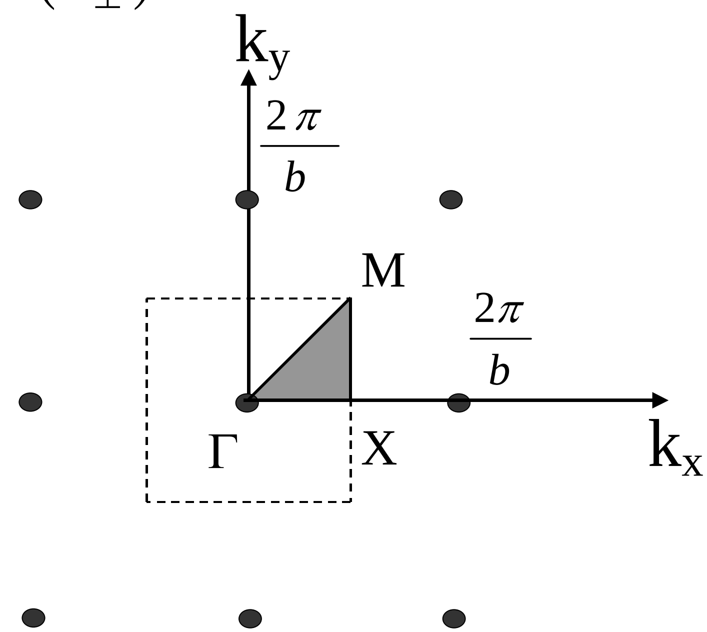




Periodicity of the square lattice

$$\sigma(\mathbf{x}_\perp + \mathbf{T}_{mn}) = \sigma(\mathbf{x}_\perp)$$



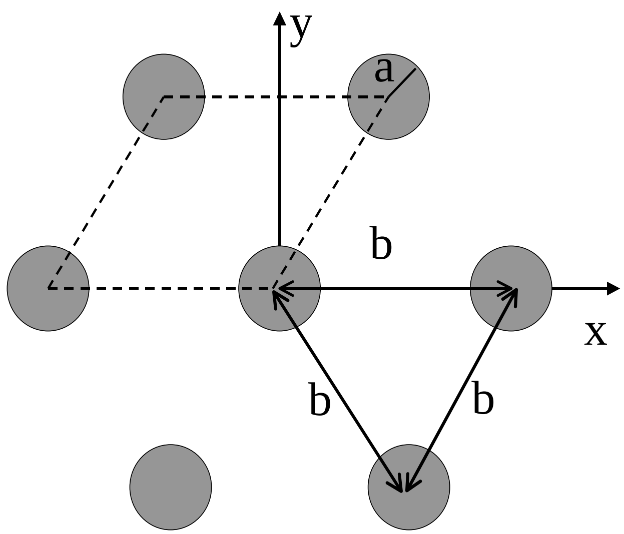


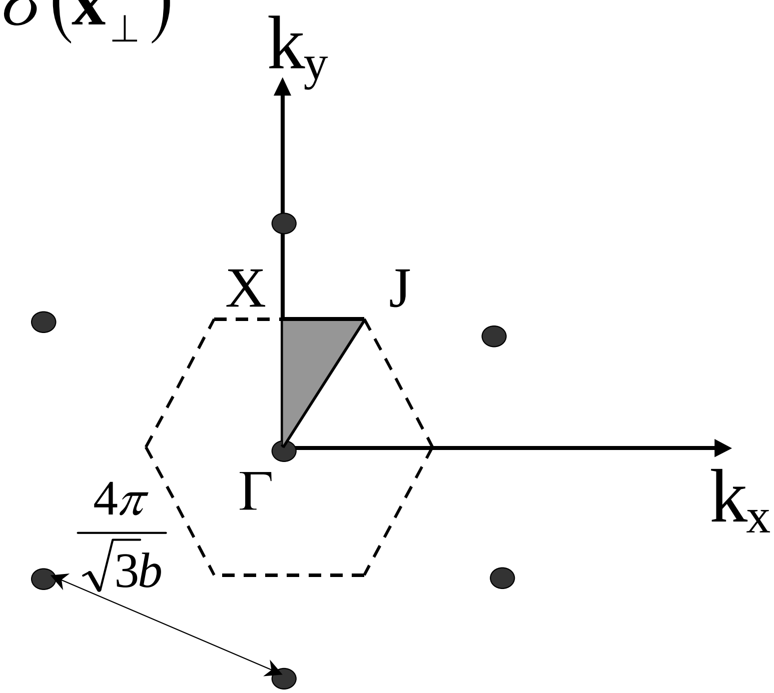
$$\mathbf{T}_{mn} = m\,b\,\hat{\mathbf{e}}_x + n\,b\,\hat{\mathbf{e}}_y$$

$$\mathbf{G}_{mn} = \frac{2\pi}{b}\left(\hat{\mathbf{e}}_x m + \hat{\mathbf{e}}_y n\right)$$

Periodicity of the triangular lattice

$$\sigma\left(\mathbf{x}_\perp + \mathbf{T}_{mn}\right) = \sigma\left(\mathbf{x}_\perp\right)$$





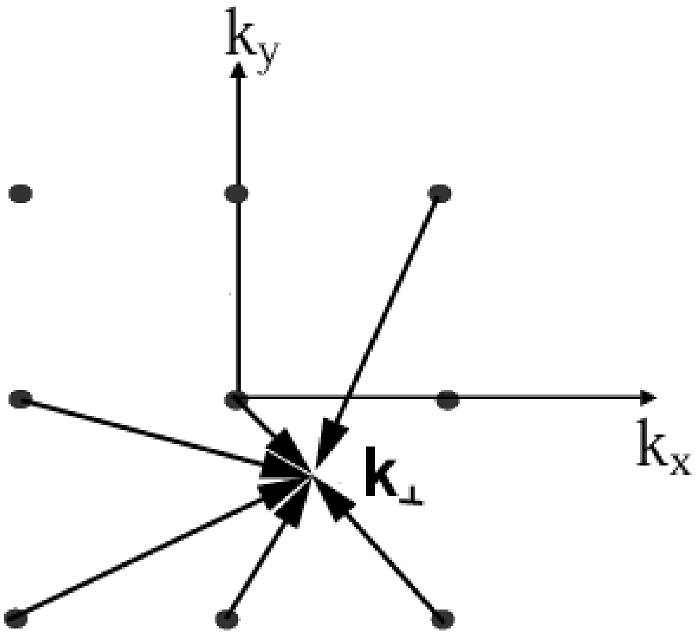
$$\mathbf{T}_{m,n} = \left(n + \frac{m}{2}\right) b\,\hat{\mathbf{e}}_x + \frac{\sqrt{3}}{2} mb\,\hat{\mathbf{e}}_y$$

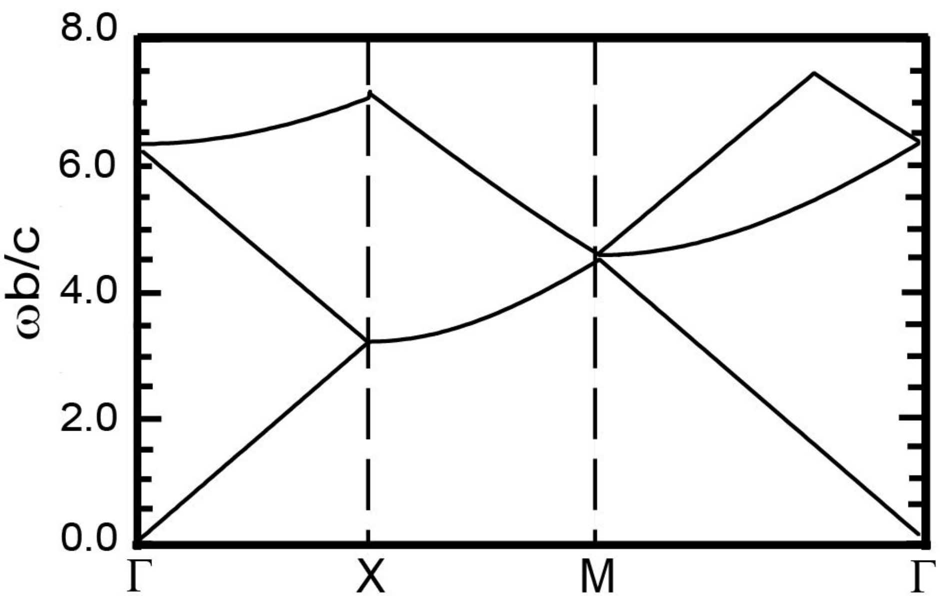
$$G_{m,n} = \frac{2\pi}{b}\left[\left(\hat{\boldsymbol{e}}_x - \frac{1}{\sqrt{3}}\hat{\boldsymbol{e}}_y\right) m + \frac{2}{\sqrt{3}}\hat{\boldsymbol{e}}_y n\right]$$

Plane wave approximation

$$\psi(\vec{x}_\perp) = e^{i\vec{k}_\perp\cdot\vec{x}_\perp}\sum_{m,n}\psi_{mn}e^{i\vec{G}_{mn}\cdot\vec{x}_\perp}$$

$$\left(\frac{\omega}{c}\right)_{m,n} = \left|\mathbf{k}_\perp + \mathbf{G}_{m,n}\right|$$





Interaction at thin metal posts

$$\nabla_{\perp}^{2}\psi(\mathbf{x}_{\perp})+\kappa^{2}\psi(\mathbf{x}_{\perp})=f(\mathbf{x}_{\perp}),\ \kappa^{2}=\frac{\omega^{2}}{c^{2}}-k_{z}^{2}$$

TM mode: $\psi = E_z$ TE mode: $\psi = H_z$

$$f(\vec{\mathbf{x}}_{\perp}) = \begin{cases} i4\pi k_z \rho(\vec{\mathbf{x}}_{\perp}) - \dfrac{i4\pi\omega}{c^2} J_z(\vec{\mathbf{x}}_{\perp}) & \text{for the TM case} \\ -4\pi\left(\dfrac{1}{c}\vec{\nabla}\times\vec{J}\right)_z & \text{for the TE case} \end{cases}$$

Wave equation in the periodic structure

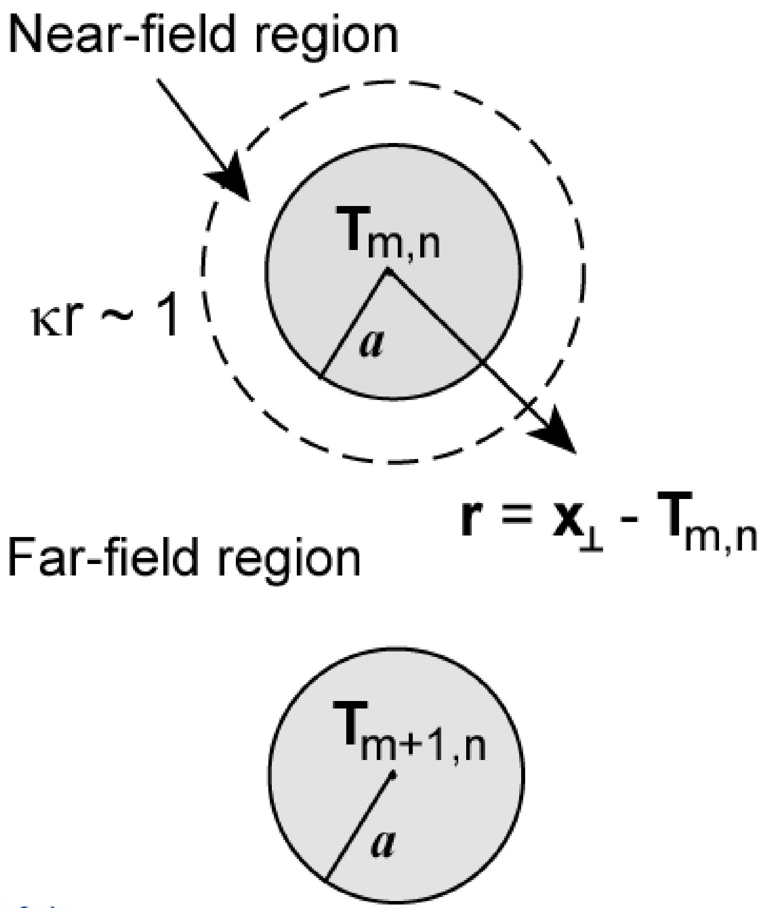
Bloch theorem:

$$\psi(\mathbf{x}_\perp) = e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} \sum_{m,n} \psi_{m.n} e^{i\mathbf{G}_{m,n} \cdot \mathbf{x}_\perp}$$

$$\sum_{m,n} \left[\kappa^2 - \left(\mathbf{k}_\perp + \mathbf{G}_{m,n}\right)^2\right] \psi_{m,n} e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp + i\mathbf{G}_{m,n} \cdot \mathbf{x}_\perp} = f(\mathbf{x}_\perp)$$

$$\left[\kappa^2 - \left(\mathbf{k}_\perp + \mathbf{G}_{m,n}\right)^2\right] \psi_{m,n} = \frac{1}{A} \int_{el.cell} f(\mathbf{x}_\perp) e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp - i\mathbf{G}_{m,n} \cdot \mathbf{x}_\perp} \mathrm{d}^2 \mathbf{x}_\perp$$

Quasistatic approximation



Near-field region:

$$\left|\nabla^2 \psi_{near}^{(m,n)}\right| \sim \psi_{near}^{(m,n)} / a^2 >> \kappa^2 \psi_{near}^{(m,n)}$$

$$\boxed{\nabla^2 \psi_{near}^{(m,n)} = 0}$$

Far-field region:

$$\left|\nabla^2 \psi_{far}^{(m,n)}\right| \sim \kappa^2 \psi_{far}^{(m,n)}$$

Matching condition:

$$\psi_{near}^{(m,n)}\Big|_{r \sim 1/\kappa} = \psi_{far}^{(m,n)}\Big|_{r \sim 1/\kappa}$$

Quasistatic approximation for the TM case

$$\vec{\nabla}^2 \psi_{near}^{m,n} = 0 \qquad \psi_{near}^{m,n} = \psi_{far}^{m,n}\left[1 - \frac{ln(\kappa r)}{ln(\kappa a)}\right]$$

The source function:

$$f^{m,n}(r) = \vec{\nabla}_{\perp}^2 \psi_{near}^{m,n} = -\frac{\psi_{far}^{m,n}}{ln(\kappa a)} \vec{\nabla}_{\perp}^2 ln(\kappa r) = -\frac{2\pi}{ln(\kappa a)} \psi_{far}^{m,n} \delta(r)$$

The wave equation with the source:

$$\vec{\nabla}_{\perp}^2 \psi(\mathbf{x}_{\perp}) + \kappa^2 \psi(\mathbf{x}_{\perp}) = -\frac{2\pi}{A ln(\kappa a)} \sum_{m,n} \psi(\mathbf{x}_{\perp})\, \delta\left(\mathbf{x}_{\perp} - \boldsymbol{T}_{m,n}\right)$$

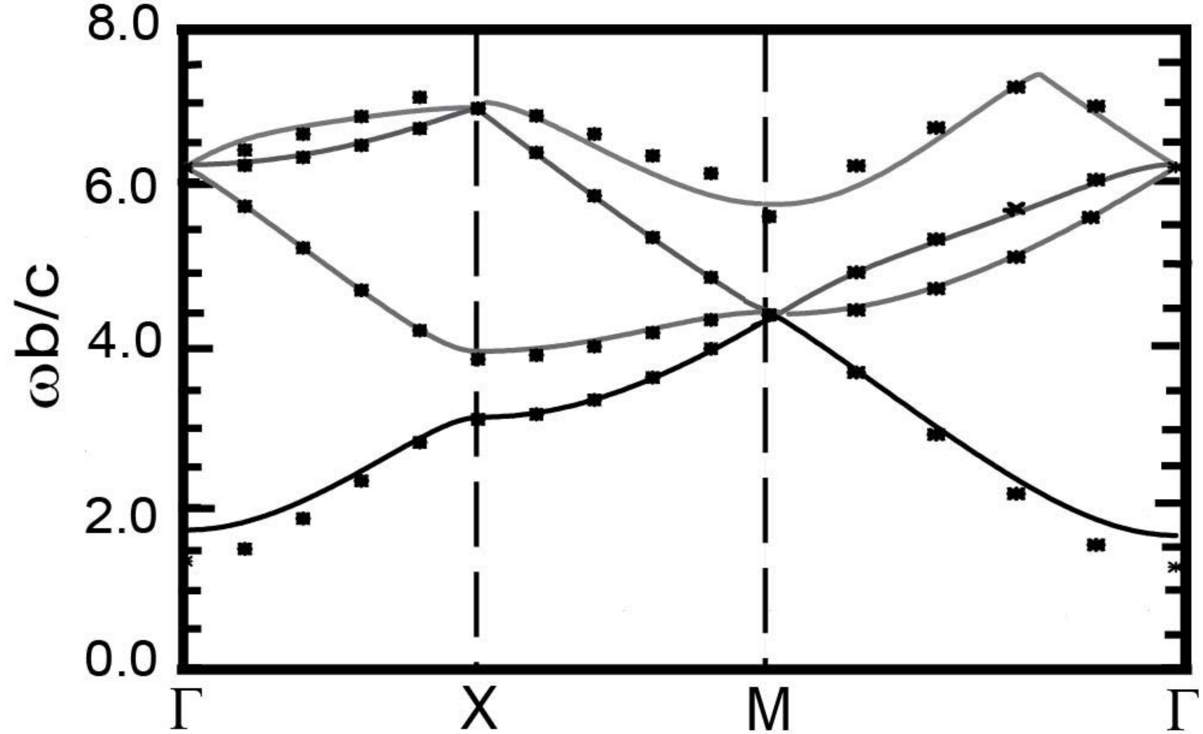
Dispersion equation for the TM case

$$\left[\kappa^2 - \left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m,n}\right)^2\right]\psi_{m,n} = -\frac{2\pi}{A\,ln(\kappa a)}\sum_{m,n}\psi_{m,n}$$

$$M = \left\|\begin{matrix} k_\perp^2 - \frac{2\pi}{A\,ln(\kappa a)} & -\frac{2\pi}{A\,ln(\kappa a)} & \cdots & -\frac{2\pi}{A\,ln(\kappa a)} & \cdots \\ -\frac{2\pi}{A\,ln(\kappa a)} & \left(\boldsymbol{k}_\perp + \boldsymbol{G}_{0,1}\right)^2 - \frac{2\pi}{A\,ln(\kappa a)} & & -\frac{2\pi}{A\,ln(\kappa a)} & \cdots \\ \vdots & & \ddots & \vdots & \\ -\frac{2\pi}{A\,ln(\kappa a)} & -\frac{2\pi}{A\,ln(\kappa a)} & \cdots & \left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m,n}\right)^2 - \frac{2\pi}{A\,ln(\kappa a)} & \cdots \\ \cdots & \cdots & & \cdots & \cdots \end{matrix}\right\|$$

Dispersion curves for the TM case

a/b=0.05



Quasistatic approximation for the TE case

$$\vec{\nabla}^2 \psi_{near}^{m,n} = 0 \qquad\qquad \psi_{near}^{m,n} = a_0 + \left(\vec{r} \cdot \vec{\nabla}_\perp\, \psi_{far}^{m,n}\right)\left[1 + \frac{a^2}{r^2}\right]$$

The source function:

$$f^{m,n}(r) = \vec{\nabla}_\perp^2 \psi_{near}^{m,n} = \left(\vec{r} \cdot \vec{\nabla}_\perp\, \psi_{far}^{m,n}\right)\vec{\nabla}_\perp^2 \frac{a^2}{r^2} = 2\pi a^2 \vec{\nabla}_\perp \psi_{far}^{m,n} \cdot \vec{\nabla}_\perp\, \delta(r)$$

The wave equation with the source:

$$\vec{\nabla}_\perp^2 \psi(\mathbf{x}_\perp) + \kappa^2 \psi(\mathbf{x}_\perp) = 2\pi a^2 \sum_{m,n} \vec{\nabla}_\perp \psi(\mathbf{x}_\perp) \cdot \vec{\nabla}_\perp\, \delta\left(\mathbf{x}_\perp - \boldsymbol{T}_{m,n}\right)$$

Dispersion equation for the TE case

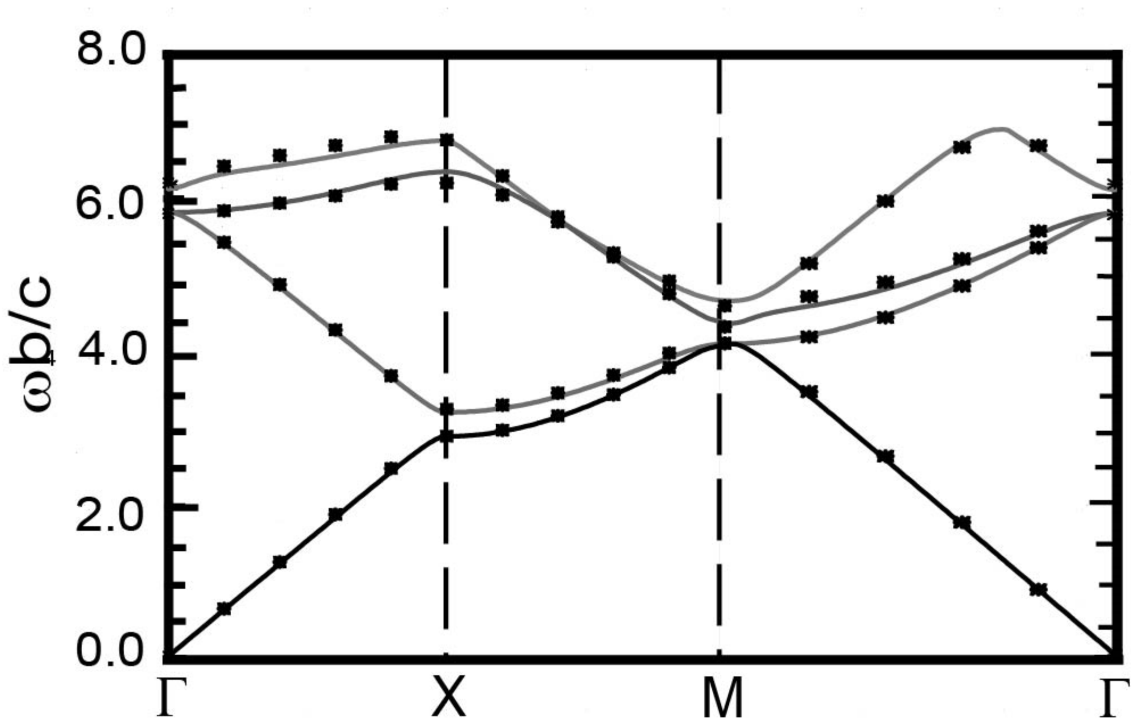
$$\left[\kappa^2 - \left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m,n}\right)^2\right]\psi_{m,n} = -\frac{2\pi a^2}{A}\sum_{m\prime,n\prime}\psi_{m\prime,n\prime}\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m\prime,n\prime}\right)\cdot\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m,n}\right)$$

M=

$$\left\|\begin{matrix} k_\perp^2 & -\frac{2\pi a^2}{A}\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{0,1}\right)\cdot\boldsymbol{k}_\perp & \cdots & -\frac{2\pi a^2}{A}\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m,n}\right)\cdot\boldsymbol{k}_\perp & \ldots \\ -\frac{2\pi a^2}{A}\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{0,1}\right)\cdot\boldsymbol{k}_\perp & \left(\boldsymbol{k}_\perp + \boldsymbol{G}_{0,1}\right)^2 & & -\frac{2\pi a^2}{A}\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m,n}\right)\cdot\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{0,1}\right) & \ldots \\ & \vdots & \ddots & \vdots & \\ -\frac{2\pi a^2}{A}\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m,n}\right)\cdot\boldsymbol{k}_\perp & -\frac{2\pi a^2}{A}\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{0,1}\right)\cdot\left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m,n}\right) & \cdots & \left(\boldsymbol{k}_\perp + \boldsymbol{G}_{m,n}\right)^2 & \ldots \\ \ldots & \ldots & & \ldots & \ldots \end{matrix}\right\|$$

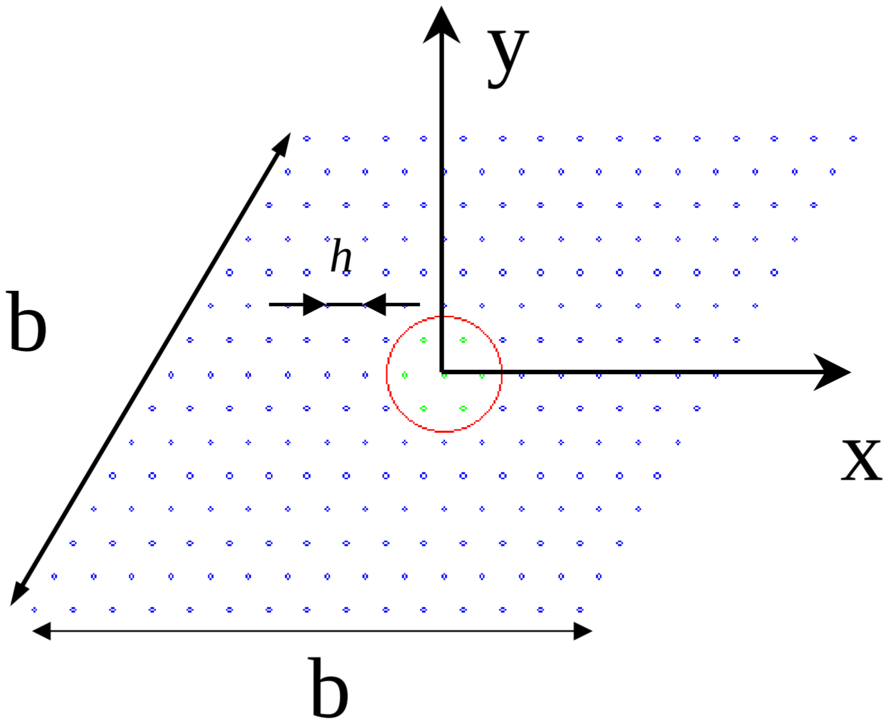
Dispersion curves for the TE case

a/b=0.1



Finite difference scheme

Maxwell's equations are solved on the grid:



Grid nodes: $x_{i,j} = h\left(i + j/2\right)$

$$y_{i,j} = h\sqrt{3}/2\, j$$

Grid step: $h = \dfrac{b}{2N+1}$

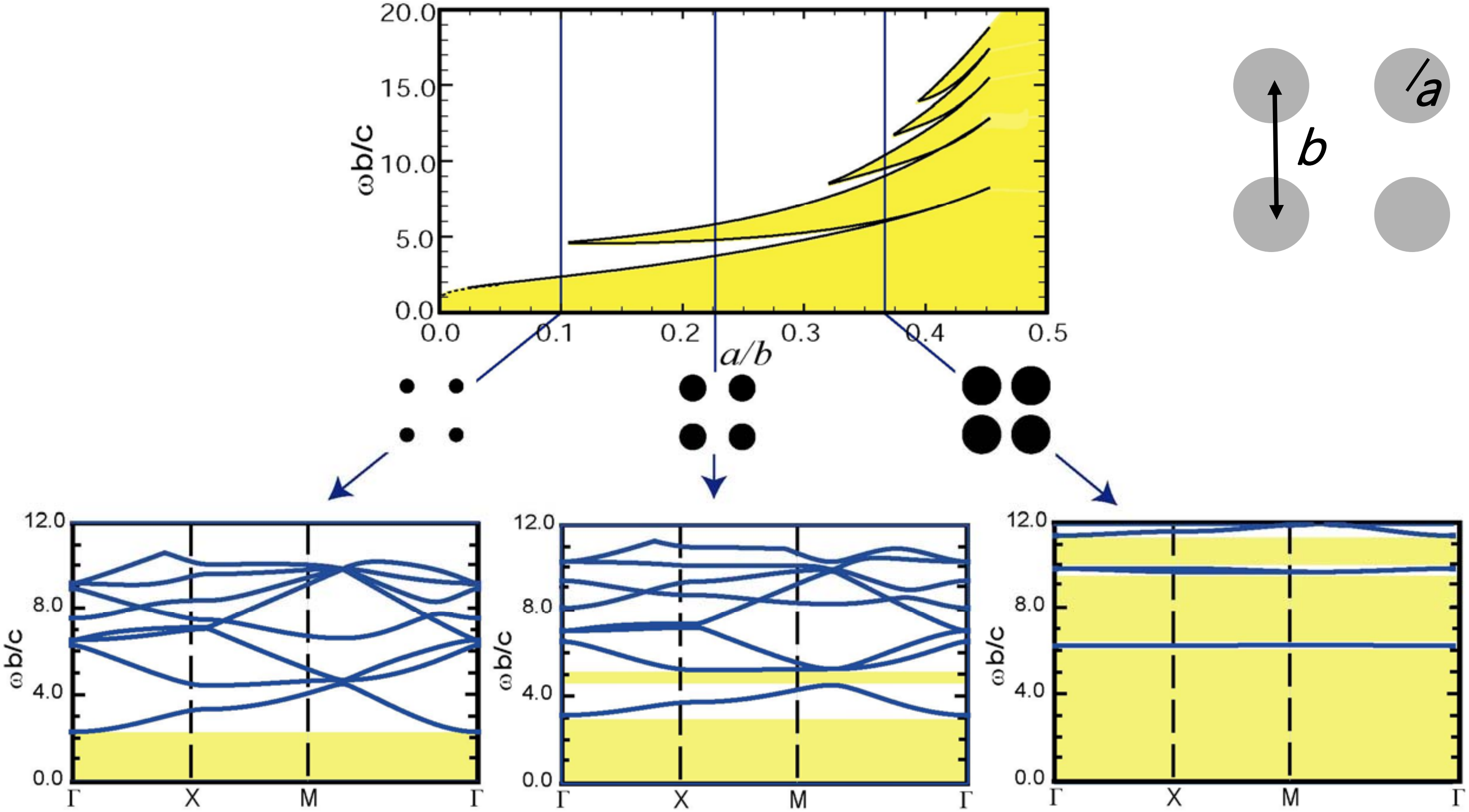
Helmholtz equation:

$$4\left(\psi_{i+1,j} + \psi_{i-1,j} + \psi_{i,j+1} + \psi_{i,j-1}\right) - \\ -\left(\psi_{i+1,j+1} - \psi_{i+1,j-1} - \psi_{i-1,j+1} + \psi_{i-1,j-1}\right) \\ = \left(3h^2\lambda + 16\right)\psi_{i,j}.$$

Periodical boundary conditions: $\begin{cases} \psi_{N+1,j} = \psi_{-N,j} e^{ik_x b}, \\ \psi_{i,N+1} = \psi_{i,-N} e^{i\frac{b}{2}\left(k_x + \sqrt{3}k_y\right)} \end{cases}.$

Global Band Gaps

Global band gap: a wave cannot propagate in either direction.
Example of a band gap diagram: square lattice of metal rods, TM waves

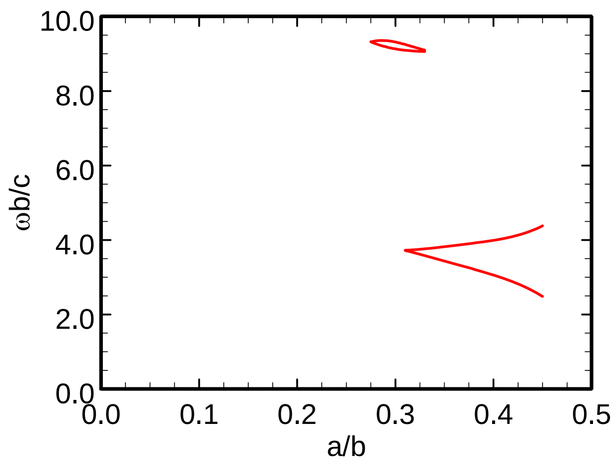


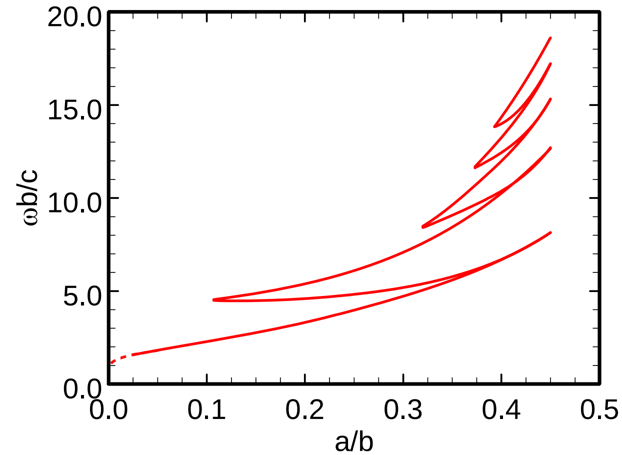
Global band gap diagrams for the structures of metal rods

The TE waves

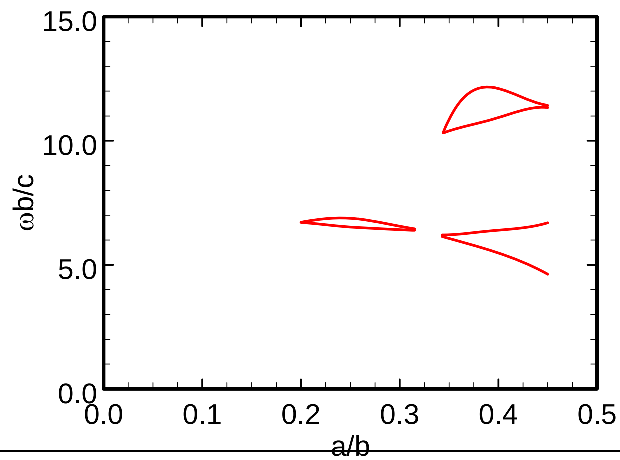
The TM waves

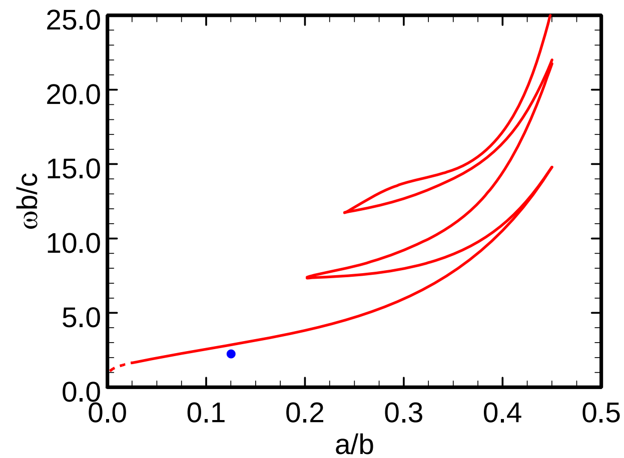
Square lattice





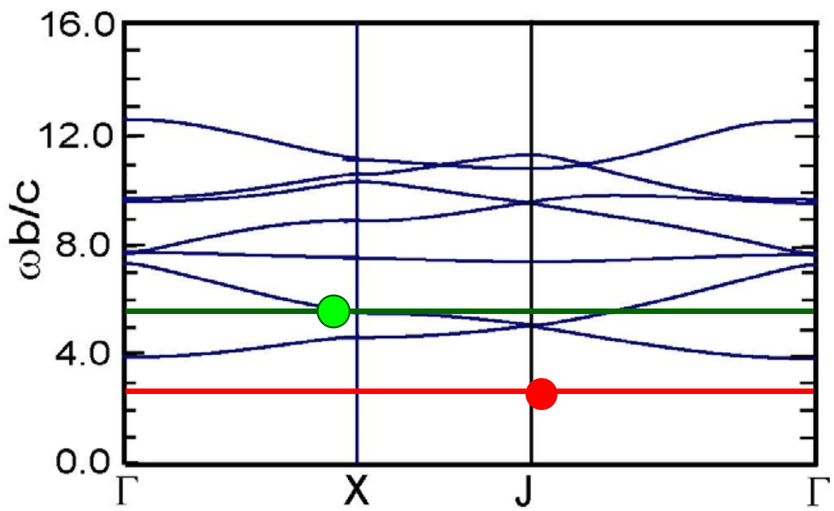
Triangular lattice

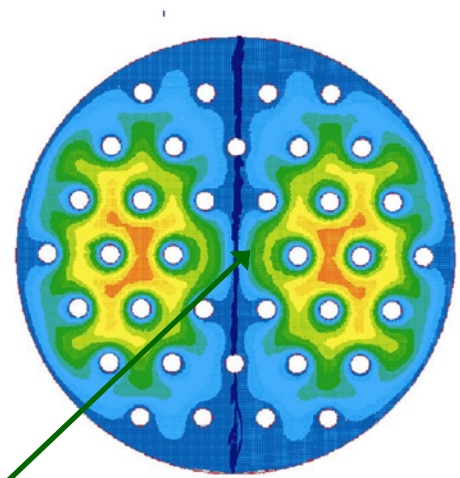




Photonic band gap resonators

A defect in a PBG structure may form a PBG resonator:





PBG cavity formed by a defect